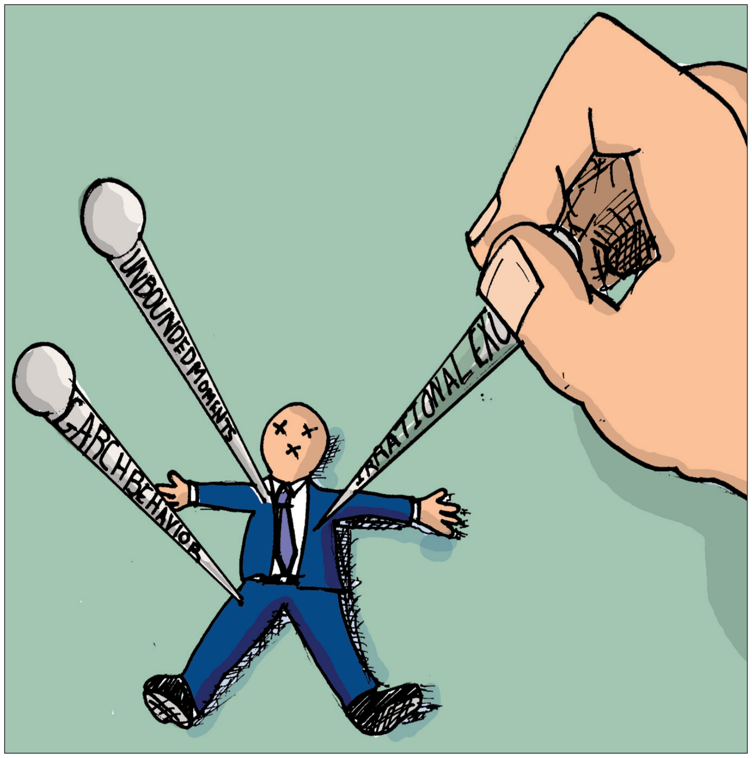
Kent Osband

FINFORMATICS:

Volatility Voodoo

How do I love thee? Let me count the ways.
I love thee to the depth and breadth and height
My soul can reach, when feeling out of sight. . .
I love thee with the passion put to use
In my old griefs, and with my childhood's faith.



When I read these words, written over a century and a half ago, I marvel at how presciently Elizabeth Barrett Browning anticipated the modern enthusiasm for financial market volatility. For some, vol is a fantastic lottery, providing an adrenaline rush most Powerball players can only dream of. For others, it's proof of man's inherent wickedness and/or the inherent injustice of life.

Even scientists are enthralled. Economists use the allegedly excess volatility of market prices relative to fundamentals to demonstrate the deep-seated irrationality of rich investors. Fellow academics, reassured that their shortcomings in wealth vouch for their smarts, heap accolades on them. As for derivatives modelers, unevenly excess vol is a goldmine: the greatest boost to the curve-fitting industry since Ptolemaic epicycles.

Unfortunately, most of the pious pronouncements about financial market volatility are nothing more than voodoo. Disciples turn into zombies, endlessly muttering "irrational exuberance", "unbounded moments", "GARCH behavior", "fractional Brownian motion", and other mantras of despair. Intimidated, few non-disciples dare to even look for simpler, less superstitious explanations of excess vol.

Simpler Explanation

Of course, if you've been reading *Finformatics*, you know a simpler explanation: ignorance. Rational people, knowing that what they know is likely either incomplete or partially outdated, seek to learn from new experience. When a bad firm returns higher-than-expected profits, they think that maybe, just maybe, this is not just a normal outlier but a sign the firm has turned good. The more persuaded they are that the firm has turned good, the higher the price-to-dividend ratio they're willing to value the stock at. Conversely, when a good firm returns lower-than-expected profits, investors think that maybe, just maybe, the good firm has turned bad, and so mark down the price-to-dividend ratio they're willing to pay. This positive correlation between dividends and the stock-price-to-dividend ratio makes stock prices more volatile than the dividends alone.

The relation between new dividends and dividend payout ratio adjustment is not constant but varies with one's current conviction and the perceived intensity and frequency of regime change. The less certain you are about the existing regime, the greater the contrast between different regimes, and the longer each regime is expected to last, the more weight you should effectively place on new information. As a result, excess volatility isn't fixed. It ebbs and flows in GARCH-like ways, and even "average" levels can vary considerably across assets.

Moreover, learning can also account for long-term mean reversion too subtle to show up in day-to-day measures. Theories of "fractional Brownian motion" also imply long-term mean reversion, but only at the cost of tortured rationales and tortuous application. In contrast, the "learning" explanation is very simple: in the very long run, expectations fluctuate around fundamentals, so that beliefs-based volatility recedes relative to fundamentals-based volatility.

All this is, I trust, is eminently reasonable, even to those inured to unfathomable description. And if you've been reading this column regularly, you will appreciate that rational learning can be formulated in compact mathematical terms. Still, some may wonder whether learning can account for enough excess vol and volatility of vol to make much difference in practice. To lay such doubts to rest, the next section walks thru a simulation.

A Simulation

Suppose that:

- Dividends D have 8% (annual) volatility with percentage drift $\mu_H = +8\%$ in the High regime and $\mu_L = -8\%$ in the Low regime, which can be summarized stochastically as:

$$dD/D = \pm 0.08\, dt + 0.08\, dz$$

- Regimes switch with frequency $\lambda = 0.1$ per year, which corresponds to an average life of $1/\lambda = 10$ years per regime as I showed in an earlier column.
- The risk-free discount rate $r = 4\%$.
- All investors know these parameters, update their beliefs rationally, and pay fair value for assets.

Recalling last column's derivations, the extreme price to annualized dividend ratios work out to:

$$v_H = \frac{r + 2\lambda - \mu_L}{(r - \mu_L)(r - \mu_H) + \lambda(2r - \mu_H - \mu_L)} = 100$$
$$v_L = \frac{r + 2\lambda - \mu_H}{(r - \mu_L)(r - \mu_H) + \lambda(2r - \mu_H - \mu_L)} = 50$$

so that dividend payout ratios fluctuate between 1% and 2%. Given belief p and dividend D, fair value V is given by:

$$V(p) = [(1 - p)v_L + pv_H]D = 50(1 + p)D$$

while p follows the stochastic process:

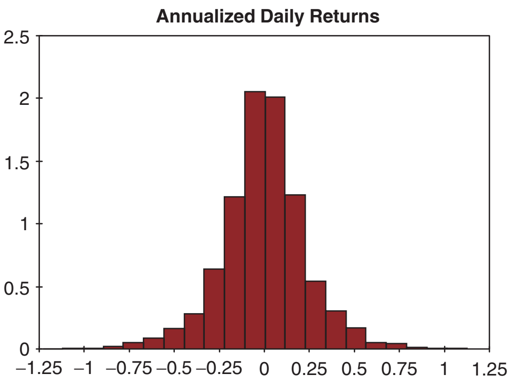
$$dp = \lambda(1 - 2p)dt + p(1 - p)\left(\frac{\mu_H - \mu_L}{\sigma}\right)\left(\frac{dD/D - [p\mu_H + (1 - p)\mu_L]dt}{\sigma}\right)$$
$$= 0.1(1 - 2p)dt + 2p\,(1 - p)(12.5\, dD/D + (1 - 2p)dt)$$

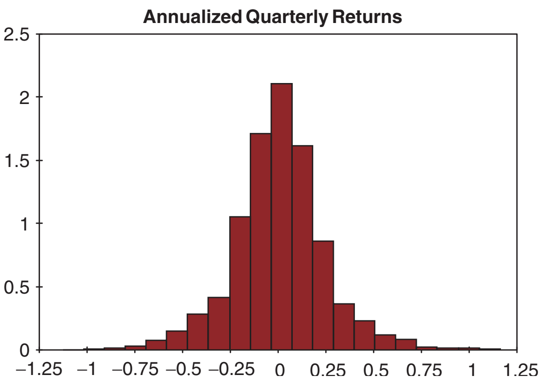
To approximate a stationary distribution of beliefs I ran Monte Carlo simulations 252 days per year for 40 years, each day calculating two random variables: one for the random walk dz component and the other for the toggle between regimes. I then continued the simulations for 1 day to 5 years longer, and calculated the corresponding percentage returns on price. In all, each Monte Carlo iteration required the generation of over 11,000 random variables and 33,000 equation updates. I then ran thru 5,000 iterations—i.e., 200,000 years of simulated daily data—to generate sample distributions.

Excess Vol and Fat Tails

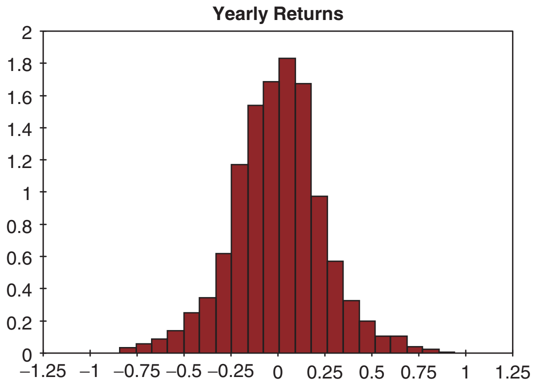
Here's what I found for 1-day returns. The odds of an overnight regime shift are only 1 in 2500, which some would round down to zero, ignoring regime shifts. In reality, or at least in virtual reality, their impact is huge. Sample volatility was 23.8%, nearly three times the volatility of daily dividends!

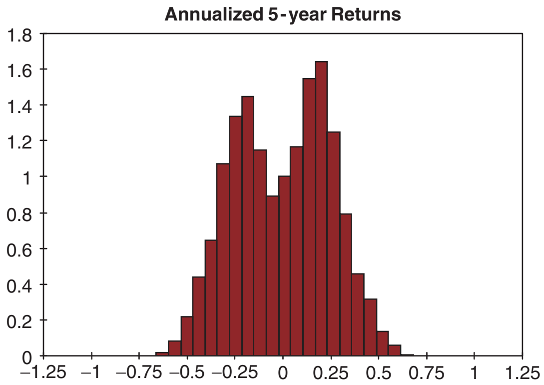
On top of that, the tails are fat. The excess kurtosis was 2.14 greater than for normal (Gaussian) distributions. Three standard deviation events occurred 1.6% of the time, 12 times as likely as for normal distributions. Four standard deviation events occurred 0.25% of the time, 80 times as likely as for normal distributions.



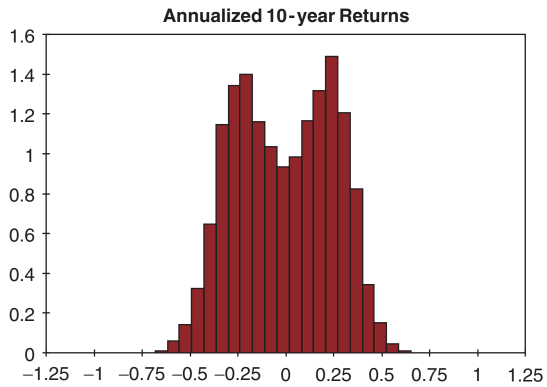


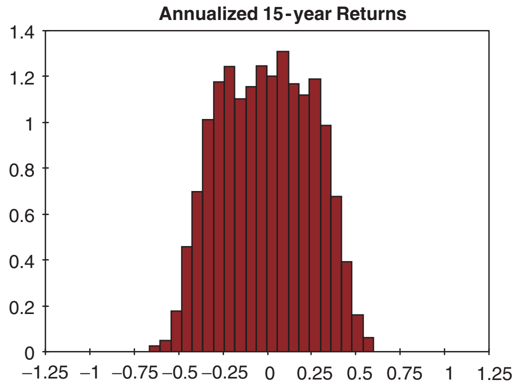
Once even a five standard deviation event occurred, which under normality should occur only once every 700 simulations. Moreover, bear in mind that this 5 standard deviation percentage price change represents nearly 15 times the standard deviation of dividends. With immutable regimes and rational expectations, such deviations should not be expected even once in the history of parallel universes, even if there are a trillion parallel universes like ours and each has lasted a trillion times as long as ours.



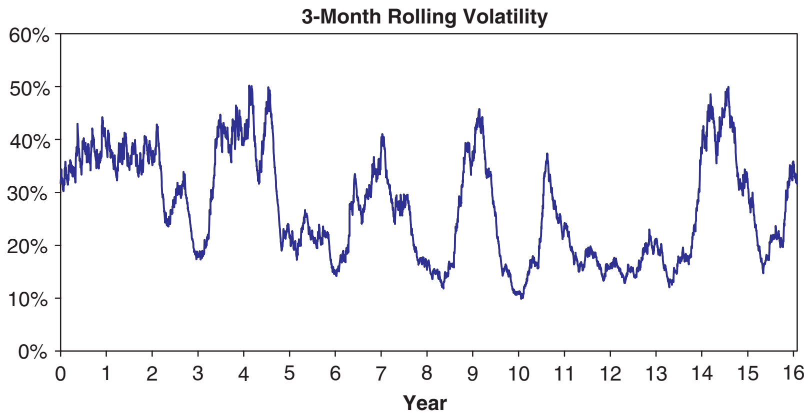


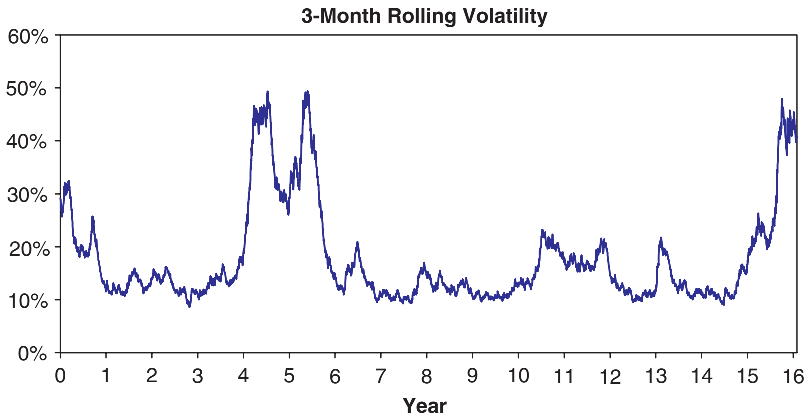
As the time period for returns lengthens from a day to a year, the annualized volatility stays about the same. At first glance this suggests zero autocorrelation, which in turn is often construed as independence. And there is little evidence of skewness either. But we know that returns can't possibly be independent, because otherwise the excess kurtosis should shrink inversely to the length of the period, mostly disappearing within a month. (I leave the proof as an exercise for the reader.) Instead, even after a quarter the excess kurtosis is 1.90, with little change in the histogram for annualized returns.

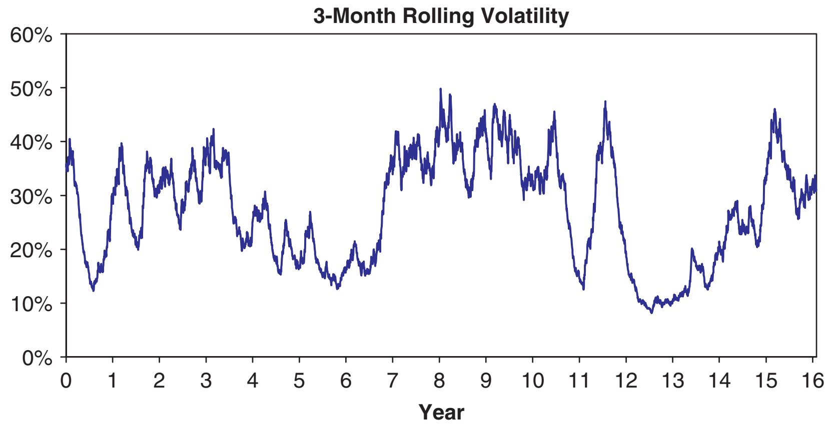




Moving from a quarter to a year, excess kurtosis shrinks much more, down to +0.9, and the tails visibly draw toward the center. Hardly any events of over 3.5 standard deviations occur during the simulation. But any suspicion that independence has kicked in is squelched once we see the histogram for 5-year returns. It has two peaks rather than one, and very few observations extend beyond the central area. Kurtosis is now 0.9 *less* than for a normal distribution, with the most extreme outlier only 2.7 standard deviations from the mean. Hence we're seeing some mean reversion over the stretch of a few years even though no reversion is visible at shorter horizons.







Why the twin peaks at five years? Because at that horizon, there are enough regime changes that many High-leaning beliefs shift down and many Low-leaning beliefs shift up. The histogram of 10-year annualized returns looks much the same. However, the twin peaks eventually must disappear, for at long horizons regimes will switch back and forth multiple times, leaving the drift clustering around the long-term pan-regime mean. In these simulations the twin peaks flatten out at a 15-year horizon, although the kurtosis remains 0.9 less than for a normal distribution.

Time Patterns of Volatility

There's so much to tell you about the time patterns of volatility, and so little time to tell you. Let me start by charting 48 years of three-month rolling volatility measured in the simulations. Each chart looks sufficiently different that the novice might suspect I altered the parameters. But I didn't. The family resemblance shows up in the range of volatility outcomes, the occasional spurts between regions of high and low vol, and in the serial correlation.

Econometricians describe this nowadays as GARCH-type behavior. GARCH stands for Generalized Auto-Regressive Conditional Heteroscedasticity. It means that the volatility is changing over time in a way that depends on its own past values and possible other elements.

Of the many GARCH-type models, one of the favorites—considered to fit market data well and to have good predictive power—is exponential GARCH(1,1) or EGARCH(1,1), which relates log volatility to its lagged value and to current news in fundamentals. So we might ask, how well does it fit the simulation data? The answer is: spectacularly well. In the various 16-year samples I tried, the t-stats averaged about 10 for news and about 750 for lagged vol. Most samples also showed a statistically significant higher response to bad news to good news, which econometricians tend to observe in market data too.

Intuitively, it's easy to see why EGARCH might fit the models well. The Learning Equation relates endogenous vol directly to prior beliefs and to unexpected news. Prior beliefs p in turn are closely linked to prior vol via the multiplier $p(1-p)$.

Indeed, the Learning Equation also explains why EGARCH tends to fit market data better than other GARCH-type models, including the asymmetric response to bad news. But that argument takes more time to develop, and time as I mentioned is running out. Let me leave you instead with a puzzle. Which well-known market phenomenon is our current learning framework least able to explain, and how might we most easily address it without voodoo? Join me again in the next issue for the answer. W