Oberseminar p-adische Arithmetik, Wintersemester 2018/19

Integral p-adic Hodge theory, after Bhatt-Morrow-Scholze

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The aim of the seminar is to understand the formulation and proof of the comparison theorem of [1] between integral p-adic étale and integral crystalline cohomology (and de Rham cohomology).

1) **Perfectoid rings, I**
Present the content of [1] §3.1 and §3.2 up to Corollary 3.18 and Remark 3.19 (i.e. without the comparison of the notion of a perfectoid ring as in Definition 3.5 with Scholzes original definition in [3]). See also [2, §3] for a summary of the important statements. (The case of a field [1, §3.3] will be treated in the second talk.) If necessary, recall (without proofs!) the elementary properties of the cotangent complex, needed for Lemma 3.14. (See e.g. [5]Tag 08P5, section 84.2,84.9,84.15).

2) **Perfectoid rings, II**
Review the definition of perfectoid algebras and the tilting equivalence as in [3, §5]. Compare this notion to the notion of the preceding talk [1, Lemma 3.20]. Then finish [1, §3].

3) **Almost Purity**
Introduce perfectoid spaces and the notion of étale and finite étale morphisms [3, §§6,7]. Explain the almost purity theorem [3, Theorem 7.12] and the almost vanishing of higher cohomology [3, Proposition 7.13].

4) **The pro-étale site and its sheaves**
Define the pro-étale site [4, §3], [2, §§4.1,4.2] and [1, §5.4]. In particular, discuss the Push-forward $X_{\mathrm{proet}} \to X_{\mathrm{et}}$ and define the sheaves $\mathcal{O}_X$, $\hat{\mathcal{O}}_X$ and $\mathbb{A}_{\mathrm{inf},X}$. Explain how to compute pro-étale cohomology following [2, §4.3].

5) **Rational p-adic Hodge theory**
Proof the basic comparison isomorphism [4, Theorem 5.1] (we can stick to the case of the trivial local system is enough). Explain as much as possible about the version with $\mathbb{Z}_p$-coefficients, [1, Theorem 5.6].

6) **Breuil-Kisin-Fargues modules**
Discuss Breuil-Kisin-Fargues modules and their properties [1, §4].

7) **The décalage functor $L\eta$**
Introduce the functor $L\eta$ and discuss its properties [1, 6] and [2, §2].

8) **The complex $\tilde{\Omega}_{\mathfrak{X}}$**
Present the content of [1, §8].

9) **The complex $A\Omega_{\mathfrak{X}}$**
Present the content of [1, §9].

10) **The relative de Rham-Witt complex**
Present the content of [1, §10].

11) **The comparison with de Rham-Witt complexes**
Present the content of [1, §11].

12) **The comparison with crystalline cohomology over A_{crys}**
    Present the content of [1, §12].

13) **Rational p-adic Hodge theory, revisited**
    Present the content of [1, §13].

14) **The main theorem**
    Present the content of [1, §14]. If time permits discuss examples (see e.g. [1, §2]).

References

[1] B. Bhatt, M. Morrow, P. Scholze, *Integral p-adic Hodge theory*, available at `https://arxiv.org/abs/1602.03148`.
[2] M. Morrow, *Notes on the $\mathbb{A}_{\mathrm{inf}}$-cohomology of Integral p-adic Hodge theory*, available at `https://webusers.imj-prg.fr/~matthew.morrow/`.
[3] P. Scholze, *Perfectoid Spaces*, Publ. Math. IHES **116** (2012), pp. 245-313.
[4] P. Scholze, *p-adic Hodge theory for rigid analytic varieties*, Forum Math. Pi **1** (2013).
[5] The stacks project, `https://stacks.math.columbia.edu`.