Cardinality, starter pack

We will use our defined notion of, "counting numbers" or "inductive numbers", $\mathbb{N}$ to *count* other sets. This is *cardinality*. In this section, we fix sets X, Y.

Definition 0.1. A function $f : X \to Y$ is

- *injective* if ???
- *surjective* if ???
- *bijective* if ???

Definition 0.2. Two sets X, Y have *equal cardinality* if there is a bijection

$$X \simeq Y$$

- A set is said to have *cardinality* n if

  $$\{i \in \mathbb{N} : 1 \leq i \leq n\} \simeq X$$

  In this case, we say X is *finite*. Otherwise, X is *infinite*.

- A set X is *countably infinite*[1] if it has same cardinality with $\mathbb{N}$.

Definition 0.3. We denote the *cardinality of a set* X by $|X|$. [2]

Historically, some take *cardinal numbers* as i.e. the equivalence class of bijective sets as the primitive notion.

Definition 0.4. Let X, Y be sets: We denote

- $|X| \leq |Y|$ if there is an injection from X to Y.
- $|X| = |Y|$ if there is a bijection between X and Y.

One of the beautiful results in Set theory is the Schroeder Bernstein theorem.

Theorem 0.5. The $\leq$ relation on cardinality, is reflexive: if $|X| \leq |Y|$ and $|Y| \leq |X|$ then $|X| = |Y|$. [3]

Problems on next page:

[1] Or *countable*. Sometimes countable means (finite *and* countably infinite).

[2] This definition does *not make sense yet!*. What if a set has two cardinality? Let us assume this is well-defined first. See question 2.

[3] why is this not obvious?