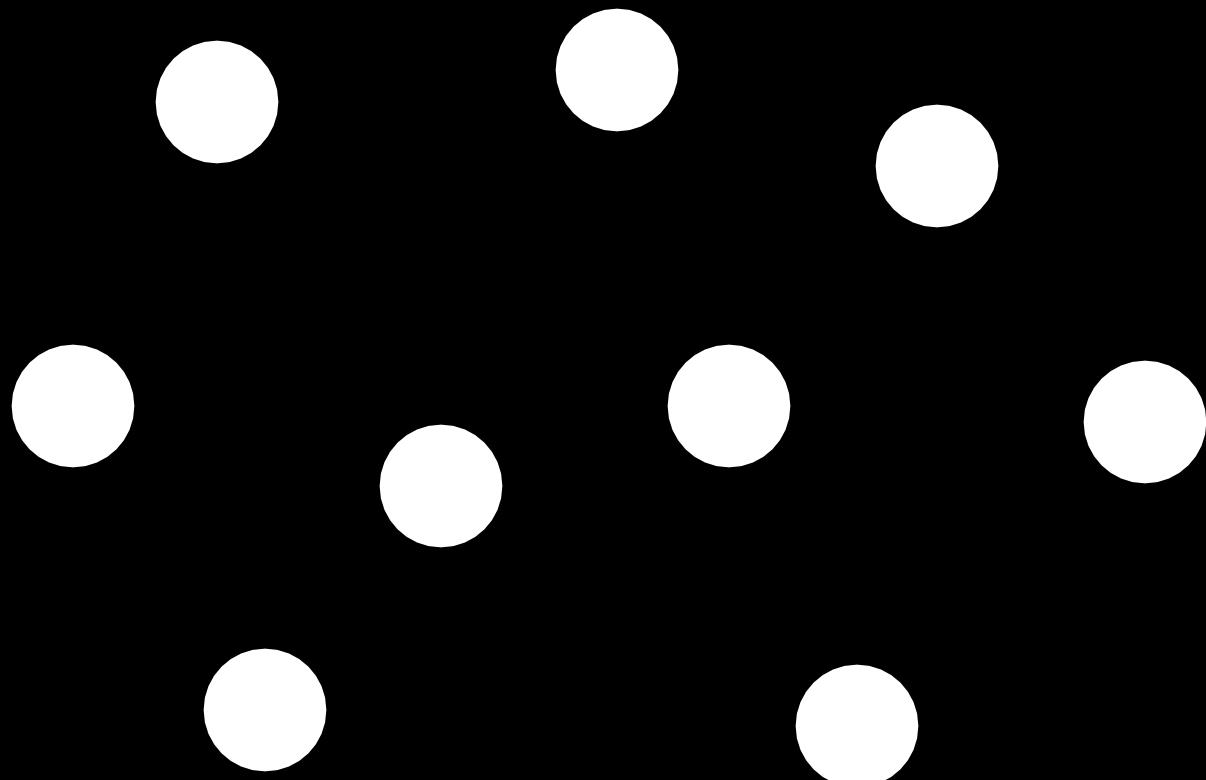


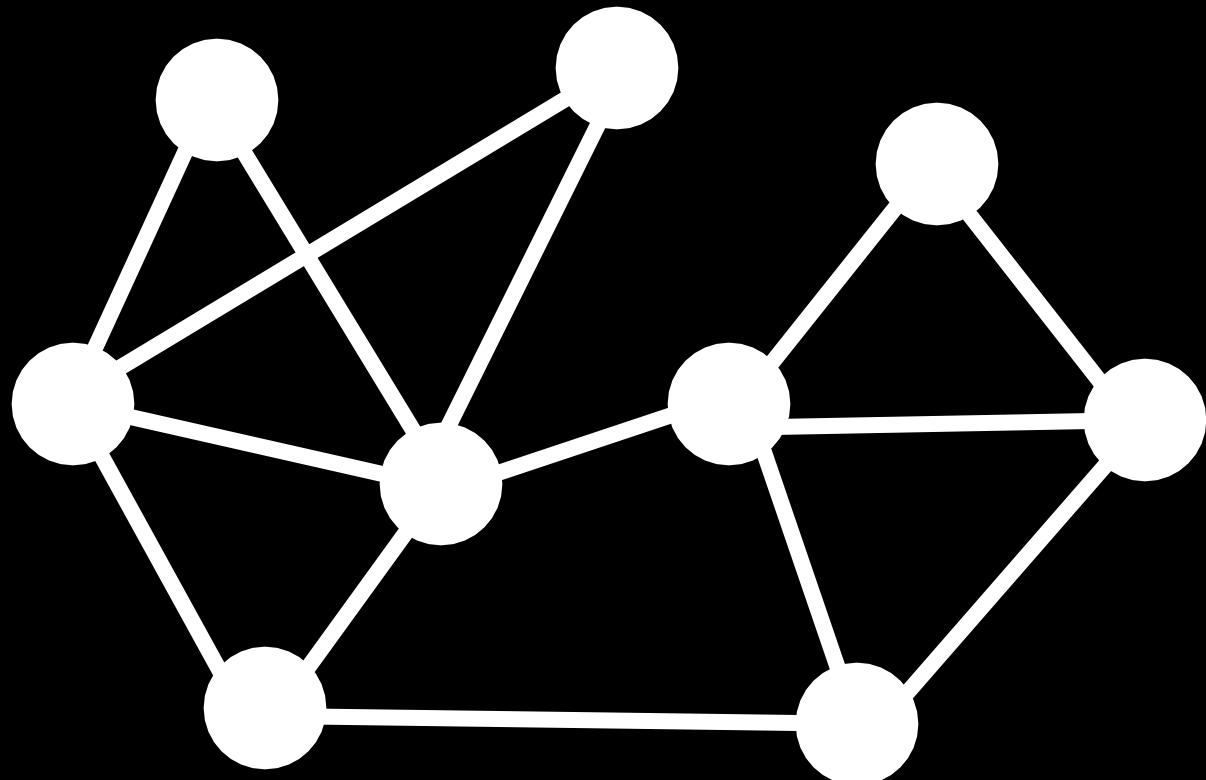
Introduction and Graph Models

Acknowledgment: Course lecture slides are from Jure Leskovec. Some slides are revised.

Why Networks?

Networks are a general
language for describing
complex systems



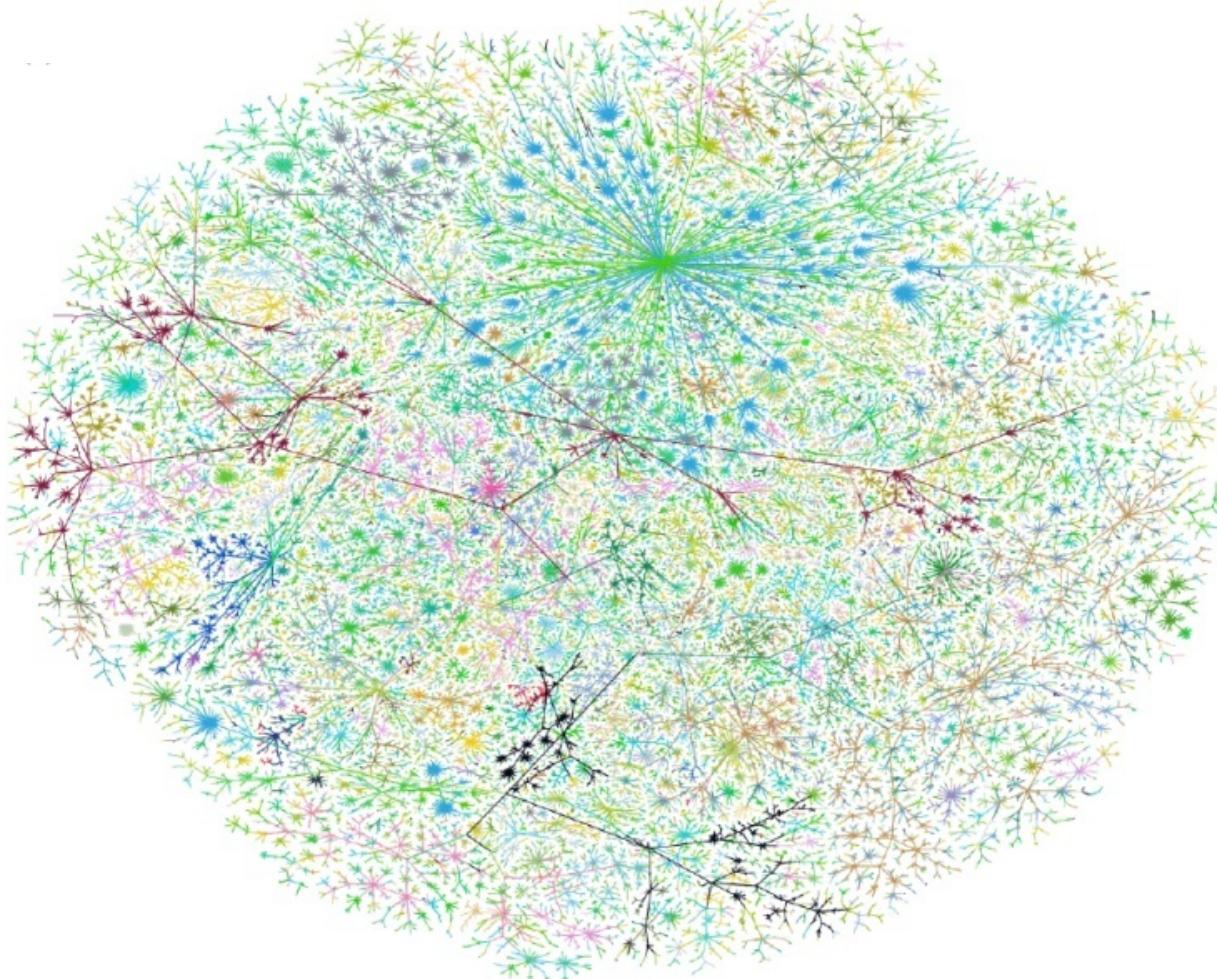


Interactions!

Networks & Complex Systems

- **Complex systems are all around us:**
 - **Society** is a collection of six billion individuals
 - **Communication systems** link electronic devices
 - **Information** and **knowledge** is organized and linked
 - Interactions between thousands of **genes/proteins** regulate life
 - Our **thoughts** are hidden in the connections between billions of neurons in our brain

What do these systems have in common?
How can we represent them?



The Network!

Networks!!

Behind many systems there is an intricate wiring diagram, **a network**, that defines the **interactions** between the components

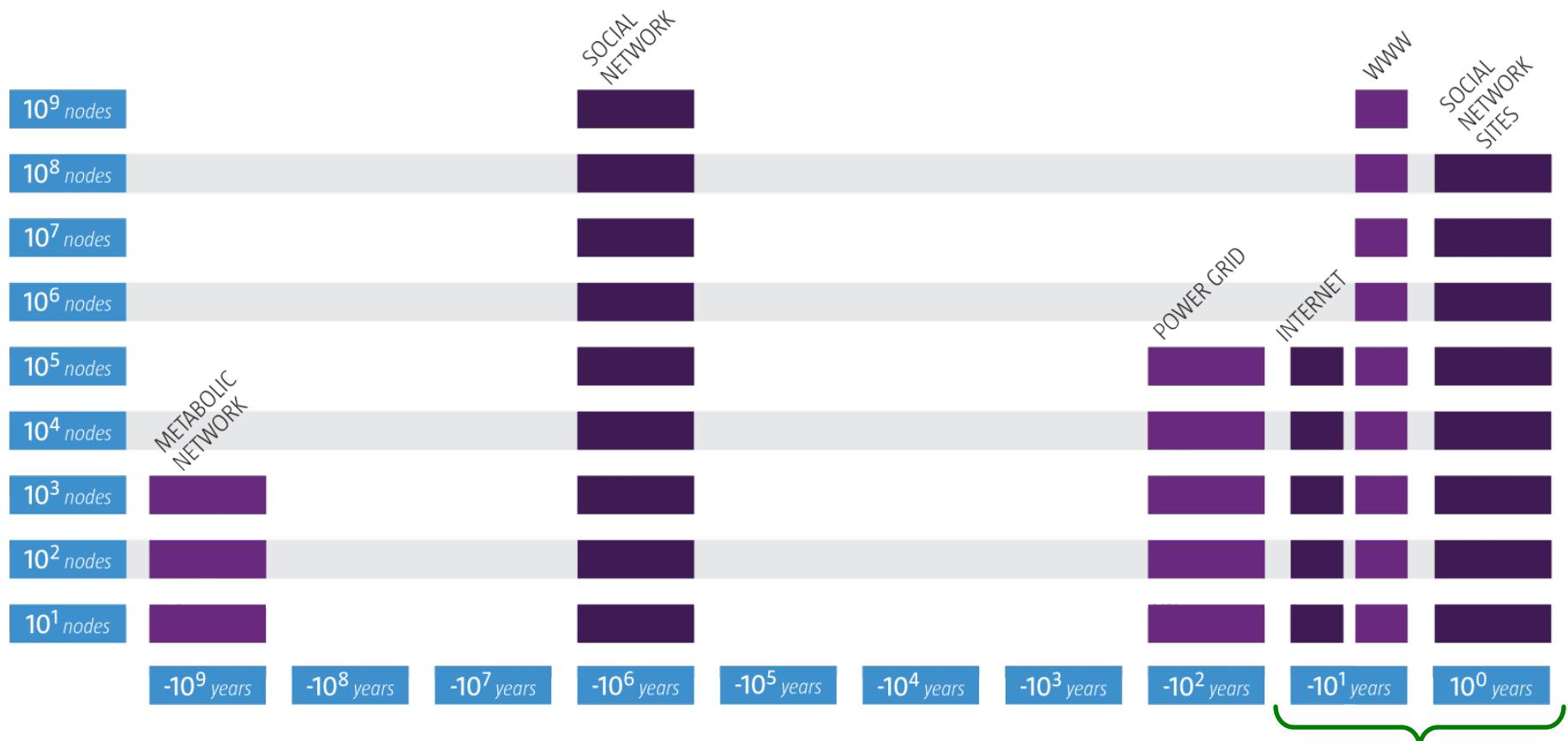
We will never understand these systems unless we understand the networks behind them!

**But why should I
care about
networks?**

Why Networks? Why Now?

- **Universal language for describing complex data**
 - Networks from science, nature, and technology are more similar than one would expect
- **Shared vocabulary between fields**
 - Computer Science, Social science, Physics, Economics, Statistics, Biology
- **Data availability (/computational challenges)**
 - Web/mobile, bio, health, and medical
- **Impact!**
 - Social networking, Social media, Drug design

Networks: Why Now?



Age and size of networks

CS!!

Web – The Lab for Humanity



Networks and Applications

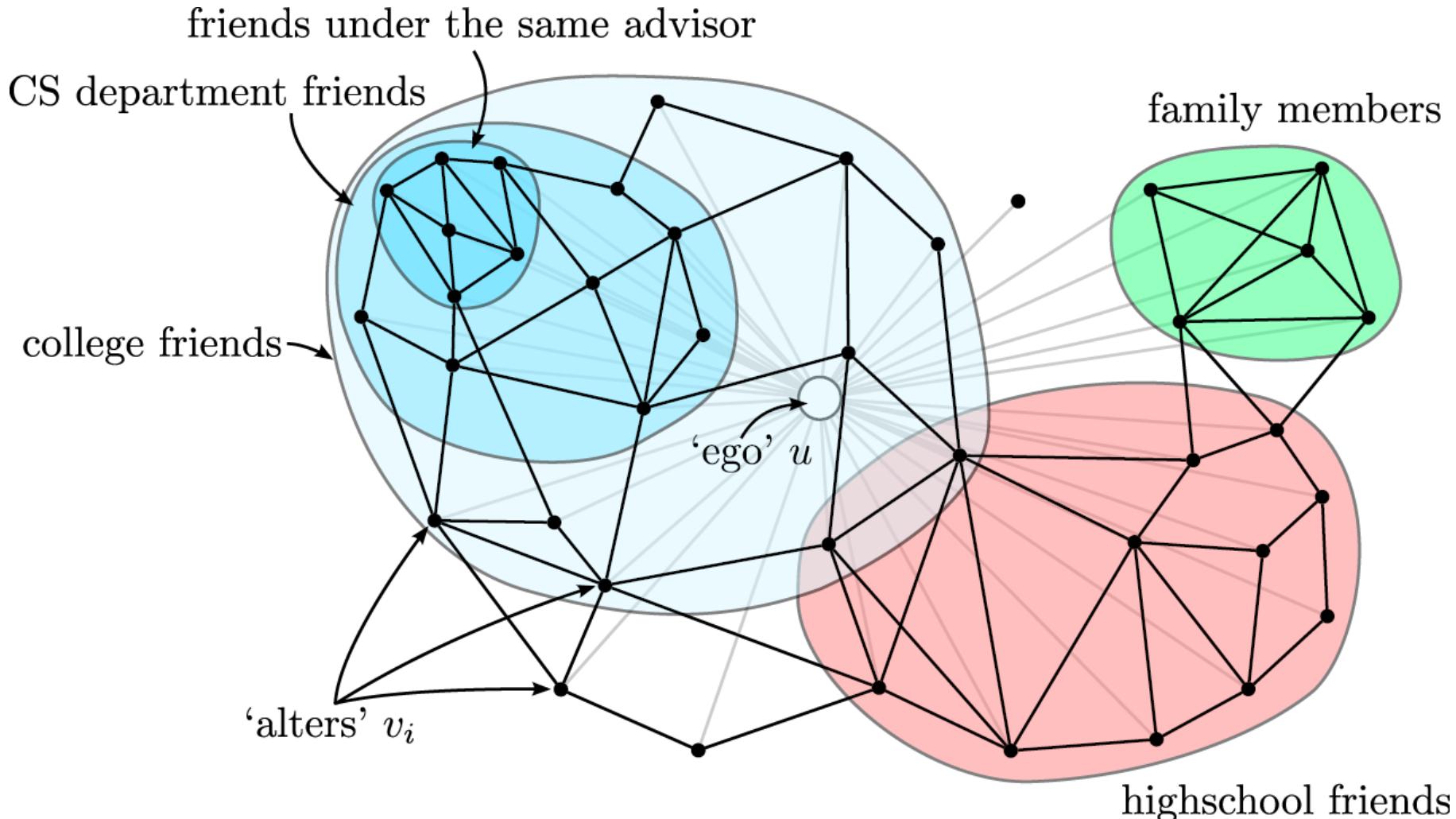
(1) Networks: Social



Facebook social graph

4-degrees of separation [Backstrom-Boldi-Rosa-Ugander-Vigna, 2011]

Application: Social Circle Detection



Discover circles and why they exist

(2) Networks: Infrastructure



Water supply distribution networks



Airline networks

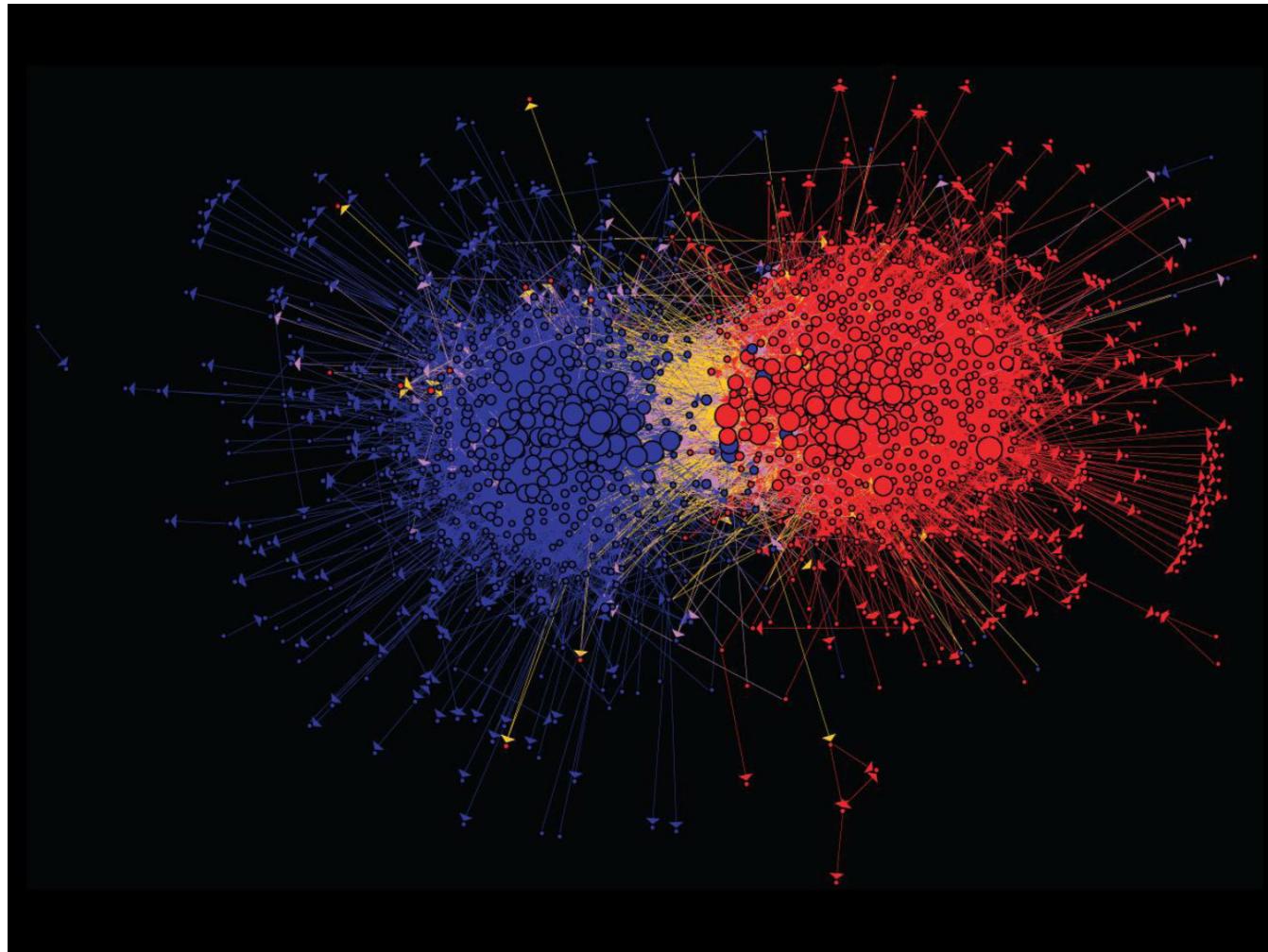
Application: Modeling Epidemics

- Infrastructure networks are crucial for modeling epidemics



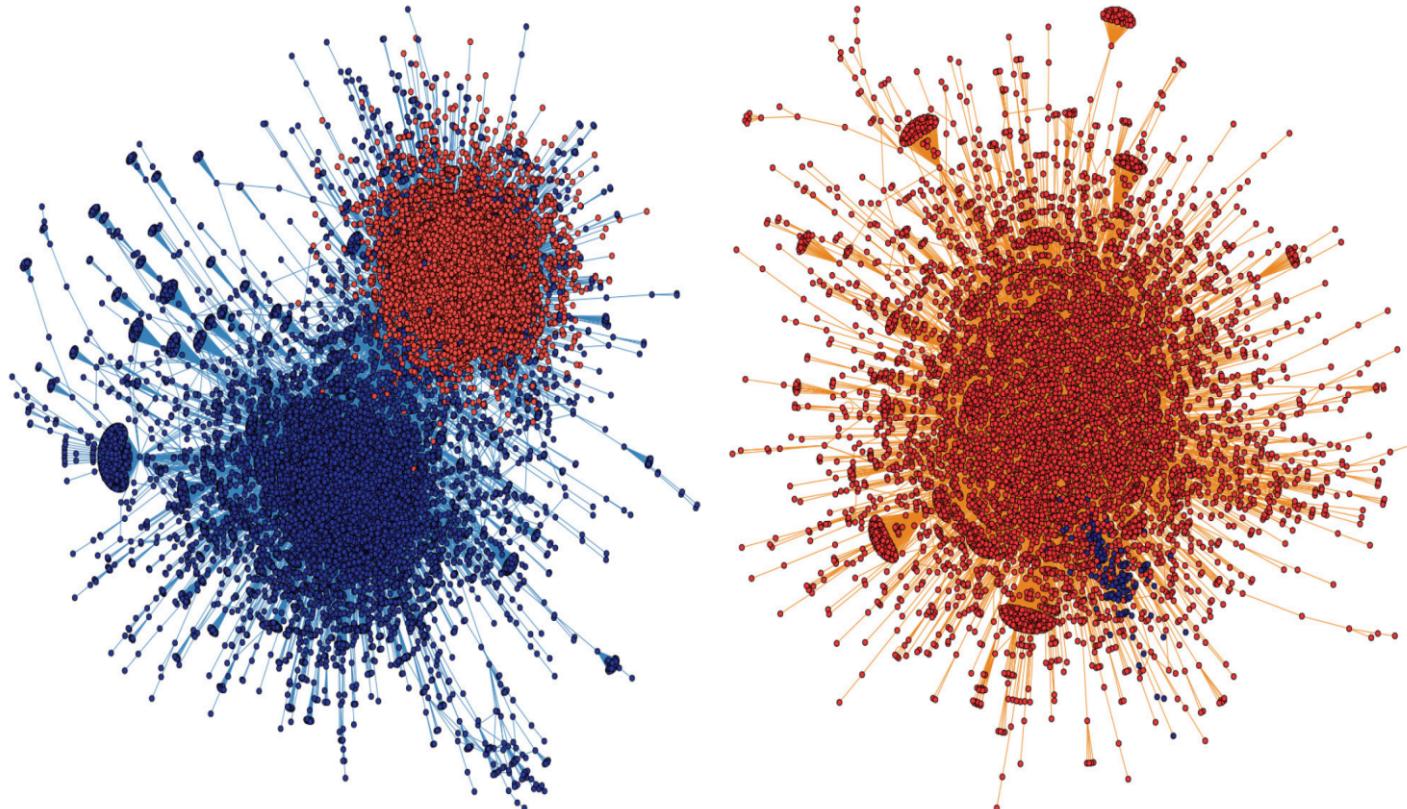
<http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0040961>

(3) Networks: Online Media



Connections between political blogs
Polarization of the network [Adamic-Glance, 2005]

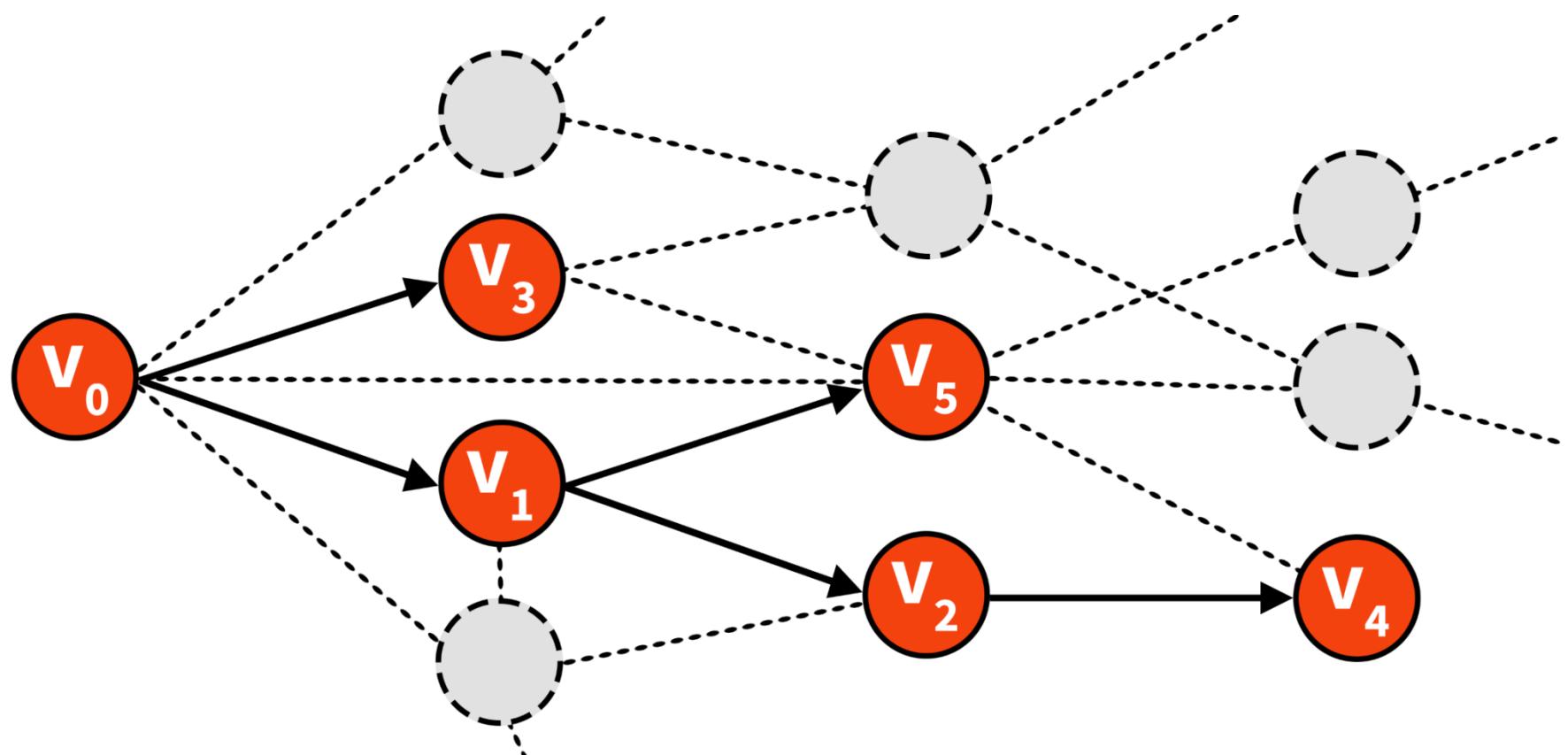
Application: Polarization on Twitter



- **Retweet networks:**
Polarized (left), Unpolarized (right)

Conover, M., Ratkiewicz, J., Francisco, M. R., Gonçalves, B., Menczer, F., & Flammini, A. "Political Polarization on Twitter." (2011)

Application: Information Diffusion



Information cascade in a social network

Facebook Information Cascades

Timeline Photos

[Back to Album](#) · I fucking love science's Photos · I fucking love science's Page

[Previous](#) · [Next](#)

Thickness a ————— Radius z



$$V = \pi z^2 a$$

$$V = \text{Pi}(z*z)a$$



I fucking love science

Seriously. If you have a pizza with radius "z" and thickness "a", its volume is $\text{Pi}(z*z)a$.

3,311 likes · Lina-von Der Stein, Iman Khallaf, 周明佳 and 73,191 others like this.

27,761 shares

1,000 comments

46 of 1,470

[archive](#)

Album: Timeline Photos

Shared with: Public

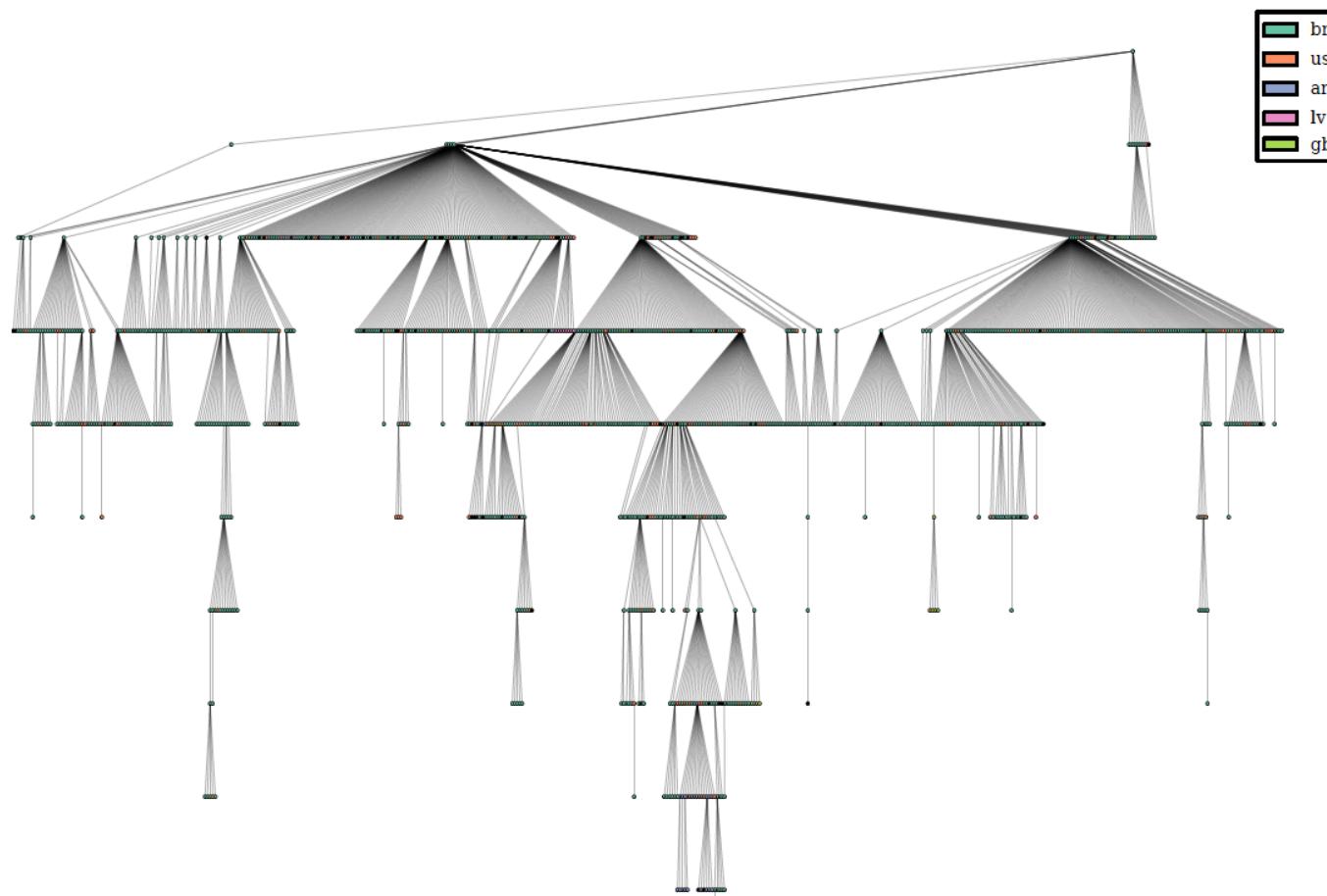
[Open Photo Viewer](#)

[Download](#)

[Embed Post](#)

Can cascades be predicted? Cheng et al., WWW '14.

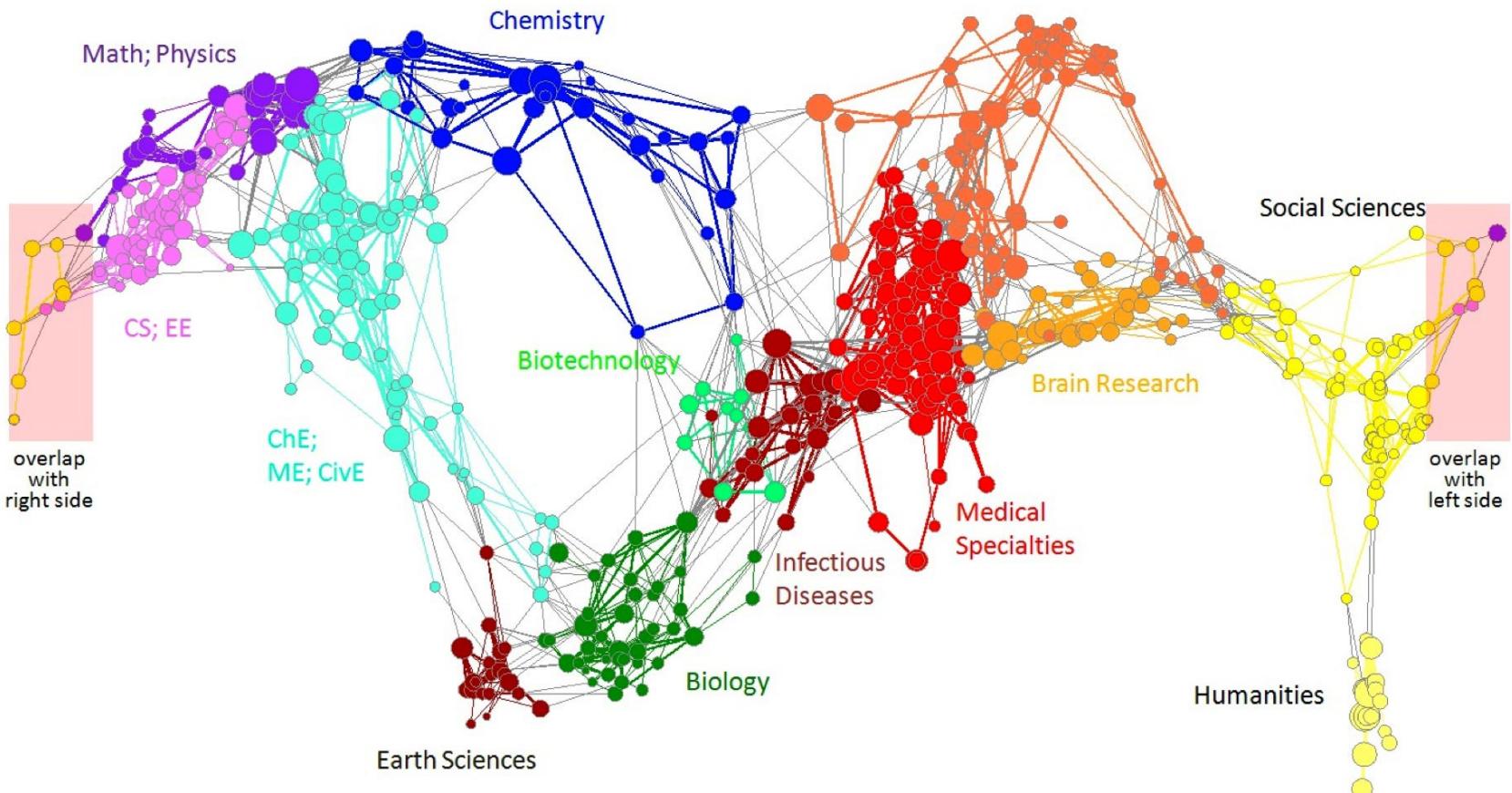
Application: Product Adoption



60-90% of LinkedIn users signed up due to an invitation from another user.
[Global Diffusion via Cascading Invitations: Structure, Growth, and Homophily](#).

Anderson et al., WWW '15.

(4) Networks: Information, Knowledge

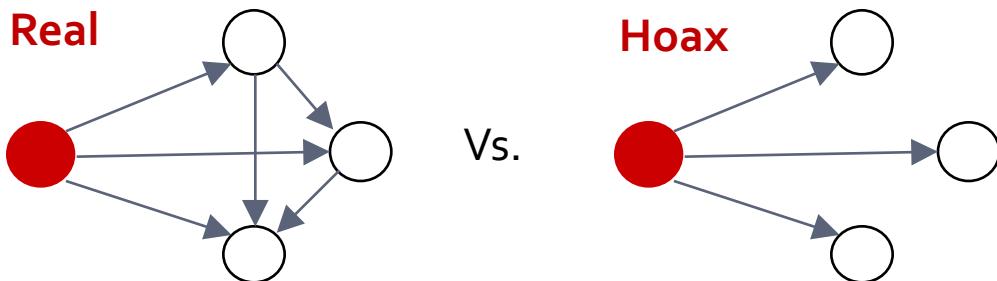


Citation networks and maps of science
[Börner et al., 2012]

Application: Misinformation

- Q: Is a given Wikipedia article a hoax?

- Real articles link more coherently:



Hoax article detection performance:

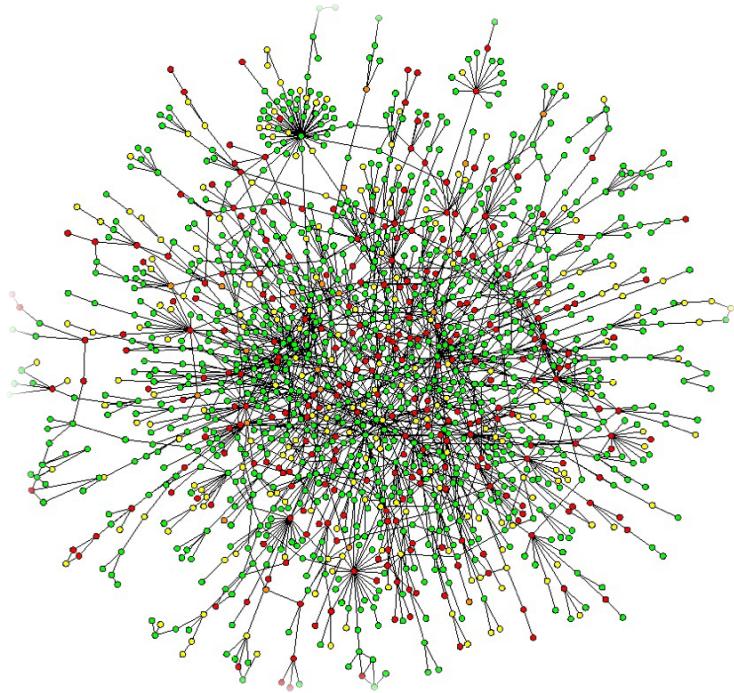
50%	66%	86%
Random	Human	WWW '16

Disinformation on the Web: Impact, Characteristics, and Detection of Wikipedia Hoaxes. Kumar et al. WWW '16.

This screenshot shows a Wikipedia page for 'Balboa French Creole'. The page title is 'Wikipedia:List of hoaxes on Wikipedia/Balboa French Creole'. A red box highlights a warning message: 'This article does not cite any references (sources). Please help improve this article by adding citations to reliable sources. Unsourced material may be challenged and removed. (January 2010)'.

The page content describes Balboa French Creole as a language used in Balboa Island, California, originating from a blend of French, English, Spanish, and German. It notes that the language is highly incomprehensible to most French speakers and is virtually extinct, with only 14 people remaining. A sidebar provides details about the language's native speakers, region, and language family.

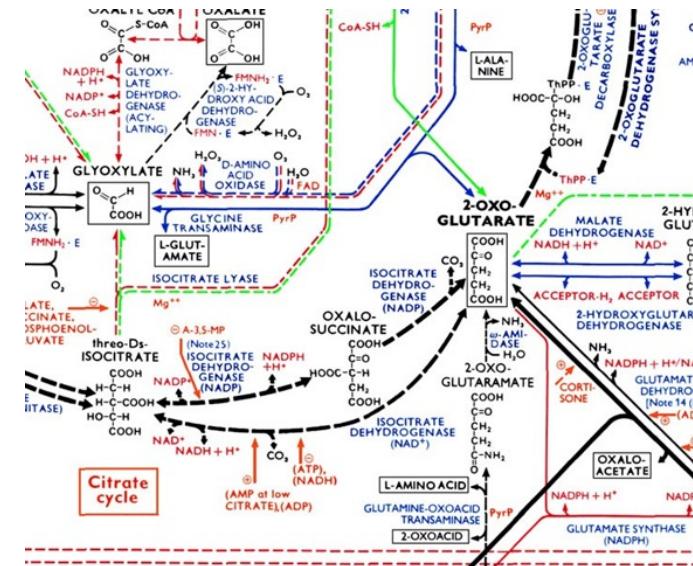
(5) Networks: Biology



Protein-protein interaction (PPI) networks:

Nodes: Proteins

Edges: 'Physical' interactions



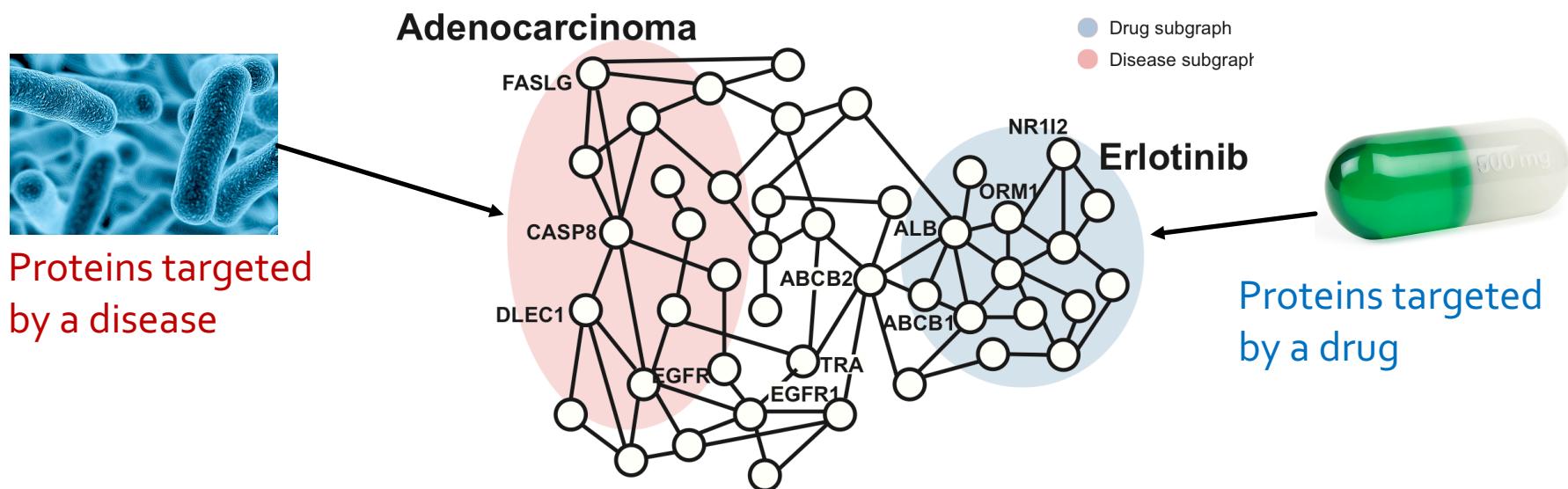
Metabolic networks:

Nodes: Metabolites and enzymes

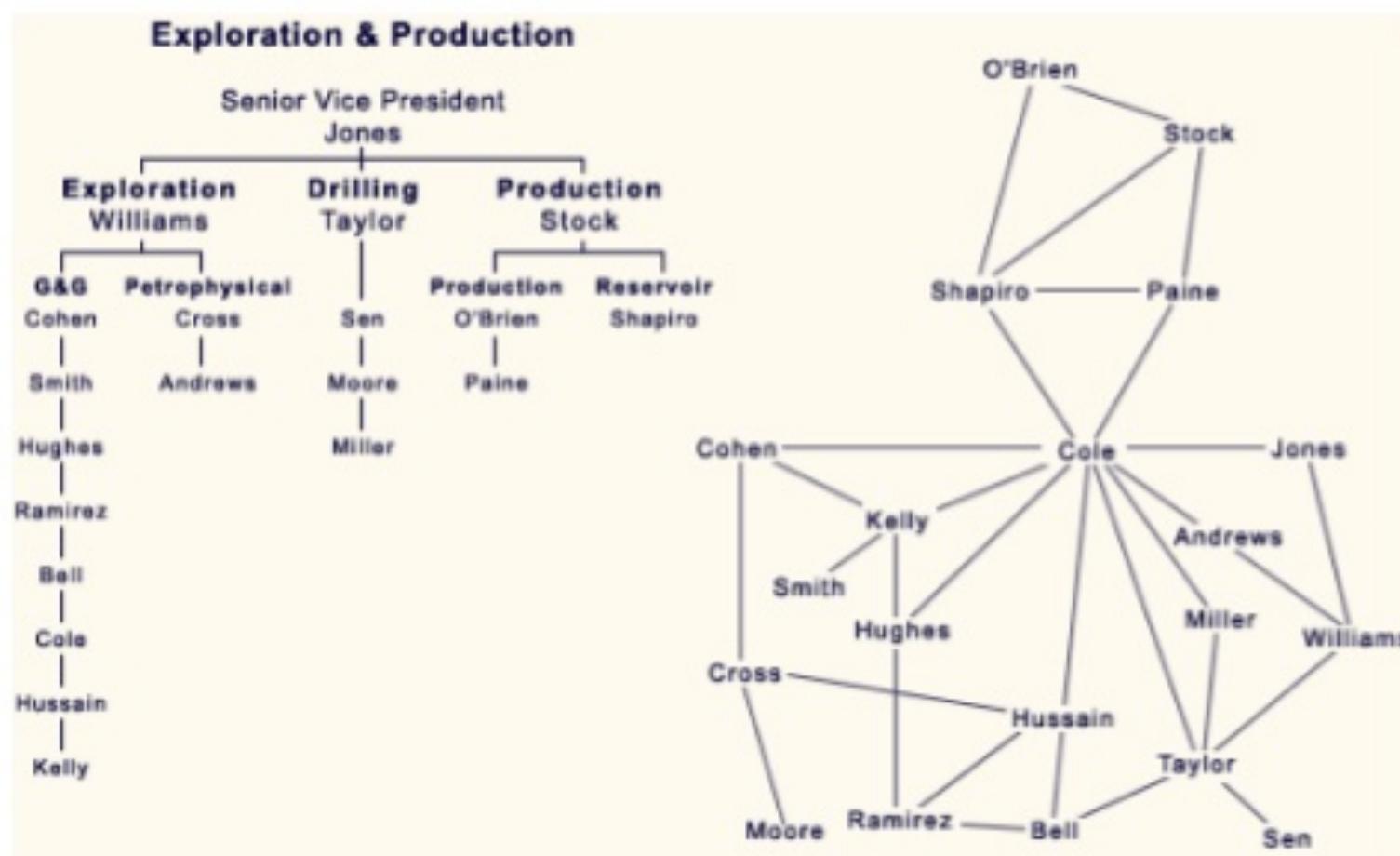
Edges: Chemical reactions

Application: Drug Repurposing

- Q: Can we predict therapeutic uses of a drug?
- **Insight:** Proteins are worker molecules in a cell.
Protein interaction networks capture how the cell works.

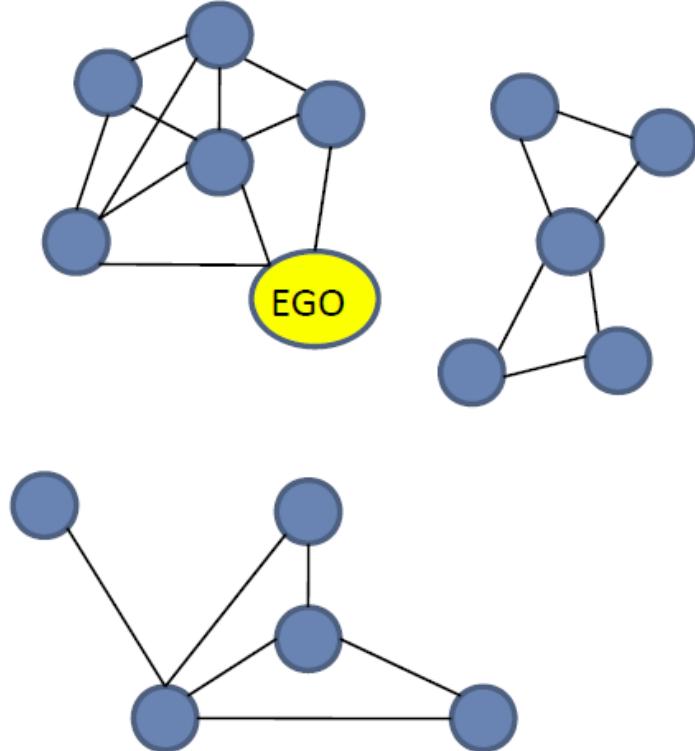


(6) Networks: Organizations

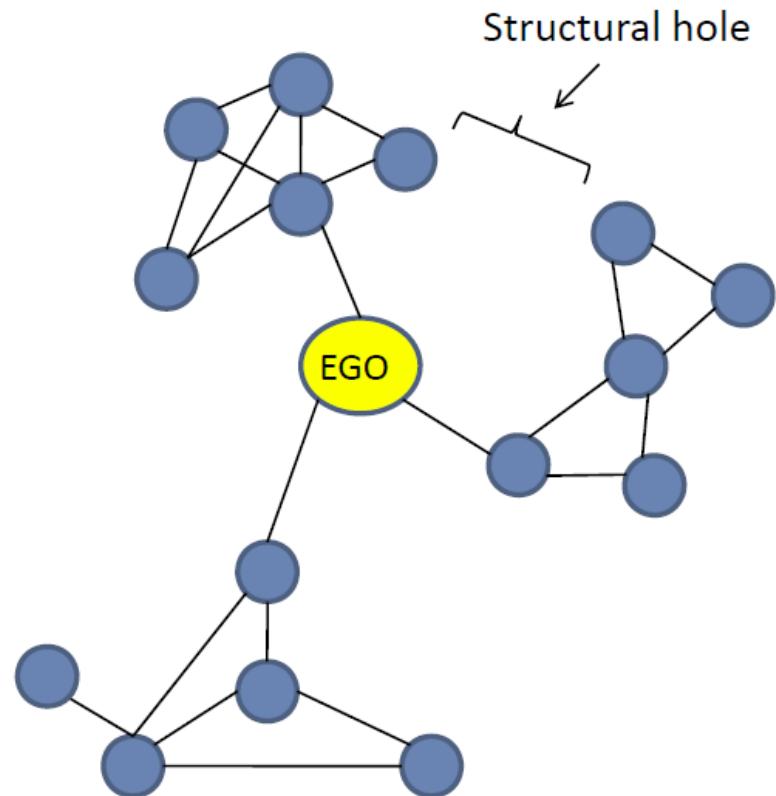


Who are the central nodes in the organization? [Krebs, 2002]

Application: Employee Success



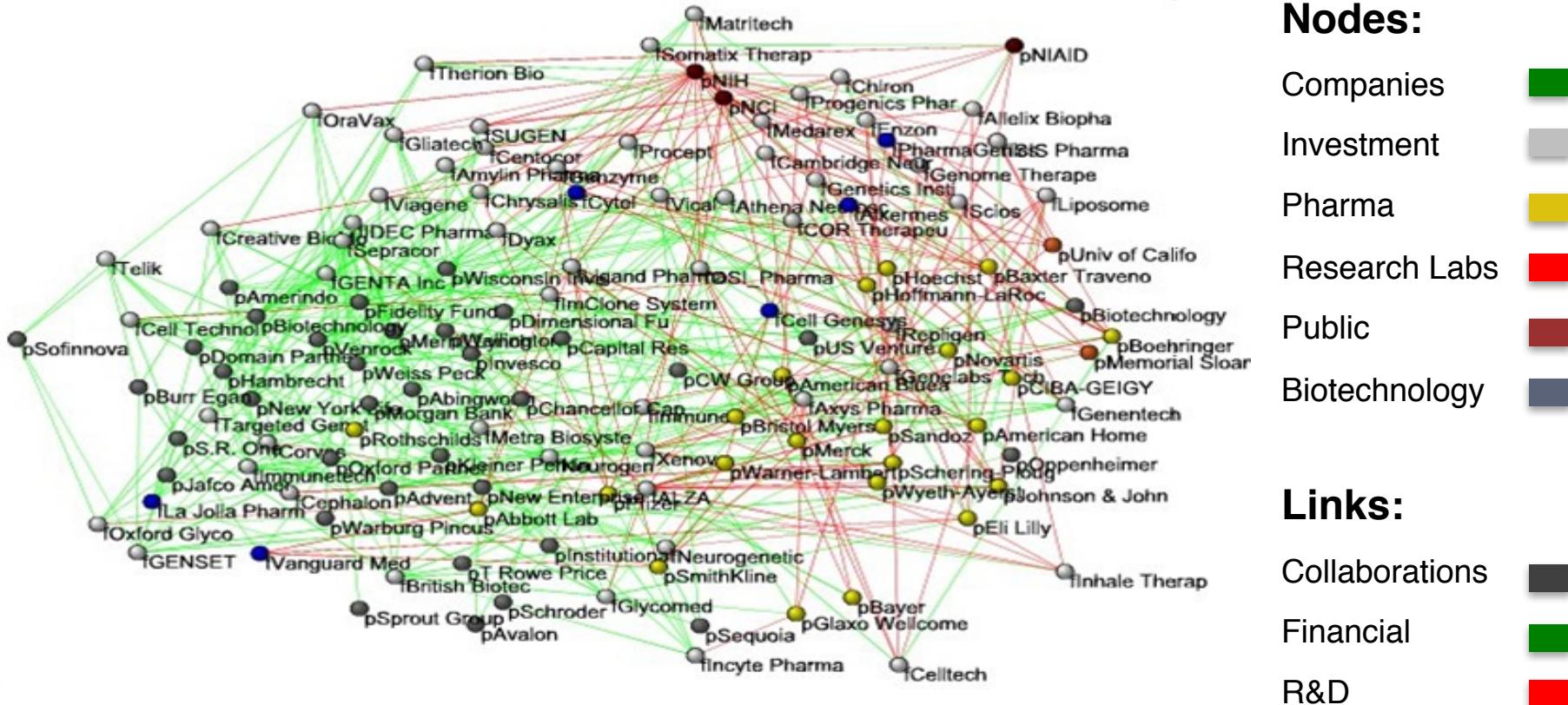
Few structural holes



Many structural holes

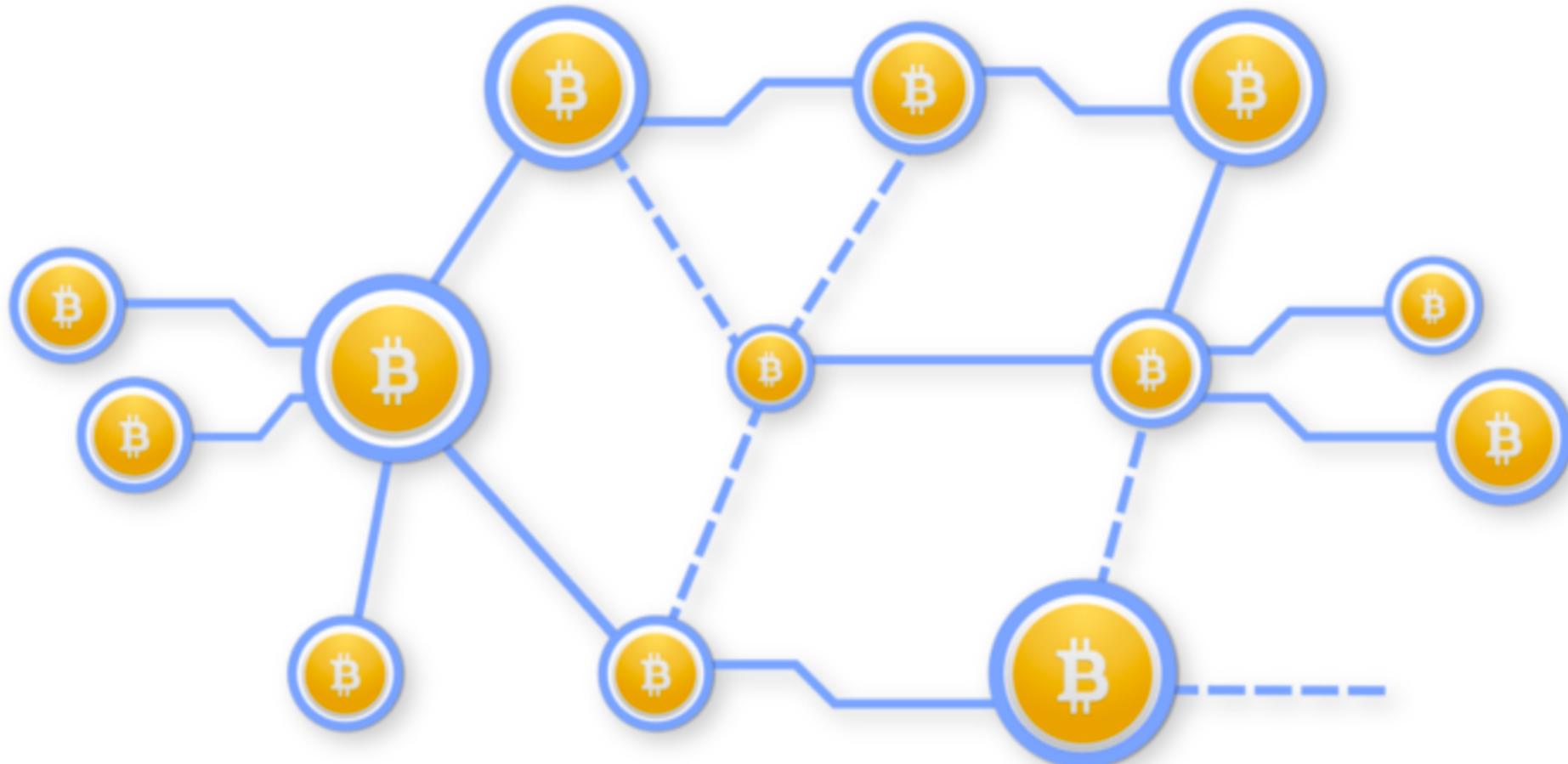
Structural Holes provide ego with access
to novel information, power, freedom

Networks: Economic Networks



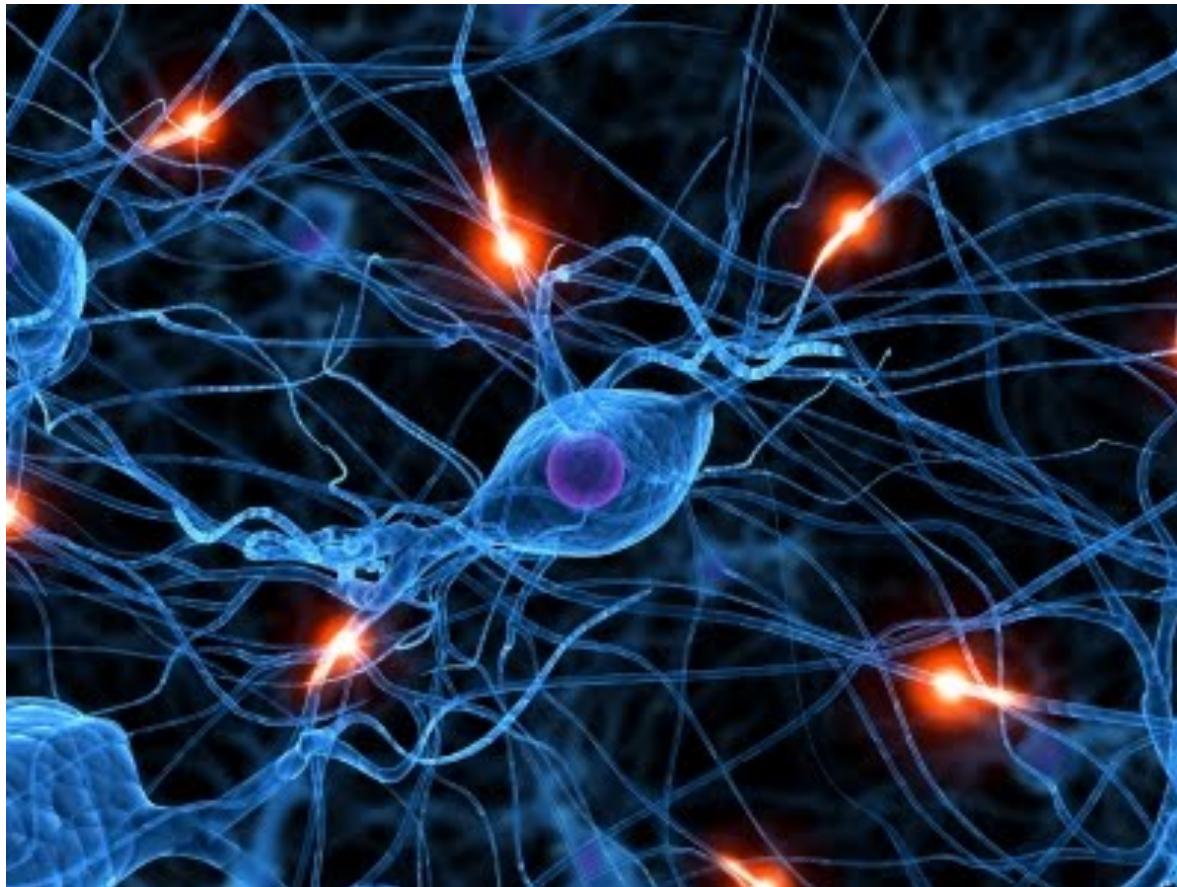
Bio-tech companies: Why companies succeed?
[Powell-White-Koput, 2002]

Networks: Transactions



- Detecting fraud and money laundering

Networks: Brain



**Human brain has between
10-100 billion neurons**
[Sporns, 2011]

Networks Really Matter

- If you want to understand the spread of diseases, **you need to figure out who will be in contact with whom**
- If you want to understand the structure of the Web, **you have to analyze the ‘links’.**
- If you want to understand dissemination of news or evolution of science, **you have to follow the flow.**

Reasoning about Networks

- What do we hope to achieve from studying networks?
 - Patterns and statistical **properties** of network data
 - **Design principles** and **models**
 - **Understand** why networks are organized the way they are
 - Predict behavior of networked systems

Reasoning about Networks

- **How do we reason about networks?**
 - **Empirical:** Study network data to find organizational principles
 - How do we measure and quantify networks?
 - **Mathematical models:** Graph theory and statistical models
 - Models allow us to understand behaviors and distinguish surprising from expected phenomena
 - **Algorithms** for analyzing graphs
 - Hard computational challenges

Networks: Structure & Process

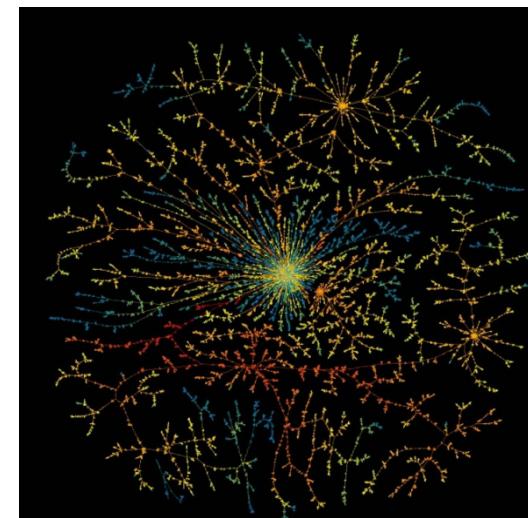
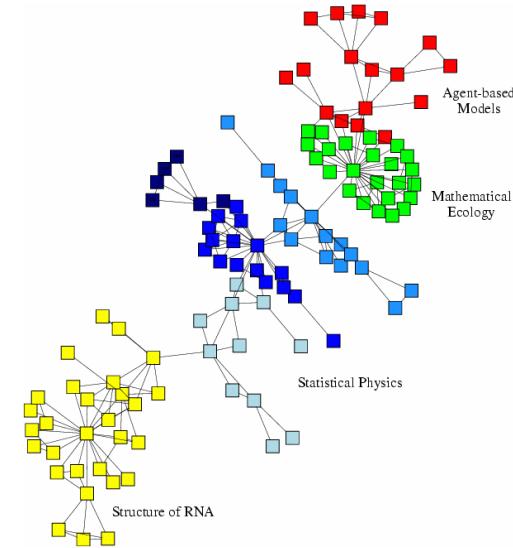
What do we study in networks?

■ Structure and evolution:

- What is the structure of a network?
- Why and how did it come to have such structure?

■ Processes and dynamics:

- Networks provide “skeleton” for spreading of information, behavior, diseases
- How do information and diseases spread?



How It All Fits Together

Properties

Small diameter,
Edge clustering

Scale-free

Strength of weak ties,
Core-periphery

Densification power law,
Shrinking diameters

Patterns of signed edge
creation

Information virality,
Memetracking

Models

Small-world model,
Erdős-Renyi model

Preferential attachment,
Copying model

Kronecker Graphs

Microscopic model of
evolving networks

Structural balance,
Theory of status

Independent cascade model,
Game theoretic model

Algorithms

Decentralized search

PageRank, Hubs and
authorities

Community detection:
Girvan-Newman, Modularity

Link prediction,
Supervised random walks

Models for predicting
edge signs

Influence maximization,
Outbreak detection, LIM

Prerequisites

- **No single topic in the course is too hard by itself**
- **But we will cover and touch upon many topics and this is what makes the course hard**

- **Good background in:**
 - Algorithms and graph theory
 - Probability and statistics
 - Linear algebra
- **Programming:**
 - You should be able to write non-trivial programs (in Python)

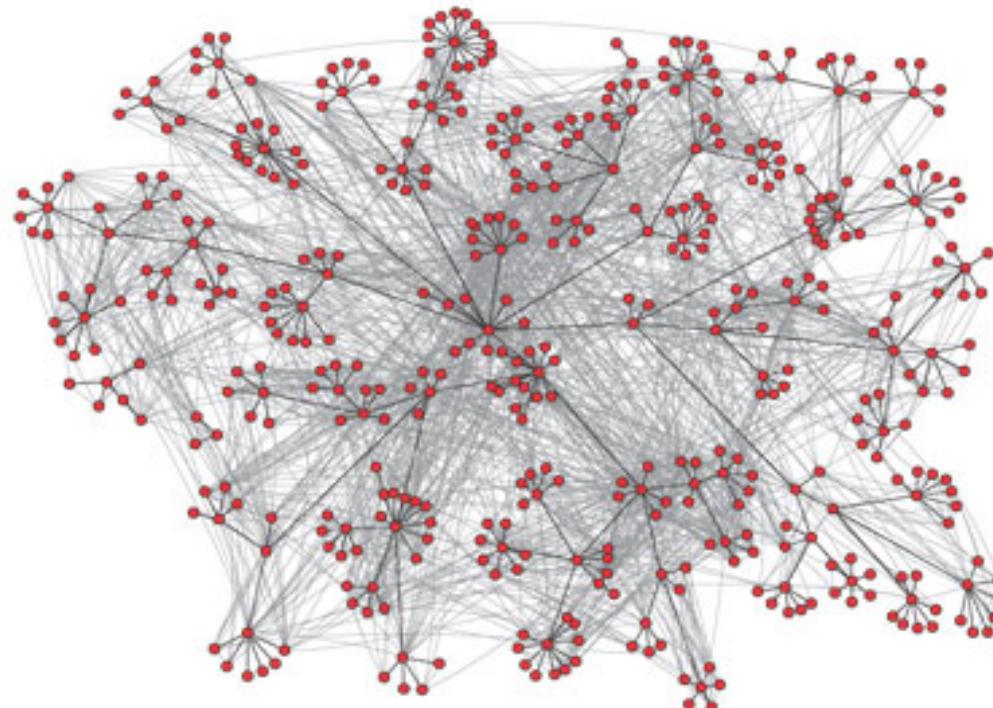
Network Analysis Tools

- **We highly recommend SNAP:**
 - **SNAP.PY:** Python ease of use, most of C++ scalability
 - HW1 asks you to do some very basic network analysis with `snap.py`
 - If you find HW1 difficult, this class is probably not for you
 - **SNAP C++:** more challenging but more scalable
 - Other tools include NetworkX, iGraph

Starter Topic:

Structure of Graphs

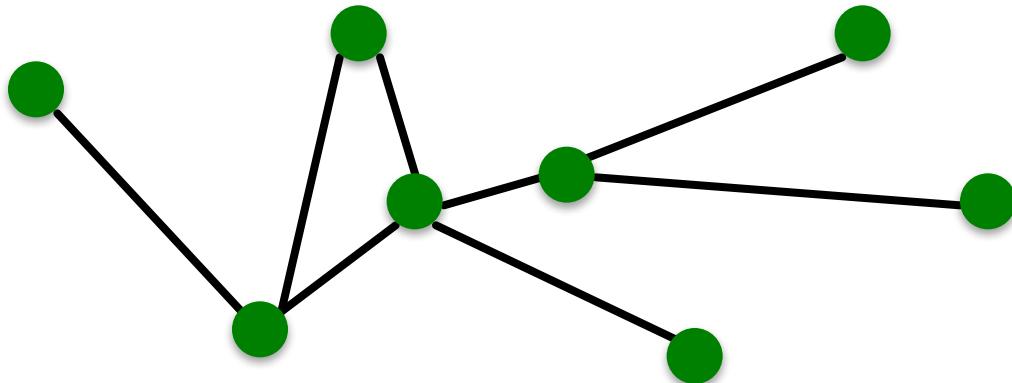
Structure of Networks?



A network is a collection of objects where some pairs of objects are connected by links

What is the structure of the network?

Components of a Network



- **Objects:** nodes, vertices N
- **Interactions:** links, edges E
- **System:** network, graph $G(N,E)$

Networks or Graphs?

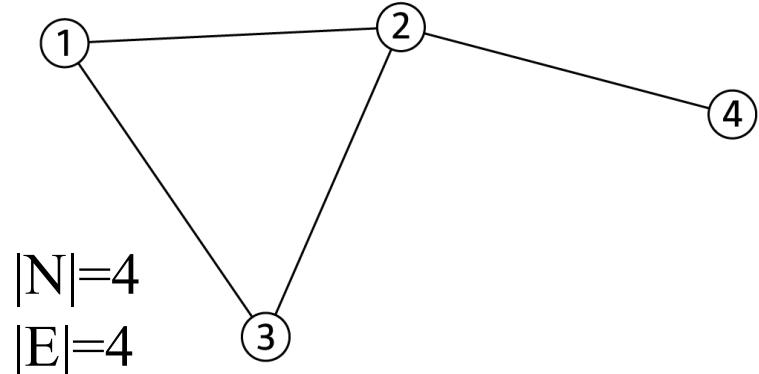
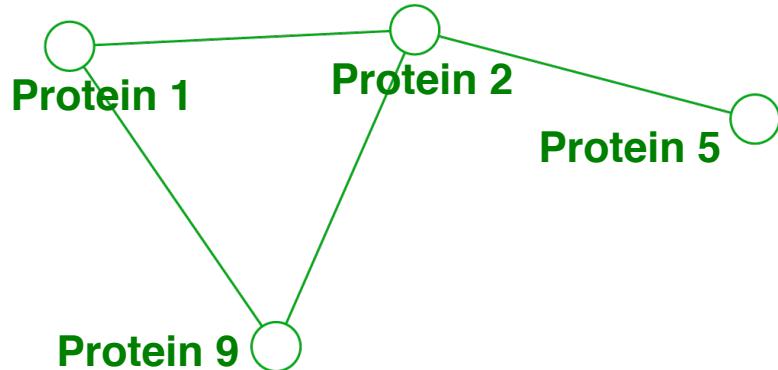
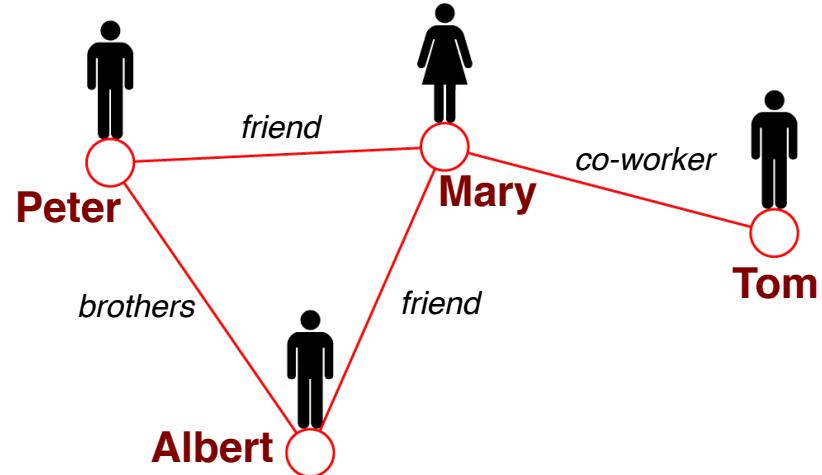
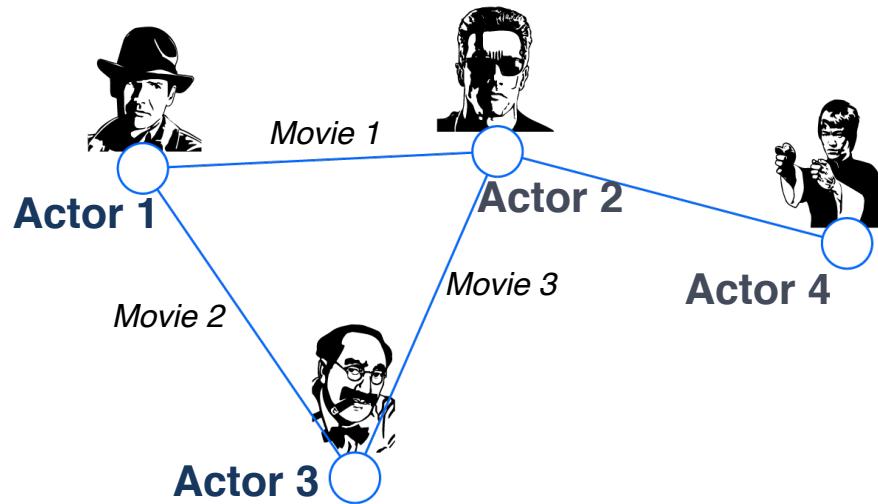
- **Network** often refers to real systems
 - Web, Social network, Metabolic network

Language: Network, node, link
- **Graph** is a mathematical representation of a network
 - Web graph, Social graph (a Facebook term)

Language: Graph, vertex, edge

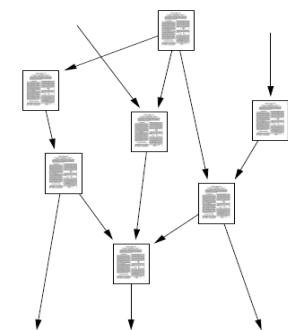
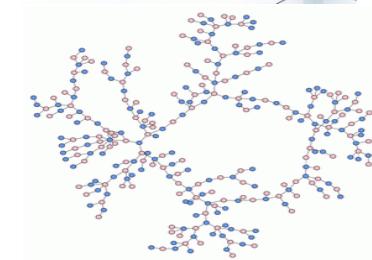
We will try to make this distinction whenever it is appropriate, but in most cases we will use the two terms interchangeably

Networks: Common Language



Choosing Proper Representations

- If you connect individuals that work with each other, you will explore a **professional network**
- If you connect those that have a sexual relationship, you will be exploring **sexual networks**
- If you connect scientific papers that cite each other, you will be studying the **citation network**
- **If you connect all papers with the same word in the title, you will be exploring what?** It is a network, nevertheless



How do you define a network?

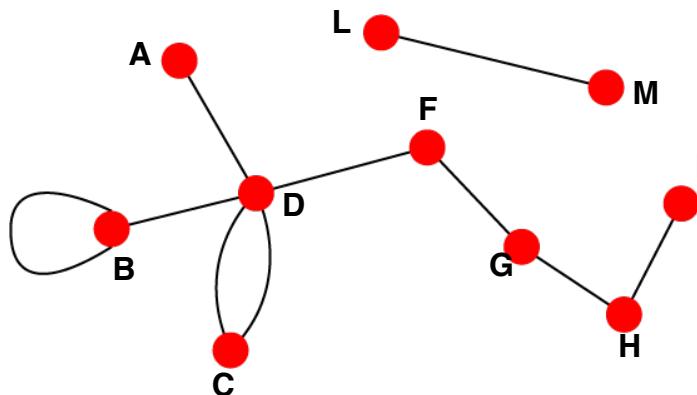
- **How to build a graph:**
 - What are nodes?
 - What are edges?
- **Choice of the proper network representation of a given domain/problem determines our ability to use networks successfully:**
 - In some cases there is a unique, unambiguous representation
 - In other cases, the representation is by no means unique
 - The way you assign links will determine the nature of the question you can study

Choice of Network Representation

Directed vs. Undirected Graphs

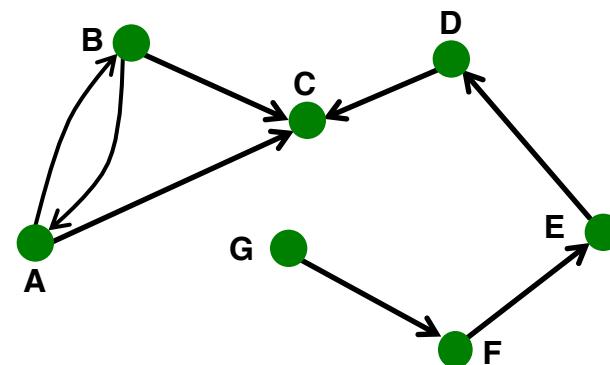
Undirected

- Links: undirected
(symmetrical, reciprocal)



Directed

- Links: directed
(arcs)



Examples:

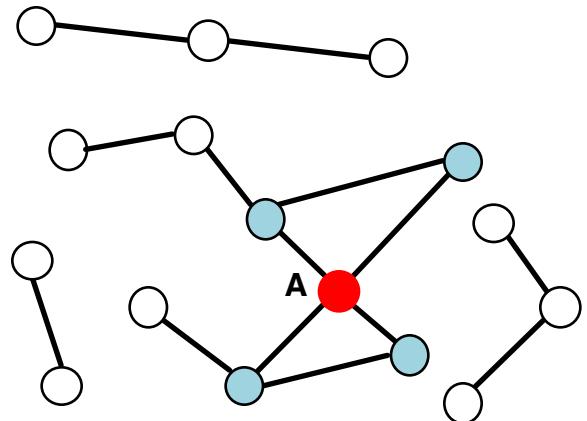
- Collaborations
- Friendship on Facebook

Examples:

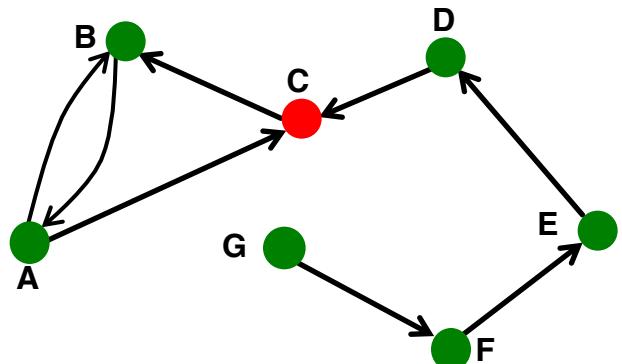
- Phone calls
- Following on Twitter

Node Degrees

Undirected



Directed



Source: Node with $k^{in} = 0$

Sink: Node with $k^{out} = 0$

Node degree, k_i : the number of edges adjacent to node i

$$k_A = 4$$

Avg. degree: $\bar{k} = \langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i = \frac{2E}{N}$

In directed networks we define an **in-degree** and **out-degree**. The (total) degree of a node is the sum of in- and out-degrees.

$$k_C^{in} = 2 \quad k_C^{out} = 1 \quad k_C = 3$$

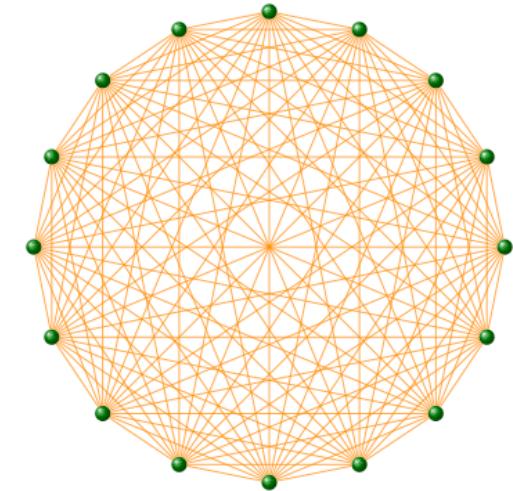
$$\bar{k} = \frac{E}{N}$$

$$\bar{k}^{in} = \bar{k}^{out}$$

Complete Graph

The **maximum number of edges** in an undirected graph on N nodes is

$$E_{\max} = \binom{N}{2} = \frac{N(N-1)}{2}$$



An undirected graph with the number of edges $E = E_{\max}$ is called a **complete graph**, and its average degree is $N-1$

Bipartite Graph

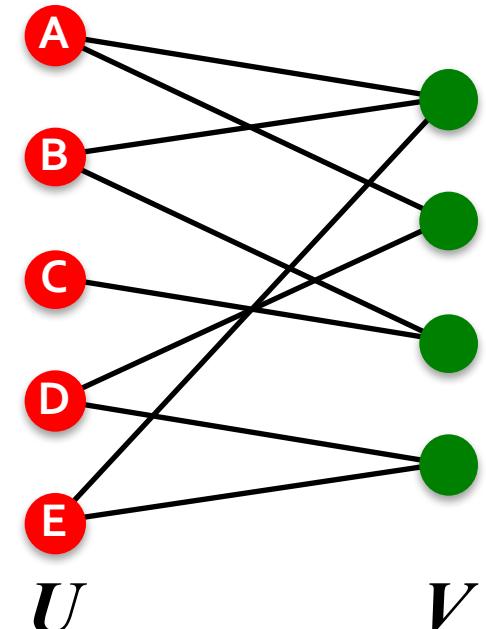
- **Bipartite graph** is a graph whose nodes can be divided into two disjoint sets U and V such that every link connects a node in U to one in V ; that is, U and V are **independent sets**

- **Examples:**

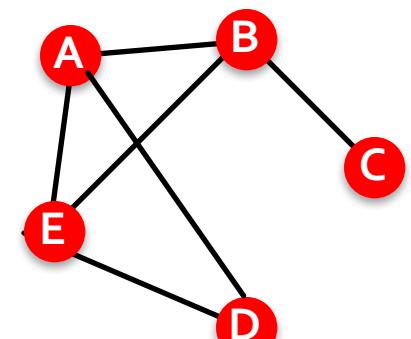
- Authors-to-papers (they authored)
- Actors-to-Movies (they appeared in)
- Users-to-Movies (they rated)

- **“Folded” networks:**

- Author collaboration networks
- Movie co-rating networks

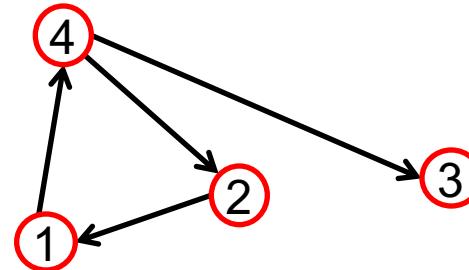
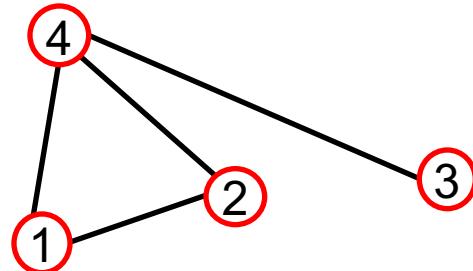


U V



Folded version of the graph above

Representing Graphs: Adjacency Matrix



$A_{ij} = 1$ if there is a link from node i to node j

$A_{ij} = 0$ otherwise

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

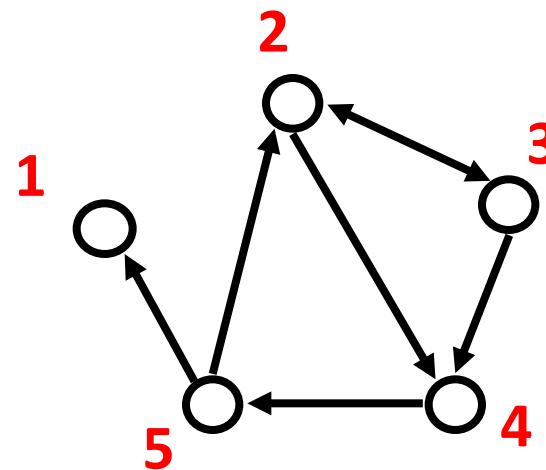
$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Note that for a directed graph (right) the matrix is not symmetric.

Representing Graphs: Edge list

- Represent graph as a set of edges:

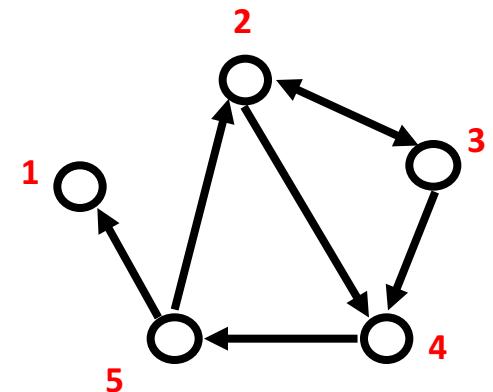
- (2, 3)
- (2, 4)
- (3, 2)
- (3, 4)
- (4, 5)
- (5, 2)
- (5, 1)



Representing Graphs: Adjacency list

■ Adjacency list:

- Easier to work with if network is
 - Large
 - Sparse
- Allows us to quickly retrieve all neighbors of a given node
 - 1:
 - 2: 3, 4
 - 3: 2, 4
 - 4: 5
 - 5: 1, 2



Networks are Sparse Graphs

Most real-world networks are **sparse**

$$E \ll E_{\max} \text{ (or } \bar{k} \ll N-1)$$

WWW (Stanford-Berkeley):	$N=319,717$	$\langle k \rangle = 9.65$
Social networks (LinkedIn):	$N=6,946,668$	$\langle k \rangle = 8.87$
Communication (MSN IM):	$N=242,720,596$	$\langle k \rangle = 11.1$
Coauthorships (DBLP):	$N=317,080$	$\langle k \rangle = 6.62$
Internet (AS-Skitter):	$N=1,719,037$	$\langle k \rangle = 14.91$
Roads (California):	$N=1,957,027$	$\langle k \rangle = 2.82$
Proteins (S. Cerevisiae):	$N=1,870$	$\langle k \rangle = 2.39$

(Source: Leskovec et al., Internet Mathematics, 2009)

Consequence: Adjacency matrix is filled with zeros!

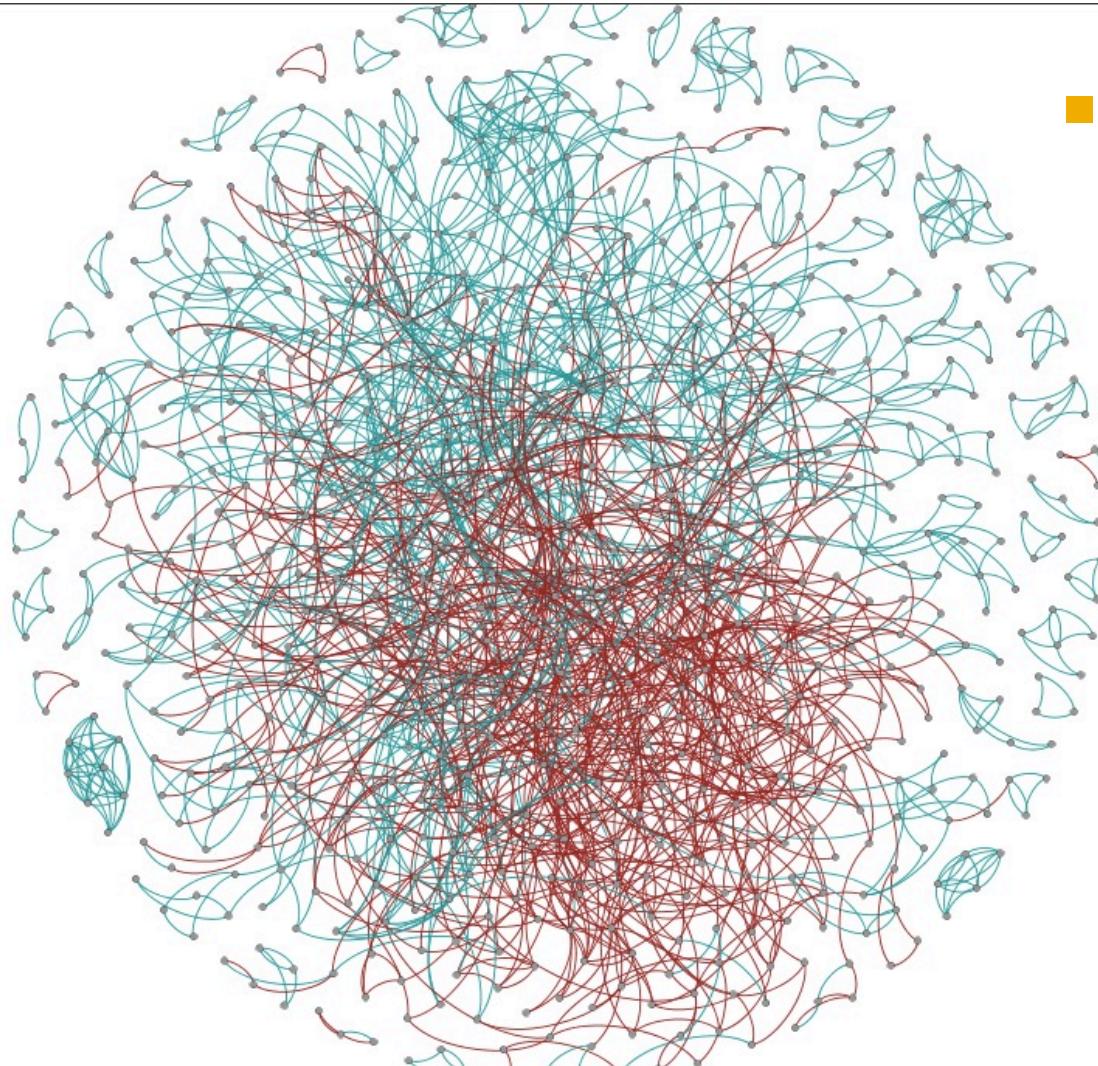
(Density of the matrix (E/N^2): WWW= 1.51×10^{-5} , MSN IM = 2.27×10^{-8})

Edge Attributes

Possible options:

- Weight (e.g. frequency of communication)
- Ranking (best friend, second best friend...)
- Type (friend, relative, co-worker)
- Sign: Friend vs. Foe, Trust vs. Distrust
- Properties depending on the structure of the rest of the graph: number of common friends

Positive and Negative Weights



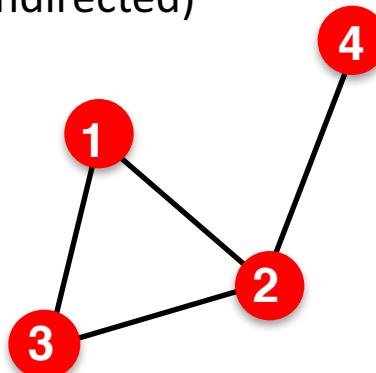
sample of positive & negative ratings from Epinions network

- One person trusting/distrusting another
 - Research challenge: How does one ‘propagate’ negative feelings in a social network? Is my enemy’s enemy my friend?

More Types of Graphs

■ Unweighted

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0$$

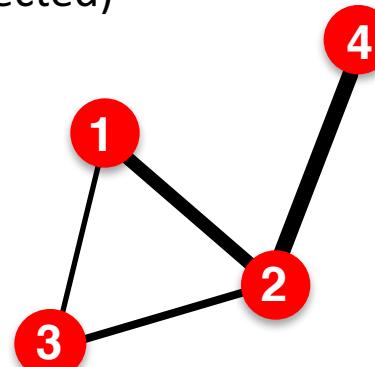
$$A_{ij} = A_{ji}$$

$$E = \frac{1}{2} \sum_{i,j=1}^N A_{ij} \quad \bar{k} = \frac{2E}{N}$$

Examples: Friendship, Hyperlink

■ Weighted

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0$$

$$A_{ij} = A_{ji}$$

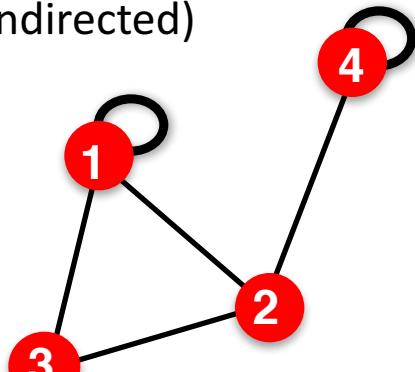
$$E = \frac{1}{2} \sum_{i,j=1}^N \text{nonzero}(A_{ij}) \quad \bar{k} = \frac{2E}{N}$$

Examples: Collaboration, Internet, Roads

More Types of Graphs

■ Self-edges (self-loops)

(undirected)



$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$A_{ii} \neq 0$$

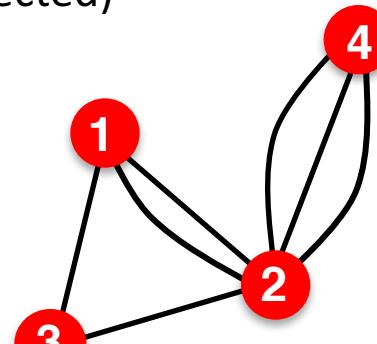
$$A_{ij} = A_{ji}$$

$$E = \frac{1}{2} \sum_{i,j=1, i \neq j}^N A_{ij} + \sum_{i=1}^N A_{ii}$$

Examples: Proteins, Hyperlinks

■ Multigraph

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0$$

$$E = \frac{1}{2} \sum_{i,j=1}^N \text{nonzero}(A_{ij})$$

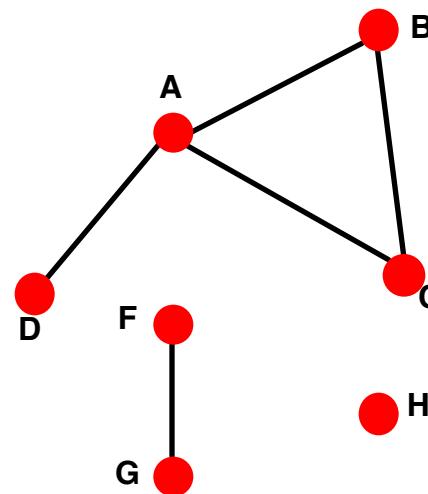
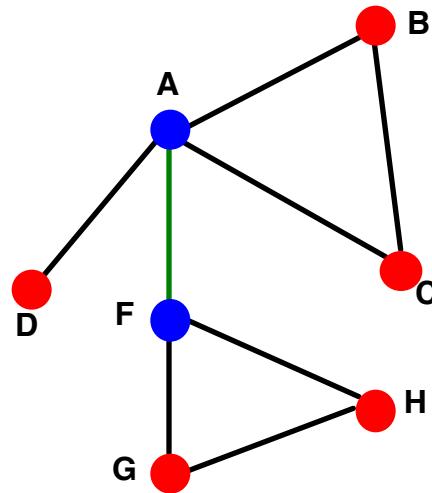
$$A_{ij} = A_{ji}$$

$$\bar{k} = \frac{2E}{N}$$

Examples: Communication, Collaboration

Connectivity of Undirected Graphs

- **Connected (undirected) graph:**
 - Any two vertices can be joined by a path
- A disconnected graph is made up by two or more connected components



Largest Component:
Giant Component

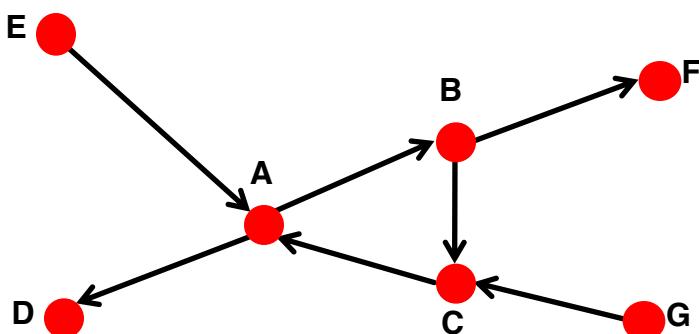
Isolated node (node H)

Bridge edge: If we erase the **edge**, the graph becomes disconnected.

Articulation node: If we erase the **node**, the graph becomes disconnected

Connectivity of Directed Graphs

- **Strongly connected directed graph**
 - has a path from each node to every other node and vice versa (e.g., A-B path and B-A path)
- **Weakly connected directed graph**
 - is connected if we disregard the edge directions



Graph on the left is connected but not strongly connected (e.g., there is no way to get from F to G by following the edge directions).

Network Representations

Email network >> directed multigraph with self-edges

Facebook friendships >> undirected, unweighted

Citation networks >> unweighted, directed, acyclic

Collaboration networks >> undirected multigraph or weighted graph

Mobile phone calls >> directed, (weighted?) multigraph

Protein Interactions >> undirected, unweighted with self-interactions