

# Deep Learning

## Lecture 10: Meta and manifold learning

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# Lecture overview

## 1 Manifold learning

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- NLDR with DNNs
- t-SNE and UMAP on DNNs
- designing tailored embeddings
- Jonker-Volgenant assignment

## 2 Meta learning

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- thinking in distributions
- the distribution of all data...
- ...and of all tasks
- definition
- the meta learning support set
- metric, optimisation and model-based

## 3 Looking forward

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- meta learning datasets
- large-scale generative models
- machine reasoning and risk
- take away points



# Manifold learning NLDR in DNNs

## Definition: NLDR in DNNs

Feature vectors in deep neural networks (DNNs) capture abstract patterns which are interesting to analyse.

We can use nonlinear dimensionality reduction (NLDR) algorithm, such as t-SNE and UMAP to examine these patterns.

The deepest (bottleneck or penultimate layer) features are often the most interesting.

## Example: bottleneck features in LeNet

```
class LeNet(nn.Module):
    def __init__(self):
        super(LeNet, self).__init__()
        self.conv1 = nn.Conv2d(1, 6, 5, padding=2)
        self.conv2 = nn.Conv2d(6, 16, 5)
        self.fc1   = nn.Linear(16*5*5, 120)
        self.fc2   = nn.Linear(120, 84)
        self.fc3   = nn.Linear(84, 10)

    def forward(self, x):
        x = F.max_pool2d(F.relu(self.conv1(x)), (2, 2))
        x = F.max_pool2d(F.relu(self.conv2(x)), (2, 2))
        x = flatten(x)
        x = F.relu(self.fc1(x))
        → f = F.relu(self.fc2(x))
        x = self.fc3(f)
        return x
```

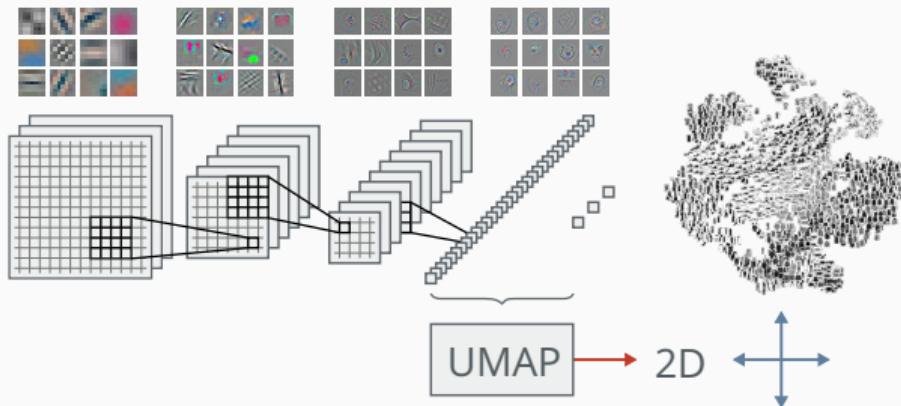
# Manifold learning example t-SNE and UMAP on DNNs

## Usage: t-SNE [1] or UMAP [2]

```
import torch
from sklearn.manifold import TSNE

# f = features for whole dataset
f = torch.randn(1000, 84, 1, 1)

# specify embedding to 2D
g = TSNE(2).fit_transform(f.squeeze())
print(g.shape) # returns (1000,2)
```

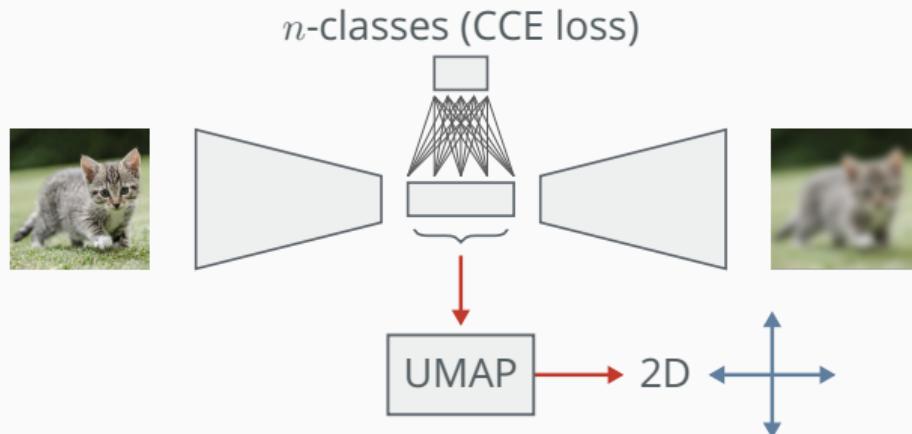


# Manifold learning designing tailored embeddings

## Example: tailored embeddings

The embedding space can be controlled by additional constraints, such as reconstruction term, additional losses (classification, regression).

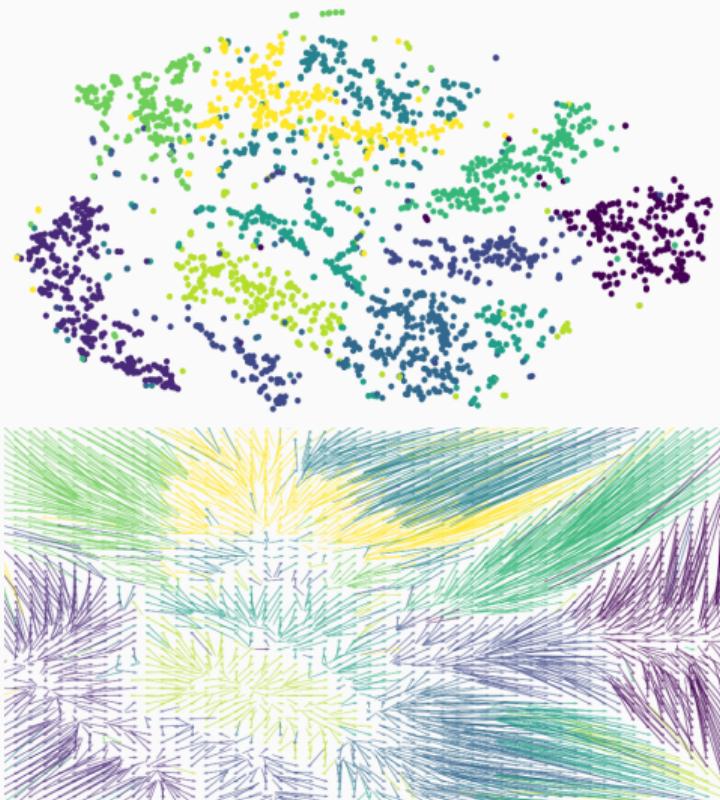
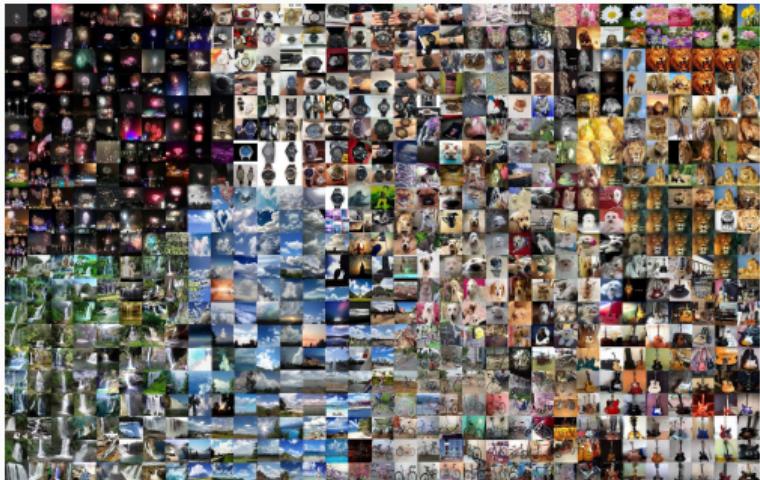
What will the 2D embedding be like for the following architecture?



# Manifold learning Jonker-Volgenant assignment

## Example: Jonker-Volgenant

A visualisation trick is to minimise an assignment cost to optimise the layout of the embeddings. The Jonker-Volgenant algorithm can be used for this, giving:

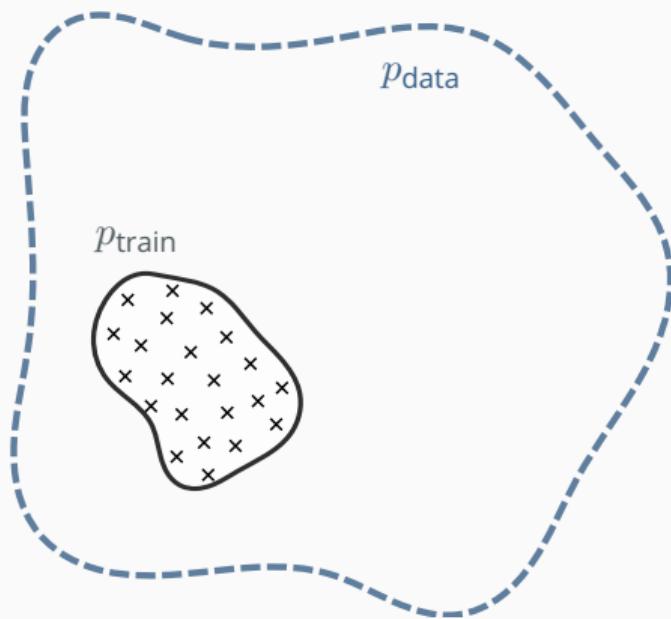




# Meta learning thinking in distributions

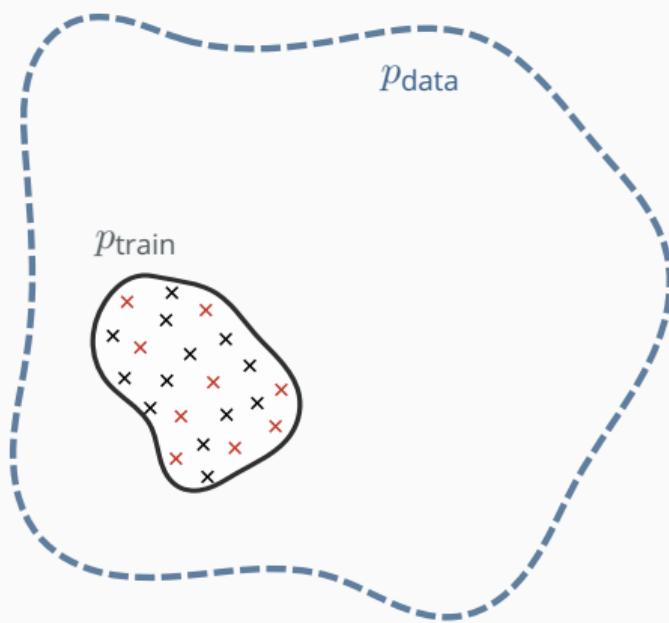
A common lie...

...is that test data  $\stackrel{\text{i.i.d.}}{\sim}$  train data (no!)



**99.7% "test accuracy"!**

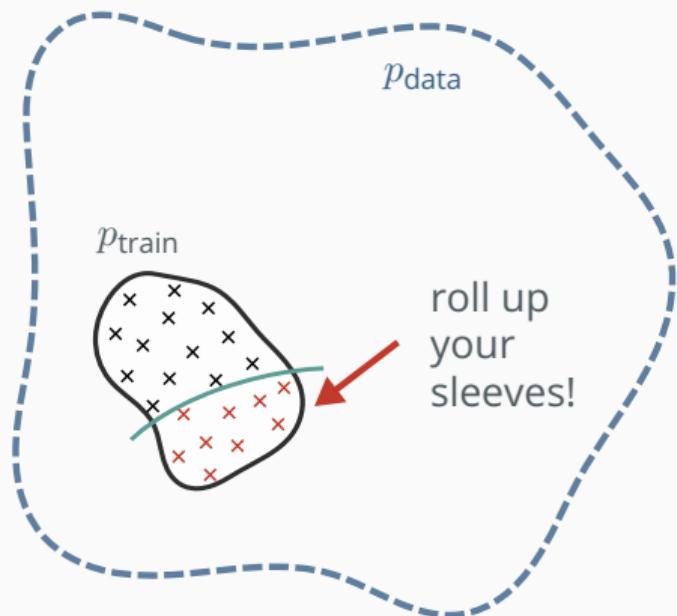
(your boss and the investors are happy)



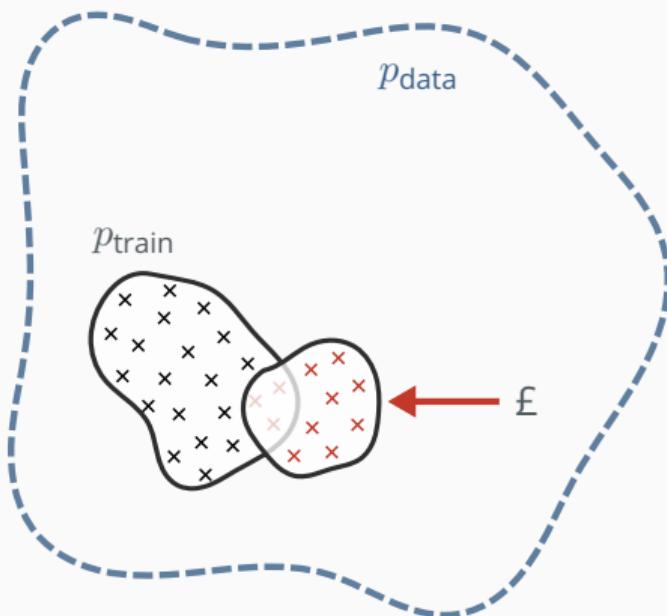


# Meta learning thinking in distributions

62% "test" accuracy  
(but closer to the truth)

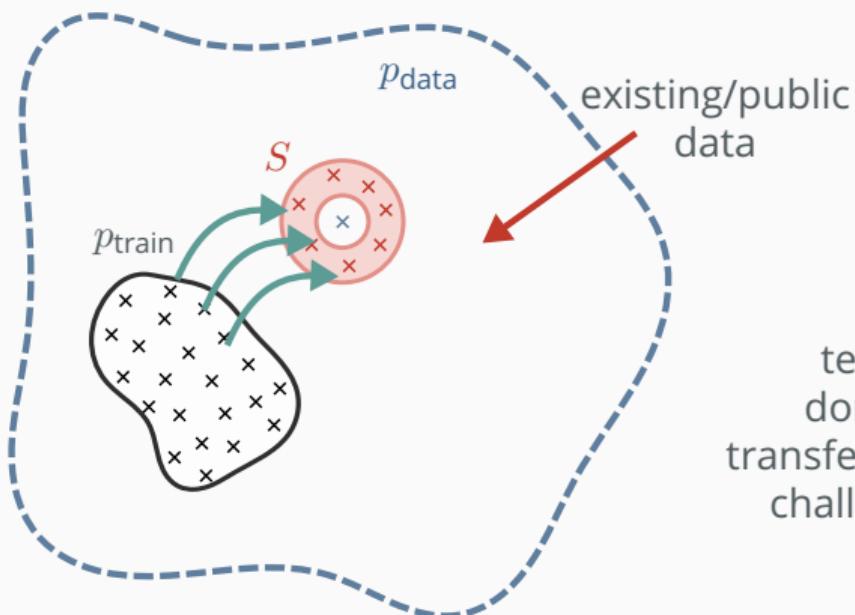


51% "test" accuracy  
(even closer to the truth)

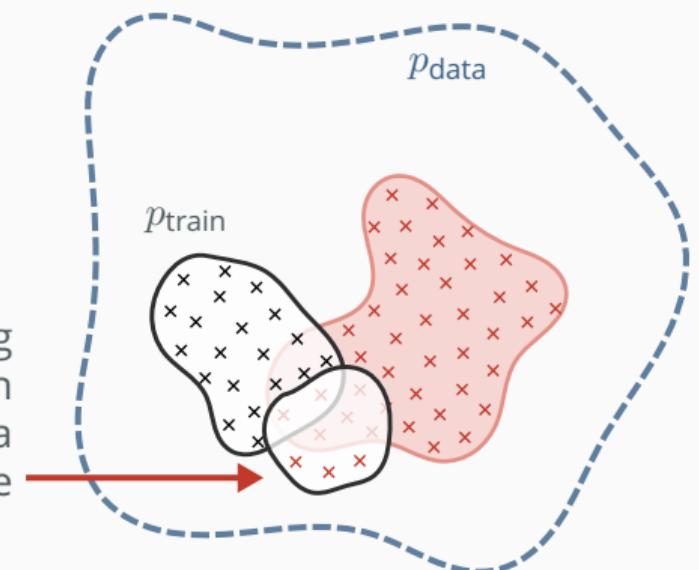


# Meta learning thinking in distributions

Meta learning (generate  $\mathbf{x}$  with related observations  $S$  from  $p_{\text{data}}$ )

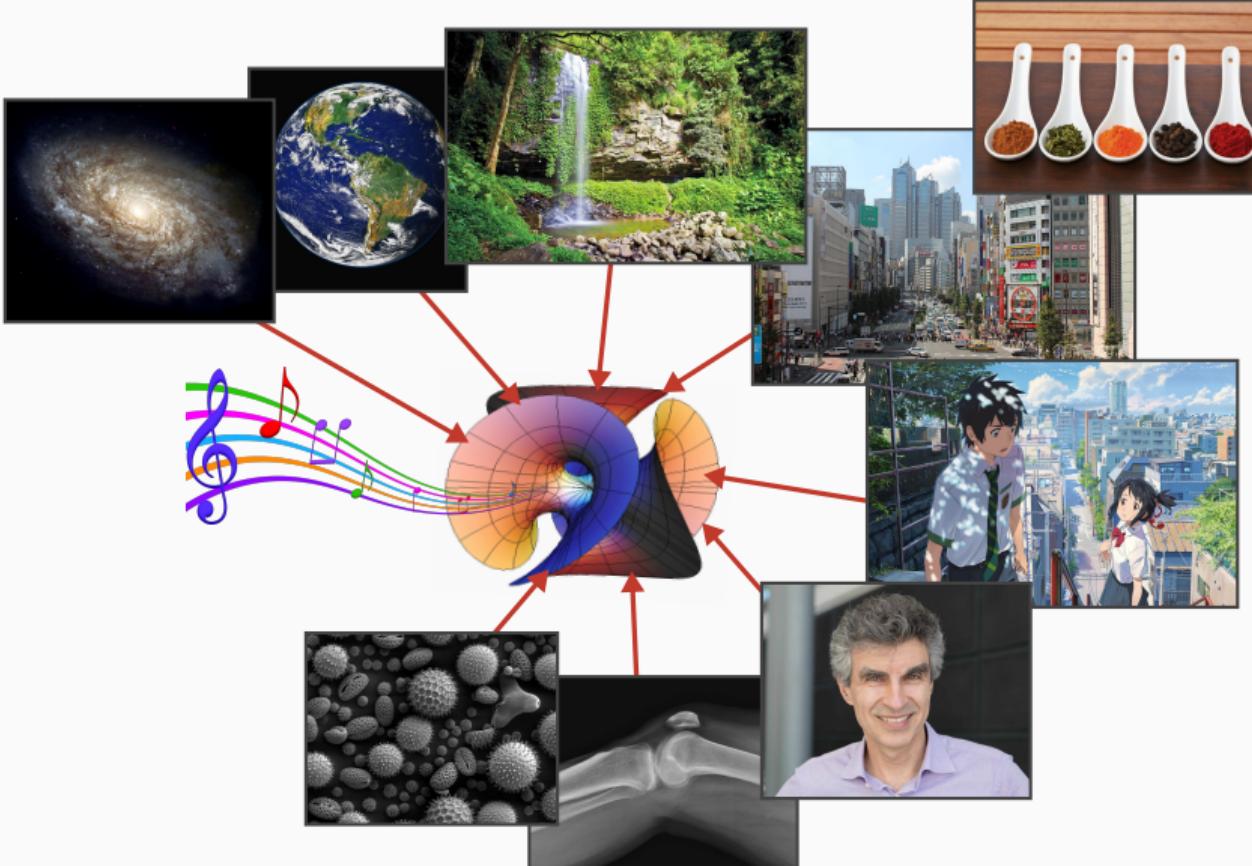


existing/public data  
testing domain transfer is a challenge





# Meta learning the distribution of all data



# Meta learning the distribution of all tasks





# Meta learning inferring new tasks

We would like to be able to generalise to unseen tasks. What do you do with these?



Solve, admire...



Play, complete, improvise...



Solve, comprehend...

1 This coil of piping has 15 turns. The diameter of the coil is about 3 m.

(a) Estimate the length of the pipe.  
(b) This is the cross-section of the pipe.  
15 cm  
3 cm

A one-metre length has mass 0.09 kg. Calculate the density of the material. State the units in your answer.

(a)  $\pi \approx 3.14$   
 $= 3.14 \times 15 \text{ cm}$   
 $= 34.5 \text{ m}$  (use 125 cm)

(b) Area of cross section =  $\pi r^2 = \pi \times 0.015^2 = 0.00707 \text{ m}^2$   
Volume of 1 metre length =  $0.00707 \times 3.14 = 0.0223 \text{ m}^3$   
 $\frac{0.0223}{0.09} = 0.24777 \text{ kg/m}^3$   
Density =  $\frac{\text{mass}}{\text{volume}}$   
 $= \frac{0.09}{0.0223} = 0.0404 \text{ kg/m}^3$   
 $= 0.404 \text{ kg/m}^3$

2 ABCD is an isosceles trapezoid. Angle ACD =  $x^\circ$ .

The size of angle ABD is  $180^\circ - 100^\circ = 80^\circ$ .  
The size of angle BCD is  $180^\circ - 80^\circ = 100^\circ$ .  
Therefore, angle ACD =  $100^\circ - x^\circ$ .

Play, measure...

Eat, taste, smell...



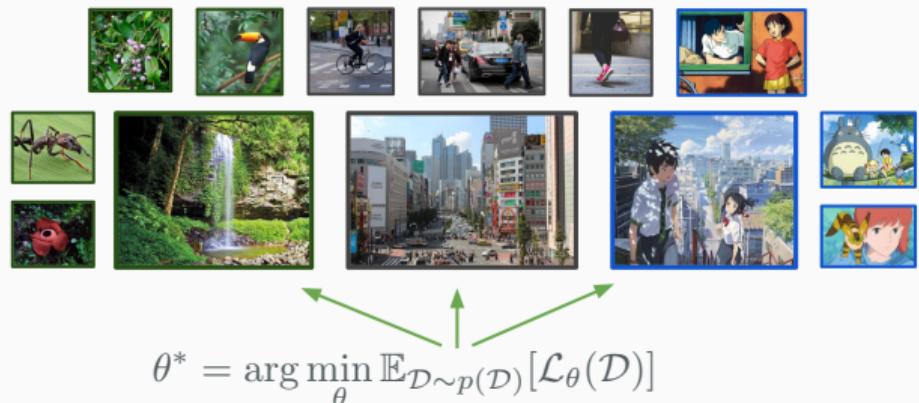
# Meta learning definition

## Definition: meta learning

Learn a distribution of (related) tasks,  
so we can infer new tasks quickly [3].

Instead of training on data samples  
 $x \sim p_{\text{data}}$  we train on datasets  $\mathcal{D} \sim p(\mathcal{D})$

$$\theta^* = \arg \min_{\theta} \mathbb{E}_{\mathcal{D} \sim p(\mathcal{D})} [\mathcal{L}_{\theta}(\mathcal{D})]$$

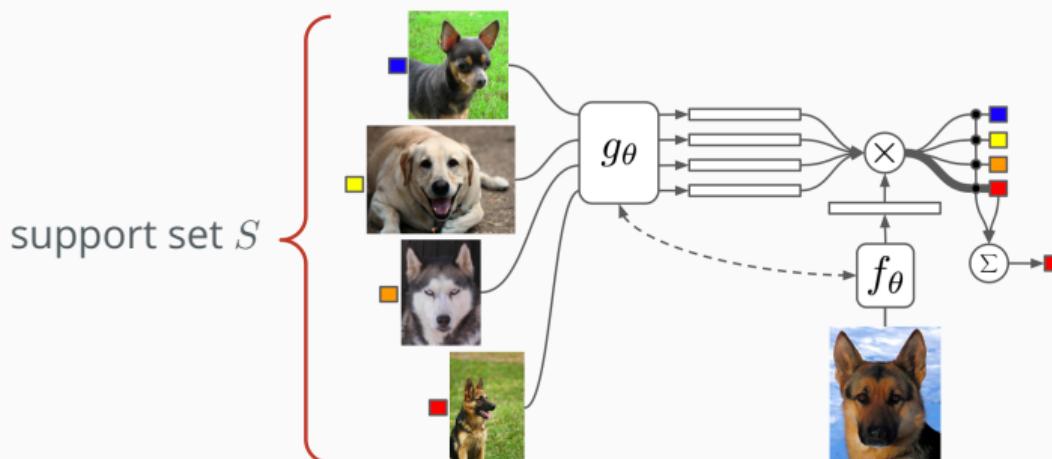


# Meta learning the meta learning support set

**Definition:** meta learning support set

Meta learners determine the task via a support set  $\mathcal{S}$

$$\theta = \arg \max_{\theta} \mathbb{E}_{L \sim \mathcal{T}} [\mathbb{E}_{\mathcal{S}^L \sim \mathcal{D}, B^L \sim \mathcal{D}} \left[ \sum_{(x,y) \in B^L} P_{\theta}(y|x, \mathcal{S}^L) \right]]$$



# Meta learning metric, optimisation and model-based meta learning

## Taxonomy: meta learning

Meta learning literature can be categorised several ways [3], such as by:

- meta-representation (what data?)
- meta-optimisation (how's it optimised?)
- meta-objective (what goal?)

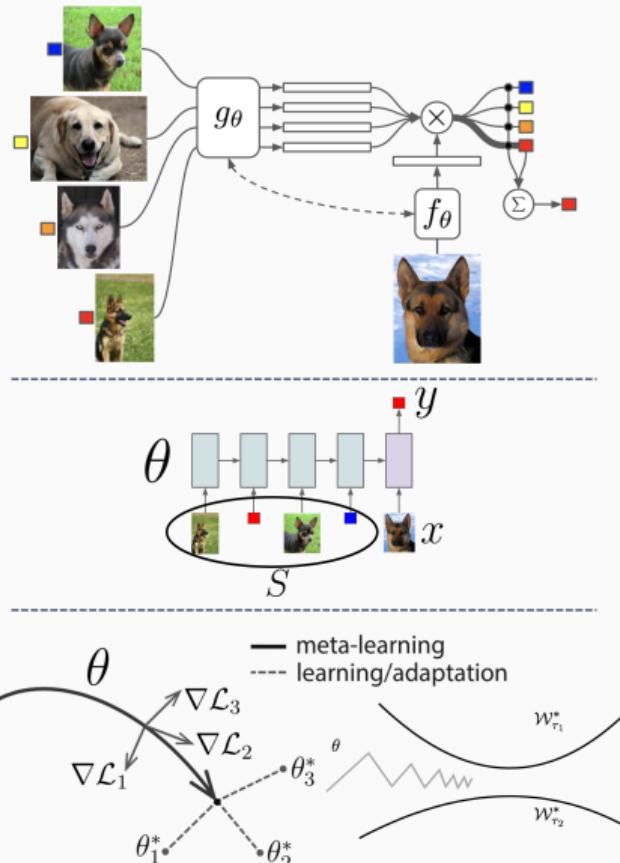
or a different taxonomy:

- metric-based  

$$P_\theta(y|\mathbf{x}, S) = \sum_{(\mathbf{x}_i, y_i) \in S} k_\theta(\mathbf{x}, \mathbf{x}_i) y_i$$
- model-based  

$$P_\theta(y|\mathbf{x}, S) = f_\theta(\mathbf{x}, S)$$
- optimisation-based  

$$P_\theta(y|\mathbf{x}, S) = P_{g_\phi(\theta, S^L)}(y|\mathbf{x})$$





# Looking forward meta learning datasets

## Omniglot

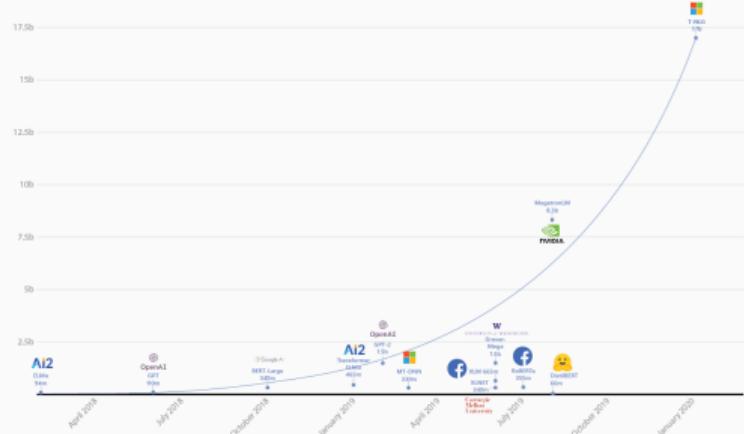
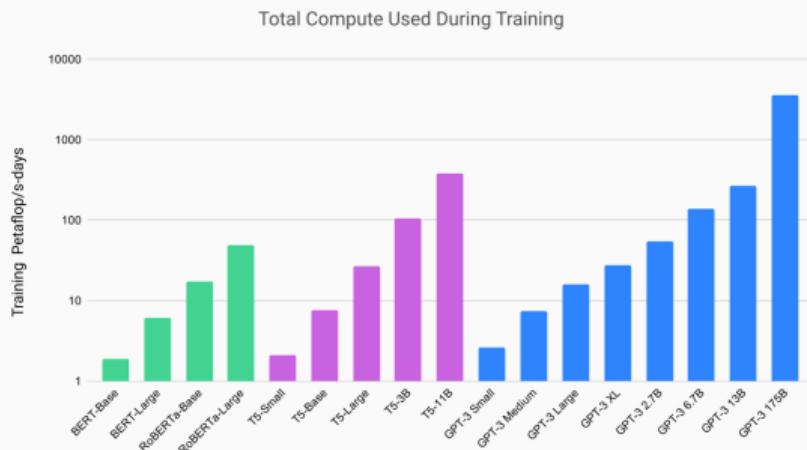
ಗ್ರಂಥಾಲಯದಲ್ಲಿ ನಿರ್ವಹಣೆ ಮತ್ತು ವಿನೋದ ಉದ್ದೇಶದಿಂದ ಪ್ರಾರಂಭಿಸಿದ ಒಂನಿಗ್ಲಾಟ್ ಹಿಂದಿನ ಕಾಲದಿಂದ ಬಳಿಕೆಯಾಗಿದೆ. ಇದು ಸಾಮಾನ್ಯ ಲಿಪಿಗಳನ್ನು ಅನುಸರಿಸಿ, ಅನುಕೂಲ ರೀತಿಯಲ್ಲಿ ಅಂತರ್ರಾಷ್ಟ್ರೀಯ ಪ್ರಾಯೋಗಿಕ ಪರಿಸರದಲ್ಲಿ ವಿನೋದ ಉದ್ದೇಶದಿಂದ ಪ್ರಾರಂಭಿಸಿದ ಒಂನಿಗ್ಲಾಟ್ ಹಿಂದಿನ ಕಾಲದಿಂದ ಬಳಿಕೆಯಾಗಿದೆ. ಇದು ಸಾಮಾನ್ಯ ಲಿಪಿಗಳನ್ನು ಅನುಸರಿಸಿ, ಅನುಕೂಲ ರೀತಿಯಲ್ಲಿ ವಿನೋದ ಉದ್ದೇಶದಿಂದ ಪ್ರಾರಂಭಿಸಿದ ಒಂನಿಗ್ಲಾಟ್ ಹಿಂದಿನ ಕಾಲದಿಂದ ಬಳಿಕೆಯಾಗಿದೆ. ಇದು ಸಾಮಾನ್ಯ ಲಿಪಿಗಳನ್ನು ಅನುಸರಿಸಿ, ಅನುಕೂಲ ರೀತಿಯಲ್ಲಿ ವಿನೋದ ಉದ್ದೇಶದಿಂದ ಪ್ರಾರಂಭಿಸಿದ ಒಂನಿಗ್ಲಾಟ್ ಹಿಂದಿನ ಕಾಲದಿಂದ ಬಳಿಕೆಯಾಗಿದೆ. ಇದು ಸಾಮಾನ್ಯ ಲಿಪಿಗಳನ್ನು ಅನುಸರಿಸಿ, ಅನುಕೂಲ ರೀತಿಯಲ್ಲಿ ವಿನೋದ ಉದ್ದೇಶದಿಂದ ಪ್ರಾರಂಭಿಸಿದ ಒಂನಿಗ್ಲಾಟ್ ಹಿಂದಿನ ಕಾಲದಿಂದ ಬಳಿಕೆಯಾಗಿದೆ.

## Mini-Imagenet





# Looking forward large-scale generative models



Figures from OpenAI and Hugging Face.

# Meta learning machine reasoning

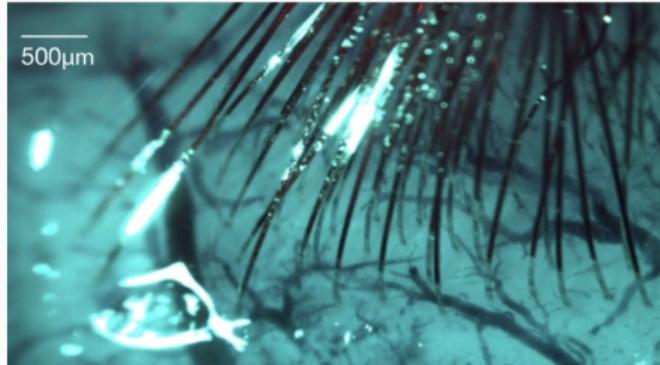
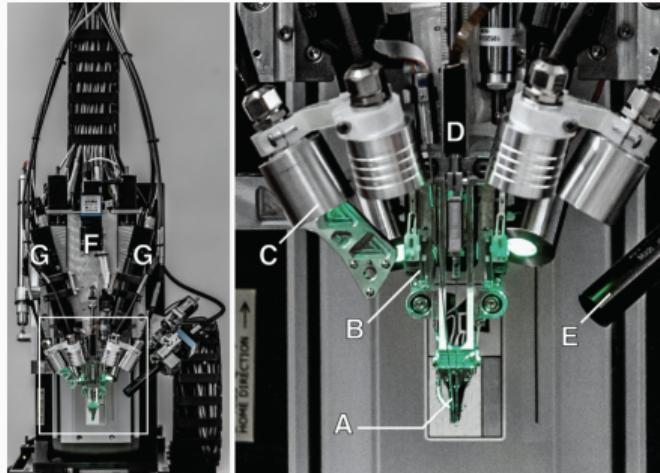
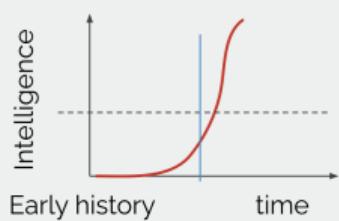
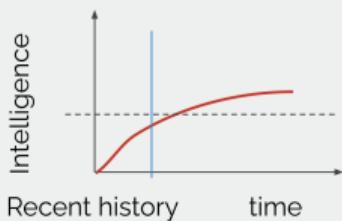
## Discussion: reasoning and risk

Machine reasoning hints at the idea that there is something beyond our current theory of generalisation. Do you agree?

$$\begin{array}{r|l} 9+7 & 6+6 \\ \hline =1 & =9 \end{array} \quad \begin{array}{r|l} 7+5 & 6+9 \\ \hline =6 & =19 \end{array} \quad \begin{array}{r|l} 3+1 & 5+7 \\ \hline =7 & =1 \end{array} \quad \begin{array}{r|l} 1+3 & \\ \hline \end{array}$$
  
$$\begin{array}{r|l} 7+5 & 9+0 \\ \hline =7 & =9 \end{array} \quad \begin{array}{r|l} 3+1 & 9+2 \\ \hline =12 & =8 \end{array} \quad \begin{array}{r|l} 6+8 & 3+8 \\ \hline =6 & =16 \end{array} \quad \begin{array}{r|l} 6+9 & \\ \hline \end{array}$$

Or is reasoning just a imitation/generative modeling with representative functions?

**Watch GPT-3 answer this for itself** ↗





# Take Away Points

## Summary

In summary, within deep learning:

- nearly all learning problems relate to generative modeling
- there's a push now towards generalising to unseen tasks
- we're heading towards a grand unification of modalities
- are reasoning and meta learning just generalisation?
- what will be the most scalable representative functions?
- humans are really bad at imagining the unknown

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# References I

- [1] Scikitlearn. Manifold learning algorithms. Available online [↗](#). 2020.
- [2] Leland McInnes, John Healy, and James Melville. "Umap: Uniform manifold approximation and projection for dimension reduction". In: arXiv preprint arXiv:1802.03426 (2018).
- [3] Timothy Hospedales, Antreas Antoniou, Paul Micaelli, and Amos Storkey. "Meta-learning in neural networks: A survey". In: arXiv preprint arXiv:2004.05439 (2020).
- [4] Elon Musk et al. "An integrated brain-machine interface platform with thousands of channels". In: Journal of medical Internet research 21.10 (2019), e16194.