MA3237 : Computer Project #3

Due on Wednsday, 4 Apr 2014

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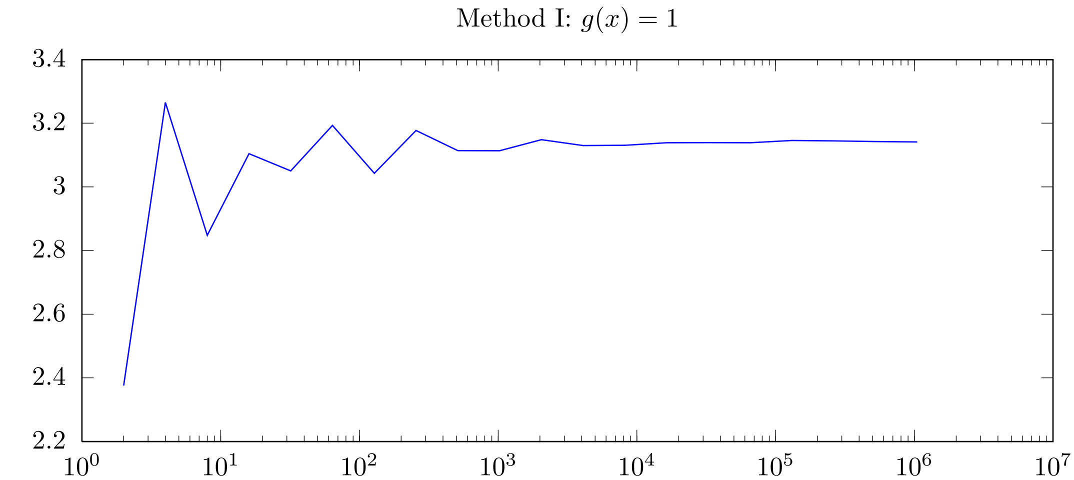
Part i

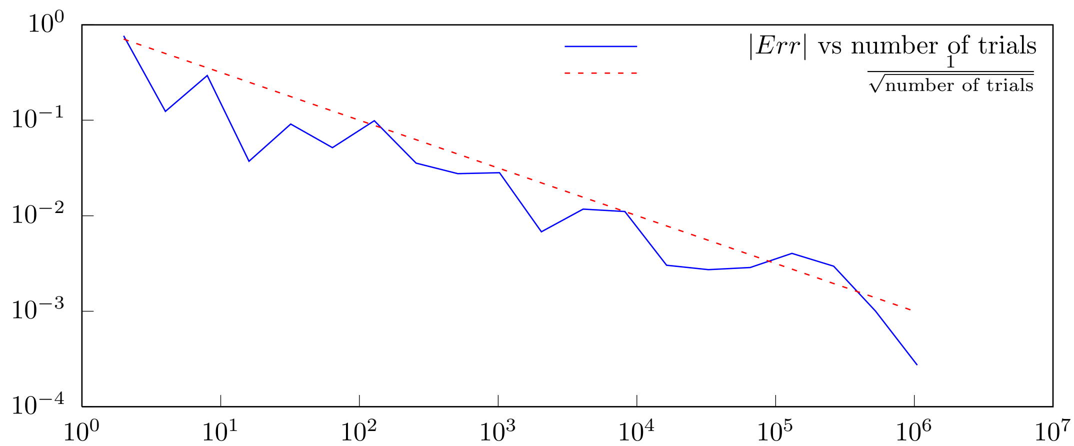
Code

```
clear;
close all;
graphics_toolkit("gnuplot");

function I = repI(i)
        I = [];
        for j = 1:i
                        I = [I,'I'];
        end
end

nTrials=2.^(1:20); m=length(nTrials);

vEst = cell(3,1);
for i = 1:3
        vEst{i} = zeros(1,m);
end

rand('state',0);
for i=1:m
        n=nTrials(i);
        x = rand(1,n);
        vEst{1}(i) = sum(4*sqrt(1-x.^2))/n;
        x = -sqrt(4 - 3*rand(1,n)) + 2;
        vEst{2}(i) = sum(12*sqrt(1-x.^2)./(4-2*x))/n;
        x = -sqrt(1 - rand(1,n)) + 1;
        vEst{3}(i) = sum(4*sqrt(1-x.^2)./(2-2*x))/n;
end

err = cell(3,1);
for i = 1:3
        err{i} = abs(vEst{i} - pi);
end

equation = cell(3,1);
equation{1} = '$g(x) = 1$';
equation{2} = '$g(x) = \frac{4-2x}{3}$';
equation{3} = '$g(x) = 2-2x$';

for i = 1:3
        the_plot = figure(i);
        subplot(2,1,1);
        semilogx(nTrials,vEst{i}, 'LineWidth', 2);
        title(['Method ',repI(i),': ', equation{i}]);
```

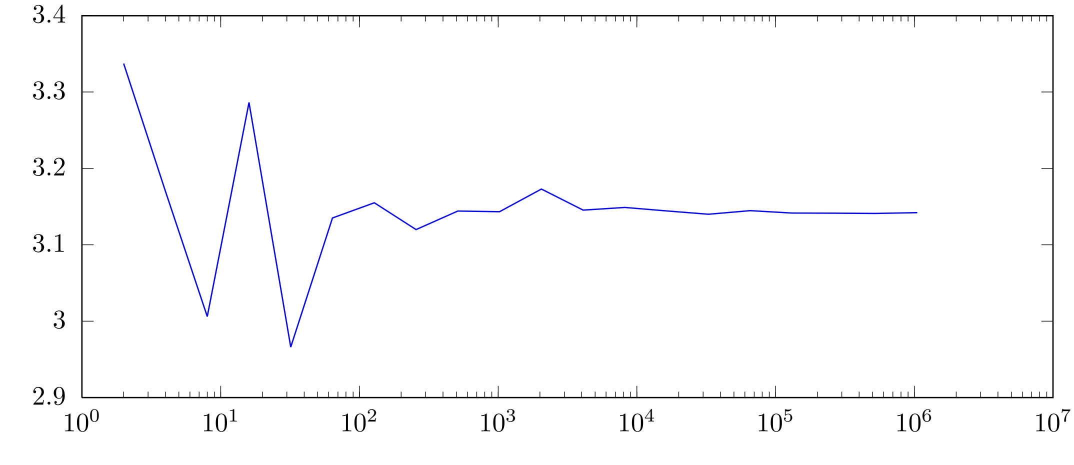
```
        subplot(2,1,2);
        hold on;
        loglog(nTrials, err{i}, 'LineWidth', 2);
        loglog(nTrials, 1./sqrt(nTrials),'r:','LineWidth', 2);
        title(['Method ', repI(i)])
        legend('$\abs{Err}$ vs number of trials','$\frac{1}{\sqrt{\text{number of trials}}}

        print(the_plot,['Method ',repI(i),'.tex'],'-S470,420','-dtex')
end

err_plot = figure(4);
err_diff = cell2mat(err).*sqrt(nTrials);
boxplot(err_diff', 1, '+', 0);
title('$\abs{Err} \times \sqrt{\text{number of trials}}$ for different methods')
set(gca(), 'ytick', [1 2 3], 'yticklabel', equation)
print(err_plot,'comparison','-S470,160','-dtex')
```

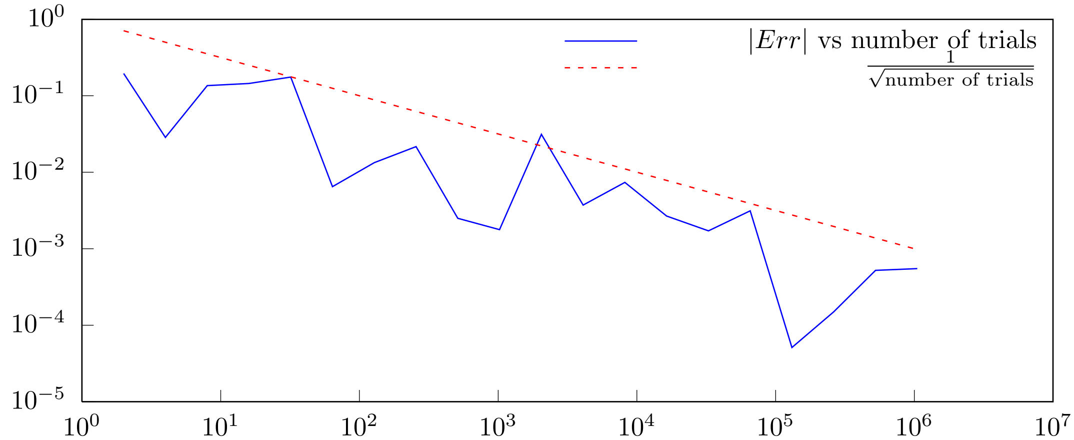
Figures


Method I: $g(x) = 1$
3.4
3.2
3
2.8
2.6
2.4
2.2
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7

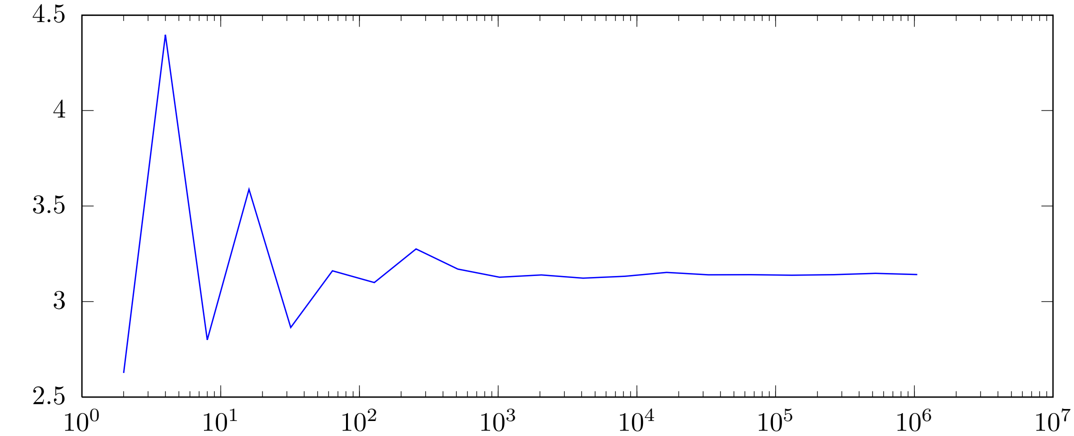

$|Err|$ vs number of trials
$\frac{1}{\sqrt{\text{number of trials}}}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7

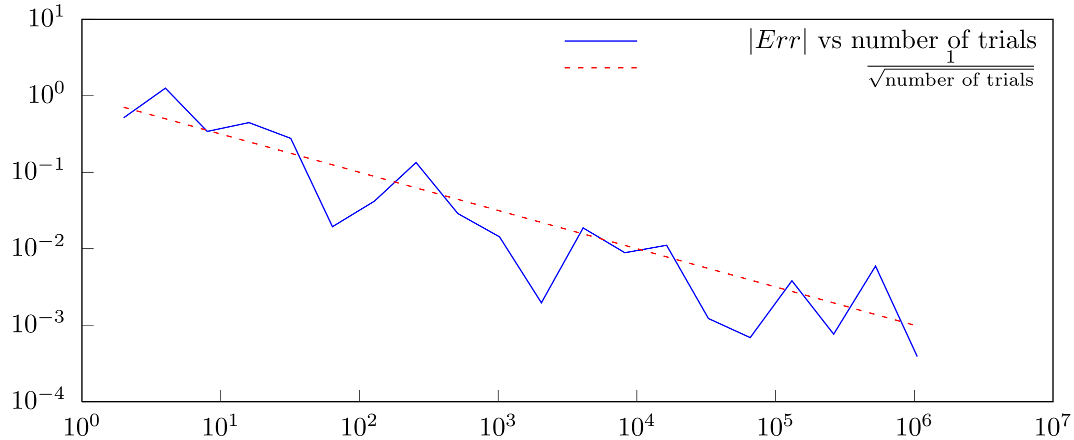
Method II: $g(x) = \frac{4-2x}{3}$



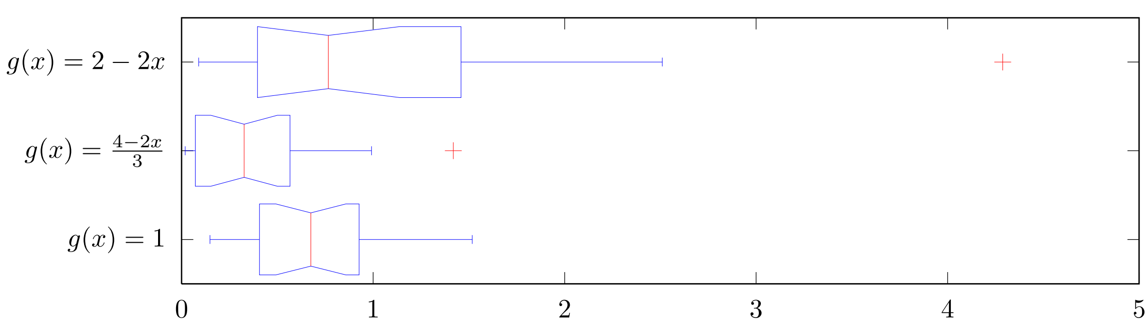


Method III: $g(x) = 2 - 2x$





$|Err| \times \sqrt{\text{number of trials}}$ for different methods



Part ii

Code

```
clear; close all;
graphics_toolkit("gnuplot");

N_bins = 101;

function M = Metro_Ising(mu, N_steps, sample_rate, N)
        x = ones(N, 1);
        M = zeros(0, N_steps/sample_rate);
        for i = 1:N_steps
                j = ceil(N*rand);
                if j == 1
                        h = exp(-2*mu*x(1)*(x(2)));
                elseif j == N
                        h = exp(-2*mu*x(N)*(x(N-1)));
                else
                        h = exp(-2*mu*x(j)*(x(j-1)+x(j+1)));
                end
                U = rand;
                if U <= h
                        x(j) = -x(j);
                end
                if mod(i, 50) == 0
                        M(i/50) = sum(x);
                end
        end
end

N_steps = 1e6;
sample_rate = 50;
N = 50;
%mesh = -N:floor(2*N/(N_bins - 1)):N;
%[n, h] = hist(M, mesh);
the_plot = figure();
M_h = Metro_Ising(1, N_steps, sample_rate, N);
M_l = Metro_Ising(2, N_steps, sample_rate, N);

h_plot = subplot(2,1,1);
hist(M_h, 101);
title(['Histogram of sum of states of high temperature ($\mu = 1$),
        '1D Ising model with $20,000$ from $1,000,000$ states']);
xlabel('Sum of states');
ylabel('Number of samples');

l_plot = subplot(2,1,2);
hist(M_l, 101);
```

Part ii continued on next page...

```
title(['Histogram of sum of states of low temperature ($\mu = 2$), ...
        '1D Ising model with $20,000$ from $1,000,000$ states']);
xlabel('Sum of states');
ylabel('Number of samples');

print(the_plot, ['MetropolisIsing','.tex'],'-S520,400','-dtex')
```

Figures