

Chapter 5 Questions.

Modal Logics and Agents.

Semantics of modal logic.

Question 1

Consider the Kripke model given thus:

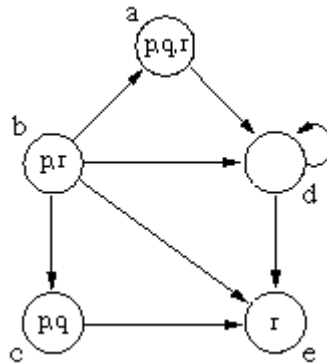
$M = (W, R, L)$, where

$W = \{a, b, c, d, e\}$,

$R = \{(b, a), (a, d), (b, d), (d, d), (b, c), (b, e), (c, e), (d, e)\}$, and

$L(a) = \{p, q, r\}$, $L(b) = \{p, r\}$, $L(c) = \{p, q\}$, $L(d) = \{\}$, $L(e) = \{r\}$,

pictured as follows:



Which of the following formulas is satisfied at the world b?

1. $\Box (\neg q \wedge \Diamond r)$
2. $\Diamond (p \wedge r)$
3. $\Box (p \vee q)$
4. $\Box \perp$
5. $\Box \Box r$

Question 2

Referring again to the Kripke model of Question 1, which of the following worlds satisfies the formula $\Diamond (p \wedge \Box \neg p)$?

1. the world a

2. the world b
3. the world c
4. the world d
5. the world e

Question 3

Which of the pairs of formulas are equivalent in the modal logic K?

1. $\Diamond p$ and $\Diamond \Box p$
2. $\Box (p \rightarrow q)$ and $\Box p \rightarrow \Box q$
3. $\Box (p \wedge q)$ and $\Box p \wedge \Box q$
4. $\Diamond \top$ and \top
5. $\Box \top$ and \top

Logic engineering.

Question 4

Which of the following expresses the correspondence condition for the formula $\Box p \vee \Box \neg p$?

1. $\forall x, y, z (R(x,y) \wedge R(x,z) \rightarrow y = z)$
2. $\forall x \forall z (\exists y (R(x,y) \wedge R(y,z)) \rightarrow x = z)$
3. $\forall x R(x,x)$
4. $\forall x, z (R(x,z) \rightarrow \exists y (R(x,y) \wedge R(y,z)))$
5. $\forall x \exists y R(x,y)$

Question 5

Which of the following formulas corresponds to the condition, $\forall x, y (R(x,y) \rightarrow x = y)$?

1. $p \rightarrow \Box \Diamond p$
2. $\Box p \rightarrow \Box \Box p$
3. $\Box p \rightarrow \Diamond p$
4. $p \rightarrow \Box p$
5. $\Box p \rightarrow p$

Logic of belief.

Question 6

Which of the following formula schemes are theorems of the logic of belief KD45 (in which $\Box p$ is read as "agent Q believes p")?

1. $\Box p \rightarrow \Diamond p$
2. $\Box (p \vee q) \rightarrow \Box p \vee \Box q$
3. $\neg \Box \perp$
4. $\Box p \rightarrow p$
5. $\Box p \vee \Box \neg p$

Logic of knowledge.

Question 7

Which of the following formula schemes are theorems of the logic of knowledge KT45 (in which $\Box p$ is read as "agent Q knows p")?

1. $\Box p \rightarrow \Diamond p$
2. $p \rightarrow \Box \Diamond p$
3. $\Diamond p \rightarrow \Box p$
4. $p \rightarrow \Box p$
5. $\Box p \vee \Box \neg p$

Reasoning about knowledge in a multi-agent system.

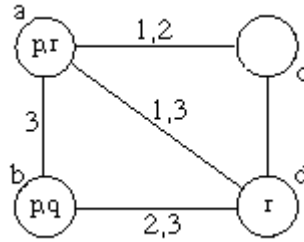
Question 8

Which of the following formulas are valid in the logic KT45ⁿ?

1. $K_2 q \rightarrow K_1 K_2 q$
2. $C_{\{1,2\}} p \leftrightarrow K_1 p \wedge K_2 p$
3. $E_G E_G p \rightarrow C_G p$
4. $C_G p \rightarrow E_G E_G p$
5. $K_1 p \wedge K_2 q \rightarrow E_{\{1,2\}} (p \vee q)$

Question 9

Which of the following assertions is true of the KT45ⁿ Kripke model given as follows:



1. $a \Vdash \neg K_1 \neg q$
2. $b \Vdash K_1 K_2 (p \vee r)$
3. $c \Vdash E_{\{1,2\}} r$
4. $a \Vdash C_{\{1,2,3\}} (q \rightarrow r)$
5. $b \Vdash \neg K_1 p$

Question 10

Referring to the same Kripke model as in the previous question, which worlds satisfy the formula $p \rightarrow E_{\{1,3\}} p$?

1. the world a
2. the world b
3. the world c
4. the world d
5. none of the worlds

Question 11

Alice and Barbara (let's call them A and B respectively) are discussing their husbands. We assume perfect reasoning and truthfulness by the two women. It is common knowledge among them (among the whole village in fact) that at least one of their husbands is unfaithful. It is also common knowledge that each woman knows whether the other woman's husband is unfaithful or not. Let "a" mean that Alice's husband is unfaithful, and "b" mean that Barbara's husband is unfaithful.

Which of the following KT45ⁿ formulas are true in this situation?

1. $a \rightarrow K_B a$
 2. $K_A b$
 3. $K_A (K_B a \vee K_B \neg a)$
 4. $C (a \wedge b)$
 5. $\neg a \rightarrow K_A \neg a$
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Question 12

Continuing the story of the two women, Alice asks Barbara "Do you know whether your husband is unfaithful?". Barbara replies that she does not know, and asks Alice if she knows about hers (Alice's) husband. Which of the following is correct?

1. Alice says she does not know.
 2. Alice says she knows her husband is faithful.
 3. Alice says she knows her husband is unfaithful.
 4. Alice says Barbara cannot have been telling the truth.
 5. We cannot predict what Alice will say.
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