RESEARCH STATEMENT: OVERVIEW

My research revolves around foundational aspects of pure mathematics and machine learning. In mathematics, I concentrate on the geometric Langlands program. In machine learning, I explore associative memory models through empirical, algebraic, and topological lenses. The first three pages of this document provide a summary, followed by detailed discussions of my work in both mathematics and machine learning.

p-adic Geometry and Langlands Program

In the geometric Langlands program, my graduate work

has focused on extensions in the mixed characteristic setting, [ILZ24], this is joint work with Ashwin Iyengar (American Mathematical Society) and Konrad Zou (Bonn Univeristy). Our work applied the framework of Zhu's perfect geometry [Zhu17] to prove the Casselman-Shalika formula [NP01] for mixed characteristics. The Casselman-Shalika formula computes the "fourier coefficients" of automorphic forms, fundamental to modern works of geometric Langlands [FR22]. Moving forward, I will continue my exploration in two directions:

(1) **Relative aspects of Langlands**, joint with Yuta Takaya (University of Tokyo), we aim to explore relative aspects of the Langlands program on the Fargues-Fontaine curve, [FS24], from recent conjectures [BSV], particularly the relationship between period sheaves and L-sheaves as [FW24].

(2) **Metaplectic aspects of Langlands**, joint with Toan Pham (Johns Hopkins University), we intend to give a geometric metaplectic Casselman–Shalika formula, building on the works of Gaitsgory and Lysenko [GL22], McNamara [McN16], and Brubaker et al. [Bru+24].

Geometry of Associative Memory Networks

In machine learning, I am interested in the foundations of modern associative memory networks [KH16]. These networks bridge biological realism, computational efficiency, and interpretable network design.

(1) **Polytopal Decomposition of Memory Networks:** In joint work with Chris Hillar (Redwood Research, Berkeley), we study the polytopal decomposition of the weight spaces of memory networks and its relation to network scaling, drawing parallels with tropical geometry [ZNL18]. Using Manin and Marcolli's recent formalism [MM24], which leverages *summing functors* and *Gamma spaces*, we explore the homotopy type of memory networks – a deeper invariant than homology[1]. This approach aims to uncover the relationship between memory capacity and homotopical invariants.

(2) **Associative Memory Models Beyond Storage Capacity:** In ongoing work with Muhan Gao (Johns Hopkins University), we empirically analyze modern energy-based memory networks for language modeling and classification tasks. Our focus is on the regime where stored memories exceed theoretical capacity [KH16, Equations (5) and (6)]. This study explores the limitations of synthetic memory networks and the relation between generalization and memory capacity.

[1]For an overview of homology in topological data analysis, see [Cur16].

References

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