## On a Logical Foundation for Explicit Substitutions

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Traditionally, calculi of explicit substitution [1] have been conceived as an implementation technique for  $\beta$ -reduction and studied with the tools of rewriting theory. This computational view has been extremely fruitful (see [2] for a recent survey) and raises the question if there may also be a more abstract underlying logical foundation.

Some forms of explicit substitution have been related to cut in the intuitionistic sequent calculus [3]. While making a connection to logic, the interpretation of explicit substitutions remains primarily computational since they do not have a reflection at the level of propositions, only at the level of proofs.

In recent joint work [4], we have shown how explicit substitutions naturally arise in the study of intuitionistic modal logic. Their logical meaning is embodied by a contextual modality which captures all assumptions a proof of a proposition may rely on. Explicit substitutions mediate between such contexts and therefore, intuitively, between worlds in a Kripke-style interpretation of modal logic.

In this talk we review this basic observation about the logical origin of explicit substitutions and generalize it to a multi-level modal logic. Returning to the computational meaning, we see that explicit substitutions are the key to a  $\lambda$ -calculus where variables, meta-variables, meta-variables, etc. can be unified without the usual paradoxes such as lack of  $\alpha$ -conversion. We conclude with some speculation on potential applications of this calculus in logical frameworks or proof assistants.

## References

- 1. Abadi, M., Cardelli, L., Curien, P.L., Lévy, J.J.: Explicit substitutions. Journal of Functional Programming 1(4), 375–416 (1991)
- 2. Kesner, D.: The theory of calculi with explicit substitutions revisited. Unpublished manuscript (October (2006)
- 3. Herbelin, H.: A lambda-calculus structure isomorphic to Gentzen-style sequent calculus structure. In: Pacholski, L., Tiuryn, J. (eds.) CSL 1994. LNCS, vol. 933, pp. 61–75. Springer, Heidelberg (1995)
- 4. Nanevski, A., Pfenning, F., Pientka, B.: Contextual modal type theory. Transactions on Computational Logic (To appear, 2007)

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