Ay190 – Worksheet 11 - Advection Equation
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1 Analytical answer

The analytical answer for the advection equation is a Gaussian with shifted center by vt. I don't know how to save the animation in python and import into LaTeX, so if you run the `Q1.py`, we can see the Gaussian moves along as time progresses.

2 Upwind Method

For upwind method, the stability zone is when $0 \leq \alpha \leq 1$. In figure 2, we can see that when alpha is out of the stability condition, the error grows significantly.

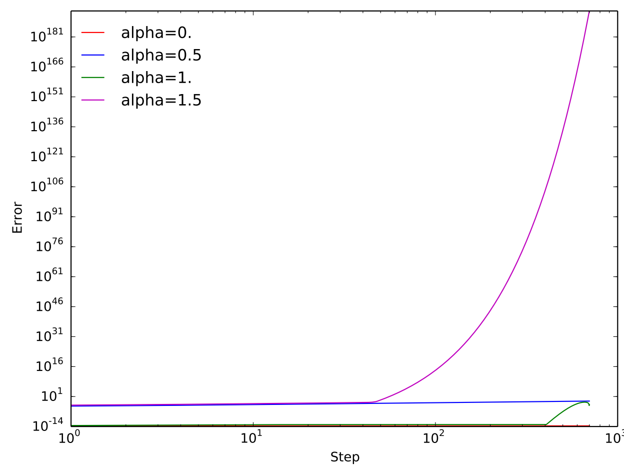

Figure 1: The progress of error with different alpha on upwind method.

When reduce the initial σ by 1/5, when perform the upwind method, as time progress, the peak of Gaussian in numerical upwind method gets smaller and smaller, and the Gaussian gets wider. Run code `Q2-2.py` to see it.

3 FTCS Method

For FTCS Method, it is always unstable. From the visualization, I can see that, start at around iteration 300, the error starts to grow, not at the peak, but before, as packages of sinusoidal beats. (See figure 2)

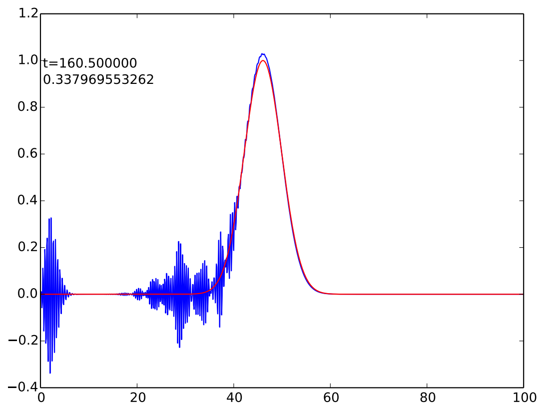

Figure 2: Error starts to grow in FTCS method after iteration of 300.

4 Lax-Friedrich Method

Run `Q4.py` to visualize. For both upwind and Lax-Friedrich Method, as time progresses, the peak of the Gaussian decreases, but the peak for Lax-Friedrich Method decreases at a faster rate.