Chapter 0
Course Introduction

Mathematical Modeling on January 12, 2017

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai
Faculty of Computer Science and Engineering
University of Technology - VNUHCM

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai

Contents

Aims

- The first part of this course introduces CSE students to the basic concepts of logic (e.g., theories, models, logical consequence, and proof).
- In the second part, students will be learned mathematical modeling through ILP, automata and formal language, Petri net, dynamical systems.
- This is the mathematical foundations for many CS areas, e.g., algorithm analysis & design, database, artificial intelligence, etc.
- Applications of logic in CSE will be highlighted.

Contents

5 chapters, 45 hours of class lectures, HW & exercices.

- Chapter 1. **Propositional Logic Review** and **(Advanced) Predicate Logic**: The need for a richer language; Predicate logic as a formal language; Proof theory of predicate logic; Semantics of predicate logic; Undecidability of predicate logic; Expressiveness of predicate logic.
- Chapter 2. **Mathematical Programming**: Constraints, objectives in ILP.
- Chapter 3. **Automata & Formal Language**: DFA, NFA, Expression, Context.
- Chapter 4. **Petri net**.
- Chapter 5. **Dynamical systems**.

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai

Grading

- 01 Assignment (Project): 20%
- Midterm (MCQ and written; 60 minutes; tentatively after 2 first chapters): 30%
- Final (MCQs + Short Answer Questions, 120 minutes): 50% (**cover all 5 chapter!**)

HW and Attendance

- The course is very intensive and will move fast. It will be very easy to become confused and to fall behind. So **reading materials in advance** and **regular attendance** should be maintained.
- After each lecture, there will be homework problems based on the reading and lecture material. HW will typically be due **6 days** after instructor hand the set out.
- All homework in this class will be **written using the mathematical typesetting program LATEX**, submitted via SAKAI.
- Doing HW is **essential** in order to successfully complete the

Nguyen An Khuong, Le Hong Trang, Huynh Tuong Nguyen, Tran Van Hoai

Course outcomes

	Course learning outcomes
L.O.1	Understanding of predicate logic
	L.O.1.1 – Give an example of predicate logic L.O.1.2 – Explain predicate logic for some real problem
L.O.2	Understanding of deterministic modeling using some discrete structures
	L.O.2.1 – Explain a linear programming (mathematical statement) L.O.2.2 – State some well-known discrete structures L.O.2.3 – Give a counter-example for a given wrong automata L.O.2.4 – Construct an automata for a simple problem
L.O.3	Be able to compute solutions, parameters of models based on data
	L.O.3.1 – Compute/Determine optimal/feasible solutions of integer linear programming models, possibly utilizing adequate libraries L.O.3.2 – Compute/ optimize solution models based on automata, . . . , possibly utilizing adequate libraries

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai

Assignment Contents

Topics change every year. It may be

- construct correct mathematical reasoning
- design digital circuits
- verify the correctness of computer programs, software verification
- distinguish between valid and invalid mathematical statement
- artificial intelligence

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai

Required Texts/Materials

Electronic copies of [2-6] are available on the WWW, or upon request to instructors.

1. Handouts (Obtained via SAKAI after classes.)
2. Michael R.A. Huth and Mark D. Ryan.*Logic in Computer Science* (2nd Ed.), Cambridge University Press, 2004. (**Chapters 1, 2**)
3. Michael R.A. Huth and Mark D. Ryan. *Logic in Computer Science: Solutions to designated exercises* (2nd Ed.), Cambridge University Press, 2004. (**Chapters 1, 2**)
4. F.R. Giordano, W.P. Fox & S.B. Horton, A First Course in Mathematical Modeling, 5th ed., Cengage, 2014.
5. K. M. Bliss K. R. Fowler B. J. Galluzzo, Math Modeling: getting started & getting solutions. Society for Industrial and Applied Mathematics (SIAM) Handbook, 2014.
6. Matousek et al. Understanding and using linear programming, Springer, 2007.
7. Peter Linz. *An Introduction to Formal Languages and Automata* (3rd Ed.) Jones and Bartlett, 2001. (**Chapters 1-6**)
8. Peter Linz. *An Introduction to Formal Languages and Automata: Instructors' Manual* (3rd Ed.) Jones and Bartlett, 2001. (**Chapters 1-6**)

Instructors

1. Nguyễn An Khương, `nakhuong@hcmut.edu.vn`
2. Lê Hồng Trang, `lhtrang@hcmut.edu.vn`
3. Huỳnh Tường Nguyên, `htnguyen@hcmut.edu.vn`
4. Trần Văn Hoài, `hoai@hcmut.edu.vn`
5. Nguyễn Văn Minh Mẫn, `mannguyen@hcmut.edu.vn`
6. Nguyễn Đức Dũng, `nddung@hcmut.edu.vn`

Tentative Schedule

Notice that the 1st week stars on Monday, January 9, 2017.

Lectures	Topic	Lecturer
01	Ch0. Intro + Demo	NAKhuong
02 - 03	Ch1. Logic	NAKhuong
04 - 06	Ch2. ILP	LHTrang/TVHoai
07	Assignment instruction	NAKhuong
08	Review and Midterm	NAKhuong
09 - 12	Ch3. Automata and Ch4. Petri Net	HTNguyen
13-14	Ch5. Dynamical systems	NAKhuong/NVMMan/NDDung
15	Assignement evaluation	NAKhuong

A Sudoku Grid and Variables

Sudoku

	1	2	3	4	5	6	7	8	9
1	5	3			7				
2	6			1	9	5			
3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
7		6					2	8	
8				4	1	9			5
9					8			7	9

Variables

$V = \{X_{ijk} \mid 1 \leq i, j, k \leq 9\}$

- X_{ijk} true iff cell at row i column j equals k.
- $|V| = 9^3 = 729$
- X_{726} is true
- X_{72k} is false for $k \neq 6$

Constraining exactly one variable to be true

Variables $= \{p, q, r, s\}$

- At least one is true:

$$\alpha = p \vee q \vee r \vee s$$

- No more than one is true:

$$\begin{aligned} \beta \quad = \quad & (\bar{p} \vee \bar{q}) \wedge (\bar{p} \vee \bar{r}) \wedge (\bar{p} \vee \bar{s}) \wedge \\ & (\bar{q} \vee \bar{r}) \wedge (\bar{q} \vee \bar{s}) \wedge (\bar{r} \vee \bar{s}) \end{aligned}$$

- Exactly one is true

$$\psi = \alpha \wedge \beta$$

Sudoku row 2 contains exactly one 8

- Row 2 must contain at least one 8

$$\alpha_{2,8} = \bigvee_{1 \le j \le 9} X_{2j8}$$

- Row 2 has at most one 8

$$\beta_{2,8} = \bigwedge_{\substack{1 \le j,m \le 9 \\ j \ne m}} \left(\bar{X}_{2j8} \vee \bar{X}_{2m8}\right)$$

- Row 2 has exactly one 8

$$\psi_{2,8} = \alpha_{2,8} \wedge \beta_{2,8}$$

Sudoku row constraints

- Row 2 contains all 9 values exactly once

$$\gamma_2 = \bigwedge_{1 \leq k \leq 9} \psi_{2,k}$$

- All 9 rows contain all 9 values exactly once

$$\begin{aligned} R &= \bigwedge_{1 \leq i \leq 9} \gamma_i = \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} \psi_{i,k} \\ &= \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} (\alpha_{i,k} \wedge \beta_{i,k}) \\ &= \bigwedge_{1 \leq i \leq 9} \bigwedge_{1 \leq k \leq 9} \left[\left(\bigvee_{1 \leq j \leq 9} X_{ijk} \right) \wedge \left(\bigwedge_{\substack{1 \leq j,m \leq 9 \\ j \neq m}} (\bar{X}_{ijk} \vee \bar{X}_{imk}) \right) \right] \end{aligned}$$

Column constraints

- All 9 columns contain all 9 values exactly once

$$C = \bigwedge_{1\leq j\leq 9}\bigwedge_{1\leq k\leq 9}\left[\left(\bigvee_{1\leq i\leq 9} X_{ijk}\right)\wedge\left(\bigwedge_{\substack{1\leq i,m\leq 9\\ i\neq m}}(\bar{X}_{ijk}\vee\bar{X}_{mjk})\right)\right]$$

3×3 box constraints

- 3×3 box containing cell $(4, 7)$ has at least one 5

$$\xi_{475} = \bigvee_{\substack{i=4,5,6 \\ j=7,8,9}} X_{ij5}$$

- 3×3 box containing cell $(4, 7)$ has at most one 5

$$\zeta_{475} = \bigwedge_{\substack{i,m=4,5,6 \\ j,n=7,8,9 \\ i \neq m \vee j \neq n}} (\bar{X}_{ij5} \vee \bar{X}_{mn5})$$

3×3 box constraints, cont...

- 3×3 box containing cell $(4, 7)$ has exactly one 5

$$\theta_{475} = \xi_{475} \wedge \zeta_{475}$$

- All 9 3×3 boxes contains exactly one 5

$$\mu_5 = \bigwedge_{\substack{i=1,4,7 \\ j=1,4,7}} \theta_{ij5}$$

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai

3×3 box constraints, cont...

- All 9 3×3 boxes contain all 9 values

$$B = \bigwedge_{1 \le k \le 9} \mu_k = \bigwedge_{1 \le k \le 9} \bigwedge_{\substack{i = 1,4,7 \\ j = 1,4,7}} \theta_{ijk}$$

Initial predefined values

$$I = X_{115} \wedge X_{123} \wedge \cdots \wedge X_{999}$$

	1	2	3	4	5	6	7	8	9
1	5	3			7				
2	6			1	9	5			
3		9	8					6	
4	8				6				3
5	4			8		3			1
6	7				2				6
7		6					2	8	
8				4	1	9			5
9					8			7	9

Contents

Sudoku Boolean Formula

$$\phi = I \wedge R \wedge C \wedge B$$

- Note that ϕ is in CNF, where
  - Conjunctive Normal Form (CNF) if it is the AND of clauses, where a clause is the OR of literals.
  - A literal is a variable or its negation.
  - A clause is an expression formed from a finite collection of literals.
- ϕ can be altered so that it contains exactly 3 literals per clause (can be fed to 3-SAT solver): See Chapter 1b.

Nguyen An Khuong,
Le Hong Trang,
Huynh Tuong Nguyen,
Tran Van Hoai

Bit strings expression

Consider the finite set of binary strings
$\{(000000), (100000), (110000), (111000), (111100), (111110), (111111), (011111), (001111), (000111), (000011), (000001)\}$
Represent such a set in a propositional formula and simplify that representation?

Solution:

- For each $0 \leq i \leq 5$, b_i is a proposition, which intuitively means that the i-th bit has value 1.
- Obviously, $\neg b_i$ means that the i-th bit does not have value 1, and thus it has value 0.
- A possible (compact) representation of the finite set of binary strings is given by the following formula:

$$\bigvee_{i=0}^{k} \left(\left(\bigwedge_{i=0}^{k} \neg b_i \wedge \bigwedge_{i=k+1}^{5} b_i \right) \bigvee \left(\bigwedge_{i=0}^{k} b_i \wedge \bigwedge_{i=k+1}^{k} \neg b_i \right) \right)$$