One Sample t Test & Two Sample t Test

The t-Statistic

- We previously, used the z-statistic in testing hypotheses about μ.
- However, the z-test requires knowing σ.
- But, we can estimate the population standard deviation using n–1.

$$s = \sqrt{\frac{SS}{n-1}}$$

- We can then use this estimate in the formula for the standard error.
- Instead of using

$$\sigma_M = \frac{\sigma}{\sqrt{n}} \quad \text{,we use} \quad s_M = \frac{s}{\sqrt{n}}$$

This is the formula for the t-statistic:

$$t_{obt} = \frac{M - \mu}{s_M}$$

<u>z-Score Formula</u>

$$z = \frac{X - \mu}{\sigma}$$

Notice that the only difference from the z-score formula is the use of the estimated standard error in the denominator in place of the actual standard error.

The t-Distribution

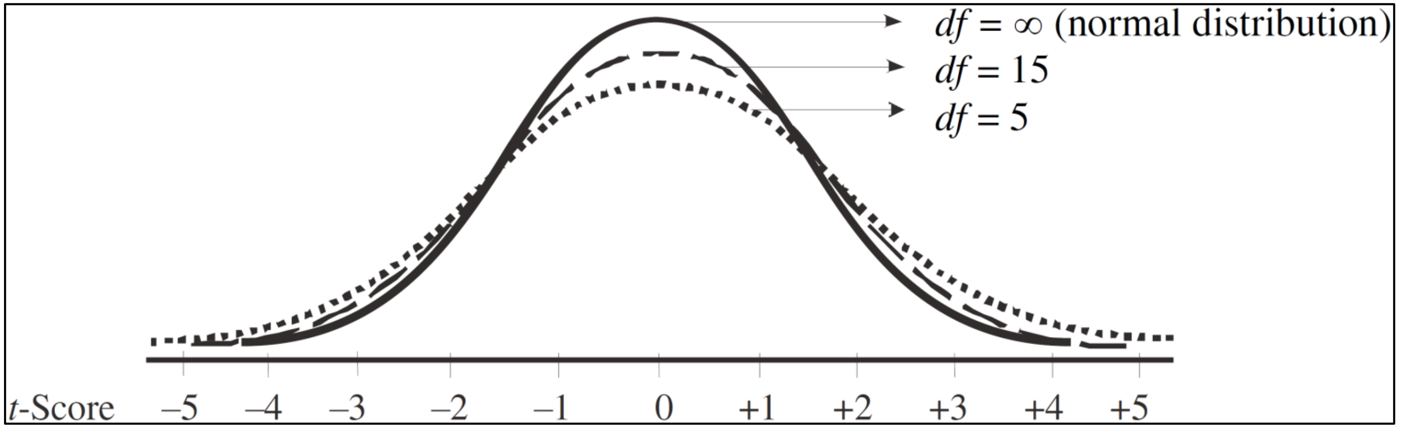
- The **t-distribution** is theoretical, symmetrical and bell-shaped.
- However, with small samples the curve is shaped differently.
- Is actually a family of curves, one for each sample size.
- The particular t-distribution that we use will be based on the degree of freedom associated with the sample.

When using the t-distribution for a single sample, df will also be n-1.

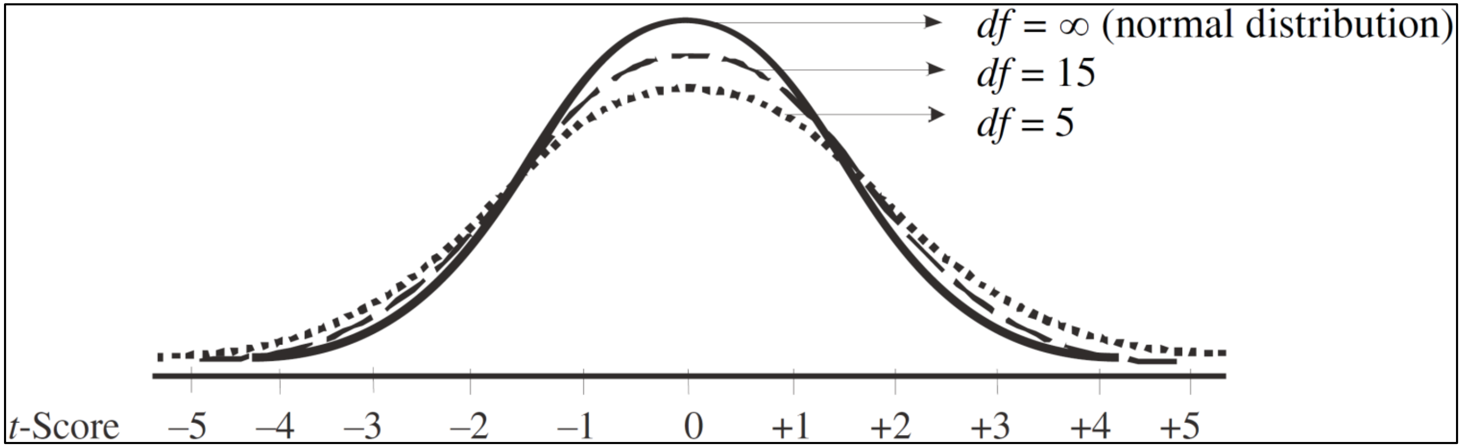
For example, if your sample size is 16, df will be 15.

Shape of the t-distribution

Illustration for 3 different df $\rightarrow$ 5, 15, and infinity



- The curve is flatter with smaller df and the t-values extend further into the tails.
- As df increase, the t-distribution looks more and more like the normal distribution.
- When df are infinite, there is no difference in shape between the t-curve and the normal distribution curve.



Since smaller samples result in a distribution that is flatter and more spread out, larger t-values will be required to reject H_0 (i.e., we have to go further into the tails to reach significance).

The t-Distribution Table

Level of significance for one-tailed test						
	.10	.05	.025	.01	.005	.0005
Level of significance for two-tailed test						
df	.20	.10	.05	.02	.01	.001
1	3.708	6.314	12.706	31.821	63.657	636.619
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460

- If the df for your particular research problem are not shown, use the critical values associated with the *next lowest* df.
- For a t test, it is not necessary to divide the proportion associated with the alpha level in half for two-tailed tests. The t-table already accommodates for this.

One Sample t-Test

- Is a test of a hypothesis about a population mean (μ) when the population standard deviation (σ) is not known.
- Use when you want to know
  - if a sample is representative of a population, and
  - if a particular treatment or condition has a significant effect.
- We use the same four-step procedure that we used for the z-test, beginning with a research problem.

For Example (Two-tailed),

Research Question: The population mean on a standardized test of critical thinking is $\mu = 53$. A group of faculty members at a small community college underwent a 10-week training program to learn techniques designed to help students develop their critical thinking skills. After the training, the new techniques were implemented in the classrooms. The mean critical thinking score for a sample of $n = 87$ students exposed to the new techniques was $M = 55$ with $SS = 6013$. Do the results suggest that the training program had a significant effect? Use a two-tailed test and $\alpha = .05$.

Step 1: Formulate Hypotheses

H_0: $\mu = 53$
H_1: $\mu \neq 53$

Step 2: Indicate the Alpha Level and Determine Critical Values:

$\alpha = .05$
df = 86
$t_{crit} = \pm 2.000$

The actual df is 86 (i.e., n – 1); however, this value is not shown in the chart. Thus, we will use the next lowest df which is 60.

Step 3: Calculate Relevant Statistics

The population standard deviation will have to be estimated.

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{6013}{87-1}} = 8.36$$

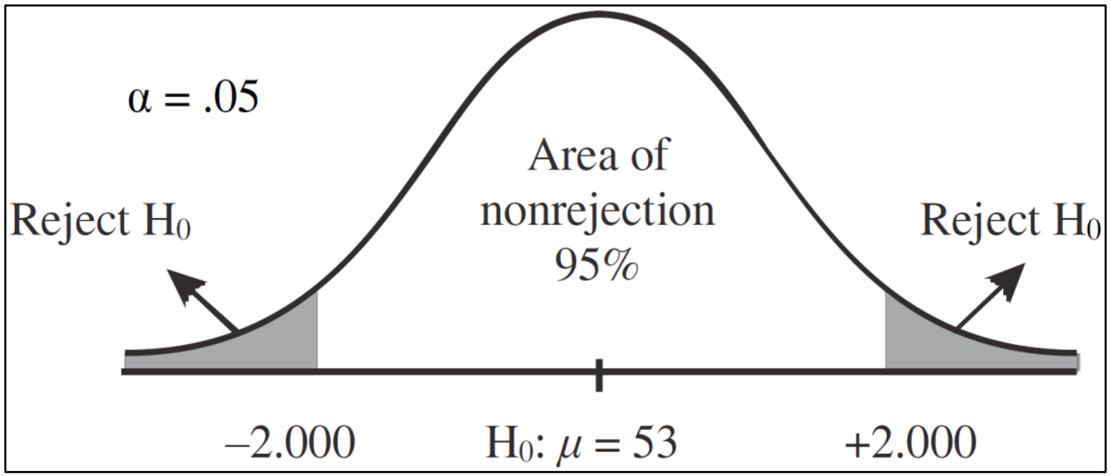
Before we can determine our obtained t-value, we first need to calculate the standard error using the estimated standard deviation above:

$$s_M = \frac{s}{\sqrt{n}} = \frac{8.36}{\sqrt{87}} = .90$$

Finally, we can calculate the t-statistic:

$$t_{obt} = \frac{M-\mu}{s_M} = \frac{55-53}{.90} = +2.22$$

Step 4: Make a Decision and Report the Results:



Students taught by the faculty who participated in the training program scored significantly higher on the critical thinking assessment than the general population. Reject H_0, $t(86) = +2.22$, $p < .05$.

Summary of Problem

Step 1: Formulate Hypotheses:

H_0: $\mu = 53$
H_1: $\mu \neq 53$

Step 2: Indicate the Alpha Level and Determine Critical Values:

$\alpha = .05$
df = 86
$t_{crit} = \pm 2.000$

Step 3: Calculate Relevant Statistics:

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{6013}{87-1}} = 8.36$$

$$s_M = \frac{s}{\sqrt{n}} = \frac{8.36}{\sqrt{87}} = .90$$

$$t_{obt} = \frac{M-\mu}{s_M} = \frac{55-53}{.90} = +2.22$$

Step 4: Make a Decision and Report the Results:

Students who were taught by faculty who participated in the training program scored significantly higher on the critical thinking assessment than the general population. Reject H_0, $t(86) = +2.22$, $p < .05$.

Effect Size of a One Sample t-Test

How significant were our results?

Cohen's d will change slightly.

For our current problem,

$$d = \frac{|M - \mu|}{s}$$

$$d = \frac{|55 - 53|}{8.36} = .24$$

Small effect size.

Another Example (One-tailed Test)

A well-known sandwich chain puts 9 grams of protein on its sandwiches. A customer complained to the home office that a particular outlet was putting “hardly any meat” on its sandwiches. A random sample of $n = 16$ sandwiches from the sandwich shop in question were weighed. The results showed a $M = 7.9$ with $s = 2.1$. Did the shop put significantly less protein on its sandwiches? Test at $\alpha = .05$, one-tailed.

Step 1: Formulate Hypotheses

$H_0: \mu \geq 9$

$H_1: \mu < 9$

Step 2: Indicate the Alpha Level and Determine Critical Values

$\alpha = .05$

$df = 15$

$t_{crit} = -2.131$

Step 3: Calculate Relevant Statistics

$$s_M = \frac{s}{\sqrt{n}} = \frac{2.1}{\sqrt{16}} = .53$$

$$t_{\text{obt}} = \frac{M - \mu}{s_M} = \frac{7.9 - 9}{.53} = -2.08$$

Step 4: Make a Decision and Report the Results

The sandwich shop did not put a significantly less amount of protein on its sandwiches. Fail to reject H_0, $t(15) = –2.08$, $p > .05$.

Assumptions

The same assumptions that apply to the z-test also apply to the t-test, with one exception. These assumptions include:

- Independent and random selection of subjects.
- The dependent variable is normally distributed in the population of interest.
- The dependent variable can be measured on an interval or ratio scale.

The z-test assumption that does not pertain to the t test is that the population standard deviation (σ) be known. Since we are estimating the population standard deviation for the t test, this requirement does not apply.

Two-Sample t Test: Independent Samples Design (Comparing Two Population Mean)

Two Sample t-Test: Independent Samples Design

- One-sample t-test → use M to test a hypothesis about μ.
- Sometimes we want to compare the means for two groups.
- In such cases, **two sample t-tests** can be used instead.
- Here, we will look at independent samples design.

The **independent samples design** (between-subjects design) involves two separate and unrelated samples for which each subject provides one score.

> A new drug for anxiety is being tested. Two samples are drawn from a population of people who suffer from generalized anxiety. Sample 1 receives the anxiety drug and Sample 2 receives a placebo. Is there enough difference between the means of the two samples to conclude significance?

With the one-sample t-test, we compared a sample mean with a population mean to see if they differed significantly ($M - \mu$). With the independent samples t-test, we will be comparing two sample means ($M_1 – M_2$) to see if they differ significantly.

Sampling Distribution of the Difference Between Means

- As always, we will compare our obtained values to a theoretical sampling distribution → the **sampling distribution of the difference between means**.
- Imagine some large population from which you draw two samples of size n_1 and size n_2. You measure each sample on some aspect of behavior and calculate a mean for each sample. You then determine the difference between those means ($M_1 – M_2$) and record it.
- You then repeat this process over and over, drawing all possible random samples of size n_1 and n_2 and recording only the difference between their means.
- These “difference between mean” values are arranged into a frequency distribution, called the sampling distribution of the difference between means.

Of course, this is not done in actuality, only in theory.

Calculations

Calculations will include a standard error of difference between means, a t test, and degrees of freedom.

The standard deviation of the sampling distribution of the difference between means is called the **standard error of difference between means.**

$$s_{M_1-M_2} = \sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

- Without σ, we can't calculate $\sigma_{M_1 - M_1}$ directly.
- But, we can estimate it using $s_{M_1 - M_1}$.
- Remember that a standard deviation is the square root of the variance. In this case, the two variances are averaged together. The pooled variance is in the parentheses on the left under the square root.
- The value that this formula produces is the estimated amount of sampling error that can be expected due to chance alone.

Formula for Independent Samples t Test

$$t_{obt} = \frac{(M_1 - M_2) - (\mu_1 - \mu_2)}{s_{M_1 - M_2}}$$

- But, the null hypothesis will specify no difference between means, $(\mu_1 - \mu_2)$ will be 0.
- Thus, we can simplify the formula as follows:

$$t_{obt} = \frac{M_1 - M_2}{s_{M_1 - M_2}}$$

- If a two-tailed test is used, t_{obt} may be either positive or negative.
- If a one-tailed test is used, the alternative hypothesis (H_1) will specify one of the means to be higher or lower than the other.
  - H_0 can be rejected only if the obtained difference between means is in the direction specified by H_1.

<u>Degrees of freedom for independent samples design</u>

$$df = n_1 + n_2 - 2$$

Hypothesis Testing

As we have learned, when means differ there are two opposing explanations for the difference observed:

- The *null hypothesis* (H_0) attributes the difference to chance, or random sampling error.
- The *alternative hypothesis* (H_1) asserts that differences between means are more than chance differences and that treatment *was* effective.

For a <u>two-tailed test</u>,

- H_0 is written as:
  - H_0: $\mu_1 - \mu_2 = 0$ (no significant difference between means), or
  - H_0: $\mu_1 = \mu_2$ (the two means are equal)
- H_1 is written as:
  - H_1: $\mu_1 \neq \mu_2$ (the two means are *not* equal)

For a <u>one-tailed test</u>, the two hypotheses will be written as:

- H_0: $\mu_1 \leq \mu_2$
  H_1: $\mu_1 > \mu_2$

Or

- H_0: $\mu_1 \geq \mu_2$
  H_1: $\mu_1 < \mu_2$

Rejecting H_0.

- For *non-directional* alternative hypothesis (two-tailed test), when a mean difference falls in the critical region in either tail of the sampling distribution, H_0 can be rejected.

- For *directional* alternative hypothesis (one-tailed test), to reject H_0 the difference value has to fall in the critical region in the direction specified in H_1.

For Example,

Research Question: A political science teacher wonders whether there is a difference in knowledge of current events between students who read newspapers and those who watch the news on television. He randomly assigns students to one of two groups. Group 1 is instructed to only read the daily newspapers for next 30 days (either online or hard copies), whereas Group 2 is instructed to only watch the nightly news. Both groups are thereafter given a test to assess their knowledge of current events. The following scores are obtained. Conduct a non-directional t test with $\alpha = .05$.

Newspaper	Television News
59	36
48	42
45	50
39	37
52	51
56	32
50	47
41	38
46	40
45	31
48	

Remember from chapter 4 that *SS* refers to sum of squares. It is the numerator in the formula for the variance. In some problems, *SS* will be provided. If it is not given, however, it will have to be calculated for each sample.

$$SS = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$

Step 1: Formulate Hypotheses:

H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

Step 2: Indicate the Alpha Level and Determine Critical Values:

$\alpha = .05$
$df = n_1 + n_2 - 2$
$= 11 + 10 - 2$
$= 19$
$t_{crit} = \pm 2.093$

X_1	X_1^2	X_2	X_2^2
59	3481	36	1296
48	2304	42	1764
45	2025	50	2500
39	1521	37	1369
52	2704	51	2601
56	3136	32	1024
50	2500	47	2209
41	1681	38	1444
46	2116	40	1600
45	2025	31	961
48	2304		
$\Sigma X_1 = 529$	$\Sigma X_1^2 = 25{,}797$	$\Sigma X_2 = 404$	$\Sigma X_2^2 = 16{,}768$

$n_1 = 11$
$M_1 = 48.09$

$n_2 = 10$
$M_2 =$
40.4

Calculate the sum of squares for each sample:

$$SS = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$

$$SS_1 = 25{,}797 - \frac{(529)^2}{11}$$
$$= 356.91$$

$$SS_2 = 16{,}768 - \frac{(404)^2}{10}$$
$$= 446.4$$

Calculate the standard error of difference:

$$s_{M_1-M_2} = \sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$
$$= \sqrt{\left(\frac{356.91+446.4}{11+10-2}\right)\left(\frac{1}{11} + \frac{1}{10}\right)}$$
$$= 2.83$$

Calculate the t-statistic:

$$t_{obt} = \frac{M_1 - M_2}{s_{M_1-M_2}}$$
$$= \frac{48.09-40.4}{2.83}$$
$$= 2.72$$

Step 4: Make a Decision and Report the Results:

Students who read the newspaper scored significantly higher on knowledge of current events than students who watched the evening news. Reject H_0, $t(19) = +2.72$, $p < .05$.

Look at the means for each sample to determine which group scored higher.

Effect Size

For an independent samples t-test, the formula for Cohen's d is as follows:

$$d = \frac{|M_1 - M_2|}{\sqrt{\frac{SS_1 + SS_2}{n_1 + n_2 - 2}}}$$

<u>For our problem,</u>

$$d = \frac{|48.09 - 40.4|}{\sqrt{\frac{356.91 + 446.4}{11 + 10 - 2}}}$$
$$= 1.18$$

Using Cohen's guidelines a *d* value of 1.18 suggests a large effect size.

Assumptions

Assumptions of the independent samples t test include:

- Independent and random selection of subjects.
- The dependent variable can be measured on an interval or ratio scale.
- The dependent variable is normally distributed in the populations of interest.
- Homogeneity of variance of the dependent variable in the populations of interest.

H**omogeneity of variance** - population variances of the dependent variable are approximately equal (i.e., $\sigma_1^2 \approx \sigma_2^2$). This assumption is necessary because the two sample variances in the standard error portion of the t-statistic were averaged together. If the variances of the population were drastically different, it would not make sense to average together our sample variances.