RESEARCH STATEMENT

MILTON LIN

My research revolves around foundational aspects of pure mathematics and machine learning. In pure mathematics, I focus on the geometric Langlands program: its metaplectic extension and its relative version. In machine learning, I aim to explore associative memory models and their scaling properties, focusing on how algebraic and topological methods can provide deeper insights into the qualitative behavior of neural networks.

Mathematics: p-adic Geometry and Langlands Program

In the geometric Langlands program, my graduate work[1] has focused on extensions in the mixed characteristic setting, [ILZ24], this is joint work with Ashwin Iyengar (American Mathematical Society) and Konrad Zou (Bonn Univeristy). Our work applied the framework of Zhu's perfect geometry [Zhu17] to prove the Casselman-Shalika formula [NP01] for mixed characteristics. The Casselman-Shalika formula computes the "fourier coefficients" of automorphic forms and is fundamental to modern works of geometric Langlands, see [FR22]. Moving forward, I will continue this research in two directions:

(1) **Relative aspects of Langlands**, joint project with Yuta Takaya (University of Tokyo), we aim to explore relative aspects of the Langlands program on the Fargues-Fontaine curve, [FS24], recent conjectures of Ben-Zvi. Sakellaridis, and Venkatesh [BSV], particularly the relationship between period sheaves and L-sheaves as in [FW24].

(2) **Metaplectic aspects of Langlands**, joint with Toan Pham (Johns Hopkins Univeristy) I intend to give a geometric metaplectic Casselman–Shalika formula, building on the works of Gaitsgory and Lysenko [GL22], McNamara [McN16], and Brubaker et al. [Bru+24].

Machine Learning: Associative Memory Networks

In machine learning, I am particularly interested[2] in foundational theories of associative memory networks, going back to the work of Hopfield and to modern-day associative memories, [KH16]. These networks serve as a bridge between biological realism and computational efficiency.

(1) **Polytopal Decomposition of Memory Networks** in joint work with Chris Hillar (former Redwood Research, currently startup on Algebraic) and Tenzin Chan (Algebraic) we focus on the polytopal decomposition of the weight spaces of memory networks and its relation to network scaling. Similar works include, [ZNL18].

(2) **Study of Homotopy Type:** To further my study with Chris Hillar, we propose to explore *Hopfield networks* using a recent formalism by Manin and Marcolli [MM24], which uses *summing functors* and *Gamma spaces* to model the allocation of resources in neural networks. The formalism allows us to study a *homotopy type* - a mathematical construct at a

Date: October 19, 2024.

[1] Details of my mathematical research: https://cwlin4916.github.io/Trees/Application/Postdoc/Research.pdf

[2] Details of my interest in memory networks https://cwlin4916.github.io/Trees/Application/Postdoc/ResearchA.pdf

deeper level than *homology*[3]. Homotopy captures invariants of the network up to continuous deformations.

(3) My joint work with Muhan Gao (Johns Hopkins University) empirically studies modern energy-based memory network transformers for language modeling. This was previous studied in the context for vision task in [Hoo+24]. This research will also highlight the limitations of synthetic memory networks, especially in their use as proxies for explaining biological networks, see also [KH21].

Stanford's Interdisciplinary Environment. Stanford's unique interdisciplinary environment is ideal for my research. The world-class expertise in pure mathematics, particularly through faculty like Professor Xinwen Zhu and Professor Daniel Bump – both inspirations for my current work – offers unparalleled opportunities. Simultaneously, Stanford's research in machine learning, along with proximity to the Wu Tsai Neurosciences Institute, provides a rich setting for interdisciplinary collaboration. I am particularly excited about the prospect of engaging with Professor Surya Ganguli Neural Dynamics and Computation Lab, whose work in theoretical neuroscience has significantly influenced my interest in computational neuroscience.

1. Timeline for Accomplishments

In the first year of my fellowship, I will complete my collaboration with Yuta Takaya on relative Langlands for the Fargues–Fontaine curve in mathematics. Concurrently, in machine learning, I will finalize my joint work with Chris Hillar on the polytopal decomposition of memory networks and with Muhan Gao on experiments involving memory networks in language modeling. Over the next three years, I plan to extend my research on p-adic aspects of the Langlands correspondence with Yashi Jain at Johns Hopkins. In machine learning, I aim to develop theoretical results supporting associative memory models and their scaling properties, striving for both mathematical rigor and practical applications in AI. To visually summarize my research timeline and collaborations, I present the following diagram: which includes the names of my co-authors.

$$
\begin{tikzcd}
& \begin{array}{c}\text{Geometric Langlands}\\ \text{Program}\end{array} \arrow[dl] \arrow[d] & \text{Memory Netowrks} \arrow[d] \arrow[dr] & \\
\begin{array}{c}\text{Relative Langlands}\\ \text{Yuta Takaya}\end{array} \arrow[dr] & \begin{array}{c}\text{Metaplectic Casselman-}\\ \text{Shalika Formula}\\ \text{Toan Pham}\end{array} & \begin{array}{c}\text{Polytopal Weight}\\ \text{Space Decomposition}\\ \text{Chris Hillar}\end{array} \arrow[d] & \begin{array}{c}\text{Energy Transformers}\\ \text{for Language Modelling}\\ \text{Muhan Gao}\end{array} \arrow[dl] \\
& \begin{array}{c}\text{Relative Langlands}\\ \text{in } p\text{-adic Langlands}\\ \text{Yashi Jain}\end{array} & \begin{array}{c}\text{Foundations for}\\ \text{Interpretable Networks}\end{array} &
\end{tikzcd}
$$

Algebraic & Geometric Formalism

In the diagram, the second row represents projects planned for the first year of my fellowship, while the third row corresponds to projects undertaken in the second year and beyond.

[3] which is commonly used in topological data analysis (TDA). For a short survey of topology and neural code, see [Cur16].

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