SFWR ENG 3DX4 Summary

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Course: SFWR ENG 3DX4

Math objects made using MathType; graphs made using Winplot.

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Note: the following summaries may be useful:

- SFWR ENG 2MX3
- ENGINEER 3N03
- TRON 3TA4

I may review to clarify or correct, but mostly I will omit those things.

Introduction to Systems

Systems can be represented by **block diagrams** to make it easier to marginalize the different parts of the systems.

Transducer: converts any form of energy to electrical signals

Laplace

Useful for…

Time begins when your signal begins

$$h(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

Initial conditions:

- $c(0)$

Time domain (t): variables are <u>lower case</u>, e.g. $f(t)$
Frequency domain (s): variables are <u>upper case</u>, e.g. $F(s)$

Transfer function:

When doing the inverse Laplace, it's useful to break your fractions up so that you can

Strictly Stable: it will eventually get back to the initial position
Marginally Stable:
Unstable: it will progressively get worse

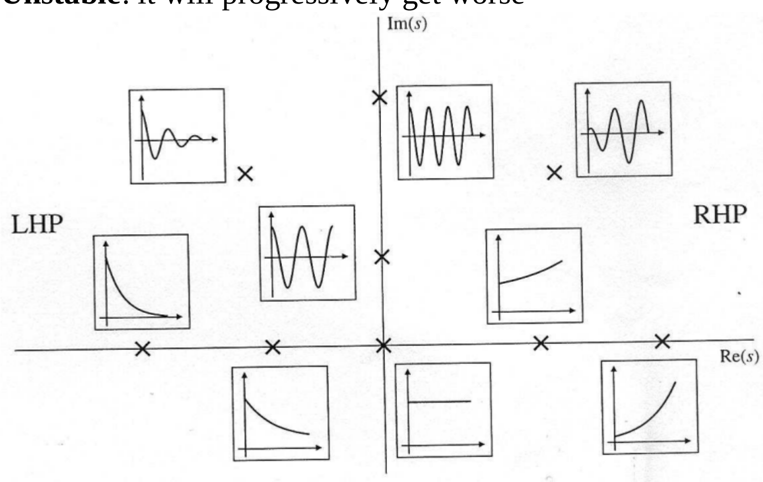


Figure 2.5 from Dorf and Bishop, *Modern Control Systems (10th Edition)*, Prentice-Hall, 2004.

Transfer Functions

Electrical

Component stuff

Impedence: $Z = \frac{V(s)}{I(s)}$

$Z_R = R$

$Z_L = sL = j\omega L$

$Z_C = \frac{1}{sC} = \frac{-j}{\omega C}$

Current

$i_R = \frac{1}{R}$

$i_L = L\frac{\mathrm{d}i}{\mathrm{d}t}$

$i_C = C\frac{\mathrm{d}v}{\mathrm{d}t}$

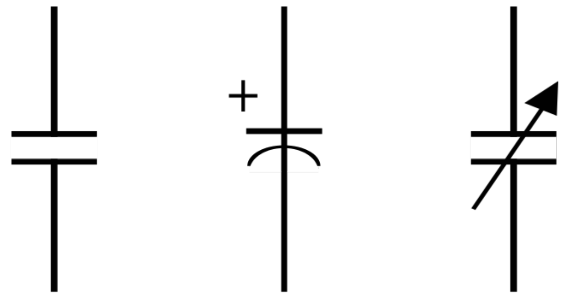
Voltage

$v_R = Ri(t)$

$v_L = L\frac{\mathrm{d}i}{\mathrm{d}t}$

$v_C = \frac{1}{C}\int_0^1 i(\tau)\mathrm{d}\tau$

Polarized capacitors: Z is positive when current is going from − to +, but negative from + to −



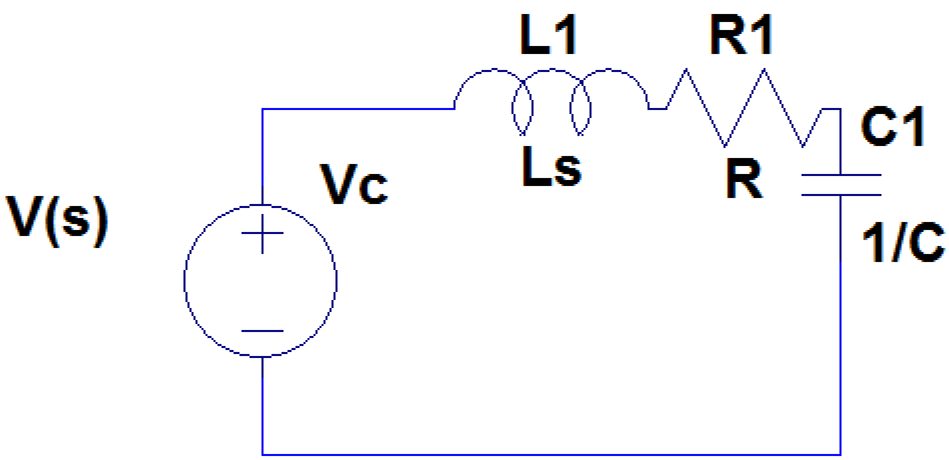
Fixed Capacitor Polarized Capacitor Variable Capacitor

admittance:

$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{R} = G$

$V_c(s) = I(s)\frac{1}{Cs}$

$I(s) = \frac{V(s)}{L_s + R + \frac{1}{Cs}}$



Mesh Analysis

Add the voltages, where V = IZ

Cramer's Rule

$$x_1 = \frac{\det(A_1)}{\det(A)}, x_2 = \frac{\det(A_2)}{\det(A)}, \ldots, x_n = \frac{\det(A_n)}{\det(A)}$$

$$V_C(s) = \overbrace{H(s)}^{\substack{\text{transfer} \\ \text{function}}} \frac{1}{Cs}$$

OP-Amps

Mechanical

Translational systems:
Rotational Systems:

Newton's Second Law of Motion: $\Sigma f = Ma$

$$Z_m(s) = \frac{F(s)}{X(x)}$$

$$f(t) = Ma(t)$$

$$= M\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

Translational Systems

For sure make a free-body diagram

Also, include "inertial force in it" = $m\frac{\mathrm{d}^2 x}{\mathrm{d}t}$, which is in the opposite direction of the applied force

e.g.

$$d_1 + 7v_1 + 2x_1 + 5v_1 = 2x_2 + 5v_2$$

$$d_2 + 2x_2 + 5v_2 = 2x_1 + 5v_1 + F(t)$$

$$v_1 = \frac{\mathrm{d}x_1}{\mathrm{d}t}$$

$$d_1 = \frac{\mathrm{d}v_1}{\mathrm{d}t}$$

$$\dot{x}_1 = v_1$$

$$\dot{v}_1 = d_1$$

$$\dot{x}_2 = v_2$$

$$\dot{v}_2 = d_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -12 & 2 & 5 \\ 0 & 0 & 0 & 1 \\ 2 & 5 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$

$$\text{Output} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

Spring

Spring is like a capacitor

Force displacement: $f(t) = Kx(t)$

Viscous Damper

Using viscous fluid to slow something down

Viscous Damper is like a resistor

Force displacement: $f(t) = f_v \frac{\mathrm{d}x(t)}{\mathrm{d}t}$

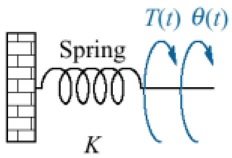
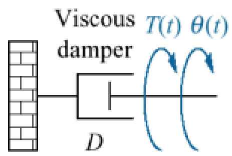
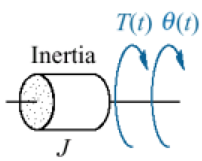
Mass

Mass is like a inductor

Force displacement: $f(t) = M \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2}$

Rotational Systems

Impedence: $Z_m(s) = \dfrac{T(s)}{\theta(s)}$

Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)\,d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

- Each θ is on an inertia block. The impedences connected to the motion at θ include the impedences directly to the left and right of the inertia block.
- When finding the sum of impedences between 2 θ's only count the impedences on wires that don't go through other θ's, i.e. 0 if no direct connection
- When there is a torque, but no inertial block, draw a fake inertial block

$$\begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{connected}\\\text{to the motion}\\\text{at }\theta_1\end{bmatrix}\theta_1(s) - \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\\theta_1\text{ and }\theta_2\end{bmatrix}\theta_2(s)$$

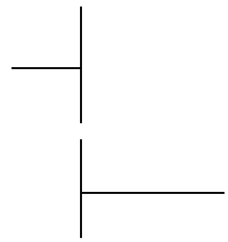
$$- \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\\theta_1\text{ and }\theta_3\end{bmatrix}\theta_3(s) = \begin{bmatrix}\text{Sum of}\\\text{applied torques}\\\text{at }\theta_1\end{bmatrix}$$

$$- \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\\theta_1\text{ and }\theta_2\end{bmatrix}\theta_1(s) + \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{connected}\\\text{to the motion}\\\text{at }\theta_2\end{bmatrix}\theta_2(s)$$

$$- \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\\theta_2\text{ and }\theta_3\end{bmatrix}\theta_3(s) = \begin{bmatrix}\text{Sum of}\\\text{applied torques}\\\text{at }\theta_2\end{bmatrix}$$

Motors and Gears

1. Pick an end of the system to use as a reference frame. Choose the easiest one and walls don't move.
2. Represent T

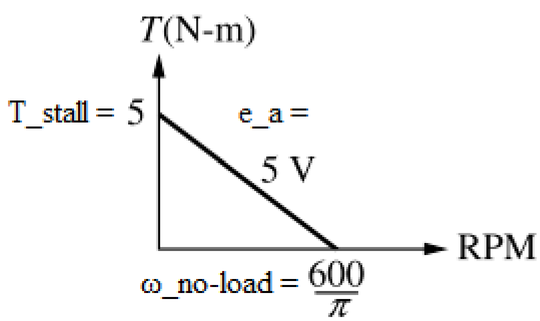


Meshing Gears are represented in the following way:
[N]: number of teeth
Let's assume var_1 = before and var_2 = after.
When gears are lined up $\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$
Applied Armature Voltage [e_a]: a.k.a. input voltage
Armature Resistance [R_a]:
Motor Torque Constant [K_t]:
Back EMF Constant [K_b]:
No load speed [$\omega_{\text{no-load}}$]: when the voltage line touches the x-axis

$$\omega_{\text{no-load}} = \frac{e_a}{K_b}$$

Stall torque[T_{stall}]: when angular velocity reaches 0, i.e. y-intercept if equation is given.

$$T_{\text{stall}} = \frac{K_t}{R_a} e_a$$



[J_a]: any J on the same line, including a motor
[J_L]: load J
[J_m]: $J_m = J_a + J_L \left(\frac{N_1}{N_2} \right)^2$

[D_m]: coefficient of viscous dampening $D_m = D_a + D_L \left(\frac{N_1}{N_2} \times \frac{N_3}{N_4} \right)^2$

$$T_e = T\left(\frac{N_2}{N_1}\right)$$

$$T(s)\left(\frac{N_2 N_4}{N_1 N_3}\right) = \theta_{destination}\left(J_{eq}s^2 + D_{eq}s\right)$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

Hints:

- If you have a spring and / or a damper in series, the wire between them rotates independently

Signals

Transducer: anything that converts energy to electrical energy
Transmitter: long distances

Unstable systems have ∞ steady state error
Steady-state error [e_∞]:

$$e_\infty = \lim_{t \to \infty} e(t)$$

Final Value Theorem

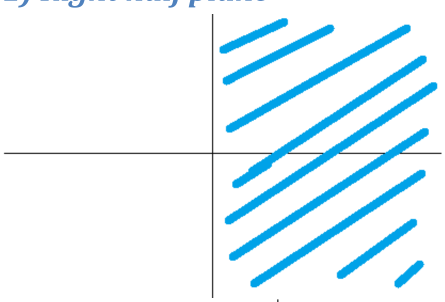
Final value theorem: finds steady state error

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

So $e_\infty = \lim_{s \to 0} sF(s)$ and you're given F(s), so just multiply by s and find the limit.

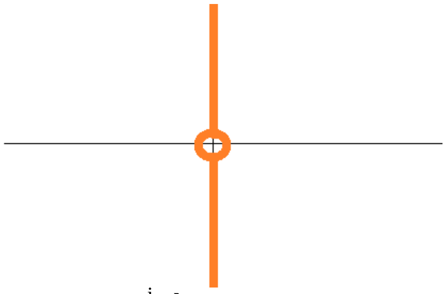
There are limitations as to where you can use this theorem. It is dependent on the location of the poles.

1) Right half plane



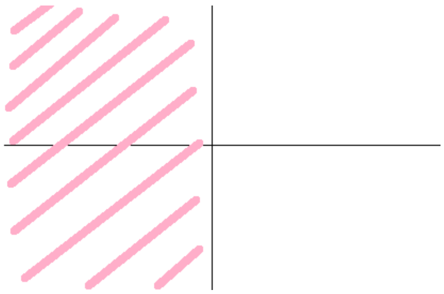
System is unstable: $e^{+} \rightarrow \infty$

2) Imaginary Axis – Origin



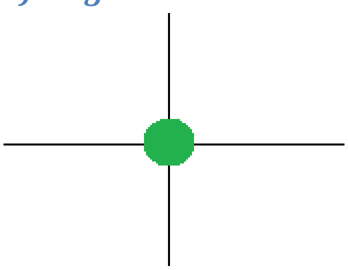
Unstable: e^{i} → Oscillatory system, so limit will be average, i.e. midpoint

3) Left Half Plane



Stable: e^{-} converges to 0, but makes transfer function 0 for every single pole

4) Origin



Stable: integrator, i.e. 1/s, so $\lim_{s\to 0} \frac{s}{s} = 1$

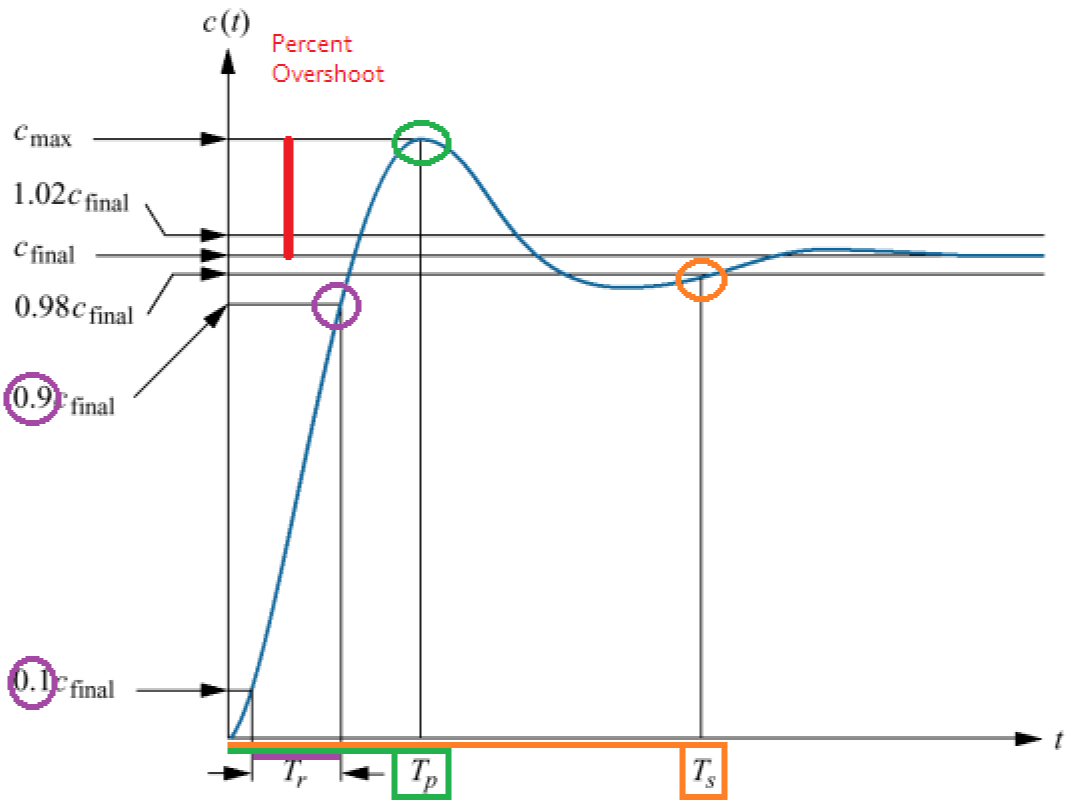
Don't use this theorem if any poles are 1 or 2.

Graph Stuff

Rise time [T_r]: time between 10% and 90% of final value (c_{final})
Peak time [T_p]: time it takes to get to highest peak ($c_{\max}$)
Settling time [T_s]: how long it takes to get to the steady state within ±2%
Percent overshoot [%OS]: how much further is the peak from the final

$$\%\text{OS} = \frac{c_{\max} - c_{\text{final}}}{c_{\text{final}}} \times 100\%$$

Time Constant [τ]: the time it takes the system's step response to reach 1 – 1/e = 63.2% of c_{final}
Second-order:



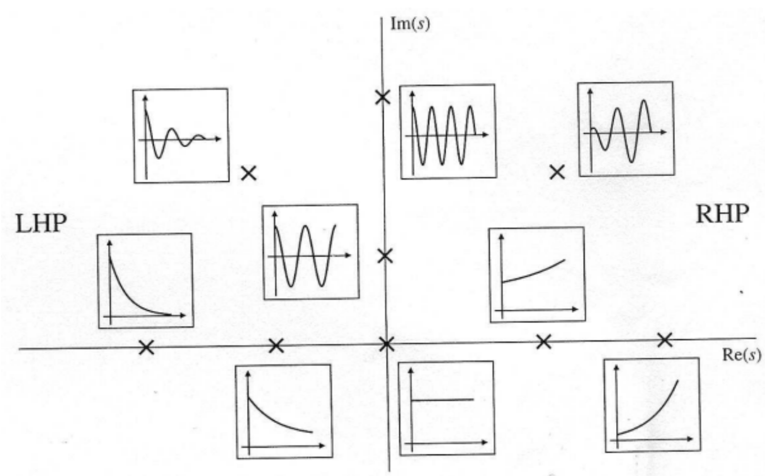


Figure 2.5 from Dorf and Bishop, *Modern Control Systems (10th Edition)*, Prentice-Hall, 2004.

$K = c_{\text{final}} \times a$

For each pole,

$$G(s) = \frac{K_1}{s+a_1} + \frac{K_2}{s+a_2} + \text{etc}, a = \frac{1}{\tau}$$

Forced response: when a = 0
Natural response: when a > 0

Nonminium-phase system: Initially the system starts in the wrong direction, then stabilizes at the right place

Non-/Linear Systems

- Op Amps are linear
- If you don't have enough voltage, your motor magnets won't have enough power to switch poles, so they require a minimum voltage

You can't model non-linear systems, until you linearize it. To do this, we find the slope and approximate the equation of the line, using y=mx+b

Proportional-Integral-Derivative (PID):
If your gears are vibrating, your PID is probably too high

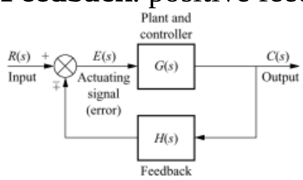
Block Diagrams

A way of representing a system

Summing junction: could be an X or +, but usually an X in this course

Cascade: subsystems in series are multiplied
Parallel: parallel subsystems have a *summing junction* at the end, so you just add everything together

Feedback: positive feedback is bad

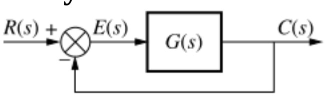


Positive: $\dfrac{G(s)}{1-G(s)H(s)}$

Negative: $\dfrac{G(s)}{1+G(s)H(s)}$

Simplification:

Unity Feedback: when the feedback path has multiplicative value of 1



State Space Equations

Yeah, you think you know them from 2MX3, but you don't really know them. Apparently the ABCD variables actually have names.

- **System Matrix [A]**:
- **Input Matrix [B]**:

- **Output Matrix [C]**:
- **Feedforward Matrix [D]**:

Transfer Function -> State Space

Phase Variable Approach:

The n state variables will consist of:

- y
- the derivatives of y