SFWR ENG 4O03

Kemal Ahmed
Dr. Deza
Fall 2015

Contents

Linear

Linear Program: an optimization problem in which the objective function is linear and each constraint is a linear inequality or equality
Decision variables: describe our choices that are under our control
Objective function: describes a criterion that we wish to max/minimize; doesn't have an in/equality
e.g. max 40x + 30y

Integer linear program: a linear program that only deals with integers

Constraints: describe the limitations that restrict our choices for our decision variables, always *inequalities*.
Free: no constraints

Basic variable: the variables corresponding to the identity matrix, usually have to be set to 0
Non-basic variable: …not basic variables

Converting constraints to equalities

Slack variable: basic variable greater than constraint, added to turn inequalities into equalities
Surplus variable: equation variable less than constraint, subtracted

Hyperplane: a hyperplane in R^x is a shape in R^{x-1}, e.g. line in R^2

Optimal Solution: either a maximum or minimum of the objective function based on constraints
Basic Solution: a solution which has as many slack variables as basic variables
Basic Feasible Solution: all basic variables are non-negative

- Unique
- obtained by setting the non-basic variables to 0

Standard form: when you take inequalities and use slack variables to turn them into equalities.

- Note: all variables need to be ≥ 0.
- All remaining constraints are expressed as equality constraints.

e.g.)

$2x_1 + 4x_2 - x_3 - x_4 \geq 1$
$2x_1 + 4x_2 - x_3 - x_4 + s = 1$

Graphical Method

1. Sketch the region corresponding to the system of constraints. The points inside or on the boundary of the region are the *feasible solutions*.
2. Find the vertices of the region.
3. Test the objective function at each of the vertices and select the values of the variables that optimize the objective function. For a bounded region, both a minimum and maximum value will exist. For an unbounded region, if an optimal solution exists, then it will occur at a vertex.

Simplex Method: Maximization

Simplex Method: useful for solving linear optimization problems cheaply

- Cannot be done with **strict inequalities**, i.e. when there is no possibility of being equal
- Can only work if your objective function is in *standard form*

Simplex Tableau: visual representation of stuff

- The *basic variables* can be identified if they have a column with one row of 1 and the rest of the rows are 0's. The value of the variable is at the row with the 1.
- The objective row is going to identify the constants for the new equation. You should see 0's in the columns that are non-basic.
- The first column (if used) is only an indicator of the existence of the variable you're trying to min/maximize, i.e. 0's for all rows, except for the objective function
- RHS must be ≥ 0

Process:

1. You'll have as many slack variables as you have constraint equations.
2. Find the column with the smallest coefficient (< 0) in the objective function. That column is called the **pivot column**. The **entering variable** is the variable with the smallest coefficient.
3. **Minimum test**: find the row with the smallest **departing variable** or **exiting variable**, i.e. RHS/x_{pivot}. That row is called the **pivot row**. x_{pivot} must be ≥ 0
4. The intersection of the pivot row & column is called the **pivot point**.
5. If your pivot point ≠ 1, divide your row out by the value of your point
6. Use row operations, i.e. Gauss-Jordan to make the other elements in the pivot column 0.
7. Go to step 2, until objective function is all ≥ 0.

Simplex: Minimization

To minimize a function, we just oppositize the problem so we can use the maximization technique on it. You'll see. Just remember that we minimize [w] & maximize [z] AND minimize is (vars ≥ 0), while maximize is (vars ≤ 0). I'll explain using an example:

e.g.)

$w = 0.12x_1 + 0.15x_2$

$60x_1 + 60x_2 \geq 300$

$12x_1 + 6x_2 \geq 36$

$10x_1 + 30x_2 \quad \geq 90$

1. Ignore slack variables for now. Make a matrix with just the variables you have.

w	x_1	x_2	
0	60	60	300
0	12	6	36
0	10	30	90
1	–0.12	–0.15	0

2. Find the transpose of this matrix

60	12	10	–0.12
60	6	30	–0.15
300	36	90	0

This gives us:

$z = 300y_1 + 36y_2 + 90y_3$

$60y_1 + 12y_2 + 10y_3 \leq 0.12$

$60y_1 + 6y_2 + 30y_3 \leq 0.15$

$300y_1 + 36y_2 + 90y_3 \leq 0$

Notice how the x's are now y's? Yeah I know you did. Well now, since you turned this into a maximization problem, what are you waiting for? Go to the maximization section!

Phase Simplex

This is useful for when you have a mix of constraints that are maximum and minimum constraints.

Artificial Variable [y]: since you can't have negative variables ($x_1, x_2 \geq 0$), you can't just use a regular slack variable

Phase I

1. Replace all negative slack variables with artificial variables
2. Replace objective function with $w = -\Sigma y_i$
3. Isolate your artificial variables in your constraint equations,
   a. e.g. $2x_1 + x_2 - x_3 - x_4 + y_2 = 10 \Rightarrow y_2 = 10 - 2x_1 - x_2 + x_3 + x_4$
4. Replace your y's in your objective function with the isolated artificial variables, then move the RHS's to the new RHS
   a. e.g. for $x_1 + x_2 - x_3 - x_4 + y_2 = 10$ & $-x_2 + x_4 + y_3 = 10$, $w - 2x_1 + x_3 = -20$
5. Treat as maximization.

Phase II

Oh no!

Bland’s Rule

Bland’s Rule: a way of guaranteeing that you don’t repeat going over the same variables (a cycle) by picking the smallest (or most negative) number

Algorithms

See SFWR ENG 2C03 Summary.