Q1. Matlab code:

```
% (1) compostie trapezoidal rule
syms x
f = sqrt(1+(sin(x))^3);
ns = [8 16 32];
integrals = [0 0 0];
for i = 1:3
   n = ns(i);
   x0 = 0;
   x1 = 1;
   h=(x1-x0)/n;
   for j=1:n
      integrals(i)=integrals(i)+vpa(subs(f,x0))+vpa(subs(f,x0+h));
      x0=x0+h;
   end;
   integrals(i)=integrals(i)*h/2;
end;
integrals
```

Result:

n	8	16	32
$\int_0^1 \sqrt{1+sin^3 x}dx$	1.0833	1.0828	1.0827

```
integrals =

    1.0833    1.0828    1.0827
```

```
% (2) compostie two-point Gauss quadrature rule
syms x c1 c2 x1 x2
f = sqrt(1+(sin(x))^3);
ns = [8 16 32];
integrals = [0 0 0];
for i=1:3
   n = ns(i);
   xa=0;
   xb=1;
   h=(xb-xa)/n;
   for j=1:n
      eq1 = c1+c2-vpa(subs(int(x^0,[xa,xa+h])));
      eq2 = c1*x1+c2*x2-vpa(subs(int(x,[xa,xa+h])));
      eq3 = c1*x1^2+c2*x2^2-vpa(subs(int(x^2,[xa,xa+h])));
      eq4 = c1*x1^3+c2*x2^3-vpa(subs(int(x^3,[xa,xa+h])));
      sol = solve(eq1,eq2,eq3,eq4);
      integrals(i) = integrals(i)+sol.c1(1)*vpa(subs(f,sol.x1(1)))+sol.c2(1)*vpa(subs(f,sol.x2(1)));
      xa = xa + h;
   end;
```

```
end;
integrals
```

Result:

n	8	16	32
$\int_0^1 \sqrt{1+sin^3 x}dx$	1.0827	1.0827	1.0827

```
integrals =

    1.0827    1.0827    1.0827
```

Q2 $\begin{cases} 4x_1 + x_2 - x_3 = 4 \\ -x_1 + 3x_2 + x_3 = -3 \\ 2x_1 + 2x_2 + 5x_3 = 6 \end{cases}$

(1) Jacobi method

First, rewrite the system of equation as:

$$\begin{cases} x_1 = -\frac{1}{4}x_2 + \frac{1}{4}x_3 + 1 \\ x_2 = \frac{1}{3}x_1 - \frac{1}{3}x_3 - 1 \\ x_3 = -\frac{2}{5}x_1 - \frac{2}{5}x_2 + \frac{6}{5} \end{cases}$$

Given $x^{(0)} = (0,0,0)^t$, after 1st iteration:

$$\begin{cases} x_1 = -\frac{1}{4}(0) + \frac{1}{4}(0) + 1 = 1 \\ x_2 = \frac{1}{3}(0) - \frac{1}{3}(0) - 1 = -1 \\ x_3 = -\frac{2}{5}(0) - \frac{2}{5}(0) + \frac{6}{5} = 1.2 \end{cases}$$

We have $x^{(1)} = (1,-1,1.2)^t$, and after 2nd iteration :

$$\begin{cases} x_1 = -\frac{1}{4}(-1) + \frac{1}{4}(1.2) + 1 = 1.55 \\ x_2 = \frac{1}{3}(1) - \frac{1}{3}(1.2) - 1 = -1.067 \\ x_3 = -\frac{2}{5}(1) - \frac{2}{5}(-1) + \frac{6}{5} = 1.2 \end{cases}$$

We have $x^{(2)} = (1.55,-1.067,1.2)^t$

(2) Gauss-Seidel method

First, rewrite the system of equation as:

$$x_1^{(k)} = -\frac{1}{4}x_2^{(k-1)} + \frac{1}{4}x_3^{(k-1)} + 1$$

$$x_2^{(k)} = \frac{1}{3}x_1^{(k)} - \frac{1}{3}x_3^{(k-1)} - 1$$

$$x_3^{(k)} = -\frac{2}{5}x_1^{(k)} - \frac{2}{5}x_2^{(k)} + \frac{6}{5}$$

Given $x^{(0)} = (0,0,0)^t$, after 1st iteration:

$$x_1^{(1)} = -\frac{1}{4}(0) + \frac{1}{4}(0) + 1 = 1$$

$$x_2^{(1)} = \frac{1}{3}(1) - \frac{1}{3}(0) - 1 = -0.667$$

$$x_3^{(1)} = -\frac{2}{5}(1) - \frac{2}{5}(-0.667) + \frac{6}{5} = 1.067$$

We have $x^{(1)} = (1, -0.667, 1.067)^t$, and after 2nd iteration :

$$x_1^{(2)} = -\frac{1}{4}(-0.667) + \frac{1}{4}(1.067) + 1 = 1.433$$

$$x_2^{(2)} = \frac{1}{3}(1.433) - \frac{1}{3}(1.067) - 1 = -0.878$$

$$x_3^{(2)} = -\frac{2}{5}(1.433) - \frac{2}{5}(-0.878) + \frac{6}{5} = 0.978$$

We have $x^{(2)} = (1.433, -0.878, 0.978)^t$

Q3. Matlab code:

```
% (1) Gauss-Seidel method
A = [0 -1/4 1/4 -1/4 -3/4; -1/4 0 1/4 1/4 -1/2; 1/5 1/5 0 -1/5 1; -1/3 1/3 -1/3 0 2/3];
x = [0 0 0 0 1]';
x_sol = [0 0 0 0]';
epsilon = 1e-3;
for i=1:4
   x(i) = dot(A(i,:),x);
end;
x_sol(:,2) = x(1:4);
c = 2;
while norm(x_sol(:,c)-x_sol(:,c-1),Inf)>epsilon
   for i=1:4
      x(i) = dot(A(i,:),x);
   end;
   c=c+1;
   x_sol(:,c)=x(1:4);
end;
x_sol(1:4,:)
```

Result:

8 iterations had been done with $x^{(8)} = (-0.7531, 0.0410, 0.7193, 0.6916)^t$

```
ans =

         0   -0.7500   -0.6125   -0.7085   -0.7361   -0.7473   -0.7512   -0.7526   -0.7531
         0   -0.3125   -0.0125    0.0213    0.0349    0.0389    0.0403    0.0408    0.0410
         0    0.7875    0.7650    0.7402    0.7271    0.7221    0.7202    0.7195    0.7193
         0    0.5500    0.6117    0.6632    0.6813    0.6880    0.6904    0.6913    0.6916
```

```
% (2) SOR method
A = [0 -1/4 1/4 -1/4 -3/4; -1/4 0 1/4 1/4 -1/2; 1/5 1/5 0 -1/5 1; -1/3 1/3 -1/3 0 2/3];
x = [0 0 0 0 1]';
x_sol = [0 0 0 0]';
w=1.2;
epsilon = 1e-3;
for i=1:4
   x(i) = (1-w)*x(i)+w*dot(A(i,:),x);
end;
x_sol(:,2) = x(1:4);
c = 2;
while norm(x_sol(:,c)-x_sol(:,c-1),Inf)>epsilon
   for i=1:4
      x(i) = (1-w)*x(i)+w*dot(A(i,:),x);
   end;
   c=c+1;
   x_sol(:,c)=x(1:4);
end;
x_sol(1:4,:)
```

Result:

7 iterations had been done with $x^{(7)} = (-0.7535, 0.0410, 0.7192, 0.6918)^t$

```
ans =

         0   -0.9000   -0.5494   -0.7832   -0.7500   -0.7564   -0.7527   -0.7535
         0   -0.3300    0.1021    0.0281    0.0484    0.0394    0.0414    0.0410
         0    0.9048    0.7518    0.7180    0.7171    0.7189    0.7193    0.7192
         0    0.6661    0.6266    0.7120    0.6901    0.6928    0.6914    0.6918
```