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PS #2

1. a) When the mean and variance of x_i is constant for all t

b) $E(x_t) = \sum_{j=0}^{4} \alpha_j \underbrace{E(\varepsilon_{t-j})}_{=0} = 0$

Constant mean and variance so it is weakly stationary

$Var(x_t) = Var\left(\sum_{j=0}^{4} \alpha_j \underbrace{\varepsilon_{t-j}}_{=\sigma^2}\right) = \sigma_\varepsilon^2 \sum_{i=0}^{4} \alpha_j^2$

c) $E(x_t) = \sum_{i=0}^{n} \alpha_j E(\varepsilon_{t-j}) = 0$

Constant mean and variance so it is weakly stationary

$Var(x_t) = Var\left(\sum_{i=0}^{n} \alpha_j \varepsilon_{t-j}\right) = \sigma_\varepsilon^2 \sum_{i=0}^{n} \alpha_j^2$

d) $E(x_t) = \sum_{i=0}^{\infty} \alpha_j E(\varepsilon_{t-j}) = 0$

The variance of x_t must be less than infinity. This is only true if $\sum_{j=0}^{\infty} \alpha_j^2$ or $\sum_{j=0}^{\infty} \alpha_j^2 \leq C < \infty$

$Var(x_i) = \sigma_\varepsilon^2 \sum_{i=0}^{\infty} \alpha_j^2$

2. a) $y_t = \sum_{j=0}^{\infty} \phi_1^j \varepsilon_{t-j}$ $\quad \alpha_j = \phi_1^j$ $\quad \alpha_j = (0.5)^j$

y_t is weakly stationary if $\sum_{j=0}^{\infty} \alpha_j^2 < \infty$

$\sum_{j=0}^{\infty} (0.5)^{2j} = \sum_{j=0}^{\infty} = (0.25)^j = \frac{1}{1-0.25} = 1.\bar{3}$

So y_t is weakly stationary

b) $E(y_t) = \sum_{j=0}^{\infty} \phi_1^j \underbrace{E(\varepsilon_{t-j})}_{=0} = 0$

c) $Var(y_t) = Var\left(\sum_{j=0}^{\infty} \phi_1^j \varepsilon_{t-j}\right) = \sum_{j=0}^{\infty} (\phi_1^j)^2 = \sum_{j=0}^{\infty} (0.25)^j = \boxed{\frac{4}{3}}$

d) $\gamma(h) = cov(x_t, x_{t-h}) = cov(\phi x_{t-1} + \varepsilon_t, x_{t-h}) = \phi\, \gamma(h-1)$

$\rho(h) = \phi_1^h = (0.5)^h$

3. (a) is weakly ~~stati~~ stationary because it is the only autoregressive process where all $|\phi_i| < 1$.