RESEARCH STATEMENT

MILTON LIN

My research revolves around foundational aspects of pure mathematics and machine learning. In mathematics, I concentrate on the geometric Langlands program, particularly its metaplectic and relative extensions. My recent interests in machine learning explore associative memory models and scaling properties, where algebraic and topological methods reveal more profound insights into neural network behavior.

Main research: p-adic Geometry and Langlands Program

I split my current and future projects into two categories: **core projects**, where I am primarily focused on advancing mixed characteristic and metaplectic aspects of the Langlands program, and additional **ongoing work in related areas**, including categorical deformations, motivic aspects, and stacky approaches.

Core proejcts. In the geometric Langlands program, my graduate work [1] has focused on extensions in the mixed characteristic setting, [ILZ24], this is joint work with Ashwin Iyengar (American Mathematical Society) and Konrad Zou (Bonn Univeristy). Our work applied the framework of Zhu's perfect geometry [Zhu17] to prove the Casselman-Shalika formula [NP01] for mixed characteristics. The Casselman-Shalika formula computes the "fourier coefficients" of automorphic forms and is fundamental to modern works of geometric Langlands, see [FR22]. Moving forward, I will continue this research in two directions:

(1) **Relative aspects of Langlands**, joint project with Yuta Takaya (University of Tokyo), we aim to explore relative aspects of the Langlands program on the Fargues-Fontaine curve, [FS24], recent conjectures of Ben-Zvi. Sakellaridis, and Venkatesh [BSV], particularly the relationship between period sheaves and L-sheaves as in [FW24].

(2) **Metaplectic aspects of Langlands**, joint with Toan Pham (Johns Hopkins Univeristy) I intend to give a geometric metaplectic Casselman–Shalika formula, building on the works of Gaitsgory and Lysenko [GL22], McNamara [McN16], and Brubaker et al. [Bru+24].

Related works. In addition to my primary projects in the relative and metaplectic aspects of Langlands, I am equally committed to three other areas of study, each of which contributes to the broader foundation of representation theory:

(1) **Categorical deformations of representation category** This builds upon my current research on the Whittaker category, from the point of view of deformation theory. We will first document a careful proof of Lurie's theorem, [Lur10, Thm 10.10], which describe *formal deformation of categories*, as gerbes see [Lur10, Ch.8-10] for definitions. Then, we will explore deformations of representation of Lusztig's small quantum group, using recent advances in quantum geometric Langlands. Ultimately, this research aims to contribute to the foundations of categorical deformations.

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[1] Details of my mathematical research: https://cwlin4916.github.io/Trees/Application/Postdoc/Research.pdf

(2) **Motivic aspects of Langlands** Building on [RS20], I aim to define a Whittaker category in motivic setting, obtaining a similar equivalence at [FGV01]. To begin, we prove the same statement in [NP01] in the category of mixed Tate motives. The difficulty is that the original argument requires characteristic 0 coefficients, whilst in positive characteristic, there are problems of extensions.

(3) **Stacky Approaches to Periods** I use recent advancements in formalism of stacks to study periods. I have proven that the unipotent fundamental group associated to a pointed scheme can be recovered via a *stacky approach*, see [GR14], [Toë06], [MR23]: for a given scheme X, there exists a natural stack, $\mathbf{U}(X_{\mathrm{dR}})$ whose fundamental group coincides with the unipotent de Rham fundamental group as studied in [Bro14]. By similar techniques of [Bha23], we should recover Haine's theorem: the pro-unipotent completion of de Rham fundamental group admits a mixed Hodge structure. [2] We hope that such work can spark new techniques and new phenomena, such as those used in p-adic integration theory, [Vol01].

Machine Learning: Geometry of Associative Memory Networks

In machine learning, I am particularly interested [3] in foundational theories of associative memory networks, going back to the work of Hopfield and to modern-day associative memories, [KH16]. These networks serve as a bridge between biological realism and computational efficiency.

(1) **Polytopal Decomposition of Memory Networks** in joint work with Chris Hillar (former Redwood Research, currently startup on Algebraic) and Tenzin Chan (Algebraic) we focus on the polytopal decomposition of the weight spaces of memory networks and its relation to network scaling. Similar works include, [ZNL18].

(2) **Study of Homotopy Type:** To further my study with Chris Hillar, we propose to explore *Hopfield networks* using a recent formalism by Manin and Marcolli [MM24], which uses *summing functors* and *Gamma spaces* to model the allocation of resources in neural networks. The formalism allows us to study a *homotopy type* - a mathematical construct at a deeper level than *homology*[4]. Homotopy captures invariants of the network up to continuous deformations. The relationship between memory capacity and homotopical invariants would be the main subject of study.

(3) **Emprical Study of Assocative Memory Models beyond the storage capcity** My joint work with Muhan Gao (Johns Hopkins University) empirically studies modern energy-based memory network transformers, such as [KH16], for language modeling and classification tasks. This was previously studied in the context of vision task in [Hoo+24]. We further explore these model in the regime where the stored memories is beyond the theoretical capacity, see Equation (5) and (6) of [KH16]. This research will also highlight the limitations of synthetic memory networks, especially in their use as proxies for explaining biological networks, see also [KH21].

References

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[2] This is important in Brown's approach, where he reduced his study of motivic periods to mixed Hodge periods [Bro17, p. 3].

[3] Details of my interest in memory networks `https://cwlin4916.github.io/Trees/Application/Postdoc/ResearchA.pdf`

[4] which is commonly used in topological data analysis (TDA). For a short survey of topology and neural code, see [Cur16].

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