数理解析研究所講究録
第 1871 巻 2013 年 28-36

THE TWISTED SATAKE ISOMORPHISM AND CASSELMAN-SHALIKA FORMULA

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ABSTRACT. For an arbitrary split adjoint group we identify the unramified Whittaker space with the space of skew-invariant functions on the lattice of coweights and deduce from it the Casselman-Shalika formula.

1. INTRODUCTION

The Casselman-Shalika formula is a beautiful formula relating values of special functions on a p-adic group to the values of finite dimentional complex representations of its dual group. Further, the formula is particularly useful in the theory of automorphic forms for studying L-functions.

In this note we provide a new approach to (and a new proof of) Casselman-Shalika formula for the value of spherical Whittaker functions.

To state our results we fix some notations. Let G be a split adjoint group over a local field F. We fix a Borel subgroup B with unipotent radical N, and consider its Levi decomposition $B=NT$, where T is the maximal split torus. Furthermore, we fix a maximal compact subgroup K.

Let Ψ be a non-degenerate complex character of N. For an irreducible representation π of G it is well known that $\dim \mathrm{Hom}_G(\pi, \mathrm{ind}_N^G \Psi) \leq 1$ and in case it is non-zero we say that the representation π is *generic*. The Whittaker model of such a generic irreducible representation π of G is the image of an embedding

$$W:\pi\hookrightarrow \mathrm{Ind}_N^G\Psi.$$

Let now π be a generic irreducible representation. Recall that π is called *unramified* if $\pi^K\neq\{0\}$ and that in this case $\pi^K=\mathbb{C}\cdot v_0$. Here v_0 is a spherical vector. The explicit formula for the function $W(v_0)$ was given in [CS] and is commonly called *the Casselman-Shalika formula.*

Recall that there is a bijection between irreducible unramified representations of G and the spectrum of the spherical Hecke algebra $H_K = C_c(K\backslash G/K)$. This commutative algebra admits the following description. Let Λ be the coweight lattice of G. Recall that Λ is canonically identified with $T/(T\cap K)$. The Weyl group W acts naturally on the lattice Λ. Denote by $\mathbb{C}[\Lambda]^W$ the algebra of W-invariant elements in $\mathbb{C}[\Lambda]$.

The Satake isomorphism

$$S: H_K\simeq \left(\mathrm{ind}_{T\cap K}^T 1\right)^W=\mathbb{C}[\Lambda]^W$$

is defined by

$$S(f)(t) = \delta_B^{-1/2}(t) \int\limits_N f(nt)\, dn$$

The main result of the paper is a description of the Whittaker spherical space $\left(\mathrm{Ind}_N^G \Psi\right)^K$ as a concrete $H_K \simeq \mathbb{C}[\Lambda]^W$-module. Namely, it is identified with the space $\mathbb{C}[\Lambda]^{W,-}$ of functions on the lattice of coweights, that are skew-invariant under the action of the Weyl group W. More formally,

Theorem 1.1. *There is a canonical isomorphism*

$$j : \left(\mathrm{ind}_N^G \Psi\right)^K \to \mathbb{C}[\Lambda]^{W,-},$$

compatible with Satake isomorphism $S : H_K \simeq \mathbb{C}[\Lambda]^W$.

From this result it easily follows that the twisted Satake map

$$S_\Psi : C_c(G/K) \to \left(\mathrm{ind}_N^G \Psi\right)^K, \quad S_\Psi(f)(t) = \int\limits_N f(nt)\overline{\Psi(n)}\, dn$$

sends the spectral basis of the spherical Hecke algebra $H_K = C_c(K\backslash G/K)$ to the basis of characteristic functions of $\left(\mathrm{ind}_N^G \Psi\right)^K$. In [FGKV], it is explained that this latter result is equivalent to the Casselman-Shalika formula in [CS] (see section 6 for a full account). Thus, we obtain a proof of the Casselman-Shalika formula that does not use the uniqueness of the Whittaker model.

Let us now quickly describe our proof of the main result. It was inspired by a new simple proof [S] by Savin of the Satake isomorphism of algebras

$$S : H_K \simeq \left(\mathrm{ind}_{T\cap K}^T 1\right)^W = \mathbb{C}[\Lambda]^W.$$

Savin has observed that the Satake map $S : C_c(G/K) \to \left(\mathrm{ind}_N^G 1\right)^K$ restricted to

$$C_c(I\backslash G/K) = \left(\mathrm{ind}_I^G 1\right)^K,$$

where I is an Iwahori subgroup, defines an explicit isomorphism

$$\left(\mathrm{ind}_I^G 1\right)^K \simeq \mathrm{ind}_{T\cap K}^T 1 = \mathbb{C}[\Lambda].$$

Restricting S_Ψ to $\left(\mathrm{ind}_I^G 1\right)^K$, we prove there there exists an isomorphism

$$j : \left(\mathrm{ind}_N^G \Psi\right)^K \to \mathbb{C}[\Lambda]^{W,-}$$

of $H_K \simeq \mathbb{C}[\Lambda]^W$-modules, making the following diagram

$$\begin{tikzcd}
\left(\mathrm{ind}_I^G 1\right)^K \arrow[r, "S_\Psi"] \arrow[d, "S"'] & \left(\mathrm{ind}_N^G \Psi\right)^K \arrow[d, "j"] \\
\mathbb{C}[\Lambda] \arrow[r, "alt"] & \mathbb{C}[\Lambda]^{W,-}
\end{tikzcd}$$

commutative. Here the space $\mathbb{C}[\Lambda]^{W,-}$ is a space of W skew-invariant elements of $\mathbb{C}[\Lambda]$ and the alternating map *alt* are defined in the section 3.

Acknowlegdements. I wish to thank Gordan Savin for explaining his proof of the Satake isomorphism and encouraging me to prove the Casselman-Shalika formula. I am most grateful to Joseph Bernstein for his attention and his help in formulating the main result. I thank Roma Bezrukavnikov for answering my questions and Eitan Sayag for his suggestions for improving the presentation. Part of this work was done during the workshop in RIMS. I wish to thank the organizers for their hospitality. My research is partly supported by ISF grant 1691/10.

2. Notations

Let F be a local non-archimedean field and let q be a characteristic of its residue field. Let G be a split adjoint group defined over F. Denote by B a Borel subgroup of G, by N its unipotent radical, by $\bar{N}$ the opposite unipotent radical, by T the maximal split torus and by W the Weyl group.

Denote by R the set of positive roots of G and by Δ the set of simple roots. For each $\alpha \in R$ let $x_\alpha : F \to N$ denote the one parametric subgroup corresponding to the root α and $N_\alpha^k = \{x_\alpha(r) : |r| \leq q^{-k}\}$.

Let Ψ be a non-degenerate complex character of N of conductor 1, i.e. for any $\alpha \in \Delta$

$$\Psi|_{N_\alpha^0} \neq 1, \Psi|_{N_\alpha^1} = 1.$$

Let K be a maximal compact subgroup of G. Then $T_K = T \cap K$ is a maximal compact subgroup of T. Choose an Iwahori subgroup $I \subset K$ such that $I \cap N = N_\alpha^1$ for all $\alpha \in R$. In particular $\Psi_{N \cap I} = 1$, but $\Psi|_{N_\alpha^0} \neq 1$ for any $\alpha \in \Delta$.

We fix a Haar measure on G normalized such that the measure of I is one.

The coweight lattice Λ of G is canonically identified with T/T_K. For any $\lambda \in \Lambda$ denote by $t_\lambda \in T$ its representative. The coweight ρ denotes the half of all the positive coroots. Since G is adjoint one has $\rho \in \Lambda$. We denote by Λ^+ the set of dominant coweights.

Let ${}^LG$ be the complex dual group of G. Then Λ is also identified with the lattice of weights of ${}^LG$. For a dominant weight λ we denote by V_λ the highest weight module of ${}^LG$ and by $wt(V_\lambda)$ the multiset of all the weights of this module.

3. Functions on lattices

Consider the algebra $\mathbb{C}[\Lambda] = Span\{e^\nu : \nu \in \Lambda\}$. The Weyl group W acts naturally on the lattice Λ. We denote by $\mathbb{C}[\Lambda]^W$ the algebra of W-invariant elements in $\mathbb{C}[\Lambda]$. The character map defines an isomorphism of algebras

$$Rep({}^LG) \simeq \mathbb{C}[\Lambda]^W.$$

For an irreducible module V_λ denote

$$a_\lambda = char(V_\lambda) = \sum_{\nu \in wt(V_\lambda)} e^\nu.$$

The elements $\{a_\lambda | \lambda \in \Lambda^+\}$ form a basis of $\mathbb{C}[\Lambda]^W$. The algebra $\mathbb{C}[\Lambda]^W$ acts on the space $\mathbb{C}[\Lambda]$ by multiplication.

The element $f \in \mathbb{C}[\Lambda]$ is called *skew-invariant* if $w(f) = (-1)^{l(w)} f$, where $l(w)$ is the length of the element w. Denote by $\mathbb{C}[\Lambda]^{W,-}$ the space of W skew-invariant elements. The algebra $\mathbb{C}[\Lambda]^W$ acts on $\mathbb{C}[\Lambda]^{W,-}$ by multiplication. Note that the action is torsion free.

Define the alternating map

$$alt : \mathbb{C}[\Lambda] \to \mathbb{C}[\Lambda]^{W,-}, \quad alt(e^\mu) = \sum_{w \in W} (-1)^{l(w)} e^{w\mu} : \quad \mu \in \Lambda$$

It is a map of $C[\Lambda]^W$ modules. The elements

$$\{r_{\mu+\rho} = \sum_{w \in W} (-1)^{l(w)} e^{w(\mu+\rho)} : \quad \mu \in \Lambda^+\}$$

form a basis of $\mathbb{C}[\Lambda]^{W,-}$. Note that for any $\lambda \in \Lambda^+$

$$alt(e^{\lambda+\rho}) = r_{\lambda+\rho} = r_\rho \cdot a_\lambda = alt(a_\lambda),$$

where the second equality is the Weyl character formula.

4. Hecke algebras

4.1. The spherical Hecke algebra. The spherical Hecke algebra $H_K = C_c(K \backslash G/K)$ is the algebra of locally constant compactly supported bi-K invariant functions with the multiplication given by convolution $*$. It has identity element 1_K - the characteristic function of K divided by $[K : I]$.

Consider the Satake map

$$S : C_c(G/K) \to C(N \backslash G/K) = C(T/T_K)$$

defined by

$$S(f)(t) = \delta_B^{-1/2}(t) \int_N f(nt) dn.$$

The famous Satake theorem claims that the restriction of S to H_K defines an isomorphism of algebras $S : H_K \simeq \mathbb{C}[\Lambda]^W$. Denote by A_λ the element of H_K corresponding to a_λ under this map. Thus $H_K = \mathrm{Span}\{A_\lambda : \lambda \in \Lambda^+\}$.

4.2. The Iwahori-Hecke algebra. The Iwahori-Hecke algebra $H_I = C_c(I \backslash G/I)$ is the algebra of locally constant compactly supported bi-I invariant functions with the multiplication given by convolution. Below we remind the list of properties of H_I, all can be found in [HKP].

(1) The algebra H_I contains a commutative algebra $A \simeq \mathbb{C}[\Lambda]$.

$$A = \mathrm{Span}\{\theta_\mu |\ \mu \in \Lambda\},$$

where

$$\theta_\mu = \begin{cases} \delta_B^{1/2} 1_{It_\mu I} & \mu \in \Lambda^+; \\ \theta_{\mu_1} * \theta_{\mu_2}^{-1} & \mu = \mu_1 - \mu_2, \quad \mu_1, \mu_2 \in \Lambda^+ \end{cases}.$$

The center Z_I of the algebra H_I is $A^W \simeq \mathbb{C}[\Lambda]^W$.

(2) The finite dimensional Hecke algebra $H_f = C(I\backslash K/I)$ is a subalgebra of H_I. The elements $t_w = 1_{IwI}$, where $w \in W$ form a basis of H_f. Multiplication in H induces a vector space isomorphism

$$H_f \otimes_{\mathbb{C}} A \to H_I$$

In particular the elements $t_w \theta_\mu$ where $w \in W, \mu \in \Lambda$ form a basis of H_I.

(3) The algebra H_K is embedded naturally in H_I. One has $H_K = Z_I * 1_K$ and

$$A_\lambda = \left(\sum_{\nu \in wt(V_\lambda)} \theta_\nu \right) * 1_K$$

(4) For the simple reflection $s \in W$ corresponding to a simple root α and a coweight μ one has

$$t_s \theta_\mu = \theta_{s\mu} t_s + (1-q) \frac{\theta_{s\mu} - \theta_\mu}{1 - \theta_{-\alpha}}.$$

In particular t_s commutes with $\theta_{k\alpha} + \theta_{-k\alpha}$ for any $k \geq 0$. In addition t_s commutes with θ_μ whenever $s\mu = \mu$.

4.3. **The intermediate algebra.** Finally consider the space $H_{I,K}$ defined by

$$H_K \subset H_{I,K} = H_I * 1_K = C_c(I\backslash G/K) \subset H_I.$$

It has a structure of right H_K module. The space $H_{I,K}$ plays a crucial role in Savin's paper [S]. The Satake map restricted to it is the isomorphism of $H_K \simeq \mathbb{C}[\Lambda]^W$ modules:

$$S : H_{I,K} \simeq \mathbb{C}[\Lambda], \quad S(\theta_\mu * 1_K) = e^\mu.$$

In particular, it is shown that the elements $\{\theta_\mu^K = \theta_\mu * 1_K,\ \mu \in \Lambda\}$ form a basis of $H_{I,K}$.

5. The Whittaker space $\left(\mathrm{ind}_N^G \Psi\right)^K$

Let Ψ be a non-degenerate character of conductor 1. Consider the space $\left(\mathrm{ind}_N^G \Psi\right)^K$ of complex valued functions on G that are (N, Ψ)-equivariant on the left, right K-invariant functions and are compactly supported modulo N.

The space $\left(\operatorname{ind}_N^G \Psi\right)^K$ has a structure of right H_K module by

$$(\phi * f)(x) = \int_G \phi(xy^{-1})f(y)dy, \quad \phi \in \left(\operatorname{ind}_N^G \Psi\right)^K, f \in H_K.$$

Any function ϕ on $\left(\operatorname{ind}_N^G \Psi\right)^K$ is determined by its values on $t_\lambda : \lambda \in \Lambda$ and $\phi(t_\lambda) = 0$ unless $\lambda \in \Lambda^+ \rho$.

The space $\left(\operatorname{ind}_N^G \Psi\right)^K$ has a basis of characteristic functions $\{\phi_\lambda : \lambda \in \Lambda^+ + \rho\}$ where

$$\phi_\lambda(ntk) = \begin{cases} \delta_B^{1/2}(t)\Psi(n) & t \in Nt_\lambda K \quad \lambda \in \Lambda^+ + \rho; \\ 0 & \text{otherwise} \end{cases}.$$

The main theorem of this paper is the description of $\left(\operatorname{ind}_N^G \Psi\right)^K$ as H_K module.

Theorem 5.1. *Let Ψ be a character of conductor* 1. *Then there is an isomorphism*

$$j : \left(\operatorname{ind}_N^G \Psi\right)^K \simeq \mathbb{C}[\Lambda]^{W,-}$$

compatible with $H_K \simeq \mathbb{C}[\Lambda]^W$.

5.1. **The twisted Satake isomorphism.** For a fixed character Ψ of N, consider a twisted Satake map

$$S_\Psi : C_c(G/I) \to \left(\operatorname{ind}_N^G \Psi\right)^I$$

defined by

$$S_\Psi(f)(t) = \int_N f(nt)\overline{\Psi(n)}\, dn.$$

Corrolary 5.2. *The restriction of S_Ψ to the right H_K submodule $\theta_\rho^K * H_K$ defines an isomorphism*

$$S_\Psi : \theta_\rho^K * H_K \simeq \left(\operatorname{ind}_N^G \Psi\right)^K$$

such that $S_\Psi(\theta_\rho^K * A_\lambda) = \phi_{\lambda+\rho}$.

Proof. By Weyl character formula $r_{\lambda+\rho} = r_\rho \cdot a_\lambda$. Hence

$$j(\phi_{\lambda+\rho}) = r_{\lambda+\rho} = r_\rho \cdot a_\lambda = j(\phi_\rho * A_\lambda),$$

and thus $\phi_\rho * A_\lambda = \phi_{\lambda+\rho}$.

Restricting S_Ψ to $H_{I,K}$, we obtain

$$S_\Psi(\theta_\rho^K * A_\lambda) = S_\Psi(\theta_\rho^K) * A_\lambda = \phi_\rho * A_\lambda = \phi_{\lambda+\rho}.$$

Since A_λ and $\phi_{\lambda+\rho}$ are bases of H_K and $\left(\operatorname{ind}_N^G \Psi\right)^K$ respectively, the map S_Ψ is an isomorphism. $\square$

To prove the theorem we shall need two lemmas. The first one ensures surjectivity of the map S_Ψ and the second one describes its kernel.

Lemma 5.3. *$S_\Psi(\theta_\mu^K) = \phi_\mu$ for all $\mu \in \Lambda^+ + \rho$. In particular the map*

$$S_\Psi : H_{I,K} \to \left(\mathrm{ind}_N^G \Psi\right)^K$$

is surjective.

Proof. It is enough to compute $S_\Psi(\theta_\mu * 1_K)(t_\gamma)$ for $\gamma \in \Lambda^+$.

Since μ is dominant one has

$$\theta_\mu^K = \delta_B^{1/2}(t_\mu)1_{It_\mu K}$$

and hence

$$S_\Psi(\theta_\mu^K)(t_\gamma) = \delta_B^{1/2}(t_\gamma) \int\limits_{N_{\gamma,\mu}} \overline{\Psi(n)}\, dn,$$

where

$$N_{\gamma,\mu} = \{n \in N : nt_\gamma \in It_\mu K\}.$$

The set

$$N_{\gamma,\mu} = \begin{cases} \emptyset & \gamma \neq \mu \\ N \cap K & \gamma = \mu \end{cases}$$

Indeed, since $\mu \in \Lambda^+$ one has $It_\mu K = (N \cap I)t_\mu K$. One inclusion is obvious. For another inclusion use the Iwahori factorization

$$I = (I \cap N)T_K(I \cap \bar{N})$$

to represent any $g \in It_\mu K$ as

$$g = na_0\bar{n}t_\mu k = nt_\mu a_0(t_\mu^{-1}\bar{n}t_\mu)k,$$

where $n \in N \cap I, a_0 \in T_K, \bar{n} \in \bar{N}, k \in K$. Since μ is dominant one has $(t_\mu^{-1}\bar{n}t_\mu) \in K$. So $g \in (N \cap I)t_\mu K$. Hence $N_{\gamma,\mu} = \emptyset$ unless $\gamma = \mu$ and $N_{\mu,\mu} = (N \cap I)t_\mu(N \cap K)t_\mu^{-1} = N \cap I$ since $\mu \in \Lambda^+ + \rho$. In particular $\Psi|_{N_{\mu,\mu}} = 1$. Hence

$$S_\Psi(\theta_\mu * 1_K) = \phi_\mu.$$

□

Lemma 5.4. *Let $\alpha \in \Delta$, s be a simple reflection corresponding to α and $\iota_\alpha = 1_I + t_s$ be the characteristic function of a parahoric subgroup I_α corresponding to α.*

(1) $S_\Psi(\iota_\alpha) = 0$.
(2) $S_\Psi(\theta_\mu^K + \theta_{s.\mu}^K) = 0$ for all $\mu \in \Lambda$.

Proof. 1)

$$S_\Psi(\iota_\alpha)(tw) = \int\limits_N \iota_\alpha(ntw)\overline{\Psi(n)}\, dn = \int\limits_{N \cap I_\alpha(tw)^{-1}} \overline{\Psi(n)}\, dn$$

The set $N \cap I_\alpha(tw)^{-1}$ is empty unless $w \in \{e, s\}$ and $t \in T_K$, in which case

$$S_\Psi(\iota_\alpha)(tw) = \int\limits_{N \cap I_\alpha} \overline{\Psi(n)}\, dn = 0$$

since the integral contains an inner integral over N^0_α on which Ψ is not trivial.

2) Let us represent any $\mu = \mu' + k\alpha$ where $(\mu', \alpha) = 0$. Then $s\mu = \mu' - k\alpha$. In particular

$$\theta^K_\mu + \theta^K_{s\mu} = \theta_{\mu'}(\theta_{k\alpha} + \theta_{-k\alpha})\iota_\alpha 1_K$$

By the results in 4.2 the element i_α commutes with $\theta_{\mu'}(\theta_{k\alpha} + \theta_{-k\alpha})$ and hence the above equals

$$\iota_\alpha \theta_{\mu'}(\theta_{k\alpha} + \theta_{-k\alpha})1_K = \iota_\alpha(\theta^K_\mu + \theta^K_{s\mu})$$

By part (1) it follows that $S_\Psi(\theta^K_\mu + \theta^K_{s\mu}) = 0$. □

Proof. of 5.1. We have shown that the map S_Ψ is surjective onto $(\mathrm{ind}^G_N \Psi)^K$ and

$$\mathrm{Ker}\, S_\Psi = \mathrm{Span}\{\theta_\mu - (-1)^{l(w)}\theta_{w\mu} | \mu \in \Lambda, w \in W\}.$$

Another words

$$(\mathrm{ind}^G_N \Psi)^K \simeq H_{I,K}/\mathrm{Ker}\, S_\Psi = \mathbb{C}[\Lambda]^{W,-}$$

as $H_K \simeq \mathbb{C}[\Lambda]^W$-modules. □

6. Casselman-Shalika Formula

Let (π, G, V) be an irreducible smooth generic unramified representation and denote by $\gamma \in {}^LT/W$ its Satake conjugacy class. Choose a spherical vector v_0 and normalize the Whittaker functional $W_\gamma \in \mathrm{Hom}_G(\pi, \mathrm{Ind}^G_N \bar\Psi)$ such that $W_\gamma(t_\rho v_0) = 1$.

The Casselman-Shalika formula reads as follows:

Theorem 6.1.

$$W_\gamma(v_0)(t_{\lambda+\rho}) = \begin{cases} \delta_B^{1/2}(t_{\lambda+\rho})\,\mathrm{tr}\, V_\lambda(t_\gamma) & \lambda \in \Lambda^+ \\ 0 & \text{otherwise} \end{cases}$$

It is shown in [FGKV], that Theorem 5.2 that the formula (6.1) implies the Corollary 5.2 and it is mentioned that the two statements are equivalent. Let us now prove the other direction.

Proof. We deduce the formula 6.1 from 5.2. Let π be a generic unramified representation with the Satake parameter $\gamma \in {}^LT$ and a spherical vector v_0 and the Whittaker model $W_\gamma : \pi_\gamma \to \mathrm{Ind}^G_N \bar\Psi$ such that $W_\gamma(v_0)(t_\rho) = 1$. Define the map $\chi_\gamma : H_K \to \mathbb{C}$ by

$$\pi(f)v_0 = \int_G f(g)\pi(g)v_0\, dg = \chi_\gamma(f)v_0$$

and the map $r_\gamma : \mathrm{ind}^G_N \Psi \to \mathbb{C}$ by

$$r_\gamma(\phi) = \int_{N\backslash G} W_\gamma(v_0)(g)\phi(g)\, dg.$$

Then

$$r_\gamma(S_\Psi(\theta_\rho^K * A_\lambda)) = \int\limits_{N\backslash G} \int\limits_{N} (\theta_\rho^K * A_\lambda)(ng)\bar{\Psi}(n) W_\gamma(v_0)(g)\, dn\, dg =$$

$$\int\limits_{G} \int\limits_{G} (\theta_\rho^K * A_\lambda)(g) W_\gamma(v_0)(g)\, dg =$$

$$\int\limits_{G} \int\limits_{G} \theta_\rho^K(gx^{-1}) A_\lambda(x) W_\gamma(gx^{-1} \cdot x \cdot v_0)(1)\, dx\, dg = \chi_\gamma(A_\lambda) W_\gamma(v_0)(t_\rho).$$

Under the identification $H_K \simeq \mathrm{Rep}(^L G)$ the homomorphism χ_γ sends an irreducible representation V to $\operatorname{tr} V(\gamma)$. In particular $\chi_\gamma(A_\lambda) = \operatorname{tr} V_\lambda(\gamma)$.

$$\operatorname{tr} V_\lambda(\gamma) = \chi_\gamma(A_\lambda) W(v_0)(t_\rho) =$$

$$r_\gamma(S_\Psi(\theta_\rho^K * A_\lambda)) = r_\gamma(\phi_{\lambda+\rho}) = \delta_B^{-1/2}(t_{\lambda+\rho}) W_\gamma(v_0)(t_{\lambda+\rho})$$

Hence

$$W_\gamma(v_0)(t_{\lambda+\rho}) = \delta_B^{1/2}(t_{\lambda+\rho}) \operatorname{tr} V_\lambda(\gamma).$$

□

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