

Chapter 0

Course Introduction

Mathematical Modeling on January 22, 2018

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai
Faculty of Computer Science and Engineering
University of Technology - VNUHCM

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- The first part of this course introduces CSE students to the basic concepts of logic (e.g., theories, models, logical consequence, and proof).



Aims

- The first part of this course introduces CSE students to the basic concepts of logic (e.g., theories, models, logical consequence, and proof).
- In the second part, students will be learned mathematical modeling through ILP, automata and formal language, Petri net, dynamical systems.



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- This is the mathematical foundations for many CS areas, e.g., algorithm analysis & design, database, artificial intelligence, etc.



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- In the second part, students will be learned mathematical modeling through ILP, automata and formal language, Petri net, dynamical systems.
- This is the mathematical foundations for many CS areas, e.g., algorithm analysis & design, database, artificial intelligence, etc.
- Applications of logic in CSE will be highlighted.



5 chapters, 45 hours of class lectures, HW & exercises.

- Chapter 1. **Propositional Logic Review** and **(Advanced) Predicate Logic**: The need for a richer language; Predicate logic as a formal language; Proof theory of predicate logic; Semantics of predicate logic; Undecidability of predicate logic; Expressiveness of predicate logic.



5 chapters, 45 hours of class lectures, HW & exercises.

- Chapter 1. **Propositional Logic Review** and **(Advanced) Predicate Logic**: The need for a richer language; Predicate logic as a formal language; Proof theory of predicate logic; Semantics of predicate logic; Undecidability of predicate logic; Expressiveness of predicate logic.
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- Chapter 4. **Petri net**.



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- Chapter 3. **Automata & Formal Language**: DFA, NFA, Expression, Context.
- Chapter 4. **Petri net**.
- Chapter 5. **Dynamical systems**.

Grading

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- 01 Assignment (Project): 20%

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- Midterm (MCQ and written; 60 minutes; tentatively after 2 first chapters): 30%

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- Final (MCQs + Short Answer Questions, 120 minutes): 50%
(**cover all 5 chapter!**)

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HW and Attendance

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- The course is very intensive and will move fast. It will be very easy to become confused and to fall behind. So **reading materials in advance** and **regular attendance** should be maintained.

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- The course is very intensive and will move fast. It will be very easy to become confused and to fall behind. So **reading materials in advance** and **regular attendance** should be maintained.
- After each lecture, there will be homework problems based on the reading and lecture material. HW will typically be due **6 days** after instructor hand the set out.

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- All homework in this class will be **written using the mathematical typesetting program LATEX**, submitted via SAKAI.

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- After each lecture, there will be homework problems based on the reading and lecture material. HW will typically be due **6 days** after instructor hand the set out.
- All homework in this class will be **written using the mathematical typesetting program LATEX**, submitted via SAKAI.
- Doing HW is **essential** in order to successfully complete the

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Course learning outcomes

L.O.1	Understanding of predicate logic
	L.O.1.1 – Give an example of predicate logic
	L.O.1.2 – Explain predicate logic for some real problem
L.O.2	Understanding of deterministic modeling using some discrete structures
	L.O.2.1 – Explain a linear programming (mathematical statement)
	L.O.2.2 – State some well-known discrete structures
	L.O.2.3 – Give a counter-example for a given wrong automata
	L.O.2.4 – Construct an automata for a simple problem
L.O.3	Be able to compute solutions, parameters of models based on data
	L.O.3.1 – Compute/Determine optimal/feasible solutions of integer linear programming models, possibly utilizing adequate libraries
	L.O.3.2 – Compute/ optimize solution models based on automata, . . . , possibly utilizing adequate libraries

Assignment Contents

Topics change every year. It may be

- construct correct mathematical reasoning

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Assignment Contents

Topics change every year. It may be

- construct correct mathematical reasoning
- design digital circuits

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Assignment Contents

Topics change every year. It may be

- construct correct mathematical reasoning
- design digital circuits
- verify the correctness of computer programs, software verification

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Assignment Contents

Topics change every year. It may be

- construct correct mathematical reasoning
- design digital circuits
- verify the correctness of computer programs, software verification
- distinguish between valid and invalid mathematical statement

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Assignment Contents

Topics change every year. It may be

- construct correct mathematical reasoning
- design digital circuits
- verify the correctness of computer programs, software verification
- distinguish between valid and invalid mathematical statement
- artificial intelligence

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Sudoku

	1	2	3	4	5	6	7	8	9
1	5	3			7				
2	6			1	9	5			
3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
7		6					2	8	
8				4	1	9			5
9					8			7	9

A Sudoku Grid and Variables



Sudoku

	1	2	3	4	5	6	7	8	9
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3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
7		6					2	8	
8				4	1	9			5
9					8			7	9

Variables

$$V = \{X_{ijk} \mid 1 \leq i, j, k \leq 9\}$$

- X_{ijk} true iff cell at row i column j equals k .

A Sudoku Grid and Variables



Sudoku

	1	2	3	4	5	6	7	8	9
1	5	3			7				
2	6			1	9	5			
3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
7		6					2	8	
8				4	1	9			5
9					8			7	9

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$$V = \{X_{ijk} \mid 1 \leq i, j, k \leq 9\}$$

- X_{ijk} true iff cell at row i column j equals k .
- $|V| = 9^3 = 729$

A Sudoku Grid and Variables



Sudoku

	1	2	3	4	5	6	7	8	9
1	5	3			7				
2	6			1	9	5			
3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
7		6					2	8	
8				4	1	9			5
9					8			7	9

Variables

$$V = \{X_{ijk} \mid 1 \leq i, j, k \leq 9\}$$

- X_{ijk} true iff cell at row i column j equals k .
- $|V| = 9^3 = 729$
- X_{726} is true

A Sudoku Grid and Variables



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1	5	3			7				
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7		6					2	8	
8				4	1	9			5
9					8			7	9

Variables

$$V = \{X_{ijk} \mid 1 \leq i, j, k \leq 9\}$$

- X_{ijk} true iff cell at row i column j equals k .
- $|V| = 9^3 = 729$
- X_{726} is true
- X_{72k} is false for $k \neq 6$

Constraining exactly one variable to be true

Variables = $\{p, q, r, s\}$

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Constraining exactly one variable to be true

Variables = $\{p, q, r, s\}$

- At least one is true:



Constraining exactly one variable to be true

Variables = $\{p, q, r, s\}$

- At least one is true:

$$\alpha = p \vee q \vee r \vee s$$



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Constraining exactly one variable to be true

Variables = $\{p, q, r, s\}$

- At least one is true:

$$\alpha = p \vee q \vee r \vee s$$

- No more than one is true:

$$\begin{aligned}\beta = & (\bar{p} \vee \bar{q}) \wedge (\bar{p} \vee \bar{r}) \wedge (\bar{p} \vee \bar{s}) \wedge \\ & (\bar{q} \vee \bar{r}) \wedge (\bar{q} \vee \bar{s}) \wedge (\bar{r} \vee \bar{s})\end{aligned}$$



Constraining exactly one variable to be true

Variables = $\{p, q, r, s\}$

- At least one is true:

$$\alpha = p \vee q \vee r \vee s$$

- No more than one is true:

$$\begin{aligned}\beta = & (\bar{p} \vee \bar{q}) \wedge (\bar{p} \vee \bar{r}) \wedge (\bar{p} \vee \bar{s}) \wedge \\ & (\bar{q} \vee \bar{r}) \wedge (\bar{q} \vee \bar{s}) \wedge (\bar{r} \vee \bar{s})\end{aligned}$$

- Exactly one is true

$$\psi = \alpha \wedge \beta$$



Sudoku row 2 contains exactly one 8

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Sudoku row 2 contains exactly one 8

- Row 2 must contain at least one 8

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Sudoku row 2 contains exactly one 8

- Row 2 must contain at least one 8

$$\alpha_{2,8} = \bigvee_{1 \leq j \leq 9} X_{2j8}$$



Sudoku row 2 contains exactly one 8

- Row 2 must contain at least one 8

$$\alpha_{2,8} = \bigvee_{1 \leq j \leq 9} X_{2j8}$$

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Sudoku row 2 contains exactly one 8

- Row 2 must contain at least one 8

$$\alpha_{2,8} = \bigvee_{1 \leq j \leq 9} X_{2j8}$$

- Row 2 has at most one 8

$$\beta_{2,8} = \bigwedge_{\substack{1 \leq j, m \leq 9 \\ j \neq m}} (\bar{X}_{2j8} \vee \bar{X}_{2m8})$$



Sudoku row 2 contains exactly one 8

- Row 2 must contain at least one 8

$$\alpha_{2,8} = \bigvee_{1 \leq j \leq 9} X_{2j8}$$

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- Row 2 has exactly one 8

$$\psi_{2,8} = \alpha_{2,8} \wedge \beta_{2,8}$$



Sudoku row constraints

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- Row 2 contains all 9 values exactly once

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Sudoku row constraints

- Row 2 contains all 9 values exactly once

$$\gamma_2 = \bigwedge_{1 \leq k \leq 9} \psi_{2,k}$$



Sudoku row constraints

- Row 2 contains all 9 values exactly once

$$\gamma_2 = \bigwedge_{1 \leq k \leq 9} \psi_{2,k}$$

- All 9 rows contain all 9 values exactly once

$$R = \bigwedge_{1 \leq i \leq 9} \gamma_i =$$



Sudoku row constraints

- Row 2 contains all 9 values exactly once

$$\gamma_2 = \bigwedge_{1 \leq k \leq 9} \psi_{2,k}$$

- All 9 rows contain all 9 values exactly once

$$\begin{aligned} R &= \bigwedge_{1 \leq i \leq 9} \gamma_i = \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} \psi_{i,k} \\ &= \end{aligned}$$



Sudoku row constraints

- Row 2 contains all 9 values exactly once

$$\gamma_2 = \bigwedge_{1 \leq k \leq 9} \psi_{2,k}$$

- All 9 rows contain all 9 values exactly once

$$\begin{aligned} R &= \bigwedge_{1 \leq i \leq 9} \gamma_i = \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} \psi_{i,k} \\ &= \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} (\alpha_{i,k} \wedge \beta_{i,k}) \\ &= \end{aligned}$$



Sudoku row constraints

- Row 2 contains all 9 values exactly once

$$\gamma_2 = \bigwedge_{1 \leq k \leq 9} \psi_{2,k}$$

- All 9 rows contain all 9 values exactly once

$$\begin{aligned} R &= \bigwedge_{1 \leq i \leq 9} \gamma_i = \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} \psi_{i,k} \\ &= \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} (\alpha_{i,k} \wedge \beta_{i,k}) \\ &= \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} \left[\left(\bigvee_{1 \leq j \leq 9} X_{ijk} \right) \wedge \left(\bigwedge_{\substack{1 \leq j, m \leq 9 \\ j \neq m}} (\bar{X}_{ijk} \vee \bar{X}_{imk}) \right) \right] \end{aligned}$$



Column constraints

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Column constraints

- All 9 columns contain all 9 values exactly once

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Column constraints

- All 9 columns contain all 9 values exactly once

$$C =$$





- All 9 columns contain all 9 values exactly once

$$C = \bigwedge_{1 \leq j \leq 9} \bigwedge_{1 \leq k \leq 9} \left[\left(\bigvee_{1 \leq i \leq 9} X_{ijk} \right) \wedge \left(\bigwedge_{\substack{1 \leq i, m \leq 9 \\ i \neq m}} (\bar{X}_{ijk} \vee \bar{X}_{mjk}) \right) \right]$$

3×3 box constraints

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3×3 box constraints

- 3×3 box containing cell $(4, 7)$ has at least one 5



3×3 box constraints

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$$\xi_{475} =$$



3×3 box constraints

- 3×3 box containing cell $(4, 7)$ has at least one 5

$$\xi_{475} = \bigvee_{\substack{i = 4, 5, 6 \\ j = 7, 8, 9}} X_{ij5}$$



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3×3 box constraints

- 3×3 box containing cell $(4, 7)$ has at least one 5

$$\xi_{475} = \bigvee_{\substack{i=4,5,6 \\ j=7,8,9}} X_{ij5}$$

- 3×3 box containing cell $(4, 7)$ has at most one 5

$$\zeta_{475} = \bigwedge_{\substack{i,m=4,5,6 \\ j,n=7,8,9 \\ i \neq m \vee j \neq n}} (\bar{X}_{ij5} \vee \bar{X}_{mn5})$$



3×3 box constraints, cont. . .

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3×3 box constraints, cont. . .

- 3×3 box containing cell $(4, 7)$ has exactly one 5



3×3 box constraints, cont. . .

- 3×3 box containing cell $(4, 7)$ has exactly one 5

$$\theta_{475} =$$



3×3 box constraints, cont. . .

- 3×3 box containing cell $(4, 7)$ has exactly one 5

$$\theta_{475} = \xi_{475} \wedge \zeta_{475}$$



3×3 box constraints, cont. . .

- 3×3 box containing cell $(4, 7)$ has exactly one 5

$$\theta_{475} = \xi_{475} \wedge \zeta_{475}$$

- All 9 3×3 boxes contains exactly one 5

$$\mu_5 = \bigwedge_{\substack{i=1,4,7 \\ j=1,4,7}} \theta_{ij5}$$



3×3 box constraints, cont. . .

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3×3 box constraints, cont. . .

- All 9 3×3 boxes contain all 9 values



3×3 box constraints, cont. . .

- All 9 3×3 boxes contain all 9 values

$$B =$$



3×3 box constraints, cont. . .

- All 9 3×3 boxes contain all 9 values

$$B = \bigwedge_{1 \leq k \leq 9} \mu_k =$$



3×3 box constraints, cont. . .

- All 9 3×3 boxes contain all 9 values

$$B = \bigwedge_{1 \leq k \leq 9} \mu_k = \bigwedge_{1 \leq k \leq 9} \bigwedge_{\substack{i = 1, 4, 7 \\ j = 1, 4, 7}} \theta_{ijk}$$



Initial predefined values

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Initial predefined values

$$I = X_{115} \wedge X_{123} \wedge \cdots \wedge X_{999}$$

	1	2	3	4	5	6	7	8	9
1	5	3			7				
2	6			1	9	5			
3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
7		6					2	8	
8				4	1	9			5
9					8			7	9



Sudoku Boolean Formula

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Sudoku Boolean Formula

$$\phi = I \wedge R \wedge C \wedge B$$

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Sudoku Boolean Formula

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- Note that ϕ is in CNF, where



Sudoku Boolean Formula

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- Note that ϕ is in CNF, where
 - Conjunctive Normal Form (CNF) if it is the AND of clauses, where a clause is the OR of literals.
 - A literal is a variable or its negation.
 - A clause is an expression formed from a finite collection of literals.
- ϕ can be altered so that it contains exactly 3 literals per clause (can be fed to 3-SAT solver): See Chapter 1b.

Bit strings expression

Consider the finite set of binary strings

$\{(000000), (100000), (110000), (111000), (111100), (111110), (111111), (011111), (001111), (000111), (000011), (000001)\}$

Represent such a set in a propositional formula and simplify that representation?



Bit strings expression

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Solution:

- For each $0 \leq i \leq 5$, b_i is a proposition, which intuitively means that the i -th bit has value 1.



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Solution:

- For each $0 \leq i \leq 5$, b_i is a proposition, which intuitively means that the i -th bit has value 1.
- Obviously, $\neg b_i$ means that the i -th bit does not have value 1, and thus it has value 0.



Bit strings expression

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- A possible (compact) representation of the finite set of binary strings is given by the following formula:



Bit strings expression

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- For each $0 \leq i \leq 5$, b_i is a proposition, which intuitively means that the i -th bit has value 1.
- Obviously, $\neg b_i$ means that the i -th bit does not have value 1, and thus it has value 0.
- A possible (compact) representation of the finite set of binary strings is given by the following formula:

$$\bigvee_{k=0}^5 \left(\left(\bigwedge_{i=0}^k \neg b_i \wedge \bigwedge_{i=k+1}^5 b_i \right) \vee \left(\bigwedge_{i=0}^k b_i \wedge \bigwedge_{i=k+1}^5 \neg b_i \right) \right)$$

