Christopher Wong
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MAT 128A
Programming Project 2
11/9/2016

Problem 1

Secant_Approx_P1.m

```
% Problem 1
% clear
format long e
clear eval_x_func x y deriv_approx;

% cd 'C:\Users\Christopher\Desktop\MAT 128A';

eval_x_func = input('Evaluate the polynomial at x = ');
x = input('Input a vector of x values: ');
y = input('Input a vector of y values: ');

% Calculates the derivative at the specified x values using the simplest
% formula for a derivative: a secant line through the two closest points

% Input checks
n = numel(x) - 1;

if numel(x) ~= numel(y)
  error('Your vectors are not of same length');
else
  if numel(unique(x)) ~= (n+1)
    error('x must be a vector of distinct values');
  end
end

if (max(eval_x_func) > max(x) || min(eval_x_func) < min(x))
  error('Points of evaluation need to be within the given domain');
end

% Setting up vectorized variables
orig_x = x;
display_x = eval_x_func;
copies = numel(eval_x_func);
eval_x_func = repmat(eval_x_func, 1, numel(x));
x = repelem(x, copies);
% To find the interval the point is between, check the signs of the
% differences. A change in sign means it lies between those two points.
token = x - eval_x_func;

for i = 1:copies
  j = i:copies:numel(token);
  token_sub = token(j);
  for k = 1:numel(j)
    % Special case when it is equal to x_0
```

```
    if (token_sub(k) == 0 && k == 1)
      indices_j(i) = k+1;
    else
      if (token_sub(k) >= 0)
        indices_j(i) = k;
        % These indices are the j's for each element in eval_x such that
        % eval_x(i) is in the interval [x_(j-1), x_j]
        break
      end
    end
  end
end

for i = 1:copies
  deriv_approx(i) = (y(indices_j(i)) - y(indices_j(i)-1)) / ...
    (orig_x(indices_j(i)) - orig_x(indices_j(i)-1));
end

% Formatting output
output = horzcat(display_x', deriv_approx');
fprintf('f''(%d) = %d\n', output');
```

Problem 2

Interpolating_polynomial_approx_P2.m

```
% Problem 2
% clear
format long e
clear eval_x_func x y w deriv_approx;

% cd 'C:\Users\Christopher\Desktop\MAT 128A';

eval_x_func = input('Evaluate the polynomial at x = ');
x = input('Input a vector of x values: ');
y = input('Input a vector of y values: ');
w = input('Input a vector of scaled barycentric weights: ');

% Calculates the derivative at the specified x values using an
% approximation to a unique polynomial of degree at most n that
% interpolates the data points (x_i, y_i)

% Input checks
n = numel(x) - 1;

if (numel(x) ~= numel(y) || numel(x) ~= numel(w) || numel(y) ~= numel(w))
  error('Your vectors are not of same length');
else
  if numel(unique(x)) ~= (n+1)
    error('x must be a vector of distinct values');
  end
end

if (max(eval_x_func) > max(x) || min(eval_x_func) < min(x))
  error('Points of evaluation need to be within the given domain');
end

% Setting up vectorized variables
orig_x = x;
display_x = eval_x_func;
copies = numel(eval_x_func);
eval_x_func = repmat(eval_x_func, 1, numel(x));
x = repelem(x, copies);
d = eval_x_func - x;

for i = 1:copies
  j = i:copies:numel(d);
  d_sub = d(j);
  % Finding the j such that |x-x_j| is minimized
  [value, smallest_diff_ind] = min(abs(d_sub));
```

```
  d_j = d_sub(smallest_diff_ind);
  if (d_j == 0)
    % Calculate using simpler formula
    for k = setdiff(1:numel(orig_x), smallest_diff_ind)
      p(k) = w(k)*((y(smallest_diff_ind)-y(k)) / (orig_x(smallest_diff_ind) - ...
        orig_x(k)));
    end
    p(smallest_diff_ind) = 0;
    p_j = sum(p);
    deriv_approx(i) = -p_j / w(smallest_diff_ind);
  else
    % Calculate according to derived formula in class
    for k = setdiff(1:numel(orig_x), smallest_diff_ind)
      q(k) = (w(k)*y(k)) / (eval_x_func(i) - orig_x(k));
      r(k) = (w(k)) / (eval_x_func(i) - orig_x(k));
      s(k) = (w(k)*y(k)) / (eval_x_func(i) - orig_x(k))^2;
      t(k) = (w(k)) / (eval_x_func(i) - orig_x(k))^2;
    end
    y_j = y(smallest_diff_ind);
    q_j = sum(q);
    r_j = sum(r);
    s_j = sum(s);
    t_j = sum(t);
    deriv_approx(i) = (w(smallest_diff_ind)*(q_j-y_j*r_j+d_j*(y_j*t_j-s_j)) + ...
       (d_j)^2*(q_j*t_j-r_j*s_j)) / (w(smallest_diff_ind)+d_j*r_j)^2;
  end
end

% Formatting output
output = horzcat(display_x', deriv_approx');
fprintf('f''(%d) = %d\n', output');
```

Problem 3

Cubic_spline_approx_P3.m

```
% Problem 3
% clear
format long e
clear eval_x_func x y deriv_approx;

% cd 'C:\Users\Christopher\Desktop\MAT 128A';

eval_x_func = input('Evaluate the polynomial at x = ');
x = input('Input a vector of x values: ');
y = input('Input a vector of y values: ');

% Calculates the derivative at the specified x values using the derivative
% of the not-a-knot cubic spline that interpolates the data points
% (x_i,y_i)

% Input checks
n = numel(x) - 1;

if (numel(x) ~= numel(y) || numel(x) ~= numel(w) || numel(y) ~= numel(w))
  error('Your vectors are not of same length');
else
  if numel(unique(x)) ~= (n+1)
    error('x must be a vector of distinct values');
  end
end

if (max(eval_x_func) > max(x) || min(eval_x_func) < min(x))
  error('Points of evaluation need to be within the given domain');
end

% From lecture, we were shown to use these functions
pp = spline(x, y);
breaks = pp.breaks;
coefs = pp.coefs;
dcoefs = [3*coefs(:,1), 2*coefs(:,2), coefs(:,3)];
ppd = mkpp(breaks, dcoefs);
yyd = ppval(ppd, eval_x_func);

% Formatting output
output = horzcat(eval_x_func', yyd');
fprintf('f''(%d) = %d\n', output');
```

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Problem 4

a = -1, b = 2;

$x_i = \frac{1}{2}(a+b) + \frac{1}{2}(b-a)cos\frac{\pi(n-i)}{n}, \quad i = 0, 1, .., n;$
$y_i = f(x_i), \quad i = 0, 1, ..., n, \quad \text{where} \quad f(x) = e^x sin(5x);$

n = 5

x	f'(x)	c(x)	$p'_n(x)$	s'(x)
-1	8.745359576372146e-01	-5.246716814879859e-01	-2.464188857107013e+01	-2.482326239914257e+00
-0.5	-2.792582532694237e+00	-1.915057015627597e-02	2.292913728592137e+00	1.513319573792451e+00
0	5.000000000000000e+00	-1.915057015627597e-02	-3.674820028671666e+00	-3.674873907882845e+00
1	1.248742390143257e+00	9.067957670504891e+00	3.293812221088603e+00	1.118659649585994e+01
2	-3.501953550417127e+01	-2.867393229388423e+01	-1.448410520583428e+02	-4.938356795879422e+01

x	$\lvert f'(x) - c(x) \rvert$	$\lvert f'(x) - p'_n(x) \rvert$	$\lvert f'(x) - s'(x) \rvert$
-1	1.399207639125200e+00	2.551642452870735e+01	3.356862197551471e+00
-0.5	2.773431962537961e+00	5.085496261286374e+00	4.305902106486688e+00
0	5.019150570156276e+00	8.674820028671666e+00	8.674873907882844e+00
1	7.819215280361633e+00	2.045069830945345e+00	9.937854105716685e+00
2	6.345603210287038e+00	1.098215165541715e+02	1.436403245462296e+01

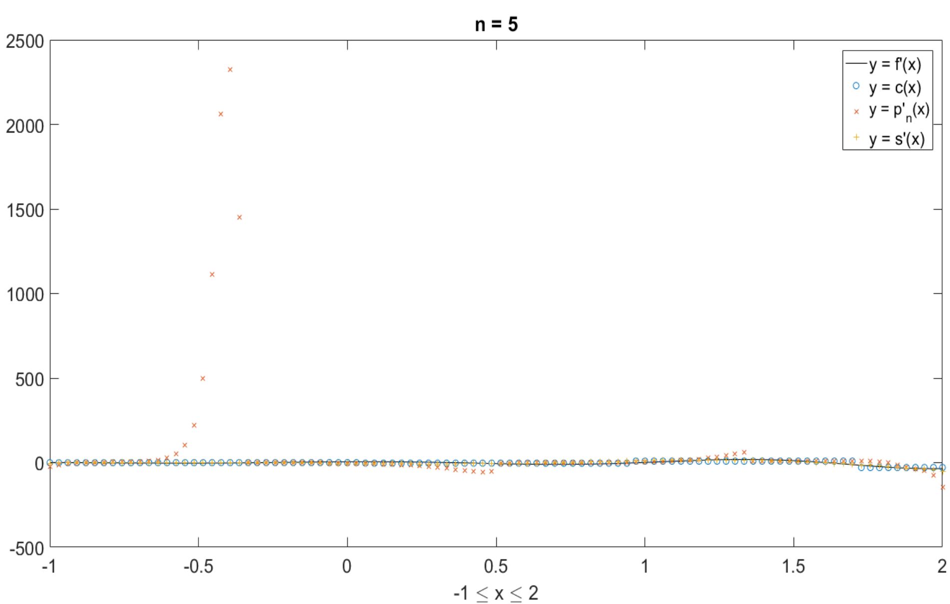

FIGURE 1.

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n = 10

x	f'(x)	c(x)	$p_n'(x)$	s'(x)
-1	8.745359576372146e-01	5.705305450635049e-01	3.235163535210087e+02	8.935738138837546e-01
-0.5	-2.792582532694237e+00	-2.551298423618298e+00	-5.670379818940499e+00	-2.742904352948967e+00
0	5.000000000000000e+00	1.990373344139357e+00	5.624026310399677e+00	4.174280287788108e+00
1	1.248742390143257e+00	1.180595593164605e+01	2.800506058973974e-01	3.867149742653975e+00
2	-3.501953550417127e+01	-3.542814723904549e+01	-5.147135995339083e+02	-3.560134496080646e+01

x	$\lvert f'(x) - c(x)\rvert$	$\lvert f'(x) - p_n'(x)\rvert$	$\lvert f'(x) - s'(x)\rvert$
-1	3.040054125737097e-01	3.226418175633715e+02	1.903785624653998e-02
-0.5	2.412841090759388e-01	2.877797286246262e+00	4.967817974526989e-02
0	3.009626655860643e+00	6.240263103996773e-01	8.257197122118924e-01
1	1.055721354150279e+01	9.686917842458598e-01	2.618407352510718e+00
2	4.086117348742206e-01	4.796940640297371e+02	5.818094566351917e-01

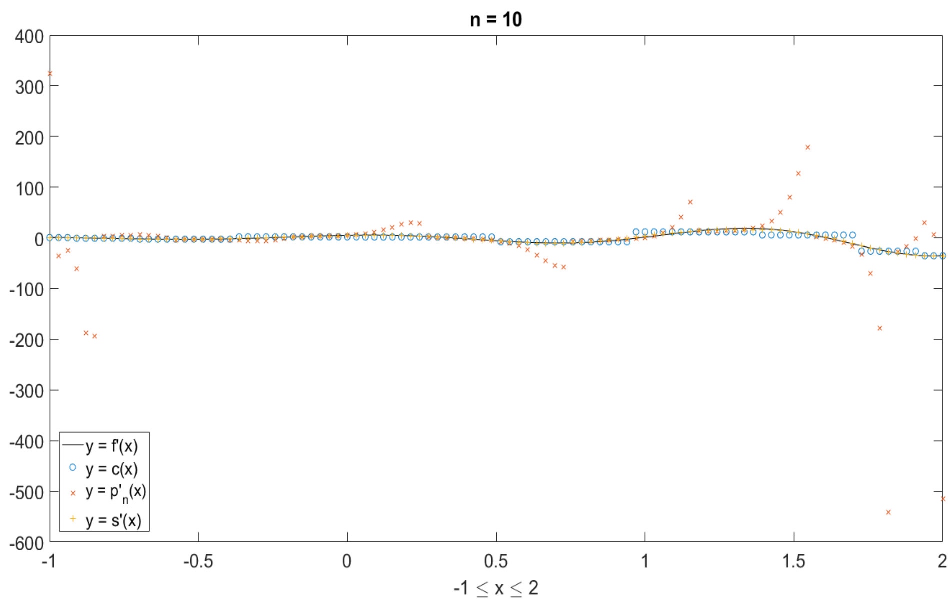

FIGURE 2.

n = 20

x	f'(x)	c(x)	$p'_n(x)$	s'(x)
-1	8.745359576372146e-01	8.038904882876280e-01	-2.652842558491021e+02	8.750089811994209e-01
-0.5	-2.792582532694237e+00	-2.540660536049228e+00	-2.786630405152687e+00	-2.799023894469834e+00
0	5.000000000000000e+00	3.882521455477203e+00	4.391758963003517e+00	4.937598854797884e+00
1	1.248742390143257e+00	6.453554797381901e+00	1.295969300099680e+00	1.455932549586021e+00
2	-3.501953550417127e+01	-3.528250361368088e+01	-2.174351096102031e+02	-3.502445356228202e+01

x	$\lvert f'(x) - c(x)\rvert$	$\lvert f'(x) - p'_n(x)\rvert$	$\lvert f'(x) - s'(x)\rvert$
-1	7.064546934958660e-02	2.661587918067393e+02	4.730235622063539e-04
-0.5	2.519219966450090e-01	5.952127541549945e-03	6.441361775596732e-03
0	1.117478544522797e+00	6.082410369964828e-01	6.240114520211648e-02
1	5.204812407238643e+00	4.722690995642220e-02	2.071901594427634e-01
2	2.629681095096146e-01	1.824155741060318e+02	4.918058110753520e-03

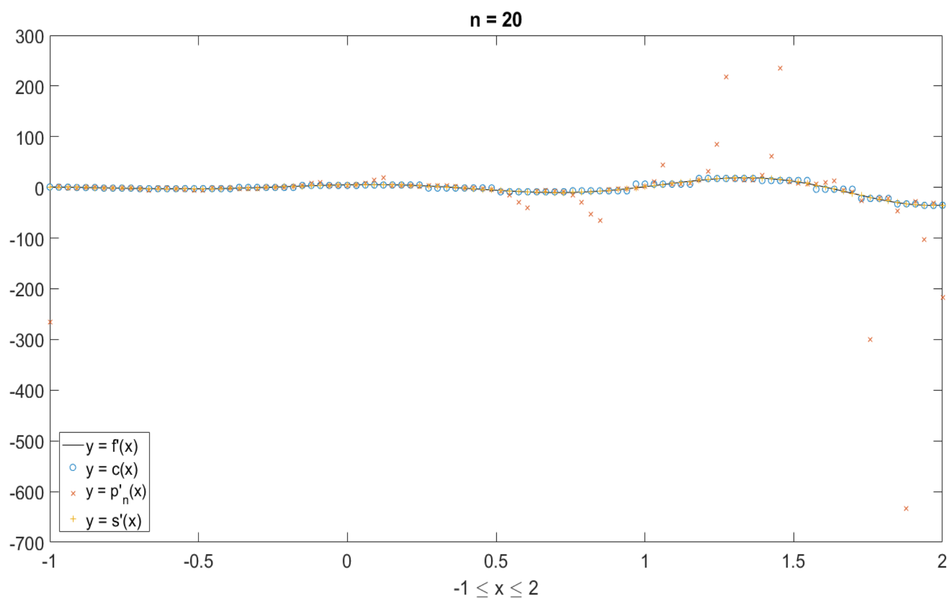

Figure 3.

n = 40

x	f'(x)	c(x)	$p'_n(x)$	s'(x)
-1	8.745359576372146e-01	8.572407504582557e-01	8.855269240222583e-01	8.745426266809122e-01
-0.5	-2.792582532694237e+00	-2.823003267080257e+00	-2.792633585283169e+00	-2.792194024255538e+00
0	5.000000000000000e+00	4.742304069138739e+00	4.495111165169681e+00	4.995671528043381e+00
1	1.248742390143257e+00	2.623971738061652e+00	1.295106876469427e+00	1.262927855955148e+00
2	-3.501953550417127e+01	-3.509575763138733e+01	-3.516484411027182e+01	-3.501959829647231e+01

x	$\lvert f'(x) - c(x)\rvert$	$\lvert f'(x) - p'_n(x)\rvert$	$\lvert f'(x) - s'(x)\rvert$
-1	1.729520717895894e-02	1.099096638504371e-02	6.669043697615784e-06
-0.5	3.042073438601944e-02	5.105258893145503e-05	3.885084386991977e-04
0	2.576959308612610e-01	5.048888348303189e-01	4.328471956618785e-03
1	1.375229347918395e+00	4.636448632617007e-02	1.418546581189095e-02
2	7.622212721606303e-02	1.453086061005493e-01	6.279230104411226e-05

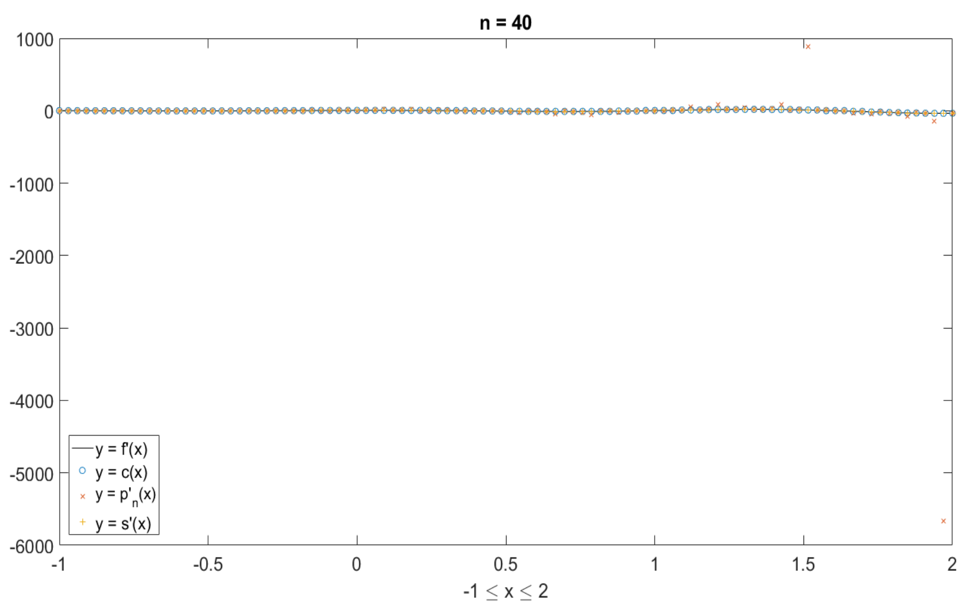

FIGURE 4.

n = 80

x	f'(x)	c(x)	$p_n'(x)$	s'(x)
-1	8.745359576372146e-01	8.702354922244022e-01	-1.139311942015228e+08	8.745360589450784e-01
-0.5	-2.792582532694237e+00	-2.770805780763892e+00	-2.792582532596967e+00	-2.792614233067023e+00
0	5.000000000000000e+00	5.067618145400889e+00	5.210830991030323e+00	5.000503783324358e+00
1	1.248742390143257e+00	6.608441943543891e-01	1.222466311196764e+00	1.247068672092272e+00
2	-3.501953550417127e+01	-3.503925194784352e+01	1.070894432007678e+09	-3.501953642819532e+01

x	$\lvert f'(x) - c(x)\rvert$	$\lvert f'(x) - p_n'(x)\rvert$	$\lvert f'(x) - s'(x)\rvert$
-1	4.300465412812371e-03	1.139311950760588e+08	1.013078637646103e-07
-0.5	2.177675193034556e-02	9.726974781187892e-11	3.170037278588112e-05
0	6.761814540088906e-02	2.108309910303232e-01	5.037833243575207e-04
1	5.878981957888682e-01	2.627607894649331e-02	1.673718050984929e-03
2	1.971644367225167e-02	1.070894467027213e+09	9.240240501640074e-07

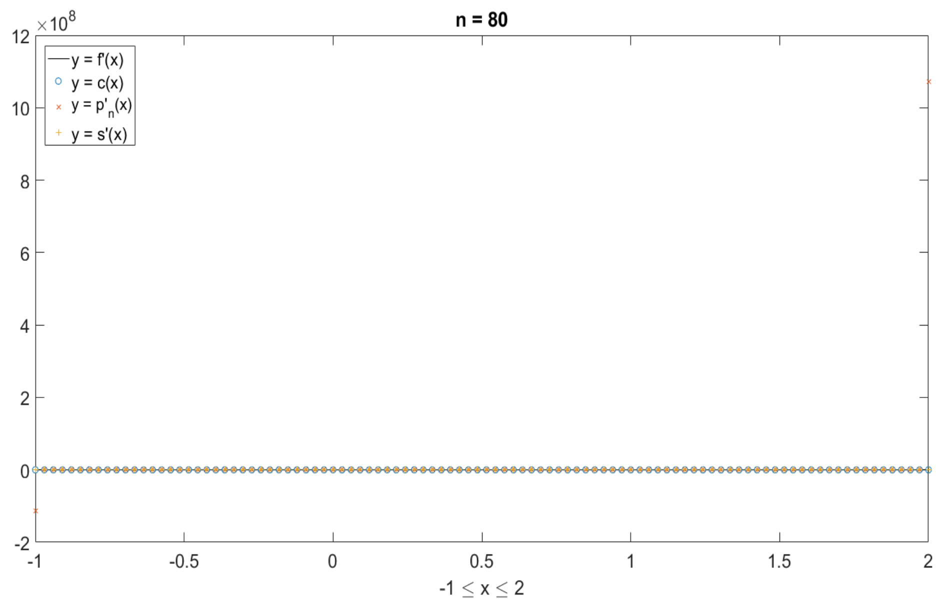

FIGURE 5.