COMP3667: Reinforcement learning practical 6

Temporal-Difference Learning

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This is the version with answers!

1 Overview

Welcome to the sixth reinforcement learning practical. In this practical, we will be experimenting with different TD methods to better understand their characteristics, advantages, and drawbacks. In particular, we will

- \bullet evaluate on-policy SARSA(0) and off-policy Q-Learning
- ullet experiment with n-step TD learning
- compare the performance of these methods in different environments
- understand when/how/why they may fail to work and how to fix them.

2 Setup

For this practical, you will need a basic Python environment with numpy, matplotlib, and OpenAI gym (version 0.20.0). You can will not need a GPU. You find a colab notebook with the complete setup code here.

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```
pip install 'gym[box2d] == 0.20.0'
import gym
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.font_manager
import itertools
from IPython import display
```

As before, we will use different versions of the frozen lake gym environment:

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```
name = 'FrozenLake-v1'
1
    env = gym.make(name, is_slippery=False)
                                                               # 4x4
    env = gym.make(name, map_name="8x8", is_slippery=False) # 8x8
    env = gym.make(name, desc=["SFHH",
                                "HFFH",
                                "HHFF"
6
                                "HHHG",], is_slippery=False) # custom
7
    env.seed(742)
8
    env.action_space.seed(742)
    LEFT, DOWN, RIGHT, UP = 0, 1, 2, 3
10
```

You can use this function (slightly modified from previous practicals) for plotting the environment

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```
def plot(env, v=None, policy=None, col_ramp=1, dpi=175, draw_vals=False, mark_ice=True):
    # set up plot

plt.rcParams['figure.dpi'] = dpi
    plt.rcParams.update({'axes.edgecolor': (0.32,0.36,0.38)})

plt.rcParams.update({'font.size': 4 if env.env.nrow == 8 else 7})

gray = np.array((0.32,0.36,0.38))
```

```
plt.figure(figsize=(3, 3))
7
        ax = plt.gca()
        ax.set_xticks(np.arange(env.env.ncol)-.5)
        ax.set_yticks(np.arange(env.env.nrow)-.5)
10
        ax.set_xticklabels([])
11
        ax.set_yticklabels([])
12
        plt.grid(color=(0.42,0.46,0.48), linestyle=':')
13
        ax.set_axisbelow(True)
14
        ax.tick_params(color=(0.42,0.46,0.48),
15
                        which='both', top='off', left='off', right='off', bottom='off')
        # use zero value as dummy if not provided
        if v is None:
18
            v = np.zeros(env.nS)
19
        # plot values
20
        plt.imshow(1-v.reshape(env.env.nrow,env.env.ncol)**col_ramp,
21
                    cmap='gray', interpolation='none',
22
                   clim=(0,1), zorder=-1)
23
        # go through states
        for s in range(env.nS):
25
            x, y = s % env.env.nrow, s // env.env.ncol
26
            # print numeric values
27
            if draw_vals and v[s] > 0:
28
                vstr = '\{0:.1e\}'.format(v[s]) if env.env.nrow == 8 else '\{0:.6f\}'.format(v[s])
29
                plt.text(x - 0.45, y + 0.45, vstr, color=(0.2, 0.8, 0.2), fontname='Sans')
30
            # mark ice, start, goal
31
            if env.desc.tolist()[y][x] == b'F':
32
                 plt.text(x-0.45,y-0.3, 'ice', color=(0.5, 0.6, 1), fontname='Sans')
33
                if mark_ice:
34
                     ax.add_patch(plt.Circle((x, y), 0.2, color=(0.7, 0.8, 1), zorder=0))
35
            elif env.desc.tolist()[y][x] == b'S':
                plt.text(x-0.45,y-0.3, 'start',color=(0.2,0.5,0.5), fontname='Sans',
37
                          weight='bold')
38
            elif env.desc.tolist()[y][x] == b'G':
39
                plt.text(x-0.45,y-0.3, 'goal', color=(0.7,0.2,0.2), fontname='Sans',
40
                          weight='bold')
41
                 continue # don't plot policy for goal state
42
            else:
43
                continue # don't plot policy for holes
44
            # plot policy
45
            def plot_arrow(x, y, dx, dy, v, scale=0.4):
46
                plt.arrow(x, y, scale * float(dx), scale * float(dy), color=gray+0.2*(1-v),
                           head_width=0.1, head_length=0.1, zorder=1)
48
            if policy is not None:
49
                if policy[s, 0] > 0.0: plot_arrow(x, y, -policy[s, 0],
                                                                                0., v[s]) # left
50
                 if policy[s, 1] > 0.0: plot_arrow(x, y,
                                                                 0., policy[s, 1], v[s]) # down
                 if policy[s, 2] > 0.0: plot_arrow(x, y, policy[s, 2],
                                                                                0., v[s]) # right
52
                 if policy[s, 3] > 0.0: plot_arrow(x, y,
                                                                0., -policy[s, 3], v[s]) # up
53
        plt.show()
```

and these two helper classes to define hard-coded policies or policies using Q-values

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```
class QPolicy:
    def __init__(self, Q, epsilon, values=False):
        self.Q = Q
        self.epsilon = epsilon
        self.values = values
    def sample(self, state):
        if np.random.rand() > self.epsilon:
        best_actions = np.argwhere(self.Q[state]==np.max(self.Q[state])).flatten()
        return np.random.choice(best_actions)
```

```
else:
10
                 return env.action_space.sample()
11
        def __getitem__(self, item):
12
             state, action = item
             if self.values:
14
                 return self.Q[state, action] / (self.Q[state].sum() + 1e-10)
15
             else:
16
                 best_actions = np.argwhere(self.Q[state] == np.max(self.Q[state])).flatten()
                 p = int(action in best_actions) / len(best_actions)
18
                 return (1 - self.epsilon) * p + self.epsilon / len(self.Q[state])
19
    class HardCodedPolicy:
20
        def __init__(self, state_action_map):
21
             self.state_action_map = state_action_map
22
        def sample(self, state):
23
             if state in self.state_action_map:
24
                 return np.random.choice(self.state_action_map[state])
25
26
                 return np.random.choice(4)
27
        def __getitem__(self, item):
             state, action = item
             if state in self.state_action_map:
30
                 if action in self.state_action_map[state]:
31
                     return 1 / len(self.state_action_map[state])
32
                 else:
33
                     return 0
34
             else:
35
                 return 1 / 4
```

Finally, these functions can be used to plot the learning progress

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```
# (using global variables in functions)
1
    def update_plot(mod):
        reward_list[experiment_id].append(reward_sum)
3
        aoc[experiment_id] += reward_sum
        test_reward_list[experiment_id].append(test_reward_sum)
        test_aoc[experiment_id] += test_reward_sum
        if episode % mod == 0:
            plot_data[experiment_id].append([episode,
                                               np.array(reward_list[experiment_id]).mean(),
                                               np.array(test_reward_list[experiment_id]).mean()])
10
            reward_list[experiment_id] = []
11
            test_reward_list[experiment_id] = []
12
            for i in range(len(plot_data)):
                 color=next(plt.gca()._get_lines.prop_cycler)['color']
                plt.plot([x[0] for x in plot_data[i]],
15
                          [x[1] for x in plot_data[i]],
16
                          '-', color=color,
17
                          label=f"{plot_labels[i]}, AOC: {aoc[i]}|{test_aoc[i]}")
                plt.plot([x[0] for x in plot_data[i]],
19
                          [x[2] for x in plot_data[i]], '--', color=color)
20
            plt.xlabel('Episode number')
            plt.ylabel('Episode reward')
22
            plt.legend()
23
            display.clear_output(wait=True)
24
            plt.show()
    def next_experiment():
26
        reward_list.append([])
27
        aoc.append(0)
28
        test_reward_list.append([])
        test_aoc.append(0)
30
```

```
plot_data.append([])
return experiment_id + 1
```

using the following global variables (re-evaluate this cell to reset them)

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```
reward_list = [[]]
aoc = [0]
test_reward_list = [[]]
test_aoc = [0]
plot_data = [[]]
plot_labels = []
experiment_id = 0
```

3 On-policy and off-policy learning with TD(0)

Recap TD Learning

Remember our 0-step temporal difference (TD) targets from the lecture, which can be computed for any (also partial) episodes

$$\begin{split} V_{\pi}(s_{t}) &= \sum_{a_{t} \in \mathcal{A}} \pi(a_{t} \mid s_{t}) \sum_{s_{t+1} \in \mathcal{S}} p(s_{t+1} \mid s_{t}, a_{t}) \Big[\mathcal{R}^{a_{t}}_{s_{t}s_{t+1}} + \gamma \, V_{\pi}(s_{t+1}) \Big] \\ &= \mathbb{E}_{a_{t} \sim \pi(a_{t} \mid s_{t})} \mathbb{E}_{s_{t+1} \sim p(s_{t+1} \mid s_{t}, a_{t})} \Big[\mathcal{R}^{a_{t}}_{s_{t}s_{t+1}} + \gamma \, V_{\pi}(s_{t+1}) \Big] \\ &\approx \underbrace{\mathcal{R}^{a_{t}}_{s_{t}s_{t+1}} + \gamma \, V_{\pi}(s_{t+1})}_{\text{TD}(0) \text{ target}} \\ Q_{\pi^{*}}(s_{t}, a_{t}) &= \sum_{s_{t+1} \in \mathcal{S}} p(s_{t+1} \mid s_{t}, a_{t}) \Big[\mathcal{R}^{a_{t}}_{s_{t}s_{t+1}} + \gamma \sum_{a_{t+1} \in \mathcal{A}} \pi^{*}(a_{t+1} \mid s_{t+1}) \, Q_{\pi^{*}}(s_{t+1}, a_{t+1}) \Big] \\ &= \mathbb{E}_{s_{t+1} \sim p(s_{t+1} \mid s_{t}, a_{t})} \Big[\mathcal{R}^{a_{t}}_{s_{t}s_{t+1}} + \gamma \, \mathbb{E}_{a_{t+1} \sim \pi^{*}(a_{t+1} \mid s_{t+1})} Q_{\pi^{*}}(s_{t+1}, a_{t+1}) \Big] \\ &\approx \underbrace{\mathcal{R}^{a_{t}}_{s_{t}s_{t+1}} + \gamma \, Q_{\pi^{*}}(s_{t+1}, a_{t+1})}_{\text{TD}(0) \text{ target}} \, . \end{split}$$

At a particular time t we are in state s_t and take action $a_t \sim \pi(a_t \mid s_t)$ sampled from the policy π . We then end up in state $s_{t+1} \sim p(s_{t+1} \mid s_t, a_t)$ based on the environment's transition function. If we are interested in learning state-action values (i.e. solving the control problem), we additionally need to consider the following action $a_{t+1} \sim \pi^*(a_{t+1} \mid s_{t+1})$ based on the policy π^* that we want to learn or evaluate.

The TD(0) targets are noisy "snapshots" of how the values should look like based on the current transition at time t. The difference between the TD target and our current value estimate gives us a noisy TD error signal that tells us "how far off" our estimates are. This can be use to update our value estimates with a learning rate α (similar to SGD) to improve them

$$V_{\pi}(s_t) \leftarrow V_{\pi}(s_t) + \alpha \left[\underbrace{\mathcal{R}_{s_t s_{t+1}}^{a_t} + \gamma V_{\pi}(s_{t+1})}_{\text{TD}(0) \text{ target}} - V_{\pi}(s_t)\right]$$

$$Q_{\pi^*}(s_t, a_t) \leftarrow Q_{\pi^*}(s_t, a_t) + \alpha \left[\underbrace{\mathcal{R}_{s_t s_{t+1}}^{a_t} + \gamma Q_{\pi^*}(s_{t+1}, a_{t+1})}_{\text{TD}(0) \text{ target}} - Q_{\pi^*}(s_t, a_t)\right].$$

On-policy SARSA(0)

We can evaluate and improve our policy "on the go". This is called *on-policy* learning and it means that the policy π we use for sampling is the same as the policy π^* we are evaluating and learning.

Exercise

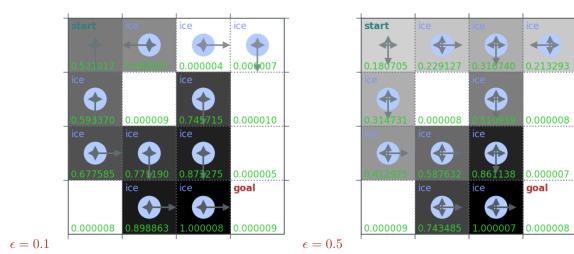
• Implement SARSA(0) by filling in the TD(0) targets and updates in the skeleton below. (The rest of the skeleton is for collecting episodes, evaluating the learned policy and plotting everything. The dashed line is the reward for the learned policy, the solid line is for the sampling policy.)

• Run a couple of evaluations with different values for the learning rate alpha and the exploration epsilon on the 4x4 environment. Use both "noisy" and "neutral" initialisations for Q (commenting in/out the respective lines in the code). What effects do you observe? With what parameters does the agent learn best?

Generally, learning occurs as a "jump". Once the agent improves, it jumps directly to the level of its final performance (this is because there is only one simple goal state; either it knows how to get there or not). "Good" learning performance means that the agent makes that jump early (with a high probability). The best performance is achieved with intermediate values for both learning rate and exploration.

Learning rate alpha: Very large (approaching 1) and very low (below 0.01) learning rates are detrimental. Large learning rates make the convergence unstable; small learning rates are not a problem with a "neutral" initialisation but delay learning with a "noisy" initialisation.

Exploration epsilon: Generally, higher exploration results in a lower final performance because the agent does not act optimally. With "neutral" initialisation, little exploration is not detrimental (the policy chooses random actions if all values are zero; if they are not zero anymore, the agent has found the solution and we do not need to explore; this is again due to having only a single goal state). With a "noisy" initialisation, little exploration delays or prevents learning. Little exploration may also results in some states of the environment not being explored (if they are not on the found "solution path") and thus not having a good policy for these states.



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```
# parameters
1
    num_episodes = 3000
2
    alpha = 0.1
3
    gamma = 0.9
4
    epsilon = 0.5
5
6
    Q = np.random.uniform(0, 1e-5, [env.observation_space.n, env.action_space.n]) # noisy
7
    Q = np.zeros([env.observation_space.n, env.action_space.n])
                                                                                       # neutral
8
    V = np.zeros([env.observation_space.n])
9
10
    # policies
11
    sample_policy = QPolicy(Q, epsilon)
12
    learned_policy = sample_policy
13
    plot_labels.append(f"SARSA (alpha={alpha}, epsilon={epsilon})")
14
15
    for episode in range(num_episodes):
16
        state = env.reset()
17
        reward_sum = 0
18
        # learning a policy
19
        for t in itertools.count():
20
            action = sample_policy.sample(state)
21
            next_state, reward, done, _ = env.step(action)
22
            next_action = learned_policy.sample(next_state)
23
```

```
# TD(0) targets
24
                                               # FILL IN HERE!
             v_target = ...
25
                                               # FILL IN HERE!
             q_target = ...
             # updates
27
             s, a = state, action
28
             V[s] += ...
                                               # FILL IN HERE!
29
                                               # FILL IN HERE!
             Q[s, a] += \dots
30
31
             reward_sum += reward
32
             if done:
33
                 break
34
             state = next_state
35
36
         # testing the learned policy
37
        state = env.reset()
38
        test_reward_sum = 0
39
        while True:
40
             action = learned_policy.sample(state)
41
             next_state, reward, done, _ = env.step(action)
^{42}
             test_reward_sum += reward
43
             state = next_state
44
             if done:
45
                 break
46
47
        update_plot(int(np.ceil(num_episodes / 20)))
48
    env.close()
50
    experiment_id = next_experiment()
51
    print("Sampling policy and values")
52
    plot(env, v=V, policy=sample_policy, draw_vals=True)
    print("Learned policy and optimal/max values")
54
    plot(env, v=Q.max(axis=1), policy=learned_policy, draw_vals=True)
55
       Answer:
                                                                                            </> copy code
```

```
# TD(0) targets
v_target = reward + gamma * V[next_state]
q_target = reward + gamma * Q[next_state, next_action]
# updates
s, a = state, action
V[s] += alpha * (v_target - V[s])
Q[s, a] += alpha * (q_target - Q[s, a])
```

Off-policy Q-Learning

Sometimes, we would like to generate samples with one policy π (typically an exploratory policy) but then use the samples to learn another policy π^* (typically a near-optimal policy). This is possible with *off-policy* methods, such as Q-Learning.

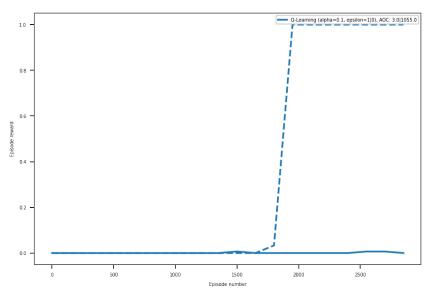
3.0.1 Exercise

- Implement Q-Learning by using a different policy for the learned_policy (with lower epsilon) then for the sample_policy.
 - Note: Strictly speaking, the sample_policy does not change in Q-Learning. You can achieve this by using epsilon=1 in the sample_policy to select actions randomly. For other values of epsilon, the sample_policy "peeks" into the values of the learned policy, which lets it profit from that learning (but is not a clean implementation).
 - Bonus exercise: Learn the sample_policy using SARSA(0) while learning the learned_policy with Q-Learning (this requires maintaining two copies of values estimates, performing separate updates for both etc).

```
# policies
sample_policy = QPolicy(Q, epsilon)
td_epsilon = 0
learned_policy = QPolicy(Q, td_epsilon)
plot_labels.append(f"Q-Learning (alpha={alpha}, epsilon={epsilon})")
```

• Run the evaluation several times on the 4x4 and 8x8 environment. Try to get the fastest and most reliable learning by tweaking epsilon. How does the performance (episode reward) of the sampling policy (solid line) compare to that of the learned policy (dashed lines)? How is this different from SARSA(0)?

In SARSA(0) both policies performed equally well (up to noise). Here, the learned policy performs much better than the sampling policy. Even with a completely exploratory sampling policy, the learned policy performs optimally.



Expected TD(0) targets

The normal TD targets to estimate state-action values are computed by sampling from the policy π^* that is to be learned. This sampling step increases the noise in the TD error signal (in addition to the noise we already have due to sampling episodes). Instead, one can use the *expected* TD targets

$$\underbrace{\mathcal{R}^{a_t}_{s_t s_{t+1}} + \gamma \sum_{a_{t+1} \in \mathcal{A}} \pi^*(a_{t+1} \mid s_{t+1}) \, Q_{\pi^*}(s_{t+1}, a_{t+1})}_{\text{expected TD(0) target}}.$$

Exercise

• In the q_target replace the sampled value with an expectation over possible actions the learned_policy could take. *Note:* learned_policy[next_state, a] is giving you the probability of taking action a in state next_state.

Replace

```
q_target = reward + gamma * Q[next_state, next_action]
with

expected_Q = sum(learned_policy[next_state, a] * Q[next_state, a] for a in range(4))
q_target = reward + gamma * expected_Q
```

• Compare the performance when using the expected TD(0) target to using the sampled one.

There should be a slight performance gain, that is, on average the agent jumps to the optimal policy earlier. However, this could be difficult to spot due to the noise of when exactly that jump happens.

4 TD(n)

In TD(0) we do not look ahead and receiving a reward does only affect the value estimate of the current state and action. All the future expected rewards are approximated by using the current value estimates. However, we can improve on that by instead look n steps ahead (in hindsight) and taking the n next steps into account for computing TD targets. These TD(n) targets are

$$V_{\pi}(s_t) \approx \underbrace{\frac{R_{s_t s_{t+1}}^{a_t} + \gamma \, \mathcal{R}_{s_{t+1} s_{t+2}}^{a_{t+1}} + \ldots + \gamma^n \, \mathcal{R}_{s_{t+n} s_{t+n+1}}^{a_{t+n}}}_{\text{TD}(n) \text{ target}} + \gamma^{n+1} \, V_{\pi}(s_{t+n+1}) \underbrace{Q_{\pi^*}(s_t, a_t) \approx \underbrace{R_{s_t s_{t+1}}^{a_t} + \gamma \, \mathcal{R}_{s_{t+1} s_{t+2}}^{a_{t+1}} + \ldots + \gamma^n \, \mathcal{R}_{s_{t+n} s_{t+n+1}}^{a_{t+n}}}_{\text{TD}(n) \text{ target}} + \gamma^{n+1} \, Q_{\pi^*}(s_{t+n+1}, a_{t+n+1})}.$$

To compute them in hindsight, we have to store a trace of the last n+1 transitions (including the current transition that is also used in TD(0)). The modified skeleton below is providing a trace of length n+1 containing the last n steps and the current transition (for n=0 this reduces to the TD(0) case of only considering the current transition).

⟨⟩ copy code

```
# parameters
    num_episodes = 3000
    alpha = 0.1
    gamma = 0.9
    epsilon = 0.5
5
    n = 0
6
    Q = np.random.uniform(0, 1e-5, [env.observation_space.n, env.action_space.n]) # noisy
    Q = np.zeros([env.observation_space.n, env.action_space.n])
9
    V = np.zeros([env.observation_space.n])
10
11
    # policies
12
    sample_policy = QPolicy(Q, epsilon)
13
    learned_policy = sample_policy
14
    plot_labels.append(f"SARSA (n={n}, alpha={alpha}, epsilon={epsilon})")
15
16
    for episode in range(num_episodes):
17
        state = env.reset()
        reward_sum = 0
19
        trace = np.zeros((n + 1, 4), dtype=int)
20
        done_n = 0
21
         # learning a policy
22
        for t in itertools.count():
23
             action = sample_policy.sample(state)
24
             next_state, reward, done, _ = env.step(action)
25
             next_action = learned_policy.sample(next_state)
26
             trace[-1] = (state, action, reward, next_action)
             if t > n:
28
                 # n-step targets
29
                 n_step_return = ... # FILL IN HERE!
30
                 v_target = ... # FILL IN HERE!
q_target = ... # FILL IN HERE!
31
32
                 # updates
33
                 s, a, _, _ = trace[0]
                 V[s] += alpha * (v_target - V[s])
35
                 Q[s, a] += alpha * (q_target - Q[s, a])
36
37
             reward_sum += reward
             done_n += done
39
             if done_n > n:
40
                 break
41
```

```
state = next_state
42
             trace = np.roll(trace, shift=-1, axis=0)
43
44
         # testing the learned policy
45
        state = env.reset()
        test_reward_sum = 0
47
        while True:
48
             action = learned_policy.sample(state)
49
            next_state, reward, done, _ = env.step(action)
50
             test_reward_sum += reward
51
             state = next state
52
             if done:
                 break
54
55
        update_plot(int(np.ceil(num_episodes / 20)))
56
57
    env.close()
58
    experiment_id = next_experiment()
59
    print("Sampling policy and values")
60
    plot(env, v=V, policy=sample_policy, draw_vals=True)
61
    print("Learned policy and optimal/max values")
62
    plot(env, v=Q.max(axis=1), policy=learned_policy, draw_vals=True)
63
```

SARSA(n)

Exercise

ullet Implement SARSA(n) by filling in the n_step_return, v_target and q_target in the skeleton.

Answer:

</pre

```
# n-step targets
n_step_return = sum(gamma ** i * r for i, (_, _, r, _) in enumerate(trace))
v_target = n_step_return + gamma ** (n + 1) * V[next_state]
q_target = n_step_return + gamma ** (n + 1) * Q[next_state, next_action]
```

• Run this on the 8x8 environment and compare different values of n in terms of performance (how quickly the agent learns) and run time (roughly how fast/slow everything is running). What effects do you observe?

Generally, the larger n the better the agent's learning performance i.e. it jumps to an optimal (up to exploration) policy earlier. This is because observed rewards "travel" n steps at a time. However, larger n also considerably slow down the entire process (as longer traces have to be maintained and evaluated).

Q-Learning

For Q-Learning there is a problem because we want to evaluate a different policy (π^*) than the one used for sampling (π) . That means that the n-step return was sampled with the "wrong" policy (π^*) might never take some of the actions sampled from π) and so is not representative for π^* . This bias can be corrected by adding an importance sampling factor (a general technique in Monte Carlo methods to correct for sampling from a "wrong" distribution) to the learning rate α

$$\rho_t = \frac{\pi^*(a_t \mid s_t) \, \pi^*(a_{t+1} \mid s_{t+1}) \, \dots \, \pi^*(a_{t+n} \mid s_{t+n})}{\pi(a_t \mid s_t) \, \pi(a_{t+1} \mid s_{t+1}) \, \dots \, \pi(a_{t+n} \mid s_{t+n})} = \frac{\prod_{k=0}^n \pi^*(a_{t+k} \mid s_{t+k})}{\prod_{k=0}^n \pi(a_{t+k} \mid s_{t+k})}$$

$$\alpha_t = \alpha \rho_t .$$

Exercise

- Implement TD(n) Q-Learning by
 - using different sample_policy and learned_policy

- computing the importance sampling factor ρ (policy[s, a] is giving you the probability of taking action a in state s).
- modifying the updates accordingly.

Use different policies:

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# policies
sample_policy = QPolicy(Q, epsilon)
td_epsilon = 0.1
learned_policy = QPolicy(Q, td_epsilon)
plot_labels.append(f"Q-Learning (n={n}, alpha={alpha}, epsilon={epsilon}|{td_epsilon})")

Compute importance sampling factor and change updates:

# importance sampling factor

the = np_prod([learned_policy[s_a] / sample_policy[s_a] for s_a = in_trace])
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# importance sampling factor

rho = np.prod([learned_policy[s, a] / sample_policy[s, a] for s, a, _, _ in trace])

# updates

s, a, _, _ = trace[0]

V[s] += alpha * rho * (v_target - V[s])

Q[s, a] += alpha * rho * (q_target - Q[s, a])
```

• Test TD(n) Q-Learning with different values for n and exploration in the sampling policy on the 8x8 environment. *Hint:* Use some small non-zero value for **epsilon** in the learned policy to make sure the importance sampling factors are not (almost) all zero. What do you observe?

TD(n) Q-Learning is less stable compared to SARSA(n). With epsilon=1 (i.e. full exploration) in the sampling policy, even a value of n=1 results in instabilities and the agent not learning a good policy. This is due to the very different probabilities between the two policies, which increases the noise in the TD targets. Additionally, full exploration does not provide many successful runs ending with a reward to learn from. Using smaller learning rates alpha can help increase stability.

With intermediate values (e.g. epsilon=0.5) in the sampling policy, learning is more reliable for small values of n. However, for larger n (≈ 10) learning success is only intermittent: it starts quickly but then degenerates again. This is again because of the noise due to the differences in the two policies. Moreover, since the sampling policy is "cheating" by using the Q values from the learned policy, it also is affected by errors due to the noise.

