UNIVERSITY OF TECHNOLOGY FACULTY OF CSE	FINAL EXAM		Semester/Academic year	1	2020-2021
			Date		28-01-2021
	Course title	Mathematical Modeling			
	Course ID	CO2011			
	Duration	80 mins	Question sheet code		2811

Notes:
- **One single sheet (both sides) of A4 paper of hand-written notes is allowed.**
- Stu. ID and Stu. Fullname fields at the bottom of the question sheet **must be filled in**.
- Submit the answer sheet together with the question sheet when finishing the test.
- Mark the correct answers in the answer sheet.
- The test consists of **25** multi-choice questions, each of which has the score of **0.4**.

In this final examiniation, for all questions concerning dynamical systems, we consider the following initial-value problem

$$\begin{cases} \dot{x}(t) = f(x), & t > t_0 \geq 0, \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where x is a real-value function dependent on t and f is a real-value function dependent on x.

Use the following information to answer **the questions 1–5**. The following initial-value problem is a model of vapour pressure of water in the air of a greenhouse

$$\begin{cases} \dot{x}(t) = \dfrac{U\phi}{Ah} + \dfrac{k(C-x)}{h}, & t > t_0 \geq 0, \\ x(t_0) = x_0. \end{cases} \tag{2}$$

a) $x(t)$ (pa) represents the vapour pressure in the greenhouse air at time t, $x_0 = x(t_0)$ is a constant, and $\dot{x}$, the time derivative of x, is the rate change of vapour pressure.

b) The constants $C = 2986$ (pa), $h = 1.94$ (kg·m·J^{-1}), $A = 100$ (m^2), $U = 85\%$ (dimensionless), and $\phi = 1.39$ (kg/s) are respectively the saturated vapour pressure at the canopy temperature, the capacity of the greenhouse air to store water vapour, the area of the greenhouse floor, the control value of the fogging system, and the capacity of the fogging system.

c) k (kg·m^{-2}·pa^{-1}·s^{-1}), the vapour exchange coefficient dependent on x (pa), satisfies

$$k = \frac{1}{8x^2 - 47713x + 79259056}. \tag{3}$$

Noting that every coefficient on the right-hand side of (3) has its own unit, which is omitted here for simplicity.

d) For pressure, 1 (bar) = 100000 (pa).

Question 1. (L.O.2.1)
Which of the following units is equivalent to pa?

Ⓐ J/m^3. Ⓑ kg/J. Ⓒ J/m. Ⓓ bar/s.

Question 2. (L.O.2.1)
At a given time, what is the value of vapour exchange coefficient k (kg·m^{-2}·bar^{-1}·s^{-1}) if the vapour pressure of water in the greenhouse air is 0.01 (bar)?

Ⓐ 0.00516 Ⓑ 0.00253 Ⓒ 0.00158 Ⓓ 0.00216

Question 3. (L.O.2.1)
What is the rate change (pa/s) of vapour pressure in the greenhouse air at a given time if the vapour pressure in the greenhouse air at that time is 2500 (pa)?

(A) 0.00725 (B) 0.00231 (C) 0.00611 (D) 0.00815

Question 4. (L.O.2.4)
Asssume $x_0 = 2500$ (pa), what is the approximate of x (pa) in the next 10 minutes using forward Euler's method with two time steps?

(A) 2503.67 (B) 2503.22 (C) 2503.10 (D) 2503.91

Question 5. (L.O.2.4)
The 1/2-rule for solving problem (1) is given as follows.

$$\begin{aligned} \text{Step 1: } x_{n+\frac{1}{2}} &= x_n + \frac{1}{2} f(x_n)\Delta t. \\ \text{Step 2: } x_{n+1} &= x_n + f\left(x_{n+\frac{1}{2}}\right)\Delta t. \end{aligned}$$

Here, $\Delta t > 0$ is the time step and x_n is an approximate of $x(t_n)$ for $n = 0, 1, 2, \ldots$

Consider problem (2). Asssume $x_0 = 2500$ (pa), what is the approximate of x (pa) in the next 10 minutes using the 1/2-rule with one time step only.

(A) 2503.22 (B) 2503.10 (C) 2503.91 (D) 2503.67

Use the following information to answer **the questions 6–9**. The 3/2-rule for solving problem (1) is given by

$$\begin{aligned} \text{Step 1: } x_{n+\frac{3}{2}} &= x_n + \frac{3}{2} f(x_n)\Delta t. \\ \text{Step 2: } x_{n+1} &= x_n + \left[\frac{2}{3} f(x_n) + \frac{1}{3} f\left(x_{n+\frac{3}{2}}\right)\right]\Delta t. \end{aligned}$$

Here $\Delta t > 0$ is the time step and x_n is an approximate of $x(t_n)$ for $n = 0, 1, 2, \ldots$

Question 6. (L.O.2.1)
The 1/2-rule is

(A) an implicit method. (B) an explicit method.

Question 7. (L.O.2.1)
For $z \in \mathbb{C}$, the stability function of 3/2-rule is

(A) $\Phi(z) = 1 + z^3$.
(B) $\Phi(z) = \dfrac{2-z}{2+z}$ for $z \neq -2$.
(C) $\Phi(z) = 1 + z + \dfrac{z^2}{2}$.
(D) $\Phi(z) = \dfrac{1}{1-z}$ for $z \neq 1$.

Question 8. (L.O.2.1)
Is 3/2-rule an A-stable method?

(A) No. (B) Yes.

Question 9. (L.O.2.4)
Consider the problem (1) where $f(x) = -3x + 2$ and $x_0 = 1$. What is the approximate value of x_2 using the 3/2-rule with $\Delta t = 0.15$?

(A) 0.91 (B) 0.95 (C) 1.22 (D) 1.24

Question 10. (L.O.2.4)
For given time step $\Delta t > 0$, the local truncation error of the backward Euler method is proportional to

(A) Δt^4. (B) Δt^2. (C) Δt^5. (D) Δt^3.

Question 11. (L.O.2.2)

Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least?

(A) 2^N. (B) N^2. (C) $N!$. (D) N.

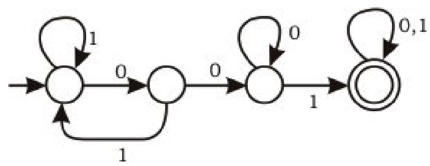
Question 12. (L.O.2.2)

Let S and T be language over $\{a, b\}$ represented by the regular expressions $(a+b^*)^*$ and $(a+b)^*$, respectively. Which of the following is true?

(A) $S \subset T$. (B) $S = T$. (C) $T \subset S$. (D) $S \cap T = \emptyset$.

Question 13. (L.O.3.2)

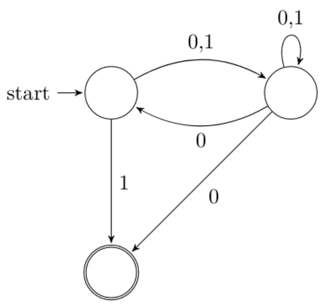
Consider the following deterministic finite state automaton M. Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is



(A) 8. (B) 5. (C) 7. (D) 10.

Question 14. (L.O.2.2)

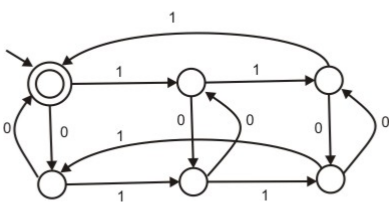
Consider the NFA M shown below. Let the language accepted by M be L. Let L_1 be the language accepted by the NFA M_1, obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statements is true?



(A) $L_1 = \{0, 1\}^* \setminus L$. (B) $L_1 \subseteq L$. (C) $L_1 = L$. (D) $L_1 = \{0, 1\}^*$.

Question 15. (L.O.3.2)

The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively.



(A) divisible by 3 and 2.
(B) old and even.
(C) even and odd.
(D) divisible by 2 and 3.

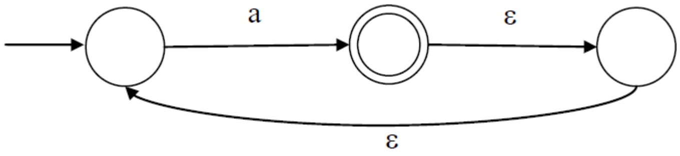
Question 16. (L.O.2.2)

Consider the languages $L_1 = \emptyset$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$?

(A) $\emptyset$. (B) $\{\epsilon\}$. (C) $\{a^*\}$. (D) $\{\epsilon, a\}$.

Question 17. (L.O.2.2)

What is the complement of the language accepted by the NFA shown below?

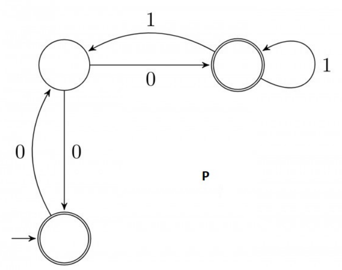


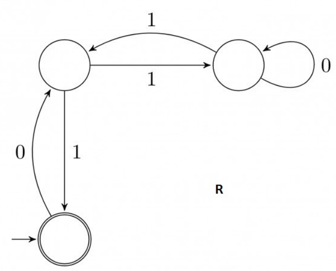
Ⓐ $\emptyset$. Ⓑ $\{a, \epsilon\}$. Ⓒ $\{\epsilon\}$. Ⓓ $\{a^*\}$.

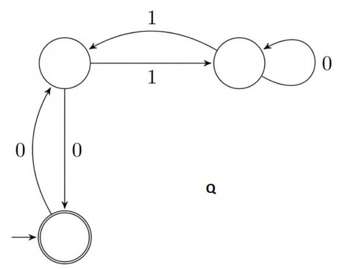
Question 18. (L.O.3.2)

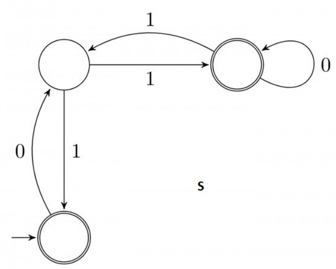
Match the following NFAs with the regular expressions they correspond to:

1. $\epsilon + 0\,(01^*1 + 00)^*\,01^*$
2. $\epsilon + 0\,(10^*1 + 00)^*\,0$
3. $\epsilon + 0\,(10^*1 + 10)^*\,1$
4. $\epsilon + 0\,(10^*1 + 10)^*\,10^*$









Ⓐ P - 2, Q - 1, R - 3, S - 4. Ⓑ P - 1, Q - 3, R - 2, S - 4.
Ⓒ P - 3, Q - 2, R - 1, S - 4. Ⓓ P - 1, Q - 2, R - 3, S - 4.

Question 19. (L.O.2.2)

Reduce the following expression $\epsilon + 1^*(011)^*(1^*(011)^*)^*$?

Ⓐ $(1 + 011)^*$. Ⓑ $(1^*(011)^*)$. Ⓒ $(1 + (011)^*)^*$. Ⓓ $(1011)^*$.

Question 20. (L.O.2.2)

Which one of the following language is regular?

Ⓐ $\{a^n b^n \mid n \geq 0\}$.
Ⓑ $\{w \mid w$ has $(3k + 1)$ characters b's, for some $k \in \mathbb{N}$ with $\Sigma = \{a, b\}\}$.
Ⓒ $\{a^n \mid n$ is prime $\}$.
Ⓓ $\{ww \mid w \in \Sigma^*$ with $\Sigma = \{0, 1\}\}$.

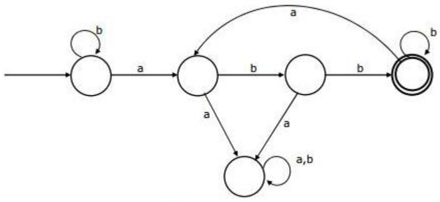
Question 21. (L.O.2.3)

If s is a string over $(0 + 1)^*$ then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is NOT regular?

Ⓐ $L = \{s \in (0 + 1)^* \mid n_0(s)$ is a 3-digit prime $\}$.
Ⓑ $L = \{s \in (0 + 1)^* \mid$ for every prefix s' of $s \mid n_0(s') - n_1(s') \mid \leq 2\}$.
Ⓒ $L = \{s \in (0 + 1)^* \mid |n_0(s) - n_1(s)| \leq 4\}$.
Ⓓ $L = \{s \in (0 + 1)^* \mid n_0(s) \mod 7 = n_1(s) \mod 5 = 0\}$.

Question 22. (L.O.3.2)

Consider the machine M given below. The language recognized by M is:



(A) $\{w \in \{a,b\}^* |$ every $a \in w$ is followed by exactly two b's$\}$.
(B) $\{w \in \{a,b\}^* \mid w$ contains the substring "abb"$\}$.
(C) $\{w \in \{a,b\}^* \mid w$ does not contain "aa" as a substring$\}$.
(D) $\{w \in \{a,b\}^* |$ every $a \in w$ is followed by at least two b's$\}$.

Question 23. (L.O.2.2)

Given two following languages

$$L_1 = \{x \in \Sigma^* | x \text{ contains an even number of bits } 0\},$$

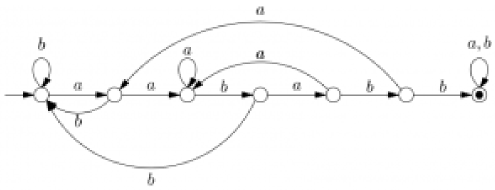
$$L_2 = \{x \in \Sigma^* | x \text{ contains an odd number of bits } 1\}.$$

Then the number of final states in $L_1 \cup L_2$ is

(A) 4. (B) 2. (C) 5. (D) 3.

Question 24. (L.O.2.2)

Consider the following Deterministic Finite Automata. Which of the following is true?



(A) It only accepts strings with substring as "aababb".
(B) It only accepts strings with prefix as "aababb".
(C) It only accepts strings with suffix as "aababb".
(D) None of the others.

Question 25. (L.O.2.2)

Which of the following transformations on automata use eliminations of states?

(A) Convert DFA to NFA.
(B) Convert NFA to DFA.
(C) Convert DFA to regular expression.
(D) All three conversions in three other choices need to use eliminations of states.