

## TABLE OF CONTENTS OF EGA I–IV

### Chapter 0. Preliminaries

(In Volume I)

- §1. Rings of fractions
  - 1.0 Rings and algebras
  - 1.1 Radical of an ideal; nilradical and radical of a ring
  - 1.2 Modules and rings of fractions
  - 1.3 Functorial properties
  - 1.4 Change of multiplicative set
  - 1.5 Change of ring
  - 1.6 The module  $M_f$  as a direct limit
  - 1.7 Support of a module
- §2. Irreducible and Noetherian spaces
  - 2.1 Irreducible spaces
  - 2.2 Noetherian spaces
- §3. Supplement on sheaves
  - 3.1 Sheaves with values in a category
  - 3.2 Presheaves on a base of open sets
  - 3.3 Gluing sheaves
  - 3.4 Direct images of presheaves
  - 3.5 Inverse images of presheaves
  - 3.6 Constant and locally constant sheaves
  - 3.7 Inverse images of presheaves of groups or rings
  - 3.8 Pseudo-discrete sheaves of spaces
- §4. Ringed spaces
  - 4.1 Ringed spaces, sheaves of  $\mathcal{A}$ -modules,  $\mathcal{A}$ -algebras
  - 4.2 Direct image of an  $\mathcal{A}$ -module
  - 4.3 Inverse image of an  $\mathcal{A}$ -module
  - 4.4 Relation between direct and inverse images
- §5. Quasi-coherent and coherent sheaves
  - 5.1 Quasi-coherent sheaves
  - 5.2 Sheaves of finite type
  - 5.3 Coherent sheaves
  - 5.4 Locally free sheaves
  - 5.5 Sheaves on a locally ringed space
- §6. Flatness
  - 6.1 Flat modules
  - 6.2 Change of rings
  - 6.3 Local nature of flatness
  - 6.4 Faithfully flat modules

- 6.5 Restriction of scalars
- 6.6 Faithfully flat rings
- 6.7 Flat morphisms of ringed spaces

§7.  $I$ -adic rings

- 7.1 Admissisble rings
- 7.2  $I$ -adic rings and projective limits
- 7.3 Pre- $I$ -adic Noetherian rings
- 7.4 Quasi-finite modules over local rings
- 7.5 Restricted formal series rings
- 7.6 Completed rings of fractions
- 7.7 Completed tensor products
- 7.8 Topologies on Hom modules

(In Volume III, Part 1)

§8. Representable functors

- 8.1 Representable functors
- 8.2 Algebraic structures in categories

§9. Constructible sets

- 9.1 Constructible sets
- 9.2 Constructible subsets of Noetherian spaces
- 9.3 Constructible functions

§10. Supplement on flat modules

- 10.1 Relations between free and flat modules
- 10.2 Local flatness criteria
- 10.3 Existence of flat extensions of local rings

§11. Supplement on homological algebra

- 11.1 Reminder on spectral sequences
- 11.2 Spectral sequence of a filtered complex
- 11.3 Spectral sequences of a double complex
- 11.4 Hypercohomology of a functor on a complex  $K^\bullet$
- 11.5 Inductive limits in hypercohomology
- 11.6 Hypercohomology of a functor on a complex  $K_\bullet$
- 11.7 Hypercohomology of a functor on a double complex  $K_{\bullet\bullet}$
- 11.8 Supplement on cohomology of simplicial complexes
- 11.9 A lemma on complexes of finite type
- 11.10 Euler-Poincaré characteristic of a complex of finite-length modules

§12. Supplement on sheaf cohomology

- 12.1 Cohomology of sheaves of modules on a ringed space
- 12.2 Higher direct images
- 12.3 Supplement on Ext of sheaves
- 12.4 Hypercohomology of the direct image functor

§13. Projective limits in homological algebra

- 13.1 Mittag-Leffler condition
- 13.2 Mittag-Leffler condition for abelian groups
- 13.3 Application to cohomology of a projective limit of sheaves
- 13.4 Mittag-Leffler condition and graded objects associated to projective systems
- 13.5 Projective limits of spectral sequences of filtered complexes
- 13.6 Spectral sequence of a functor relative to a finitely filtered object
- 13.7 Derived functors on projective limits

(In Volume IV, Part 1)

§14. Combinatorial dimension of a topological space

- 14.1 Combinatorial dimension of a topological space
- 14.2 Codimension of a closed subset
- 14.3 Chain condition

§15.  $M$ -regular and  $\mathcal{F}$ -regular sequences

- 15.1  $M$ -regular and  $M$ -quasi-regular sequences
- 15.2  $\mathcal{F}$ -regular sequences

§16. Dimension and depth of Noetherian local rings

- 16.1 Dimension of a ring
- 16.2 Dimension of a semi-local Noetherian ring
- 16.3 Systems of parameters in a Noetherian local ring
- 16.4 Depth and co-depth
- 16.5 Cohen-Macaulay modules

§17. Regular rings

- 17.1 Definition of regular ring
- 17.2 Reminder on projective and injective dimension
- 17.3 Cohomological theory of regular rings

§18. Supplement on extension of algebras

- 18.1 Inverse images of augmented rings
- 18.2 Extension of ring by a bi-module
- 18.3 The group of  $A$ -extension classes
- 18.4 Extensions of algebras
- 18.5 Case of topological rings

§19. Formally smooth algebras and Cohen rings

- 19.0 Introduction
- 19.1 Formal epi- and monomorphisms
- 19.2 Formally projective modules
- 19.3 Formally smooth algebras
- 19.4 First criteria for formal smoothness
- 19.5 Formal smoothness and associated graded rings
- 19.6 Case of algebras over a field

- 19.7 Case of local homomorphisms; existence and uniqueness theorems
- 19.8 Cohen algebras and  $p$ -rings; structure of complete local rings
- 19.9 Relatively formally smooth algebras
- 19.10 Formally unramified and formally étale algebras

## §20. Derivations and differentials

- 20.1 Derivations and extensions of algebras
- 20.2 Functorial properties of derivations
- 20.3 Continuous derivations of topological rings
- 20.4 Principal and differential subsets
- 20.5 Basic functorial properties of  $\Omega_{B/A}^1$
- 20.6 Imperfection modules and characteristic homomorphisms
- 20.7 Generalizations to topological rings

## §21. Differentials in rings of characteristic $p$

- 21.1 Systems of  $p$ -generators and  $p$ -bases
- 21.2  $p$ -bases and formal smoothness
- 21.3  $p$ -bases and imperfection modules
- 21.4 Case of a field extension
- 21.5 Application: separability criteria
- 21.6 Admissible fields for an extension
- 21.7 Cartier's identity
- 21.8 Admissibility criteria
- 21.9 Completed modules of differentials for formal power series rings

## §22. Differential criteria for smoothness and regularity

- 22.1 Lifting of formal smoothness
- 22.2 Differential characterization of formally smooth local algebras over a field
- 22.3 Application to relations between certain local rings and their completions
- 22.4 Preliminary results on finite extensions of local rings in which  $\mathfrak{m}^2 = 0$ .
- 22.5 Geometrically regular and formally smooth algebras
- 22.6 Zariski's Jacobian criterion
- 22.7 Nagata's Jacobian criterion

## §23. Japanese rings

- 23.1 Japanese rings
- 23.2 Integral closure of a Noetherian local domain

# Chapter I. The language of schemes

## (Volume I)

### §1. Affine schemes

- 1.1 Prime spectrum ( $\text{Spec}$ ) of a ring
- 1.2 Functorial properties of  $\text{Spec}$
- 1.3 Sheaf associated to a module
- 1.4 Quasi-coherent sheaves on  $\text{Spec}$

- 1.5 Coherent sheaves on Spec
- 1.6 Functorial properties of quasi-coherent sheaves on Spec
- 1.7 Characterization of morphisms of affine schemes
- (Volume II addenda)
- 1.8 Morphisms of ringed spaces and local rings of affine schemes
- §2. Preschemes and their morphisms
  - 2.1 Definition of prescheme
  - 2.2 Morphisms of preschemes
  - 2.3 Gluing preschemes
  - 2.4 Local schemes
  - 2.5 Preschemes over a prescheme
- §3. Product of preschemes
  - 3.1 Disjoint union of preschemes
  - 3.2 Product of preschemes
  - 3.3 Formal properties of the product; change of base prescheme
  - 3.4 Points of a prescheme with values in a prescheme; geometric points
  - 3.5 Surjections and injections
  - 3.6 Fibers
  - 3.7 Application: reduction of a prescheme mod  $\mathcal{I}$
- §4. Sub-preschemes and immersions
  - 4.1 Sub-preschemes
  - 4.2 Immersions
  - 4.3 Product of immersions
  - 4.4 Inverse image of a prescheme
  - 4.5 Local immersions and local isomorphisms
- §5. Reduced preschemes; separatedness
  - 5.1 Reduced preschemes
  - 5.2 Existence of sub-prescheme with a given underlying space
  - 5.3 Diagonal; graph of a morphism
  - 5.4 Separated morphisms and preschemes
  - 5.5 Criteria for separatedness
- §6. Finiteness conditions
  - 6.1 Noetherian and locally Noetherian preschemes
  - 6.2 Artinian preschemes
  - 6.3 Morphisms of finite type
  - 6.4 Algebraic preschemes
  - 6.5 Local determination of a morphism
  - 6.6 Quasi-compact morphisms and morphisms locally of finite type
- §7. Rational maps
  - 7.1 Rational maps and rational functions

- 7.2 Domain of definition of a rational map
- 7.3 Sheaf of rational functions
- 7.4 Torsion sheaves and torsion-free sheaves

## §8. Chevalley schemes

- 8.1 Allied local rings
- 8.2 Local rings of an integral scheme
- 8.3 Chevalley schemes

## §9. Supplement on quasi-coherent sheaves

- 9.1 Tensor product of quasi-coherent sheaves
- 9.2 Direct image of a quasi-coherent sheaf
- 9.3 Extension of sections of quasi-coherent sheaves
- 9.4 Extension of quasi-coherent sheaves
- 9.5 Closed image of a prescheme; closure of a sub-prescheme
- 9.6 Quasi-coherent sheaves of algebras; change of structure sheaf

## §10. Formal schemes

- 10.1 Affine formal schemes
- 10.2 Morphisms of affine formal schemes
- 10.3 Ideals of definition of a formal affine scheme
- 10.4 Formal preschemes and their morphisms
- 10.5 Ideals of definition of formal preschemes
- 10.6 Formal preschemes as inductive limits of schemes
- 10.7 Product of formal schemes
- 10.8 Formal completion of a prescheme along a closed subset
- 10.9 Extension of morphisms to completions
- 10.10 Application to coherent sheaves on formal schemes

## Chapter II. Basic global properties of some classes of morphisms.

### (Volume II)

## §1. Affine morphisms

- 1.1  $S$ -preschemes and  $\mathcal{O}_S$ -algebras
- 1.2 Preschemes affine over a prescheme
- 1.3 Affine prescheme over  $S$  associated to an  $\mathcal{O}_S$ -algebra
- 1.4 Quasi-coherent sheaves on a prescheme affine over  $S$
- 1.5 Change of base prescheme
- 1.6 Affine morphisms
- 1.7 Vector bundle associated to a sheaf of modules

## §2. Homogeneous prime spectra

- 2.1 Generalities on graded rings and modules
- 2.2 Rings of fractions of a graded ring
- 2.3 Homogeneous prime spectrum of a graded ring
- 2.4 The scheme structure of  $\text{Proj}(S)$

- 2.5 Sheaf associated to a graded module
- 2.6 Graded  $S$ -module associated to a sheaf on  $\text{Proj}(S)$
- 2.7 Finiteness conditions
- 2.8 Functorial behavior
- 2.9 Closed sub-preschemes of  $\text{Proj}(S)$
- §3. Homogeneous prime spectrum of a sheaf of graded algebras
  - 3.1 Homogeneous prime spectrum of a graded, quasi-coherent  $\mathcal{O}_Y$ -algebra
  - 3.2 Sheaf on  $\text{Proj}(\mathcal{S})$  associated to a sheaf of graded  $\mathcal{S}$ -modules
  - 3.3 Sheaf of graded  $\mathcal{S}$ -modules associated to a sheaf on  $\text{Proj}(\mathcal{S})$
  - 3.4 Finiteness conditions
  - 3.5 Functorial behavior
  - 3.6 Closed sub-preschemes of  $\text{Proj}(\mathcal{S})$
  - 3.7 Morphisms from a prescheme to a Proj
  - 3.8 Criteria for immersion into a Proj
- §4. Projective bundles; ample sheaves.
  - 4.1 Definition of projective bundles
  - 4.2 Morphisms from a prescheme to a projective bundle
  - 4.3 The Segre morphism
  - 4.4 Immersions in projective bundles; very ample sheaves
  - 4.5 Ample sheaves
  - 4.6 Relative ample sheaves
- §5. Quasi-affine, quasi-projective, proper and projective morphisms
  - 5.1 Quasi-affine morphisms
  - 5.2 Serre's criterion
  - 5.3 Quasi-projective morphisms
  - 5.4 Universally closed and proper morphisms
  - 5.5 Projective morphisms
  - 5.6 Chow's lemma
- §6. Integral and finite morphisms
  - 6.1 Preschemes integral over another
  - 6.2 Quasi-finite morphisms
  - 6.3 Integral closure of a prescheme
  - 6.4 Determinant of an endomorphism of a sheaf of  $\mathcal{O}_X$ -modules
  - 6.5 Norm of an invertible sheaf
  - 6.6 Application: criteria for ampleness
  - 6.7 Chevalley's theorem
- §7. Valuative criteria
  - 7.1 Reminder on valuation rings
  - 7.2 Valuative criterion for separatedness
  - 7.3 Valuative criterion for properness
  - 7.4 Algebraic curves and function fields of dimension 1

- §8. Blowup schemes; projective cones; projective closure
  - 8.1 Blowup preschemes
  - 8.2 Preliminary results on localization of graded rings
  - 8.3 Projective cones
  - 8.4 Projective closure of a vector bundle
  - 8.5 Functorial behavior
  - 8.6 A canonical isomorphism for pointed cones
  - 8.7 Blowing up projective cones
  - 8.8 Ample sheaves and contractions
  - 8.9 Grauert's ampleness criterion: statement
  - 8.10 Grauert's ampleness criterion: proof
  - 8.11 Uniqueness of contractions
  - 8.12 Quasi-coherent sheaves on projective cones
  - 8.13 Projective closure of sub-sheaves and closed subschemes
  - 8.14 Supplement on sheaves associated to graded  $\mathcal{S}$ -modules

### Chapter III. Cohomological study of coherent sheaves

(Volume III Part 1)

- §1. Cohomology of affine schemes
  - 1.1 Reminder on the exterior algebra complex
  - 1.2 Čech cohomology of an open cover
  - 1.3 Cohomology of an affine scheme
  - 1.4 Application to cohomology of general preschemes
- §2. Cohomological study of projective morphisms
  - 2.1 Explicit calculation of some cohomology groups
  - 2.2 Fundamental theorem on projective morphisms
  - 2.3 Application to sheaves of graded algebras and modules
  - 2.4 Generalization of the fundamental theorem
  - 2.5 Euler-Poincaré characteristic and Hilbert polynomial
  - 2.6 Application: criteria for ampleness
- §3. Finiteness theorem for proper morphisms
  - 3.1 “Dévissage” lemma
  - 3.2 Finiteness theorem for ordinary schemes
  - 3.3 Generalization of the finiteness theorem
  - 3.4 Finiteness theorem for formal schemes
- §4. Fundamental theorem on proper morphisms, and applications
  - 4.1 The fundamental theorem
  - 4.2 Special cases and variations
  - 4.3 Zariski's connectedness theorem
  - 4.4 Zariski's “main theorem”
  - 4.5 Completion of Hom modules



- 4.6 Relations between ordinary and formal morphisms
- 4.7 An ampleness criterion
- 4.8 Finite morphisms of formal preschemes
- §5. An existence theorem for coherent sheaves
  - 5.1 Statement of the theorem
  - 5.2 Proof in the projective & quasi-projective case
  - 5.3 Proof in the general case
  - 5.4 Application: comparison between morphism of ordinary and formal schemes; algebraisable formal schemes
  - 5.5 Decomposition of certain schemes
- (Volume III Part 2)
- §6. Local and global Tor, Künneth formula
  - 6.1 Introduction
  - 6.2 Hypercohomology of complexes of sheaves of modules on a prescheme
  - 6.3 Hypertor of two complexes
  - 6.4 Local hypertor for quasi-coherent complexes, affine case
  - 6.5 Local hypertor for quasi-coherent complexes, general case
  - 6.6 Global hypertor for quasi-coherent complexes and Künneth spectral sequence, case of an affine base
  - 6.7 Global hypertor for quasi-coherent complexes and Künneth spectral sequence, general case
  - 6.8 Associativity spectral sequence for global hypertor
  - 6.9 Base-change spectral sequence for global hypertor
  - 6.10 Local nature of certain cohomological functors
- §7. Base change for homological functors on sheaves of modules
  - 7.1 Functors on  $A$ -modules
  - 7.2 Characterization of the tensor product functor
  - 7.3 Exactness criteria for homological functors on modules
  - 7.4 Exactness criteria for the functors  $H_{\bullet}(P_{\bullet} \otimes_A M)$
  - 7.5 Case of Noetherian local rings
  - 7.6 Descent of exactness properties; semi-continuity theorem and Grauert's exactness criterion
  - 7.7 Application to proper morphisms: I. Exchange property
  - 7.8 Application to proper morphisms: II. Cohomological flatness criteria
  - 7.9 Application to proper morphisms: III. Invariance of Euler-Poincaré characteristic and Hilbert polynomial
- Chapter IV. Local study of schemes and morphisms
  - (Volume IV Part 1)
  - §1. Relative finiteness conditions; constructible sets in preschemes
    - 1.1 Quasi-compact morphisms

- 1.2 Quasi-separated morphisms
- 1.3 Morphisms locally of finite type
- 1.4 Locally finitely presented morphisms
- 1.5 Morphisms of finite type
- 1.6 Finitely presented morphisms
- 1.7 Improvements of preceding results
- 1.8 Finitely presented morphisms and constructible sets
- 1.9 Pro- and ind-constructible morphisms
- 1.10 Application to open morphisms

(Volume IV Part 2)

§2. Base change and flatness

- 2.1 Flat sheaves of modules on preschemes
- 2.2 Faithfully flat sheaves on preschemes
- 2.3 Topological properties of flat morphisms
- 2.4 Universally open morphisms and flat morphisms
- 2.5 Persistence of properties of sheaves under faithfully flat descent
- 2.6 Persistence of set-theoretic and topological properties under faithfully flat descent
- 2.7 Persistence of various properties of morphisms under faithfully flat descent
- 2.8 Preschemes over a regular base scheme of dimension 1; closed subschemes in the

closure of the generic fiber

§3. Associated prime cycles and primary decomposition

- 3.1 Associated prime cycles of a sheaf of modules
- 3.2 Irredundant decompositions
- 3.3 Relations with flatness
- 3.4 Properties of sheaves of the form  $\mathcal{F}/t\mathcal{F}$

§4. Change of ground field for algebraic preschemes

- 4.1 Dimension of algebraic preschemes
- 4.2 Associated prime cycles on algebraic preschemes
- 4.3 Reminder on tensor products of fields
- 4.4 Irreducible and connected preschemes over an algebraically closed field
- 4.5 Geometrically irreducible and connected preschemes
- 4.6 Geometrically reduced preschemes
- 4.7 Multiplicities in primary decomposition on an algebraic prescheme
- 4.8 Fields of definition
- 4.9 Subsets defined over a field

§5. Dimension, depth, and regularity for locally Noetherian preschemes

- 5.1 Dimension of preschemes
- 5.2 Dimension of algebraic preschemes
- 5.3 Dimension of the support of a sheaf; Hilbert polynomial
- 5.4 Dimension of the image of a morphism
- 5.5 Dimension formula for a morphism of finite type

- 5.6 Dimension formula and universally catenary rings
- 5.7 Depth and property  $(S_k)$
- 5.8 Regular preschemes and property  $(R_k)$ ; Serre's criterion for normality
- 5.9  $Z$ -pure and  $Z$ -closed sheaves of modules
- 5.10 Property  $(S_2)$  and  $Z$ -closure
- 5.11 Coherence criteria for sheaves  $\mathcal{H}_{X/Z}^0(\mathcal{F})$
- 5.12 Relations between the properties of a Noetherian local ring  $A$  and a quotient  $A/tA$ .
- 5.13 Properties that persist under inductive limits
- §6. Flat morphisms of locally Noetherian preschemes
  - 6.1 Flatness and dimension
  - 6.2 Flatness and projective dimension
  - 6.3 Flatness and depth
  - 6.4 Flatness and property  $(S_k)$
  - 6.5 Flatness and property  $(R_k)$
  - 6.6 Transitivity properties
  - 6.7 Application to change of base for algebraic preschemes
  - 6.8 Regular, normal, reduced and smooth morphisms
  - 6.9 Theorem on generic flatness
  - 6.10 Dimension and depth of a sheaf normally flat along a closed sub-prescheme
  - 6.11 Criteria for  $U_{S_n}(\mathcal{F})$  and  $U_{C_n}(\mathcal{F})$  to be open
  - 6.12 Nagata's criteria for  $\text{Reg}(X)$  to be open
  - 6.13 Criteria for  $\text{Nor}(X)$  to be open
  - 6.14 Base change and integral closure
  - 6.15 Geometrically unibranched preschemes
- §7. Noetherian local rings and their completions; excellent rings
  - 7.1 Formal equidimensionality and formally catenary rings
  - 7.2 Strictly formally catenary rings
  - 7.3 Formal fibers of Noetherian local rings
  - 7.4 Persistence of properties of formal fibers
  - 7.5 A criterion for  $P$ -morphisms
  - 7.6 Application I: Locally Japanese rings
  - 7.7 Application II: Universally Japanese rings
  - 7.8 Excellent rings
  - 7.9 Excellent rings and resolution of singularities
- (Volume IV Part 3)
- §8. Projective limits of preschemes
  - 8.1 Introduction
  - 8.2 Projective limits of preschemes
  - 8.3 Constructible subsets of a projective limit of preschemes
  - 8.4 Irreducibility and connectedness criteria for projective limits of preschemes

- 8.5 Finitely presented sheaves of modules on a projective limit of preschemes
- 8.6 Finitely presented subschemes of a projective limit of preschemes
- 8.7 Criteria for a projective limits of preschemes to be a reduced (resp. integral) prescheme
- 8.8 Preschemes finitely presented over a projective limit of preschemes
- 8.9 Initial applications to elimination of Noetherian hypotheses
- 8.10 Properties of morphisms persistent under projective limits
- 8.11 Application to quasi-finite morphisms
- 8.12 Another proof and generalization of Zariski's "main theorem"
- 8.13 Translation into the language of pro-objects
- 8.14 Characterization of a prescheme locally finitely presented over another, in terms of the functor it represents
- §9. Constructible properties
  - 9.1 Principle of finite extension
  - 9.2 Constructible and ind-constructible properties
  - 9.3 Constructible properties of morphisms of algebraic preschemes
  - 9.4 Constructibility of certain properties of sheaves of modules
  - 9.5 Constructibility of topological properties
  - 9.6 Constructibility of certain properties of morphisms
  - 9.7 Constructibility of the properties of separability, and geometric irreducibility and connectedness
  - 9.8 Primary decomposition in the neighborhood of a generic fiber
  - 9.9 Constructibility of local properties of fibers
- §10. Jacobson preschemes
  - 10.1 Very dense subsets of a topological space
  - 10.2 Quasi-homeomorphisms
  - 10.3 Jacobson spaces
  - 10.4 Jacobson preschemes and rings
  - 10.5 Noetherian Jacobson preschemes
  - 10.6 Dimension of Jacobson preschemes
  - 10.7 Examples and counterexamples
  - 10.8 Rectified depth
  - 10.9 Maximal spectra and ultra-preschemes
  - 10.10 Algebraic spaces in the sense of Serre
- §11. Topological properties of finitely presented flat morphisms; flatness criteria.
  - 11.1 Flatness loci (Noetherian case)
  - 11.2 Flatness of a projective limit of preschemes
  - 11.3 Application to elimination of Noetherian hypotheses
  - 11.4 Descent of flatness by arbitrary morphisms: case of a prescheme over an Artinian base
  - 11.5 Descent of flatness by arbitrary morphisms: general case

11.6 Descent of flatness by arbitrary morphisms: case of a prescheme over a unibranched base

11.7 Counterexamples

11.8 Valuative criterion for flatness

11.9 Separated and universally separated families of homomorphisms of sheaves of modules

11.10 Schematically dominant families of morphisms and schematically dense families of sub-preschemes

§12. Fibers of finitely presented flat morphisms

12.0 Introduction

12.1 Local properties of the fibers of a locally finitely presented flat morphism

12.2 Local and global properties of the fibers of a proper, flat, finitely presented morphism

12.3 Local cohomological properties of the fibers of a locally finitely presented flat morphism

§13. Equidimensional morphisms

13.1 Chevalley's semi-continuity theorem

13.2 Equidimensional morphisms: case of dominant morphisms of irreducible preschemes

13.3 Equidimensional morphisms: general case

§14. Universally open morphisms

14.1 Open morphisms

14.2 Open morphisms and dimension formula

14.3 Universally open morphisms

14.4 Chevalley's criterion for universally open morphisms

14.5 Universally open morphisms and quasi-sections

§15. Fibers of a universally open morphism

15.1 Multiplicities of fibers of a universally open morphism

15.2 Flatness of universally open morphisms with geometrically reduced fibers

15.3 Application: criteria for reducedness and irreducibility

15.4 Supplement on Cohen-Macaulay morphisms

15.5 Separable rank of the fibers of a quasi-finite and universally open morphism; application to geometrically connected components of the fibers of a proper morphism

15.6 Connected components of fibers along a section

15.7 Appendix: local valuative criteria for properness

(Volume IV Part 4)

§16. Differential invariants; differentially smooth morphisms

16.1 Normal invariants of an immersion

16.2 Functorial properties of normal invariants

16.3 Basic differential invariants of a morphism of preschemes

16.4 Functorial properties of differential invariants

- 16.5 Relative tangent sheaves and bundles; derivations
- 16.6 Sheaves of  $p$ -differentials and exterior differentials
- 16.7 The sheaves  $\mathcal{P}_{X/S}^n(\mathcal{F})$
- 16.8 Differential operators
- 16.9 Regular and quasi-regular immersions
- 16.10 Differentially smooth morphisms
- 16.11 Differential operators on a differentially smooth  $S$ -prescheme
- 16.12 Characteristic 0 case: Jacobian criterion for differentially smooth morphisms
- §17. Smooth, unramified and étale morphisms
  - 17.1 Formally smooth, unramified and étale morphisms
  - 17.2 General differential properties
  - 17.3 Smooth, unramified and étale morphisms
  - 17.4 Characterization of unramified morphisms
  - 17.5 Characterization of smooth morphisms
  - 17.6 Characterization of étale morphisms
  - 17.7 Properties of descent and passage to the limit
  - 17.8 Criteria for smoothness and unramification in terms of fibers
  - 17.9 Étale morphisms and open immersions
  - 17.10 Relative dimension of a prescheme smooth over another
  - 17.11 Smooth morphisms of smooth preschemes
  - 17.12 Smooth subschemes of a smooth prescheme; smooth and differentially smooth morphisms
  - 17.13 Transverse morphisms
  - 17.14 Local and infinitesimal characterizations of smooth, unramified and étale morphisms
  - 17.15 Case of preschemes over a field
  - 17.16 Quasi-sections of flat and smooth morphisms
- §18. Supplement on étale morphisms; Henselian local rings and strictly local rings
  - 18.1 A remarkable equivalence of categories
  - 18.2 Étale covers
  - 18.3 Finite étale algebras
  - 18.4 Local structure of unramified and étale morphisms
  - 18.5 Henselian local rings
  - 18.6 Henselization
  - 18.7 Henselization and excellent rings
  - 18.8 Strictly local rings and strict Henselization
  - 18.9 Formal fibers of Noetherian Henselian rings
  - 18.10 Preschemes étale over a geometrically unbranched or normal prescheme
  - 18.11 Application to complete Noetherian local algebras over a field
  - 18.12 Applications of étale localization to quasi-finite morphisms (generalizations of preceding results)

## §19. Regular immersions and normal flatness

- 19.1 Properties of regular immersions
- 19.2 Transversally regular immersions
- 19.3 Relative complete intersections (flat case)
- 19.4 Application: criteria for regularity and smoothness of blowups
- 19.5 Criteria for  $M$ -regularity
- 19.6 Regular sequences relative to a filtered quotient module
- 19.7 Hironaka's criterion for normal flatness
- 19.8 Properties of projective limits
- 19.9  $\mathcal{F}$ -regular sequences and depth

## §20. Meromorphic functions and pseudo-morphisms

- 20.0 Introduction
- 20.1 Meromorphic functions
- 20.2 Pseudo-morphisms and pseudo-functions
- 20.3 Composition of pseudo-morphisms
- 20.4 Properties of domains of definition of rational functions
- 20.5 Relative pseudo-morphisms
- 20.6 Relative meromorphic functions

## §21. Divisors

- 21.1 Divisors on a ringed space
- 21.2 Divisors and invertible fractional ideal sheaves
- 21.3 Linear equivalence of divisors
- 21.4 Inverse images of divisors
- 21.5 Direct images of divisors
- 21.6 1-codimensional cycle associated to a divisor
- 21.7 Interpretation of positive 1-codimensional cycles in terms of sub-preschemes
- 21.8 Divisors and normalization
- 21.9 Divisors on preschemes of dimension 1
- 21.10 Inverse and direct images of 1-codimensional cycles
- 21.11 Factoriality of regular rings
- 21.12 Van der Waerden's purity theorem for the ramification locus of a birational

morphism

- 21.13 Parafactorial pairs; parafactorial local rings
- 21.14 The Ramanujan-Samuel theorem
- 21.15 Relative divisors