Oberseminar über Arakelov-Theorie

ADIC SPACES

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Venue: Tuesday, 14.15, room M 102

The development of a theory of analytic spaces over non-archimedean fields has been wished since the fifties of last century, when Tate asked a doubtful Grothendieck about the existence of global p-adic spaces in the setting of elliptic curves. The desire for the connectedness of the closed unit ball, in contrast to the poor topological properties of naively defined non-archimedean manifolds, led Tate himself to introduce the notion of *rigid analytic spaces* at the beginning of the sixties. Since then, Tate's theory has found many applications in the setting of Galois representations and more generally in number theory, such as the solution of the local Langlands conjecture for the general linear group by Harris and Taylor, and the proof of Abyhankar's conjecture for fundamental groups of curves in positive characteristic by Raynaud and Harbater. In the following decades, other approaches to non-archimedean analytic geometry have been proposed, among which the ones developed by Raynaud, Fujiwara-Kato and Berkovich.
At the beginning of the nineties, in a series of papers written while he was working at the Universität Regensburg, Roland Huber introduced a natural category of non-archimedean analytic varieties, called *adic spaces*, generalizing both locally noetherian formal schemes and rigid analytic spaces. His insight in [Hub93a][Hub93b][Hub94] was to consider a class of topological rings, nowadays called *Huber rings*, which contain both the noetherian adic rings used in formal geometry and the Tate algebras cooking up rigid analytic spaces. Inspired by the classical constructions of schemes, one can associate to a Huber ring A the set $\operatorname{Spa} A$ of continuous valuations on it, then equip it with a topology and, under certain condition of noetherianity, with a sheaf of topological rings with local stalks. In the subsequent paper [Hub96], Huber proved that the étale cohomology of a rigid analytic variety is equivalent to the one of the corresponding adic space, which is instead of easier access, granting useful applications of the theory to non-archimedean geometry.
Despite their strength and simplification, adic spaces went almost unnoticed until Peter Scholze realised that they were providing the correct theoretical framework for his definition of *perfectoid spaces* [Sch12]; after establishing new cases of the weighted monodromy conjecture by the use of these new objects, Scholze received in 2018 a Fields Medal for "transforming arithmetic algebraic geometry over p-adic fields" and claimed back the attention of mathematicians to the rich cues offered by the theory of adic spaces.

The goal of this oberseminar is to enter the theory of adic spaces by understanding

the basic definitions and constructions, by exploring the relation with other non-archimedean theories and the role played in Scholze's work.

In recent years, several learning seminars about adic spaces have been organised, and the lecture notes that have sprung from them provide nowadays an easier access to Huber's work. Among these activities, we can cite the lecture series given at the University of Stanford in Fall 2014 by Brian Conrad (see `http://virtualmath1.stanford.edu/~conrad/Perfseminar/`), the MSRI lecture of the same author [Con18] and the learning seminar organized by Bhargav Bhatt at the University of Michigan during Winter 2017 (see `http://www-personal.umich.edu/~stevmatt/adic_spaces.pdf`). Our primary references and leading sources during the seminar will be the unpublished notes by Sophie Morel [Mor19] and the ones by Torsten Wedhorn [Wed12]. These two texts contain many details and a lot of preparation from the theory of valuations; our choice, however, is to start right away with the hot material and come back to the preliminary results whenever needed.

A USER GUIDE

Every section of these notes concerns a talk for the learning seminar and, after a quick presentation of the main subject of the lecture, it features a **"Condensed abstract"** and a **"Tips"** paragraph. The first one is intended to be a one-sentence motto for the lecture, containing the primary goal that the speaker should aim to for a successful talk. The second one is a collection of suggestions intended to guide the speaker in the preparation of their exposition: as long as the goal is accomplished and the time schedule respected (1h30 per talk), the final choice of propositions, examples and proofs to present is left to their taste, as well as the depth of details to enter. Also, the participants are free to choose their favourite references among the one proposed above (or further ones!); however, for a better global coordination, it is suggested to stick to the notation of [Mor19] and [Wed12], the only exception being the use of "Huber ring" instead of "f-adic ring" and of "Huber pair" instead of "affinoid ring". In any case, I stay at disposal for discussions on how to organize any specific lecture or for any doubts concerning the outlines of the talks given here. This document is available online at `https://homepages.uni-regensburg.de/~gur23971/documents/OBER19_program.pdf`.

Contents

Welcome session (15.10, *Roberto Gualdi*)

The goal of this first meeting is to give a very vague presentation of the learning seminar, sum up the contents of the talks and agree on a planned schedule among participants by finding an answer to the question "*Who talks when?*". Whoever is interested in participating to the oberseminar is heartily invited and will be warmly welcomed, with no discrimination on age, sex or academical position. However, for a fruitful participation to the lectures, a previous knowledge in algebra, valuation theory and scheme theory is recommended.

◄ The valuation spectrum of a ring ►

1. Valuation spectra (22.10, *Martino Stoffel*)

The aim of this first talk is to introduce the notion of the *valuation spectrum* $\mathrm{Spv}\, A$ of a commutative ring A. Roughly speaking, this is the set of equivalence classes of the valuations definable over A, endowed with a suitable topology. In adic geometry, a similar space plays the role that in algebraic geometry is covered by the topological space underlying $\mathrm{Spec}\, A$ (to which moreover it is continuously mapped), and it rejoices pleasant topological properties. Moreover, analogously to the classical case, the association

$$A \mapsto \mathrm{Spv}\, A$$

gives a contravariant functor between the category of rings and the one of spectral spaces.

Condensed abstract: the definition of the space $\mathrm{Spv}\, A$ and its first topological properties should be explained; presenting examples is strongly encouraged, as well as the notion of valuation spectrum associated to a scheme.

Tips: the speaker is invited to follow [Wed12, section 4.1 and section 4.3], taking [Wed12, Definition 4.1 and Proposition 4.7] as the main goals of the talk; it would be useful to start by briefly recalling the notions of valuations over a ring and their equivalence relation, see [Wed12, section 1.2]. The topological properties appearing in the statement of [Wed12, Proposition 4.7] are explained in great details in [Wed12, chapter 3]; the speaker is referred to this for definitions and results. Other useful source for the talk are [Mor19, I.2] and [Bha17, Theorem 7.4.8] for the fundamental Hochster's criterion for spectrality.

2. Specialization in valuation spectra (29.10, *Thomas Fenzl*)

Recall that if x is a point of a topological space, any point contained in the closure of $\{x\}$ is said to *specialize* x. In the case of Spv A, there are two main types of specialization. The first one, called *vertical*, arises when the two involved points of the valuation spectrum share the same associated prime ideal of A. A specialization is instead called *horizontal* if one valuation is obtained from another by "forgetting" some of the values it take. It turns out that all specialization in Spv A can be seen as a combination of a vertical and a horizontal one, justifying the possibility to restrict to such two cases.

Condensed abstract: the notions of vertical and horizontal specializations in Spv A should be presented, showing that they suffice to describe any specialization in valuation spectra.

Tips: the material for the talk is [Wed12, section 4.2], with milestones being [Wed12, Definition 4.12, Definition 4.16 and Proposition 4.21]. The notion of convexity in totally ordered groups is presented in [Wed12, section 1.1] and should be recalled briefly. The corresponding treatment in Morel's note is done in [Mor19, I.3].

◀ Huber and Tate rings ▶

3. Huber and Tate rings (05.11, *Christian Vilsmeier*)

The goal is this lecture is to properly define the notions of *Huber rings* (or, as they were originally called in [Hub93a] and [Hub93b], f-adic rings) and of *Tate rings*. Precisely speaking, a Huber ring A is a topological ring containing an open subring A_0 whose subspace topology is induced by a finitely generated ideal; the subring A_0 is called a *ring of definition* for A. If moreover A has a topologically nilpotent unit, that is a unit whose sequence of powers converges to 0_A, the ring is called a Tate ring. In the theory of Huber rings, boundedness also plays an important role and it allows for instance to characterize the union of all rings of definition of a given Huber ring, and to properly define *adic morphisms*.

Condensed abstract: after recalling the needed notions about topological rings, the definitions of Huber rings, Tate rings and adic morphisms are given and their first properties are discussed.

Tips: the speaker should present the content of [Mor19, II.1.1, II.1.2 and II.1.3]. It is also recommended they have a look at [Wed12, section 6.1, 6.2 and 6.5], leaving aside examples and [Wed12, Remark 6.8 and Proposition 6.9]. It would be useful to have the feeling of some proofs, for example the ones of [Mor19, Proposition II.1.2.4], [Mor19, Lemma II.1.2.7], [Mor19, Proposition II.1.3.3.(iii)] and [Mor19, Proposition II.1.3.4]. In any case, the denomination "Huber rings" instead of the original "f-adic ring" is suggested. To correctly state definitions and claims, a good basis on topological rings is required: it would hence be advisable to start the talk by recalling the necessary ingredients from [Wed12, sections 5.1, 5.2 and 5.3].

4. Examples of Huber and Tate rings (19.11, *Peter Wieland*)

Having introduced the notions of Huber and Tate rings, it is time to present a zoo of examples which will be useful to have in mind during the remaining of the seminar. The most basic example is the one of a field k equipped with the topology induced by a valuation of rank 1; in such a case, any normed k-algebra is a Tate ring, such as the *Tate algebra of strictly convergent power series in*

n variables equipped with the Gauß norm.
More generally, if A is a Huber ring (resp., a Tate ring), there are natural topologies on the polynomial ring over A, on a certain subring of the ring of formal power series over A and on the topological localization of A at a multiplicative subset which confers them the structure of Huber rings (resp., Tate rings).

Condensed abstract: examples of Huber and Tate rings should be presented, with particular attention reserved to the case of polynomial rings, power series and topological localizations.

Tips: the speaker should aim at two goals during the talk. The first one is to give an overview of examples of Huber rings, making use of [Wed12, Example 6.5, Example 6.12, Example 6.13] and [Mor19, II.1.4]; in doing that, it would be better to introduce the notion of a microbial valuation on a ring, see [Wed12, section 5.5] or [Mor19, I.1.5]. The second goal is to explain [Wed12, Proposition 6.21]. Before that, the constructions in [Wed12, section 5.6] need to be presented; as a general motivation of the talk, or as an application of the main result, the case of Tate algebras from [Wed12, section 5.7] should be discussed. It could be useful to have a look at [Mor19, II.3.3 and II.3.4] for another source on the same topics.

5. Further constructions with Huber rings (26.11, *Stefan Stadlöder*)

Other admissible constructions in the category of Huber rings are completions and tensor products: again, there are natural topologies that one can define over the ring completion and the tensor product of Huber rings to likewise equip them with a Huber ring structure. Finally, it would be useful to define *strongly noetherian Tate rings*: they are Tate rings which are "topologically virtually noetherian", in the sense that every Tate ring which is topologically of finite type over them is noetherian. Their relevance for our seminar is that their associated adic space is particularly pleasant.

Condensed abstract: the Huber structure on completions and tensor products should be presented, before moving to the discussion of the notion of strongly noetherian Tate rings.

Tips: the speaker should cover the material explained in [Mor19, II.3.1, II.3.2] and [Wed12, section 6.6 and 6.7]. Regarding completions, one could also have a look at [Wed12, section 5.4, Remark 6.8 and Proposition 6.9], while for strong noetherianity [Mor19, Definition IV.1.1.1] and [Mor19, IV.1.2] should be integrated in the talk. If time permits and the speaker wants, they can give ideas on the proof of [BGR84, Theorem 1 of section 5.2.6], asserting that any complete non-archimedean field is strongly noetherian.

◀ **Adic spaces** ▶

6. Spectra of continuous valuations (03.12, *César Martínez*)

Recall that in the first talk of the seminar we treated the notion of the valuation spectrum of a commutative ring. If now we consider a topological ring A, it makes sense to restrict to valuations on A whose induced topology is coarser than the original one, obtaining a topological subspace $\operatorname{Cont} A$ of $\operatorname{Spv} A$. The situation we are most interested in is when A is a Huber ring. In such a case, $\operatorname{Cont} A$ is again a spectral space and it is stable under horizontal specializations and non-trivial vertical generizations. One further useful notion is the one of *analytic points*, which are points of $\operatorname{Cont} A$ whose support is not open; recalling the characterization of open ideals in a Huber ring it is immediate to see that, when A is a Tate ring, every point of $\operatorname{Cont} A$ is analytic.

Condensed abstract: the spectrum of continuous valuations associated to a topological ring is introduced, as well as its analytic points; more focus is given to the case of Huber rings and Tate rings.

Tips: the speaker should integrate [Mor19, II.2.1, II.2.2, II.2.3 and II.2.4] with [Wed12, sections 7.1 and 7.2] and [Wed12, section 7.5] (but ignoring in this last section all the claims referred to affinoid rings and $\operatorname{Spa} A$). Also, it is suggested they present [Mor19, Remark II.2.5.7 and Proposition II.3.1.12.(iii)] which were not treated in the previous lectures. Seeing the proof of some of the results, for example of [Mor19, Theorem II.2.2.1 and Corollary II.2.2.3], [Mor19, Proposition II.2.3.1] and [Mor19, Proposition II.2.4.2] may be useful.

7. The adic spectrum of a Huber pair (10.12, *Philipp Jell*)

An *Huber pair* (called affinoid ring in Huber's original work) is the datum (A, A^+) of a Huber ring A and of an open and integrally closed subring of A made of power-bounded elements. To such a pair, one can associate its *adic spectrum* $\operatorname{Spa}(A, A^+)$: it is the space of continuous valuations on A with respect to which all the elements of A^+ are integral. This space is again spectral, it is dense in $\operatorname{Cont} A$ and it has a basis of quasi-compact open subsets, called *rational subsets*, which is stable under finite intersection.

Condensed abstract: the notion of the adic spectrum of an Huber pair should be presented, and its first topological properties discussed, with particular attention to its rational subsets.

Tips: the speaker should refer to [Wed12, sections 7.3, 7.4 and 7.5] and [Mor19, III.1, III.2 and III.3], the main goals being the explanation of [Mor19, Proposition III.1.5.(i), Definition III.1.7, Definition III.2.1, Corollary III.2.4 and Theorem III.3.1]. For [Wed12, section 7.5], many of the claims have already been stated in the more general case of $\operatorname{Cont} A$, so that it would be enough to underline the refined properties enjoyed by analytic points of adic spectra. For this reason, it is suggested that the speakers of this lecture and of the previous one agree in advance on how to treat [Wed12, section 7.5].

8. Properties and constructions with adic spectra (17.12, *Miriam Prechtel*)

In principle, the adic spectrum of a Huber pair (A, A^+) could be empty or lack analytic points. However, simple criteria can be given to avoid these pathological cases; for instance, if the ideal $\{0\}$ is not dense in A, then $\operatorname{Spa}(A, A^+) \neq \emptyset$ and if A is complete but not discrete, then $\operatorname{Spa}(A, A^+)$ has at least one analytic point. Giving a precise characterization of these non-emptiness conditions and considering its consequences is the first goal of the talk. The second one is the description of the behaviour of the adic spectrum under quotient, completion and localization of the corresponding Huber pair. For instance, the adic spectrum of the completion of a Huber ring (A, A^+) is homeomorphic to $\operatorname{Spa}(A, A^+)$.

Condensed abstract: after characterizing the non-emptiness of the adic spectra and of the set of its analytic points, the behaviour of the functor Spa under quotient, completion and localization should be considered.

Tips: the goal of the talk is to present the content of [Mor19, III.4]. It would be nice to explore the details of the proofs of [Mor19, Corollary III.4.2.2 and Proposition III.4.4.1] and of some of the consequences of the second result; the proofs are relatively simple, but offer the excuse to dive into the recurring technical steps appearing in adic geometry, without getting lost in the general arguments. Part of this material, such as the characterization of emptiness and the behaviour of Spa under localization, is also discussed in [Wed12, section 7.6 and Proposition 8.2].

9. Adic points and other examples of adic spectra (07.01, *Roberto Gualdi*)

In classical algebraic geometry, points of a scheme X correspond bijectively to equivalence classes of morphisms from spectra of fields to X, as can be seen by considering their residue fields. In the setting of Huber's theory, an *adic point* is the adic spectra of a Huber pair made by a non-archimedean complete field and an open valuation subring; as a topological space it can have more than one point, but it has a unique closed point and a unique generic point. Similarly to the classical algebraic geometrical case, it turns out that the analytic points of an adic spectrum $\mathrm{Spa}(A, A^+)$ are in bijection with equivalence classes of adic morphisms from adic points to $\mathrm{Spa}(A, A^+)$; this is obtained by considerations on the *completed residue field* at points of the adic spectrum. Another interesting example of adic spectra is the one arising from a Huber algebra which is topologically of finite type over a non-archimedean complete field.

Condensed abstract: two examples of adic spectra should be considered: the first one comes from non-archimedean fields, the second one from algebras topologically of finite type over them.

Tips: it is suggested that the speaker starts by quickly recalling the definition of the adic spectrum of a Huber pair given before the holidays. Then, they are invited to refer to [Mor19, III.5.1] for the discussion on adic points, with main goals being [Mor19, Lemma III.5.1.8 and Proposition III.5.1.9]. It would be important to underline how this last result is an adic analogue of a classical property for schemes, which is explained for instance in [Sta19, Lemma 25.13.3]. For the part on adic spectra defined from algebras topologically of finite type over an archimedean ring, the references are [Wed12, Definition 7.56 and Example 7.58]; even if the speaker can very quickly quote [Wed12, Example 7.57 and Example 7.59], notice that these examples will be treated in details only in a later lecture.

10. Adic spaces (14.01, *Miriam Prechtel*)

Let (A, A^+) be a Huber pair and $X := \mathrm{Spa}(A, A^+)$ its adic spectrum. The behaviour of the functor Spa under localization allows to identify rational subsets of X with adic spectra of certain completed localizations of A. In analogy with what is done for schemes, this can be exploited to define a presheaf $\mathscr{O}_X$ of complete topological rings on X, with local stalks. An *adic space* is a triple $(X, \mathscr{O}_X, (v_x)_{x\in X})$ of a topological space, a sheaf of complete topological rings with local stalks and a valuation on each stalk $\mathscr{O}_{X,x}$, which is locally isomorphic to the adic spectrum of a Huber pair.

Condensed abstract: the notion of adic space should be introduced; this requires the definition of the structure presheaf on an adic spectrum and the discussion of the properties of its stalks.

Tips: the speaker should cover the material of [Mor19, III.6.1, III.6.2, III.6.3, III.6.4 and III.6.5], ignoring the part on Tate rings (that is, from [Mor19, Definition III.6.3.3] till the end of the section). The speaker may decide to prove some of the properties of stalks in [Mor19, Proposition III.6.3.1], with the proof of the locality being highly recommended. The corresponding treatment in Wedhorn's notes is [Wed12, section 8.1] and the first part of [Wed12, section 8.2]. Also, [Bha17, section 7.5] represents a further useful reference.

11. When is $\mathrm{Spa}(A, A^+)$ an adic space? (21.01, *Klaus Künnemann*)

Remark that for an adic spectrum $X := \mathrm{Spa}(A, A^+)$ to be an adic space one requires that the presheaf $\mathscr{O}_X$ defined in the previous lecture is a sheaf, which is actually not the case for any choice of the Huber pair (A, A^+). However, Huber gave in [Hub94] some criteria on A to assure the adicity of the corresponding spectrum: this holds for instance when the completion of A is discrete or when A is a strongly noetherian Tate ring. In such situations, he could also study the Čech cohomology of the structure sheaf and show that, as it is expected by the analogy with the affine schematic case, all the higher cohomology groups of $\mathscr{O}_X$ on rational domains vanish.

Condensed abstract: sufficient conditions for $\mathrm{Spa}(A, A^+)$ to be an adic space should be given, and examples when they can be applied provided.

Tips: a motivation for the talk could be taken from [Hub93a, end of section 3.2], where an example of a Tate ring for which $\mathscr{O}_X$ is not a sheaf of rings is exhibited. The speaker should then cover the content of [Mor19, chapter IV], focusing on the statement of the main result [Mor19, Theorem IV.1.1.5] and on the proof of [Mor19, Theorem IV.1.3.5], which is interesting for perfectoid algebras. The specific choice of the treatment of the proof of [Mor19, Theorem IV.1.1.5] is left to their own taste: the speaker could present a detailed proof for one of the cases or give an idea of the arguments for a couple of them. A reading of the more concise [Wed12, section 8.2] could be helpful, while an overview of the necessary bases in Čech cohomology can also be found in [Wed12, appendix A].

12. Fiber products of adic spaces (28.01, *Philipp Jell*)

In constrast to the case of schemes, fiber products are not always defined in the category of adic spaces. In fact, to assure that a fiber product is again an adic space, some hypotheses on the structure morphisms should be assumed. This is the case, for instance, if f and g are *locally of weakly finite type*, that is they locally induce homomorphisms on Huber rings which are topologically of finite type. Another useful construction is the one of the adic space defined, via an universal property, as the fiber product between an adic space and a scheme; most remarkably, if X is an algebraic variety defined over a non-archimdean field, this allows to define the *adic space X^{ad} associated to X.*

Condensed abstract: after studying morphisms between adic spaces, fiber products in the category of adic spaces should be discussed, as well as the construction of the adic space associated to a scheme.

Tips: the speaker could start the talk by presenting the different properties of morphisms between adic spaces defined in [Wed12, sections 8.4 and 8.5]. In doing so, the analytic points of an adic space play a role and they deserve to be recalled or complemented following [Wed12, section 8.3]. Finally, [Wed12, sections 8.6 and 8.7] contain the treatment of fiber products; it is suggested to present the proofs of [Wed12, Theorem 8.55], at least in the case of both morphisms being locally of weakly finite type, and of [Wed12, Proposition and Definition 8.60]. Some further motivation for the talk can be found in [Hub93a, section 3.10] or at `https://algebrateahousejmath.wordpress.com/2017/02/02/adic-spaces-ii-analytification-and-fiber-products/`.

◀ Additional material ▶

13. Relations with other non-archimedean analytic theories (04.02, *Walter Gubler*)

Huber's theory is only one of the several different approaches to non-archimedean analytic geometry and it is natural to wonder how it relates to some of the others. Similarly to what happens in the setting of *Berkovich analytic geometry*, the adic space associated to a scheme is constructed by enlarging the underlying topological space instead of considering a Grothendieck topology on it; however, Huber's theory allows points corresponding to valuations of higher rank, giving a larger space than the one built up by means of Berkovich's approach. The situation can be well-understood by considering the example of the Huber pair of convergent power series in one variables over a non-archimedean field. On the other side, restricting to trivial valuations give a fully faithful functor from the category of *locally noetherian formal schemes* to the category of adic spaces.

Condensed abstract: the comparison of Huber's theory with Berkovich geometry and formal geometry is explored, with the help of illustrative examples.

Tips: a useful categorical reference for the comparison with Berkovich geometry is offered by [Hen16]. Also, it is highly recommended to explain such a relation by working out in details the example of the closed unit ball, see [Mor19, III.5.2] and [Wed12, Example 7.57]. The links between adic spaces and formal geometry are discussed very quickly in [Mor19, III.5.3] and [Wed12, Example 7.59]; it is suggested to have a look to the more extensive treatment of [Wed12, chapter 9] and to resume its content. Finally, [Wed12, chapter 10] gives two remarks on the way Huber's theory is linked to the one of rigid analytic spaces.

14. An overview of perfectoid spaces (*bonus talk*)

In recent years, Scholze introduced the notion of a *perfectoid Tate ring* A, which, loosely speaking, is a Tate ring with an extra requirement on a certain associated Frobenius map; for example, when A is of positive characteristic, the condition is equivalent to the fact that A is complete and perfect, while any algebraically closed complete non-archimedean field turns out to be a perfectoid field. A *perfectoid Huber pair* is a Huber pair made by a perfectoid Tate ring and its ring of integral elements, and a *perfectoid space* is an adic space which is locally isomorphic to the adic spectrum of a perfectoid Huber pair. The category of perfectoid spaces admits fiber products and a fundamental *tilting operation* which allows to reduce to the case of a given positive characteristic. Thanks to the proof of a strong form of Faltings's *almost purity theorem* showing the invariance of the étale site of a perfectoid space under tilting, Scholze was able to establish new special cases of Deligne's weighted monodromy conjecture, opening the door for future promising research.

Condensed abstract: by giving an overview on the subject, the definition of perfectoid spaces should be given, and the almost purity theorem stated.

Tips: the speaker could give a rapid survey of [Mor19, chapter V], with a first focus on [Mor19, Definition V.1.1.1, Proposition V.1.2.1 and Definition V.2.1]; as a general piece of advice, proofs should not be done in details and more emphasis should be put on an ideas-focused presentation of the material. If possible, the statement of the almost purity theorem [Mor19, Theorem V.3.1.3 and Corollary V.3.1.5] should be given and its powerful consequences underlined. The original article by Scholze [Sch12] is of course a fundamental reference, with [Bha17, chapters 9 and 10] and [Fon13] being good companions.

References

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