COMP4901V Homework 2 Result

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2.1.1

As the problem stated, the second camera is a pure translation that is parallel to the x-axis. So, the translateion can be written as :

$$p_2 = T_x p_1$$

where p_1 and p_2 are corresponding points in two cameras, and T_x is the translation matrix with pure translation that parallel to x-axis and t in matrix represent the unit of translateion:

$$T_x = \begin{bmatrix} t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If we take a point p_1 in the first camera, its epipolar line in the second camera is given by:

$$l_2 = F p_1$$

where F is the fundamental matrix:

$$F = K_2^{-T} T_x R K_1^{-1}$$

R is an Identity matrix in this case as there are no rotation in this pure translation. So,

$$F = K_2^{-T} T_x K_1^{-1}$$
$$T_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t \\ 0 & t & 0 \end{bmatrix}$$

K_1 and K_2 are the camera calibration matrices for the two camera respectively where where f_1 and f_2 are the focal lengths, and (c_{x_1}, c_{y_1}) and (c_{x_2}, c_{y_2}) are the principal points of the two cameras.

$$K_n = \begin{bmatrix} f_n & 0 & c_{x_n} \\ 0 & f_n & c_{y_n} \\ 0 & 0 & 1 \end{bmatrix}$$

Substituting the values of T_x and K_1 and K_2, we get:

$$
\begin{aligned}
K_2^{-T} &= (K_2^{-1})^T = \begin{bmatrix} \frac{1}{f_2} & 0 & 0 \\ 0 & \frac{1}{f_2} & 0 \\ -\frac{c_{x_2}}{f_2} & -\frac{c_{y_2}}{f_2} & 1 \end{bmatrix} \\
K_1^{-1} &= \begin{bmatrix} \frac{1}{f_1} & 0 & -\frac{c_{x_1}}{f_1} \\ 0 & \frac{1}{f_1} & -\frac{c_{y_1}}{f_1} \\ 0 & 0 & 1 \end{bmatrix} \\
F &= \begin{bmatrix} \frac{1}{f_2} & 0 & 0 \\ 0 & \frac{1}{f_2} & 0 \\ -\frac{c_{x_2}}{f_2} & -\frac{c_{y_2}}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t \\ 0 & t & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{f_1} & 0 & -\frac{c_{x_1}}{f_1} \\ 0 & \frac{1}{f_1} & -\frac{c_{y_1}}{f_1} \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{t}{f_2} \\ 0 & t & t\frac{c_{y_2}}{f_2} \end{bmatrix} \begin{bmatrix} \frac{1}{f_1} & 0 & -\frac{c_{x_1}}{f_1} \\ 0 & \frac{1}{f_1} & -\frac{c_{y_1}}{f_1} \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{t}{f_2} \\ 0 & \frac{t}{f_1} & \frac{tc_{y_2}}{f_2} - \frac{tc_{y_1}}{f_1} \end{bmatrix}
\end{aligned}
$$

So, the epipolar line in camera 2 corresponding to a point p_1:

$$
\begin{aligned}
l_2 &= Fp_1 \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{t}{f_2} \\ 0 & \frac{t}{f_1} & \frac{tc_{y_2}}{f_2} - \frac{tc_{y_1}}{f_1} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ -\frac{t}{f_2} \\ \frac{y_1 t}{f_1} + \frac{tc_{y_2}}{f_2} - \frac{tc_{y_1}}{f_1} \end{bmatrix}
\end{aligned}
$$

The second coordinate is constant, so the epipolar line is parallel to x-axis in the second camera. Similarly, the first camera did the same work. So, we can conclude that the epipolarlines in both camera is parallel to x-axis

2.1.2

Effective rotation and translation between two frames at different time stamps:

$$
\begin{aligned}
R_{rel} &= R_2 R_1^T \\
t_{rel} &= t_2 - R_{rel} t_1
\end{aligned}
$$

Essential matrix:

$$E = [t_{rel}] \times R_{rel}$$

Fundamental matrix:

$$F = K^{-T}[t_{rel}] \times R_{rel} K^{-1}$$

2.1.3

The reflection point of x is x'=$R_f x$, where R_f is the reflection.

R_f=$H \begin{bmatrix} \Lambda & 0 \\ 0^T & 1 \end{bmatrix} H^-1$ and $H = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$ and $\Lambda = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

At Camera C, the points will be C_x and C'_x=$CR_f x$ which is equivalent to x by two camera C and CR_f

$$\begin{aligned} C &= K[I|0] \\ CR_f &= K[I|0]H\Lambda H^-1 \\ &= K[R\Lambda R^T| - R\Lambda R^T t] \\ &= K[R\Lambda R^T|R\Gamma R^T t] \end{aligned}$$

where

$$\Gamma = I - \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The corresponding canonical cameras are $[I|0]$ and $K[R\Lambda R^T|R\Gamma R^T t]$.
From previous part, we got $F = K^{-T}[t_{rel}] \times R_{rel}K^{-1}$ where t_{rel} is the relative translation between the two cameras and R_{rel} is the relative rotation matrix.

We can rewrite F in terms of the two cameras C and CR_f as follows:

$$\begin{aligned} F &= K^{-T}[t_{rel}] \times R_{rel}K^{-1} \\ &= K^{-T}[R\Gamma R^T t] \times RK^{-1} \\ &= K^{-T}[R\Gamma R^T t] \times RK^{-T}KK^{-1} \\ &= K^{-T}[R\Gamma R^T t] \times RK^{-T}C^{-1}CR_f^{-1} \\ &= (CR_f^{-T})^{-1}\left[R\Gamma R^T t\right] \times (CR_f^{-1})^{-1} \\ &= \left[CR_f^{-T} t \times R\Gamma R_f^{-1}\right]_\times \end{aligned}$$

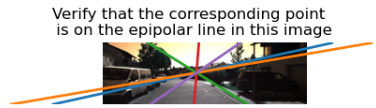
where $t_\times$ is the skew-symmetric matrix associated with the translation vector t. We can see that F is skew-symmetric, which is a necessary condition for it to be a fundamental matrix.

Therefore, we have shown that the situation where a camera views an object and its reflection in a plane mirror is equivalent to having two images of the object which are related by a skew-symmetric fundamental matrix F.

2.3.1

2.3.2

$$\text{F}=\begin{bmatrix} -1.30724119e-09 & -4.31399380e-07 & 7.32353517e-05 \\ 4.29598356e-07 & 7.49867998e-10 & -2.52749514e-04 \\ -7.14434163e-05 & 2.58007162e-04 & -1.32813462e-03 \end{bmatrix}$$



2.4.1

$$E=\begin{bmatrix} -1.69071897e-01 & 1.33884605e-01 & 5.80582647e-03 \\ -1.32435582e-01 & -1.69139297e-01 & 2.28499567e-03 \\ -3.64977290e-03 & -2.91751291e-03 & 7.73717309e-05 \end{bmatrix}$$