Examples for Ax.2

Ex: $\{0, 1, 2\}$.

Ax.1: is satisfied. $0 \in \{0,1,2\}$.

Ax.2: • if $n \in \mathbb{N}$

$$S(0) = 0$$
$$S(1) = 2$$
$$S(2) = 1.$$

← does not satisfy Ax.3.

"S is a rule which assigns to each $n \in \mathbb{N}$ a new number in $\mathbb{N}$".

• if $n \in \mathbb{N}$

$$S(0) = 1$$
$$S(1) = 2$$
$$\boxed{S(2) = 0.}$$

← does not satisfy axiom 3.

• Another way to state is

$$S : \mathbb{N} \to \mathbb{N}.$$

Ex: • $\{0, 1\}$

• $S(0) = 1$ ✓

$\boxed{S(1) = 0}$ ← does not satisfy ax. 3.

Ex: $\bullet\ \{0, 1, 2\}$.

$\bullet$ $S(0) = 1$.
$S(1) = 1$
$S(2) = 1$.

$$S(0) = 1,\quad S(1) = 1 \quad \overset{\text{ax. 4}}{\underset{\text{"implies"}}{\Longrightarrow}} \quad 0 = 1.$$

Ex: $\{0, \frac{1}{2} = 0.5, 1, \frac{3}{2} = 1.5, 2, \frac{5}{2}, 3, \frac{7}{2}, \dots\}$.

$\bullet$ $S(n) = n+1$.

$S(0) = 1$	$S(1) = 2$	$P'(0) = T$
$S(0.5) = 1.5$	$S(2.5) = 2.5$.	$P'(0.5) = F$
		$P'(1) = T$.
		$P'(1.5) = F$
		$\vdots$

the axioms so far can't exclude this.

Ex: $\{0, A, AA, AAA, \dots\}$

rule: $S(A \dots A) = A \dots A\text{"}A\text{"}$ ← adding a letter A.
$S(A) = AA$
$S(AA) = AAA$.

Ex: $\{0, 0.5, 1, 1.5, 2, 2.5, 3, \ldots\}$ " = " $\mathbb{N}$.

- $S(n) = n+1$

$P(n)$ be the statemet "n has decimal places".

5a. $P(0)$ is T.
5b. $P(n)$ is T then $P(n+1)$ is T.

"if n+1 has decimal places then $(n+1)+1$ has no decimal places".

n must be one of those whole numbers

n+2 must be also a whole number.

therefore.

$\therefore$ By principle of induction.

$P(n)$ is T for all n.

But this is contradiction!
for $n = 0.5$, $P(0.5)$ is F.

Prop: $1 \neq 0$.

Pf: • $1 := S(0)$.

• By axiom 3, $S(0) \neq 0$.

Prop: $5 \neq 2$

Pf $5 := S(4)$ $2 := S(1)$.

$S(4) \neq S(1)$

Ax. 4: (if $S(4) = S(1)$ then $4 = 1$).

STS: $4 \neq 1$. $4 := S(3)$, $1 = S(0)$

Ax. 4 ie STS: $3 \neq 0$.

Is it $3 \neq 0$?

$3 = S(2)$.,

WTS: $S(2) \neq 0$. , by Ax. 3 this T.

$$\boxed{\begin{array}{l} \text{First, } 3 \neq 0 \text{ by Ax. 3.} \\ \text{this implies} \\ \qquad 4 \neq 1, \text{ by Ax. 4.} \\ \text{this implies} \\ \qquad 5 \neq 2 \text{ by Ax. 4.} \end{array}}$$

"

M is a set of $\underline{2023}$ elemets.

↑ we will induct on.

induct on the size of M.

cardinaley.

what happens when $|M| = 1$ $= \{1\}$.

ϕ, $\{1\}$.

- let N : $0 \leq N \leq 2^1$ ← is the size of set

$N = 0$: ✓

$N = 1$: doesn't matter where we put

$N = 2$: put everything white.

- $|M| = 2$.

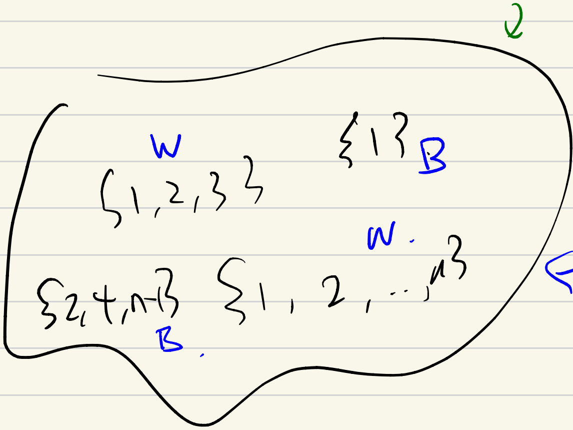
let N : $0 \leq N \leq 2^2$

$N = 0$: ✓

$N = 1$: ✓

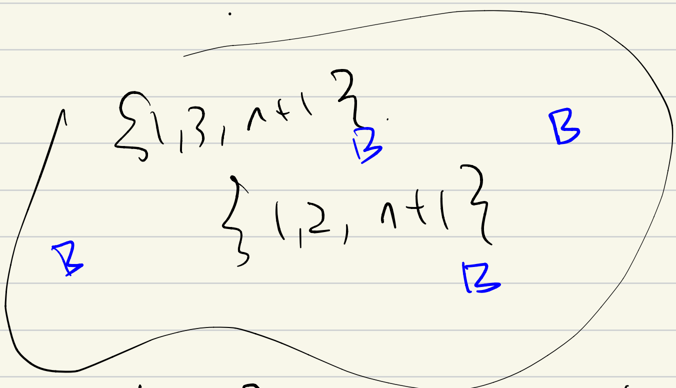
$N = 2$.

"closed under union"



← 2^n. subset.

1. use coloring scheme from hypothesis on this coll'n



$A : n+1 \in A$

2^n ~~subsets~~

$n+1$

2. declare B on everything else.

• On whole 2^{n+1} subsets is (a) ✓ satisfied?

(b) two blacks in sets w/o $n+1$

Two blacks, one has $n+1$.