

Deep Learning

Lecture 10: Meta and manifold learning

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Lecture overview

1 Manifold learning

- NLDR with DNNs
- t-SNE and UMAP on DNNs
- designing tailored embeddings
- Jonker-Volgenant assignment

2 Meta learning

- thinking in distributions
- the distribution of all data...
- ...and of all tasks
- definition
- the meta learning support set
- metric, optimisation and model-based

3 Looking forward

- meta learning datasets
- large-scale generative models
- machine reasoning and risk
- take away points



Manifold learning NLDR in DNNs

Definition: NLDR in DNNs

Feature vectors in deep neural networks (DNNs) capture abstract patterns which are interesting to analyse.

We can use nonlinear dimensionality reduction (NLDR) algorithm, such as t-SNE and UMAP to examine these patterns.

The deepest (bottleneck or penultimate layer) features are often the most interesting.

Example: bottleneck features in LeNet

```
class LeNet(nn.Module):
    def __init__(self):
        super(LeNet, self).__init__()
        self.conv1 = nn.Conv2d(1, 6, 5, padding=2)
        self.conv2 = nn.Conv2d(6, 16, 5)
        self.fc1   = nn.Linear(16*5*5, 120)
        self.fc2   = nn.Linear(120, 84)
        self.fc3   = nn.Linear(84, 10)

    def forward(self, x):
        x = F.max_pool2d(F.relu(self.conv1(x)), (2, 2))
        x = F.max_pool2d(F.relu(self.conv2(x)), (2, 2))
        x = flatten(x)
        x = F.relu(self.fc1(x))
        → f = F.relu(self.fc2(x))
        x = self.fc3(f)
        return x
```

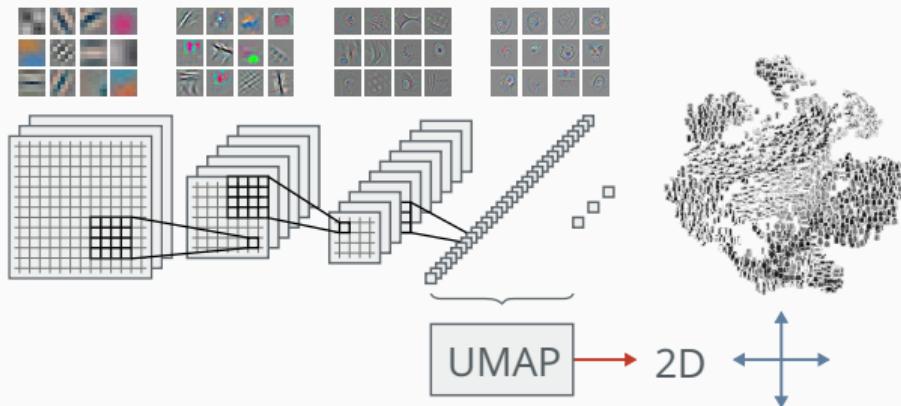
Manifold learning example t-SNE and UMAP on DNNs

Usage: t-SNE [1] or UMAP [2]

```
import torch
from sklearn.manifold import TSNE

# f = features for whole dataset
f = torch.randn(1000, 84, 1, 1)

# specify embedding to 2D
g = TSNE(2).fit_transform(f.squeeze())
print(g.shape) # returns (1000,2)
```

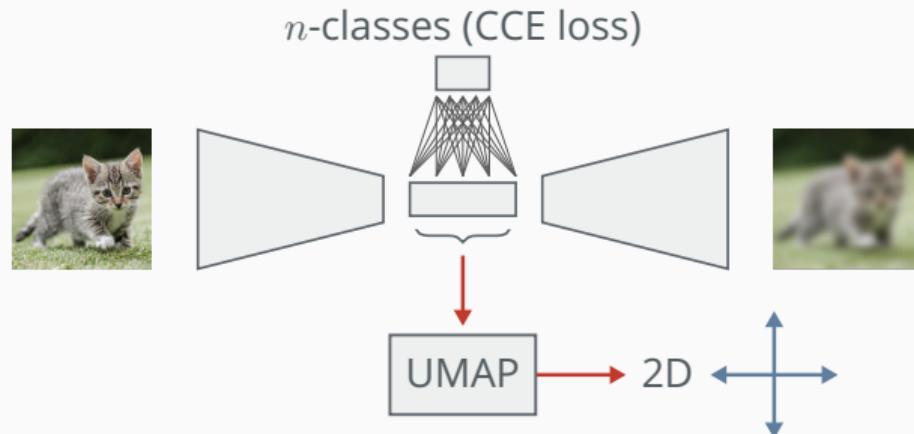


Manifold learning designing tailored embeddings

Example: tailored embeddings

The embedding space can be controlled by additional constraints, such as reconstruction term, additional losses (classification, regression).

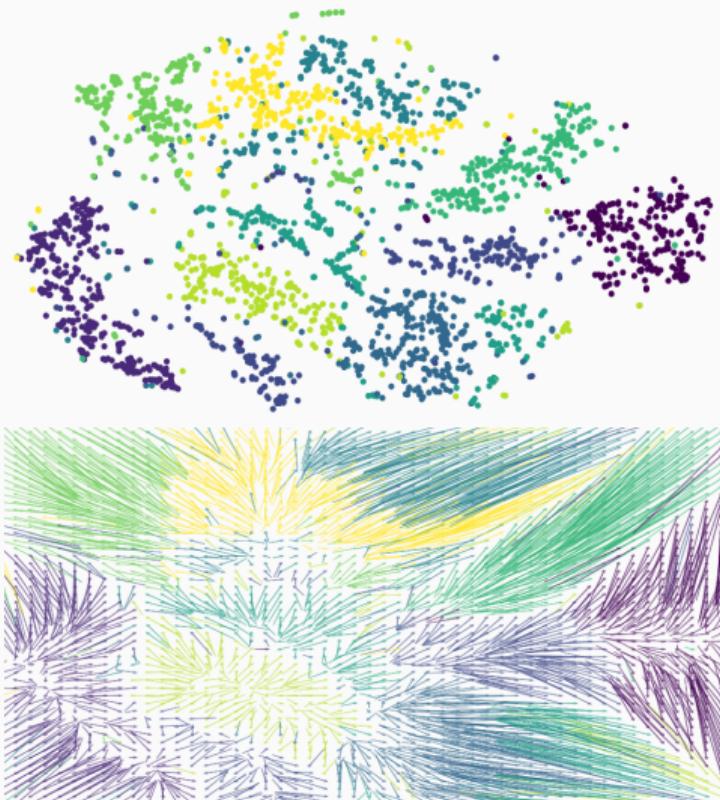
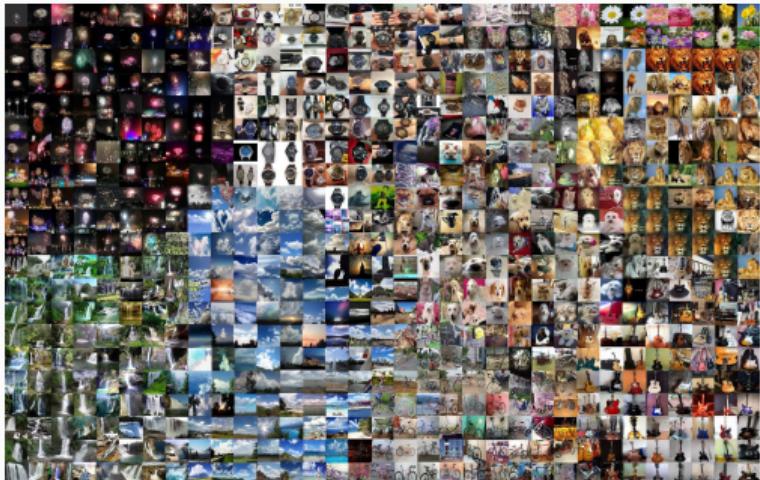
What will the 2D embedding be like for the following architecture?



Manifold learning Jonker-Volgenant assignment

Example: Jonker-Volgenant

A visualisation trick is to minimise an assignment cost to optimise the layout of the embeddings. The Jonker-Volgenant algorithm can be used for this, giving:

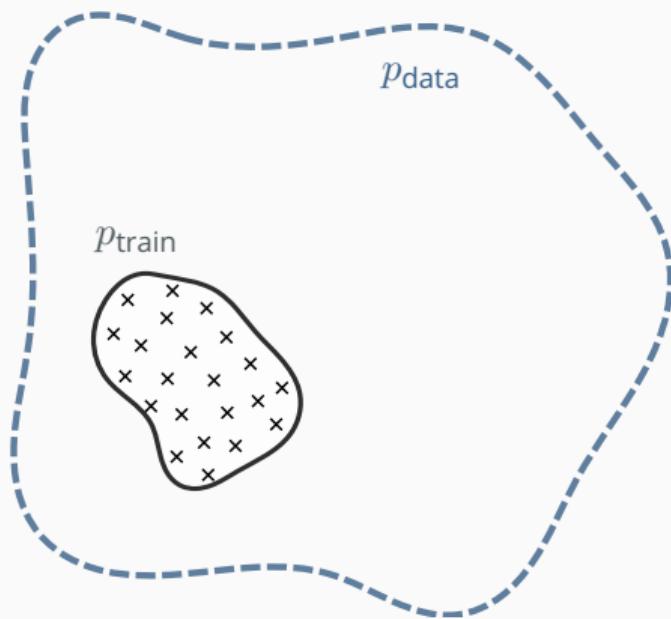




Meta learning thinking in distributions

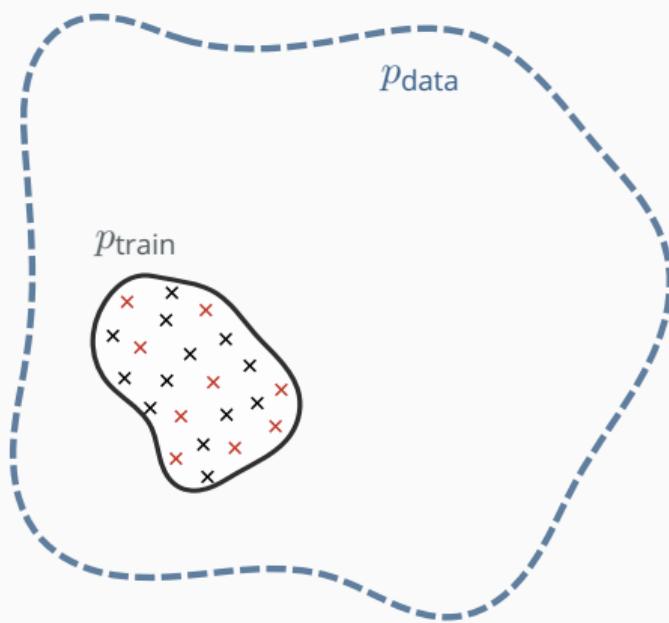
A common lie...

...is that test data $\stackrel{\text{i.i.d.}}{\sim}$ train data (no!)



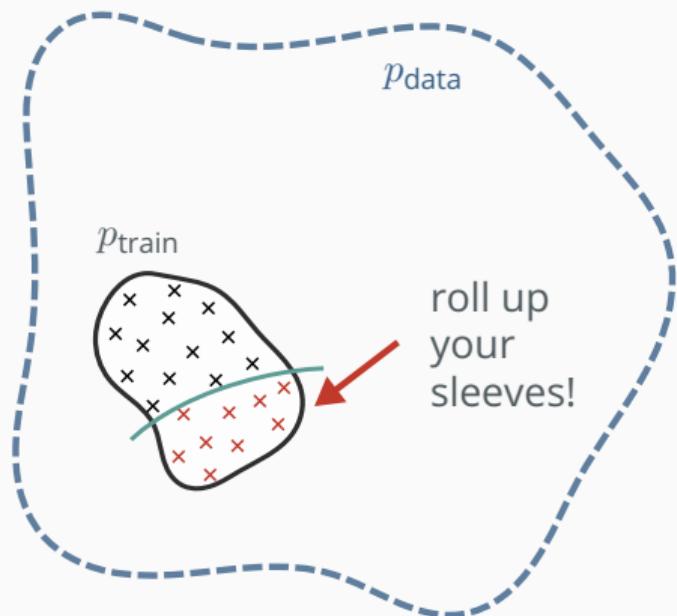
99.7% "test accuracy"!

(your boss and the investors are happy)

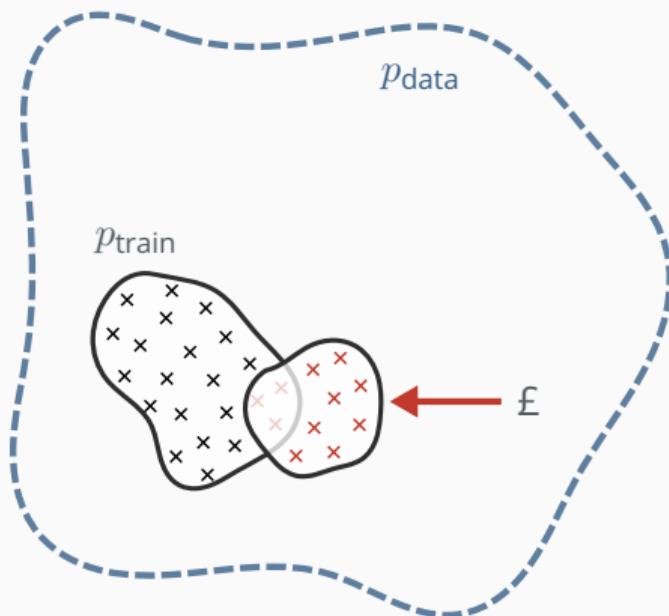


Meta learning thinking in distributions

62% "test" accuracy
(but closer to the truth)

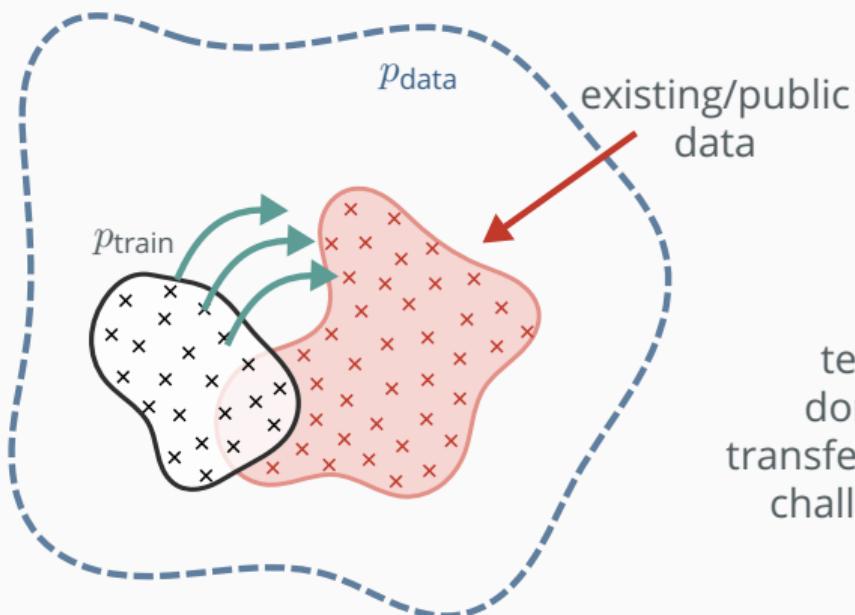


51% "test" accuracy
(even closer to the truth)

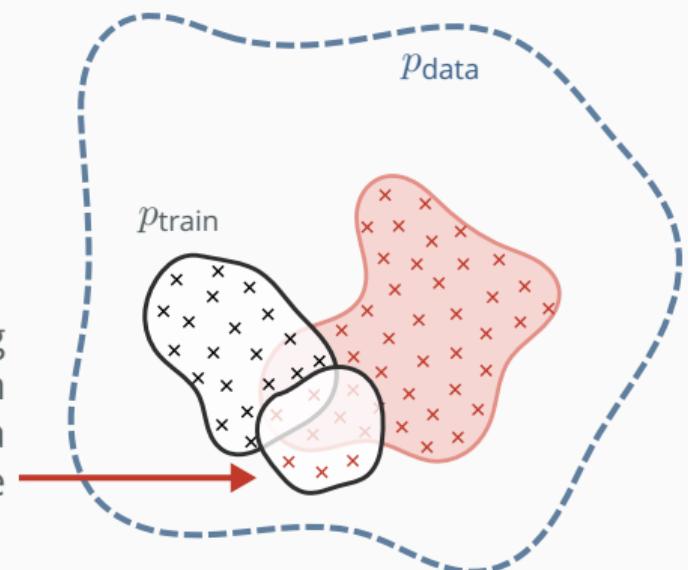


Meta learning thinking in distributions

Generative models (e.g. domain adaptation, transfer and meta learning)

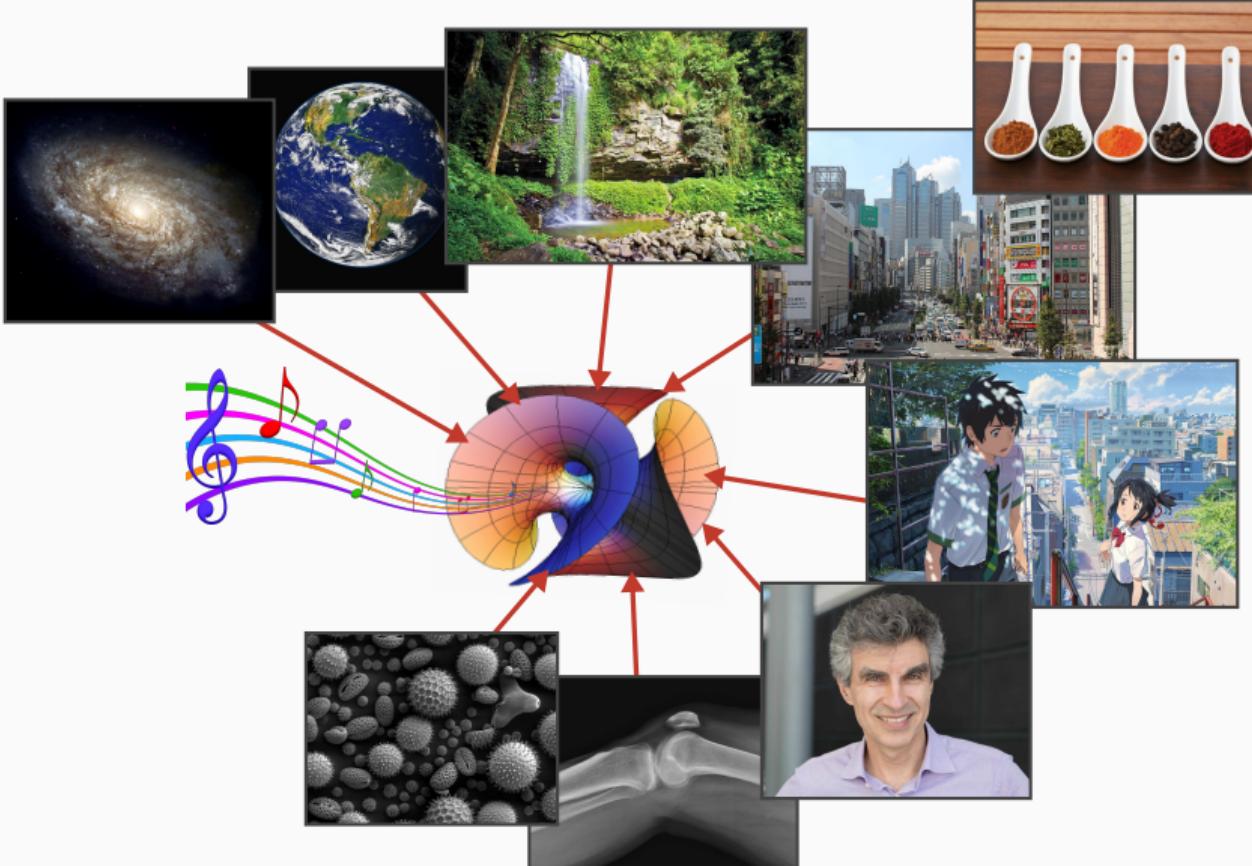


testing
domain
transfer is a
challenge





Meta learning the distribution of all data



Meta learning the distribution of all tasks





Meta learning inferring new tasks

We would like to be able to generalise to unseen tasks. What do you do with these?



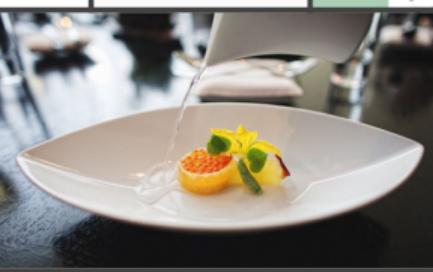
Solve, admire...



Play, complete, improvise...



Solve, comprehend...



Play, measure...

Eat, taste, smell...

1 This coil of piping has 15 turns. The diameter of the coil is about 3 m.

(a) Estimate the length of the pipe.
(b) This is the cross-section of the pipe. 15 cm.

2 A cone-metre length has mass 0.09 kg. Calculate the density of the material. State the units in your answer.

(a) $\pi = 3.14$
 $= 343 \text{ m}$ (use 15 cm)

(b) Area of cross section = $\pi r^2 = \pi (0.15)^2 = 0.0707 \text{ m}^2$
Volume of 1 metre length = $0.0707 \times 3.14 = 0.221 \text{ m}^3$
 $\frac{0.09}{0.221} = 0.404 \text{ kg/m}^3$
Density = $\frac{0.09}{0.221} = 0.404 \text{ kg/m}^3$

3 ABCD is an isosceles trapezoid. Angle ACD = x° .

(a) Angle ABC = $180^\circ - 2x^\circ$
Angle ADC = $180^\circ - 2x^\circ$
Angle BCD = $180^\circ - 2x^\circ$
Angle BAD = $180^\circ - 2x^\circ$
Hence ABCD is a cyclic quadrilateral

Meta learning definition

Definition: meta learning

Learn a distribution of (related) tasks,
so we can infer new tasks quickly [3].

Instead of training on data samples
 $x \sim p_{\text{data}}$ we train on datasets $\mathcal{D} \sim p(\mathcal{D})$

$$\theta^* = \arg \min_{\theta} \mathbb{E}_{\mathcal{D} \sim p(\mathcal{D})} [\mathcal{L}_{\theta}(\mathcal{D})]$$



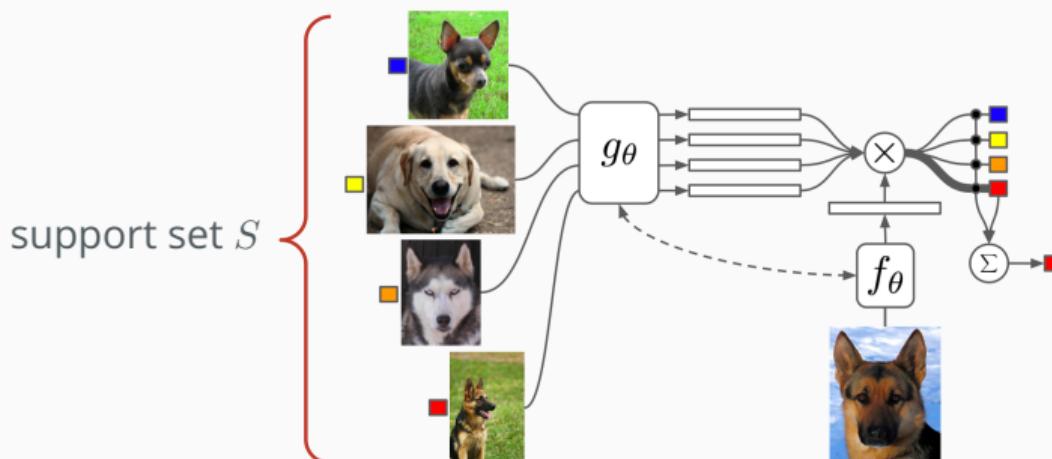
$$\theta^* = \arg \min_{\theta} \mathbb{E}_{\mathcal{D} \sim p(\mathcal{D})} [\mathcal{L}_{\theta}(\mathcal{D})]$$

Meta learning the meta learning support set

Definition: meta learning support set

Meta learners determine the task via a support set \mathcal{S}

$$\theta = \arg \max_{\theta} \mathbb{E}_{L \sim \mathcal{T}} [\mathbb{E}_{\mathcal{S}^L \sim \mathcal{D}, B^L \sim \mathcal{D}} \left[\sum_{(x,y) \in B^L} P_{\theta}(y|x, \mathcal{S}^L) \right]]$$



Meta learning metric, optimisation and model-based meta learning

Taxonomy: meta learning

Meta learning literature can be categorised several ways [3], such as by:

- meta-representation (what data?)
- meta-optimisation (how's it optimised?)
- meta-objective (what goal?)

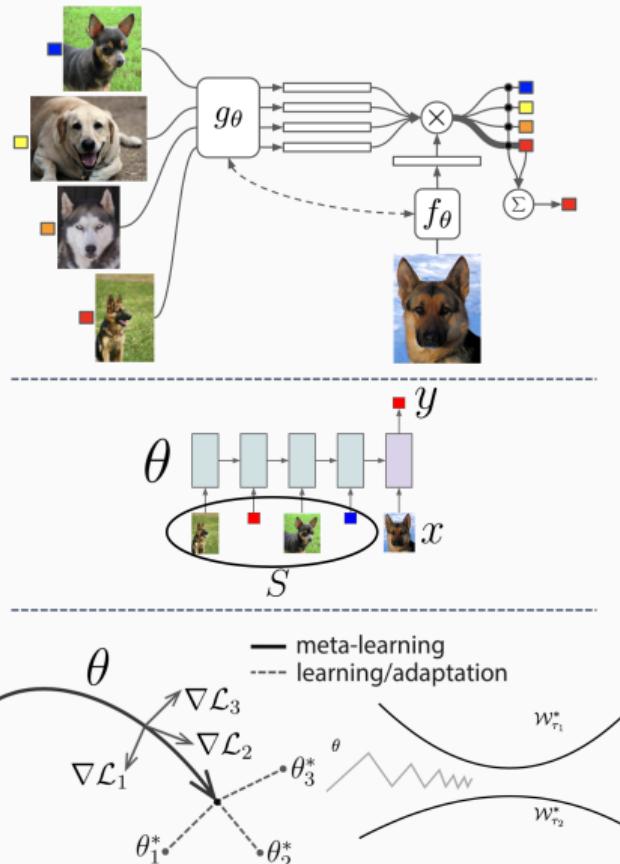
or a different taxonomy:

- metric-based

$$P_\theta(y|\mathbf{x}, S) = \sum_{(\mathbf{x}_i, y_i) \in S} k_\theta(\mathbf{x}, \mathbf{x}_i) y_i$$
- model-based

$$P_\theta(y|\mathbf{x}, S) = f_\theta(\mathbf{x}, S)$$
- optimisation-based

$$P_\theta(y|\mathbf{x}, S) = P_{g_\phi(\theta, S^L)}(y|\mathbf{x})$$





Looking forward meta learning datasets

Omniglot

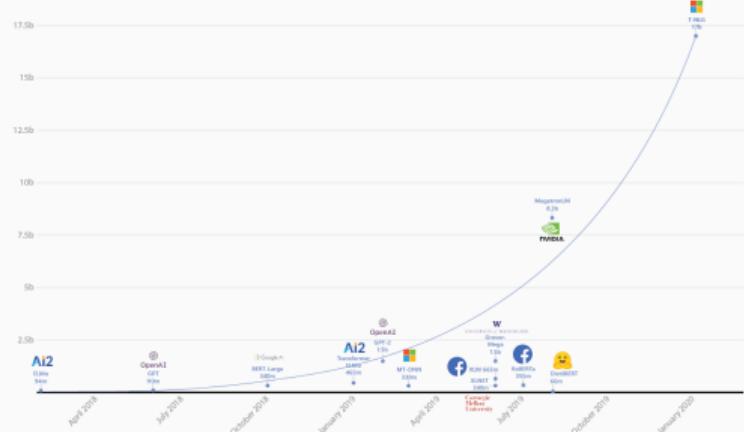
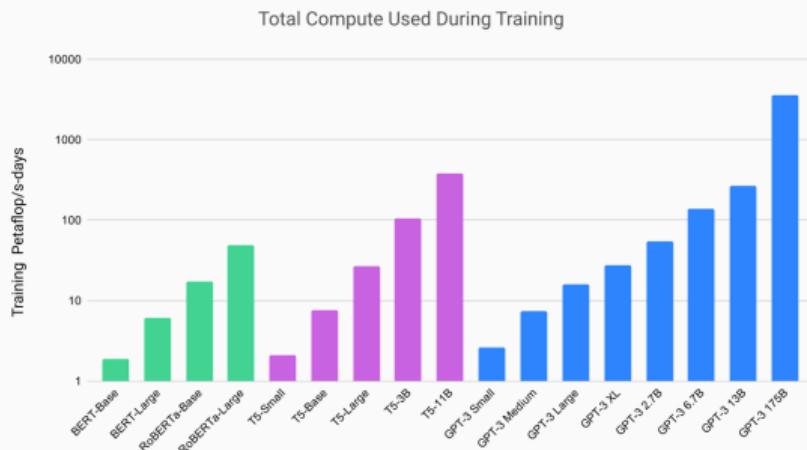
ಗ್ರಂಥಾಲಯದಲ್ಲಿ ನಿರ್ವಹಣೆ ಮತ್ತು ವಿನೋದ ಉದ್ದೇಶದಿಂದ ಪ್ರಾರಂಭಿಸಿದ ಒಂನಿಗ್ಲಾಟ್ ಹಿಂದಿನ ಕಾಲದಿಂದ ಬಳಿಕೆಯಾಗಿ ವಿಳಿದುಕೊಂಡಿರುತ್ತಿದ್ದ ಅಂತರರಾಷ್ಟ್ರೀಯ ಲಿಪಿಗಳ ಶಿಕ್ಷಣ ಮತ್ತು ಪರಿಶೋಧನೆಯ ಸಾಮಾನ್ಯ ಪ್ರಯೋಜನವಾಗಿದೆ. ಈ ಡಿಟಾಸೆಟ್ ಅನೇಕ ಲಿಪಿಗಳ ಅನುಕೂಲ ರೂಪದಲ್ಲಿ ಲಿಪಿಗಳ ಶಿಕ್ಷಣ ಮತ್ತು ಪರಿಶೋಧನೆಯ ಸಾಮಾನ್ಯ ಪ್ರಯೋಜನವಾಗಿದೆ. ಈ ಡಿಟಾಸೆಟ್ ಅನೇಕ ಲಿಪಿಗಳ ಅನುಕೂಲ ರೂಪದಲ್ಲಿ ಲಿಪಿಗಳ ಶಿಕ್ಷಣ ಮತ್ತು ಪರಿಶೋಧನೆಯ ಸಾಮಾನ್ಯ ಪ್ರಯೋಜನವಾಗಿದೆ. ಈ ಡಿಟಾಸೆಟ್ ಅನೇಕ ಲಿಪಿಗಳ ಅನುಕೂಲ ರೂಪದಲ್ಲಿ ಲಿಪಿಗಳ ಶಿಕ್ಷಣ ಮತ್ತು ಪರಿಶೋಧನೆಯ ಸಾಮಾನ್ಯ ಪ್ರಯೋಜನವಾಗಿದೆ. ಈ ಡಿಟಾಸೆಟ್ ಅನೇಕ ಲಿಪಿಗಳ ಅನುಕೂಲ ರೂಪದಲ್ಲಿ ಲಿಪಿಗಳ ಶಿಕ್ಷಣ ಮತ್ತು ಪರಿಶೋಧನೆಯ ಸಾಮಾನ್ಯ ಪ್ರಯೋಜನವಾಗಿದೆ. ಈ ಡಿಟಾಸೆಟ್ ಅನೇಕ ಲಿಪಿಗಳ ಅನುಕೂಲ ರೂಪದಲ್ಲಿ ಲಿಪಿಗಳ ಶಿಕ್ಷಣ ಮತ್ತು ಪರಿಶೋಧನೆಯ ಸಾಮಾನ್ಯ ಪ್ರಯೋಜನವಾಗಿದೆ.

Mini-Imagenet





Looking forward large-scale generative models



Figures from OpenAI and Hugging Face.

Meta learning machine reasoning

Discussion: reasoning and risk

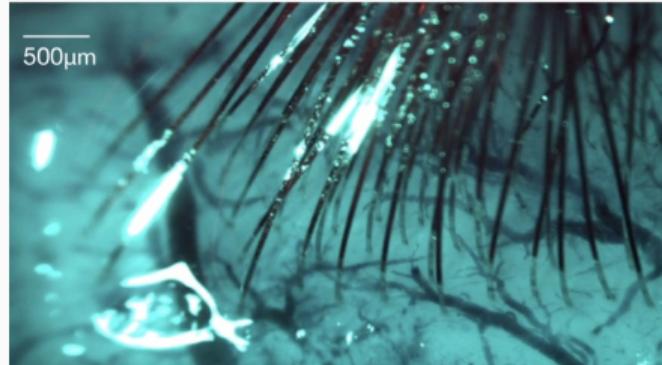
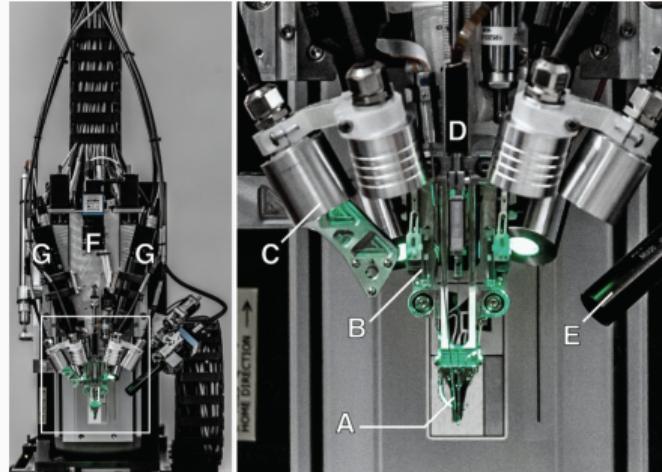
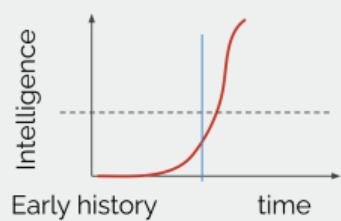
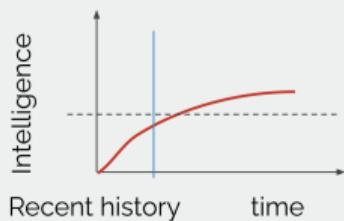
Machine reasoning hints at the idea that there is something beyond our current theory of generalisation. Do you agree?

$$\begin{array}{r|l} 9+7 & 6+6 \\ \hline =1 & =9 \end{array} \quad \begin{array}{r|l} 7+5 & 6+9 \\ \hline =6 & =19 \end{array} \quad \begin{array}{r|l} 3+1 & 5+7 \\ \hline =7 & =1 \end{array} \quad \begin{array}{r|l} 1+3 & \\ \hline \end{array}$$

$$\begin{array}{r|l} 7+5 & 9+0 \\ \hline =7 & =9 \end{array} \quad \begin{array}{r|l} 3+1 & 9+2 \\ \hline =12 & =8 \end{array} \quad \begin{array}{r|l} 6+8 & 3+8 \\ \hline =6 & =16 \end{array} \quad \begin{array}{r|l} 6+9 & \\ \hline \end{array}$$

Or is reasoning just a imitation/generative modeling with representative functions?

Watch GPT-3 answer this for itself ↗





Take Away Points

Summary

In summary, within deep learning:

- nearly all learning problems relate to generative modeling
- there's a push now towards generalising to unseen tasks
- we're heading towards a grand unification of modalities
- are reasoning and meta learning just generalisation?
- what will be the most scalable representative functions?
- humans are really bad at imagining the unknown

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References I

- [1] Scikitlearn. Manifold learning algorithms. Available online [↗](#). 2020.
- [2] Leland McInnes, John Healy, and James Melville. "Umap: Uniform manifold approximation and projection for dimension reduction". In: arXiv preprint arXiv:1802.03426 (2018).
- [3] Timothy Hospedales, Antreas Antoniou, Paul Micaelli, and Amos Storkey. "Meta-learning in neural networks: A survey". In: arXiv preprint arXiv:2004.05439 (2020).
- [4] Elon Musk et al. "An integrated brain-machine interface platform with thousands of channels". In: Journal of medical Internet research 21.10 (2019), e16194.