

Chapter 1 Questions.

Propositional Logic.

Declarative sentences.

Question 1

Which of the following propositional formulas represents the sentence, '*He will come on the 8:15 or the 9:15 train; if the former, he will have time to visit us*', where

p means '*He will come on the 8:15*'

q means '*He will come on the 9:15*'

r means '*He will have time to visit us*'

1. $\neg p \rightarrow q \vee r$
 2. $p \vee q \rightarrow r$
 3. $(p \rightarrow q) \wedge (p \vee r)$
 4. $p \vee \neg q \rightarrow r$
 5. $(p \vee q) \wedge (p \rightarrow r)$
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Question 2

Which of the following sentences has the logical form $(p \wedge q) \rightarrow r$

1. If you miss the party, then Jane, who is already upset, will be angry with you.
 2. Handel is great, and the same goes for Vivaldi.
 3. If inflation is up and an election is approaching, then public borrowing goes up.
 4. He will come on the 8:15 or the 9:15 train; if the former, he will have time to visit us.
 5. Heavy traffic and rain have put him in a bad mood.
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Semantics.

Question 3

Which of the following propositional formulas is satisfied by the valuation which assigns T to P, and F to q and r.

1. $(\neg p \vee \neg r) \rightarrow q$
2. $\neg(\neg r \rightarrow (p \wedge q))$
3. $\neg r \rightarrow (p \wedge q)$
4. $\neg p \vee q \vee r$
5. $(p \leftrightarrow q) \wedge \neg r$

Recall that $p \leftrightarrow q$ is an abbreviation for $(p \rightarrow q) \wedge (q \rightarrow p)$.

Question 4

Which of the following propositional formulas is a tautology?

Recall that a tautology is a formula that evaluates to T for all possible assignments of truth values.

1. $(\neg p \vee r) \rightarrow (p \vee \neg r)$
2. $\neg(p \rightarrow (p \wedge q))$
3. $r \rightarrow (p \wedge \neg r)$
4. $\neg p \wedge T$
5. $(p \leftrightarrow q) \vee (p \leftrightarrow \neg q)$

Recall that $p \leftrightarrow q$ is an abbreviation for $(p \rightarrow q) \wedge (q \rightarrow p)$.

Question 5

Which of the following entailments is valid?

1. $(p \vee q) \rightarrow r, r \models \neg p$
2. $p, \neg p \leftrightarrow q \models \neg q$
3. $(p \wedge q) \rightarrow r, \neg r \models p \vee q$
4. $\neg q \models \neg p$
5. $p \wedge q \models p \leftrightarrow \neg q$

Question 6

Which of the columns below represents the truth table for the formula,
 $(p \rightarrow q) \vee \neg(r \wedge \neg q)$?

p	q	r	1	2	3	4	5
T	T	T	F	T	T	T	T
T	T	F	T	T	T	T	F
T	F	T	F	F	F	T	T
T	F	F	T	T	T	T	T

F T T F T T T F
F T F T T T T F
F F T F F T T T
F F F T T T T T

More declarative sentences.

Question 7

Which of the following formulas represents the sentence *'If Smith has installed central heating, then he has sold his car or he has not paid his mortgage'*, where;

p means *'Smith has installed central heating'*

q means *'Smith has sold his car'*

r means *'Smith has paid his mortgage'*.

1. $\neg p \rightarrow q$
 2. $p \rightarrow q \vee r$
 3. $(p \rightarrow q) \vee \neg r$
 4. $p \rightarrow q \vee \neg r$
 5. $p \vee q \rightarrow \neg r$
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Question 8

Which of the following formulas represents the sentence, *'Share prices will go up, and if interest rates go up too, there will be a recession'*, where;

p means *'share prices will go up'*

q means *'interest rates will go up'*

r means *'there will be a recession'*.

1. $p \wedge q \rightarrow r$
 2. $p \wedge (q \rightarrow r)$
 3. $p \rightarrow q \wedge r$
 4. $(p \rightarrow q) \vee r$
 5. $p \wedge q \wedge r$
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Question 9

Which of the following sentences could be written as $p \vee (q \wedge r)$, for suitable p, q, and r ?

1. If inflation goes up and an election is approaching, then public borrowing goes up.
2. You can go swimming, or use the sauna and the shower.
3. You need to buy bread, cheese, and wine.
4. Heavy traffic and rain put him in a bad mood.
5. Plants need water and food, but not talking to.

Syntax.

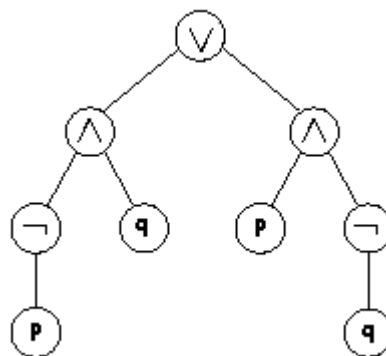
Question 10

According to the standard convention about binding priorities, the formula, $\neg p \rightarrow \neg q \wedge r$, is implicitly one of the following. Which?

1. $((\neg p) \rightarrow (\neg p)) \wedge r$
2. $(\neg p) \rightarrow \neg(q \wedge r)$
3. $\neg(p \rightarrow \neg(q \wedge r))$
4. $(\neg p) \rightarrow ((\neg q) \wedge r)$
5. $\neg(p \rightarrow ((\neg q) \wedge r))$

Question 11

Which of the following formulas has the parse tree:



1. $(\neg p \wedge q) \vee (p \wedge \neg q)$.
2. $(p \wedge \neg q) \vee (\neg p \wedge q)$.
3. $(\neg p \vee q) \wedge (p \vee \neg q)$.
4. $(p \vee \neg q) \wedge (\neg p \vee q)$.
5. None of the above.

Natural deduction.

Question 12

Which of the following is a **correct proof** of,
 $p \rightarrow q \wedge r \vdash p \rightarrow q$?

1. 1 $p \rightarrow q \wedge r$ premise
 2 $p \rightarrow q$ $\wedge e$ 1

2. 1 $p \rightarrow q \wedge r$ premise
 2 p $\rightarrow e$ 1
 3 q $\wedge e$ 1
 4 $p \rightarrow q$ $\rightarrow i$ 2-3

3. 1 $p \rightarrow q \wedge r$ premise
 2 p assumption
 3 $q \wedge r$ $\rightarrow e$ 1, 2
 4 q $\wedge e$ 3
 5 $p \rightarrow q$ $\rightarrow i$ 2-4

4. 1 $p \rightarrow q \wedge r$ premise
 2 p assumption
 3 $q \wedge r$ assumption
 4 q $\wedge e$ 3
 5 $q \wedge r \rightarrow q$ $\rightarrow i$ 3-4
 6 q $\rightarrow e$ 5, 3
 7 $p \rightarrow q$ $\rightarrow i$ 2-7

5. 1 $p \rightarrow q \wedge r$ premise
 2 $q \wedge r$ assumption
 3 q $\wedge e$ 2
 4 p $\rightarrow e$ 1, 2
 5 $p \rightarrow q$ $\rightarrow i$ 2-4

Question 13

Which of the sequents below are **valid**, i.e. have a natural deduction proof?
 (You do not need to state a proof.)

1. $p \rightarrow q \wedge r, q \vdash r$
2. $p \rightarrow q \wedge r, \neg q \vdash p$
3. $p \rightarrow q \wedge r, \neg q, \neg r \vdash \neg p$
4. $p \rightarrow q \vdash q \rightarrow p$
5. $p \rightarrow q \vdash p \wedge q$

Question 14

Which of the following statements about Natural Deduction is **true** ?

1. $p \vee \neg p$ cannot be proved in natural deduction.
2. The rules *Modus Tollens* (MT) and *Reductio ad absurdum* (RAA) cannot be derived from the rules $\wedge i$ and $\wedge e$ alone.
3. Boxes are not used to delineate the scope of assumptions.
4. Contradictions don't play an important role in natural deduction.
5. None of the above.

Question 15

Which of the following are valid sequents?

1. $p \wedge \neg q \rightarrow r, \neg p, \neg r \vdash q$.
2. $p \wedge \neg q \rightarrow r, p, q \vdash \neg r$.
3. $p \wedge \neg q \rightarrow r, q, \neg r \vdash p$.
4. $p \wedge \neg q \rightarrow r, q, \neg r \vdash \neg p$.
5. None of the above.

Question 16

The following is allegedly an incomplete proof of the sequent $p \rightarrow (q \rightarrow r), q, \neg r \vdash \neg p$.

1	$p \rightarrow (q \rightarrow r)$	premise
2	q	premise
3	$\neg r$	premise
4	<div style="border: 1px solid black; padding: 10px; display: inline-block;"> p \perp </div>	assumption
	$\neg p$	$\neg i$

1. The proof so far is correct; and a possible next step is to derive r by $\rightarrow e$ on lines 1 and 2.
2. The proof so far is correct; and a possible next step is to derive $q \rightarrow r$ by $\rightarrow e$ on lines 1 and 4.
3. The proof so far is incorrect, because the \perp at the end of the box should be $q \rightarrow r$.
4. The proof so far is incorrect, because the assumption of p at line 4 should be an assumption of $q \rightarrow r$.
5. None of the above.

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