Ay190 – Worksheet 03
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1 Integration via Newton-Cotes Formulae

1.1 Integration Over Sin(x) Function

Do the numerical integration of sin(x) from x = 0 to x = π with three different methods, midpoint, trapezoid, and Simpson's rule. The analytical answer is 2. Figure 1 shows the absolute error. We can see that the error for midpoint and trapezoid method goes down quadratically, and error for SImpson's rule goes down with h^4, as expected.

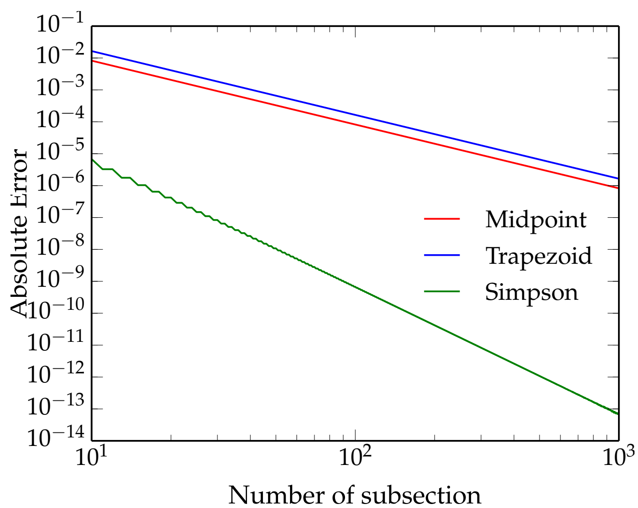

Figure 1: Absolute error from integration of sin(x) from x = 0 to x = π with three different methods, midpoint, trapezoid, and Simpson's rule.

1.2 Integration Over x*Sin(x) Function

Similar to above section, but now the integration is of x*sin(x) from x = 0 to x = π. Again, plot the absolute error to observe the convergent rate of three different methods. Figure 2 show that the convergent rate (i.e. the slope of the line) is the same as in figure 1.

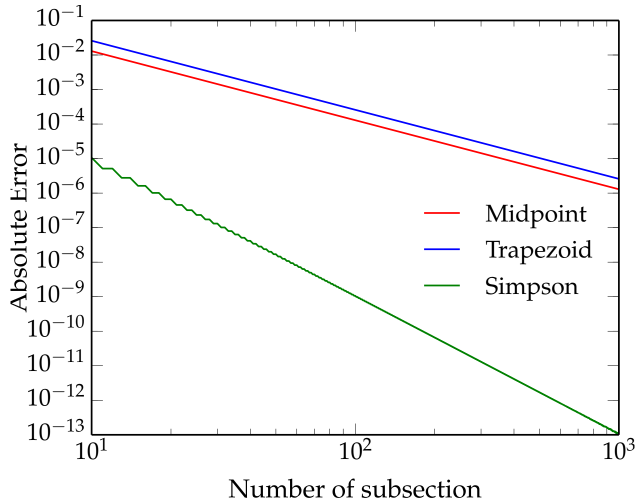

Figure 2: Absolute error from integration of x*sin(x) from x = 0 to x = π with three different methods, midpoint, trapezoid, and Simpson's rule.

2 Gaussian Quadrature

2.1 Gauss-Laguerre

Modify the integral to get the correct weight function for the Gauss-Laguerre quadrature.

$$
\begin{aligned}
n_{e^{\pm}} &= \frac{8\pi(k_B T)^3}{(2\pi\bar{h}c)^3}\int_0^\infty \frac{x^2}{e^x+1}dx \\
&= \frac{8\pi(k_B T)^3}{(2\pi\bar{h}c)^3}\int_0^\infty \frac{1}{e^x}\frac{e^x x^2}{e^x+1}dx \\
&= \frac{8\pi(k_B T)^3}{(2\pi\bar{h}c)^3}\int_0^\infty W(x)\frac{e^x x^2}{e^x+1}dx
\end{aligned}
$$

The constant in front of the integral is approximately $8.4384 \times 10^{33} m^{-3}$. In figure 3, it seems that our answer is bounded by accuracy, even at low n. In fact, when I try to use high n as 500, python spits out an overflow error.

2.2 Gauss-Legendre

Modify the integral in terms of energy, $E = pc$, by substitute $x = \beta E$.

$$
\begin{aligned}
n_{e^{\pm}} &= \frac{8\pi(k_B T)^3}{(2\pi\bar{h}c)^3}\int_0^\infty \frac{x^2}{e^x+1}dx \\
&= \frac{8\pi}{(2\pi\bar{h}c)^3}\int_0^\infty \frac{E^2}{e^{\beta E}+1}dE
\end{aligned}
$$

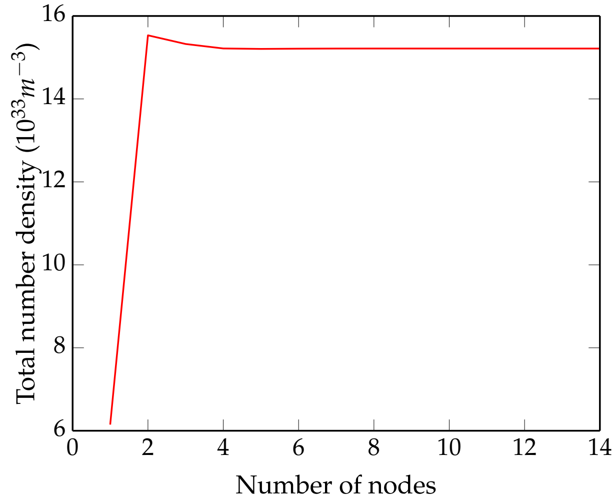

Figure 3: The number density calculated with different number of nodes of Laguerre function.

We shall divide the integral into bins, with width of $\Delta E = 5MeV$, up until $E \approx 150MeV$.

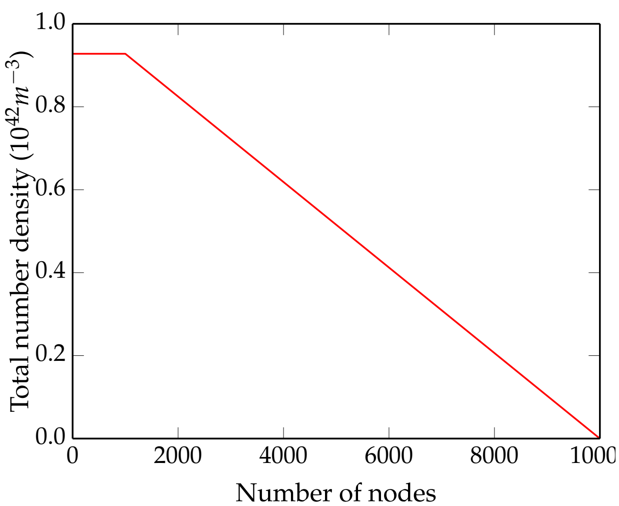

Figure 4: The number density calculated with different number of nodes of Legendre function.

I don't know why the answer is not the same as in previous subsection, but, in figure 4, at least we can see the convergence.