The International Association for the Properties of Water and Steam

Berlin, Germany
September 2008

Release on the IAPWS Formulation 2008 for the Viscosity of Ordinary Water Substance

This release contains 9 pages, including this cover page.

This release has been authorized by the International Association for the Properties of Water and Steam (IAPWS) at its meeting in Berlin, Germany, 7-12 September 2008, for issue by its Secretariat. The members of IAPWS are: Argentina and Brazil, Britain and Ireland, Canada, the Czech Republic, Denmark, France, Germany, Greece, Italy, Japan, Russia, the United States of America, and associate member Switzerland.

This release replaces the "Revised Release on the IAPS Formulation 1985 for the Viscosity of Ordinary Water Substance," issued in 2003.

Further information concerning this release and other documents issued by IAPWS can be obtained from the Executive Secretary of IAPWS or from http://www.iapws.org.

Contents

1. Introductory Remarks

This release provides a correlating equation for the shear viscosity of pure water substance over an extensive range of fluid states. A discussion of the background, development, and validation of this formulation is presented in Ref. [1].

Section 2 of this release contains the correlating equation, necessary constants, range of validity of the equation, and estimates of the uncertainty of the correlation. Section 3 concerns the industrial application of the viscosity correlation, and Section 4 presents selected values of the correlation at specific state points to enable computer verification of an implementation of the correlation.

2. Recommended Correlating Equation

2.1.   Nomenclature
        T denotes absolute temperature on the International Temperature Scale of 1990
        ρ denotes density
        p denotes pressure
        μ denotes viscosity

2.2.   Reference constants
        The reference constants used in this formulation for temperature, pressure, and density agree with presently accepted values of the critical temperature, pressure, and density of water recommended by IAPWS [2], while the reference constant for viscosity has no physical significance.

reference temperature: $\quad T^* = 647.096$ K $\qquad\qquad\qquad$ (1)
reference density: $\quad \rho^* = 322.0$ kg·m^{-3} $\qquad\qquad\qquad$ (2)
reference pressure: $\quad p^* = 22.064$ MPa $\qquad\qquad\qquad$ (3)
reference viscosity: $\quad \mu^* = 1.00 \times 10^{-6}$ Pa·s $\qquad\qquad\qquad$ (4)

2.3. <u>Dimensionless variables</u>

temperature: $\quad \overline{T} = T/T^*$ $\qquad\qquad\qquad$ (5)
density: $\quad \overline{\rho} = \rho/\rho^*$ $\qquad\qquad\qquad$ (6)
pressure: $\quad \overline{p} = p/p^*$ $\qquad\qquad\qquad$ (7)
viscosity: $\quad \overline{\mu} = \mu/\mu^*$ $\qquad\qquad\qquad$ (8)

2.4. <u>Range of validity</u>

Equation (10) below is recommended for computation of the viscosity for all thermodynamically stable fluid states in the following ranges of pressure p and temperature T:

$$
\begin{array}{lll}
0 < p < p_t & \text{and} & 273.16 \text{ K} \leq T \leq 1173.15 \text{ K} \\
p_t \leq p \leq 300 \text{ MPa} & \text{and} & T_m(p) \leq T \leq 1173.15 \text{ K} \\
300 \text{ MPa} < p \leq 350 \text{ MPa} & \text{and} & T_m(p) \leq T \leq 873.15 \text{ K} \\
350 \text{ MPa} < p \leq 500 \text{ MPa} & \text{and} & T_m(p) \leq T \leq 433.15 \text{ K} \\
500 \text{ MPa} < p \leq 1000 \text{ MPa} & \text{and} & T_m(p) \leq T \leq 373.15 \text{ K}
\end{array}
\qquad (9)
$$

In Eq. (9), T_m is the pressure-dependent melting temperature and p_t is the triple-point pressure as given in Refs. [3,4]. For most applications, the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use [5,6] should be used to determine the densities used as input to Eq. (10), when the state point under consideration is defined by pressure and temperature or by other thermodynamic variables instead of density and temperature.

In addition, IAPWS makes the following statements about extrapolation of Eq. (10) outside the range of validity given above:

- For vapor states at temperatures below the triple-point temperature of 273.16 K and pressures less than or equal to the sublimation pressure, the viscosity calculation is dominated by the dilute-gas term, and this behaves in a physically reasonable manner down to at least 250 K.
- For stable fluid states outside the range of validity of Eq. (10) but within the range of validity of the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use [5,6], the extrapolation behavior of Eq. (10) is physically reasonable.
- At high temperatures, the extrapolation of the dilute-gas portion of Eq. (10) is physically reasonable up to at least 2500 K.
- For the metastable subcooled liquid at atmospheric pressure, Eq. (10) is in fair agreement (within 5 %) with available data down to 250 K.

2.5.   Estimated uncertainty

The uncertainties in this formulation are summarized in Figure 1; they can be considered as estimates of a combined expanded uncertainty with a coverage factor of two. Thus the viscosity at any state point can be expressed as $\mu \pm \delta$ where δ is the applicable value in Figure 1. The formulation reproduces the ISO recommended value of the viscosity at 20 °C (293.15 K) and standard atmospheric pressure within the number of digits given in Ref. [7]; it also agrees with all values from 288.15 to 313.15 K at atmospheric pressure in Ref. [7] within the stated uncertainty of 0.17 % at 293.15 K.
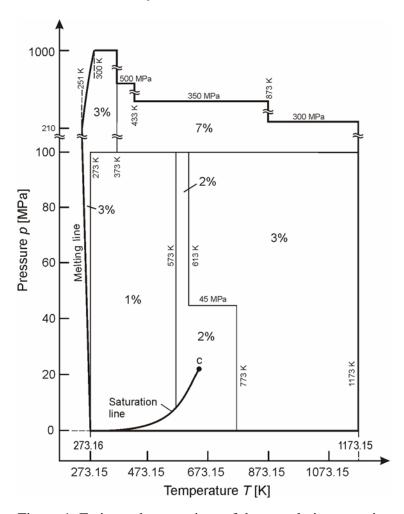

Figure 1. Estimated uncertainty of the correlating equation.

2.6.   Correlating equation

The viscosity is represented by the equation

$$\overline{\mu} = \overline{\mu}_0(\overline{T}) \times \overline{\mu}_1(\overline{T},\overline{\rho}) \times \overline{\mu}_2(\overline{T},\overline{\rho}). \tag{10}$$

The first factor $\overline{\mu}_0$ of the product represents the viscosity in the dilute-gas limit and is given by

$$\overline{\mu}_0(\overline{T}) = \frac{100\sqrt{\overline{T}}}{\displaystyle\sum_{i=0}^{3}\frac{H_i}{\overline{T}^i}} \quad , \tag{11}$$

with coefficients H_i given in Table 1.

Table 1. Coefficients H_i for $\overline{\mu}_0(\overline{T})$

i	H_i
0	1.67752
1	2.20462
2	0.6366564
3	−0.241605

The second factor $\overline{\mu}_1$ represents the contribution to viscosity due to finite density:

$$\overline{\mu}_1(\overline{T},\overline{\rho}) = \exp\left[\overline{\rho}\sum_{i=0}^{5}\left(\frac{1}{\overline{T}}-1\right)^i\sum_{j=0}^{6}H_{ij}(\overline{\rho}-1)^j\right] , \tag{12}$$

with coefficients H_{ij} given in Table 2. The third factor $\overline{\mu}_2$ represents the critical enhancement of the viscosity.

Table 2. Coefficients H_{ij} for $\overline{\mu}_1(\overline{T},\overline{\rho})$

i	j	H_{ij}
0	0	5.20094×10^{-1}
1	0	8.50895×10^{-2}
2	0	-1.08374
3	0	-2.89555×10^{-1}
0	1	2.22531×10^{-1}
1	1	9.99115×10^{-1}
2	1	1.88797
3	1	1.26613
5	1	1.20573×10^{-1}
0	2	-2.81378×10^{-1}
1	2	-9.06851×10^{-1}
2	2	-7.72479×10^{-1}
3	2	-4.89837×10^{-1}
4	2	-2.57040×10^{-1}
0	3	1.61913×10^{-1}
1	3	2.57399×10^{-1}
0	4	-3.25372×10^{-2}
3	4	6.98452×10^{-2}
4	5	8.72102×10^{-3}
3	6	-4.35673×10^{-3}
5	6	-5.93264×10^{-4}

Note: Coefficients H_{ij} omitted from Table 2 are identically equal to zero.

2.7   Critical enhancement

The critical enhancement is only significant in a very small region in density and temperature around the critical point. Although exactly at the critical point the viscosity is infinite, the enhancement term contributes an amount greater than 2 % of the full viscosity only within the following boundaries:

$$645.91 \text{ K} < T < 650.77 \text{ K}, \quad 245.8 \text{ kg·m}^{-3} < \rho < 405.3 \text{ kg·m}^{-3} . \tag{13}$$

Thus, the critical enhancement is significant only within the boundaries specified by Eq. (13); outside this region, the enhancement is always less than the uncertainty in the formulation. This allows simplification for certain calculations (see Sections 2.8 and 3).

The function $\bar{\mu}_2$ is defined over the entire range of states by:

$$\bar{\mu}_2 = \exp(x_\mu Y) , \tag{14}$$

where x_μ is the critical exponent for viscosity and the function Y is defined for two ranges of correlation length ξ. For $0 \le \xi \le 0.3817016416$ nm

$$Y = \frac{1}{5} q_C \xi \, (q_D \xi)^5 \left(1 - q_C \xi + (q_C \xi)^2 - \frac{765}{504}(q_D \xi)^2 \right) , \tag{15}$$

while for $\xi > 0.3817016416$ nm

$$\begin{aligned} Y = {} & \frac{1}{12} \sin(3\psi_D) - \frac{1}{4 q_C \xi} \sin(2\psi_D) + \frac{1}{(q_C \xi)^2}\left[1 - \frac{5}{4}(q_C \xi)^2 \right]\sin(\psi_D) \\ & - \frac{1}{(q_C \xi)^3}\left\{ \left[1 - \frac{3}{2}(q_C \xi)^2 \right]\psi_D - \left| (q_C \xi)^2 - 1 \right|^{3/2} L(w) \right\} \end{aligned} , \tag{16}$$

with

$$\psi_D = \arccos\left[\left(1 + q_D^2 \xi^2 \right)^{-1/2} \right] \tag{17}$$

and with the function $L(w)$ given by

$$L(w) = \begin{cases} \ln\dfrac{1+w}{1-w}, & \text{for } q_C \xi > 1 \\ 2 \arctan|w|, & \text{for } q_C \xi \le 1 \end{cases} . \tag{18}$$

The variable w is defined by

$$w = \left| \frac{q_C \xi - 1}{q_C \xi + 1} \right|^{1/2} \tan\left(\frac{\psi_D}{2} \right). \tag{19}$$

The critical enhancement of viscosity given by Eqs. (14)–(19) is a function of the correlation length ξ:

$$\xi = \xi_0 \left(\frac{\Delta\bar{\chi}}{\Gamma_0} \right)^{\nu/\gamma},$$

(20)

in terms of $\Delta\bar{\chi}$ (≥ 0) which is defined by

$$\Delta\bar{\chi} = \bar{\rho} \left[\varsigma(\bar{T}, \bar{\rho}) - \varsigma(\bar{T}_R, \bar{\rho}) \frac{\bar{T}_R}{\bar{T}} \right]$$

(21)

$$\varsigma = \left(\frac{\partial\bar{\rho}}{\partial\bar{p}} \right)_{\bar{T}}.$$

(21a)

When $\Delta\bar{\chi}$ calculated by Eq. (21) is less than zero, it must be set to zero for calculations to proceed.[1]

The constants needed to compute the critical enhancement, $\bar{\mu}_2$, are provided in Table 3.

Table 3. Critical-Region Constants

Constant	Value
x_μ	0.068
q_C^{-1}	1.9 nm
q_D^{-1}	1.1 nm
ν	0.630
γ	1.239
ξ_0	0.13 nm
Γ_0	0.06
$\bar{T}_R$	1.5

---

[1] Due to the numerical implementation of the equation of state, the calculated singularity in the first derivative in Eq. (21) may not occur exactly at T_c and ρ_c as it should. Therefore, calculated values of $\bar{\mu}_2$ may behave unphysically at points extremely close to the critical point (approximately within 0.01 kg·m^{-3} of ρ_c on the critical isotherm). The formulation should be used with caution in this very small region.

2.8   Simplified use outside the critical region

Because the critical enhancement is insignificant except in a small region around the critical point (described approximately by Eq.(13)), complexity and computing time may be reduced by omitting the critical enhancement for applications outside this region.[2] This can be done by setting $\bar{\mu}_2 = 1$.

3. Recommendation for Industrial Application

For industrial uses where greater computing speed is needed, Eq. (10) should be simplified by setting $\bar{\mu}_2 = 1$. In addition, the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam [8,9], known as IAPWS-IF97, should be used to determine the density for use in Eq. (10) when the state point is defined by the temperature and pressure or other state variables. When IAPWS-IF97 is used to calculate densities, the error introduced by the use of the different thermodynamic formulation is smaller than the uncertainty of the viscosity correlation, provided the point is within the range of validity of IAPWS-IF97, except for points close to and inside the near-critical region described by Eq. (13).

4. Computer-Program Verification

The following tables are provided to assist the user in computer-program verification. The viscosity calculations are based on the tabulated temperatures and densities.

Table 4. Sample points for computer-program verification
of the correlating equation, Eq. (10), with $\bar{\mu}_2 = 1$.

T (K)	ρ (kg·m^{-3})	μ (µPa·s)
298.15	998	889.735100
298.15	1200	1437.649467
373.15	1000	307.883622
433.15	1	14.538324
433.15	1000	217.685358
873.15	1	32.619287
873.15	100	35.802262
873.15	600	77.430195
1173.15	1	44.217245
1173.15	100	47.640433
1173.15	400	64.154608

---

[2] If a calculation is performed that uses the critical enhancement, Eq. (14), near the critical point but omits it far from the critical point, some discontinuity is inevitable. This discontinuity is less than 0.0051 % for single-phase states outside a region near the critical point bounded by the equation

$$T/(1\,\text{K}) = \sum_{i=0}^{3} a_i \left(\rho/(1\,\text{kg}\cdot\text{m}^{-3})\right)^i,$$

where $a_0 = 457.95895935062$, $a_1 = 1.68077273385305$, $a_2 = -3.24405775203984\times10^{-3}$, and $a_3 = 1.43032446173023\times10^{-6}$.

Table 5. Sample points for computer-program verification of the correlating equation, Eq. (10), in the region near the critical point.

T (K)	ρ (kg·m^{-3})	ξ (nm)	$\overline{\mu}_2$	μ (µPa·s)
647.35	122	0.309247	1.00000289*	25.520677
647.35	222	1.571405	1.00375120	31.337589
647.35	272	5.266522	1.03416789	36.228143
647.35	322	16.590209	1.09190440	42.961579
647.35	372	5.603768	1.03665871	45.688204
647.35	422	1.876244	1.00596332	49.436256

* Correlation length ξ < 0.3817016416 nm so Y is evaluated with Eq. (15).

5. References

[1]   Huber, M. L., R. A. Perkins, A. Laesecke, D. G. Friend, J. V. Sengers, M. J. Assael, I. N. Metaxa, E. Vogel, R. Mareš, and K. Miyagawa, J. Phys. Chem. Ref. Data **38**, 101 (2009).

[2]   International Association for the Properties of Water and Steam, *Release: Values of Temperature, Pressure and Density of Ordinary and Heavy Water Substances at their Respective Critical Points*, available at http://www.iapws.org (1992).

[3]   International Association for the Properties of Water and Steam, *Revised Release on the Pressure along the Melting and Sublimation Curves of Ordinary Water Substance*, available at http://www.iapws.org (2011).

[4]   Wagner, W., T. Riethmann, R. Feistel, and A. H. Harvey, J. Phys. Chem. Ref. Data **40**, 043103 (2011).

[5]   Wagner, W., and A. Pruß, J. Phys. Chem. Ref. Data **31**, 387 (2002).

[6]   International Association for the Properties of Water and Steam, *Revised Release on the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use*, available at http://www.iapws.org (2009).

[7]   International Organization for Standardization (ISO), *Viscosity of Water*, ISO/TR Technical Report 3666: 1998(E), Geneva.

[8]   Wagner, W., J. R. Cooper, A. Dittmann, J. Kijima, H.-J. Kretzschmar, A. Kruse, R. Mareš, K. Oguchi, H. Sato, I. Stöcker, O. Šifner, Y. Takaishi, I. Tanishita, J. Trübenbach, and Th. Willkommen, J. Eng. Gas Turbines & Power **122**, 150 (2000).

[9]   International Association for the Properties of Water and Steam, *Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam,* available at http://www.iapws.org (2007).