PH.D. THESIS SUMMARY

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My Ph.D. thesis explores the intersection of the Geometric Langlands program, p-adic geometry, and categorical deformation theory, building upon the advancements of Fargues-Scholze [FS24] and Lurie's work on higher algebra [Lur09; Lur18]. The research employs combinatorial algebraic geometry, homotopy theory, higher category theory, deformation theory, and p-adic geometry.

The thesis comprises the following key contributions, some of which result from collaborative efforts:

(1) **Casselman-Shalika Formula in Mixed Characteristics**: (Completed) In collaboration with Ashwin Iyengar (American Mathematical Society) and Konrad Zou (Bonn University), we applied Zhu's perfect geometry framework [Zhu17] to establish the Casselman-Shalika formula [NP01] for mixed characteristics. This formula computes the Fourier coefficients of automorphic forms and is fundamental to modern developments in the geometric Langlands program [FR22]. The resulting work [ILZ24] is submitted to

(2) **Relative Langlands Program in p-adic Geometry**: (Aspects of this should be completed by March) In collaboration with Yuta Takaya (University of Tokyo), we focus on extending the relative Langlands program, as outlined by Ben-Zvi, Sakellaridis, and Venkatesh [BSV], within the context of p-adic geometry.

(3) **Categorical Deformation Theory**: (Aspects of this should be completed by April) We study, as proposed in Lurie's ICM talk [Lur10], deformations of categories, applying modern theories to further representation theory.

The research statement provides more details.

I would like to note that during my final academic year, my research interests have expanded beyond my thesis work to include foundational aspects of machine learning. I have focused on applying categorical and algebraic techniques to better understand neural networks, leading to an ongoing collaboration with Chris Hillar from Redwood Research on the polytopal decomposition of weight spaces in memory networks. I view joining Max Plank Institute for Mathematics in the Sciences as an exceptional opportunity to further these interdisciplinary efforts, using my expertise in pure mathematics to tackle fundamental challenges in machine learning.

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References

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