DISSERTATION PRIZE FELLOWSHIP PROGRAM APPLICATION

Dear Dissertation Prize Committee,

I am applying for the Dissertation Prize Fellowship to complete my dissertation on advancing the Geometric Langlands Program in the mixed characteristic setting. My dissertation explores and extends the Casselman-Shalika formula, a foundational result in the theory of automorphic forms, in various contexts. This fellowship will enable me to finalize my thesis in Spring 2025.

More details to my work

My graduate research focuses on advancing the Geometric Langlands Program in mixed characteristic settings. As a preliminary result, joint with Ashwin Iyengar (Amermican Mathematical Society) and Konrad Zou (Bonn University) we extended the geometric Casselman-Shalika formula using Zhu's perfect geometry framework [ILZ24]. This is essential to modern advancements in geometric representation theory [FR22]. Building on this I have three ongoing projects:

(1) **Relative aspects of Langlands in mixed characteristics:** In collaboration with Yuta Takaya (University of Tokyo), we explore relative aspects of the Langlands program on the Fargues-Fontaine curve [FS24] and recent conjectures by Ben-Zvi, Sakellaridis, and Venkatesh [BSV], particularly concerning period sheaves and L-sheaves [FW24]. We have obtained partial results on the automorphic side.

(2) **Categorical deformations of representation categories:** Building on the geometric Casselman-Shalika equivalence in [FGV01], this project aims to provide a detailed proof of Lurie's description of categorical deformations as gerbes [Lur10]. This will contribute towards understanding the geometric deformation of quantum representations, conjectured in op.cit Remark 10.12.

(3) **Metaplectic aspects of geometric Langlands:** I aim to develop a metaplectic Casselman-Shalika formula, extending works by Gaitsgory, Lysenko, McNamara, and others [GL22; McN16; Bru+24].

The first project will be a key component to my final thesis. If time permits, I plan to work further on Projects 2 and 3.

Best, Milton Lin
Ph.D. Candidate in Pure Mathematics

Date: October 25, 2024.

References

[Bru+24] Brubaker, Ben, Buciumas, Valentin, Bump, Daniel, and Gustafsson, Henrik P. A. *Metaplectic Iwahori Whittaker functions and supersymmetric lattice models*. 2024. arXiv: 2012.15778 [math.RT]. URL: https://arxiv.org/abs/2012.15778 (cit. on p. 1).

[BSV] Ben-Zvi, David, Sakellaridis, Yiannis, and Venkatesh, Akshay. "Relative Langlands duality". In: () (cit. on p. 1).

[FGV01] Frenkel, E., Gaitsgory, D., and Vilonen, K. "Whittaker patterns in the geometry of moduli spaces of bundles on curves". In: *Ann. of Math. (2)* 153.3 (2001), pp. 699–748. ISSN: 0003-486X,1939-8980. URL: https://doi.org/10.2307/2661366 (cit. on p. 1).

[FR22] Faergeman, Joakim and Raskin, Sam. *Non-vanishing of geometric Whittaker coefficients for reductive groups*. 2022. arXiv: 2207.02955 [math.RT]. URL: https://arxiv.org/abs/2207.02955 (cit. on p. 1).

[FS24] Fargues, Laurent and Scholze, Peter. "Geometrization of the local Langlands correspondence". In: *arXiv e-prints*, arXiv:2102.13459 (Feb. 2024), arXiv:2102.13459. arXiv: 2102.13459 [math.RT] (cit. on p. 1).

[FW24] Feng, Tony and Wang, Jonathan. *Geometric Langlands duality for periods*. 2024. arXiv: 2402.00180 [math.NT]. URL: https://arxiv.org/abs/2402.00180 (cit. on p. 1).

[GL22] Gaitsgory, D. and Lysenko, S. *Parameters and duality for the metaplectic geometric Langlands theory*. 2022. arXiv: 1608.00284 [math.AG]. URL: https://arxiv.org/abs/1608.00284 (cit. on p. 1).

[ILZ24] Iyengar, Ashwin, Lin, Milton, and Zou, Konrad. *Geometric Casselman-Shalika in mixed characteristic*. 2024. arXiv: 2408.07953 [math.AG]. URL: https://arxiv.org/abs/2408.07953 (cit. on p. 1).

[Lur10] Lurie, Jacob. "Moduli problems for ring spectra". In: *Proceedings of the International Congress of Mathematicians. Volume II*. Hindustan Book Agency, New Delhi, 2010, pp. 1099–1125. ISBN: 978-81-85931-08-3; 978-981-4324-32-8; 981-4324-32-9 (cit. on p. 1).

[McN16] McNamara, Peter J. "The metaplectic Casselman-Shalika formula". In: *Trans. Amer. Math. Soc.* 368.4 (2016), pp. 2913–2937. ISSN: 0002-9947,1088-6850. URL: https://doi.org/10.1090/tran/6597 (cit. on p. 1).