CS189: Machine Learning

Homework 6: Neural Nets

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Problem 1: Derivations. For stochastic gradient descent we need to compute $\nabla_W L$ and $\nabla_V L$ where $L = -\sum_{j=1}^{K} y_i ln(z_i) + (1-y_i)ln(1-z_i)$ where K is the number of classes. We will derive the gradients by their rows first.

$$\nabla_{W_j} L = \frac{\partial L}{\partial z_j}\frac{\partial z_j}{\partial w_j} = \frac{\partial L}{\partial z_j}\frac{\partial}{\partial w_j} s(W_j \cdot h)$$

$$\nabla_{W_j} L = \frac{\partial L}{\partial z_j} s(W_j \cdot h)(1 - s(W_j \cdot h)h = \frac{\partial L}{\partial z_j} z_j(1-z_j)h;$$

So we still need $\frac{\partial L}{\partial z_j}$, we'll come back and plug that in later.

$$\nabla_{V_J} L = \frac{\partial L}{\partial h_j}\frac{\partial h_j}{\partial V_j} = \frac{\partial L}{\partial h_j}\frac{\partial}{\partial V_j}(tanh(V_j \cdot x)) = \frac{\partial L}{\partial h_j}(1 - tanh^2(V_j \cdot x)) = \frac{\partial L}{\partial h_j}(1-h_j^2)x;$$

So we still need $\frac{\partial L}{\partial h_j}$. Let's derive $\frac{\partial L}{\partial z_j}$ and come back to that.

$$\frac{\partial L}{\partial z_j} = \frac{1-y_j}{1-z_j} - \frac{y_j}{z_j}.$$

So

$$\nabla_{W_j} L = (z_j - y_j)h$$

and thus,

$$\nabla_W L = (z-y)h^T.$$

Let's derive $\frac{\partial L}{\partial h_j}$ now:

$$\frac{\partial L}{\partial h_j} = \frac{\partial}{\partial h_j}(-\sum_{i=1}^{K} y_i ln(s(W_i \cdot h)) + (1-y_i)ln(1 - s(W_i \cdot h)))$$

$$\frac{\partial L}{\partial h_j} = -\Big[\sum_{i=1}^{K}\Big(\frac{1-y_j}{1-z_i} - \frac{y_i}{z_i}\Big) z_i(1-z_i)\Big]\frac{\partial}{\partial h_j}(W_i \cdot h)$$

$$\frac{\partial L}{\partial h_j} = -\sum_{i=1}^{K}(y_i - z_i)\frac{\partial}{\partial h_j}(W_{i,1}h_1 + W_{i,2}h_2 \dots W_{i,m}h_m)$$

$$\frac{\partial L}{\partial h_j} = -\sum_{i=1}^{K}(y_i - z_i)W_{i,j}$$

$$\frac{\partial L}{\partial h_j} = \sum_{i=1}^{K}(z_i - y_i)W_{i,j}.$$

Plugging into the equation above, we have:

$$\nabla_{V_j} L = \Big(\sum_{i=1}^{K}(z_i - y_i)W_{i,j}\Big)(1 - h_j^2)x.$$

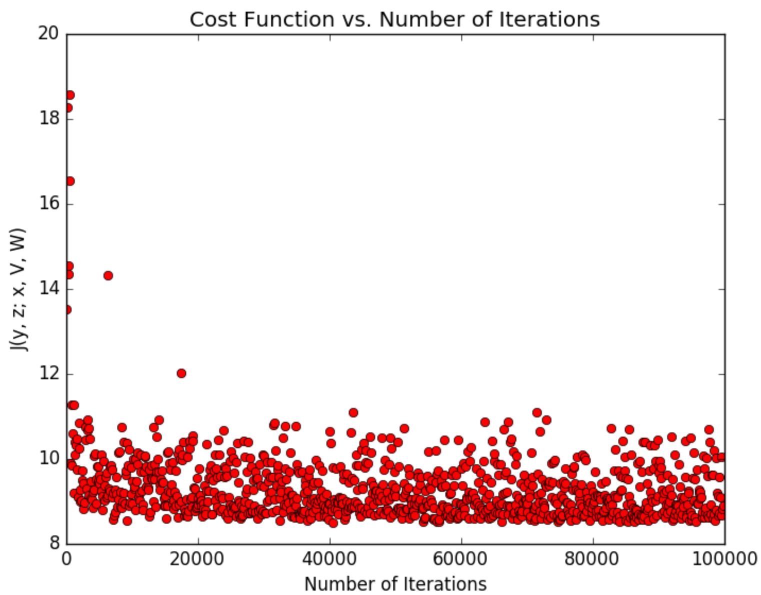
For $\nabla_{V_j} L$, this says the j^{th} row of $\nabla_V L$ is the dot product of $z - y$ and W_j multiplied by $1 - h_j^2$ all scalar multiplied into the vector x. In matrix vector form, that is

$$\nabla_V L = [W^T(z - y) \odot (1 - h^2)]x^T$$

where $\odot$ represents component-wise scalar multiplication.

Problem 2: Implementation. I tuned the target values, the learning rates for V and W, the decay rates for V and W, and the batch size for mini-batch gradient descent. For my kaggle submission I settled on a batch size of 25 samples, target values 0.1 and 0.9, learning rates 0.1 and 0.01, and decay rates 0.9 and 0.6 for V and W respectively which decayed every 4 epochs.
After 8 epochs I was able to achieve a training accuracy of 91% and a validation accuracy of 86%.

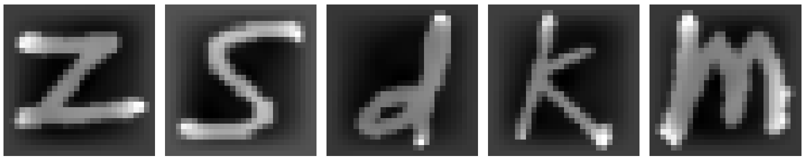
This is a graph representing the value of the cost function of the predicted values at the i^{th} iteration using stochastic gradient descent.




```
Kaggle Display Name: scoobyd
Kaggle Score: 0.86154
```

Problem 3: Visualization. Using a Neural Net that classified 88% of the traning set images correctly and 86% of the validation set images correctly here are 2 sets of 5 validation images that were classified.

5 images classified correctly:



5 images classfied incorrectly.

Problem 4: Bells and Whistles.

A

NeuralNetwork.py

```
import numpy as np
import scipy as sci

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

# A Neural Network class that holds a trained Neural Network.
class NeuralNet:

# IMAGES is a matrix of n samples points with 784 features, LABELS
# is an array of n labels for each sample point and WEIGHT_DECAY
# is an optional paramter used for regularization.
def __init__(self, images, labels, weight_decay=None):
self.images = np.concatenate((images, np.ones((len(images[:,0]), 1))), axis=1)
self.labels = labels
self.weight_decay = weight_decay
self.input_layer_size = 784
self.hidden_layer_size = 200
self.output_layer_size = 26
mu = 0.0
sigma_v = np.sqrt(1/self.input_layer_size)
sigma_w = np.sqrt(1/self.hidden_layer_size)
self.V = np.random.normal(mu, sigma_v, (self.hidden_layer_size, self.input_layer_size +
self.W = np.random.normal(mu, sigma_w, (self.output_layer_size, self.hidden_layer_size +

# The log loss function. Z,Y are predicted lables and lables respectively.
def costFunction(self, z, y):
for i in range(len(z)):
if (z[i] == 0):
z[i] == 10**-12
elif (z[i] == 1):
z[i] = 1 - 10**-12
return -1*(np.dot(y.T, np.log(z)).ravel() + np.dot((1-y).T, np.log(1-z)).ravel())

# The forward step of the Neural Network. Takes in a sample point, or batch of
# sample points X and return a hidden layer H and predicted values Z.
def forward(self, X):
h = np.tanh(np.dot(self.V, X))
h = np.insert(h, len(h), values=1, axis=0)
z = sci.special.expit((np.dot(self.W, h)))
```

```
return h, z

# The backwards step of the Neural Network, takes in sample point, or batch
# of sample points X, labels Y, hidden units H, and predicted values Z and
# returns dJ/DV and dJdW (the gradients of the cost function w.r.t our weights.
def backward(self, X, y, h, z):
dJdV = np.dot(np.delete((np.dot(self.W.T, (z-y)) * (1 - h**2)), 200, axis=0), X.T)
dJdW = np.dot((z-y), h.T)
return dJdV, dJdW

# The learn method is the gradient descent step. Takes in DJDV, DJDW,
# the gradients of the the loss function w.r.t V and W. V_LEARNING_RATE
#  and W_LEARNING_RATE are the learning rates for V and W respectively.
def learn(self, dJdV, dJdW, v_learning_rate, w_learning_rate):
self.V = self.V - (v_learning_rate*dJdV)
self.W = self.W - (w_learning_rate*dJdW)

# Train method. Takes in V_LEARNING_RATE, W_LEARNING_RATE and trains
# the data using those learning rates performing one full epoch through
# the data.
def train(self, v_learning_rate, w_learning_rate, randomized=False):
for i in range(len(self.images)):
if (randomized==True):
j = np.random.randint(len(self.images))
else:
j = i
x = (self.images[j, :]).reshape((len(self.images[0]), 1))
y = self.labels[j,:].T.reshape(len(self.labels[0]), 1)
h, z = self.forward(x)
dJdV, dJdW = self.backward(x, y, h, z)
self.learn(dJdV, dJdW, v_learning_rate=v_learning_rate, w_learning_rate=w_learning_rate)

# TrainMini method. Takes in BATCH_SIZE, V_LEARNING_RATE, and W_LEARNING_RATE
# training the data using those learning rates performing one full epoch
# through the data.
def trainMini(self, batch_size, v_learning_rate, w_learning_rate):
splits = len(self.images)//batch_size
X = (self.images[:batch_size,:]).T
Y = (self.labels[:batch_size,:]).T
h,z = self.forward(X)
dJdV, dJdW = self.backward(X, Y, h, z)
self.learn((1/batch_size)*dJdV, (1/batch_size)*dJdW, v_learning_rate=v_learning_rate,
w_learning_rate=w_learning_rate)
for i in range(2,splits):
X = (self.images[batch_size*(i-1):batch_size*i,:]).T
```

```
Y = (self.labels[batch_size*(i-1):batch_size*i,:]).T
h,z = self.forward(X)
dJdV, dJdW = self.backward(X, Y, h, z)
self.learn((1/batch_size)*dJdV, (1/batch_size)*dJdW, v_learning_rate=v_learning_rate,
w_learning_rate=w_learning_rate)
X = (self.images[batch_size*splits:,:]).T
Y = (self.labels[batch_size*splits:,:]).T
h,z = self.forward(X)
dJdV, dJdW = self.backward(X, Y, h, z)
self.learn(dJdV*(1/batch_size), dJdW*(1/batch_size), v_learning_rate=v_learning_rate,
w_learning_rate=w_learning_rate)

# Same as the train method but returns two arrays, x_plot and y_plot where
# the x_plot is the number of iterations and y_plot are the values of the
# cost function at those iterations using the labels obtained by the Neural
# Network at those points.
def trainPlot(self, v_learning_rate, w_learning_rate, randomized=False):
x_plot = list()
y_plot = list()
for i in range(len(self.images)):
if (randomized==True):
j = np.random.randint(len(self.images))
else:
j = i
x = (self.images[j, :]).reshape((len(self.images[0]), 1))
y = self.labels[j,:].T.reshape(len(self.labels[0]), 1)
h, z = self.forward(x)
dJdV, dJdW = self.backward(x, y, h, z)
if (i % 100 == 0 and i != 0):
x_plot.append(i)
y_plot.append(self.costFunction(z, y))
self.learn(dJdV, dJdW, v_learning_rate=v_learning_rate, w_learning_rate=w_learning_rate)
return x_plot, y_plot

# The classify method takes in a sample point X and returns a class 1-26.
def classify(self, x):
if (len(x) != len(self.V[0,:])):
x = np.insert(x, len(x), values=1, axis=0)
h = np.append(np.tanh(np.dot(self.V, x.T)), 1)
z = sci.special.expit(np.dot(self.W, h))
return np.argmax(z) + 1

# The classifyAll method takes a matrix of images _IMAGES and returns
# an list of classes 1-26 for each images.
def classifyAll(self, _images):
```

```
ret_arr = list()
for i in range(len(_images)):
ret_arr.append(self.classify(_images[i]))
return ret_arr

# Updates the Neural Networks images and labels. Used during training to
# so that the training data can be reshuffled randomly and be fed back
# to the Neural Network.
def updateData(self, images, labels):
self.images = np.concatenate((images, np.ones((len(images[:,0]), 1))), axis=1)
self.labels = labels
```

B

trainer.py

```
from NeuralNetwork import NeuralNet
import numpy as np
import sklearn.preprocessing as skp
import matplotlib.pyplot as plt

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

def main(target_values=None, epochs=8, v_learning_rate=0.1,
w_learning_rate=0.01, v_decay_rate=1, w_decay_rate=1, randomized=False):

_v_learning_rate = v_learning_rate
_w_learning_rate = w_learning_rate
images = np.load('data/images.npy')
labels = np.load('data/vec_labels.npy')
data = np.concatenate((images, labels), axis=1)
np.random.shuffle(data)


train_images = data[:round(.8*len(data)), :len(images[0])]
train_labels = data[:round(.8*len(data)), len(images[0]):]
validation_images = data[round(.8*len(data)):, :len(images[0])]
validation_labels = data[round(.8*len(data)):, len(images[0]):]

scaler = skp.StandardScaler()
train_images = scaler.fit_transform(train_images)
validation_images = scaler.transform(validation_images)

# Set target values in our labels matrix(e.g. to 0.15 and 0.85).
if target_values is not None:
for i in range(len(train_labels[:,0])):
for j in range(len(train_labels[0,:])):
if train_labels[i,j] < 0.5:
train_labels[i,j] = target_values[0]
else:
train_labels[i,j] = target_values[1]

print('\n============\n  SETTINGS\n============')
print('Target Values: ', target_values)
print('Number of Epochs: ', epochs)
```

```
print('V Learning Rate: ', v_learning_rate)
print('W Learning Rate: ', w_learning_rate)
print('V Decay Rate: ', v_decay_rate)
print('W Decay Rate: ', w_decay_rate)
print('Randomized: ', randomized)

NN = NeuralNet(train_images, train_labels)

for i in range(epochs):
if (i > 0):
data = np.concatenate((train_images, train_labels), axis=1)
np.random.shuffle(data)
train_images = data[:, :len(images[0])]
train_labels = data[:, len(images[0]):]
NN.updateData(train_images, train_labels)
if ((i % 5) == 0):
_v_learning_rate = v_decay_rate*v_learning_rate
_w_learning_rate = w_decay_rate*w_learning_rate
NN.train(v_learning_rate=_v_learning_rate, w_learning_rate=_w_learning_rate,
randomized=randomized)

print('\n============\n  EPOCH:', i, '\n============')

# TRAINING ACCURACY
y_hat = NN.classifyAll(train_images)
test_correct = 0
test_size = len(NN.images)
for j in range(test_size):
if (y_hat[j] == np.argmax(NN.labels[j]) + 1):
test_correct += 1
else:
continue
print('\nTest Classfication Complete:')
print('Test Set Error: ', 1 - (test_correct / test_size))

# VALIDATION ACCURACY
total_correct = 0
validation_size = len(validation_images)
z = NN.classifyAll(validation_images)
for j in range(len(z)):
if (z[j] == np.argmax(validation_labels[j]) + 1):
total_correct += 1
else:
continue
print('\nValidation Classification Complete:')
```

```
print('Validation Error Rate: ', 1 - (total_correct/validation_size), '\n')

if __name__ == '__main__':
main(argv)
```

C practice.py

```
from trainer import main

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

main(v_learning_rate=.1, w_learning_rate=.01, epochs=10)
```

D batch_trainer.py

```
from NeuralNetwork import NeuralNet
import numpy as np
import sklearn.preprocessing as skp

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

def main(batch_size = 25, target_values=[0.1,0.9], epochs=7,
v_learning_rate=0.1, w_learning_rate=0.01, v_decay_rate=1, w_decay_rate=1,
decay_frequency=4):
_v_learning_rate = v_learning_rate
_w_learning_rate = w_learning_rate
images = np.load('data/images.npy')
labels = np.load('data/vec_labels.npy')
data = np.concatenate((images, labels), axis=1)
np.random.shuffle(data)

train_images = data[:round(.8*len(data)), :len(images[0])]
train_labels = data[:round(.8*len(data)), len(images[0]):]
validation_images = data[round(.8*len(data)):, :len(images[0])]
validation_labels = data[round(.8*len(data)):, len(images[0]):]

scaler = skp.StandardScaler()
train_images = scaler.fit_transform(train_images)
validation_images = scaler.transform(validation_images)


# Set target values in our labels matrix(e.g. to 0.15 and 0.85).
if target_values is not None:
for i in range(len(train_labels[:,0])):
for j in range(len(train_labels[0,:])):
if train_labels[i,j] < 0.5:
train_labels[i,j] = target_values[0]
else:
train_labels[i,j] = target_values[1]

print('\n============\n  SETTINGS\n============')
print('Batch Size: ', batch_size)
print('Target Values: ', target_values)
print('Number of Epochs: ', epochs)
```

```
print('V Learning Rate: ', v_learning_rate)
print('W Learning Rate: ', w_learning_rate)
print('V Decay Rate: ', v_decay_rate)
print('W Decay Rate: ', w_decay_rate)
print('Decay Frequency: ', decay_frequency)

NN = NeuralNet(train_images, train_labels)

for i in range(epochs):
if (i > 0):
data = np.concatenate((train_images, train_labels), axis=1)
np.random.shuffle(data)
train_images = data[:, :len(images[0])]
train_labels = data[:, len(images[0]):]
NN.updateData(train_images, train_labels)
if ((i % decay_frequency) == 0):
_v_learning_rate = v_decay_rate*_v_learning_rate
_w_learning_rate = w_decay_rate*_w_learning_rate

NN.trainMini(batch_size=batch_size,v_learning_rate=_v_learning_rate,
w_learning_rate=_w_learning_rate)

print('\n============\n  EPOCH:', i, '\n============')

# TRAINING ACCURACY
y_hat = NN.classifyAll(train_images)
test_correct = 0
test_size = len(NN.images)
for j in range(test_size):
if (y_hat[j] == np.argmax(NN.labels[j]) + 1):
test_correct += 1
else:
continue
print('\nTest Classfication Complete:')
print('Test Set Error: ', 1 - (test_correct / test_size))

# VALIDATION ACCURACY
total_correct = 0
validation_size = len(validation_images)
z = NN.classifyAll(validation_images)
for j in range(len(z)):
if (z[j] == np.argmax(validation_labels[j]) + 1):
total_correct += 1
else:
continue
```

```
print('\nValidation Classification Complete:')
print('Validation Error Rate: ', 1 - (total_correct/validation_size), '\n')

if __name__ == '__main__':
main(argv)
```

E practice.py

```
from batch_trainer import main

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

main(target_values=[0.05,0.95], epochs=48, v_decay_rate=0.9,
w_decay_rate=0.6, decay_frequency=8)
```

F visualize.py

```
from NeuralNetwork import NeuralNet
import numpy as np
import sklearn.preprocessing as skp
import matplotlib.pyplot as plt
import matplotlib.cm as cm

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

images = np.load('data/images.npy')
labels = np.load('data/vec_labels.npy')
data = np.concatenate((images, labels), axis=1)
np.random.shuffle(data)

train_images = data[:round(.8*len(data)), :len(images[0])]
train_labels = data[:round(.8*len(data)), len(images[0]):]
validation_images = data[round(.8*len(data)):, :len(images[0])]
validation_labels = data[round(.8*len(data)):, len(images[0]):]

scaler = skp.StandardScaler()
train_images = scaler.fit_transform(train_images)
validation_images = scaler.transform(validation_images)

# Set target values in our labels matrix to 0.15 and 0.85
for i in range(len(train_labels[:,0])):
for j in range(len(train_labels[0,:])):
if train_labels[i,j] < 0.5:
train_labels[i,j] = 0.1
else:
train_labels[i,j] = 0.9

# CREATE NeuralNet Object
NN = NeuralNet(train_images, train_labels)

# TRAIN NeuralNet Object
for i in range(5):
if (i > 0):
data = np.concatenate((train_images, train_labels), axis=1)
np.random.shuffle(data)
train_images = data[:, :len(images[0])]
```

```
train_labels = data[:, len(images[0]):]
NN.updateData(train_images, train_labels)
NN.trainMini(batch_size=25, v_learning_rate=0.1, w_learning_rate=0.01)

correct_list = list()
incorrect_list = list()

# TRAINING ACCURACY
y_hat = NN.classifyAll(train_images)
test_correct = 0
test_size = len(NN.images)
for i in range(test_size):
if (y_hat[i] == np.argmax(NN.labels[i]) + 1):
test_correct += 1
else:
continue
print('\nTest Classfication Complete:')
print('Test Set Error: ', 1 - (test_correct / test_size))

# VALIDATION ACCURACY
total_correct = 0
validation_size = len(validation_images)
z = NN.classifyAll(validation_images)
for i in range(len(z)):
if (z[i] == np.argmax(validation_labels[i]) + 1):
total_correct += 1
correct_list.append(validation_images[i])
else:
incorrect_list.append(validation_images[i])
continue
print('\nValidation Classification Complete:')
print('Validation Error Rate: ', 1 - (total_correct/validation_size), '\n')

c_map = cm.get_cmap('Greys_r')

for i in range(5):
plt.imsave('images/correct'+str(i), correct_list[i].reshape(28,28), cmap=c_map)
plt.imsave('images/incorrect'+str(i), incorrect_list[i].reshape(28,28), cmap=c_map)
```

G cost_plot.py

```
from NeuralNetwork import NeuralNet
import scipy.io as sio
import numpy as np
import sklearn.preprocessing as skp
import matplotlib.pyplot as plt

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

images = np.load('data/images.npy')
labels = np.load('data/vec_labels.npy')
data = np.concatenate((images, labels), axis=1)
np.random.shuffle(data)


train_images = data[:round(.8*len(data)), :len(images[0])]
train_labels = data[:round(.8*len(data)), len(images[0]):]
validation_images = data[round(.8*len(data)):, :len(images[0])]
validation_labels = data[round(.8*len(data)):, len(images[0]):]

scaler = skp.StandardScaler()
train_images = scaler.fit_transform(train_images)
validation_images = scaler.transform(validation_images)

# Set target values in our labels matrix to 0.15 and 0.85
for i in range(len(train_labels[:,0])):
for j in range(len(train_labels[0,:])):
if train_labels[i,j] < 0.5:
train_labels[i,j] = 0.1
else:
train_labels[i,j] = 0.9

# CREATE NeuralNet Object
NN = NeuralNet(train_images, train_labels)

# TRAIN NeuralNet Object
for i in range(1):
x_plot, y_plot = NN.trainPlot(v_learning_rate=0.01, w_learning_rate=0.001)
# Plot of cost function vs number of iterations.
plt.plot(x_plot, y_plot, 'ro')
```

```
plt.xlabel('Number of Iterations')
plt.ylabel('J(y, z; x, V, W)')
plt.title('Cost Function vs. Number of Iterations')
plt.savefig('Cost_Function.png', bbox_inches='tight')

# TRAINING ACCURACY
y_hat = NN.classifyAll(train_images)
test_correct = 0
test_size = len(NN.images)
for i in range(test_size):
if (y_hat[i] == np.argmax(NN.labels[i]) + 1):
test_correct += 1
else:
continue
print('\nTest Classfication Complete:')
print('Test Set Error: ', 1 - (test_correct / test_size))

# VALIDATION ACCURACY
total_correct = 0
validation_size = len(validation_images)
z = NN.classifyAll(validation_images)
for i in range(len(z)):
if (z[i] == np.argmax(validation_labels[i]) + 1):
total_correct += 1
else:
continue
print('\nValidation Classification Complete:')
print('Validation Error Rate: ', 1 - (total_correct/validation_size), '\n')
```

H kaggle_submission.py

```
from NeuralNetwork import NeuralNet
import scipy.io as sio
import numpy as np
import sklearn.preprocessing as skp

'''
Written by Cameron W.J. Abrams 4/13/2017
'''

images = np.load('data/images.npy')
labels = np.load('data/vec_labels.npy')
data = np.concatenate((images, labels), axis=1)
np.random.shuffle(data)

train_images = data[:, :len(images[0])]
train_labels = data[:, len(images[0]):]

scaler = skp.StandardScaler()
train_images = scaler.fit_transform(train_images)
test_data = np.load('data/test_data.npy')
test_data = scaler.transform(test_data)

# Set target values in our labels matrix(e.g. to 0.15 and 0.85).
for i in range(len(train_labels[:,0])):
for j in range(len(train_labels[0,:])):
if train_labels[i,j] < 0.5:
train_labels[i,j] = .05
else:
train_labels[i,j] = .95

NN = NeuralNet(train_images, train_labels)

for i in range(8):
if (i > 0):
data = np.concatenate((train_images, train_labels), axis=1)
train_images = data[:, :len(images[0])]
train_labels = data[:, len(images[0]):]
NN.updateData(train_images, train_labels)
NN.trainMini(batch_size=50,v_learning_rate=0.1, w_learning_rate=0.01,
v_decay_rate=1, w_decay_rate=1)
```

```
f = open('kaggle_submission.csv', 'w')
header = 'Id,Category\n'
f.write(header)

predictions = NN.classifyAll(test_data)
for i in range(len(test_data)):
predicted_class = predictions[i]
S = str(i + 1) + ',' + str(predicted_class) + '\n'
f.write(S)
f.close()
```