SFWR ENG 4E03

Kemal Ahmed

Fall 2015

Note: material covered in Stats 3Y03 Summary will not be covered in this summary

Contents

Statistics

Expected Value [μ]: definition of expected (NOT RIGHT!!)

Poisson parameter [λ]:

Exponential distribution: not always for time

Probability Distribution Function (PDF):

Cumulative Distribution Function (CDF):

Uniform Distribution: no memoryless property

Exponential Distribution:

- Memoryless
- Either CDF or PDF of original equation $F = 1 - e^{-\lambda x}$

Think chemistry, i.e. cancelling units

Variance

$$\text{var}(X) = E\left[X^2\right] - \left(E[X]\right)^2$$

- Don't change probability, but square X for calculation only
- For discrete: use the sum of the X's, so $E[X] = \Sigma(P(X=i)\cdot X_i)$ and $E[X^2] = \Sigma(P(X=i)\cdot X_i^2)$

Continuous Random Variable (CRV):

$$Var(x) = E\left[(X-\mu)^2\right]$$
$$= \sigma^2$$
$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,\mathrm{d}x$$
$$= \int x^2 f(x)\,\mathrm{d}x - \mu^2$$

Exponential

- **Mean** [E[X]]: 1/λ
- **Variance** [E[X]]: a.k.a. Expected value
- **Probability Distribution Function (PDF)** [P(X)]: $\lambda e^{-\lambda x}$
- **Cumulative Distribution Function (CDF)** [f(x)]: CDF = ∫PDF, i.e. $1 - e^{-\lambda x}$

Uniform

- **Mean**: $(b-a)^2/12$
- **Variance**: (a+b)/2
- **PDF**: 1 / (b–a) , a ≤ x ≤ b
- **CDF**: 1

Operations Analysis

Device [i]: units that are in terms of *i* are specific to an individual device or node within a system
Total devices [k]:

Service Time [S]: time per specific job

Visitation [V]: given or projected visits/jobs (closed system); cannot be calculated; basically a probability
[E(V)]: calculated visit/job ratio
P(visit)·total visits in previous node

Demand [D]: total service time for all jobs

$$D_i = E[S_i] \cdot V_i$$

$$D = \sum_{i=0}^{k} D_i$$

Time in system [T]: time the job is in the system

$$E[T] = \frac{N}{X}$$

$$E[T] \geq \max\left(D, ND_{\max} - E[Z]\right)$$

Response Time [R]: time the job is *being processed* in the system

Users [M]:
Optimal users [M*]:

$$M^* = \frac{D + E[Z]}{D_{\text{bottleneck}}}$$

Total Jobs [N]: N=M in a closed system

$$E[N]=\lambda E[T],\lambda=X$$

If E[Z] = 0, R = N

$$E[N]=\lambda E[R],\lambda=X$$

Think time [Z]: time it takes the user to put a request in and start, it's kinda like the frequency that users put in requests (seconds / request)

$$E[Z]=E[T]-E[R]$$

Throughput [X]: out-rate, jobs / hour of full system

$$X \le \min\left(\frac{1}{D_{\max}},\frac{N}{D+E[Z]}\right)$$

Note: $\frac{1}{D_{\max}}$ and $\frac{N}{D+E[Z]}$ converge at their lowest point, so equate them

$$X_i = E[V_i]X$$

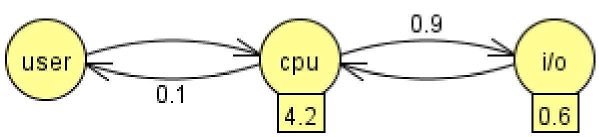
Utilization [ρ]: ratio that the time is busy

$$\rho_i = X_i E[S_i]$$

$$\rho_i = XD_i$$

Visitation Trick

If determining visitation at a node, establish a reference node from one of the incoming nodes, usually the user node, that has a returning percentage



$V_{user} = 1 = 0.1 \cdot V_{CPU}$

DTMC

Discrete Time Markov Chains (DTMC):

Geometric Series: $\sum_{i=0}^{\infty} r^i = \frac{r}{1-r}$

Geometric Sequence: $S_n = \sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$

$$S_n = \sum_{i=1}^{n} r^i = \frac{r\left(1-r^n\right)}{1-r}$$