Ay190 – Worksheet 08
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1 ODE Integration: Simplified Stellar Structure

1.1 Code Explanation

- Function `tov_RHS` returns the value of $\frac{dP}{dr}$ and $\frac{dM}{dr}$, i.e. the right hand side of the ODE, respectively.
- Function `tov_integrate_FE` takes in current values of parameters we want to determine, calls function `tov_RHS` to calculate the RHS, and adds that to the current values in order to get the next value for pressure and mass.
- Then, we setup grid of radius with desired resolution, i.e. number of points. Set initial value for density, pressure, and mass. Set the termination criterion to be minimum pressure of $10^{-10} \times P_o$. And, importantly, set a parameter $nsurf$ to zero, which will keep track of whether we hit the surface of the star or not.
- Run the for loop entire radius grid, except the last one. Then, update the value of the parameters by calling function `tov_integrate_FE`.
- For each time in the for loop, check that if the next pressure value is less than the minimum pressure or not (`press[n+1] < press_min`), and check whether we have already hit the surface before or not (`nsurf==0`), If both criterion are satisfied, that means this is the first that we have hit the surface, so let $nsurf$ equal to the current index in the for loop.
- The next if statement (`if(n+1 > nsurf and nsurf > 0):`) checks that we have already hit the surface or not. If yes, then the value shouldn't be calculated by the FE integrator, but it will be equal to the value at the surface.
- Before the end of for loop, update the new value of density, using the equation of state.

1.2 Testing FE Code

After I fill the appropriate codes into `[ FILL IN CODE ]`, I ran the code and get the result to be, $M = 1.45069351877 M_{\odot}$ and $radius = 1501.5015015 km$, which are close to what we expected.

1.3 Convergent Plot

Plot the convergent rate of final mass of different methods. Since I don't know the analytical answer to the final mass, I just use the final mass calculated by the most resolution. That is why, for every lines in figure 1, the line dips fast at the right hand end. However, if we ignore that dipping, the line for FE, RK2, and RK3, has the slope of -1, -2, and -3, respectively, as expected. When I tried to plot the result for RK4, it yields similar result to those of RK3, so I shift it down by 10^{-2}, so we can see both RK3 and RK4 at the same time. RK4 somehow has the same slope as RK3.

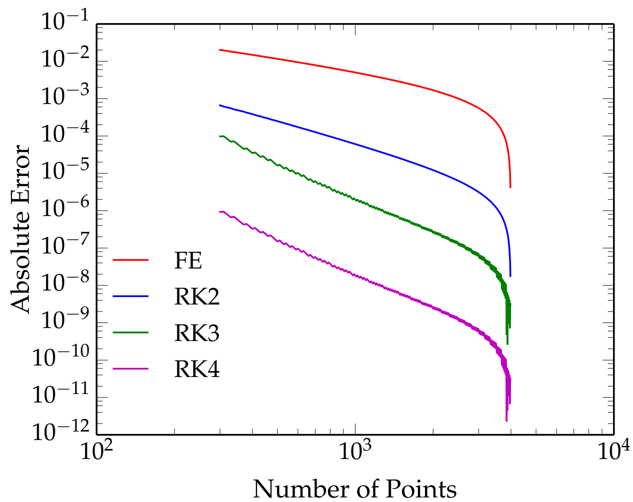

Figure 1: Convergent Plot of different methods on final mass

1.4 Plot of density, pressure, and mass, versus radius

Plot $\rho(r)$, $P(r)$, and $M(r)$, using RK2. I scale down mass and pressure to $\frac{Mass}{FinalMass}$ and $\frac{Pressure}{CentralPressure}$, in order to fit both of them on the same axis. When I try to plot with RK3, and RK4, they yield the same answer, or, at least, with unnoticeable differences.

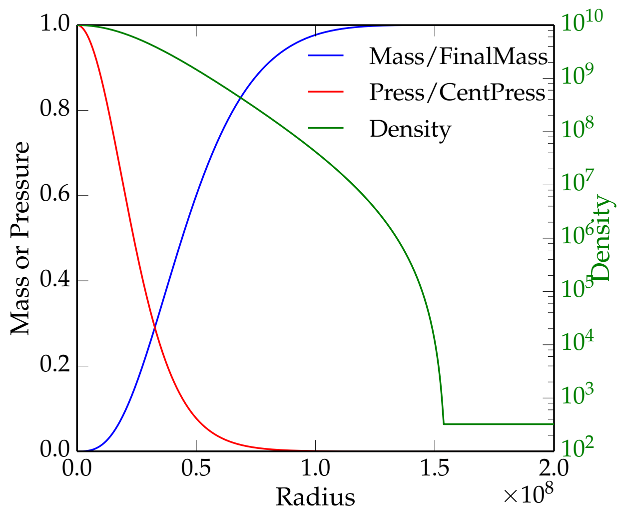

Figure 2: Evolution of mass, pressure, and density with radius.