Problem Set 1
ECON 1150 Fall 2018

Due **in class** Thursday September 13

1. A few years ago the news magazine *The Economist* listed some of the stranger explanations used in the past to predict presidential election outcomes. These included whether or not the hemlines of women's skirts went up or down, stock market performances, baseball World Series wins by an American League team, etc. Thinking about this problem more seriously (i.e. looking for causality rather than correlation), you decide to analyze whether or not the presidential candidate for a certain party did better if his party controlled the house. Accordingly you collect data for 34 presidential elections. You should think of this data as comprising a population which you want to describe, rather than a sample from which you want to infer behavior of a larger population. You generate the accompanying table:

   Joint Distribution of Presidential Party Affiliation and Party Control of House of Representatives, 1860-1996

	Dem. Control of House	Rep. Control of House	Total
	$(Y = 0)$	$(Y = 1)$	
Dem. President $(X = 0)$	0.412	0.030	0.442
Rep. President $(X = 1)$	0.176	0.382	0.558
Total	0.588	0.412	1.00

   (a) Interpret one of the joint probabilities and one of the marginal probabilities.

   (b) Compute $E(X)$. How does this differ from $E(X \mid Y = 0)$? Explain..

   (c) If you picked one of the Republican presidents at random, what is the probability that during his term the Democrats had control of the House?

   (d) What would the joint distribution look like under independence? Check your results by calculating the two conditional distributions and compare these to the marginal distribution.

2. Find the following probabilities:

   (a) Y is distributed χ^2_4. Find $\Pr(Y > 9.49)$.

   (b) Y is distributed t_∞. Find $\Pr(Y > -0.5)$.

   (c) Y is distributed $F_{4,\infty}$. Find $\Pr(Y < 3.32)$.

   (d) Y is distributed $N(500, 10000)$. Find $\Pr(Y > 696 \text{ or } Y < 304)$.

3. X and Z are two random variables. X is equal to 1 with probability 0.3, and equal to 0 with probability 0.7. You also know that $E[Z \mid X = 1] = 10$, and $E[Z \mid X = 0] = 1$. Compute $E[Z]$.

4. This exercise should help you understanding the properties of *summations.* Remember that

$$\sum_{i=1}^{n} X_i = X_1 + X_2 + ... + X_{n-1} + X_n$$

   Consider the following sequences of variables

$$\begin{array}{lll} X_1 = 1 & Y_1 = 1 & Z_1 = 3 \\ X_2 = 0 & Y_2 = 2 & Z_2 = 3 \\ X_3 = 2 & & Z_3 = 3 \\ & & Z_4 = 3 \\ & & Z_5 = 3 \end{array}$$

You should show each of the following things two ways, first using the formulas and then using the actual numbers.

(a) Show that

$$\sum_{i=1}^{5} Z_i = 5Z_1$$

(b) Show that

$$\sum_{i=1}^{3}\sum_{j=1}^{2} X_i Y_j = \left(\sum_{i=1}^{3} X_i\right)\left(\sum_{j=1}^{2} Y_j\right) = \sum_{j=1}^{2}\left(Y_j \sum_{i=1}^{3} X_i\right)$$

(c) Show that

$$\sum_{i=1}^{3}\left(\frac{X_i}{\sum_{j=1}^{2} Y_j}\right) = \frac{\sum_{i=1}^{3} X_i}{\sum_{j=1}^{2} Y_j}$$

(d) Show that

$$\sum_{i=1}^{3}\sum_{j=1}^{4} X_i Z_j = 12\sum_{i=1}^{3} X_i$$