The Normal Distribution

History of the Normal Distribution

- Binomial distribution

  Useful to answer questions like

  If a fair coin is flipped 100 times, what is the probability of getting 60 or more heads?

Probability with Binomial Distribution

$$P(x) = \frac{N!}{x!(N-x)!}\pi^{x}(1-\pi)^{N-x}$$

$$P(60) + P(61) + P(62) + \ldots$$

Probability with Binomial Distribution

$$P(x) = \frac{N!}{x!(N-x)!}\pi^{x}(1-\pi)^{N-x}$$

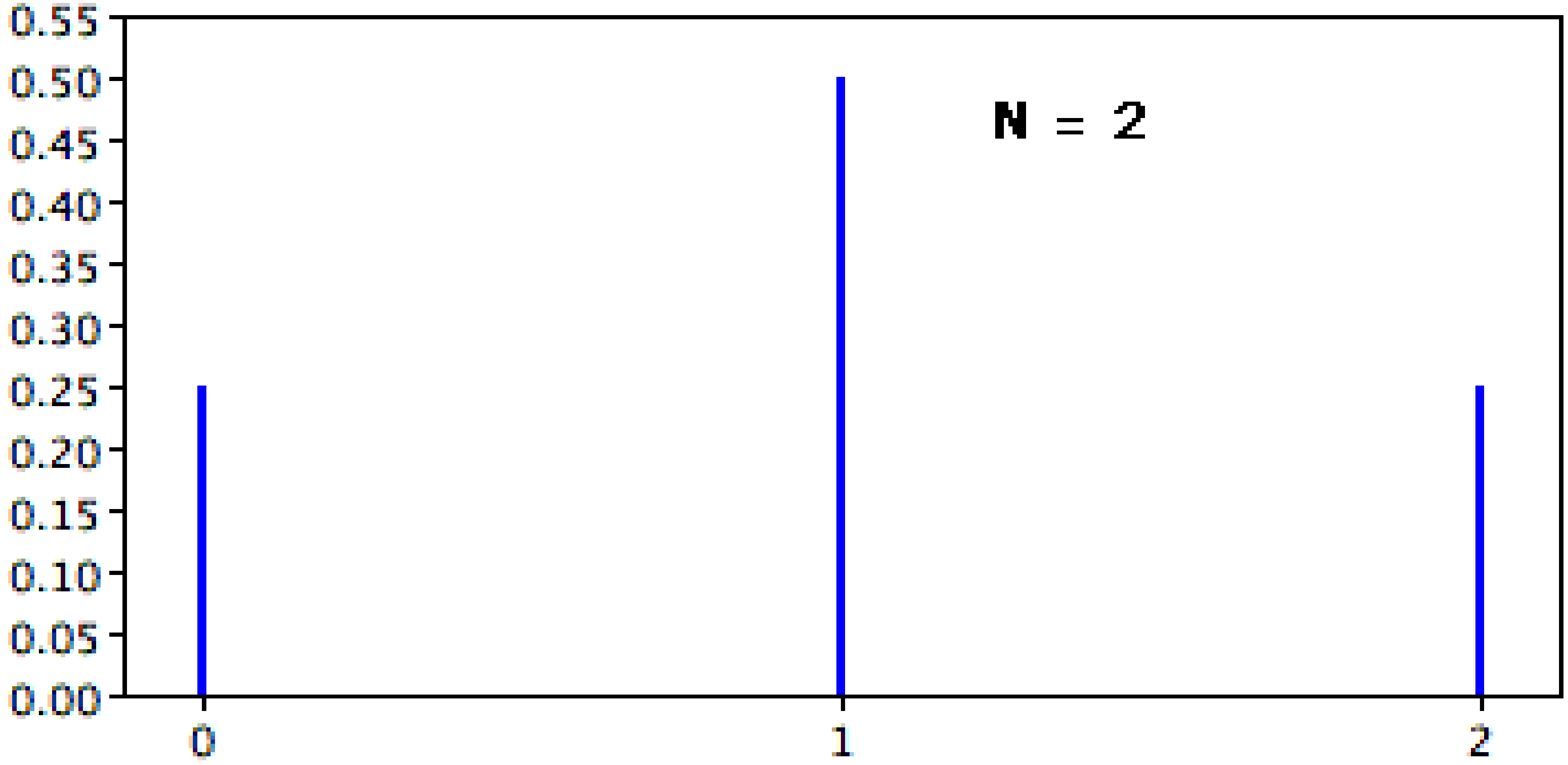
$$P(60) + P(61) + P(62) + \ldots$$

Whew!

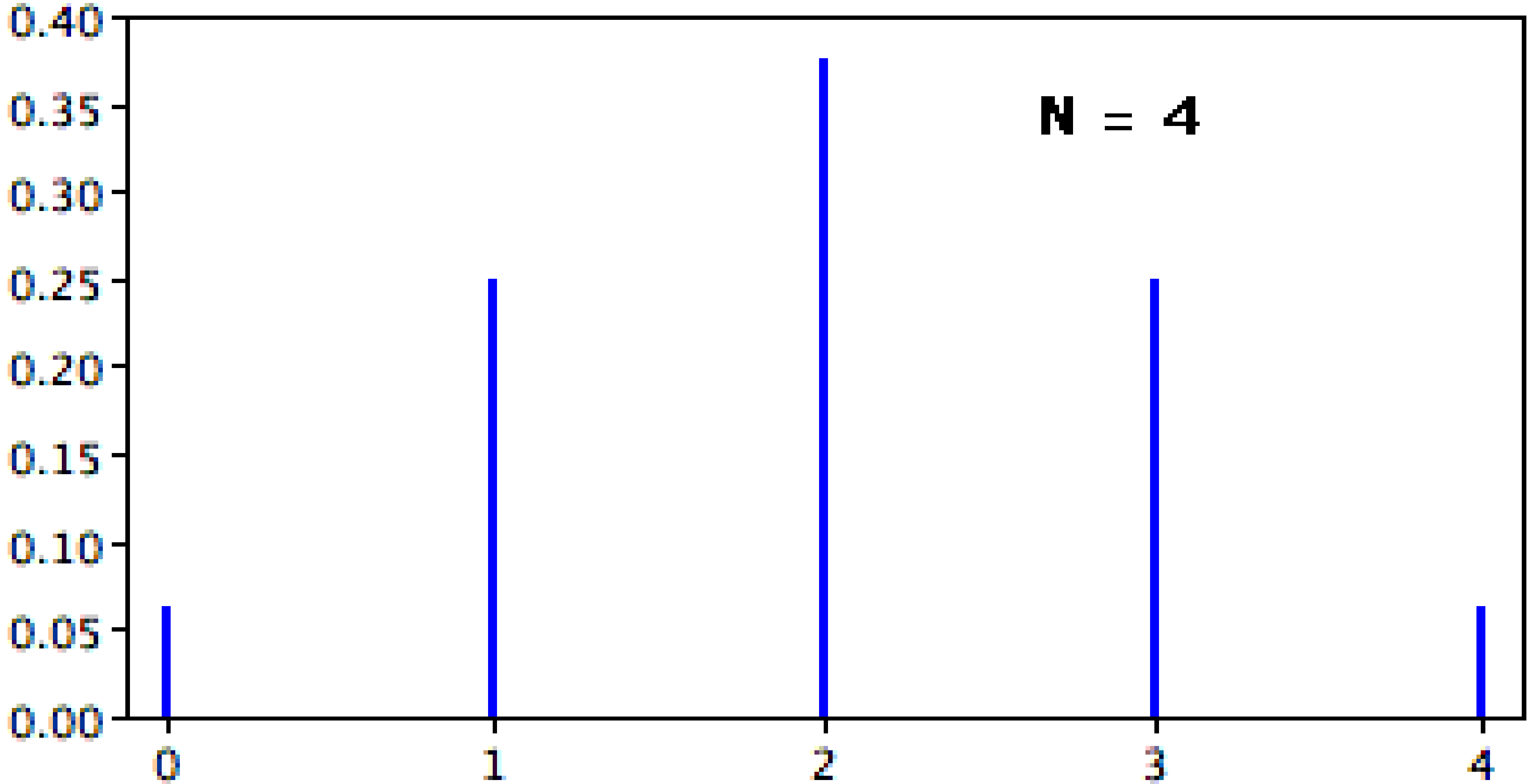
Important to Gamblers

- Abraham de Moivre
  - Consultant to gamblers
  - Noticed that as the number of events increased, the distribution approached a smooth curve.

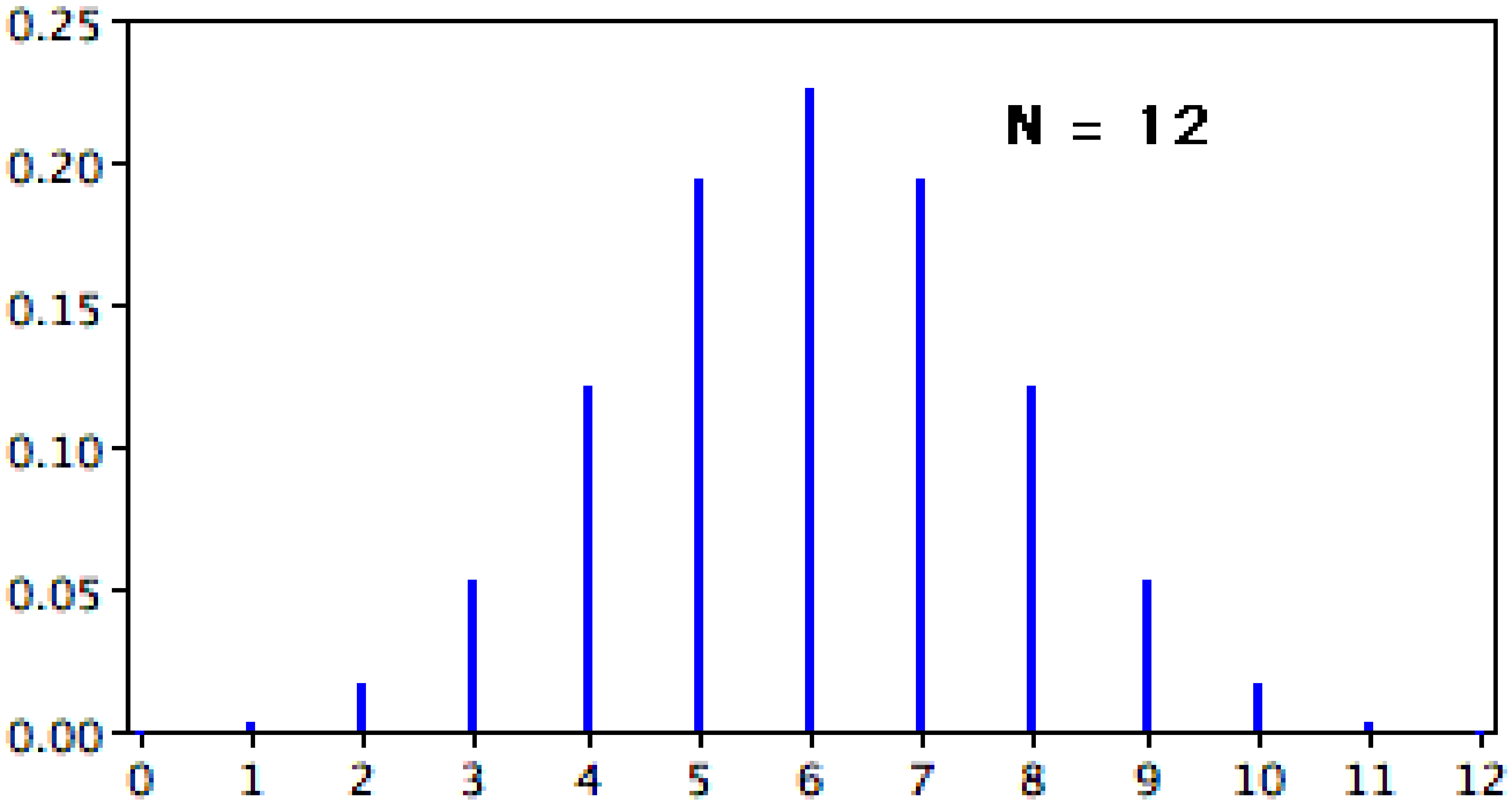
Binomial Distribution



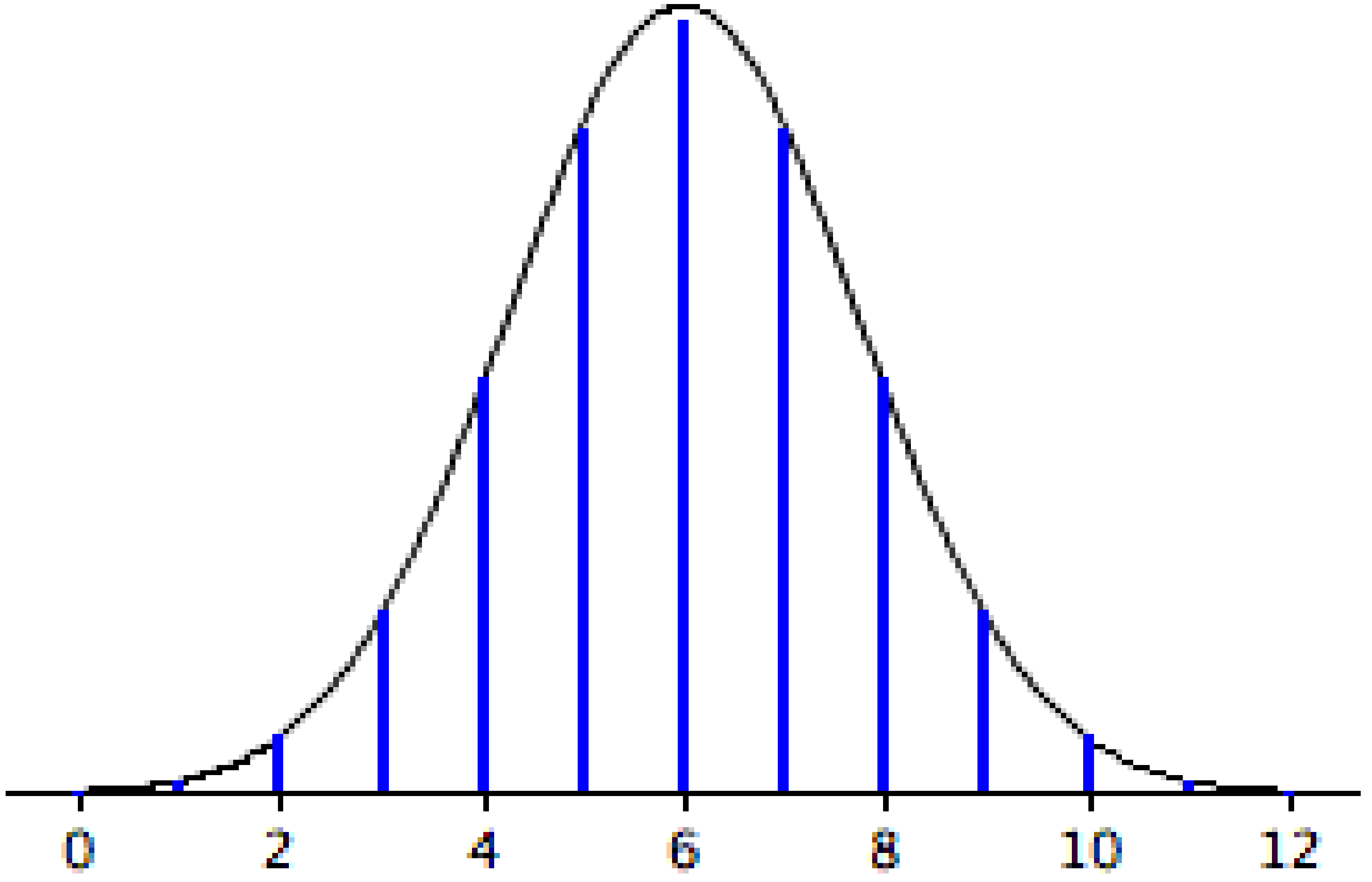
Binomial Distribution



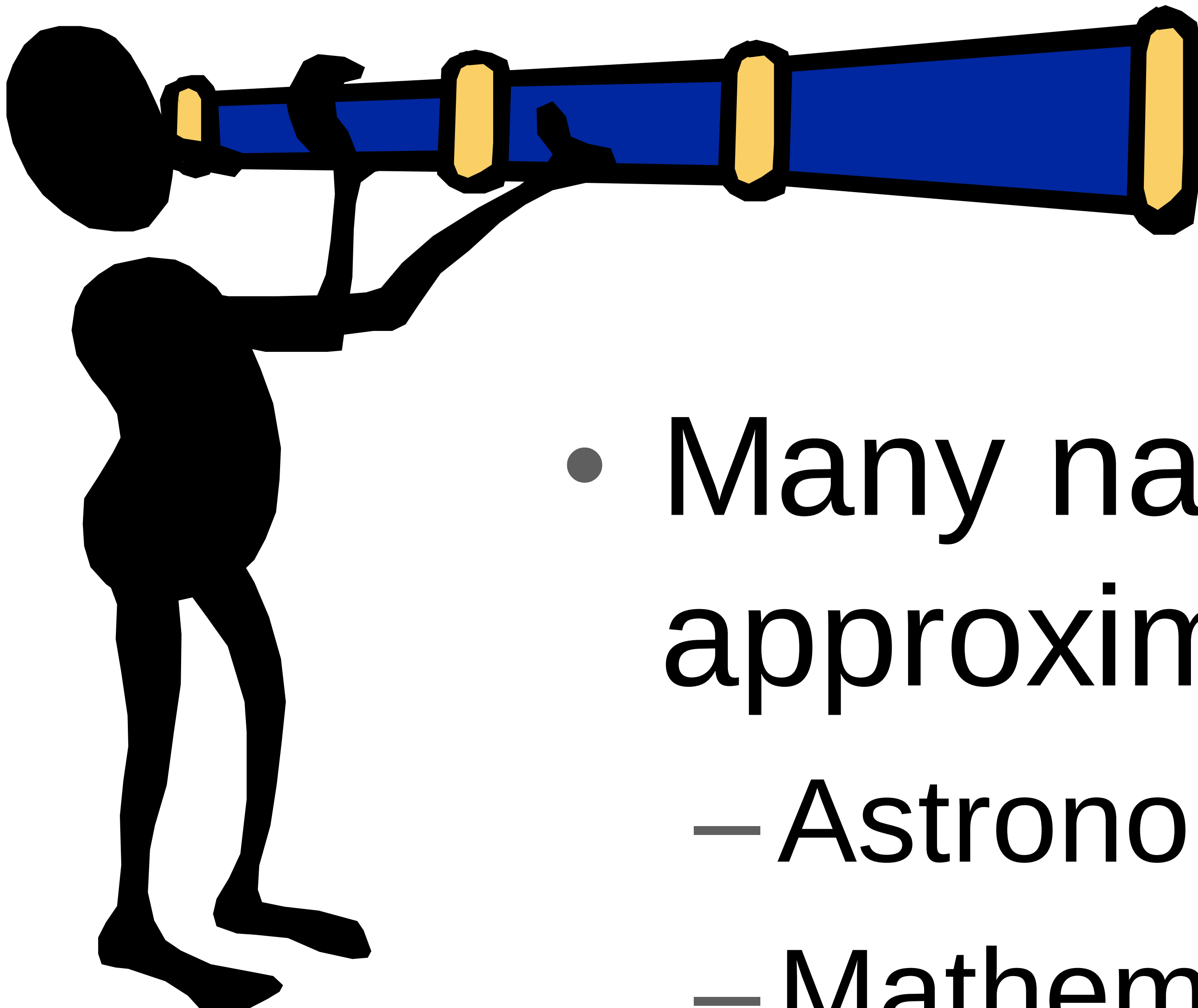
Binomial Distribution



Discovery of the Normal Curve



Importance of the Normal Curve



- Many natural phenomena are approximately normal
  - Astronomical observations
  - Mathematics

Central Limit Theorem

- Laplace discovered the same distribution in 1778.
  - Derived the “Central Limit Theorem”
  - Distribution of means is approximately normal

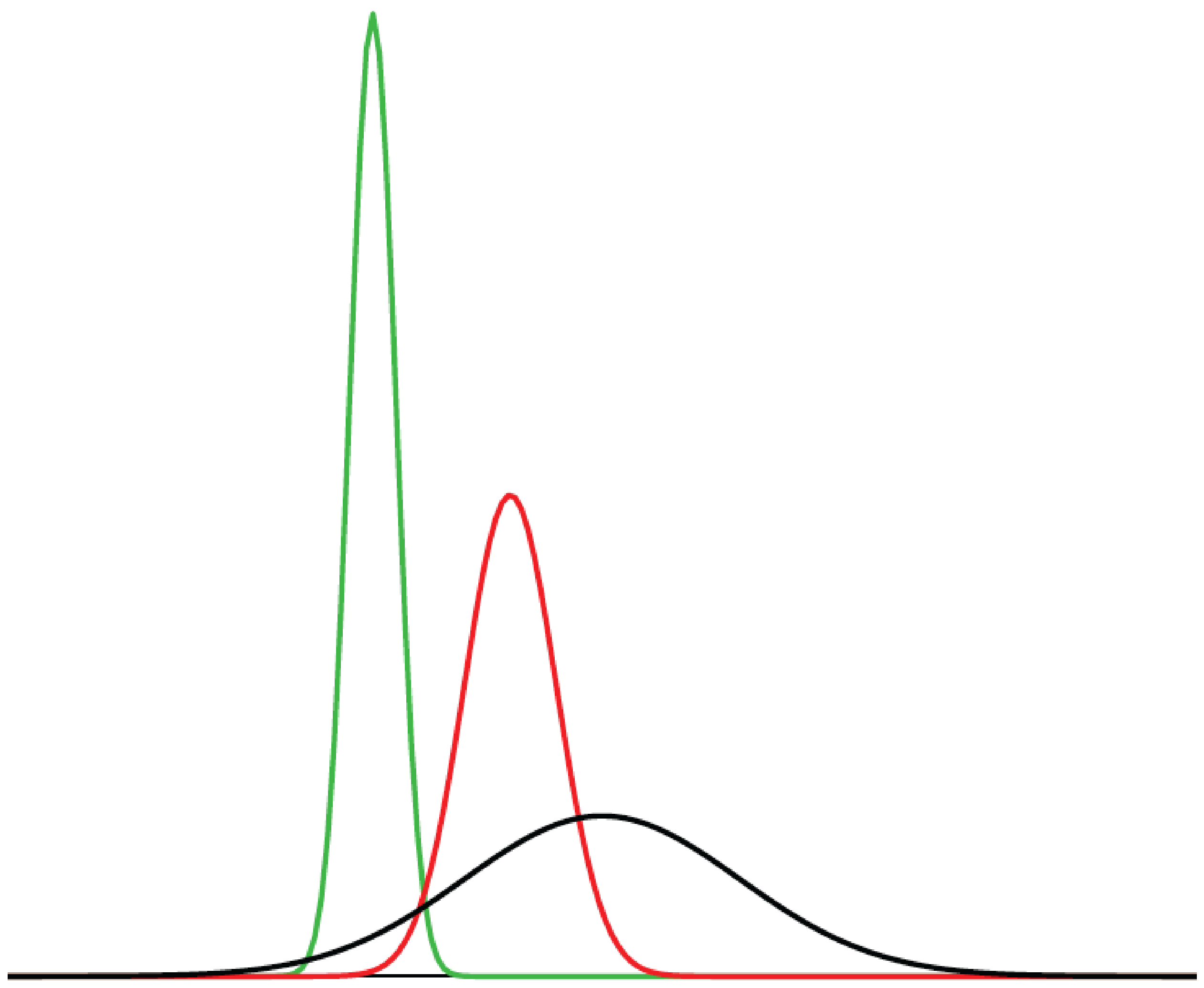
Application to Humans

- Quételet was the first to apply the normal distribution to human characteristics
- Noted that characteristics such as height, weight, and strength were normally distributed

Introduction to Normal Distributions

- The most important distribution in statistics
- The most widely used distribution in statistics

Naming the Normal Distribution



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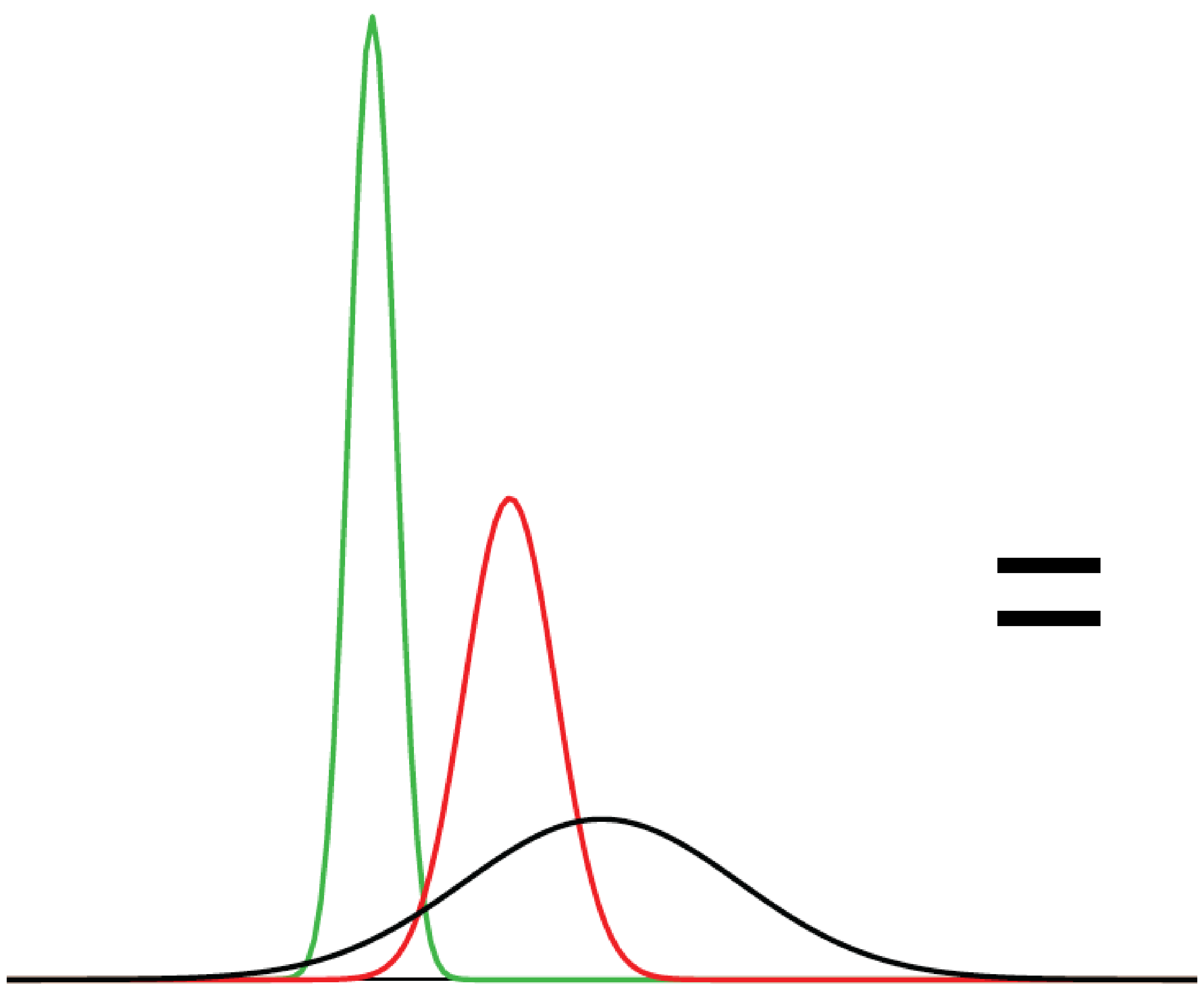
Normal Distribution = Bell Curve

Naming the Normal Distribution



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Gaussian Curve

Normal Curve = Bell Curve = Gaussian Curve

Naming the Normal Distribution

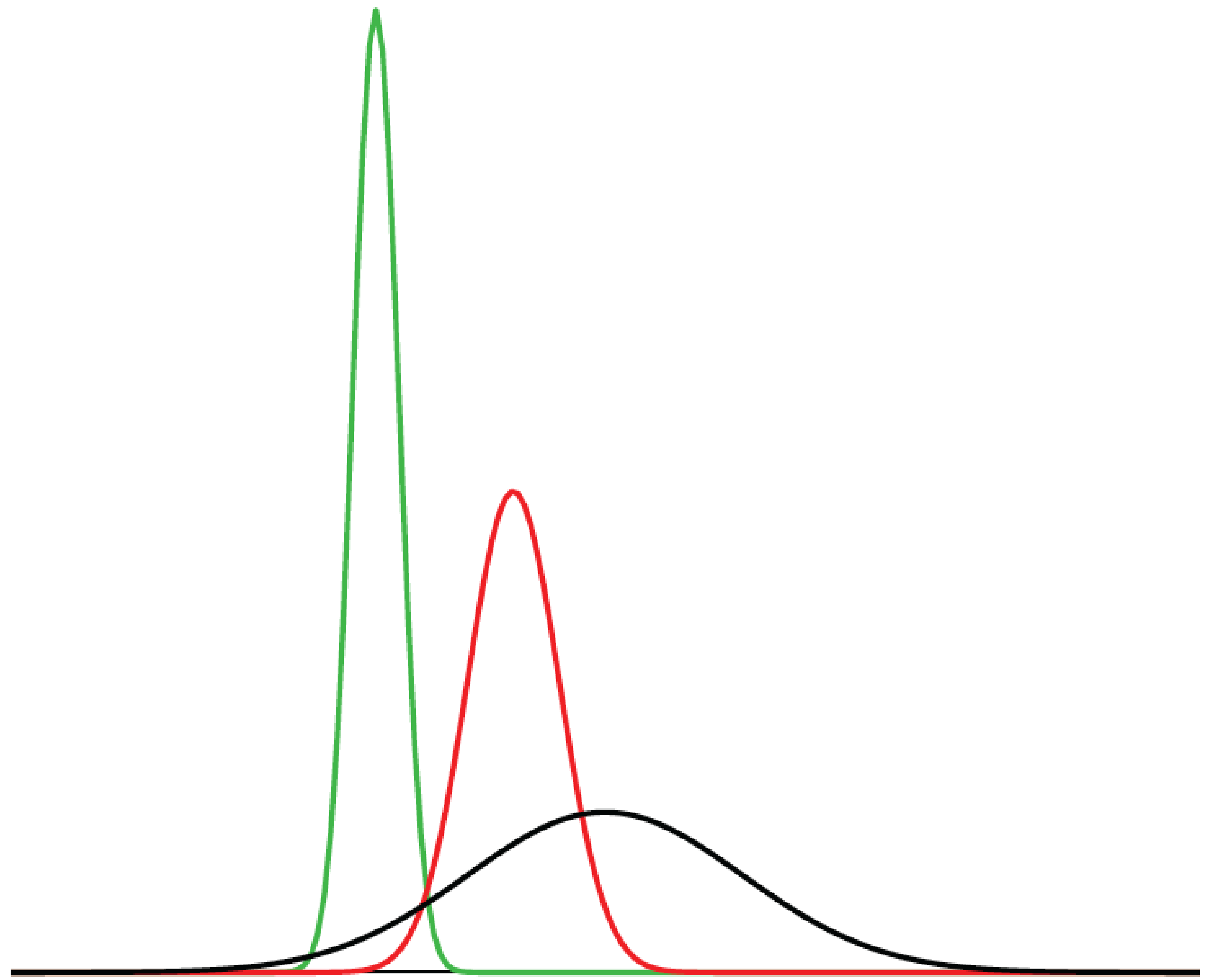
Varieties of Normal Distributions

- It is not correct to talk about “**the** normal distribution”
- There are many normal distributions

Varieties of Normal Distributions

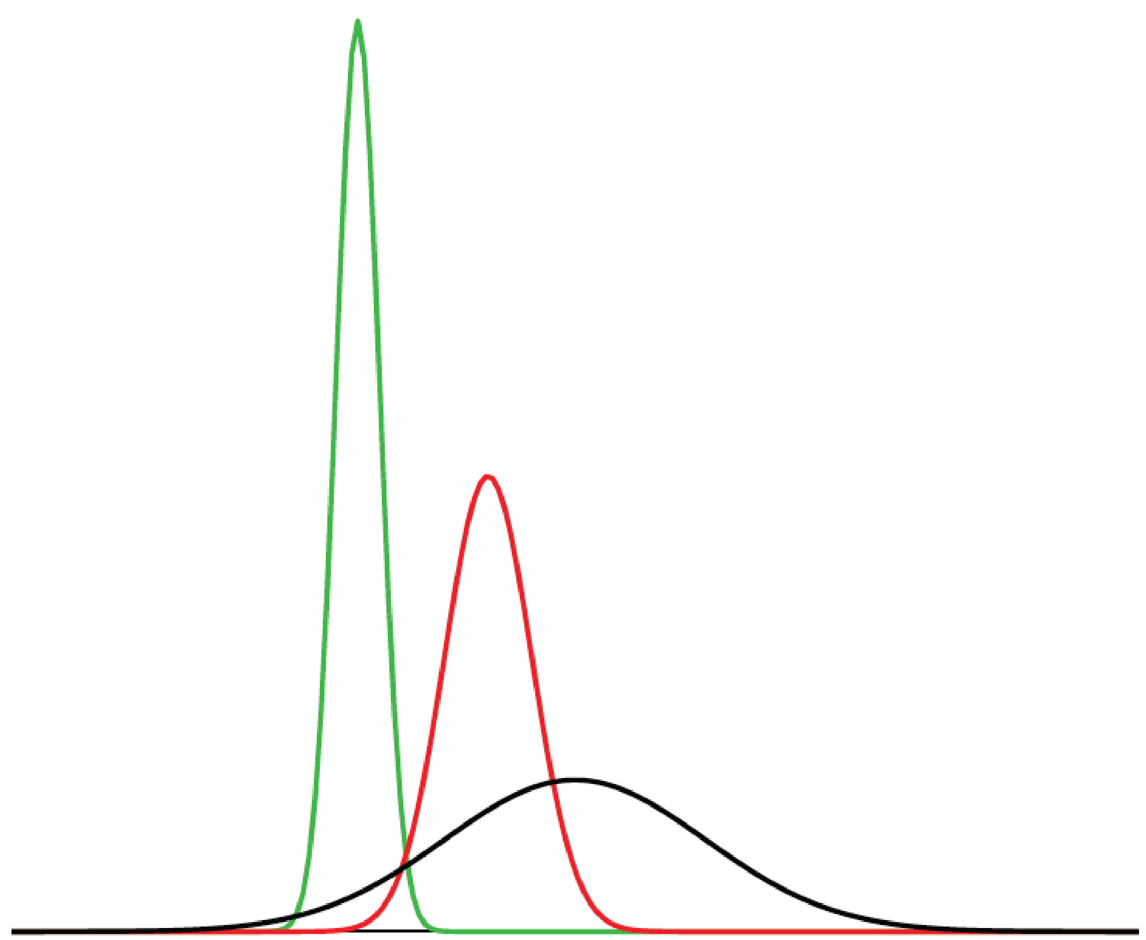
- Normal distributions can differ in their means
- Normal distributions can differ in their standard deviations

Varieties of Normal Distributions



Normal Distributions

- All three distributions are symmetric
- All have relatively more values at the center of the distribution and relatively few in the tails



Formula for the Density of the Normal Distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

- This formula can be thought of as the height for a given value on the X axis.

Formula for the Density of the Normal Distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

μ is the mean

σ is the standard deviation

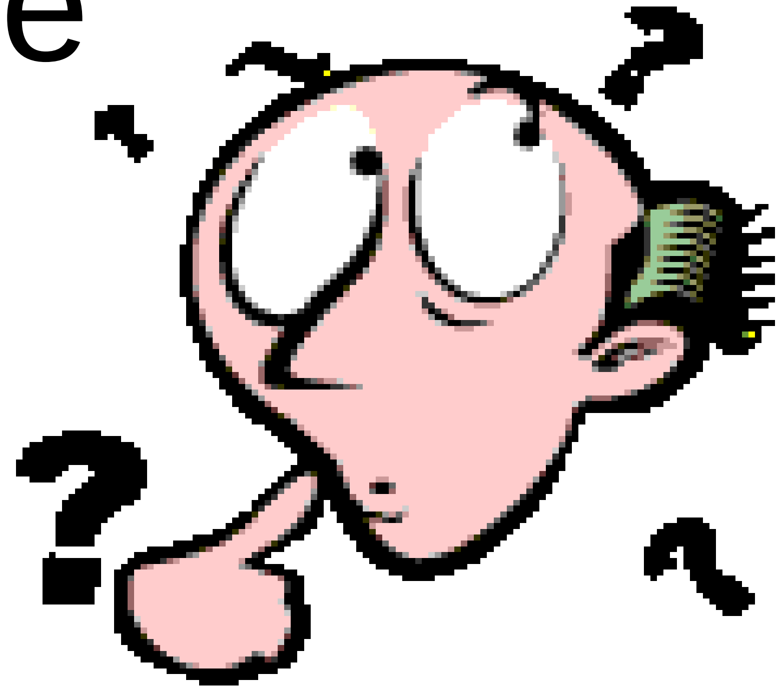
e is the base of the natural logarithm

π is the constant pi

Formula for the Density of the Normal Distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

- Don't worry if this expression confuses you. We will **not** be referring back to it.



Features of Normal Distributions

1. Normal distributions are symmetric around their mean.
2. The mean, median, and mode of a normal distribution are equal.
3. The area under the normal curve is equal to 1.0.

Features of Normal Distributions

4. Normal distributions are denser in the center and less dense in the tails.
5. Normal distributions are defined by two parameters, the mean (μ) and the standard deviation(σ).
6. 68% of the area of a normal distribution is within one standard deviation of the mean.

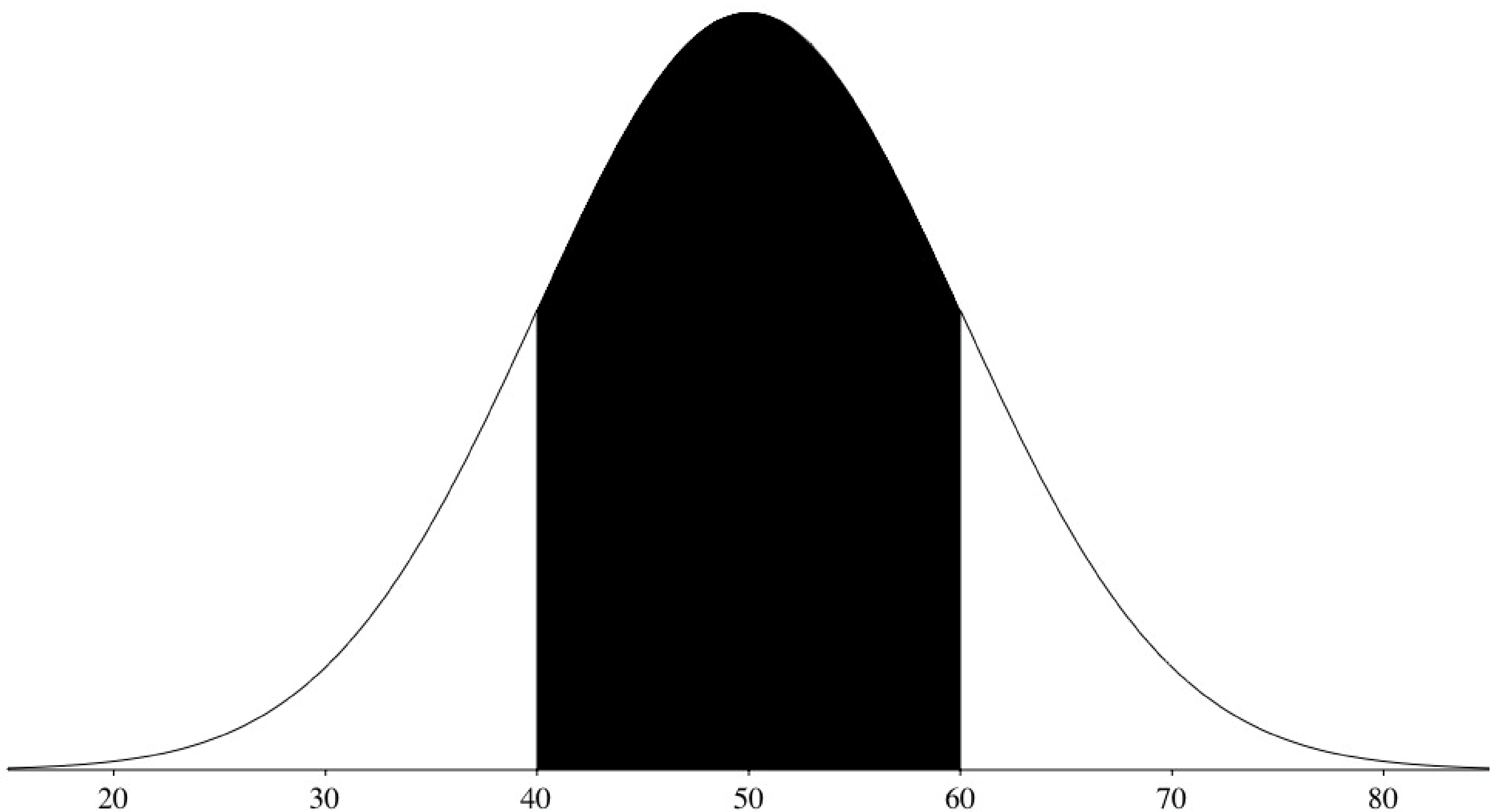
Areas Under Normal Distributions

- Areas under portions of a normal distribution can be computed using calculus.

- We are going to use computer programs and tables to find the area under normal distributions.

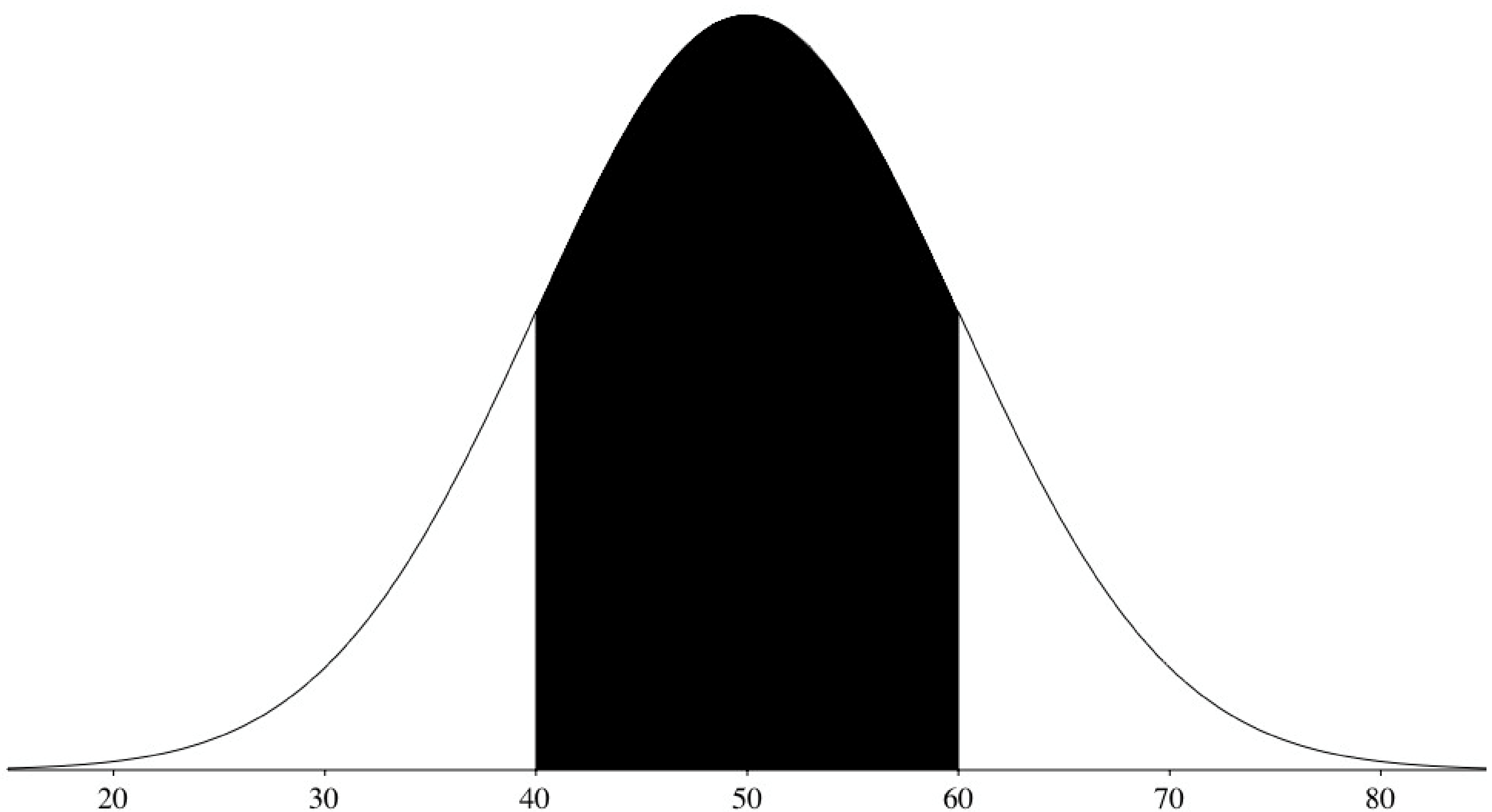
Areas Under Normal Distributions

- The normal distribution shown here has a mean of 50 and a standard deviation of 10.

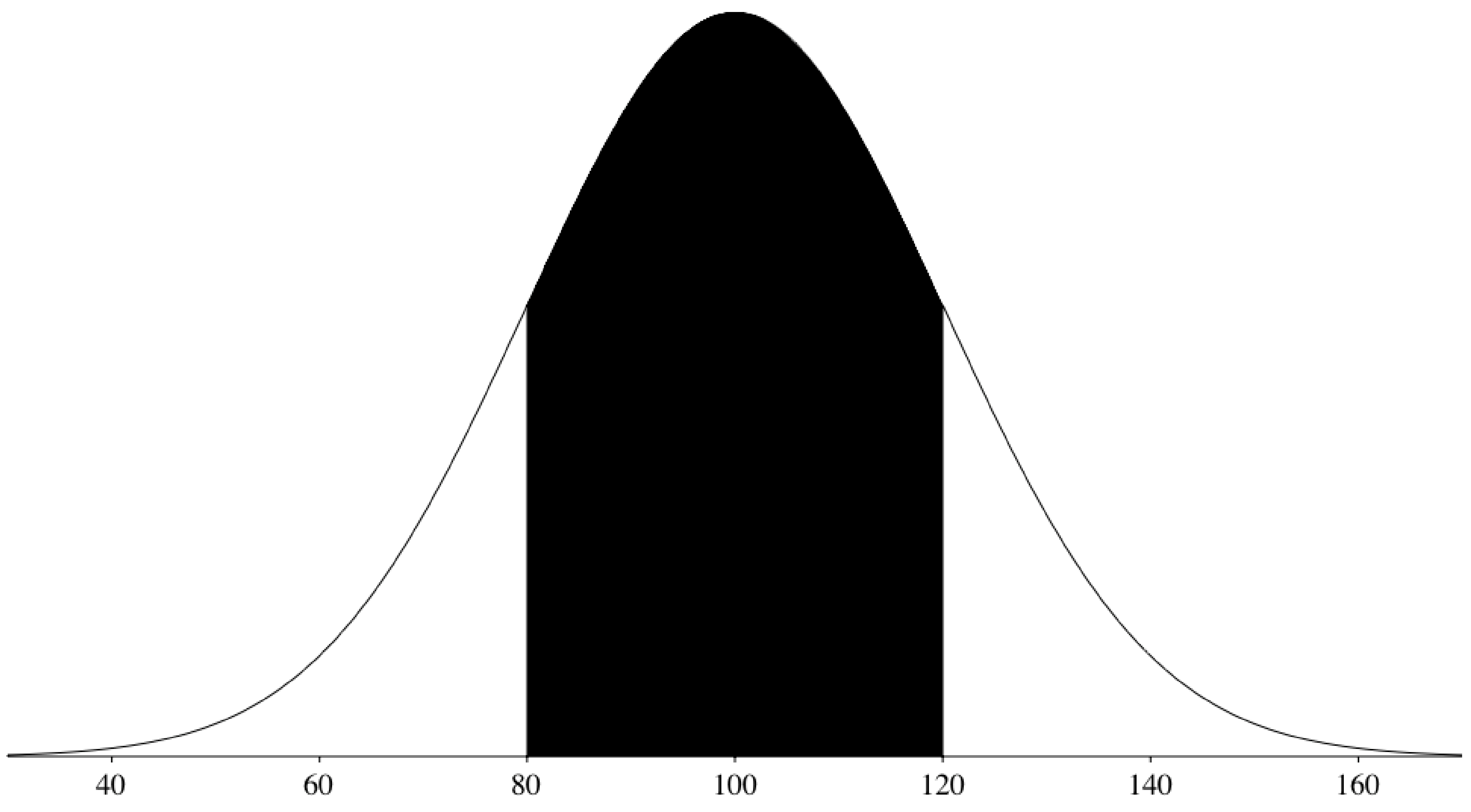


One Standard Deviation from the Mean

- The shaded area between 40 and 60 contains 68% of the distribution

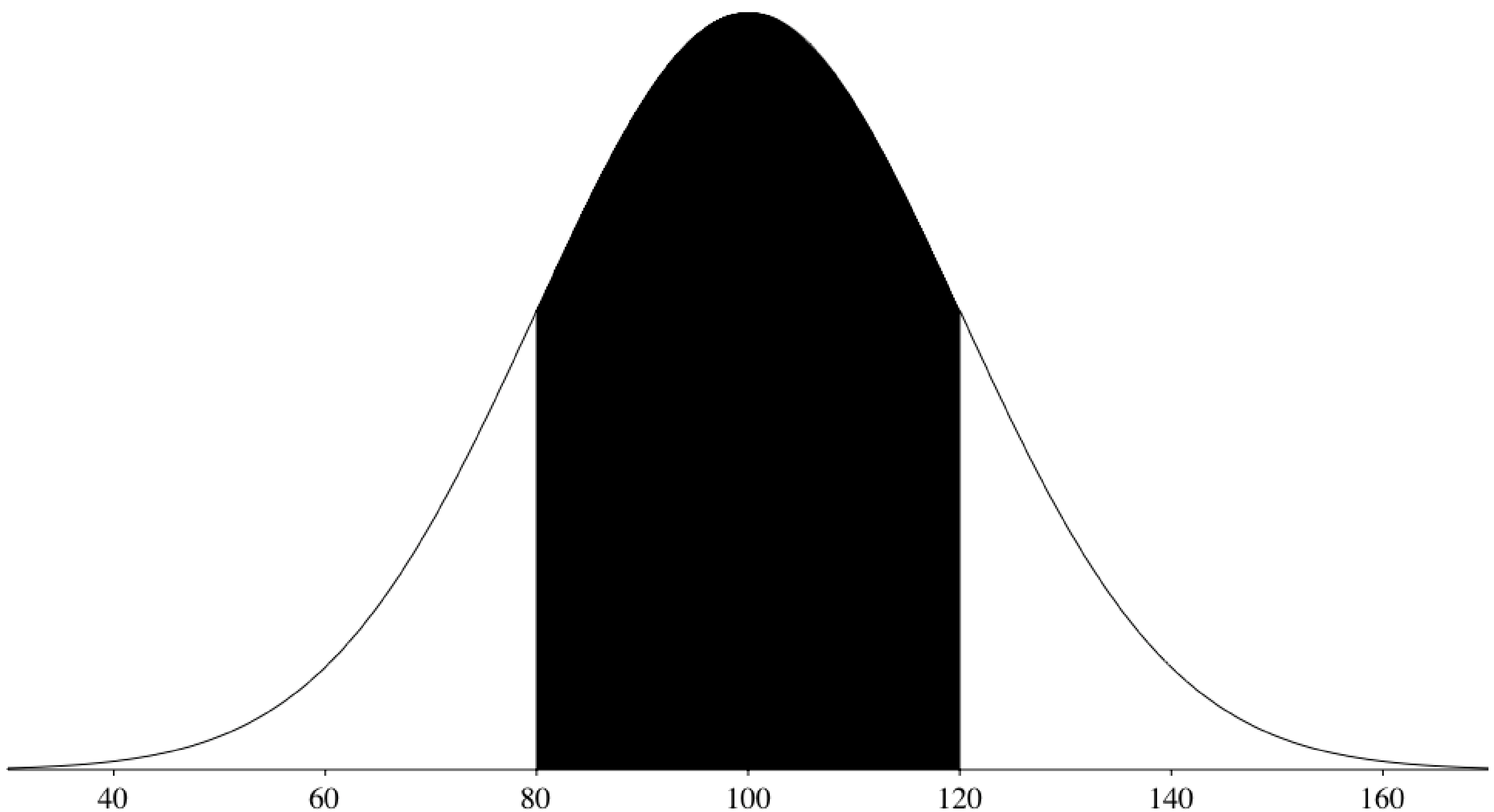


Normal Distribution with a Mean of 100 an SD of 20



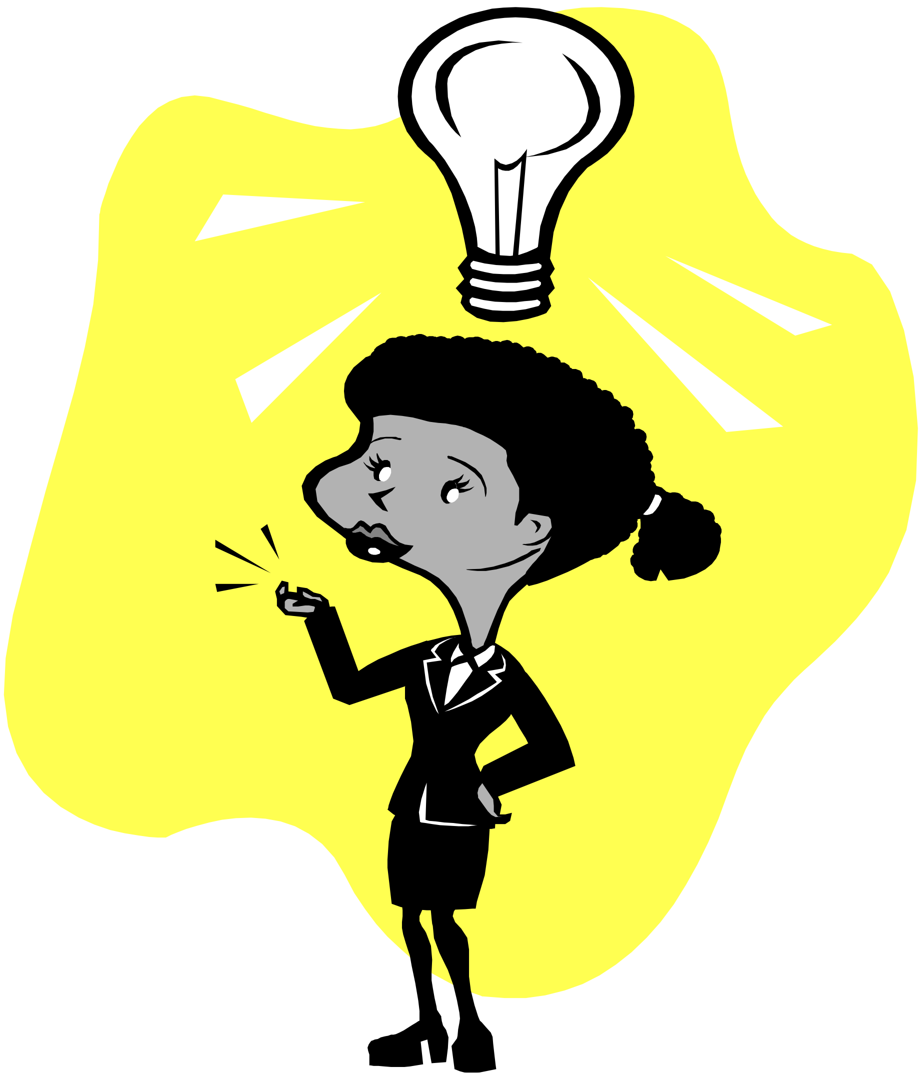
One Standard Deviation from the Mean

68% of the area

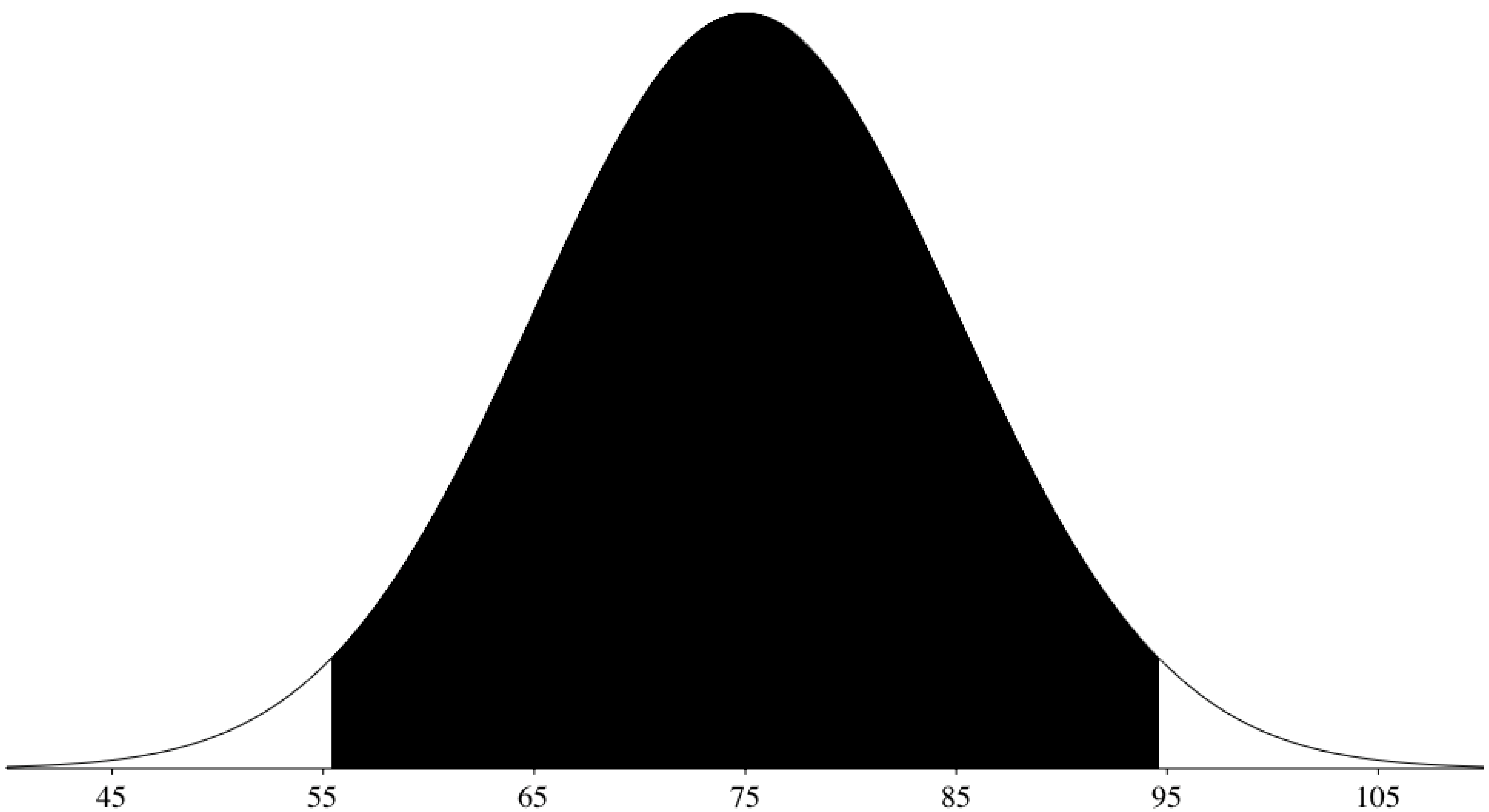


One Standard Deviation from the Mean

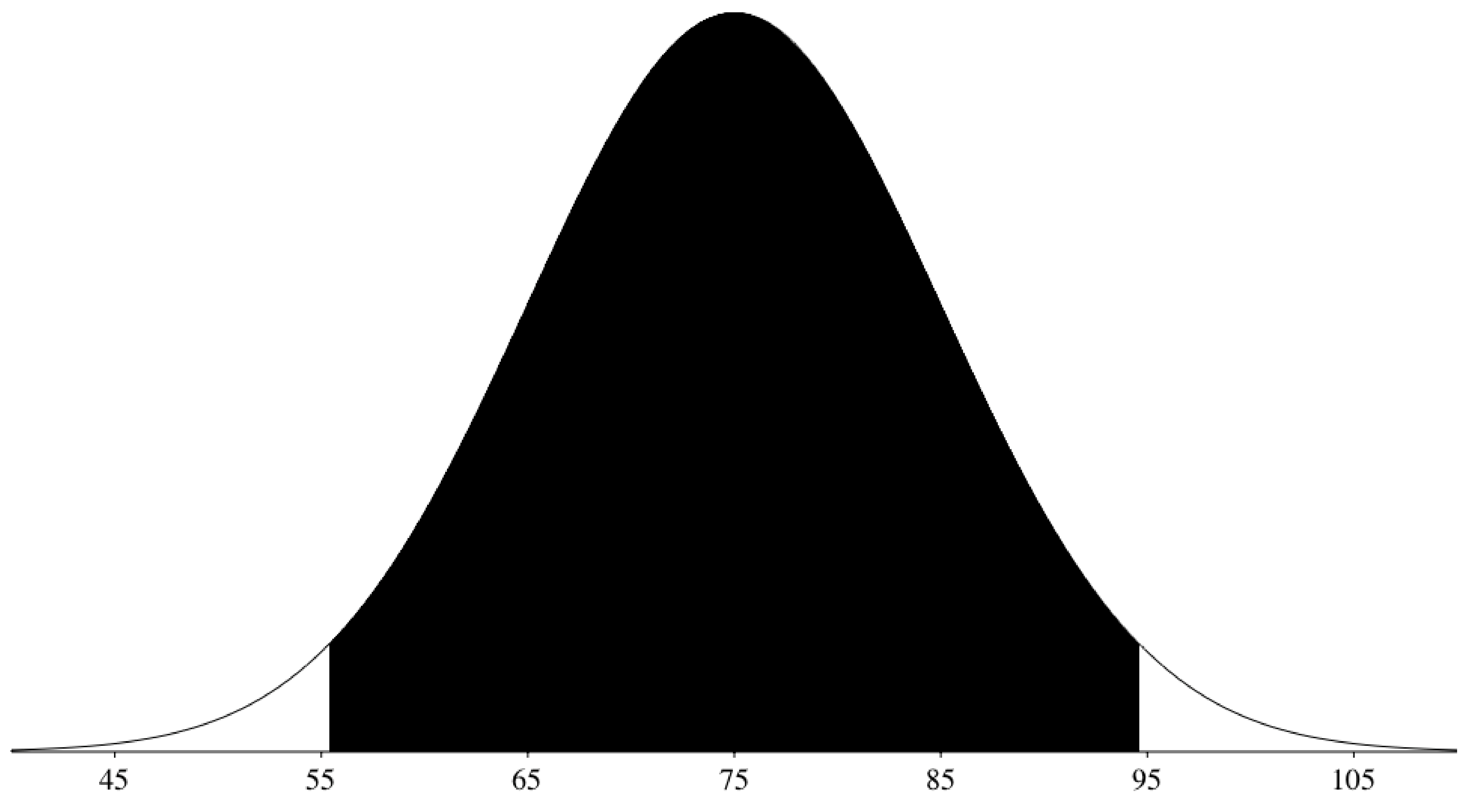
- 68% of the area of **any** normal distribution is within one standard deviation of the mean.



1.96 Standard Deviations from the Mean



1.96 Standard Deviations from the Mean



Two Standard Deviations from the Mean

- For normal distributions 95% of the area is within 1.96 standard deviations of the mean
- For an approximation 2 can be used

Standard Normal Distribution

- A normal distribution with a mean of 0 and a standard deviation of 1 is called a standard normal distribution

Transforming to a Standard Normal Distribution

- A value from any normal distribution can be transformed into its corresponding value on a standard normal distribution using this formula:

$$Z = (X-\mu)/\sigma$$

Transforming to a Standard Normal Distribution

$$Z = (X-\mu)/\sigma$$

- Z is the value on the standard normal distribution
- X is the value on the original distribution
- μ is the mean of the original distribution
- σ is the standard deviation of the original distribution

An Example

- What portion of a normal distribution with a mean of 50 and a standard deviation of 10 is below 26?

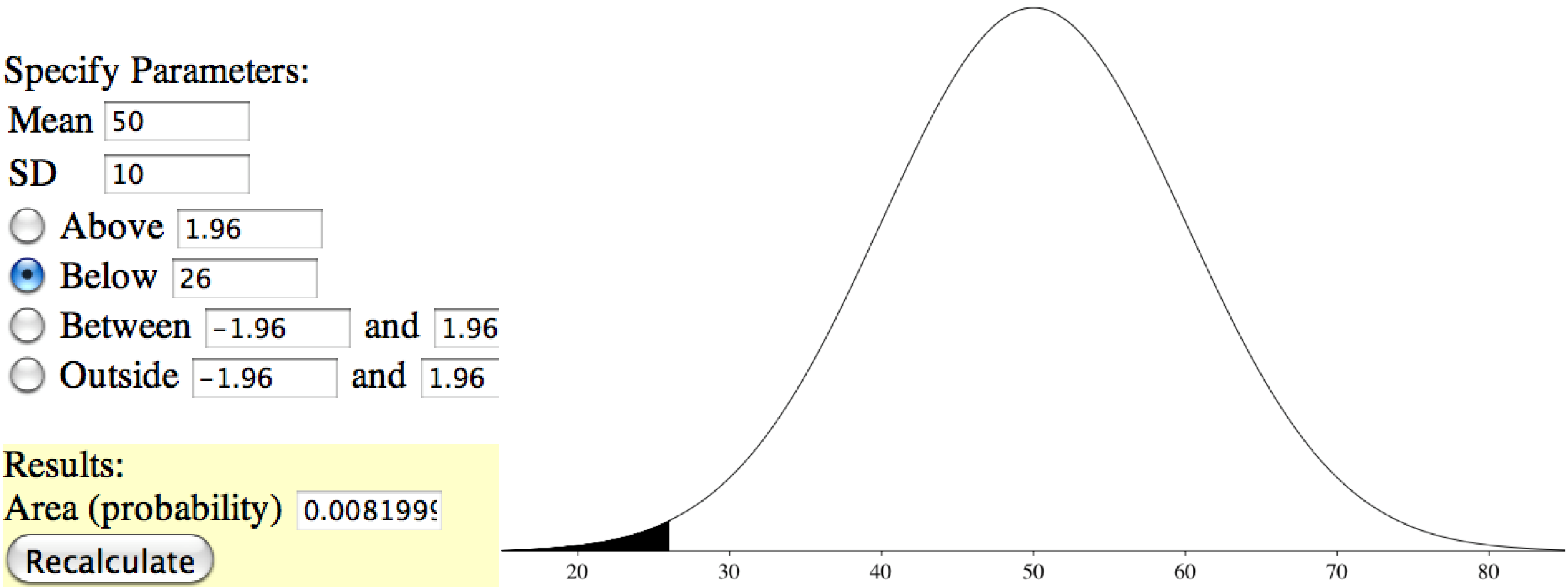
$$
\begin{aligned}
Z &= (X-\mu)/\sigma \\
&= (26 - 50)/10 \\
&= -2.4
\end{aligned}
$$

An Example

- From the table we can see that 0.0082 of the distribution is below -2.4.

Z	Area below Z
-2.50	0.0062
-2.49	0.0064
-2.48	0.0066
-2.47	0.0068
-2.46	0.0069
-2.45	0.0071
-2.44	0.0073
-2.43	0.0075
-2.42	0.0078
-2.41	0.0080
-2.40	0.0082
-2.39	0.0084
-2.38	0.0087
-2.37	0.0089
-2.36	0.0091
-2.35	0.0094
-2.34	0.0096
-2.33	0.0099
-2.32	0.0102

Standard Normal Distribution



Standard Normal Distribution

- If all values transformed to Z scores, then the distribution will have a mean of 0 and a standard distribution
- This process is called standardizing the distribution