12/01/21.

Defn: $A \in \mathrm{Top}^{\mathrm{Rng}}$ is Huber if.

i) $A_0 \hookrightarrow A$ open.

ii) Adic wrt fg. ideal $I \triangleleft A_0$

called a ideal of def'n. IOD.

- A_0 is ring of defin., ROD.
- (A_0, I) a couple of defn for A.

Ref: Hub93. A generalization of fml sch. and rigid anlytc varieties.
Hub92. cts vals.
Mor19: Adic spcs.

i.e. A_0 is I-adic.

- Hub93. Goal is to cst a natrl cat. $\mathcal{A}$ ctng
1. loc noeth. frml sch. $\overset{loc.}{\cong}$ Spf A. A is noethn. adic.
2. Rigid anlytc, varieties.

- Analyze the "local" of 1+2.

- Frml affine: $A = A_0$. I is IOD.

Rigid spcs: $K\langle \xi_1, .., \xi_n \rangle$ has rng of Defn.

$A_0 = \mathcal{O}_K\langle \xi_1, .., \xi_n \rangle$, $\mathcal{O}_K$ is ring of integers of K,

if let $t \in K$ st. $0 < |t| < 1$

the $\{ t^n A_0 \}$ forms an nbhd basis. of A_0.

(using the Gauss norm).

- K has cpl. nonarc. non trivial val'n.
$\rightsquigarrow$ such t $\exists$.

Defn: A Huber pair: (A, A^+)

- $A \in$ HubRng
- $A^+ \subset A^\circ$ is open + int. clsd subring. of A.

- Adic geometry book Scholze. J.W. [Berkeley lectures 2014), 3.

Goal: To associate a TopSpc. to a Huber pair (A, A^+),

- $B \subseteq A$ is bdd if $\forall$ $0 \in U \subseteq A$ $\exists V \subseteq A$ st. $\underbrace{V.B}_{\text{set}} \subseteq U$.
- $A^\circ :=$ subset of pwbdd elmt.
$= \{ x \in A : \{ x^n \mid n > 0 \} \text{ is bdd} \}$
- $T.A$ = subgp genrtd by T.
$= \{ \sum t_i a_i : t_i \in T, a_i$

Defn. let $s \in A$. $T \subseteq A$. finite. $T.A \subset A$ is open.

$R(\frac{T}{s}) := \{ x \in X \mid |t|_x \leq |s|_x \neq 0 \quad \forall t \in T \}$.

$X = \mathrm{Spa}(A, A^+) = \mathrm{cts}(|\cdot| : A \to T_{\cup} 0 \ \mathrm{st} \ \{ : |A^+| \leq 1 \} / \sim$.

- S is invertible if A is cpl
$\rightsquigarrow |t/s|_x \leq 1 \Rightarrow t/s$ is pwbdd.

These are the rational subsets.

$\rightsquigarrow$ These form a qc. open basis of $\mathrm{Spa}(A, A^+)$

which is a spectral spc i.e. $\cong$ Spec B

for some ring B.

Defn: $A \in$ Huber ring, A is cplt if $A \in \mathrm{Top}^{\mathrm{Res}}_{\beta}$.

Cstr.
$$\mathrm{HubRing}^{\mathrm{cpl}} \underset{\perp}{\rightleftarrows} \mathrm{HubRing}$$
$$\hat{A} \longleftarrow A$$

- Pick some COD, (A_0, I) and
  1. $\hat{A} \cong \varprojlim A/I^n$ as abelian grps
  2. $\hat{A}$ is $I\hat{A}_0$-adically cplt.

Cstr
$$\mathrm{HubPair}^{\mathrm{cpl}} \underset{\perp}{\rightleftarrows} \mathrm{HubPair}$$
$$(\hat{A}, \hat{A}^+) \longleftarrow (A, A^+)$$

- $\hat{A}^+ :=$ integral closure of image of $A^+ \hookrightarrow A \to \hat{A}$.

- show. rat. subsets are themselves adic spectra:

Hub94. 1.3. $\mathcal{U} \underset{\text{rat}}{\subseteq} \mathrm{Spa}(A, A^+)$ $\exists$ cpl. Huber $(\mathcal{O}_X(\mathcal{U}), \mathcal{O}_X^+(\mathcal{U})) \in \mathrm{HubPair}$.
$(A, A^+) \to (\mathcal{O}_X(\mathcal{U}); \mathcal{O}_X^+(\mathcal{U}))$ st.

i)
$$\begin{tikzcd} \mathrm{Spa}(\mathcal{O}_X(\mathcal{U}), \mathcal{O}^+_X(\mathcal{U})) \arrow[r] \arrow[dr] & \mathrm{Spa}(A, A^+) \\ & \mathcal{U} \arrow[u] \end{tikzcd}$$

ii) is universal for such maps.

Pf: Setup. Consider any $(A, A^+) \xrightarrow{\varphi} (B, B^+)$, $(B, B^+) \in \mathrm{HubPair}$.
st.
$$\begin{tikzcd} \mathrm{Spa}(B, B^+) \arrow[r, "\varphi"] \arrow[dr] & \mathrm{Spa}(A, A^+) \\ & \mathcal{U} \arrow[u] \end{tikzcd}$$

x, $|\cdot|_x$, φ_*, $|\cdot|_{\varphi_* x} = \varphi \circ |\cdot|_x$.

$R(\frac{T}{s}) := \{ x \in \mathrm{Spa}(A, A^+) : |t|_x \leq |s|_x \neq 0 \ \forall t \in T \}$

Step1 Unwinding defn of "factoring through."

Morin III.4.4.4: i) $f \in A^+ \iff |f|_x \leq 1 \ \forall x \in \mathrm{Spa}(A, A^+)$. ①

ii) if A is cpl $f \in A^\times \iff |f|_x \neq 0 \ \forall x \in \mathrm{Spa}(A, A^+)$.

f is invtible.

Pf: Morin III.4.4.3: let T be a subset of A. $(A, A^+) \in \mathrm{HubRing}^{\mathrm{cpl}}$.

if $T \cdot A = A$ (i.e. the ideal generated by T).

T finite. then $(R(\frac{T}{t}))_{t \in T}$ covers $\mathrm{Spa}(A, A^+)$.

let $T = \{f\}$.: $f \cdot A = A$ iff f is a unit.

- Prove in III.3.11

if $A \in \mathrm{Top}^{\mathrm{Ab}}$.

$i: A \to \hat{A}$ cpl map.

map bijection

$\{$ opens of $A \} \leftrightarrow \{$ opens of $\hat{A} \}$

$A \mapsto \overline{i(A)}$.

- regarding A as an R_0-module.
- refer to SP. on cpl.

- $R(\frac{T}{s}) =: \mathcal{U} \subseteq \mathrm{Spa}(A, A^+)$.

WTS: $\mathrm{Spa}(B, B^+) \simeq R(\frac{T}{s})$

for some $(B, B^+) \in \mathrm{HubPair}$.

- $\mathrm{HubPair} \to \mathrm{TopSpc}^{\mathrm{spec}}$

$(A, A^+) \mapsto \mathrm{Spa}(A, A^+)$. $|\cdot|_x : A \to \Gamma \cup \{0\}$

$\uparrow \varphi$ $\quad$ $\downarrow$ $\quad$ $\Phi \ |\cdot|_x \circ \varphi$

(B, B^+) $\quad$ $\mathrm{Spa}(B, B^+)$.

$B \xrightarrow{\varphi} A \xrightarrow{|\cdot|_x} \Gamma \cup \{0\}$.

- i.e. $\forall$ cpl Huber pair. (B, B^+).

St. $\mathrm{Spa}(B, B^+) \longrightarrow \mathrm{Spa}(A, A^+)$, $\to \mathcal{U} \hookrightarrow$.

$\exists$ $(\mathcal{O}_X(\mathcal{U}), \mathcal{O}_X^+(\mathcal{U})) \to (B, B^+)$

st.
$$\begin{tikzcd} \mathrm{Spa}(B, B^+) \arrow[r] \arrow[dr] & \mathrm{Spa}(A, A^+) \\ & \mathrm{Spa}(\mathcal{O}_X(\mathcal{U}), \mathcal{O}_X^+(\mathcal{U})) \arrow[u] \end{tikzcd}$$

- $A^+ \subset A^\circ$, open int. closed in A

① $(\Leftarrow)$ By defn, $|f|_x \leq 1 \ \forall x \in \mathrm{Spa}(A, A^+)$.

A^+ is defined X

Step1: Factor thr $\mathcal{U}$ $\iff$ invert. s. $U = R(\frac{T}{s})$, $s \in A$.

1a. $\varphi(s) \in B$. $|\cdot|_x \in \mathrm{Spa}(B, B^+)$.

$|\varphi(s)|_x = |s|_{\varphi_* x} \neq 0$ as $\varphi_* x \in \mathcal{U}$.

$\Rightarrow \varphi(s) \in B^\times$. as B is cpl. by III.4.4.4 ii.

where do $\varphi(t), \varphi(s)$ get mapped to? $t \in T$.

1b. $\underbrace{|\varphi(t)|_x \le |\varphi(s)|_x \neq 0}_{\text{factoring thrgh } \mathcal{U}}$ $\Rightarrow$ $|\varphi(t)\varphi(s)^{-1}|_x \le 1$ $\Rightarrow$ $\varphi(t)\varphi(s)^{-1} \in B^+$

1a + 1b $\leadsto$ we want to cstr. a ring a ring B, which, $A \xrightarrow{\tilde{\varphi}} B$ universally,

i) inverts $s \in A$.

ii) $\tilde{\varphi}(t)\tilde{\varphi}(s)^{-1}$ is pbdd in B.

Step2: Translate this to a universal cstr.

Mar 19

II.3.4.1/6. let $A \in$ HubRing.

- let $T = (T_i)_{i \in I}$ be family of subsets. Satisfying a "Scentry cond'n". II.3.3.1.
- $S = (S_i)_{i \in I}$ family of el. in A

R = multiplicative subset generated by s_i.

Then $\exists$ unique narc top. ring on $R^{-1}A$ st. $\widehat{R^{-1}A}$ has the desired UP. of Step2.

$\therefore$ As a ring. $(\mathcal{O}_X(U), \mathcal{O}^+_X(U)) = \left(\widehat{A[\tfrac{T}{s}]}, \widehat{A^+[\tfrac{T}{s}]^\sim}\right)$

$:= (A\langle\tfrac{T}{s}\rangle, A\langle\tfrac{T}{s}\rangle^+)$.

i.e. $(A, A^+) \overset{\text{invt } s}{\leadsto} (A[\tfrac{1}{s}], A^+[\tfrac{1}{s}]^\sim)$

$\overset{\text{cpl.}}{\leadsto} \left(\widehat{A[\tfrac{1}{s}]}, \text{int. clo of im}\left(A^+[\tfrac{1}{s}]^\sim \to \widehat{A[\tfrac{1}{s}]}\right)\right)$

in page 2.

$\square$.

- (B, B^+) we refer B^+ as ring of integral elements.
- narc. top. ring: $A \in$ Top Ring. has an open basis of nbhd given subgps of $(A, +)$.
- $\sim$ is integral closure.
- Narc. Top. Ring. : if 0 has a basis of nbhoods consisting of subgp of $(A, +)$.
- $\sim$ denotes int. closure.

Str. Presh.f

Mar 19
III.6.2.1 : $(A,A^+) \in \mathrm{HubPair}$ $X = \mathrm{Spa}(A,A^+)$.

- $\mathcal{O}_X$: $U = R(\frac{T}{s}) \longmapsto \mathcal{O}_X(U) = A\langle\frac{T}{s}\rangle$. $(= \widehat{A[\frac{1}{s}]})$
  U arbitrary $\mapsto \varprojlim_{U' \subset U} \mathcal{O}_X(U') =: \mathcal{O}_X(U)$
  give proj. lim. top.

  $\leadsto$ $\mathcal{O}_X$ is valued in cpl. top. spc.
  - presheaf on $\mathrm{Spa}(A,A^+)$.

- $\mathcal{O}_X^+$: $U \longmapsto \{ f \in \mathcal{O}_X(U) : \forall x \in U,\ |f|_x \le 1 \}$.

$\leadsto$ if U is rational
$$(\mathcal{O}_X(U), \mathcal{O}^+_X(U)) \simeq (A\langle\tfrac{T}{s}\rangle, A\langle\tfrac{T}{s}\rangle^+)$$
Pf: Uses UP. and III.4.4.4 in above pf.

$\leadsto$ By UP. on each rational subset.
if $x \in U = R(\frac{T}{s})$. $\exists$ a map. (loc + cpl.).
$$\text{via}: (A, A^+) \to (A\langle\tfrac{T}{s}\rangle, A\langle\tfrac{T^+}{s}\rangle) = (\mathcal{O}_X(U), \mathcal{O}_X^+(U))$$
$|\cdot|_x$ extends to a unique val'n on $A\langle\frac{T}{s}\rangle$.

- $\mathcal{O}_{X,x} := \varinjlim_{\substack{x \in U \\ U \subset_{\text{op}} X}} \mathcal{O}_X(U) \simeq \varinjlim_{\substack{x \in U \\ U \subset_{\text{rat}} X}} \mathcal{O}_X(U)$ ($\because$ U ac. open basis.).

- $|\cdot|_x : \mathcal{O}_{X,x} \longrightarrow \Gamma_x \cup \{0\}$, for each $x \in X$.

- $(A,A^+) \in \mathrm{HubPair} \leadsto (\mathrm{Spa}(A,A^+), \mathcal{O}_X, \{|\cdot|_x\}_{x \in X})$
  where $|\cdot|_x \in \mathrm{Spv}\, \mathcal{O}_{X,x}$ (not necessarily cts).

- (A,A^+) is sheafy. if $\mathcal{O}_X$ is a sheaf

Mar 19
- III.2.4 : the rat subsets.
  are qc-open form ($\simeq \mathrm{Spec} B$)
  basis for spectral space
  $\mathrm{Spa}(A,A^+)$.
- Ex. : $\mathcal{O}_X(X) = \hat{A}$.

i) $f \in A^+ \iff |f|_x \le 1\ \forall\ x \in \mathrm{Spa}(A,A^+)$.

$x \in \mathrm{Spa}(A,A^+)$
$$x : A \longrightarrow \Gamma_x \cup \{0\}.$$
$$A \searrow A\langle\tfrac{T}{s}\rangle \nearrow \qquad A\langle\tfrac{T}{s}\rangle = \widehat{A[\tfrac{T}{s}]}$$

- also imply $\mathcal{O}^+_X$ is.

- $\mathrm{Spv}\, A$ = val'n on A.

Defn: Hub94.2. let $\mathcal{A}$

obj: $(X, \mathcal{O}_X, \{|\cdot|_x\}_{x\in X})$

i) $X \in$ Top ii) $\mathcal{O}_X$ is shf of cpl. topological rngs

iii) $\{|\cdot|_x\}_{x\in X}$, $|\cdot|_x \in \mathrm{Spv}\,\mathcal{O}_{X,x}$

Morph: (of top. loc. ring spcs). $(X, \mathcal{O}_X, \{|\cdot|_x\}) \to (Y, \mathcal{O}_Y, (|\cdot|_y)_{y\in Y})$.

- $f: X \to Y$. $\varphi: \mathcal{O}_Y \to f_*\mathcal{O}_X$
- $\varphi_x: \mathcal{O}_{Y,f(x)} \to \mathcal{O}_{X,x}$ is compatible with val'n

- $\mathrm{Spv}\,\mathcal{O}_{X,x}$ is just a val'n.

Hub92. 2.

- morphisms of top rngs i.e. $\mathcal{O}_Y(U) \to \mathcal{O}_X(f^{-1}(U))$ are mor in cpl Top rng.
- i.e. $\varphi_x(|\cdot|_x) = |\cdot|_{f(x)}$.

Hub94.2: $\mathrm{HubPair}^{\mathrm{shfy}} \longrightarrow \mathcal{A}$ is ff.

$(A, A^+) \mapsto \mathrm{Spa}(A, A^+)$ $\quad (*)$

Defn:
- ess'n image of $(*)$ = affinoid adic spc
- $(X, \mathcal{O}_X, \{|\cdot|_x\}_{x\in X}\}$ is adic spc if locally is affinoid adic.

- Examples of sheafy Huber pairs.

We return. to our goal Hub94.4.

- k is nona cplt valuation field.

$t: \mathcal{L}e \hookrightarrow \mathcal{A}$ $\qquad A \mapsto \mathrm{Spa}(A, A)$ (noetherian adic rng.)

$r_k: \mathcal{R}_k \hookrightarrow \mathcal{A}$. $\qquad A \mapsto \mathrm{Spa}(A, A^\circ)$. (Tate k-alg of tfe, ; set of pwbdd elmts of A.)

This is well-approved as:

$\mathcal{L}e$ = locally noetherian formal schs

$\mathcal{R}_k$ = rigid analytic spaces $/k$.

- Hub94.2.2. : $(A, A^+) \in \mathrm{HubRing}^{\mathrm{cpl}}$. if A
  - i) Noetherian adic ring.
  - ii) Tate k-alg of tfe

  Then it is sheafy. $\square$

- Sch12. (R, R^+) is of form
  $R = k\langle \xi_1, \dots, \xi_n \rangle / I$.
  $R^+ = R^\circ$ (ring of pwbdd elts).

Ref: Bos14.: Non-arc. geometry. 1-7.

Fml schs: Its "affine" models. Bos14.7., EGA I.10.

• let A be an adic rng with $\mathfrak{a} \triangleleft A$ its IOD.
↳ $\{\mathfrak{a}^n : n \in \mathbb{N}_{\geq 0}\}$ is an opn basis of A.

$\mathrm{Spf}\, A := \mathrm{Spec}\, A/\mathfrak{a} = \{\text{open primes of } A\}$

$\mathcal{O}_{\mathrm{Spf} A} :=$ on basis opens:

$D(f) \mapsto A\langle f^{-1}\rangle := \varprojlim (A/\mathfrak{a}^n[f^{-1}])$. is a shf.

⇝ Can define an Zariski of. $\mathrm{Spec}\, A/\mathfrak{a}$.

↳ These are sheaves in top. rngs.

• Further $\mathcal{O}_{\mathrm{Spf} A} \simeq \varprojlim \mathcal{O}_{\mathrm{Spf} \underset{\text{discrete rngs}}{A_n}}$ $A_n := A/\mathfrak{a}^n$ with discrete topology.

• This is the aff. fml sch of A.

⇝ Can generalize to defn of fml sch X.

Problem: We want this defn to be local.
If $f \in A$, $U = D(f) \subset \mathrm{Spf}\, A$.
We want: $(U, \mathcal{O}_X|_U)$ is also
This is not true in general.

• Two solns:

Bos14. a) lem if $\mathfrak{a} \subset R$ is fg. then $\hat{R}$ is adic with $\mathfrak{a}\hat{R}$ topology.

⇝ Restrict to $\text{Adic Rngs}^{\text{cpl, sep, fg. IOD}} =:$ for Aff.

EGA sol'n b). Consider more general type of rings called ad. rngs

• We take approach a.

• Morphisms: are mph of locally top. rnged spcs.
i.e. all rng homo are cts.

• $f: X \to Y$, $\varphi: \mathcal{O}_Y \to f_*\mathcal{O}_X$
$\mathcal{O}_Y(U) \to \mathcal{O}_X(f^{-1}(U))$ ct cts maps
$\varphi_x : \mathcal{O}_{Y, f(x)} \to \mathcal{O}_{X,x}$ maps of local rngs.

• Summary: local models: A adic rng, with IOD, $\mathfrak{a}$.[1]
st. $\mathfrak{a}$ is fg.

$A \rightsquigarrow (\mathrm{Spf}\, A, \mathcal{O}_{\mathrm{Spf} A})$.
$= \mathrm{Spec}\, A/\mathfrak{a}$
$= \{\text{open prime ideals}\}$.

• Bos14, 7.2.

• An adic rng: is always cpl+sep.

• $\mathfrak{p} \triangleleft A$ is open
iff $\mathfrak{a}^n \subseteq \mathfrak{p}$ for some n
iff $\mathfrak{a} \subseteq \mathfrak{p}$. rad.

• The diagram
$A\langle f^{-1}\rangle \to \prod A\langle f_i^{-1}\rangle \rightrightarrows \prod A\langle f_i f_j^{-1}\rangle$
is exact.

it is given by lim of exact diagrams.

• Fml sch X: rng sp locally is aff. fml.

• $A\langle f^{-1}\rangle$ is not necs. adic.

• Rmk 7.1/8 Bos14.

Admissible rngs: are not adic.
i) lin. top. has basis of ideals $(I_\lambda)_{\lambda \in \Lambda}$
ii) has ideal of defn: $\exists$ open $\mathfrak{a} \subset A$ st. $\mathfrak{a}^n \to 0$.
iii) A is sep. + cpl.

[1] A has open basis $\{\mathfrak{a}^n : n \in \mathbb{Z}\}$

Cor: $\mathrm{Hom}_{\mathrm{For}\,\mathrm{Aff}}(A, B) \xrightarrow{\sim} \mathrm{Hom}_{\mathrm{loc.\,Top\,rng.\,spc}}(\mathrm{Spf}\, B, \mathrm{Spf}\, A)$.

Ex:
$$\begin{tikzcd} \mathrm{Spf}(A \hat{\otimes}_R B) \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & \mathrm{Spf}\, A \arrow[d] \\ \mathrm{Spf}\, B \arrow[r] & \mathrm{Spf}\, R \end{tikzcd}$$

- $A \hat{\otimes}_R B := \varprojlim (A/\mathfrak{a}^n \otimes_R B/\mathfrak{b}^n)$ $\mathfrak{a}, \mathfrak{b}$ fg. ideals.
  has. IOD image of $\mathfrak{a} \otimes_R B + A \otimes_R \mathfrak{b}$.

- $\mathfrak{a} \otimes_R B + A \otimes_R \mathfrak{b} \hookrightarrow$
  $A \otimes_R B \longrightarrow A/\mathfrak{a}^n \otimes_R B/\mathfrak{b}^n$

— — — — — skip — — — — — —

Cst formal cpl alg. $Y \overset{i}{\hookrightarrow} X$.

- $Y \leftrightsquigarrow \mathcal{Y} \in \mathrm{Qcoh}(X)$.
  $\mathcal{O}_Y := i^*(\varprojlim \mathcal{O}_X/\mathcal{Y}^n))$
  $\rightsquigarrow (Y, \mathcal{O}_Y)$ is loc top rng spc.

- locally: $X = \mathrm{Spec}\, A$, $\mathcal{Y} = \mathfrak{a} \subset A$.
  $(Y, \mathcal{O}_Y) = \mathrm{Spf}(\varprojlim A/\mathfrak{a}^n) = \mathrm{Spf}\, \hat{A}$.

Ex: if $A = R[x_1, \ldots, x_n]$. R a cplt. dvr. height 1.
$X = \mathbb{A}^n_R$, $Y = \mathfrak{a} = (t)$ where $t \in R^\times$ is uniformizer.

- ht 1 $\rightsquigarrow$ $\mathfrak{m}$ has no prime lying below.

- $\varprojlim (R/t^n)[x_1, \ldots, x_n] = R\langle x_1, \ldots, x_n \rangle$

$R\langle x_1, \ldots, x_n \rangle = \sum_{\underline{v} \in \mathbb{N}^n} a_{\underline{v}} x^{\underline{v}}$ st. $\lim_{|\underline{v}| \to \infty} |a_{\underline{v}}| = 0$.

- Then observe: $R\langle x_1, \ldots, x_n \rangle \otimes_R K$ (base change to generic fibre)
  $\simeq K\langle x_1, \ldots, x_n \rangle$. (restricted power series).
  is a Tate algebra $/K$. cf J.

— — — — — — — — — — — —

Rigid Analytic Spcs: its "affinoid" models. Fix K.

Defn: A is an affinoid K-alg. if $\exists$ $\alpha: T_n \twoheadrightarrow A$.
epi of K-algebra $T_n := K\langle \xi_1, \dots, \xi_n \rangle$, $n \in \mathbb{N}$.

- restricted pwr series $f \in K[[\xi_1, \dots, \xi_n]]$
strictly conv't ps. $\sum a_\nu x^\nu$ s.t. $\lim_{|\nu| \to \infty} |a_\nu| \to 0$

Defn: $\operatorname{Spm} A := \{$ maximal ideals in $A \}$.

- This Groth. top: For a set X. a G-topological spc / G-site.
is a site on a fullsubcat of $\operatorname{Poset}(X)$.
- morphisms is gvn by inclusions.

$\rightsquigarrow$ Can associate a strong G-top. on $\operatorname{Sp} A$
with shf of rgs $(X, \mathcal{O}_X)$. it is
a G-site on X satisfy cpl. cndn 5.1/5/Bos14.

- Defn: A rigid alytic K spc. : is $(X, \mathcal{O}_X)$.
a loc. rgs G-spc st.
i) G-site on X satisfies cpl. ness cndtn 5.1/5.
ii) X admits a covering (in the topology) X_i
st. $(X_i, \mathcal{O}_X|_{X_i})$ are affinoid K-spc.

$\rightsquigarrow$ morph. of rigid K-spc. is mph of loc. rgd topoi.

Cor. 5.3/7/Bos14. Y is affinoid K spc $Y = (\operatorname{Sp} A, \mathcal{O}_{\operatorname{Sp} A})$.
$\operatorname{Hom}_{\text{rig}.K}(X, Y) \simeq \operatorname{Hom}_{\text{cts. rng homo.}}(\mathcal{O}_Y(Y), \mathcal{O}_X(X))$.

Summary: A affinoid K-alg $\rightsquigarrow$ $((\operatorname{Sp} A)_{\text{Strg } G\text{-top}}, \mathcal{O}_{\operatorname{Sp} A})$

- K. cpl. nonarc. non trivial val'n.
- Piotr Achinger lecture notes has conventions.
- AWS, Brian Conrad, Sev. app. Nonarchimedean geometry.
- $\nu \in \mathbb{N}^n$, $|\nu| = \sum \nu_i$
- 5.2/5. Bos14
- X has a G-site. A a Groth-top on $A \subset \operatorname{Poset}(X)$.
- loc. ringed obj in cat. shf on a G-site of a set X.
- $(\operatorname{Spm} A, \mathcal{O}_{\operatorname{Spm} A})$
$\operatorname{Spm} A$ is given the Strg G-topology: G-site with a cpl. cdtion.

Fml Sch$_{,R}$ $\longrightarrow$ Rig$_K$. 7.3 Bos14

- R is cpl. valn. rng. of height 1, its field of fractions. K.
- assume that all fml R schs are locally of topological finite type. i.e. locally Spf A_i. st. $A_i \cong R\langle \xi_1, \dots, \xi_n \rangle / \mathfrak{a}$ is endowed with the I-adic topology. , $\mathfrak{a} \triangleleft K\langle \xi_1, \dots \xi_n \rangle$.
  "where $I \triangleleft R$ is a rdy of definition"

7.3 Bos14. $\text{Formal}_R^{\text{loc.ft.}} \longrightarrow \text{Rig}_K$

$$\text{Spf } A \longmapsto \text{Spm } (A \otimes_R K)$$

$$X \longmapsto X_{rig}.$$

This is referred to as the generic fibre. of fml sch.

Q. Given $X_K \in \text{Rig}_K$. What are the fml R-schs that send to X_K under the generic fibre map?

8, Bos14.

- In general K is a adic val'n ring with fg. ID.

(i.e. if $\exists$ open affine Covering U_i st. $U_i = \text{Spf } A_i$ A_i is an R-alg top. of finite type)

if M is a module over a ring R, $\mathfrak{a} \triangleleft R$ can endow M with the $\mathfrak{a}$-adic topology. $\mathfrak{a}^n M$ is a basis nbhd of 0.