Exercise for chapter 4 (Part 1)
Automata

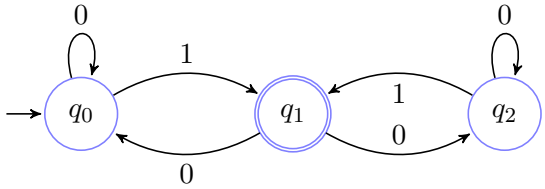
1 Introduction

In this exercise, we will be familiar with some basic concepts and definitions in automata: language, regular expression, finite automata and NFA (nondeterministic finite automata). Students should review the slide and related theoretical documents before doing the exercises below.

2 Example

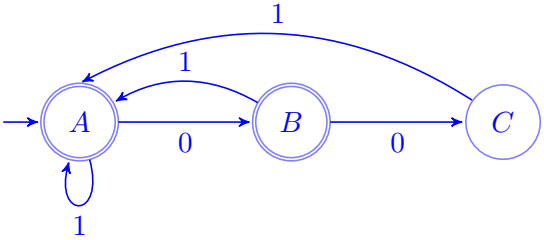
Question 1.
Let $\Sigma = \{a, b\}$ and $L = \{ab, aa, baa\}$.
Which of the following strings are in L^*: *abaabaaabaa*, *aaaabaaaa*, *baaaaabaaaab*, *baaaaabaa*?
Solution.
abaabaaabaa, aaaabaaaa, baaaaabaa □

Question 2.
Which of the strings 0001, 01001, 0000110 are accepted by the following automata:



Solution.
0001, 01001 □

Question 3.
Consider the set of strings on $\{0, 1\}$ in which every 00 is followed immediately by 1. For example 101, 0010, 0010011001 are in the language, but 0001 and 00100 are not. Construct an accepting automata.
Solution.



□

3 Homework

Question 4.
Let $\Sigma = \{a, b\}$.
Find all strings in $L = ((a + b)^*b(a + ab)^*)$ of length less than four.

Question 5.
Let $\Sigma = \{a, b\}$.

For which language it is true that $L = L^*$?

a) $L = a^n b^{n+1} : n \geq 0$

b) $L = w : n_a(w) = n_b(w)$

Question 6.
Give a finite automata for the language $L = \{a, ba, aba, bab, bbba\}$.

Question 7.
Let $\Sigma = \{a\}$. Give finite automata for the sets consisting of

a) all strings with exactly one a.

b) all strings with no more than three a's.

Question 8.
Let $\Sigma = \{a, b\}$.
Give finite automata for the sets consisting of

a) all strings with exactly one a.

b) all strings with no more than three a's.

Question 9.
Let $\Sigma = \{a, b, c\}$. Give finite automata for the sets consisting of

a) all strings with exactly one a.

b) all strings with no more than three a's.

c) all strings with no more than three a's and at least one b.

Question 10.
Give an automata for the language $L = \{ab^5wb^4 : w \in \{a, b\}^*\}$.

Question 11.
Find automatas for the following languages on $\Sigma = \{a, b\}$

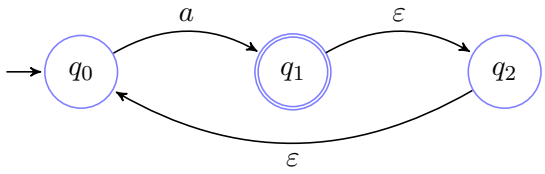
a) $L_1 = \{w : |w| \ mod \ 3 = 0\}$

b) $L_2 = \{w : |w| \ mod \ 5 \neq 0\}$

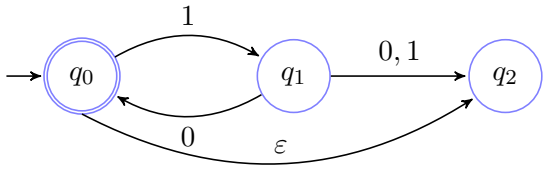
c) $L_3 = \{w : n_a(w) \ mod \ 3 > 1\}$

Question 12.
Show that the language $L = a^n : n \geq 0, n \neq 4$ is regular.

Question 13.
Find $\delta^*(q_0, a)$ and $\delta^*(q_1, \varepsilon)$ for the following automata



Question 14.
For the following automata, find $\delta^*(q_0, 1010)$ and $\delta^*(q_1, 00)$.



Question 15.
Find an automata with three states that accepts the language $\{ab, abc\}^*$

Question 16.
Let $\Sigma = \{a, b, c\}$.
Give complet automatas for the sets consisting of

a) all strings with exactly one 'a'.

b) all strings of even length.

c) all strings which the number of appearances of 'b' is divisible by 3.

d) all strings ending with 'a'.

e) all strings not ending with 'a'.

f) all non-empty strings not ending with 'a'.

g) all strings with at least one 'a'.

h) all strings with at most one 'a'.

i) all strings without any 'a'.

j) all strings including at least one a and whose the first appearance of 'a' is not followed by a 'c'.

Complet automata: a finite automata in which from each state, it is defined precisely when receiving any event.