Chapter 2 Questions.

Predicate logic.

Declarative sentences in predicate logic.

Question 1

In this question, assume the following predicate and constant symbols:

W(x,y) : x wrote y
L(x,y) : x is longer than y
N(x) : x is a novel
h : Hardy
a : Austen
j : Jude the Obscure
p : Pride and Predjudice

Given these specifications, which of the predicate logic formulas below represent the sentence, *'Hardy wrote a novel which is longer than any of Austen's'* in predicate logic?

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1. $\forall x\ (W(h,x) \rightarrow L(x,a))$
2. $\forall x\ \exists y\ (L(x,y) \rightarrow W(h,y) \wedge W(a,x))$
3. $\forall x\ \forall y\ (W(h,x) \wedge W(a,y) \rightarrow L(x,y)))$
4. $\exists x\ (N(x) \wedge W(h,x) \wedge \forall y\ (N(y) \wedge W(a,y) \rightarrow L(x,y)))$
5. $\exists x\ \forall y\ (W(h,x) \rightarrow W(a,y) \wedge L(x,y))$

Question 2

In this question, assume the following predicate and constant symbols.

W(x,y) : x wrote y
L(x,y) : x is longer than y
h : Hardy
a : Austen
j : Jude the Obscure
p : Pride and Predjudice

Which of the following represents the sentence, *"Jude the Obscure' is not longer than 'Pride and Predjudice"*, in predicate logic?

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1. $W(h,j) \wedge W(a,p)$
2. $\neg\forall x\ \forall y\ \neg L(x,y)$

3. L(p,j)
4. ¬L(j,p)
5. L(h,a)

Semantics.

Question 3

Working with a unary predicate sysmbol, P, a binary predicate symbol Q and a unary function sysmbol f, which of the following formulas are satisfied in the model M, given by,

A = {a,b,c,d};
P^M = {a,b}
Q^M = {(a,b), (b,b), (c,b)}
f^M (a) = b, f^M (b) = b, f^M (c) = a, and f^M (d) = c

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1. $\forall x\ (P(x) \rightarrow \exists y\ Q(y,x))$
2. $\forall x\ Q(f(x),x)$
3. $\forall x\ (Q(f(x),x) \rightarrow Q(x,x))$
4. $\forall x\ \forall y\ (Q(x,y) \rightarrow P(x))$
5. $\forall x\ \exists y\ (Q(x,y) \vee Q(y,x))$

Question 4

Which of the following sets of sentences is satisfiable? (Recall that a "sentence" is a predicate logic formula that has no free variables.)

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1. $\{ \exists x\ Q(x), \forall x\ (Q(x) \rightarrow R(x)), \forall x\ \neg\ R(x)\}$.
2. $\{ \exists y\ \forall x\ P(x,y), \forall x\ \neg P(x,x)\}$.
3. $\{ \forall x\ \forall y\ (P(x,y) \rightarrow P(y,y)), \forall x\ \neg P(x,x), \exists x\ \exists y\ (P(x,y)\}$.
4. $\{ \forall x\ \exists y\ P(x,y), \forall x\ \neg P(x,x)\}$.
5. $\{ \exists x\ Q(x), \forall x\ \neg Q(x)\}$.

Question 5

Which of the following semantic entailments are valid in predicate logic?

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1. $\forall x\ (P(x) \vee Q(x)) \models \forall x\ P(x) \vee \forall x\ Q(x)$
2. $\forall x\ (P(x) \rightarrow Q(x)) \models \forall x\ P(x) \rightarrow \forall x\ Q(x)$

3. $\forall x\ P(x) \rightarrow \forall x\ Q(x) \models \forall x\ (P(x) \rightarrow Q(x))$
4. $\neg \forall x\ (P(x) \wedge Q(x)) \models \exists x\ \neg P(x) \wedge \exists x\ \neg Q(x)$
5. $\exists x\ P(x) \wedge \exists x\ Q(x) \models \exists x\ (P(x) \wedge Q(x))$

More declarative sentences.

Question 6

In this question assume the specifications

F(x,y) : x is the father of y
M(x,y) : x is the mother of y.

Which of the formulas in predicate logic below express the sentence *'Everybody has a mother'*?

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1. $\forall x\ \forall y\ M(x, y)$
2. $\exists x\ \forall y\ M(x, y)$
3. $\exists y\ \forall x\ M(x, y)$
4. $\forall x\ \exists y\ M(x, y)$
5. $\forall y\ \exists x\ M(x, y)$

Question 7

In this question assume only the specifications

F(x,y) : x is the father of y
M(x,y) : x is the mother of y.

Which of the formulas in predicate logic below express the sentence *'Anyone who is a father is not a maternal grandmother'*?

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1. $\forall x\ (\exists y\ F(x, y) \rightarrow \neg \exists y\ \exists z\ (M(x, y) \wedge M(y, z)))$
2. $\forall x\ (\exists y\ F(x, y) \rightarrow \neg \exists y\ M(x, M(y)))$
3. $\forall x\ (\exists y\ F(x, y) \rightarrow \neg \exists y\ G(x, y))$
4. $\exists x\ \exists y\ F(x, y) \rightarrow \neg \exists x\ \exists y\ M(x, y)$
5. $\exists x\ (F(x) \rightarrow \neg M(M(x)))$

Question 8

For this question and the following one, assume the specifications below:
The predicate symbols

S(x,y) : x is y's sister
B(x,y) : x is y's brother
H(x,y) : x is y's husband and
O(x,y) : x is older than y

and the constant symbols

j : John,
c : Carl, and
m : Monique.

Which of the following formulas express the sentence *'Carl is Moniques brother-in-law'*?

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1. $B(c,m) \wedge H(c,m)$.
2. $B(c,m) \vee H(c,m)$.
3. $\forall x \forall y (S(x,m) \wedge H(c,y) \rightarrow x = y)$.
4. $\exists x ((S(x,m) \wedge H(c,x)) \vee (H(x,m) \wedge B(c,x)))$.
5. None of the above.

Question 9

Assuming the same specifications as for Question 8, which of the following sentences are expressed by the predicate logic formula
$\forall x \forall y (S(x,j) \wedge S(y,j) \rightarrow O(x,y))$?.

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1. John has an older sister.
2. All of John's sisters are older than him.
3. John is older than his sisters.
4. One of John's sisters is older than another.
5. None of the above.

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