Christopher Wong
999234204
MAT 128A
Programming Project 1 10/26/2016

Problem 1

Divided_differences_P1.m

```
% Problem 1
% clear
format long e

% cd 'C:\Users\Christopher\Desktop\MAT 128A\Project 1';

n = input('Specify the power of the desired polynomial: ');
x = input('Input a vector of x values: ');
y = input('Input a vector of y values: ');

% Calculates the divided differences and returns them in a along with
% divided differences table f

if numel(x) ~= (n+1) || numel(y) ~= (n+1)
  error('Your vectors are not of length n+1');
else
  if numel(unique(x)) ~= (n+1)
    error('x must be a vector of distinct values');
  end
end
a = zeros(1, (n+1));
f = zeros((n+1), (n+1));
f(:, 1) = y';
for i = 2:(n+1)
  for j = i:(n+1)
    f(j, i) = (f(j, (i-1)) - f((j-1), (i-1))) / (x(j) - x(j-i+1));
  end
end
a = diag(f);
```

Problem 2

Horners_Rule_P2.m

```
% Problem 2
% clear
format long e

% cd 'C:\Users\Christopher\Desktop\MAT 128A\Project 1';

% Use Divided_differences_P1 to get the vector a
Divided_differences_P1;

% Since we already call Divided_differences_P1 some arguments are already
% provided.

% n = input('Specify the power of the desired polynomial: ');
eval_x = input('Evaluate the polynomial at x = ');
% x = input('Input a vector of x values: ');
% a = input('Input the coefficients of the Newton form of p: ');

if numel(x) == (n+1)
  xn = x(1:n);
end

if numel(a) ~= (n+1)
  error('Your vectors are not of length n+1');
else
  if numel(unique(xn)) ~= (n)
    error('x must be a vector of distinct values');
  end
end

eval_y = repelem(a(n+1), numel(eval_x));
for j = n:-1:1
  for i = 1:(numel(eval_x))
    eval_y(i) = a(j) + (eval_x(i) - xn(j)).*eval_y(i);
  end
end

output = horzcat(eval_x', eval_y');
fprintf('f(%d) = %d\n', output');
```

Christopher Wong
999234204

Problem 3

(a)

The barycentric interpolation formula of $p_n(x)$ is given by

$$p_n(x) = \frac{\sum_{i=0}^{n} \frac{w_i}{x-x_i} y_i}{\sum_{i=0}^{n} \frac{w_i}{x-x_i}}.$$

We have that $p_n(x) \in C^{n+1}[a,b]$ because it is a degree n polynomial so the theorem on error of polynomial interpolations applies. That is

$$f(x) - p_n(x) = \frac{1}{(n+1)} f^{n+1}(\xi_x) \prod_{j=0}^{n} (x - x_j).$$

Taking the limit as $x \to x_j$ on both sides we obtain

$$\begin{aligned} \lim_{x \to x_j} f(x) - p_n(x) &= \lim_{x \to x_j} \frac{1}{(n+1)} f^{n+1}(\xi_x) \prod_{j=0}^{n} (x - x_j) \\ &= 0. \end{aligned}$$

So we have

$$\begin{aligned} \lim_{x \to x_j} f(x) &= \lim_{x \to x_j} p_n(x) \\ &\Leftrightarrow \\ y_j &= \lim_{x \to x_j} p_n(x), \qquad \text{j = 0, 1, ..., n.} \end{aligned}$$

(b)

No, this is not a viable remedy. Consider $x - x_j = 10^{-15}$. Since this is less than machine precision, the if statement will evaluate. However, the subtraction of nearly equal numbers is ill-conditioned. As a result, the barycentric interpolation formula will return $p_n(x) = y_j$ even though $x - x_j \neq 0$.

(c)

For equally-spaced x_i's in $[a, b]$, we have

$$w_i = c(n, a, b)\hat{w}_i, \quad \text{where} \quad \hat{w}_i, \quad i = 0, 1, ..., n.$$

Using the barycentric interpolation formula for $p_n(x)$ and the equation above, we have

$$\begin{aligned} p_n(x) &= \frac{\sum_{i=0}^{n} \frac{c(n,a,b)\hat{w}_i}{x-x_i} y_i}{\sum_{i=0}^{n} \frac{c(n,a,b)\hat{w}_i}{x-x_i}} \\ &= \frac{c(n,a,b) \sum_{i=0}^{n} \frac{\hat{w}_i}{x-x_i} y_i}{c(n,a,b) \sum_{i=0}^{n} \frac{\hat{w}_i}{x-x_i}} \\ &= \frac{\sum_{i=0}^{n} \frac{\hat{w}_i}{x-x_i} y_i}{\sum_{i=0}^{n} \frac{\hat{w}_i}{x-x_i}} \end{aligned}$$

For the Chebyshev interpolation points, we have

$$w_i = d(n,a,b)\hat{w}_i, \quad \text{where} \quad \hat{w}_i = (-1)^i \cdot \begin{cases} \frac{1}{2}, & i = 0, n \\ 1, & i = 1, 2, ..., n-1 \end{cases}$$

Using the barycentric interpolation formula for $p_n(x)$ and the equation above, we have

$$\begin{aligned} p_n(x) &= \frac{\sum_{i=0}^{n} \frac{d(n,a,b)\hat{w}_i}{x - x_i} y_i}{\sum_{i=0}^{n} \frac{d(n,a,b)\hat{w}_i}{x - x_i}} \\ &= \frac{d(n,a,b) \sum_{i=0}^{n} \frac{\hat{w}_i}{x - x_i} y_i}{d(n,a,b) \sum_{i=0}^{n} \frac{\hat{w}_i}{x - x_i}} \\ &= \frac{\sum_{i=0}^{n} \frac{\hat{w}_i}{x - x_i} y_i}{\sum_{i=0}^{n} \frac{\hat{w}_i}{x - x_i}} \end{aligned}$$

Problem 4

Barycentric_interpolation_P4.m

```
% Problem 4
% clear
format long e

% cd 'C:\Users\Christopher\Desktop\MAT 128A\Project 1';

% Use Divided_differences_P1 to get the vector a
% Use Horners_Rule_P2 to get y = f(x(j))

n = input('Specify the power of the desired polynomial: ');
eval_x = input('Evaluate the polynomial at x = ');
x = input('Input a vector of x values: ');
y = input('Input a vector of y values: ');
w = input('Input a vector of w values: ');

if numel(x) ~= (n+1) || numel(y) ~= (n+1) || numel(w) ~= (n+1)
  error('Your vectors are not of length n+1');
else
  if numel(unique(x)) ~= (n+1)
    error('x must be a vector of distinct values');
  end
end

display_x = eval_x;
copies = numel(eval_x);
eval_x = repmat(eval_x, 1, numel(x));
x = repelem(x, copies);
y = repelem(y, copies);
w = repelem(w, copies);
token = eval_x - x;
for i = 1:copies
  for j = i:copies:numel(token)
    if token(j) ~= 0
      j = i:copies:numel(token);
      eval_y(i) = (sum((w(j)./(eval_x(j) - x(j))).*y(j))) / (sum(w(j)./(eval_x(j) - x(j))));
    else
      if token(j) == 0
        eval_y(i) = y(j);
        break
      end
    end
  end
end
output = horzcat(display_x', eval_y');
```

```
fprintf('f(%d) = %d\n', output');
```

Problem 5

(a)

$x_i = -\pi + \frac{2\pi}{n} i, \quad i = 0, 1, .., n;$
$y_i = f(x_i), \quad i = 0, 1, ..., n, \quad$ where $\quad f(x) = sinx;$

n = 5

x	f(x)	$p_n^{(1)}(x)$	$\left\|f(x) - p_n^{(1)}(x)\right\|$
$-\pi$	-1.224646799147353e-16	-1.224646799147353e-16	0
π	1.224646799147353e-16	5.751090196869910e-16	4.526443397722557e-16
$-\pi + \frac{\pi}{5}$	-5.877852522924733e-01	-5.659550824061890e-01	2.183016988628428e-02
$-\pi + \frac{3\pi}{5}$	-9.510565162951535e-01	-9.549555391042396e-01	3.899022809086050e-03
0	0	2.263221698861278e-16	2.263221698861278e-16
$-\pi + \frac{7\pi}{5}$	9.510565162951535e-01	9.549555391042401e-01	3.899022809086605e-03

x	$p_n^{(2)}(x)$	$\left\|f(x) - p_n^{(2)}(x)\right\|$
$-\pi$	-1.224646799147353e-16	0
π	1.224646799147353e-16	0
$-\pi + \frac{\pi}{5}$	-5.659550824061888e-01	2.183016988628450e-02
$-\pi + \frac{3\pi}{5}$	-9.549555391042400e-01	3.899022809086494e-03
0	1.435132967750804e-18	1.435132967750804e-18
$-\pi + \frac{7\pi}{5}$	9.549555391042398e-01	3.899022809086272e-03

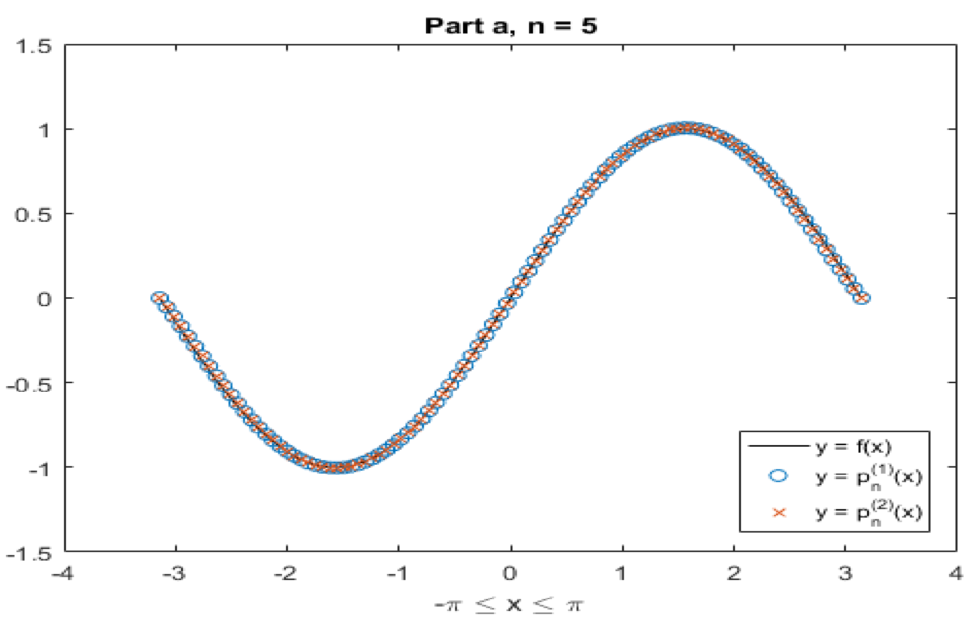

FIGURE 1.

n = 10

x	f(x)	$p_n^{(1)}(x)$	$\left\lvert f(x) - p_n^{(1)}(x) \right\rvert$
$-\pi$	-1.224646799147353e-16	-1.224646799147353e-16	0
π	1.224646799147353e-16	-1.224646799147353e-16	2.449293598294706e-16
$-\pi + \frac{\pi}{10}$	-3.090169943749475e-01	-3.089771607884714e-01	3.983358647613455e-05
$-\pi + \frac{3\pi}{10}$	-8.090169943749475e-01	-8.090234146182949e-01	6.420243347404764e-06
$-\pi + \frac{15\pi}{10}$	1.000000000000000e+00	9.999980820745196e-01	1.917925480432459e-06
$-\pi + \frac{17\pi}{10}$	8.090169943749475e-01	8.090234146182943e-01	6.420243346849652e-06

x	$p_n^{(2)}(x)$	$\left\lvert f(x) - p_n^{(2)}(x) \right\rvert$
$-\pi$	-1.224646799147353e-16	0
π	1.224646799147353e-16	0
$-\pi + \frac{\pi}{10}$	-3.089771607884719e-01	3.983358647557944e-05
$-\pi + \frac{3\pi}{10}$	-8.090234146182950e-01	6.420243347515786e-06
$-\pi + \frac{15\pi}{10}$	9.999980820745196e-01	1.917925480432459e-06
$-\pi + \frac{17\pi}{10}$	8.090234146182950e-01	6.420243347515786e-06

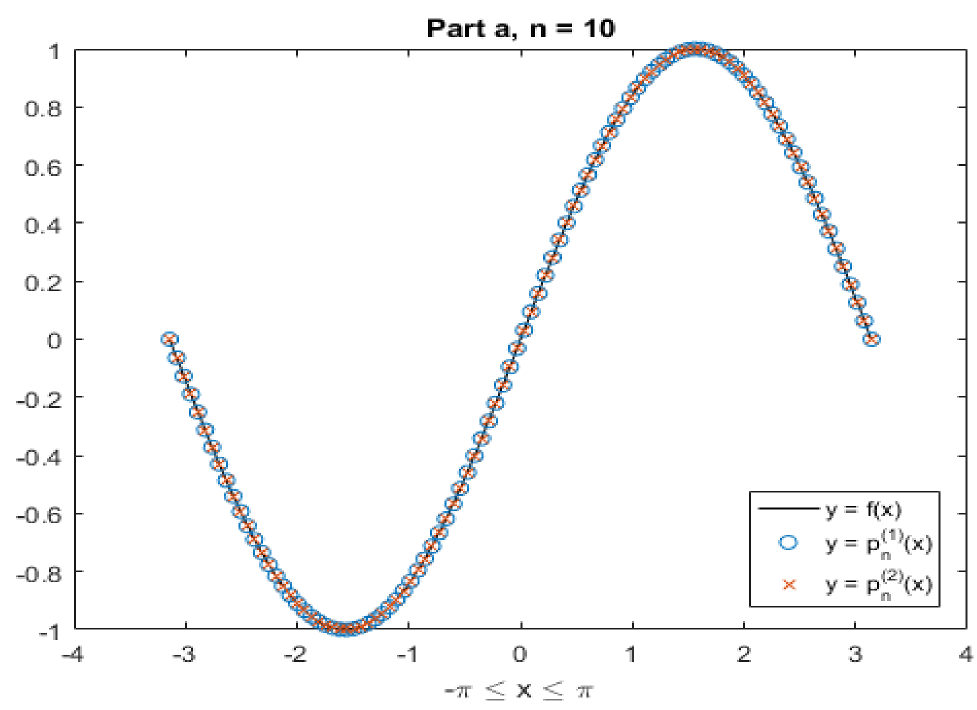

FIGURE 2.

n = 20

x	f(x)	$p_n^{(1)}(x)$	$\left\lvert f(x) - p_n^{(1)}(x) \right\rvert$
$-\pi$	-1.224646799147353e-16	-1.224646799147353e-16	0
π	1.224646799147353e-16	2.667830118492170e-15	2.545365438577435e-15
$-\pi + \frac{\pi}{20}$	-1.564344650402310e-01	-1.564344650403139e-01	8.287814878826794e-14
$-\pi + \frac{3\pi}{20}$	-4.539904997395469e-01	-4.539904997395404e-01	6.494804694057166e-15
$-\pi + \frac{35\pi}{20}$	7.071067811865476e-01	7.071067811865508e-01	3.219646771412954e-15
$-\pi + \frac{37\pi}{20}$	4.539904997395469e-01	4.539904997395407e-01	6.161737786669619e-15

x	$p_n^{(2)}(x)$	$\left\lvert f(x) - p_n^{(2)}(x) \right\rvert$
$-\pi$	-1.224646799147353e-16	0
π	1.224646799147353e-16	0
$-\pi + \frac{\pi}{20}$	-1.564344650401648e-01	6.622480341889059e-14
$-\pi + \frac{3\pi}{20}$	-4.539904997395377e-01	9.214851104388799e-15
$-\pi + \frac{35\pi}{20}$	7.071067811865461e-01	1.443289932012704e-15
$-\pi + \frac{37\pi}{20}$	4.539904997395405e-01	6.328271240363392e-15

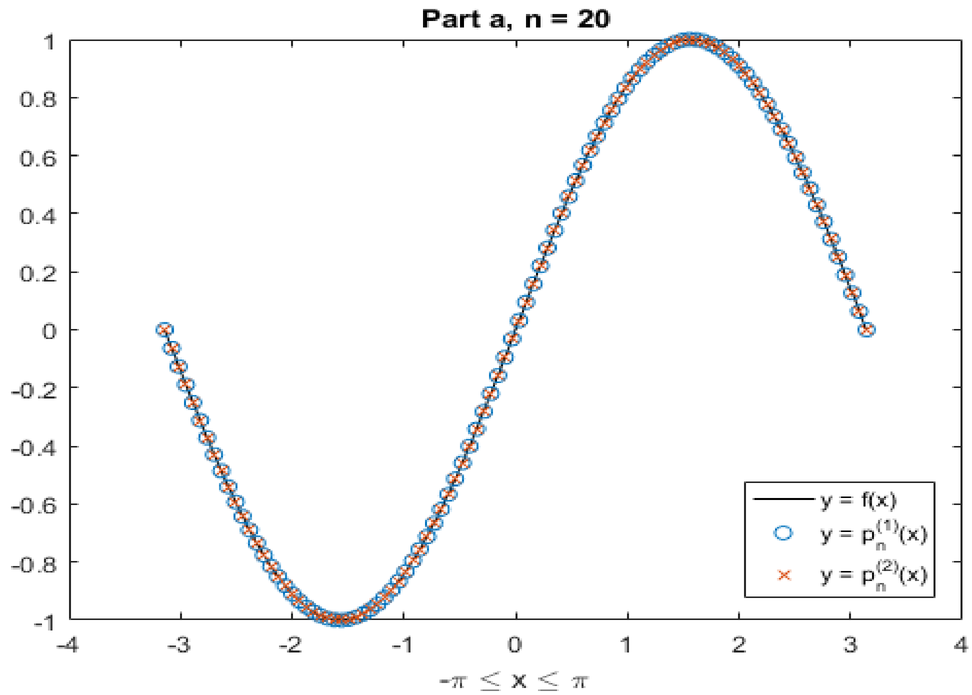

FIGURE 3.

n = 40

x	f(x)	$p_n^{(1)}(x)$	$\left\|f(x) - p_n^{(1)}(x)\right\|$
$-\pi$	-1.224646799147353e-16	-1.224646799147353e-16	0
π	1.224646799147353e-16	-4.307906877525093e-15	4.430371557439828e-15
$-\pi + \frac{\pi}{40}$	-7.845909572784507e-02	-7.845908904192023e-02	6.685924841542956e-09
$-\pi + \frac{3\pi}{40}$	-2.334453638559055e-01	-2.334453641401451e-01	2.842395763202887e-10
$-\pi + \frac{75\pi}{40}$	3.826834323650903e-01	3.826834323442731e-01	2.081718131208277e-11
$-\pi + \frac{77\pi}{40}$	2.334453638559051e-01	2.334453641401489e-01	2.842437674122067e-10

x	$p_n^{(2)}(x)$	$\left\|f(x) - p_n^{(2)}(x)\right\|$
$-\pi$	-1.224646799147353e-16	0
π	1.224646799147353e-16	0
$-\pi + \frac{\pi}{40}$	-7.845908932036021e-02	6.407484862136492e-09
$-\pi + \frac{3\pi}{40}$	-2.334453626359434e-01	1.219962103560235e-09
$-\pi + \frac{75\pi}{40}$	3.826834323866440e-01	2.155370326661910e-11
$-\pi + \frac{77\pi}{40}$	2.334453640966305e-01	2.407253840708279e-10

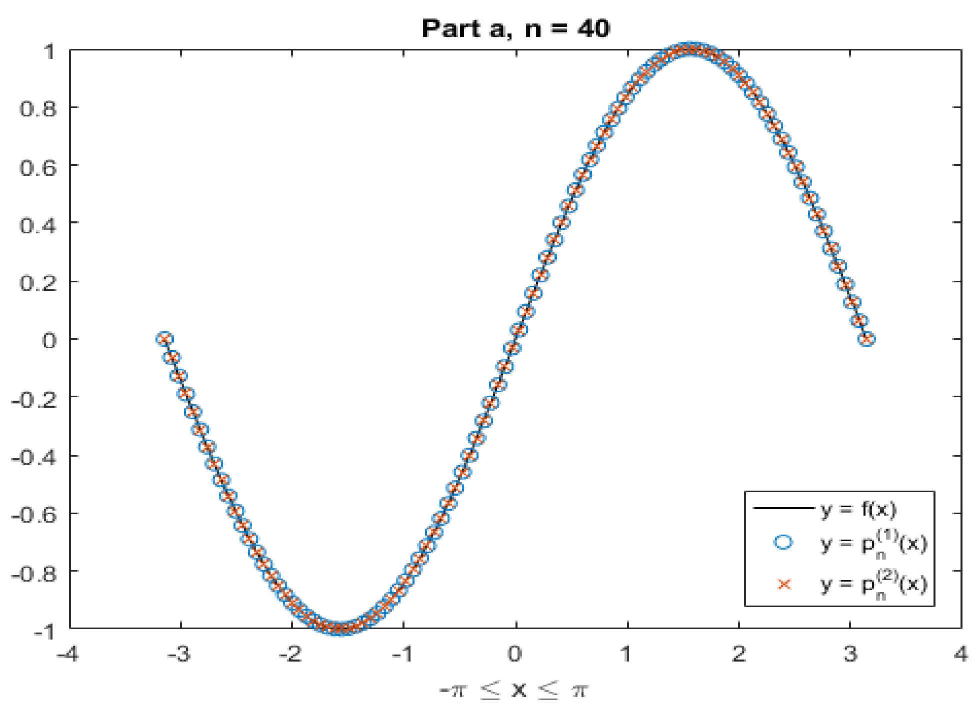

Figure 4.

(b)

$x_i = -5 + \frac{10}{5}i, \quad i = 0, 1, .., n;$
$y_i = f(x_i), \quad i = 0, 1, ..., n, \quad \text{where} \quad f(x) = \frac{1}{1+x^2};$

<u>n = 5</u>

x	f(x)	$p_n^{(1)}(x)$	$\left\lvert f(x) - p_n^{(1)}(x) \right\rvert$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.846153846153846e-02	0
-5 + $\frac{5}{5}$ = -4	5.882352941176471e-02	-4.807692307692309e-02	1.069004524886878e-01
-5 + $\frac{15}{5}$ = -2	2.000000000000000e-01	3.211538461538461e-01	1.211538461538461e-01
-5 + $\frac{25}{5}$ = 0	1.000000000000000e+00	5.673076923076924e-01	4.326923076923076e-01
-5 + $\frac{35}{5}$ = 2	2.000000000000000e-01	3.211538461538461e-01	1.211538461538461e-01

x	$p_n^{(2)}(x)$	$\left\lvert f(x) - p_n^{(2)}(x) \right\rvert$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
-5 + $\frac{5}{5}$ = -4	-4.807692307692309e-02	1.069004524886878e-01
-5 + $\frac{15}{5}$ = -2	3.211538461538461e-01	1.211538461538461e-01
-5 + $\frac{25}{5}$ = 0	1.435132967750804e-18	4.326923076923075e-01
-5 + $\frac{35}{5}$ = 2	3.211538461538462e-01	1.211538461538462e-01

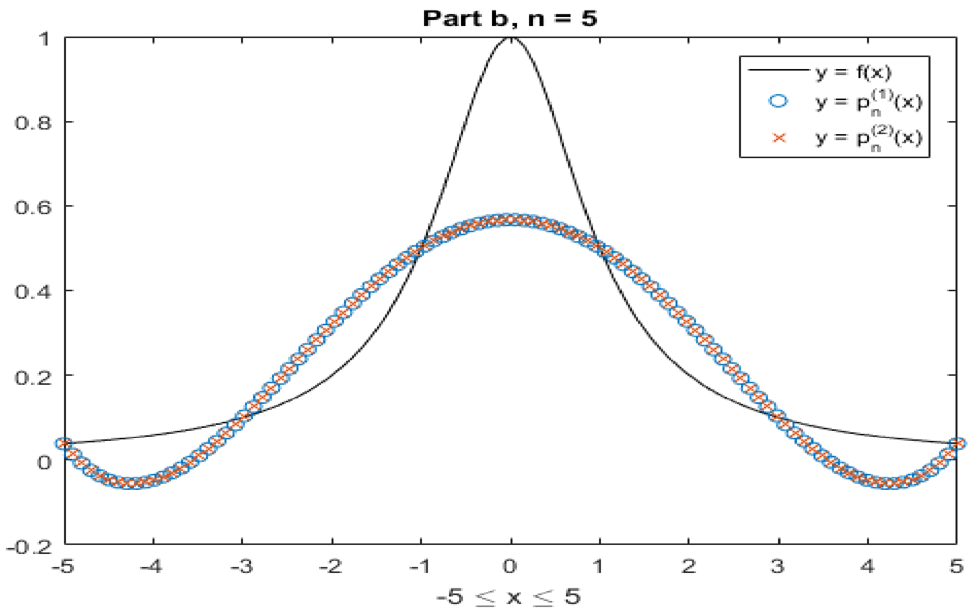

Figure 5.

n = 10

x	f(x)	$p_n^{(1)}(x)$	$\left\|f(x) - p_n^{(1)}(x)\right\|$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.846153846146172e-02	7.674416657721395e-14
$-5 + \frac{5}{10} = -4.5$	4.705882352941176e-02	1.578720990349265e+00	1.531662166819853e+00
$-5 + \frac{15}{10} = -3.5$	7.547169811320754e-02	-2.261962890625000e-01	3.016679871757075e-01
$-5 + \frac{75}{10} = 2.5$	1.379310344827586e-01	2.537554572610284e-01	1.158244227782698e-01
$-5 + \frac{85}{10} = 3.5$	7.547169811320754e-02	-2.261962890625145e-01	3.016679871757221e-01

x	$p_n^{(2)}(x)$	$\left\|f(x) - p_n^{(2)}(x)\right\|$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
$-5 + \frac{5}{10} = -4.5$	1.578720990349265e+00	1.531662166819853e+00
$-5 + \frac{15}{10} = -3.5$	-2.261962890625000e-01	3.016679871757075e-01
$-5 + \frac{75}{10} = 2.5$	2.537554572610294e-01	1.158244227782708e-01
$-5 + \frac{85}{10} = 3.5$	-2.261962890624999e-01	3.016679871757075e-01

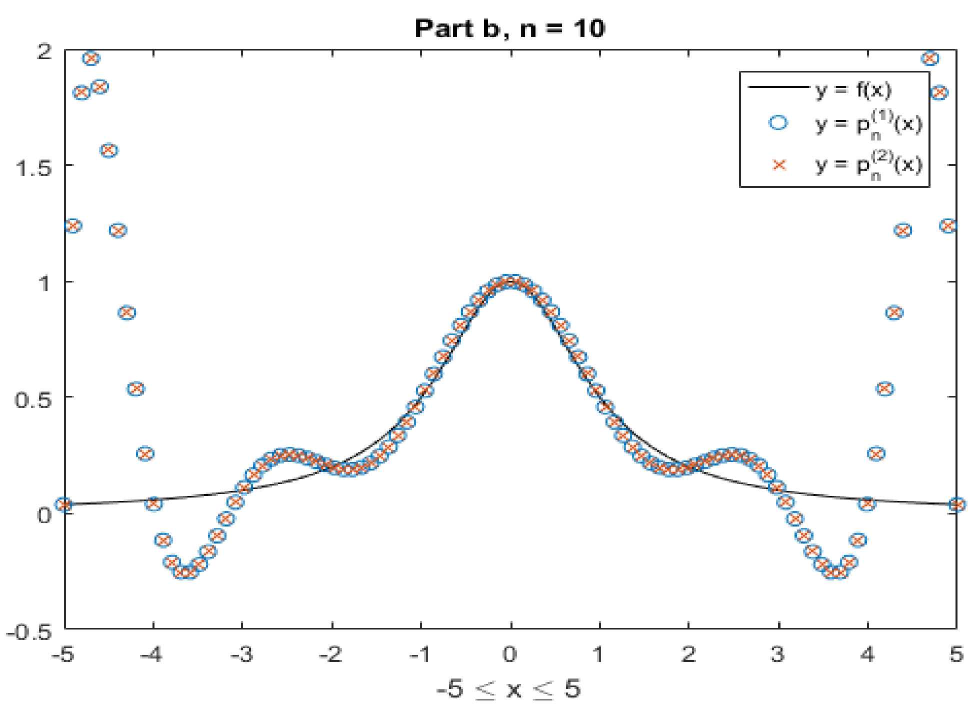

FIGURE 6.

n = 20

x	f(x)	$p_n^{(1)}(x)$	$\left\lvert f(x) - p_n^{(1)}(x) \right\rvert$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.846153838071152e-02	8.082694236133392e-11
-5 + $\frac{5}{20}$ = -4.75	4.244031830238727e-02	-3.995244903304142e+01	3.999488935134381e+01
-5 + $\frac{15}{20}$ = -4.25	5.245901639344262e-02	3.454957799864103e+00	3.402498783470661e+00
-5 + $\frac{175}{20}$ = 3.75	6.639004149377593e-02	-4.470519607099369e-01	5.134420022037128e-01
-5 + $\frac{185}{20}$ = 4.25	5.245901639344262e-02	3.454957799871618e+00	3.402498783478176e+00

x	$p_n^{(2)}(x)$	$\left\lvert f(x) - p_n^{(2)}(x) \right\rvert$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
-5 + $\frac{5}{20}$ = -4.75	-3.995244903303957e+01	3.999488935134196e+01
-5 + $\frac{15}{20}$ = -4.25	3.454957799864109e+00	3.402498783470667e+00
-5 + $\frac{175}{20}$ = 3.75	-4.470519607088353e-01	5.134420022026113e-01
-5 + $\frac{185}{20}$ = 4.25	3.454957799864113e+00	3.402498783470671e+00

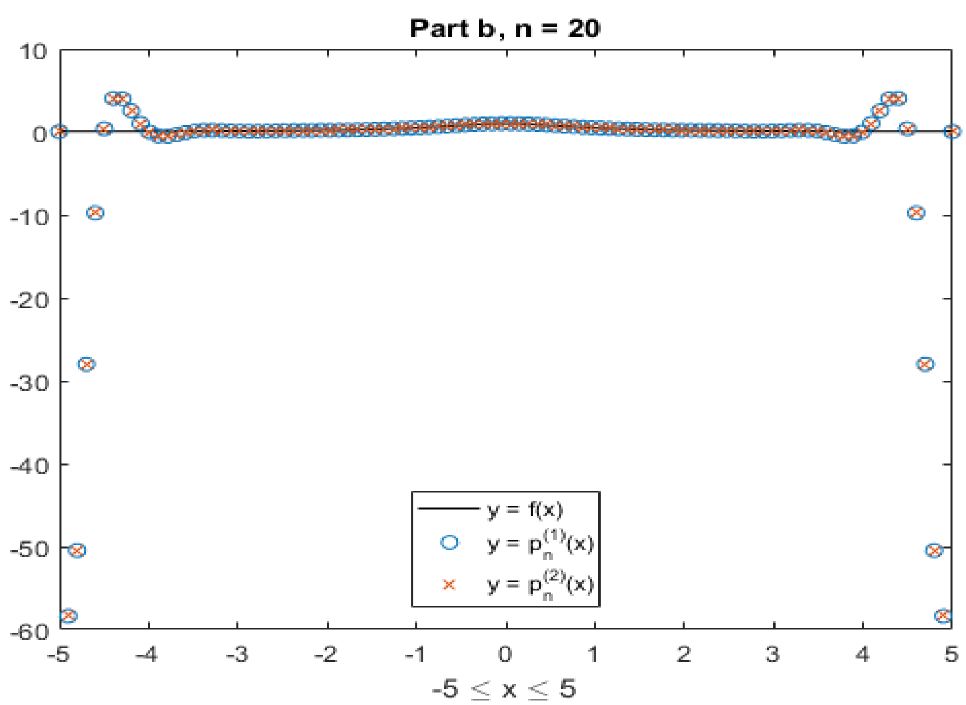

FIGURE 7.

n = 40

x	f(x)	$p_n^{(1)}(x)$	$\left\|f(x) - p_n^{(1)}(x)\right\|$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.783539850919955e-02	6.261399523389138e-04
$-5 + \frac{5}{40} = -4.875$	4.037854889589906e-02	-5.740917974225197e+04	5.740922012080087e+04
$-5 + \frac{15}{40} = -4.625$	4.466154919748779e-02	2.287728498702767e+03	2.287683837153570e+03
$-5 + \frac{375}{40} = 4.375$	4.965089216446858e-02	-1.561697587655903e+02	1.562194096577548e+02
$-5 + \frac{385}{40} = 4.625$	4.466154919748779e-02	2.287728405507201e+03	2.287683743958003e+03

x	$p_n^{(2)}(x)$	$\left\|f(x) - p_n^{(2)}(x)\right\|$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
$-5 + \frac{5}{40} = -4.875$	-5.740917956810044e+04	5.740921994664934e+04
$-5 + \frac{15}{40} = -4.625$	2.287728499091658e+03	2.287683837542461e+03
$-5 + \frac{375}{40} = 4.375$	-1.561697170404079e+02	1.562193679325723e+02
$-5 + \frac{385}{40} = 4.625$	2.287728498445287e+03	2.287683836896090e+03

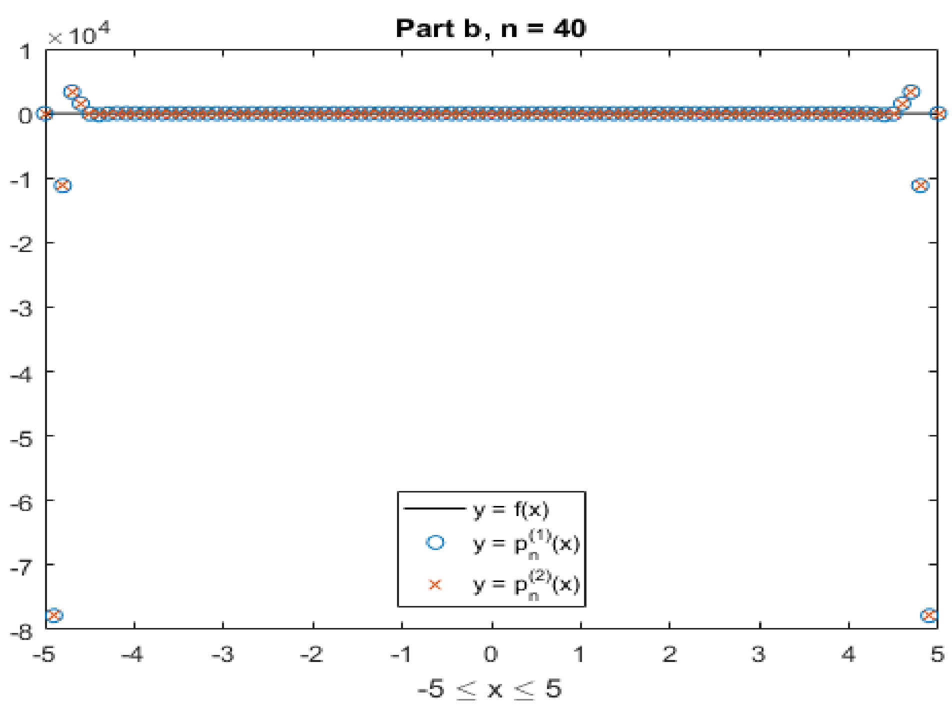

FIGURE 8.

(c)

$x_i = 5cos\frac{\pi(n-i)}{n}, \quad i = 0, 1, .., n;$
$y_i = f(x_i), \quad i = 0, 1, ..., n, \quad \text{where} \quad f(x) = \frac{1}{1+x^2};$

n = 5

x	f(x)	$p_n^{(1)}(x)$	$\left\|f(x) - p_n^{(1)}(x)\right\|$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.846153846153898e-02	5.204170427930421e-16
-5 + $\frac{5}{5}$ = -4	5.882352941176471e-02	6.053298454998771e-02	1.709455138223001e-03
-5 + $\frac{15}{5}$ = -2	2.000000000000000e-01	2.547617101283415e-01	5.476171012834147e-02
-5 + $\frac{25}{5}$ = 0	1.000000000000000e+00	3.613586201258890e-01	6.386413798741110e-01
-5 + $\frac{35}{5}$ = 2	2.000000000000000e-01	2.547617101283413e-01	5.476171012834130e-02

x	$p_n^{(2)}(x)$	$\left\|f(x) - p_n^{(2)}(x)\right\|$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
-5 + $\frac{5}{5}$ = -4	6.042195269562019e-02	1.598423283855489e-03
-5 + $\frac{15}{5}$ = -2	2.422783387446042e-01	4.227833874460416e-02
-5 + $\frac{25}{5}$ = 0	4.420828905419766e-01	5.579171094580234e-01
-5 + $\frac{35}{5}$ = 2	2.277151044988212e-01	2.771510449882123e-02

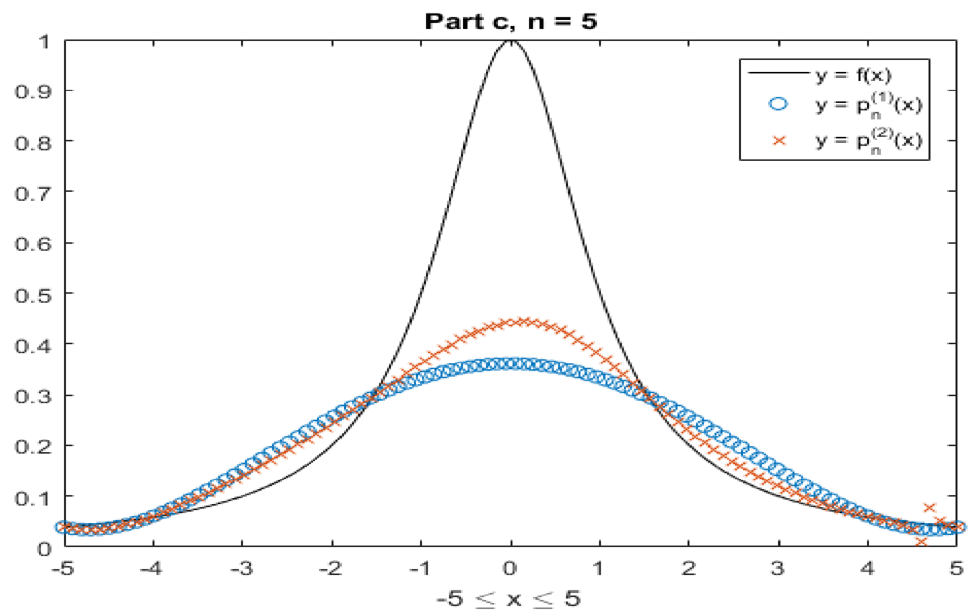

FIGURE 9.

n = 10

x	f(x)	$p_n^{(1)}(x)$	$\left\|f(x) - p_n^{(1)}(x)\right\|$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.846153846152320e-02	1.526556658859590e-14
-5 + $\frac{5}{10}$ = -4.5	4.705882352941176e-02	2.227578045912658e-02	2.478304307028519e-02
-5 + $\frac{15}{10}$ = -3.5	7.547169811320754e-02	1.269137477424124e-01	5.144204962920483e-02
-5 + $\frac{75}{10}$ = 2.5	1.379310344827586e-01	6.705169411207847e-02	7.087934037068015e-02
-5 + $\frac{85}{10}$ = 3.5	7.547169811320754e-02	1.269137477424130e-01	5.144204962920550e-02

x	$p_n^{(2)}(x)$	$\left\|f(x) - p_n^{(2)}(x)\right\|$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
-5 + $\frac{5}{10}$ = -4.5	2.190364931315244e-02	2.515517421625933e-02
-5 + $\frac{15}{10}$ = -3.5	1.233649133871124e-01	4.789321527390483e-02
-5 + $\frac{75}{10}$ = 2.5	6.332254337505151e-02	7.460849110770711e-02
-5 + $\frac{85}{10}$ = 3.5	1.543668791943819e-01	7.889518108117440e-02

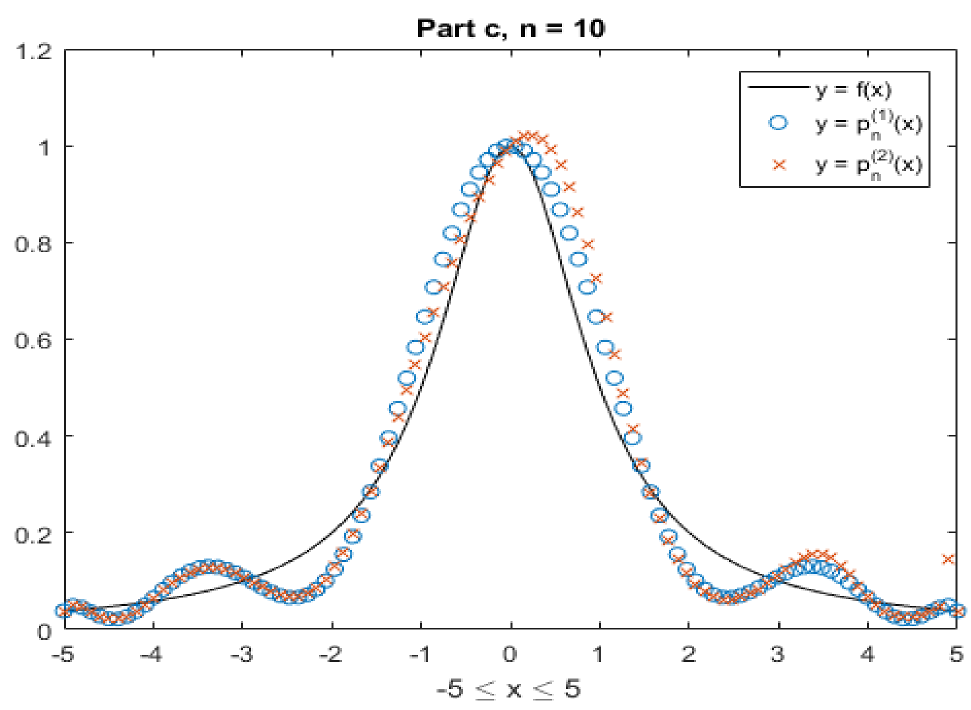

FIGURE 10.

n = 20

x	f(x)	$p_n^{(1)}(x)$	$\left\lvert f(x) - p_n^{(1)}(x) \right\rvert$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.846153846171893e-02	1.804667526528192e-13
-5 + $\frac{5}{20}$ = -4.75	4.244031830238727e-02	4.228249771970687e-02	1.578205826804000e-04
-5 + $\frac{15}{20}$ = -4.25	5.245901639344262e-02	5.676927887131598e-02	4.310262477873354e-03
-5 + $\frac{175}{20}$ = 3.75	6.639004149377593e-02	6.061946230812289e-02	5.770579185653046e-03
-5 + $\frac{185}{20}$ = 4.25	5.245901639344262e-02	5.676927886967291e-02	4.310262476230287e-03

x	$p_n^{(2)}(x)$	$\left\lvert f(x) - p_n^{(2)}(x) \right\rvert$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
-5 + $\frac{5}{20}$ = -4.75	4.228041947629929e-02	1.598988260879744e-04
-5 + $\frac{15}{20}$ = -4.25	5.703234023406498e-02	4.573323840622354e-03
-5 + $\frac{175}{20}$ = 3.75	6.380442846878059e-02	2.585613024995340e-03
-5 + $\frac{185}{20}$ = 4.25	5.404649869973156e-02	1.587482306288940e-03

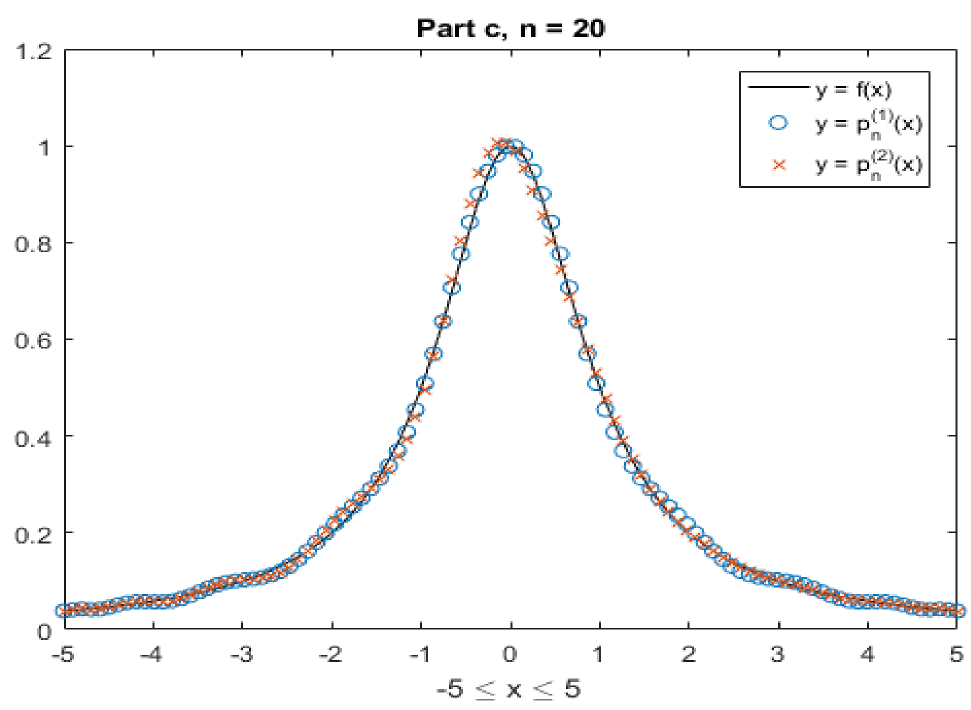

FIGURE 11.

n = 40

x	f(x)	$p_n^{(1)}(x)$	$\left\|f(x) - p_n^{(1)}(x)\right\|$
-5	3.846153846153846e-02	3.846153846153846e-02	0
5	3.846153846153846e-02	3.846726313822841e-02	5.724676689945751e-06
$-5 + \frac{5}{40} = -4.875$	4.037854889589906e-02	4.036503847749406e-02	1.351041840499251e-05
$-5 + \frac{15}{40} = -4.625$	4.466154919748779e-02	4.465516851484980e-02	6.380682637988078e-06
$-5 + \frac{375}{40} = 4.375$	4.965089216446858e-02	4.958003579809714e-02	7.085636637144122e-05
$-5 + \frac{385}{40} = 4.625$	4.466154919748779e-02	4.465636778867244e-02	5.181408815349564e-06

x	$p_n^{(2)}(x)$	$\left\|f(x) - p_n^{(2)}(x)\right\|$
-5	3.846153846153846e-02	0
5	3.846153846153846e-02	0
$-5 + \frac{5}{40} = -4.875$	4.036265608819473e-02	1.589280770432355e-05
$-5 + \frac{15}{40} = -4.625$	4.465158662449435e-02	9.962572993439667e-06
$-5 + \frac{375}{40} = 4.375$	5.074360392873475e-02	1.092711764266167e-03
$-5 + \frac{385}{40} = 4.625$	4.474854371662106e-02	8.699451913327105e-05

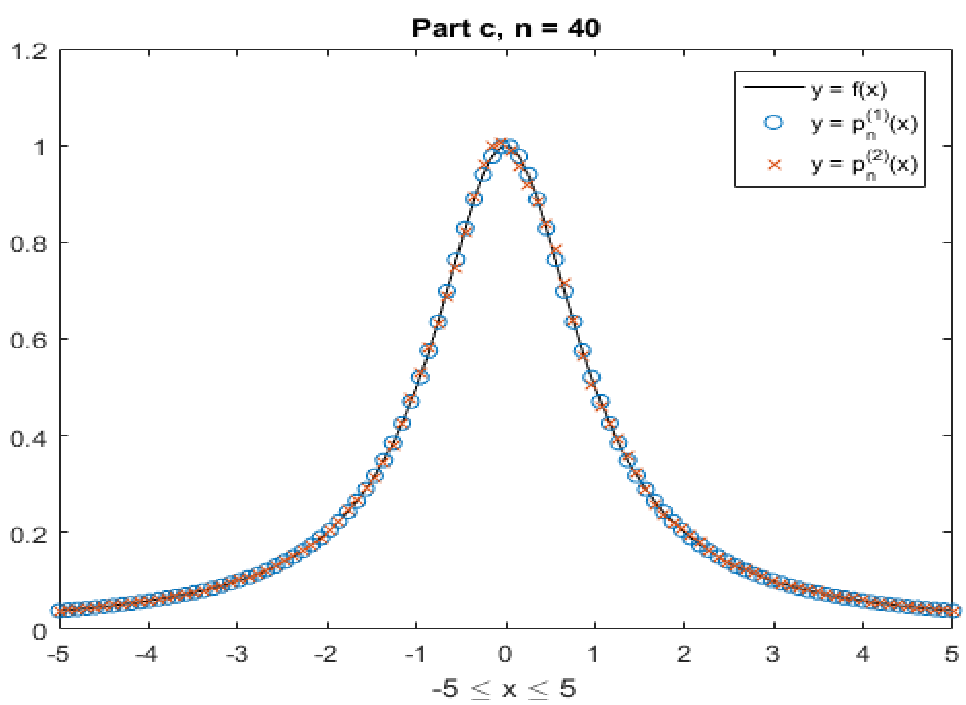

FIGURE 12.