

# Variance Risk Premium Demystified\*

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## Abstract

We study the dynamics and cross-sectional properties of the variance risk premia embedded in options on stocks and indices, approximated by the synthetic variance swap returns. Several important stylized facts and contributions arise. First, variance risk premia for indices are systematically larger (more negative) than for individual securities. Second, there are systematic cross-sectional differences in the price of variance in individual stocks. Linking variance swaps to firm size/book-to-market, and stock turnover characteristics, an investor gains access to several lucrative long-short strategies with Sharpe Ratios around 2.85. Third, principal component analysis reveals at most one important factor driving both stock and variance swap returns, which corresponds to the traditional market factor. For the remainder of the dynamics, the stock and its variance processes are nearly linearly independent. Fourth, we find the leverage effect through analysis of the relationship between the variance risk premium and stock to variance correlation. The systematic (market factor) part of the leverage effect provides additional evidence of the existence of one factor common to both variance swaps and stocks, but the contribution of the market risk premium to the total variance premium is very small. These findings stress the importance of using variance-based instruments in the portfolio of an investor.

*Keywords* : variance risk premium, variance swap, individual options, variance factors.

*JEL* : C21, G13, G14

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# 1 Introduction

The current state of finance is such that we believe in using stochastic volatility (SV) for econometric financial time series modeling and option pricing (for a review, see for example Ghysels, Harvey and Renault, 1996 and Shephard, 1996). The observed path of most financial assets reveals changing volatility (filtered out from assets returns, as in Johannes, Polson and Stroud, 2007, among others, or observed directly as realized volatility, as in Andersen et al., 2003, and many others), and stochastic volatility models generate more consistent option prices than the Black-Scholes-Merton model (Black and Scholes, 1973; Merton, 1973). Having an 'SV standard', we still do not know exactly what drives the stochastic volatility. It is not fully established if it is driven by the separate "variance" risk factor in the economy (as partially in Heston, 1993, among others) or by the same factors driving the stock process (being a function of the history of the stock returns, as in Hobson and Rogers, 1998, or of the current level of stock price, as in Dumas, Fleming and Whaley, 1998). Knowing the volatility factor structure is important for an investor to manage the risk and return profile of the portfolio containing derivatives. Possible correlation between the variance and underlying stock process may diminish the expected diversification benefits, and it is important to optimize the portfolio not with respect to the instruments, but with respect to the underlying factors.

We study the variance risk premium priced into the stock and index options in order to resolve a number of issues related to the factor structure of volatility. Using a nine-year history of daily option prices on about 100 indices and 5,000 stocks, we make the following contributions.

- First, we test the observation of Driessen, Maenhout, and Vilkov (forth.) and others that index options bear a systematically larger (more negative) risk premium than the individual stock options. We contribute additional empirical evidence based on the largest available data set of exchange-traded options for a US market, and find that options on indices bear a systematically higher variance risk premium than options on individual stocks. This is compatible with the hypothesis about the priced stochastic correlation between stocks: it contributes to the variance risk premium of the index, but is not present in the individual stock options in its orthogonal part.
- Second, we show that the mean variance risk premia (variance swap excess returns) for individual names exhibit a great deal of cross-sectional variation. One can use the underlying company characteristics (we use size, book-to-maturity, stock turnover and industry code) to construct the decile portfolios (or industry portfolios in the case of the industry characteristic) of variance swap returns. These portfolios demonstrate a nice monotonic relationship between the characteristics and returns, and using long-short decile portfolios we derive a lucrative trading strategy with a Sharpe Ratio of around 2.85.
- Third, using the standard two-stage Fama and MacBeth (1973) procedure, we test for the risk premia on a number of the self-made factors from the variance swap return space (constructed from the long-short decile portfolio strategy). We also construct the statistical factors (using principal components analysis) from the variance swap and stock returns, and check for the relationship between them. It turns out

that variance swaps and stocks have at most a common factor, which corresponds with a high probability to the market factor. The economic factors constructed from the variance swaps do not produce robust risk premia estimation except for the turnover based factor, where the risk premia is positive. This can be explained in terms of replication costs (or delta hedge costs) for options on illiquid stocks vs. liquid stocks.

- Fourth, we suggest a conceptually new method for testing the leverage effect based on the combined dynamics of variance risk premium, stock returns and realized variance. We find that though there is a pronounced leveraged effect in stock return volatility in general, and though the market factor seems to be present in both the stock returns and the variance process, the majority of the variance risk premium is explained by the factors beyond the market. It may be the case that some variance-specific factor bears a much higher premium than the market factor.

As our paper makes four different contributions, there are several areas of related literature. The systematic difference between the expensiveness of the index and individual options (in terms of the embedded variance premium) has been discussed in only a few papers. Bakshi, Kapadia, and Madan (2003) document the differential pricing of individual equity options vs. the market index, based on empirical analysis of OEX options and 30 stocks. Bollen and Whaley (2004) examine the relationship between net buying pressure and the shape of the implied volatility function (IVF) of S&P 500 index options and options on 20 individual stocks, and find a significant difference between the two classes of assets. Garleanu, Pedersen, and Poteshman (2007) try to explain it with an irrational demand misalignment between index and individual options, while Driessen, Maenhout, and Vilkov (forth.) propose a risk-based explanation for that difference in pricing (and probably demand), making the priced stochastic correlation responsible for the observed discrepancy.

This work is also related to several studies on the cross-sectional variation in variance risk premia. Pietro and Vainberg (2006) show how size and book-to-market firm characteristics are linked to the expensiveness of equity options, and check if these characteristics influence the pricing of systematic variance risk in the equity options market. They find that under the same level of expected realized variance under physical measure, options on small stocks are more expensive than options on large stocks, and that options on value stocks are more expensive than options on growth stocks. We confirm their findings for the book-to-market characteristic, but find that the size effect in the variance swap returns is not robust and depends upon the estimation period and the method used. Carr and Wu (2008) also investigate the synthesized variance risk swaps for 35 individual securities and find a large cross-sectional variation in variance risk premia.

A number of papers look at the factor structure of variance and option returns. Carr and Wu (2008) find that the market risk factor is correlated with the stock return variance, and that size, book-to-market and momentum risk factors explain only a small portion of the observed variance risk premia. They conclude that the market variance should be largely driven by risk factors independent of those driving the underlying returns. Goyal and Saretto (2007) document an economically important source of mispricing in the implied volatilities for short-term ATM options. They construct zero-cost trading strategies in options based on the variance risk premium proxy and show that these strategies yield a

significant average monthly return. Plyakha and Vilkov (2008) look at the option returns on about 5,000 underlying stocks and show how one can derive factors from the option return space to benefit the CRRA investor beyond the traditional three Fama-French (1992) and momentum (Carhart, 1997) factors for underlying stocks. Estimating the factor risk premia, they find that market and book-to-market equity factors are consistently priced in options, while size and momentum factors deliver mixed results. Bondarenko (2007) synthesizes the prices of the variance contract on the S&P 500 index over the 17-year period, and finds that the variance risk premium is negative and economically very large. Moreover, it cannot be fully explained by the known risk factors and option returns.

The asymmetric properties of the variance process are consistent with the well-known leverage effect. This important and well-documented phenomenon has attracted a lot of attention since Black (1976), Cox and Ross (1976), and Christie (1982), among others. Formal econometric analysis of stock return/latent volatility dynamics estimated from the financial series (Figlewski and Wang, 2000; Bekaert and Wu, 2000; Dennis, Mayhew, and Stivers, 2006; Yu, 2005, among others) provides some formal support to the asymmetric SV, and hence indirectly to the leverage effect, although discussion of the return-volatility relationship, its form, and causes is far from its end. For index-type securities, Bondarenko (2004) uses S&P 500 options and futures contracts to support the previous findings that the variance return and the market return are negatively correlated, and also finds that this negative correlation is responsible for less than 20% of the observed variance risk premium. Individual stock returns to variance relation is studied in Carr and Wu (2008), and the authors come to the same general conclusion that we do, although we use a larger sample and different tests.

The rest of this paper is organized as follows. Section 2 presents a general model of asset prices dynamics with stochastic volatility, and discusses the definition of the variance risk premium and its proxy in the empirical work. In this section we also talk about the potential reasons for the variance being priced by economic agents and the ways to study the variance risk premium properties. In Section 3 we define the data set used and all the filters and data manipulations we carry out for further analysis. Section 4 is devoted to tests, results, and discussion. Here we split the analysis into four different parts and discuss them separately. Then we present a short summary of results implications for investor portfolio policy. Finally, Section 5 concludes.

## 2 Variance: what's inside?

### 2.1 Variance Process and the Integrated Variance Proxy

We assume a simple setup with prespecified dynamics for the stock price. A stock follows an Ito process with jumps and stochastic volatility as follows:

$$\frac{dS_t}{S_t} = (\mu(\varphi_1, \dots, \varphi_n) - \eta E[v_t]) dt + \mathbf{1}^T \Sigma(\varphi_1, \dots, \varphi_n) dW_t^S + v_t dJ_t, \quad (1)$$

where  $\Sigma(\varphi_1, \dots, \varphi_n)$  is an  $n \times n$  diffusion matrix with elements  $\sigma_{ij}$ , the  $\varphi_1, \dots, \varphi_n$  are the state variables in the economy (some may be non-traded),  $W_t^S$  is an  $n$ -dimensional standard Wiener,  $v_t$  is the jump size, and  $\eta$  is jump intensity. The diffusion matrix is a function of state-variables, i.e., it is stochastic by itself. We do not specify its structure, but assume that it always remains positive definite<sup>1</sup>.

We want to see what is inside the diffusion matrix  $\Sigma(\varphi_1, \dots, \varphi_n)$ : a) to ascertain whether the variance is priced, i.e., to see if the diffusion term  $\Sigma(\varphi_1, \dots, \varphi_n)$  is a function of priced sources of uncertainty, and b) to determine if there are any cross-sectional regularities about the variance risk premium.

To address these questions, we analyze the process for the stock return variance. The instantaneous variance of a stock return at time  $t$  can be written as

$$\sigma_i^2(\varphi_1, \dots, \varphi_n) = \mathbf{1}^T \Sigma(\varphi_1, \dots, \varphi_n) \Sigma(\varphi_1, \dots, \varphi_n)^T \mathbf{1}. \quad (2)$$

Using Ito lemma we derive an SDE for the variance as a function of state variables<sup>2</sup>:

$$\sigma_i^2(t) = \sigma_i^2(0) + \int_0^t \left( \mu_{\sigma_i^2} + \lambda_{\sigma_i^2} \right) d\tau + \int_0^t \sum_{j=1}^n \frac{\partial \sigma_i^2(\varphi_1, \dots, \varphi_n)}{\partial \varphi_j} \{d\varphi_j - E[d\varphi_j]\}.$$

If the state variables entering the variance SDE are priced in the economy, the move from actual probability measure  $P$  to risk neutral probability measure  $Q$  results in a variance risk premium  $\lambda_{\sigma_i^2}$  being subtracted from the drift:

$$\sigma_i^2(t)^* = \sigma_i^2(0) + \int_0^t \mu_{\sigma_i^2} d\tau + \int_0^t \sum_{j=1}^n \frac{\partial \sigma_i^2(\varphi_1, \dots, \varphi_n)^*}{\partial \varphi_j} \{d\varphi_j^* - E[d\varphi_j^*]\}.$$

The instantaneous difference between the variance under actual and risk-neutral measures equals the instantaneous risk premium:

$$d\sigma_i^2(t)^* - d\sigma_i^2(t) = -\lambda_{\sigma_i^2} dt.$$

Moving to the finite period expected difference, we capture not the pure integrated risk premium but rather the integrated effect of the change of measure, i.e. the cumulative

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<sup>1</sup>A way to model it would be the continuous time counterpart of the Wishart Autoregressive Process, as in Gouriéroux and Sufana (2004).

<sup>2</sup> Note that we have an  $n$ -dimensional standard Wiener, so the correlation between each risk factor is captured by the VCV matrix  $\Sigma(\varphi_1, \dots, \varphi_n)$ .

difference in expected variance paths:

$$E_t \int_t^{t+\Delta t} [d\sigma_i^2(\tau)^* - d\sigma_i^2(\tau)] = E_t \int_t^{t+\Delta t} [\xi(\tau) \sigma_i^2(\tau) d\tau] - E_t \left[ \int_t^{t+\Delta t} \sigma_i^2(\tau) d\tau \right],$$

where  $\xi(\tau)$  is a pricing kernel.

Looking at the variance risk premium and making parametric assumptions about the size of the specific factor risk premia, one can infer the factors driving the variance process and the sign the premium investor is paying (or getting) for exposure to those factors.

One can infer the model-specific variance risk premium by calibrating a prespecified (assumed) variance process to the observed variance proxies or variance-dependent derivative instruments under the physical probability measure and under the risk-neutral probability measure. Instead, in an effort to analyze the existence, the sign, and the magnitude of the variance risk premium, we follow the approach of Bollerslev, Gibson and Zhou (2004), Bondarenko (2004), and Carr and Wu (2008), among others, by comparing the model-free proxies for integrated variance under the risk-neutral probability measure  $Q$  and the actual probability measure  $P$ .

We use the results of Dumas (1995), Carr and Madan (1998), and Britten-Jones and Neuberger (2000), who build on the seminal work of Breeden and Litzenberger (1978), which show that the integrated variance of a stock in the period  $(t, t + \Delta t)$  under the  $Q$ -measure, or  $E_t^Q [\sigma_{i,t:t+\Delta t}^2]$ , can be expressed as a function of an infinite number of options of the appropriate maturity and with strikes equal to all stock values occurring with positive probability:

$$E_t^Q [\sigma_{i,t:t+\Delta t}^2] = 2 \int_0^\infty \frac{C_i(K, t + \Delta t) - \max(S_i(t) - K, 0)}{K^2} dK. \quad (3)$$

Jiang and Tian (2005) show that the approximation to formula (3) based on a number of existing options provides an appropriate proxy to the theoretical concept of risk-neutral integrated variance. The proxy is independent of the underlying stock return model, which we call the model-free implied variance (*MFIV*). The stock process specified in (1) may jump, and the model-free implied variance in (3) translates the implied jumps into the implied variance, so we get the total implied quadratic variation of the stock return in one number.

As the proxy for quadratic variation under the actual probability measure  $P$ , we should optimally use the model-free realized variance (*MFRV*) calculated from the high-frequency data as it represents the unbiased and consistent estimator for the stock integrated variance (for a review of the extensive literature on this topic see Andersen, Bollerslev, and Diebold, 2005). However, the Monte-Carlo experiments carried out by Bollerslev, Gibson, and Zhou (2003) show that for the sake of variance risk premium estimation (they use Heston (1993) model for illustration), the use of a combination of *MFIV* and *RV*, realized variance estimated from daily returns delivers a variance risk premium estimator that is biased but still acceptable for large samples. The mean bias of

the variance risk premium estimator for the sample size of 600 under the worst scenario did not exceed 1.8% of the theoretical premium. We consider this level of accuracy to be sufficient for our purposes. The payoff on the short variance swap with notional 1 is equal to:

$$r_{t:t+\Delta t}^\sigma = E_t^Q [\sigma_{t:t+\Delta t}^2] - \sigma_{t:t+\Delta t}^2,$$

where  $\sigma_{t:t+\Delta t}^2$  is the realized variance of a stock return over a period of time  $(t, t + \Delta t)$ , and  $E_t^Q [\sigma_{t:t+\Delta t}^2]$  is the quoted price equal<sup>3</sup> to the expectation of the realized variance under the risk-neutral probability measure  $Q$ . Carr and Wu (2008) derive the trading strategy that replicates the payoff of such an instrument, and hence prove that the integrated variance risk premium belongs to the (empirical) payoff space.

The expected excess return on a variance swap at time  $t$  then gives us the quantification of the expected integrated variance risk premium:

$$E_t [r_{t:t+\Delta t}^\sigma] = \frac{E_t [\sigma_{t:t+\Delta t}^2]}{E_t^Q [\sigma_{t:t+\Delta t}^2]} - 1. \quad (4)$$

In further analysis we will be using the terms variance risk premium, integrated variance risk premium, and short variance swap return interchangeably.

## 2.2 What Drives the Variance?

One may wonder what factors affect the variance risk premium and how we can identify their effect empirically. A number of accepted models allow for a priced variance process.

In a representative agent economy with time-separable (e.g., CRRA) utility, only the factors correlated with the aggregate consumption (read: market) will be priced. Deviations from that benchmark model may allow for priced volatility risk in the part uncorrelated with aggregate consumption, and among these models are a representative agent economy with heteroscedastic consumption process and Epstein-Zin utility (e.g., Bansal and Yaron, 2004, and Tauchen, 2005), an economy where investors are not able to diversify holdings perfectly (Malkien and Xu, 2003, Jones and Rhodes-Kropf, 2003, among others), and behavioral models with unexpected utility formulations (e.g., Barberis and Huang, 2001, etc.)

The variance risk may be priced (or seem to be priced) for a different reason. People have limited potential in formalizing information processing and analyzing all the available information. Style investing and narrow framing (Barberis and Schleifer, 2003), the choice to specialize, i.e., to learn a part instead of the whole (Nieuwerburgh and Veldkamp, 2008), and the limitations alike lead us to an idea that investors may not be able to track all the characteristics of stock return distribution separately and/or consistently apply their own (in incomplete market settings) pricing kernel to move into the risk-neutral world. By analyzing individual return distribution characteristics separately, we may miss

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<sup>3</sup>Assuming no bargaining power of the market maker and no bid/ask spread.

any tradeoff between them that the investor is carrying out subconsciously (or probably without paying much attention) by investing in stocks and their derivatives.

The main question that we want to answer in this paper is, how do we identify the forces that drive the variances of individual stocks?

One can make restrictive assumptions about an underlying model, as above, and test it directly. This approach is valid, but the results are then model-specific and may not hold under different assumptions. We want to leave as many options open as possible and proceed without making any parametric assumptions on the preferences or the ways that economic agents process information. We are going to look at the dynamics of the variance risk premium expressed by the realized excess return on our synthetic variance swaps. There are several empirical directions that we plan to explore.

First, following Driessen, Maenhout, and Vilkov (forth.), we expect to see a systematic difference between the variance risk premia priced in the index-type securities (basket, or index options) and the premia in individual stock options due to the priced stochastic correlation between the stocks. This discrepancy in the variance processes is due to an additional priced risk factor in the index-based variance that is not present in the individual stock variance, namely due to the stochastic correlation between the stocks. If the variance is driven by the same factors on both individual and index levels, then simply aggregating individual securities into an index we can see that the variance risk premium implied in the index derivatives is not going to be larger than the premium in the underlying individual securities. We take the absence of systematic difference between variance risk premia as the null hypothesis and test it in the empirical section.

Second, Pietro and Vainberg (2006), and Carr and Wu (2008), among others, showed that variance is systematically mispriced in value vs. growth firms and in small vs. large firms. We would like to look at this stylized fact in more detail and to construct the factors from the variance swaps payoff space using the firm-specific characteristics. The primary objective of this exercise is the search for systematic forces driving individual stock variance, and the identification of those factors that are priced by economic agents.

Third, we want to construct the statistical factors from the variance swap excess returns using the factor (or principal component) analysis, and to see what these factors are related to. Statistically constructed factors surely explain more of the common time series variance than selected economic factors, but we hope to be able to interpret these factors and not just demonstrate their statistical importance.

## 3 Data Description

### 3.1 Option Data

We use the data for all equity-based individual and index options from OptionMetrics (OM). The daily data cover the US market from January 1996 until December 2004. OM provides the most comprehensive coverage of equity options traded in the US, and we take advantage of this by using the richest possible individual options sample in the analysis.



We select all available underlying securities, which gives us 4,884 individual stocks and more than 150 indices. We do not cut the data initially based on a number of available observations, trading volume, or any other criterion, but rather set task-specific filters later at the testing stage.

The financial markets are developing fast, and new products arrive every now and then. Variance swaps have been traded for several years in ever-increasing volumes in the OTC markets as products directly linked to the realized variance over the period. In 2004, the exchange trading of VIX futures commenced, and recently options on VIX have been introduced. The VIX options would be an invaluable source of data for testing the stochastic volatility models as they give direct access to a volatility risk-neutral assessment. However, as VIX is based on the S&P500 index, and not on the individual securities, we have no choice but to create and analyze synthetic variance swaps. Thus, from OM data we first have to calculate the variance swap returns.

As a starting point we take the Volatility Surface file that provides interpolated implied volatilities for options with standard maturities and deltas of -0.2 to -0.8 with a 0.05 increment. We select all Black-Scholes implied volatilities with 30-day maturity as short-term options tend to be more liquid, and hence interpolation does not distort the results. To construct the *MFIV* as in (3), we need a set of option prices with different strikes and the same maturity. We go through several steps to move from volatility surface data to the required option prices.

First, inverting the Black-Scholes formula, we calculate the corresponding moneyness for each given delta. The inputs for this procedure include the implied volatility, risk-free rate, and dividends. Implied volatility is taken directly from the Volatility Surface File, the risk-free rate is provided in the Zero Coupon Yield Curve File, and dividends for each day are either given in the Index Dividend Yield File (for index-type securities), or approximated by combining the forward price in the Standardized Options File with the spot rate used for the forward price calculations (in the Security Prices File). This approximate dividend rate for individual securities is different from the one used by OM as it utilizes the tree method with discrete dividends instead of the Black-Scholes formula with continuous dividends, but as we calculate the continuous analog of that discrete payment on each day separately, the distortions should be limited.

Second, we fit the piecewise cubic Hermite interpolating polynomial with moneyness as an independent variable and implied volatility as a function value to get interpolated values of implied volatility for a range of moneyness. To extrapolate beyond the given moneyness, the last known volatility value on the boundary is used. We follow the procedure described in Jiang and Tian (2005) and use 61 points within the moneyness range  $[1/1.2, 1.2]$  to approximate the integrated *MFIV*. To be included in the sample for a given day, the security must have at least four volatility surface points.

To obtain the corresponding time-series of *RV*, we use the underlying security's daily returns from the Securities File. At this stage we filter out the securities that have fewer than 16 valid return observations within each 30 calendar (or 21 trading) days to exclude very illiquid ones. The realized and model-free implied variances are annualized by being scaled up by 12 (for *MFIV*), or by 252 (for *RV*). Having the *MFIV* and *RV*, we calculate the variance swap excess return by the formula 4.

### 3.1.1 Stock Data

To investigate the cross-sectional properties of the individual variance swap returns, we utilize the company-specific information from CRSP, and the CRSP/Compustat merged database. Using several matching historical identifiers (Secid→Cusip/Date→Ncusip/Date), we merged OM with those two databases to extract information about the size of the company, book value, market value, exchange-based turnover of the stock and main industry, where the company operates. Some of the numbers are reported monthly, and we transform them to quarterly ones by aggregation. Based on these indicators, we calculate a number of characteristics.

The size characteristic is calculated as the market value of equity, the value characteristic is the relation of book to market value, and turnover is calculated as the number of shares traded within a given period (i.e., each quarter) over the average number of shares outstanding during that period.

## 4 Results and Discussion

### 4.1 Index vs. Individual Variance Risk Premium

The null hypothesis of this subsection can be summarized as follows: there is no systematic difference in the variance risk premium implied by the individual stock options and the options on traded indices. The alternative would then be that there is a systematic difference, and index variance swap returns are lower than the individual ones. The alternative hypothesis stems from the negative correlation risk premium embedded in index options, as studied by Driessen, Maenhout, and Vilkov (forth.).

To test this hypothesis, we follow the line of reasoning that if stock and index variances are driven by the same factors, then the price of being exposed to these factors in the aggregate security should be smaller than or equal to the price of being exposed to all securities separately. It follows from the diversification effect as some factor exposures (priced or systematic ones) may cancel out when we mix the securities. Thus, to test the hypothesis we need to check if the variance swap returns for indices are equal to or smaller than the variance swap returns for individual stocks. A more accurate test would be to compare the variance risk premium on each index with the variance risk premium embedded in the options based on its components only. We do not check for a mismatch between basket compositions and the individual stock universe at our disposal; however, taking into account that about 100 available indices cover almost the whole market (possibly with some overlap), we are not likely to obtain biased results.

Tables 1a and 1b provide interesting summary statistics on both index and individual variance swap returns (VSW). For the whole sample period<sup>4</sup> index VSW returns are either

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<sup>4</sup>We use daily overlapping 30-day returns, so to avoid an autocorrelation influence, we calculate the autocorrelation adjusted standard errors with 21 lags using Newey and West's (1987) methodology. To be included in the sample, a security must have at least 20% of the maximum datapoints for the nine-year interval. This leaves us with 91 indices and 3409 stocks. For each subperiod we also require at least 20% of the total datapoints present, and the number of qualified securities is given in tables.

negative (61 out of 91) or not different from zero (30 of 91). The means that the 30-day return for the years 1996-2004 is -18.51%. Various subperiods show the same picture, and only in two years (2000 and 2002) the mean VSW returns are not negative. The individual variance swaps look different. They also bring an investor negative returns on average (-4.60% monthly return for the whole period), but the individual names show us a wider cross-sectional distribution of mean time-series difference between the model-free and realized variances. In total, more than a half of the variance swaps returns are not significantly different from zero (1,739 out of 3,409), about 15% (535 out of 3,409) are significantly positive, and the rest (1,135 out of 3,409) are significantly negative. In the subperiods these proportions do not change much, and only in two years (same as for the index: 2000 and 2002) will the investor enjoy positive mean returns, and fewer stocks will be located in negative return bins (356 out of 2,575 and 376 out of 2,326 for the years 2000 and 2002 respectively). The positive returns on the variance swaps look counterintuitive: it is as if the investor likes additional uncertainty about the variance priced into the options and is ready to pay for it. However, note that we cannot make such an inference based on a limited length return path (as there is a difference between an expected and a realized path) - we may be facing the peso problem described by Bondarenko (2003a, 2003b) and extended by Branger and Schlag (2005).

To formalize the observed differences between the variance risk premia embedded in options on indices and on individual securities, we test the significance of the difference of their time series variance swap return means. We assume that the sample means for both index and individual securities are normally distributed, but do not necessarily come from the distribution with equal variances. There is no exact solution for this test known as the Behrens-Fisher problem, and we use the Satterthwaite's approximation for the effective degrees of freedom (see, for example, Kreyszig, 1970). The results of the test are given in Table 2. Using a one-tail t-test at the 2.5% significance level, we strongly reject the null in favor of the alternative hypothesis for the whole period of nine years and all subperiods (but the year 2002). Thus, index and individual variances bear systematically different risk premia, and the index risk premium is significantly more negative than the variance risk premium embedded in individual options.

## 4.2 Individual Variance Swaps: Cross-sectional Analysis

A wide cross-sectional distribution of the mean variance swap returns, seen in Table 1b and also illustrated in Figure 1, inspires further investigation of the systematic differences in variance swap returns among individual names. In this subsection we explore the following question: do the major firm- and asset-based characteristics (size, book-to-market ratio, momentum, industry classification) bear necessary information to predict the sign and/or the magnitude of the variance risk premium embedded in the exchange-traded options? To answer this question we provide several pieces of evidence.

First, we look at the cross-section of mean variance swap returns. One can see obvious patterns in the sign of expected integrated variance risk premium. Table 3 shows the monthly average returns of equally weighted short variance swap portfolios based on deciles of size, book to market ratio, stock exchange-based turnover, and industry. Portfolios are rebalanced annually based on the deciles' composition recalculated based on the

previous calendar year's information (for the first year the deciles are based on the first quarter data). The results are robust to quarterly rebalancing, but we decided to go with the annual ones as transaction costs may be an issue for illiquid stocks.

In general, there are pronounced trends for each portfolio type. The total variance risk premium is positive (as if one pays extra to get more exposure to variance uncertainty) for low BTM firms, and negative (variance risk is priced normally) for high BTM firms. The same holds for the size of the firm - small firms have a positive variance risk premium, and large firms have a negative variance risk premium. The turnover-based portfolios show an opposite trend, i.e., for the illiquid stocks the variance risk premium is negative, and for relatively more liquid ones it is positive. Industry-based portfolios also show a non-trivial variance risk premium, and it is interesting that for two industries (telecoms and hi-tech), on average the realized variance exceeds the variance priced into the plain options. One can use the sorted decile portfolios to design a trading strategy by investing in the extreme deciles with the opposite sign (selling one and buying the other). The strategies give 17.48%, 14.56%, and 24.96% monthly returns with book-to-market, size, and turnover sorting, and almost equal annualized Sharpe Ratio (Sharpe, 1964) of 2.89, 2.85, and 2.84 respectively.

Second, these facts are confirmed to a large extent by the simple cross-sectional OLS regression of the time-series mean variance swap return on the mean firm characteristics over a given period. We test the whole nine-year period and two subperiods: 1996-99 and 2000-04. The output given in Table 4 supports decreasing dynamics of variance risk premium in relation to BTM ratio, and increasing dynamics in respect to stock liquidity proxied by turnover. The additional evidence for the VSW return and size relationship is not persuasive. The size regression coefficient for nine years is not statistically significant, and it changes sign from positive in 1996-99 to negative in 2000-04. By comparing the rebalanced decile portfolio analysis and cross-sectional regression results, it can be seen that in addition to interesting cross-sectional features, the variance risk premium may exhibit a lot of time series dynamics that need to be explained.

### 4.3 Individual Variance Swaps: Economic vs. Traditional Factors

As we have seen in the previous subsection, the underlying characteristics of the firm provide us with the information about the expected variance swap returns, or about the variance risk premia. We cannot claim that some options are overpriced with respect to others, but the embedded risk premium may be a reflection of an instrument's exposure ( $\beta$ ) to the respective risk factors. Even very steady returns over only a nine-year period should not be considered proof of irrationality. In this section we intend to shed light upon the factors driving the variance of individual stocks and the premium that investors pay or demand for being exposed to them.

By construction, the factors driving the returns belong to the traded return space (at least partially with possibly some orthogonal part belonging to a non-traded space). Variance swaps provide an investor with exposure to the variance process dynamics. However,

a well-documented fact is that variance is correlated with the market and stock processes<sup>5</sup>. As a consequence, in searching for factors we need to go to both stock and variance swap payoff spaces. We will proceed in two directions. First, we will select the candidate factors based on the economic reasons and estimate the risk premia for them. The problem with this approach is that even the standard Fama-French factors are far from orthogonal, and represent a mix (or serve as proxies) of some systemic factors. Second, we perform formal factor analysis on the variance swap returns to extract the factors explaining most of the return variance and see what these factors correspond to. This approach will provide a better fit statistically, but we may have problems with factor identification.

We would like to work with the linear factors, and we assume that the orthogonal variance factors are not priced in stocks. Then, in the APT framework we can write the following linear factor model for excess returns on two types of assets (stocks and variance swaps on stocks) as follows:

$$\forall i : \begin{cases} r_{i,t}^S = \alpha_i + \sum_{n=1}^k \beta_{i,n}^S r_t^{f,n} + \varepsilon_{i,t}^S \\ r_{i,t}^\sigma = \alpha_i^\sigma + \sum_{n=1}^k \beta_{i,n}^\sigma r_t^{f,n} + \sum_{n=k+1}^N \beta_{i,n}^\sigma r_t^{f,n} + \varepsilon_{i,t}^\sigma, \end{cases} \quad (5)$$

where  $r_{i,t}^S$  and  $r_{i,t}^\sigma$  are excess returns of the stock  $i$  and variance swap on that stock at time  $t$ , and  $r_t^{f,n}$  with  $n = 1..k$  are the factors (from the stock returns space) driving both stock and variance, and with  $n = k + 1..N$  the factors (from the variance swap returns space orthogonal to the stock returns space) driving variance only. We assume that stock and variance factors are orthogonal, i.e.,  $E[r^{f,j} r^{f,l}] = 0$ ,  $\forall j = 1..k, l = k + 1..N$ .

We are not interested in the risk premia on the stock factors, and hence we estimate this model in the second equation only, identifying the factor risk premia for orthogonal variance-driving factors separately. For that we first have to remove the effect of the stock-based factors from the variance swap returns by regressing variance swap returns on these factors and taking the residuals as the input to the next procedure. We include in the stock factors three FF factors (Fama and French, 1992) and momentum (Carhart, 1997), where all are calculated with monthly frequency. To be consistent with other asset-pricing literature we use the ready-to-use factors from Kenneth R. French's website rather than the self-made ones. Then we construct three candidate variance factors from the sorted decile portfolios of variance swaps. We saw in the previous section that the size, book-to-market, and the stock turnover sorted portfolios exhibit a smooth monotonous relationship of excess return to sorting variable. In each of the three factor portfolios we invest an equal amount in the two top deciles for the respective sorting variable and take a short position for the same amounts in the two bottom deciles, thus we create standard long-short portfolios from annually rebalanced variance swap portfolios. In Table 5 one can see the correlations between the stock- and variance swap-based factors. Interesting is that size and book-to-market factors derived from variance swaps are not highly correlated with the stock factors, but the correlation is negative (with one exception of momentum,

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<sup>5</sup>Formal econometric analysis of stock return/latent volatility dynamics estimated from the financial series (Figlewski and Wang, 2000; Bekaert and Wu, 2000; Dennis, Mayhew, and Stivers, 2005; Yu, 2005; many others) provides formal support to the asymmetric stochastic volatility, and hence indirectly to the leverage effect. Later in this section we will also perform new tests of the leverage effect based on the variance risk premia dynamics.

where it is positive), while the turnover factor on average has a higher correlation to stock factors. We should also note that size and BTM factors are higher correlated with each other (correlation 0.76), and it may be problematic to use both of them in the regression simultaneously due to collinearity.

For the risk premia estimation we follow a standard two-stage Fama and MacBeth (1973) procedure using monthly data and assuming constant betas over the whole nine-year period. We build 30 portfolios from our variance swaps, using the estimated factor beta as a sorting variable. At the second stage we infer the factor risk premia with GLS (using the variance-covariance matrix of the first-stage residuals as a weighting matrix, as suggested by Shanken (1985), among others) from these variance swap portfolios. The turnover factor is the only one with the only risk premium that turns out to be significant and robust to changes in a number of portfolios, sorting, sampling frequency, and which sign and magnitude coincide with the mean return on the factor portfolio. The risk premium on the turnover factor is positive ( $\gamma_{turnover} = 0.3645$  with Shanken (1992) corrected  $t$ -statistics 9.76), i.e., a variance swap yields higher return if it is positively exposed to the turnover factor. The result is compatible with Table 3, and may be explained in terms of replication costs (or delta hedge costs) for options on illiquid stocks vs. liquid stocks<sup>6</sup>.

Thus, so far, we are able to identify only one robust linear factor for variance swaps (beyond the stock factors affecting the variance) based on economic intuition. Now we try using a different approach: with statistical methods we identify factors responsible for the most of the variance on the variance swap returns, and then make sense of them. This procedure resembles the approach by Ludvigson and Ng (2005), who used it for stock returns.

Principal component analysis (PCA) finds the factors explaining most of the total variance of the given variables as a linear combination of them. Applying PCA to variance swap and stock returns separately, we isolate in both variable's spaces the first five principal components with the top eigenvalues (first five eigenvalues for stocks range from 3.92 to 0.31, and for variance swaps between 99.99 and 12.01). The second principal component for both spaces is about 4-5 times less important than the first in terms of amount of total variance explained. Depending on the criteria for choosing the number of important principal components, we may end up with 2-3 factors for each space, but we still decided to take five factors for both stocks and variance swaps.

To see what these principal components (PC) represent, we calculate a number of pairwise correlations (in Table 6) between the principal components for variance swaps and stocks, between variance swap principal components and economic factors (3 Fama-French +1 momentum usual factors), and between stock principal components and the same economic factors. Most interesting is the left upper quadrant of Table 6. From these numbers we can gauge the factor<sup>7</sup> structure correspondence for variance swaps and stocks. The inference is straightforward - most of the variance swap return dynamics are explained by the factors linearly independent from the factors driving stock returns. Only the first variance swap PC is somewhat correlated with two stock PCs (with correlations -0.28

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<sup>6</sup> In addition to the turnover factor (variance swap-based), we tried to use the Pastor and Stambaugh (2003) liquidity factor for stocks, but the estimations of the risk premia using variance swap portfolios turned out to not be robust enough to slight regression modifications.

<sup>7</sup>For linear factors only as principal components represent some linear combinations of returns.

and 0.24), and this may hint at the known 'leverage' effect that we also test in the next subsection.

The correlations between stock and economic factors do not reveal much new information: the first PC explaining most of the stock variance (with eigenvalue 3.92) is almost perfectly correlated with the market excess return ( $\rho = 0.92$ ) and also highly correlated with the other factors (with absolute correlations ranging from 0.31 to 0.56 for PCs with eigenvalues equal to 1.11 and smaller). Such a high correlation with all factors simultaneously follows from the high correlation among the economic factors in the first place. The second and further stock PCs explain four and more times less variance than the first factor, though they are still greatly correlated with SMB and HML factors. Looking at the relationship between the variance swap PCs and economic factors, we can stress quite a low correlation between them, but it is difficult to interpret due to the mentioned common driving force in economic factors.

#### 4.4 Individual Variance Risk Premium: Leverage Effect

The integrated variance risk premium provides us with the unique opportunity to test the existence of the leverage effect - a generic name for the observed negative correlation between the variance and stock and/or market returns. Numerous studies have discussed the leverage effect. Theoretical and empirical treatment has been suggested by Cox and Ross (1976), Black (1976), and Christie (1982), among many others. An agreement in the field is that volatility and stock returns are correlated, though the exact effect of that correlation on expected stock returns is still not very clear. Bekaert and Wu (2000) reject the pure leverage model, and give support for the related volatility feedback story. They also provide a nice summary of previous work (see Table 1 in their paper).

We look at the volatility-return relation from a different angle: we test the relationship between the factors (both market part and total stock return) and integrated variance risk premium rather than just identify the combined dynamics of the series. If the leverage effect is linked to stock return or market factor exposure through their effect on the riskiness of the company equity, then the price of risk (through the changing leverage and volatility feedback), or integrated risk premium, should also be related to stock return and factor dynamics. In other words, if equity and equity variance are driven by the same factors, then they are correlated, and the variance risk premium should be related to this correlation in a logical way. We opt for several pieces of evidence for the leverage effect.

First, we look at the correlation between the monthly individual stock returns  $r_{i,j} = \frac{S_{i+1,j}}{S_{i,j}} - 1$  and the change in realized variance for a given stock  $j$  from the previous month to the current one, i.e.,  $\Delta\sigma_{i,j}^2 = \sigma_{i:i+1,j}^2 - \sigma_{i-1:i,j}^2$ . As expected, the average correlation is significantly negative for almost 90% of the underlying securities in the sample, and for this part of the sample equals  $-0.04$ . Assuming that the variance risk premium is proportional to the variance, we should anticipate the negative correlation between the stock return and the change in the variance risk premium, i.e., the correlation between the  $r_{i,j}$  and  $\Delta r_{i,j}^\sigma = r_{t:t+1,j}^\sigma - r_{t-1:t,j}^\sigma$ . This is indeed true for our sample, and we find a significant mean negative correlation ( $-0.15$ ) for all stocks, a negative correlation ( $-0.17$ ) for the subsample where the correlation between the stock return and change in realized

variance was negative, and, logically, we find a significant positive correlation (0.07) for the subsample, where the correlation between the stock return and change in realized variance was positive. This may serve as an informal proof that stock and variance processes have at least some priced common factors, and that on average, risk premia on stocks and variance are negatively correlated. Hence, being exposed to both asset classes (stocks and variance contracts) may be beneficial in terms of diversification.

Second, we look at the same relationship with respect to the one selected factor that is present in the stock returns and also affects the variance swap returns (as we have seen in the previous subsections) - the market factor. Basically, we search for a form of a systematic leverage effect instead of the total leverage effect. On the scatter plot in Figure 2 we can see that there is an evident negative relationship between the market factor to stock return correlation  $\rho_{r_m, r_j}$  and market factor to changes in variance correlation  $\rho_{r_m \Delta \sigma_j^2}$ . Running a simple regression of the form

$$\rho_{r_m \Delta \sigma_j^2} = \alpha + \beta \rho_{r_m, r_j}$$

with White (1980) standard errors, we get a significant estimation of the slope  $\beta = -0.27$  and a decent  $R^2 = 11\%$ . Hence, taking into account that the correlation between the market factor and the change in variance is mostly negative, higher exposure of the stock to the market factor basically means a higher negative beta of variance with respect to the market factor. From this observation we may formulate a working hypothesis that variance risk premium is also related to a correlation between the market factor and the individual stock variance changes. Looking at the scatter plot in Figure 3 we would most probably reject this hypothesis immediately as the variance risk premium does not seem to depend upon the mentioned correlation at all. Running the regression of variance risk premium on the constant and the correlation between the market factor and variance changes we fail to reject the hypothesis formally, achieving a slope of  $-0.07$  with a White (1980) t-stat of 2.45, but the  $R^2$  of less than 0.1% tells us that economically the market factor cannot compete with other factors that contribute to the variance risk premium. Regressing the changes in variance risk premium on the same regressors, we achieve a slightly negative and insignificant slope coefficient.

This analysis gives us one more reason to believe that although there is a pronounced leveraged effect in volatility in general, and the market factor seems to be present in both stock returns and variance process, the majority of the variance risk premium is explained by the factors beyond the market. It may be the case that some variance specific factor bears a much higher premium than the market factor.

## 4.5 Implications for Portfolio Policy

The discussion above has important implications for the portfolio policy of an investor willing to optimize her portfolio with access to both the stocks and the variance contracts (or to a wide cross-section of plain options to replicate them) on these stocks. The variance swaps provide a way to extract a significant risk premium, as has been seen from decile portfolio analysis. The annualized Sharpe Ratios of the long-short portfolio in variance



swaps based on such underlying stock characteristics as size, book-to-market, and turnover lie around the 2.84 – 2.89 level, and thus variance swaps alone (synthetically through the traded options) should be considered an interesting and lucrative investment vehicle.

What is especially important for a portfolio policy of an optimizing investor is that the variance swap returns seem to have a different factor structure from that of the underlying stocks. This confirms that variance contracts (and options that we used to approximate these) are not redundant securities. As we can see from the correlations between the variance swap-based factors and traditional stock factors, there is not much of a relationship between the two groups. The market factor is still in play, and this is confirmed by the principal component analysis - it reveals the only factor that is common to both return spaces and that is highly correlated to the market factor.

Formal estimation of the factor risk premia (on the selected factors constructed from the variance swap return space) in the variance swap returns after taking out the standard three Fama-French and a momentum factors shows that the estimations are not robust for factors based on size and book-to-market values of underlying stocks. It may be due to the short estimation period, and further investigation is necessary. The stock turnover-based factor bears a significant premium in variance swap returns, and it may reflect the transaction costs of replicating the synthetic variance swap positions. The investigation of the leverage effect reveals that even though it exists and is quite pronounced on the stock level for the total return-variance relationship and for the systematic part of the stock return<sup>8</sup>-variance relationship, the variance risk premium is mostly made up of variance-specific risk factors.

## 5 Conclusion

Potentially, many theoretical models are compatible with the priced variance on the individual stocks and the indices, and some models predict and explain the differential pricing of index and individual variance. Using a large data set on index and individual stock options in the US, we provide empirical evidence to support and extend such theories.

We show that index and individual securities are different in terms of variance prices they pass on to plain options, and that the index options are systematically more expensive than the individual ones. Variance risk in individual options is less expensive on average, but individual variance swap returns exhibit a lot of cross-sectional variation. Using underlying stock characteristics like size, book-to-equity value, and exchange-based turnover for the stock we can create portfolios of variance swaps having a monotonic return relationship to the value of a given characteristic. Interestingly, for some extreme characteristic values (small size of the firm, high exchange-based stock turnover), as well as for two industries (hi-tech and telecoms), the variance seems to be systematically underpriced with respect to its realized value. Studying the factor structure of the variance swap returns we come to a conclusion that variance has at most one important factor in common with the stock returns, and this is the market factor. Economic factors from

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<sup>8</sup> Represented by the market factor.

the stock return space (3 Fama-French + 1 momentum factors) are in general correlated among each other and with the market factor, hence it is hard to separately identify the influence that each of them has on the variance risk premium, and we make use of principal component analysis for this task. Suggesting a conceptually new method for testing the leverage effect, we confirm that there is a leverage effect for individual stocks on average and that the correlation between a stock and its variance affects the variance risk premium in a predictable way. We also find the systematic leverage effect stemming from the market factor, but the contribution of the market risk premium to the total variance risk premium is very small and economically insignificant.

This work offers comprehensive empirical analysis of the variance risk premium, and thus stimulates the work in a theoretical direction that may explain the observed stylized facts. If the difference between the index and the individual variance risk premium really stems from the stochastic correlation, it is important to model the correlation so that individual stocks can be aggregated into an index in a consistent way. It may also be interesting to work with the non-linear factors instead of the linear factors as it has been shown (Jones, 2006) that option returns are better explained by non-linear factors. Another possibility is to recognize that investors are not quite rational and cannot perform the correct comparison of latent variance processes, or there is a disagreement on the form or parametrization of such processes. This may also lead to systematic differences in variance pricing. In general, there are still many directions in which one can search for an explanation of the stylized factors documented in this work.

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## Tables 1a-1b: Indices and Individual Stocks: Variance Swap Returns

The tables report the mean variance swap returns for the US indices and individual securities with exchange-based options. The numbers are calculated from excess returns on synthetic variance swaps, as described in Section 2. To calculate the mean VSW return from the daily observations of monthly VSW returns we first compute the simple time-series average of return for each underlying with the standard errors corrected for autocorrelation with lag=21 (Newey and West, 1987), and then cross-sectionally average these time-series numbers. To be in the sample for the first stage calculation, an underlying asset should have at least 20% of the maximum number of observations in that period. We report the number of qualified assets for mean calculations, and the average number of time-series observations for these assets. The last three columns give the number of assets with the t-stat from the TS mean calculation in a given range.

*Table 1a: Indices*

Period	Mean VSW return	# of indices	Mean # of obs.	# of indices with NW t-stat		
				(- $\infty$ , -1.96)	(1.96, $\infty$ )	(-1.96, 1.96)
1996-2004	-0.1851***	91	1593	61	0	30
1996-1999	-0.2382***	76	844	53	0	23
2000-2004	-0.1591***	93	908	52	5	36
1996-1998	-0.2298***	71	654	46	0	25
1999-2001	-0.1326***	77	657	38	3	36
2002-2004	-0.2192***	87	602	60	2	25
1996	-0.3723***	60	217	48	0	12
1997	-0.2124***	68	234	32	0	36
1998	-0.1506***	73	243	30	1	42
1999	-0.2642***	75	239	53	1	21
2000	-0.0045	71	236	16	8	47
2001	-0.0985***	73	224	24	2	47
2002	0.0536	69	242	5	9	55
2003	-0.3328***	69	238	58	1	10
2004	-0.2983***	87	218	71	2	14

## Tables 1a-1b: Indices and Individual Stocks: Variance Swap Returns

The tables report the mean variance swap returns for the US indices and individual securities with exchange-based options. The numbers are calculated from excess returns on synthetic variance swaps, as described in Section 2. To calculate the mean VSW return from the daily observations of monthly VSW returns we first compute the simple time-series average of return for each underlying with the standard errors corrected for autocorrelation with lag=21 (Newey and West, 1987), and then cross-sectionally average these time-series numbers. To be in the sample for the first stage calculation, an underlying asset should have at least 20% of the maximum number of observations in that period. We report the number of qualified assets for mean calculations, and the average number of time-series observations for these assets. The last three columns give the number of assets with the t-stat from the TS mean calculation in a given range.

*Table 1b: Individual Stocks*

Period	Mean VSW return	# of stocks	Mean # of obs.	# of stocks with NW t-stat in		
				$(-\infty, -1.96)$	$(1.96, \infty)$	$(-1.96, 1.96)$
1996-2004	-0.0460***	3409	1316	1135	535	1739
1996-1999	-0.0375***	3090	706	886	410	1794
2000-2004	-0.0365***	2793	880	900	433	1460
1996-1998	-0.0138***	2772	566	699	351	1722
1999-2001	0.0148***	3167	524	739	687	1741
2002-2004	-0.1296***	2482	596	1106	75	1301
1996	-0.1461***	2006	224	872	100	1034
1997	-0.0617***	2400	222	752	168	1480
1998	0.0865***	2708	224	418	388	1902
1999	-0.0649***	2788	224	911	286	1591
2000	0.1667***	2575	214	356	811	1408
2001	-0.0254***	2298	220	579	271	1448
2002	0.0313***	2326	220	376	276	1674
2003	-0.224***	2184	228	1274	33	877
2004	-0.2049***	2362	209	1256	26	1080



Table 2: Testing the Difference in Variance Swap Returns

This table provides information for testing the null hypotheses that variance swap returns on individual and index type securities are equal vs. the alternative that index variance swap returns are lower than the individual ones. We assume that the sample means for both index and individual securities are normally distributed, but do not necessarily have equal variances. This is a Behrens-Fisher problem, and we use the Satterthwaite's approximation for the effective degrees of freedom. Using a one-tailed t-test at the 2.5% significance level, we calculate the p-values and confidence intervals for the equal means.

Period	Mean VSW returns			H0: equal means vs index < indiv		
	Index	Indiv	Index-Indiv	p-value	conf int	conf int
1996-2004	-0.2098	-0.0460	-0.1391	0	$-\infty$	-0.1079
1996-1999	-0.2445	-0.0375	-0.2007	0	$-\infty$	-0.1656
2000-2004	-0.1731	-0.0365	-0.1226	0	$-\infty$	-0.0817
1996-1998	-0.2413	-0.0138	-0.2160	0	$-\infty$	-0.1742
1999-2001	-0.1441	0.0148	-0.1474	0	$-\infty$	-0.1003
2002-2004	-0.2221	-0.1296	-0.0896	0	$-\infty$	-0.0483
1996	-0.4031	-0.1461	-0.2262	0	$-\infty$	-0.1822
1997	-0.1936	-0.0617	-0.1507	0	$-\infty$	-0.1013
1998	-0.1923	0.0865	-0.2371	0	$-\infty$	-0.1812
1999	-0.2824	-0.0649	-0.1993	0	$-\infty$	-0.1582
2000	-0.0163	0.1667	-0.1623	0	$-\infty$	-0.0932
2001	-0.1132	-0.0254	-0.0730	0.01	$-\infty$	-0.0141
2002	0.0178	0.0313	0.0223	0.81	$-\infty$	0.0716
2003	-0.3567	-0.2224	-0.1104	0	$-\infty$	-0.0641
2004	-0.3003	-0.2049	-0.0934	0	$-\infty$	-0.0538

Table 3: Variance Swap Portfolios' Returns by BM, Size, Turnover Deciles, and 10 Industry Portfolios

Table 3 exhibits the monthly excess returns on the decile portfolios formed by book-market ratio and size of the company, exchange-based turnover of the stock, and the 10 industry portfolios (as defined on Kenneth French's website). Portfolios are rebalanced annually based on the deciles composition, recalculated based on the previous calendar year's information (for the first year the deciles are based on the first quarter data). To calculate the mean returns we first calculate the cross-sectional average return for a portfolio on each day, and then calculate the time-series mean return for each portfolio over the entire nine years. As a remedy for autocorrelation we use Newey and West (1987) standard errors to get t-statistics. The returns and Sharpe Ratios on the difference portfolios (between decile 1 and 10 for BM/Size/Turnover) are calculated from the same time series of one-month variance swap returns, but without overlap, i.e., with sampling every 21 days. The SR is annualized, while the returns are monthly.

Decile	BTM		Size		Turnover		Industry		
	Mean	t-stat	Mean	t-stat	Mean	t-stat	Name	Mean	t-stat
1	4.61%	1.627	3.21%	1.352	-16.74%	-6.665	<b>NoDur</b>	-17.71%	-7.885
2	1.90%	0.769	6.55%	2.578	-17.08%	-7.034	<b>Durbl</b>	-8.56%	-3.364
3	1.24%	0.487	5.18%	2.035	-13.22%	-5.361	<b>Manuf</b>	-8.05%	-3.291
4	-3.54%	-1.432	3.18%	1.282	-11.39%	-4.687	<b>Enrgy</b>	-8.56%	-3.555
5	-6.03%	-2.550	-1.41%	-0.581	-8.11%	-3.411	<b>HiTec</b>	7.66%	2.970
6	-7.64%	-3.229	-4.30%	-1.760	-5.40%	-2.239	<b>Telcm</b>	1.40%	0.476
7	-8.36%	-3.449	-8.68%	-3.688	-2.40%	-0.970	<b>Shops</b>	-4.68%	-1.934
8	-11.49%	-4.904	-11.40%	-5.000	1.89%	0.722	<b>Hlth</b>	-3.27%	-1.389
9	-13.92%	-5.696	-12.84%	-5.108	4.89%	1.889	<b>Utils</b>	-31.25%	-10.385
10	-13.01%	-5.922	-11.65%	-4.353	8.59%	3.205	<b>Other</b>	-9.83%	-3.712
<b>10-1</b>	-17.48%	-6.978	-14.56%	-6.375	24.96%	10.672			
<b>SR</b>	2.89	-	2.85	-	2.84	-			

Table 4: Variance Swap Portfolios' Returns vs. Firm Characteristics: Cross-Sectional Analysis

This table shows the results of the cross-sectional OLS regression of the time-series mean one-month variance swap return for each company on the average characteristics of this company over the nine-year period, and two subperiods: 1996-99 and 2000-04.

	1996-2004		1996-1999		2000-2004	
Variable	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
Constant	-0.1206	-25.97	-0.1472	-24.91	-0.1069	-17.90
BTM	-0.0035	-7.44	-0.0026	-2.91	-0.0017	-8.88
Size	0.0838	1.11	0.4401	3.00	-0.1472	-2.06
Turnover	0.0045	24.86	0.0065	25.62	0.0039	18.83

Table 5: Stock vs. Variance Swap Factors Correlations

The table shows the sample time series correlation between the monthly time series of traditional stock factors (three Fama-French and momentum factor) and the variance swap factors. We derive the variance swap factors as the returns on the long-short investment in decile portfolios based on the size, book-to-market, and turnover characteristics of the underlying stock. In contrast, with stock factors, for variance swaps we always buy the highest decile and sell the lowest one.

Factors	Stock Factors				VSW Factors		
	Market	SMB	HML	MOM	Size	LMH	Turnover
Stock: Market	1						
Stock: Size	0.19	1					
Stock: BTM	-0.56	-0.52	1				
Stock: Mom	-0.23	0.17	-0.06	1			
VSW: Size	-0.18	-0.25	-0.13	0.02	1		
VSW: BTM	-0.16	-0.09	-0.14	0.07	0.76	1	
VSW: Turnover	-0.24	-0.28	0.22	-0.11	0.15	0.06	1

Table 6: PCA: Stock vs. Variance Swap vs. Traditional Factors Correlations

The table shows the sample time series correlation between the monthly time series of traditional stock factors (three Fama-French and momentum factor) and the first five Principal Components derived using the PCA from monthly variance swap returns, and the first five Principal Components derived from the monthly stock returns.

	Variance Swap Principal Components					Stocks Principal Components				
	PC1	PC2	PC3	PC4	PC5	PC1	PC2	PC3	PC4	PC5
Stocks: PC1	-0.28	-0.10	0.07	0.01	0.17					
Stocks: PC2	-0.13	0.04	-0.08	-0.14	0.03					
Stocks: PC3	0.07	-0.02	-0.01	0.05	-0.12					
Stocks: PC4	-0.06	-0.02	-0.10	-0.31	-0.02					
Stocks: PC5	0.24	-0.14	0.06	0.08	0.15					
Mkt-Rf	-0.26	-0.03	0.12	0.12	0.20	0.92	0.11	-0.05	-0.20	0.12
SMB	-0.31	0.03	0.03	-0.01	-0.03	0.43	-0.50	0.13	0.25	-0.54
HML	-0.03	-0.02	-0.26	-0.19	-0.11	-0.56	0.62	-0.01	0.12	-0.03
UMD	0.00	0.18	-0.03	0.15	-0.11	-0.31	-0.31	0.24	-0.12	-0.04

Figure 1: Mean Variance Swap Return Distribution: Individual Securities

Figure 1 shows the empirical distribution of the time-series mean variance risk premium. The variance risk premium is approximated by the variance swap return and is calculated as the relationship between realized variance and model-free implied variance over the same monthly period, minus 1. The variance swap returns for each security are overlapping and observed daily. The histogram is constructed using 30 bins, and the density function is approximated using Normal kernel. The red dotted line shows the mean of the cross-sectional distribution.

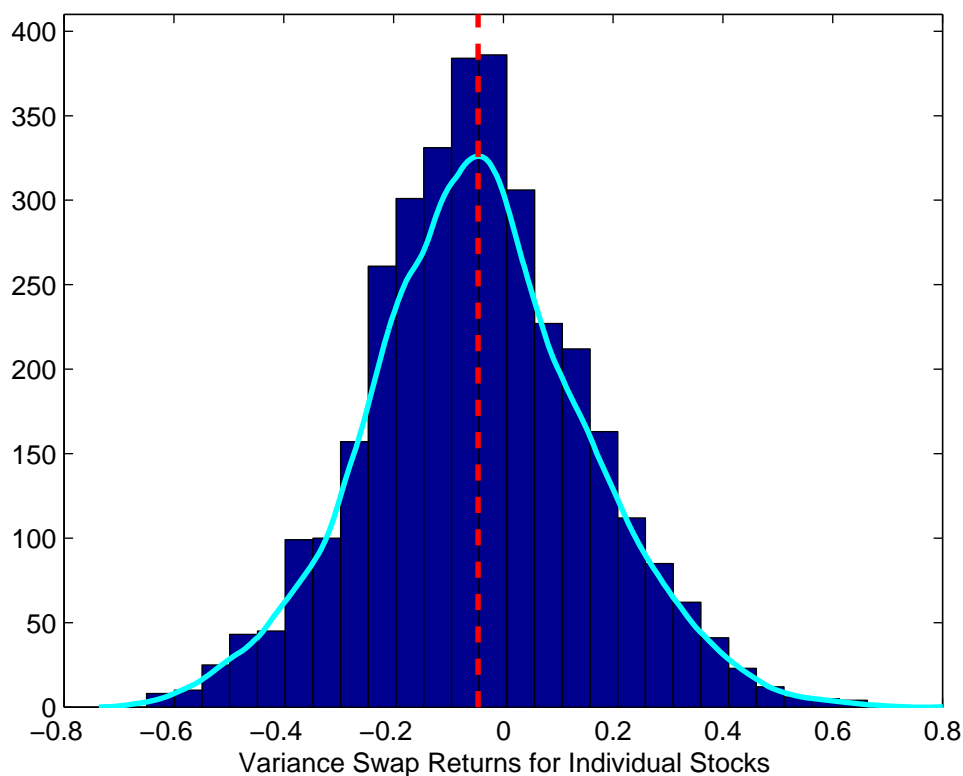


Figure 2: Market Return to Individual Variance Correlation vs. Market to Individual Returns Correlation

The figure shows the relationship between the correlations of market return with variance changes and with individual returns. The correlations are calculated using non-overlapping monthly (21 working days) market/individual stock returns and changes in monthly realized individual stock variances from daily stock returns.

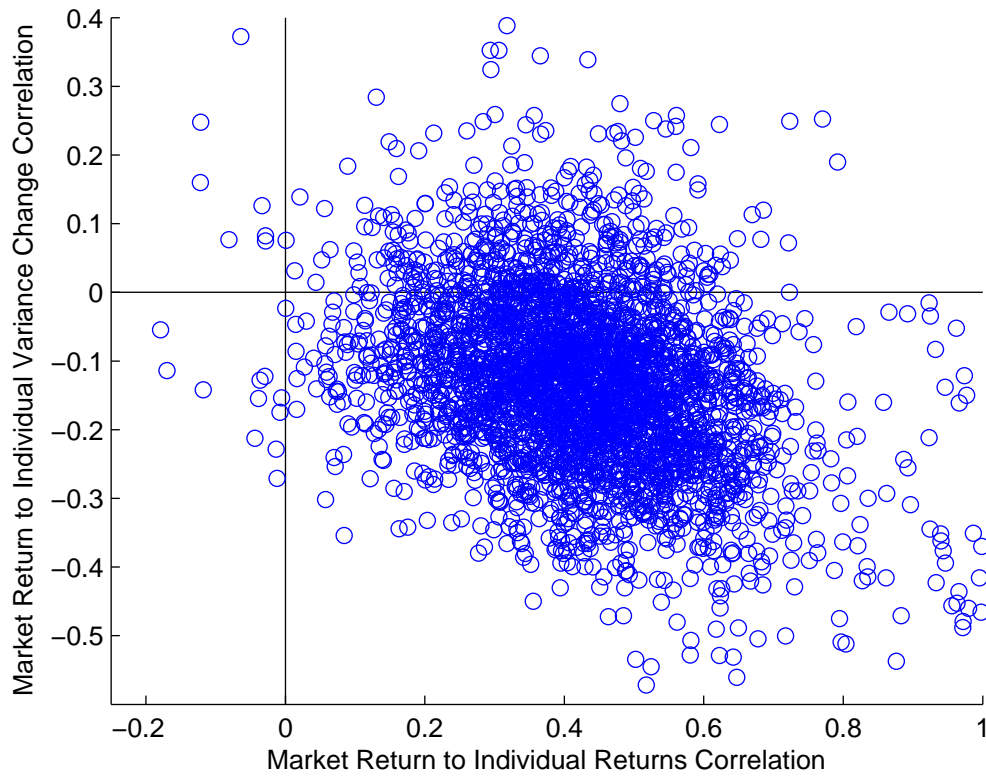


Figure 3: Market Return to Individual Variance Correlation vs. Individual Short Variance Swap Returns

The figure shows the relationship between the correlations of market return with variance changes and individual stock integrated variance risk premium (variance swap returns). The correlations are calculated using non-overlapping monthly (21 working days) observations of market returns, and changes in individual stock monthly variance from daily stock returns. The variance risk premium is approximated by the variance swap return and is calculated as the relationship of realized variance to model-free implied variance over the same monthly period, minus 1.

