

# SFWR ENG 4AA4

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Note: information from the pre-requisite, [SFWR ENG 3DX4](#) will not be included in this summary (although corrections will be).

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## Real-Time Systems

### Classifications

What happens upon failure to meet deadlines:

- **Soft:** performance is degraded but not destroyed
- **Firm:** a few times will simply degrade performance, but after may lead to system failure
- **Hard:** complete and catastrophic system failure
  - **Safety Critical:** may cause injury / death (a type of hard)
- **Controller** [C(s)]:
- **Input** [E(s)]:
- **Output** [U(s)]:
- $U(s) = C(s)E(s)$

### Difference Equations

**Forward difference method:** derivatives using  $f'(x) = \frac{f(x+h) - f(x)}{h}$

**Backwards Difference method:** derivatives using  $f'(x) = \frac{f(x) - f(x-h)}{h}$

e.g.)

$$u[n] - u[n-1] = 48e[n] - 40e[n-1]$$

$$a\dot{U} + bU = c\dot{E} + dE$$

$$a \frac{u(kT_s) - u((k-1)T_s)}{T_s} + bu(kT_s) = c \frac{e(kT_s) - e((k-1)T_s)}{T_s} + de(kT_s)$$

$$a \frac{u(kT_s)}{T_s} - \frac{au((k-1)T_s)}{T_s} + bu(kT_s) = c \frac{e(kT_s)}{T_s} + de(kT_s) - c \frac{e((k-1)T_s)}{T_s}$$

Group:

$$\left(\frac{a}{T_s} + b\right)u(kT_s) + \left(-\frac{a}{T_s}\right)u((k-1)T_s) = \left(\frac{c}{T_s} + d\right)e(kT_s) - \left(\frac{c}{T_s}\right)e((k-1)T_s)$$

Equate each section to the values from the equation:

$$\frac{a}{T_s} + b = 1$$

$$b = 1 - \frac{a}{T_s}$$

$$\boxed{b = 1 - 10a}$$

$$\frac{a}{T_s} = 1$$

$$\boxed{a = \frac{1}{10}}$$

$$\boxed{b = 0}$$

$$\frac{c}{T_s} + d = 48$$

$$\boxed{d = 48 - 10c}$$

$$-\frac{c}{T_s} = -40$$

$$\boxed{c = 4}$$

$$d = 48 - 40 = 8$$

$$\frac{1}{10}\dot{U} + 0U = 4\dot{E} + 8E$$

$$\frac{U}{E}(0.1s) = 4s + 8$$

$$\frac{U}{E} = \frac{4s + 8}{0.1s}$$

## Task optimization

**Task [T]:**  $T_i = (p_i, r_i, e_i, d_i)$

**Period [p]:** time between tasks are repeatedly released

**Release time [r]:** time it takes to release task

**Execution time [e]:** slowest time task could take to be completed (but assume the tasks will take this long no matter what)

**Deadline [d]:** when task needs to be completed

**Number of tasks [n]:**

**Feasible:** a schedule that meets all deadlines

**Optimal:** will produce a schedule every time if *feasible* schedule exists

**Processor Utilization [U]:** used as a priority level  $U = \sum_{i=1}^n \frac{e_i}{p_i}$

If  $U > 1$ , nothing is feasible

If  $r_i = 0$  and  $p_i = d_i$ , then write  $T_i = (p_i, e_i)$

## Types of Scheduling

### Static

**Static Scheduling:**

- task's priority is assigned before execution and does not change
- If a task misses its deadline, you mess up all the deadlines after it like an airport at Christmas
- A.K.A. **Fixed priority**

### FIFO

**First In First Out (FIFO):**

- Could cause problems for tasks whose execution time is significantly shorter than the rest when there are deadlines
  - E.g.  $T_1 = (100, 3)$ ;  $T_2 = (2, 1)$
- A.K.A. **First Come, First Served (FCFS)**

**Cyclic Executive:** frame-based scheduling

- When you allocate an amount of time where a task can execute
- Can have multiple executions of the same task
- Tasks might not even fill the full frame

**Schedule:** the order in which tasks will be executed

**Hyperperiod [H]:** the entire length of a cycle, least common multiple

**Harmonic:** every task period evenly divides every longer period

**Pre-empting:** splitting a task up into multiple mini tasks. Also, if a task misses its deadline, halt the task at the deadline

**Frame Size [f]:**

- The best way for computers to segment the schedule in a way that it verify that the appropriate tasks have been executed
- Process: try each see which is the largest frame size that follows all the below constraints from 1 to  $e_{\max}$ .
- Constraints:
  1.  $f \geq \max_{1 \leq i \leq n} (e_i)$
  2.  $H \% f = 0$
  3.  $2f - \gcd(p_i, f) \leq d_i$

**Least Compute Time (LCT):** tasks with smallest execution times executed first

- static
- Think *greedy*
- Works poorly; worse than RR

**Rate Monotonic (RM):** shorter period, higher priority

- Think: tasks requiring frequent attention should have higher priority
- If harmonic, feasible as long as  $U \leq 1$
- If non-harmonic, guaranteed feasible if  $U \leq n \left( 2^{\frac{1}{n}} - 1 \right)$ 
  - If the equation fails, it still might be, so draw the whole thing to be safe.

## Dynamic

**Dynamic:** each of the tasks' priorities can change. *Think:* while for static priorities it is constantly re-evaluating which task has the highest priority, dynamic scheduling also re-evaluates the actual priorities, themselves.

**Shortest Completion Time (SCT):** dynamic

The only two optimal dynamic priorities are:

- **Earliest Deadline First (EDF):**
  - more flexible, better U
  - If deadlines < periods, still optimal, but determining feasibility is NP-hard
  - Always feasible if  $U \leq 1$
- **Least Slack Theorem (LST):** not as popular as EDF

## Multiprocessor

Once you have multiple processors, neither EDF nor RM are guaranteed to work.

Look into first-fit algorithms

## Task Interactions

**Suspended:** active choice, of access prevention until algorithm allows it to

**Blocked:** as a result of waiting for a resource to be free

How to do the timing diagrams with locks:

- $S_1 = \text{lock}(S_1)$
- $S_1^{\wedge} = \text{unlock}(S_1)$

**One-shot Tasks:** non-periodic tasks

**Critical Section:** when a task tries to acquire a shared resource already locked by another task

**Deadlock:** think if you and I are at a table with one fork and one knife and you need both to eat, but you take the fork and I take the knife.

**Priority Inversion:** an issue when a lower priority task is using a resource a higher priority task wants

**Priority Inheritance Protocol (PIP):** a method of dealing with *Priority Inversion*

- a method of avoiding deadlock by telling high priority tasks to share their resources with the lower priority tasks even when it's not their turn
- Temporarily raise the priority of a task only if and when it actually blocks a higher priority task
- On leaving the critical section, the task priority reverts to its original value
- Protect the resource with a semaphore
- You can make it so that tasks can use the resource even after they release the semaphore, but you risk overwriting in that time
- Issues:
  - If only one shared resource, there's only one possible schedule
  - If more than one resource blocking:
    - Blocking time may be excessively long
    - Deadlock may occur
  - If accessing multiple resources, you can only use them in the same order

**Priority Ceiling Protocol (PCP):** tasks entering a critical section can only access the blocked resource if it has a priority higher than the priority ceiling

- **Priority Ceiling (PC):** maximum priority of all tasks ever going to access a given resource

- Only need to check PC when entering a critical section
- If any task needs priority higher than the priority ceiling of ALL of the semaphores currently locked, it's suspended
- Main advantages:
  - No locked resources, so free access
  - "The state of the art when resolving resource-contention issues"
  - "Deadlock free for an arbitrary number of tasks with an arbitrary number of resources acted upon in an arbitrary way."

## Sporadic Server

**Execution Budget** [ $e_s$ ]: periodic tasks aren't flexible...

**Execution time** [ $e_i$ ]: ...sporadic tasks are

**Deadline** [ $d_i$ ]: absolute deadline

**Release Time** [ $r_i$ ]: delay before the task is released to be executed

**Set of Sporadic Tasks** [ $\theta$ ]:

**Sporadic Task** [ $S_i$ ]:

- Non-periodic task (a.k.a. aperiodic)
- ( $r_i, e_i, d_i$ )
- Typically interrupt-driven

**Rules** [ $\rho$ ]: set of rules regulating a sporadic server

**Sporadic Server** [ $\Phi_s$ ]: ( $p_s, e_s, \theta, \rho$ )

**Periodic Task**: ( $p_s, e_s$ ) a type of sporadic server

- no expectation of when it finishes, only that a new one is queued every period

Assume:

- $\Phi_s$  scheduled with  $T_i$  according to RM

We don't use  $K_d$  because it looks at the derivative regardless of the size of the error function. If your error is a sine function with a small amplitude,  $K_d$  will only take the derivative into account and it will overcompensate.

**Open loop response**: plant with no control

**Ziegler-Nichols Tuning Rule**: a PID tuning rule

Look at the *open loop response*. It could have a longer rise time / overshoot than preferred.

1. Tangent to curve on upslope

High sample rate  $\rightarrow$  lots of high frequency noise

**Effective Utilization** [ $\delta$ ]:

$$U = U_{\text{periodic}} + \delta U_{\text{sporadic}}$$

**Error bound**  $[\epsilon]$ :

**Slack**  $[\omega]$ :

**Acceptance Test**: check of stuff

$$\omega(S_k, t) = \left\lfloor \frac{d_k - t}{p_s} \right\rfloor e_s - e_k - \sum_{S_i \in \theta: d_i < d_k} e_i - \xi_i$$

1. If  $\omega(S_k, t) < 0$ , reject task
2. If  $\omega(S_k, t) \geq 0$ , need to check if already accepted sporadic tasks are adversely affected, i.e.  $\omega(S_j, t) - e_k \geq 0$  holds for all  $S_j \in \theta$  with  $d_j \geq d_k$ .

The set  $\theta$  is maintained dynamically.

## Clocks

**Computer Clock**  $[C]$ :

**Standard Clock**  $[C_s]$ : perfect clock; has real time

Attributes:

- Correctness
- Bounded Drift
- Monotonicity
- Chronoscopia

**(EPS)**: a bounded/maximum difference between the clock time and the real time

$$|C(t) - C_s(t)| \leq \text{EPS}$$

**Reset time**  $[r]$ : the real time you set the clock to when you reset it

**Drift**  $[E]$ : rate of change of the clock value away from a perfect clock (each second)

- There's usually a reason why a clock drifts

**Drift Bound**  $[\rho]$ : maximum drift

$$\left| \frac{dC(t)}{dt} - 1 \right| \leq \rho$$

**Reset Error**  $[\epsilon]$ : error between actual time and time clock was set to at reset

**Total Error**  $[E]$ :  $E(t) = \epsilon + \text{drift\_since\_reset}$

$$\text{drift\_since\_reset} \leq \rho(C(t) - r)$$

$$E(t) = \rho(C(t) - r) + \epsilon \leq \text{EPS}$$

$$C(t) - r \leq (\text{EPS} - \epsilon)/\rho$$

Real time will be within this interval –  $[C(t) - E(t), C(t) + E(t)]$

**Monotonicity**: Clock will always have a consistent spacing and will only move in one order (forward / backwards)

SSL certs will fail signature if your clock is wrong as to ensure this

**Chronoscopicity** [ $\gamma$ ]: maximum changing drift

second derivative of stuff  $\left| \frac{d^2 C(t)}{dt^2} \right| \leq \gamma$

### Cristian's Algorithm

**Minimum Latency** [ $T_{\min}$ ]:

**Request Send Time** [ $T_0$ ]:

**Request Receive Time** [ $T_1$ ]:

**Server Time** [ $T_{\text{server}}$ ]: time returned by the server

$$T_{\text{new}} = T_{\text{server}} + \frac{T_1 - T_0}{2}$$

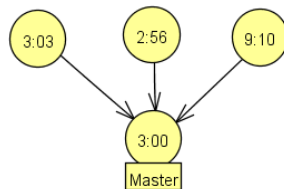
$$\text{Accuracy is } \pm \frac{T_1 - T_0}{2} - T_{\min}$$

**Round Trip Time (RTT):**

### Berkeley

Not often used, but useful for learning

1. Elect 1 node to be the **master**, the one that runs the algorithm



2. Finds the average of the nodes. However, that's probably going to find a value that isn't near any of them.
3. Eliminate the outliers:
  - a. Standard deviation: the more outliers, the harder to remove them, i.e.  $\sqrt{\frac{1}{2} \times \sum (x_i - \mu)^2}$
  - b. Median
  - c. **Maximum deviation**: maximum clock drift  $\times$  time since last synchronization; sometimes it's good to use physical limitations as the minimum check to ensure accuracy
  - d.

### PID Control

**Plant** [ $G(s)$ ]: a transfer function, e.g.  $\frac{1}{s^2 + 10s + 20}$

Remember this from 3DX4? Most of the stuff is still there, so refer to that. [More here](#).

Each of the K's represent a different error or gain

4 types of controllers [ $P(s)$ ]:



- **Proportional Controller (P),(PC):**  $\frac{K_p}{s^2 + as + (b + K_p)}, \frac{K_p}{s^2 + 10s + (20 + K_p)}$
- **Proportional Integral (PI):**  $\frac{K_p s + K_i}{s^3 + as^2 + (b + K_p s + K_i)}, \frac{K_p s + K_i}{s^3 + 10s^2 + (20 + K_p s + K_i)}$
- **Proportional Derivative (PD):**  $\frac{K_d s + K_p}{s^2 + (a + K_d)s + (b + K_p)}, \frac{K_d s + K_p}{s^2 + (10 + K_d)s + (20 + K_p)}$
- **Proportional Integral Derivative (PID):**  $\frac{K_p s + K_i + s^2 K_d}{s(s^2 + as + b) + K_p s + K_i + s^2 K_d}$

$$\frac{K_d s^2 + K_p s + K_i}{s^3 + (10 + K_d)s^2 + (20 + K_p)s + K_i}$$

$$s^3 + 10s^2 + K_d s^2 + 20s + K_p s + K_i$$

$$= s(s^2 + 10s + 20)$$

$$u(t) = K_p e(t) + K_d \dot{e}(t) + K_i \int_0^t e(v) dv$$

**Dominant pole:** largest magnitude

Finding a zero: Numerator = 0

Finding a pole: Denominator = 0

### Sampling

Closed loop: Good sampling rate is 10-20× the bandwidth

$$1\text{Hz} = 2\pi \text{ rad}$$

Find a zero from the numerator and cancel the pole closest to it

$$f_s \approx \text{chosen pole} \frac{\text{rad}}{s} / 2\pi \text{rad}$$

$$H(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

$$\omega_h \approx \frac{\omega_n}{2\zeta}$$

$$\omega_h = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}}$$

If sampling rate is given, use C(s) to get:  $\frac{C}{T_s}(u[k] - u[k-1]) + Du[k] = \frac{A}{T_s}(e[k] - e[k-1]) + Be[k]$

If not, use -3dB frequency

## Designing a PID Controller

1. Obtain an open-loop response and determine what needs to be improved
2. Add  $K_p$  to improve the rise time
3. Add  $K_d$  to improve the overshoot
4. Add  $K_i$  to eliminate the steady-state error
5. Adjust each of  $K_p$ ,  $K_d$ , and  $K_i$  until you obtain a desired overall response. You can always refer to the table below to find out which controller controls what characteristics.

Increasing this	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
$K_p$	Decrease	Increase	Small Change	Decrease
$K_d$	Small Change	Decrease	Decrease	No Change
$K_i$	Decrease	Increase	Increase	Eliminate

### Ziegler-Nichols Tuning Rule:

- a plant with neither integrators nor dominant complex-conjugate pairs
- Look at the *open loop response*. It could have a longer rise time / overshoot than preferred.
- Tangent to curve on upslope
- For PID controllers

Gain  $[H(s)]$ :

Noise frequency  $[\omega_n]$ :

Noise amplitude  $[a_n]$ :

Open loop: plant with no control

Closed loop:  $H(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$

$$H(s) = K_c \frac{\prod_{i=0}^N (s - z_i)}{\prod_{j=0}^M (s - p_j)}$$

$$C(s) = K_D s + K_P + \frac{K_I}{s}$$

So, you need to rearrange your  $H(s)$  that is in the first formula to look more like the second formula

## Jitter

**Jitter**  $[J]$ : a delay

**Relative Jitter**: difference in response time between current and previous response times

$$\max_k |R_{i,k+1} - R_{i,k}|$$

**Absolute Jitter**: difference between largest response time and smallest response time

$$\max_k R_{i,k} - \min_k R_{i,k}$$

Absolute jitter  $\geq$  relative jitter

## Fail

- **Fail-safe:** in the event of a specific type of failure, responds in a way that will cause no harm, or at least a minimum of harm, to other devices or to personnel
- **Fail-stop:** detects exceptions, but doesn't worry about handling them or raising them
  - failure in one component might not be visible until it leads to failure in another component
- **Fail-fast:** when a problem occurs, a fail-fast system fails immediately

**Passive Fault Tolerance:** fault masking, e.g. TMR

**Active Fault Tolerance:** reconfiguration, like turning off sets of servers, e.g. Pair and Spare

**Duplication:**

- primary produces an output
- secondary produces an output, compared with primary
- if outputs in disagreement, flag error

## Voting Schemes

**Plurality [k]:** number of votes needed for a majority

- **Median voter:** chooses median value as *correct* output (for this example, 2.00)
- **Majority voter:** given observations,  $d_i$ , and tolerance  $\epsilon$ , i.e. willingness for error in *correct* value:
  1. Construct sets,  $P_k: x \in P_k \leftrightarrow |x - y| \leq \epsilon$  for all  $y \in P_k$ , where  $P_k$  has all elements within  $\epsilon$  of each other,  $P_k$  is maximal, i.e. cannot add any points to it
  2. Choose  $P_k$  with largest  $|P_k|$ , where  $|P_k| = \text{len}(P_k)$ 
    - If  $|P_k| > \text{floor}(N/2)$ : choose any one of  $P_k$  as *correct* value or a combination of many
    - Else, no result
    - e.g. Choose:
      - $\epsilon = 0.1$
      - $P_1 = \{2.00, 2.01, 1.98, 2.05\} \Rightarrow |P_1| = 4 > \text{floor}(5/2) \leftarrow$  majority chooses value in  $P_k$
      - $P_2 = \{1.80\}$
    - What is the minimum value of  $\epsilon$  that leads to the majority voter outputting a value?

$\epsilon = 0.03$  (i.e. range of 2.00, 2.01, 1.98);  $d_1, d_2, d_3$  all satisfy  $|d_i - d_j| \leq 0.03$

- **K-plurality:** make a section of size  $k$
- **Pair-and-spare:** when you have 2 sets of 2

**Modular Redundancy (MR):** when you have multiple separate redundant systems

**Triple Modular Redundancy (TMR):** having 3 systems with the same purpose running together. This ensures that if a system is not working properly, the things it outputs is compared against the other 2 systems and can be verified as wrong

**Cold spare:** a redundant system that is off until needed

**Warm spare:** a redundant system that is in a standby state until needed

**Hot spare:** a redundant system that is functional, on, and actively collaborating with the primary system

**Byzantine General's Problem:** useful for sensors with noisy values

$$n \geq 3t + 1$$

[n]: number of generals

[t]: number of traitorous generals

A lot of these problems are based on binomial

### Information Redundancy

e.g. 1101100\_, '\_ ' is a **parity** or **checksum**, where the number of 1's are identified as even (1) for **even parity** and odd (0) for **odd parity**.

- Can detect single bit errors
- Cannot correct errors
- Adding more bits can also allow for correction
- Used in all Communication pRotoCols (**CRC**)

### Execution Time

Underestimate the time

**Best Case Execution Time (BCET):**

**Worst Case Execution Time (WCET):**

(BCET<sup>^</sup>): estimation of BCET

(WCET<sup>^</sup>): estimation of WCET

BCET<sup>^</sup> < BCET < WCET, WCET<sup>^</sup>

### Natural Language Standards

**Formal requirement:** If <condition>, <action> shall occur

**Soft requirement:** within <response time>, <minimum probability> of the time

**Hard requirement:** within <response time>

QoS: a functional requirement with a hard / soft requirement to a