# SUPPORTING STATEMENT AND TIMELINE FOR ACCOMPLISHMENTS

MILTON LIN

In this document I list my relevance to the selection criteria and a timeline for accomplishment on the third page.

## Completion of PhD

I expect to complete my PhD in May 2025 under the supervision of Professor David Gepner at Johns Hopkins University.

## Research Expertise and Background

My research focuses on advancing mixed characteristic and metaplectic aspects of the Langlands program, as well as categorical deformations of representation categories. These interests place me at the intersection of **homotopy theory**, **derived algebraic geometry**, and **characteristic $p$ geometry**, which are directly relevant to this position.

**Foundational Preparation.** During the initial years of my graduate studies, I developed a robust foundation in modern homotopy theory and derived algebraic geometry, guided by Lurie's works: *Higher Topos Theory*, *Higher Algebra*, and *Spectral Algebraic Geometry*. Notable seminar presentations from my CV include: limits in compactly assembled $\infty$-categories (2024), cohesive topoi and differential cohomology (2023), derived Galois deformation theory (2022), and integral models for spaces (2020).[1] This preparation has informed my current research directions and collaborations.

**Graduate Research and Relevance.** My primary research contributes to the categorical Langlands program as developed by Fargues, Scholze, and others. Highlights include:

(1) **Mixed-characteristic Casselman-Shalika formula:** Collaborative work with Ashwin Iyengar and Konrad Zou [ILZ24] established the Casselman-Shalika formula in mixed characteristic using Zhu's perfect geometry [Zhu17]. This formula is foundational for the geometric Langlands program.

(2) **Relative Langlands aspects:** Jointly with Yuta Takaya (University of Tokyo), I am exploring relative Langlands theory on the Fargues-Fontaine curve, inspired by Ben-Zvi, Sakellaridis, and Venkatesh's conjectures [BSV].

(3) **Metaplectic Langlands:** I am developing a metaplectic Casselman-Shalika formula, building on works by Gaitsgory, Lysenko, McNamara, and Brubaker et al. [GL22; Bru+24].

(4) **Categorical deformations of representation categories:** This project focuses on formal deformations of categories, as described in Lurie's work [Lur10, Thm 10.10]. It includes deformations of representations of Lusztig's small quantum group and explores connections to quantum geometric Langlands.

[1] Sample notes: https://cwlin4916.github.io/Trees/Application/Postdoc/IntegralHomotopy.pdf

**Broader Research Interests.** Additional projects include:

(1) **Motivic Langlands program:** Building on Richarz's work [RS20], I am studying Whittaker categories in the motivic setting and their applications to Langlands duality [FGV01].

(2) **Stacky approaches to periods:** Leveraging advances in stack theory [GR14; Toë06], I explore applications to periods and $p$-adic integration theory [Vol01].

**Collaborative Experience.** Collaboration has been central to my research, as demonstrated by joint papers [ILZ24; Has+24]. I have worked with mathematicians across institutions, including the University of Bonn, the University of Tokyo, and the University of Washington.

**Suitability for the Position.** I am eager to collaborate with Professor Brantner, whose work on the generalization of Lurie-Pridham correspondence in characteristic $p$ has profoundly influenced my learning of deformation theory. I aim to explore its applications to the Langlands program, particularly in the context of characteristic $p$ geometry, if given the postdoctoral position.

## Teaching and Supervision at Johns Hopkins

I enjoy teaching and also supervisory work.[2] My teaching experiences encompass both service and specialized courses.

**Serivce Courses.**

- Teaching Assistant for various courses, including Honors Linear Algebra, Graduate Algebra, Introduction to Proofs, Calculus I and II.
- Instructor for Honors Calculus I, where I developed lecture notes [3], designed course syllabus, and had my own teaching assistant.
- Instructor of SOUL (Special Opportunities for Undergraduate Learning), a self-designed 1-credit course focusing on the mathematical foundations and interpretability of neural networks.

**Directed Reading Program (DRP).** I co-organized the Directed Reading Program (DRP), which offers mentorship to a diverse student body. Notable mentorships:

- Nick Lombardi (2022): Junior mathematics major, covered basic elliptic curve properties and Chapter 5 of Silverman's *The Arithmetic of Elliptic Curves*.
- Orisis Zheng (2023): Senior physics major, explored algebraic geometry with *Sheaves in Geometry and Logic*. Now a Ph.D. student at Montana State University.
- Spencer Huang and Dev Lalwani (2023): Sophomore applied mathematics majors, focused on mechanistic interpretability of vision models [4].
- Viggy Vanchinathan (2024): Junior biomedical engineering major, collaborated on experiments related to *Progress measures for grokking via mechanistic interpretability*.

**Graduate Student Supervision.**

- I currently act as a secondary advisor to Yashi Jain (main supervisor, David Savitt). We are exploring relative Langlands in the context of $p$-adic categorical Langlands as developed by Emerton, Gee, Hellmann et al.

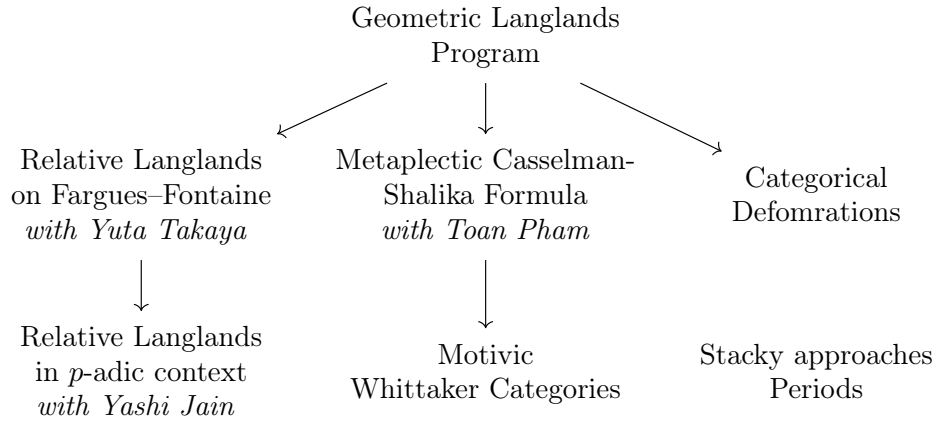
[2] My teaching statement: https://cwlin4916.github.io/Trees/Application/Postdoc/Teaching.pdf
[3] Draft: https://cwlin4916.github.io/Trees/Application/Postdoc/teach_draft_v0.pdf
[4] https://distill.pub/2020/circuits/zoom-in/

**Learning seminar.** I am deeply committed to promoting a learning environment for graduate students. I actively engage in and organize graduate seminars. Recent activities include:

- Fall 2024: I gave talks in $K$-theory seminar on the paper of dualizable categories, following recent advancement of Efimov, [Efi24] and also Number Theory learning seminar George Boxer's thesis, [Box15].
- Spring 2024: I was participant and speaker in both $E$-theory seminar which studied Gross-Hopkins Periods map and Number Theory seminar which studied the works of Francis Brown on motivic periods, [Bro14].
- Fall 2023, I was seminar organizer for Prismatic Cohomology[5]; I gave talks in the Topological Quantum Field Theory seminar ($\sigma$-models), Representation Theory seminar (uniformization theorem), and Topology Seminar (on Dieudonné modules).

## Timeline for Accomplishments

By 2025, I plan to complete my collaboration with Yuta Takaya on relative Langlands for the Fargues-Fontaine curve. Over the following two years, I aim to extend this work to the $p$-adic Langlands program, building on the framework of Emerton and Gee. Additionally, I intend to explore categorical deformation theory and advance Lurie's conjecture from his ICM talk. Below is a diagram which summarizes my provisional plans:




The second row represents projects planned to be completed by 2025, while the third and fourth row corresponds to projects undertaken beyond year 2026.

## References

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