Connor Johnson

1. a) $p(x|y,z) = \frac{p(x,y,z)}{p(y,z)} = \frac{kf(x,z)\cancel{g(y,z)}\cancel{h(z)}}{\cancel{g(y,z)}\cancel{h(z)}}$

$p(x|y,z) = kf(x,z)$

$p(x|y,z) \propto f(x,z)$ since k is a constant

b) $p(y|x,z) = \frac{p(x,y,z)}{p(x,z)} = \frac{k\cancel{f(x,z)}g(y,z)\cancel{h(z)}}{\cancel{f(x,z)}\cancel{h(z)}}$

$p(y|x,z) = kg(y,z)$ $p(y|x,z) \propto g(y,z)$ since k is a constant

c) X and Y are conditionally independent given Z iff $p(x,y|z) = p(x|z)\,p(y|z)$

$$\frac{p(x,y,z)}{p(z)} = \frac{p(x,z)}{p(z)} \cdot \frac{p(y,z)}{p(z)}$$

$$\frac{f(x,z)g(y,z)\cancel{h(z)}}{\cancel{h(z)}} = \frac{f(x,z)\cancel{h(z)}}{\cancel{h(z)}} \cdot \frac{g(y,z)\cancel{h(z)}}{\cancel{h(z)}}$$

$$f(x,z)g(y,z) = f(x,z)\,g(y,z) \quad \square$$

2. a)

	X = 0	X = 1
~~Y = 0~~		~~0.2~~
~~Y = 1~~		~~0.3~~
	~~.5~~	~~.5~~

	X = 0	X = 1	
Y = 0	0.2	0.3	0.5
Y = 1	0.3	0.2	0.5
	.5	.5	

b) $E(Y) = .5(.6) + .5(.4) = 0.5$

c) $Var(Y|X=0) = (.6)(.4) = 0.24$

$Var(Y|X=1) = (.4)(.6) = 0.24$

$Var(Y) = (.5)(.5) = 0.25$

Var(Y) is higher. This makes sense because no info is given so there is more uncertainty

d) $P_r(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$ $\quad P_r(X=0|Y=1) = \frac{P_r(Y=1|X=0)\, P_r(X=0)}{P_r(Y=1)}$

$P_r(X=0|Y=1) = \frac{.6 \cdot .5}{.5} = \boxed{.6}$

3. a) $\text{Poisson}(\theta) = \frac{\theta^k e^{-\theta}}{k!}$ $\quad p(\theta|y) = \frac{\frac{20\theta^y e^{-20\theta}}{y!} \cdot \frac{1}{90}}{\sum_{k=1}^{K} \frac{20\theta_k^y e^{-20\theta_k}}{y!}}$

$p(y|\theta) = \frac{\theta^y e^{-\theta}}{y!}$