**Problem Set 7A (due 11:59 pm, 9 December 2021)**

1. **Wealth Game of Hang Seng Index**

Write a Python program modeling agents using Wealth Game strategies and payoffs to invest in the time series of Hang Seng Index. Fix the memory size . Below are the suggested logical steps:

1. Download the time series of Hang Seng Index as it was done in Problem Set 5. This will form the time series of the prices .
2. Set the market impact factor to Starting from , the time series of the transaction price will be generated using
3. Initialize the input state at to 0 (that is, 000 for ).

Simulation of the strategies

1. For each of the 256 strategies with , initialize the virtual wealth of each strategy by
2. Initialize the virtual positions of each strategy to
3. Starting from calculate the input states of the strategies according to

where is the index of the price change from to (equal to 1 for price rise and 0 for a price drop). Refer to Problem Set 6 for this equation.

1. Calculate the decision of strategy at time with input state according to the methods used in Problem Set 6. Update the position of each strategy as long as the position of the strategy is bounded by This is implemented by the equation

where is the virtual wealth of strategy at time and is the stock price at time Note that the max and min functions are used to restrict the positions to within the limit.

1. Update the virtual wealth of the strategies using the formula

Simulation of the agents

1. For each of the agents, initialize the virtual wealth of agent by
2. Initialize the real positions of each agent to
3. For each of the agents, generate a random integer between 0 and 1. This will be used to determine the choice of strategies of each agent at . Starting from , calculate the decision of agent following the strategy with the highest virtual wealth belonging to the agent at each time step. The real positions and the real wealth of the agents are calculated using formulas similar to those in steps (7) and (8) for the strategies.

Presentation of results and discussions

1. Calculate the best, worst, and average virtual wealth of the strategies at the end of the game. Then calculate the best, worst, and average real wealth of the agents at the end of the game. To facilitate a better interpretation of your results, divide your results by the initial price.
2. Find the best 3 and worst 3 strategies and plot a graph of their virtual wealth as a function of time. For comparison, also plot the stock index multiplied by 5.
3. Find the best 3 and worst 3 agents and plot a graph of their real wealth as a function of time. For comparison, also plot the stock index multiplied by 5.
4. Write down some observations of the strategies and the agents. This is an open-ended question and I encourage you to have some analysis. For example, what makes a strategy a winning or losing one, if any? Are there any need for an agent to change strategy when the environment changes? What strategy/agent is good when the price has a trend? What strategy/agent is good when the price trend is rugged? Etc.
5. **Portfolio Management with Principal Component Analysis**

This problem should be completed with Excel using the file PS7B\_PCA.xlsm.

(1) Download from Yahoo!Finance the daily closing price of the following constituents of S&P 500:

MSFT, AAPL, AMZN, GOOGL, TSLA, GOOG, FB, NVDA, BRK-B, JPM

Also download the daily closing price of S&P 500 (^GSPC).

The period is from 20 November 2020 to 19 November 2021. (In Yahoo!Finance you need to type the end date as 20/20/2021.) Copy the prices to columns A to L. In columns N to X, calculate the return of the stock on day from 20 November 2020 to 18 November 2021 using the formula

where is the stock price on day

(2) In AA3:AJ12, calculate the correlation matrix of the 10 stocks. The useful Excel command is CORREL with the following syntax:

CORREL(array1, array2),

where array1 and array2 are the price time series of stock 1 and stock 2, respectively.

To save effort in calculating the correlation functions repeatedly, array1 and array2 can be replaced by the INDEX command that returns the price time series. Its syntax is:

INDEX(area, row number, column number),

where:

area is the array containing all the price time series.

row number should be set to 0, which enables the command to return a column from area (if it is nonzero, it will only return a single element of that row number in the area).

column number is the column number containing the price time series of a stock. It should be read from the row or column label of the correlation matrix, so that the matrix can be filled up by dragging the cells.

(3) In AA16:AJ25 and AA29:AJ38, calculate the eigenvalues and eigenvectors of the correlation matrix, respectively. The macro of this calculation can be downloaded from Google by typing:

“Download worksheet for calculating eigenvectors and eigenvalues”.

The method of calculation is explained in the worksheet “Eigencalcs”. Note that for matrix operations in Excel, one should type Ctrl-Shift-Enter after marking the area covered by the matrix.

(4) Since the cells in the matrix area cannot be edited individually, copy the eigenvalues and eigenvectors into AA42:AJ52. Use the Paste-Value command. Note that in the eigenvalue matrix AA16:AJ25, all non-diagonal elements are zero, and so the most convenient way to copy the eigenvalues to a row is to sum the columns of the matrix.

(5) In AA56:AJ66, sort the eigenvalues and eigenvectors in descending magnitude of the eigenvalues. The “Sort left to right” option in Custom Sort should be useful.

(6) Plot the eigenvalues of the eigenmodes versus the eigenmode numbers. You may find that it is difficult to determine the number of principal portfolios to be included, probably due to the small number of stocks being studied. Hence, we will use a different criterion in the next part.

(7) In AA69:AJ69, calculate the cumulative variance. Let be the number of first few principal portfolios whose cumulative variance corresponds to the 80% cutoff. Determine .

**Cluster Visualization**

(8) Plot the 10 stocks in the space of the 2nd and 3rd principal components. Comment on any clusters that you observe, and comment on the nature of the clusters.

**Diversification Strategies**

Below we investigate 4 diversification strategies.

1. **Stock investment with equal weight**

(9) In this strategy, the capital allocated to each of the 10 stocks is the same. Considering the stock price, calculate the volume of stocks to be invested. Then calculate the value of the portfolio in column BB. The Excel command SUMPRODUCT is useful here (and also in many other calculations below).

**(B) Risk Parity (or Equal Risk Contribution)**

(10) In this strategy, the risk of stock per unit of capital is estimated by its standard deviation (note that the risk correlations of the stocks are neglected). The capital allocated to each of the 10 stocks is such that the risk of each stock is the same. Hence the capital allocated to each stock is proportional to Calculate the volume of stocks to be invested. Then calculate the value of the portfolio in column BA. The Excel command STDEV is useful for calculating the standard deviation.

**(C) Equal Weight Portfolio (or 1/N Strategy)**

(11) In this strategy, the first principal portfolios are selected, and equal weight is allocated to each of them. The first step is to determine the signs of the eigenvectors to be adopted. We will consider a portfolio with equal capital allocated to the 10 stocks, and require the projection of this portfolio to the principal eigenvectors to be positive. Calculate the projections.

(12) Calculate the sign-corrected eigenvectors. Then calculate the total weight of each stock contributed from the principal portfolios, and the volume of stocks to be invested.

(13) Which stocks in this strategy are in short positions? Calculate the value of the portfolio in column BB. Note that the initial value is no longer equal to 1 due to the short-positioned stocks.

**(D) Diversified Risk Parity**

(14) In this strategy, the capital allocated to each of the principal portfolios are such that the risk of each portfolio is the same. Since the eigenvalue of each portfolio is the variance , the capital allocated to each portfolio is proportional to Calculate the volume of stocks to be acquired. Which stocks in this strategy are in short positions? Then calculate the value of the strategy in column BC.

**Comparison of the Strategies**

(15) Plot the value of the 4 strategies. Comment on any observations you make.

(16) The ability of the 4 strategies to reduce risks is compared by calculating the following parameters at the top of columns BB to BE:

Gain (the final value divided by the initial value)

Standard deviation

Minimum value (divided by the initial value)

Comment on any observations you make.

Remark: In practice, the real test of the performance of the strategies should be done with a different data set (for example, data collected after November 2021).

(17) The standard deviation is not informative of the risk when the price has a trend. Another common risk measure is the beta coefficient, or systematic risk, defined as

Calculate of the 4 strategies from: the covariance of the daily returns of the strategies and the daily returns of the market (S&P 500) averaged over the one-year period, and the variance of the daily returns of the market, also averaged over the one-year period. The Excel commands COVARIANCE.P and VAR are useful.

Comment on any observations you make.

For your further insights and interest, the names of the stocks are listed below:

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| --- | --- |
| MSFT | [Microsoft Corporation](https://www.slickcharts.com/symbol/MSFT) |
| AAPL | [Apple Inc.](https://www.slickcharts.com/symbol/AAPL) |
| AMZN | [Amazon.com Inc.](https://www.slickcharts.com/symbol/AMZN) |
| GOOGL | [Alphabet Inc. Class A](https://www.slickcharts.com/symbol/GOOGL) |
| TSLA | [Tesla Inc](https://www.slickcharts.com/symbol/TSLA) |
| GOOG | [Alphabet Inc. Class C](https://www.slickcharts.com/symbol/GOOG) |
| FB | [Meta Platforms Inc. Class A](https://www.slickcharts.com/symbol/FB) |
| NVDA | [NVIDIA Corporation](https://www.slickcharts.com/symbol/NVDA) |
| BRK-B | [Berkshire Hathaway Inc. Class B](https://www.slickcharts.com/symbol/BRK.B) |
| JPM | [JPMorgan Chase & Co.](https://www.slickcharts.com/symbol/JPM) |