

$$g(x) = \int_{-\infty}^{\infty} s_0(y)s_0(y+x)dy = \begin{cases} \int_{-3}^{3-x} \sin(2\pi y) \sin(2\pi(x+y)) dy, & \text{for } x \in [0,6] \\ \int_{-3-x}^3 \sin(2\pi y) \sin(2\pi(x+y)) dy, & \text{for } x \in [-6,0) \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} & \int_{-3}^{3-x} \sin(2\pi y) \sin(2\pi(x+y)) dy \\ &= \int_{-3}^{3-x} \sin(2\pi y) [\sin(2\pi x) \cos(2\pi y) + \cos(2\pi x) \sin(2\pi y)] dy \\ &= \int_{-3}^{3-x} \sin(2\pi x) \sin(2\pi y) \cos(2\pi y) + \cos(2\pi x) \sin^2(2\pi y) dy \\ &= \int_{-3}^{3-x} \frac{1}{2} \sin(2\pi x) \sin(4\pi y) + \frac{1}{2} \cos(2\pi x) (1 - \cos(4\pi y)) dy \\ &= \frac{-1}{8\pi} \sin(2\pi x) [\cos(4\pi y)]_{-3-x}^{3-x} + \frac{1}{8\pi} \cos(2\pi x) [4\pi y - \sin(4\pi y)]_{-3-x}^{3-x} \\ &= \frac{-1}{8\pi} \sin(2\pi x) [\cos(4\pi x) - 1] + \frac{1}{8\pi} \cos(2\pi x) [24\pi - 4\pi x + \sin(4\pi x)] \\ &= \frac{1}{8\pi} [\sin(4\pi x) \cos(2\pi x) - \sin(2\pi x) \cos(4\pi x) + \sin(2\pi x) + 24\pi \cos(2\pi x) - 4\pi x \cos(2\pi x)] \\ &= \frac{1}{8\pi} [2\sin(2\pi x) + 24\pi \cos(2\pi x) - 4\pi x \cos(2\pi x)] \\ &= \frac{1}{4\pi} \sin(2\pi x) + 3\cos(2\pi x) - \frac{1}{2} x \cos(2\pi x) \end{aligned}$$

Similarly,

$$\begin{aligned} & \int_{-3-x}^3 \sin(2\pi y) \sin(2\pi(x+y)) dy \\ &= \frac{-1}{8\pi} \sin(2\pi x) [\cos(4\pi y)]_{-3-x}^3 + \frac{1}{8\pi} \cos(2\pi x) [4\pi y - \sin(4\pi y)]_{-3-x}^3 \\ &= \frac{-1}{8\pi} \sin(2\pi x) [1 - \cos(4\pi x)] + \frac{1}{8\pi} \cos(2\pi x) [24\pi + 4\pi x - \sin(4\pi x)] \\ &= \frac{1}{8\pi} [\sin(2\pi x) \cos(4\pi x) - \sin(4\pi x) \cos(2\pi x) - \sin(2\pi x) + 24\pi \cos(2\pi x) + 4\pi x \cos(2\pi x)] \\ &= \frac{1}{8\pi} [-2\sin(2\pi x) + 24\pi \cos(2\pi x) + 4\pi x \cos(2\pi x)] \\ &= \frac{-1}{4\pi} \sin(2\pi x) + 3\cos(2\pi x) + \frac{1}{2} x \cos(2\pi x) \end{aligned}$$

Hence, we obtain:

$$g(x) = \int_{-\infty}^{\infty} s_0(y)s_0(y+x)dy = \begin{cases} \frac{1}{4\pi} \sin(2\pi|x|) + 3\cos(2\pi x) - \frac{1}{2}|x|\cos(2\pi x), & \text{for } x \in [-6,6] \\ 0 & \text{otherwise} \end{cases}$$