SFWR ENG 4O03

Kemal Ahmed
Dr. Deza
Fall 2015

Contents

Linear

Linear Program: an optimization problem in which the objective function is linear and each constraint is a linear inequality or equality
Decision variables: describe our choices that are under our control
Objective function: describes a criterion that we wish to max/minimize; doesn't have an in/equality
e.g. max 40x + 30y

Integer linear program: a linear program that only deals with integers

Constraints: describe the limitations that restrict our choices for our decision variables, always *inequalities*.
Free: no constraints

Basic variable: the variables corresponding to the identity matrix, usually have to be set to 0
Non-basic variable: …not basic variables

Converting constraints to equalities

Slack variable: basic variable greater than constraint, added to turn inequalities into equalities
Surplus variable: equation variable less than constraint, subtracted

Hyperplane: a hyperplane in R^x is a shape in R^{x-1}, e.g. line in R^2

Optimal Solution: either a maximum or minimum of the objective function based on constraints
Basic Solution: a solution which has as many slack variables as basic variables
Basic Feasible Solution: all basic variables are non-negative

- Unique
- obtained by setting the non-basic variables to 0

Standard form: when you take inequalities and use slack variables to turn them into equalities.

- Note: all variables need to be ≥ 0.
- All remaining constraints are expressed as equality constraints.

e.g.)

$2x_1 + 4x_2 - x_3 - x_4 \quad \geq 1$
$2x_1 + 4x_2 - x_3 - x_4 + s \quad = 1$

Graphical Method

1. Sketch the region corresponding to the system of constraints. The points inside or on the boundary of the region are the *feasible solutions*.
2. Find the vertices of the region.
3. Test the objective function at each of the vertices and select the values of the variables that optimize the objective function. For a bounded region, both a minimum and maximum value will exist. For an unbounded region, if an optimal solution exists, then it will occur at a vertex.

Simplex Method: Maximization

Simplex Method: useful for solving linear optimization problems cheaply

- Cannot be done with **strict inequalities**, i.e. when there is no possibility of being equal
- Can only work if your objective function is in *standard form*

Simplex Tableau: visual representation of stuff

1. The *basic variables* can be identified if they have a column with one row of 1 and the rest of the rows are 0's. The value of the variable is at the row with the 1.
2. The bottom row is going to identify the constants for the new equation. You should see 0's in the columns that are non-basic

Process:

1. You'll have as many slack variables as you have constraint equations.
2. Find the column with the "lowest z value". That column is called the **pivot column**. The **entering variable** is the smallest z.
3. **Minimum test**: find the row with the smallest **departing variable** or **exiting variable**, i.e. RHS/x_{pivot}. That row is called the **pivot row**. x_{pivot} must be ≥ 0
4. The intersection of the pivot row & column is called the **pivot point**.
5. If your pivot point $\neq 1$, divide your row out by the value of your point
6. Use row operations, i.e. Gauss-Jordan

Simplex: Minimization

To minimize a function, we just oppositize the problem so we can use the maximization technique on it. You'll see. Just remember that we minimize [w] & maximize [z] AND minimize is (vars ≥ 0), while maximize is (vars ≤ 0). I'll explain using an example:

e.g.)

$w = 0.12x_1 + 0.15x_2$
$60x_1 + 60x_2 \geq 300$

$12x_1 + 6x_2 \geq 36$

$10x_1 + 30x_2 \geq 90$

1. Ignore slack variables for now. Make a matrix with just the variables you have

60	60	300
12	6	36
10	30	90
–0.12	–0.15	0

2. Find the transpose of this matrix

60	12	10	–0.12
60	6	30	–0.15
300	36	90	0

This gives us:

$z = 300y_1 + 36y_2 + 90y_3$

$60y_1 + 12y_2 + 10y_3 \leq 0.12$

$60y_1 + 6y_2 + 30y_3 \leq 0.15$

$300y_1 + 36y_2 + 90y_3 \leq 0$

Notice how the x's are now y's? Yeah I know you did. Well now, since you turned this into a maximization problem, what are you waiting for? Go to the maximization section!

Phase Simplex

This is useful for when you have a mix of constraints that are maximum and minimum constraints.

Artificial Variable: since you can't have negative variables ($x_1, x_2 \geq 0$), you can't just use a regular slack variable

Phase I

Hi

Phase II

Oh no!

Bland's Rule

Bland's Rule: a way of guaranteeing that you don't repeat going over the same variables (a cycle) by picking the smallest (or most negative) number