Connor Johnson

1. $1 - \frac{2}{3!} = \boxed{\frac{2}{3}}$

2. Win $\frac{6}{36} + \frac{2}{36} = \frac{8}{36}$ $\boxed{.454}$

Loss $\frac{1}{36} + \frac{2}{36} + \frac{1}{36} = \frac{4}{36}$

$$\frac{24}{36} + \left(\frac{3/36}{\cancel{8}/36}\cdot\frac{3}{36} + \frac{4}{82}\cdot\frac{4}{36} + \frac{5}{83}\cdot\frac{5}{36}\right)\cdot 2$$

$.03125$ $\frac{2}{36}$ $.0868$

$.0227$ $.0\overline{37}$ $.0534$ $.055\overline{5}$

3. $\left(\frac{1}{3}\right)^4 + \left(\underbrace{{}_5C_4}_{5}\right)\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) + \left(\underbrace{{}_6C_4}_{15} - \underbrace{{}_5C_4}_{5}\right)\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + \left(\underbrace{{}_7C_4}_{35} - \underbrace{{}_6C_4}_{15} - \cancel{{}_5C_4 - 1}\right)\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^3 = \boxed{.173}$

4. $\frac{1/3}{\left(1/3 + 1/4\right)} = \boxed{.571}$

5. a)

		M		
		0	1	
I	0	.025	.45	.475
	1	.075	.45	.525
		.1	.9	1

$$P_r(M_1|B) = \frac{P_r(M_1)P_r(B|M_1)}{P_r(M_1)P_r(B|M_1) + P_r(M_1^c)P_r(B|M_1^c)}$$

$P_r(B|M_1) = {}_5C_4(.5)^4(.5)^1 = .15625$

$P_r(B|M_2) = {}_5C_4(.25)^4(.75) = .0146$

$$P_r(M_1|B) = \frac{.9 \times .15625}{.9 \times .15625 + .1 \times .0146} = \frac{.140625}{.140625 + .00146} = \frac{.140625}{.142085}$$

$\boxed{.9897}$

b) $P = \frac{.9897 \cdot 0.5}{.9897 \cdot 0.5 + .0103 \cdot .75} = \frac{.49485}{.49485 + .007725} = \frac{.49485}{.502575} = \boxed{.985}$

6.

$$a_i = \frac{\left(\frac{1-p}{p}\right)^i - 1}{\left(\frac{1-p}{p}\right)^k - 1}$$

$a_i = 0.99$
$p = 2/3$
$1-p = 1/3$
$k = i+2$

$$.99 = \frac{\left(\frac{1/3}{2/3}\right)^i - 1}{\left(1/3 / 2/3\right)^{i+2} - 1}$$

$$.99 = \frac{\left(\frac{1}{2}\right)^i - 1}{\left(1/2\right)^{i+2} - 1}$$

$$.99\left(1/2\right)^{i+2} - 0.99 = -1 + \left(\tfrac{1}{2}\right)^i$$

$$\frac{.99}{4}\left(\tfrac{1}{2}\right)^i - \left(\tfrac{1}{2}\right)^i = -0.01$$

$$\left(\tfrac{1}{2}\right)^i\left(\frac{.99}{4} - 1\right) = -0.01$$

$$2^i = 75.25$$

$$2^i \geq 75.25$$

$i = 7$

The gambler must start with \$7