The BZSO formula for Bung

6 Let $G \subset X$, and $\Sigma_B = \Sigma_{Betti} \simeq B \pi_i$.

Set
$$Bun_G := \underline{Map}(\Sigma_{Betti}, BG)$$
 mapping $Bun_G := \underline{Map}(\Sigma_{Betti}, X/G)$ stacks

 $Rep_G := \underline{Hom}_{Gp}(\pi_1, G)$, so that As well as Bung = (Repg)/G.

Let $p:\pi, \to G$ be a group map, i.e. $p:x \to \text{Rep}_G$

Induces $\pi_1 C \times$ with $\gamma \in \pi_1$ acting by $\gamma(\gamma) C \times \dots$ so set $\gamma := \chi^{\pi_1} = \chi^{\pi_1}$ fixed-pts wit this π_1 -action.

Prop: The fiber of Bung -> Bung over * Prepg -> Bung is X.

Proof: Let us first (for additional) work this out on the level of points:

 $Map_{Stk}(x, Bun_{G}^{\times} \times \{p\})$ by def of Map_Stk $(B\pi, X/G) \times \{p\}$ $Map_{Stk}(B\pi, BG)$



Stk/BT, ~ LModx, (Stk) standard equivalence by Stacks over BT, & stack, ~ $(Y/\pi, \rightarrow B\pi,) \longleftrightarrow (\pi, CY)$

 $Map_{Sk/RG}(B\pi, X/G)$

Map $Stk/B\pi$, $(B\pi, , X/\pi)$

 $Map_{LMod_{\pi_{1}}(Stk)}(*, \times)$

War Stk (*, XT)

det of XP Mapsek (*, X9)

by replacing Stk -> Stk/s for any boses (bus the argument really works in any co-topos), we obtain an equiv on the level of S-points VS, thus on equivalence of stacks.

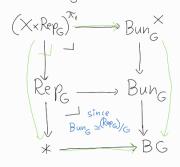
Or worked out Mop(S, Bun x x { p}) = Map(S x B \(\text{n}, \text{ X/6} \) = \(\text{f x B \(\text{n}, \text{ X/6} \)} \) = \(\text{ it's really the

Using that in the above argument, we could have taken of to be an A-pt, or one S-pt, of Rope if we would

More precisely, for any A-point PE Rep (A) we have a natural equivalence over Spec(A). Bun X x Spec(A) ~ (X x Spec(A)) recover for the representation potential pot

Applying this to A = O(RepG) since RepG = Spec(A) is affine, and the "universal repres." Puniv & RepG(A), corresponding to Repaid, Repa, ve get Bung Repg $\simeq (\times \times \text{Repg})^{P_{univ}} = (\times \times \text{Repg})^{\mathcal{T}_{i}}$

observe the diagram of pullback squares:



and the outer pullback square exhibits an equivalence of stacks

$$\operatorname{Bun}_{G}^{\times} \simeq \left(\times \operatorname{Rep}_{G} \right)^{\tau_{1}} / G$$