Chapter 2

PRAM: Matrix

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Matrix addition

A[nxn] + B[nxn] = C[nxn]

a_{11}	a_{12}	…	a_{1n}
a_{21}	a_{22}	…	a_{2n}
…	…	…	…
a_{n1}	a_{n2}	…	a_{nn}

+

b_{11}	b_{12}	…	b_{1n}
b_{21}	b_{22}	…	b_{2n}
…	…	…	…
b_{n1}	b_{n2}	…	b_{nn}

=

c_{11}	c_{12}	…	c_{1n}
c_{21}	c_{22}	…	c_{2n}
…	…	…	…
c_{n1}	c_{n2}	…	c_{nn}

C[nxn]

$c_{11} = a_{11} + b_{11}$	$c_{12} = a_{12} + b_{12}$	…	$c_{1n} = a_{1n} + b_{1n}$
$c_{21} = a_{21} + b_{21}$	$c_{22} = a_{22} + b_{22}$	…	$c_{2n} = a_{2n} + b_{2n}$
…	…	…	…
$c_{n1} = a_{n1} + b_{n1}$	$c_{n2} = a_{n2} + b_{n2}$	…	$c_{nn} = a_{nn} + b_{nn}$

Matrix addition: PRAM with nxn processors

C[nxn]

$c_{11} = a_{11} + b_{11}$	$c_{12} = a_{12} + b_{12}$	…	$c_{1n} = a_{1n} + b_{1n}$
P_{11}	P_{12}		P_{1n}
$c_{21} = a_{21} + b_{21}$	$c_{22} = a_{22} + b_{22}$	…	$c_{2n} = a_{2n} + b_{2n}$
P_{21}	P_{22}		P_{2n}
…	…	…	…
…	…	…	…
$c_{n1} = a_{n1} + b_{n1}$	$c_{n2} = a_{n2} + b_{n2}$	…	$c_{nn} = a_{nn} + b_{nn}$
P_{n1}	P_{n2}		P_{nn}

- P_{ij} processor:
  - Read: a_{ij} & b_{ij}
  - Write: C_{ij}

No overlapping data

- CRCW: O(1)
- EREW: O(1)

➢ All (*nxn*) processors run '+' in parallel: O(1) =>

Matrix addition: PRAM with n processors

- All n processors run '+' in parallel in *n* steps: O(n) ??? =>

C[nxn]

Step 1	Step 2	…	Step n
$c_{11} = a_{11} + b_{11}$	$c_{12} = a_{12} + b_{12}$	…	$c_{1n} = a_{1n} + b_{1n}$
P_1	P_1	…	P_1
$c_{21} = a_{21} + b_{21}$	$c_{22} = a_{22} + b_{22}$	…	$c_{2n} = a_{2n} + b_{2n}$
P_2	P_2	…	P_2
…	…	…	…
…	…	…	…
$c_{n1} = a_{n1} + b_{n1}$	$c_{n2} = a_{n2} + b_{n2}$	…	$c_{nn} = a_{nn} + b_{nn}$
P_n	P_n	…	P_n

- P_{ij} processor:
  - Read: a_{ij} & b_{ij}
  - Write: c_{ij}

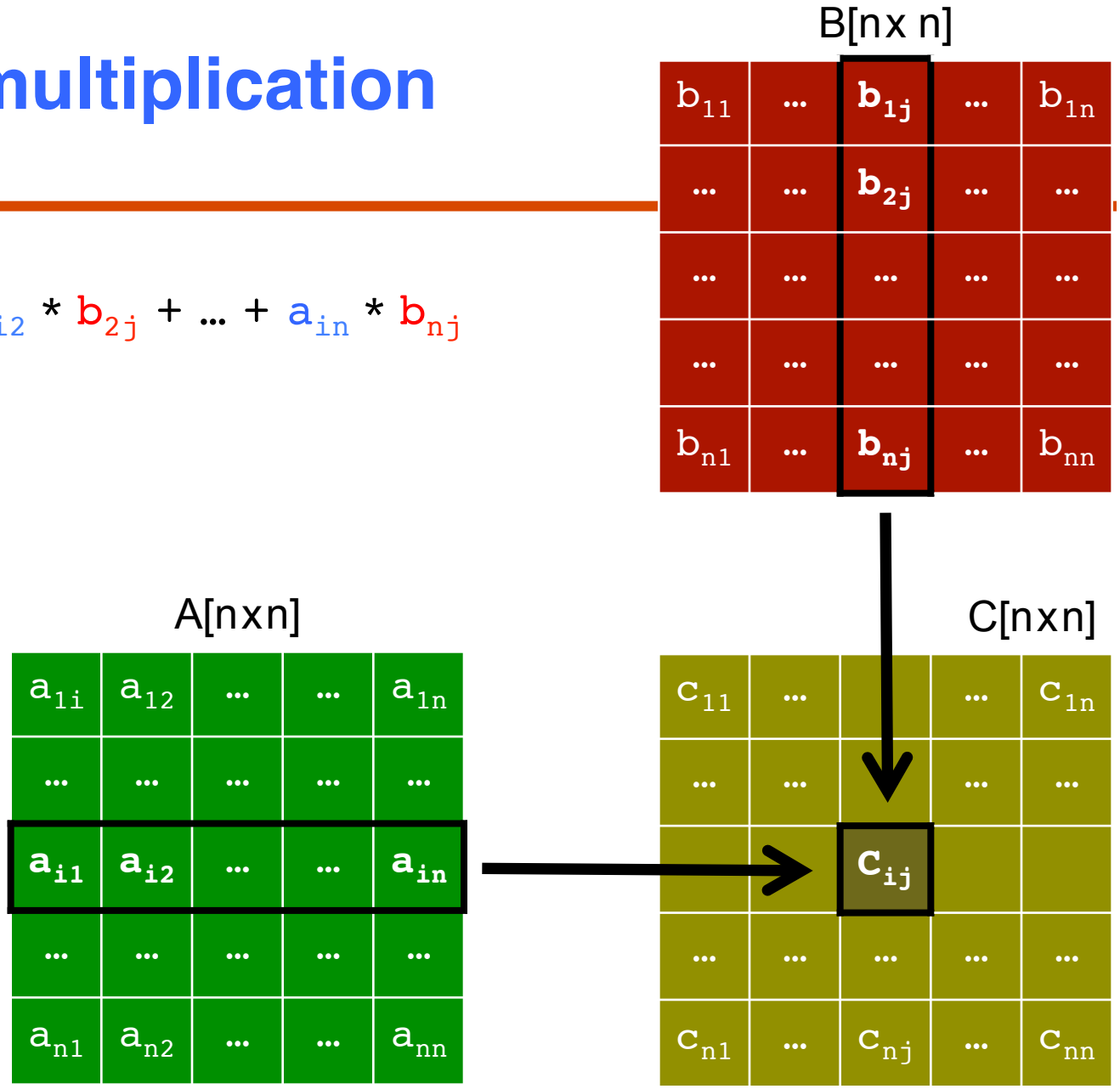
No overlapping data

  - CRCW: O(n)
  - EREW: O(n)

Your algorithm with ***k*** processors (***k*** << ***n***)?

Matrix multiplication

$$C_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + \ldots + a_{in} * b_{nj}$$

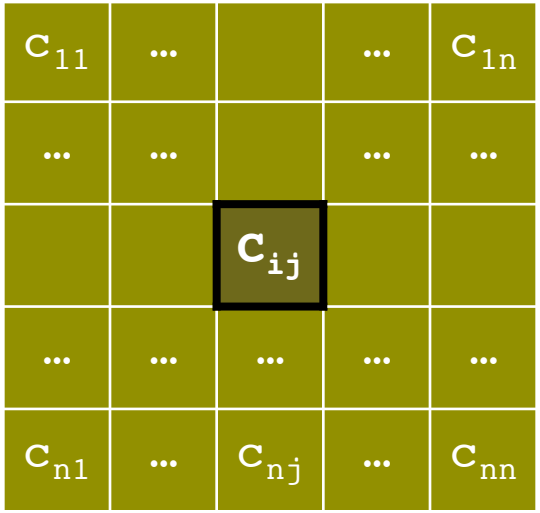


C_{ij}

- $C_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + \ldots + a_{in} * b_{nj}$
- Vector: A[i^{th} row] x B[j^{th} column]
- Number of operations = $\underbrace{n * opt('*')}_{O(1)} + \underbrace{(n-1) * opt('+')}_{O(\log(n))}$
- n processors ~ $O(\log(n))$

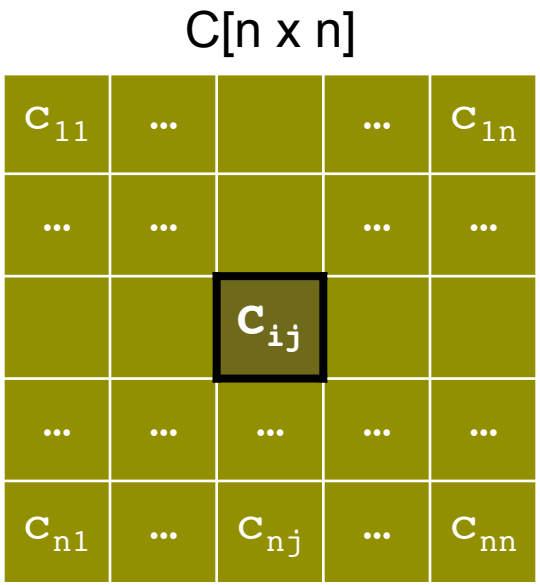
~ $O(\log(n))$

C[nxn]



PRAM
Matrix multiplication

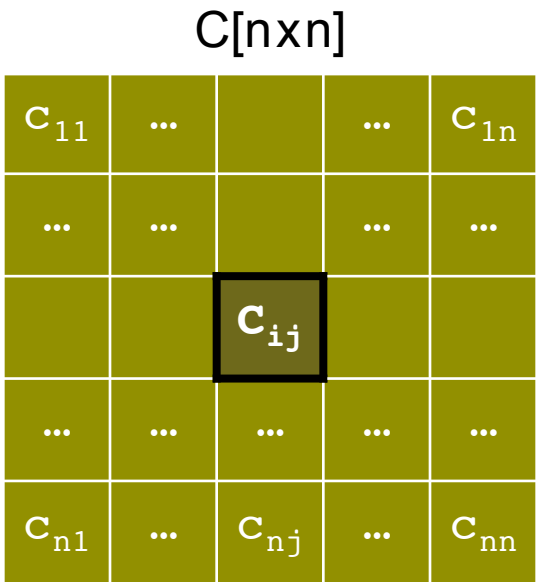
- C_{ij} using n processors: O(log(n))
- C[n x n]: (n*n)C_{ij}
  - n processors: O(n^2 * log(n))
  - n^3 processors: O(log(n))
  - n^2 processors???



PRAM

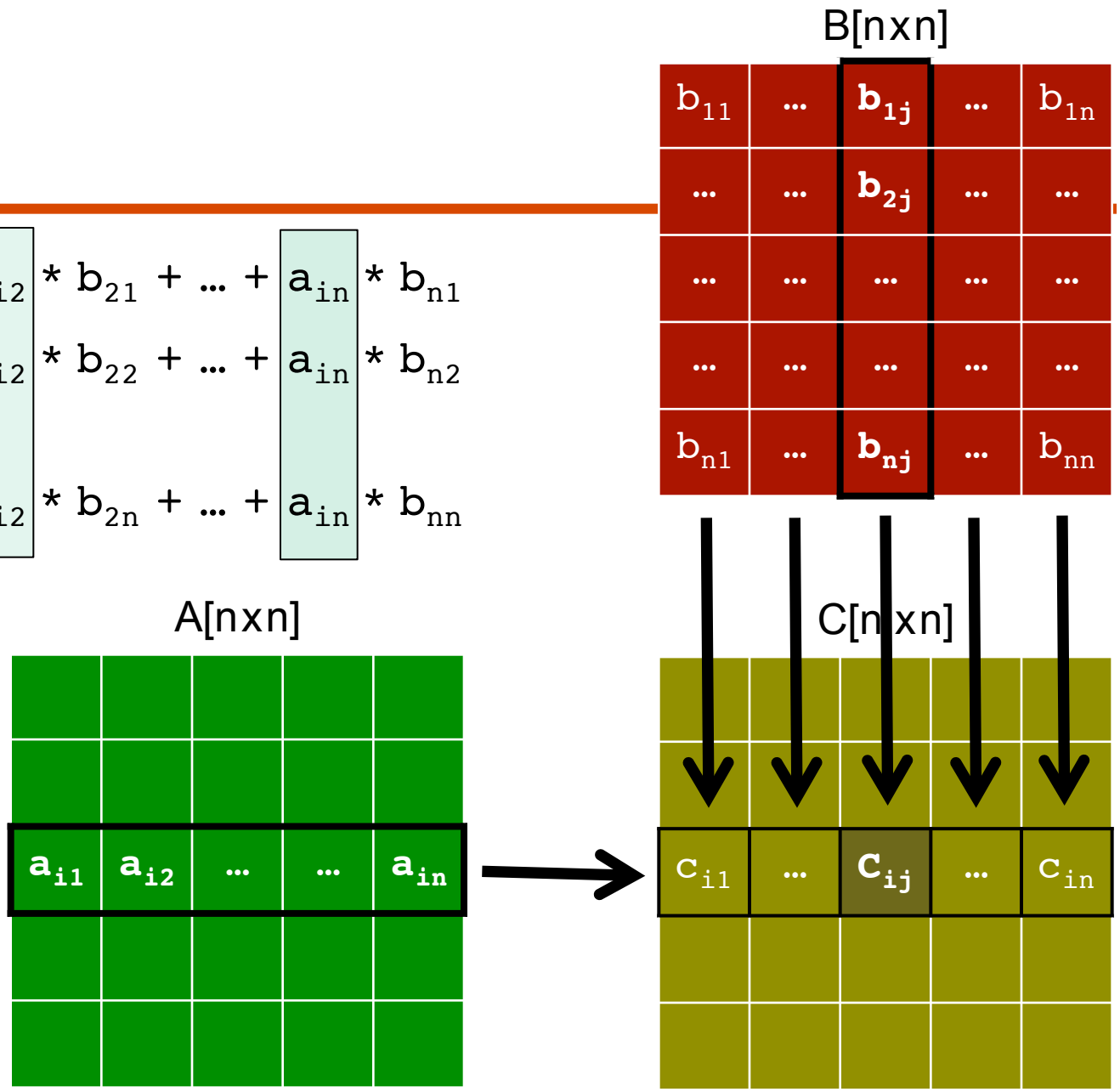
- C_{ij} using n processors: O(log(n))
- C[n x n]: (n*n)C_{ij}
  - n processors: O(n^2 * log(n))
  - n^3 processors: O(log(n))
  - n^2 processors???

=> Difference between **CRCW** & **EREW**???

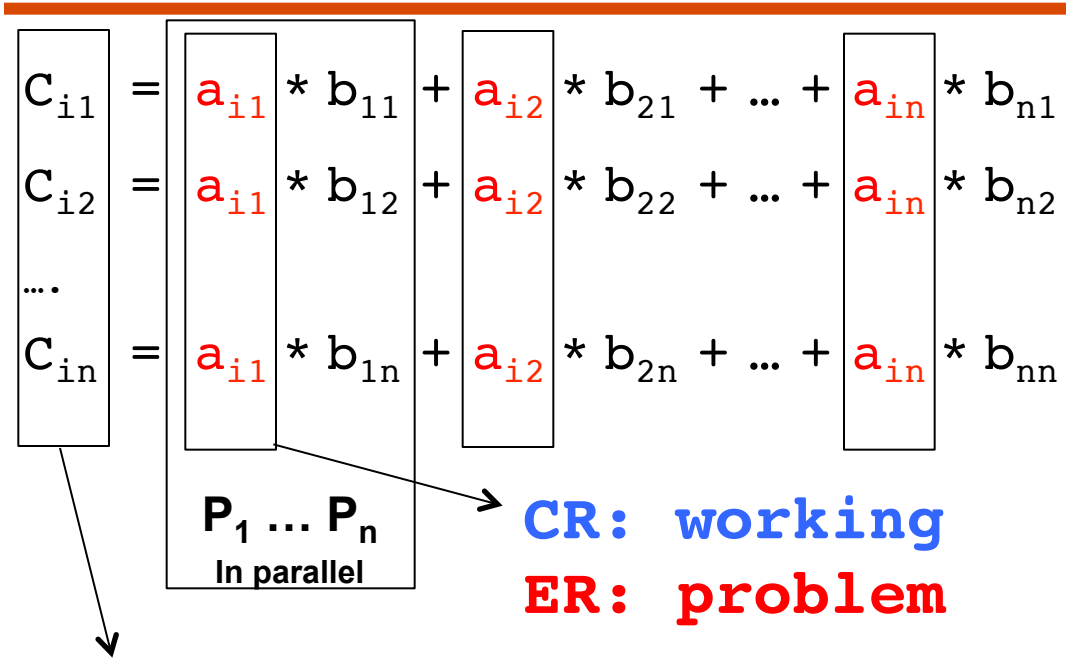


Row

$$
\begin{aligned}
C_{i1} &= a_{i1} * b_{11} + a_{i2} * b_{21} + \ldots + a_{in} * b_{n1} \\
C_{i2} &= a_{i1} * b_{12} + a_{i2} * b_{22} + \ldots + a_{in} * b_{n2} \\
&\ldots. \\
C_{in} &= a_{i1} * b_{1n} + a_{i2} * b_{2n} + \ldots + a_{in} * b_{nn}
\end{aligned}
$$

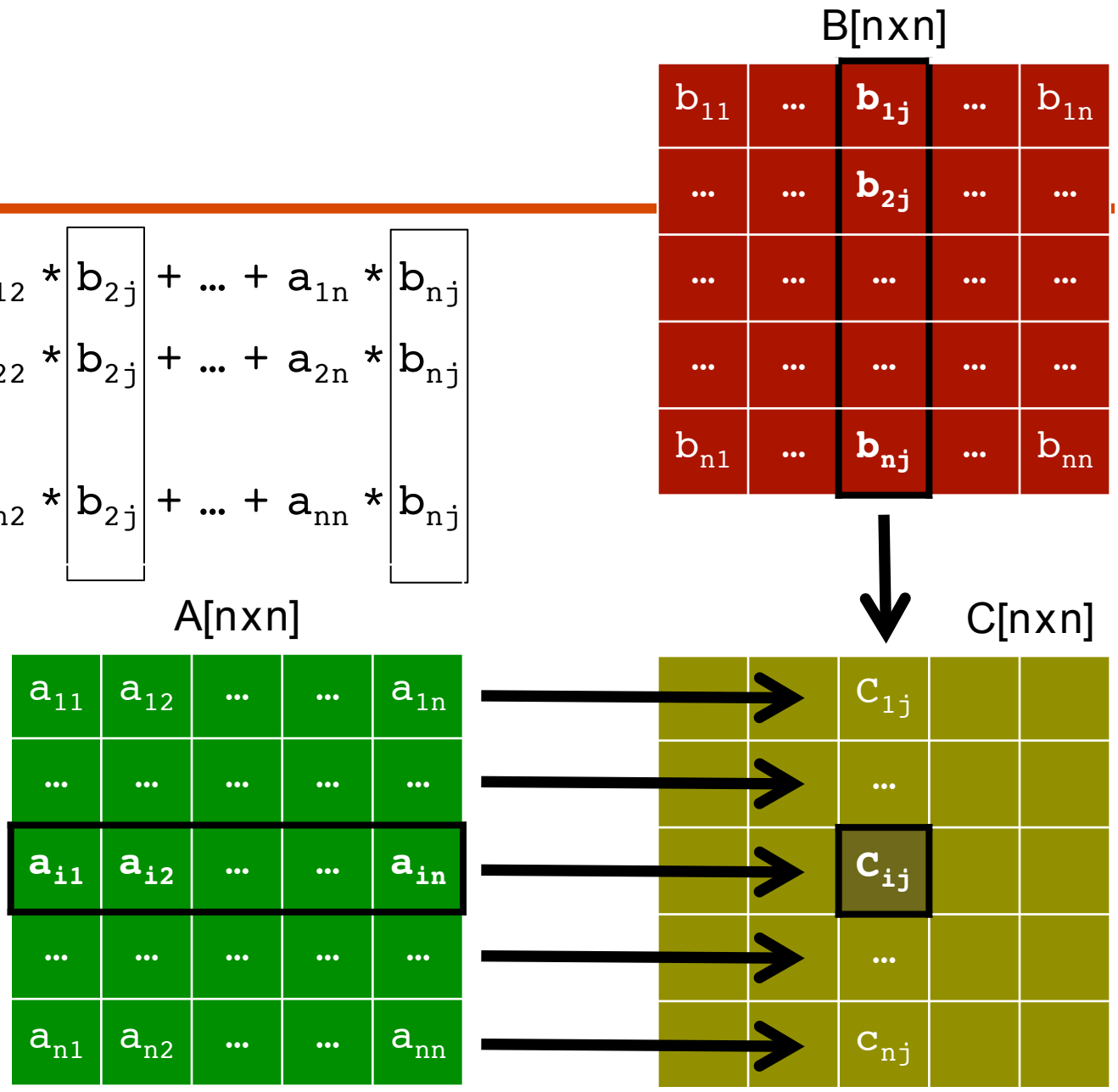


Concurrent Read: in row

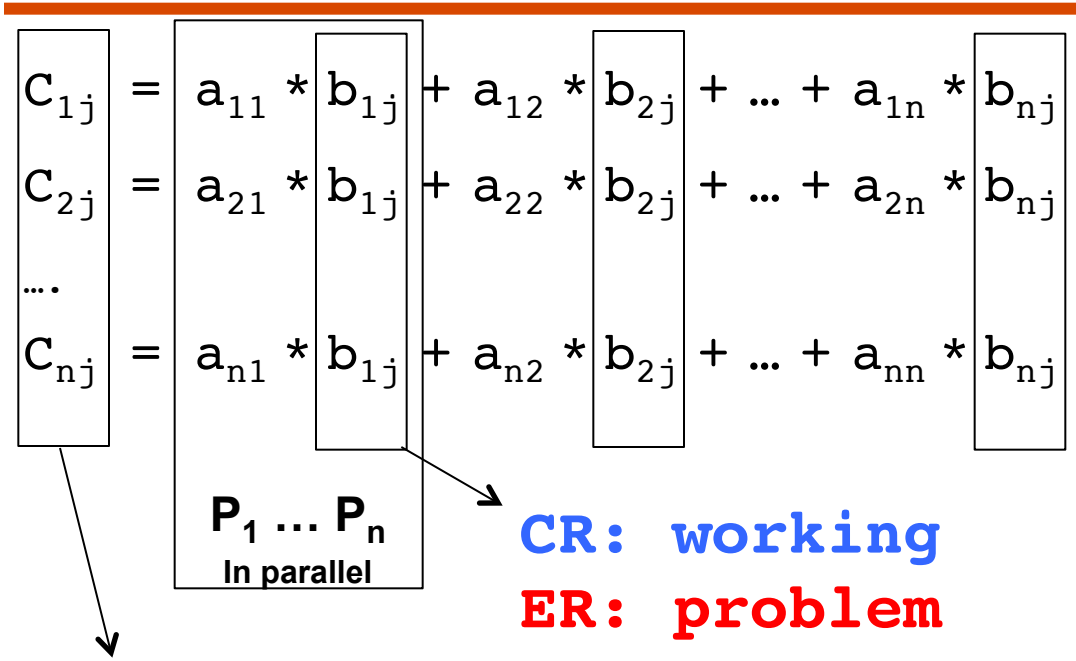


Column

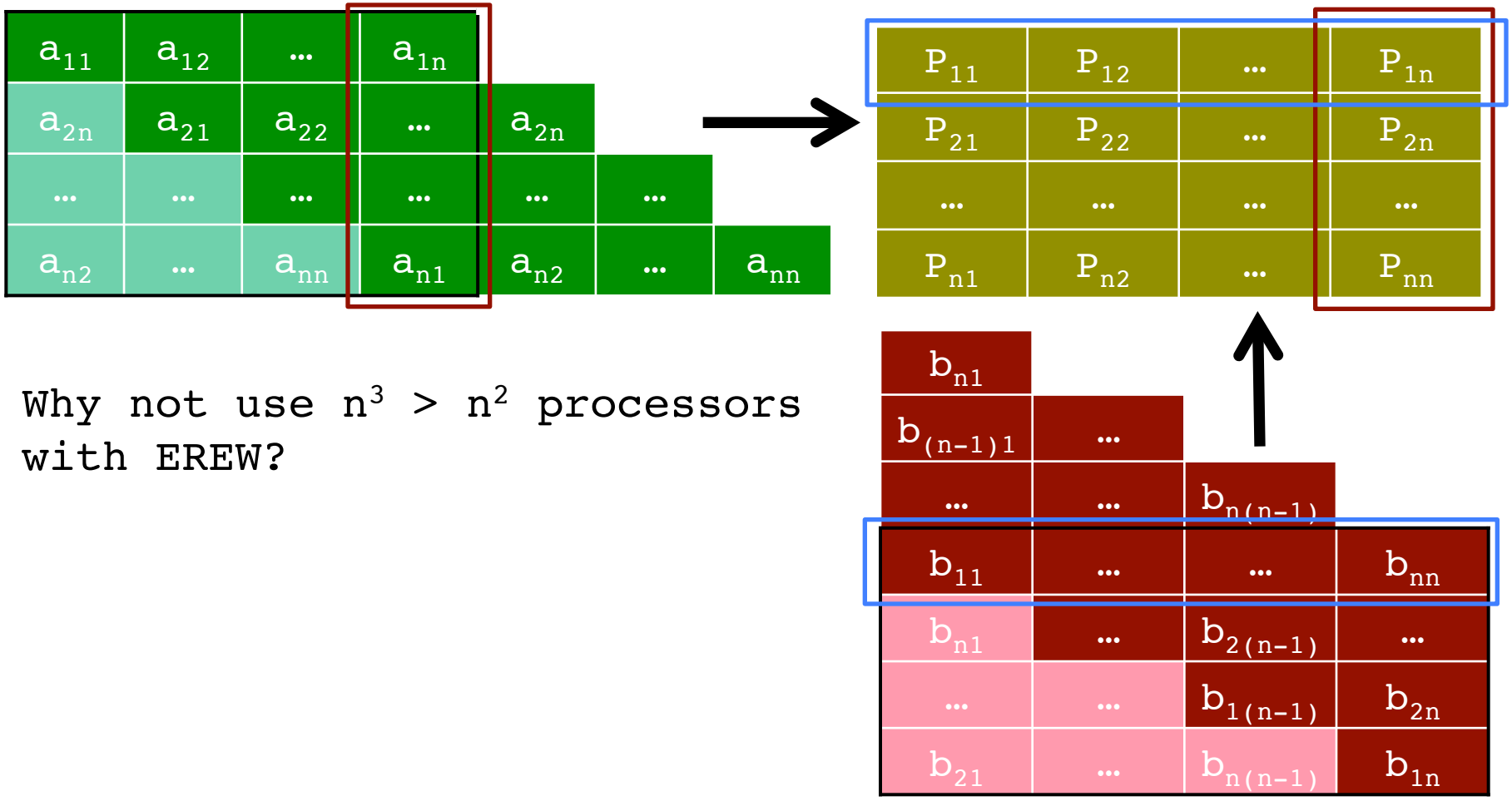
$$
\begin{aligned}
C_{1j} &= a_{11} * b_{1j} + a_{12} * b_{2j} + \ldots + a_{1n} * b_{nj} \\
C_{2j} &= a_{21} * b_{1j} + a_{22} * b_{2j} + \ldots + a_{2n} * b_{nj} \\
&\ldots. \\
C_{nj} &= a_{n1} * b_{1j} + a_{n2} * b_{2j} + \ldots + a_{nn} * b_{nj}
\end{aligned}
$$



Column

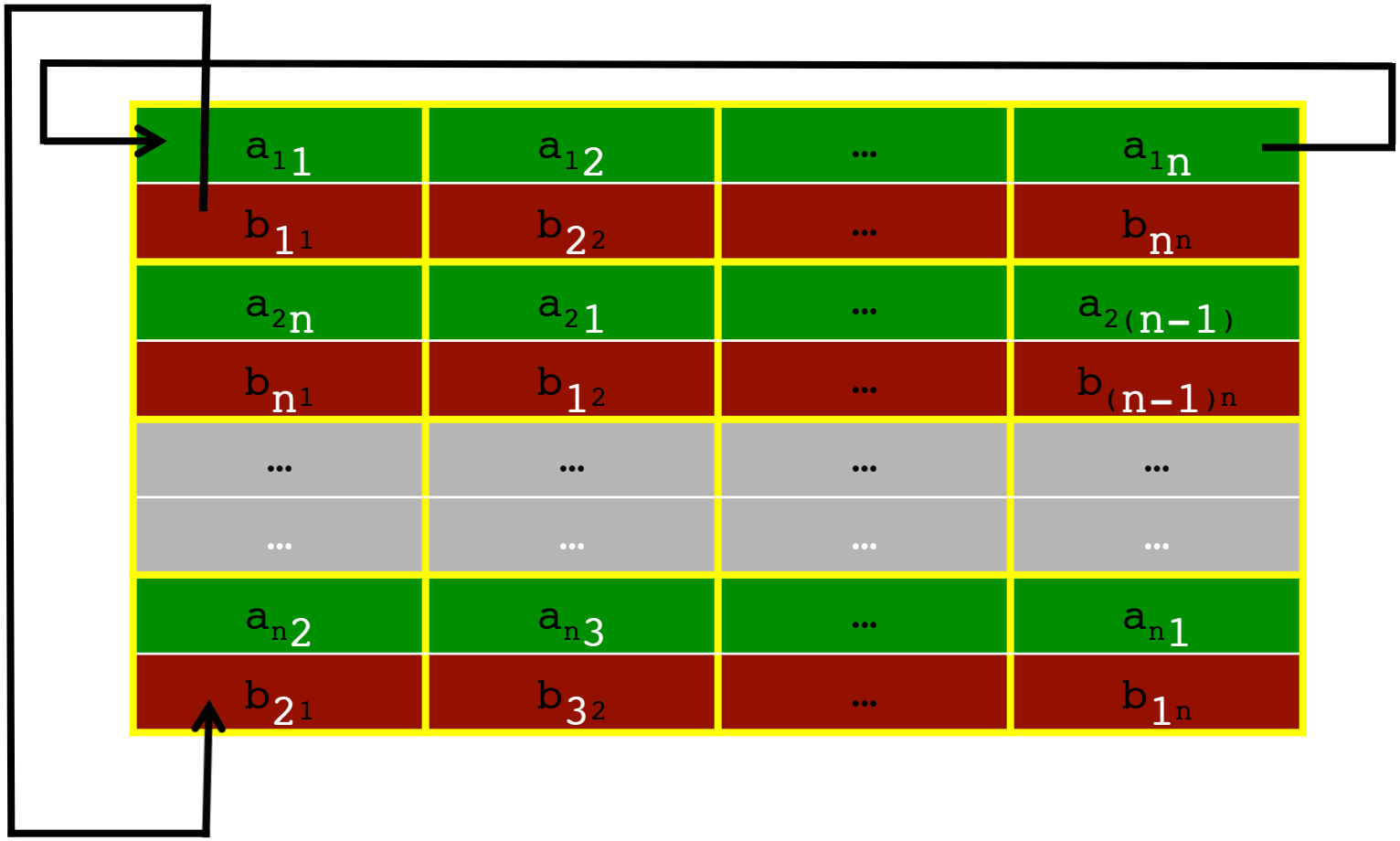

$C_{1j} = a_{11} * b_{1j} + a_{12} * b_{2j} + \ldots + a_{1n} * b_{nj}$
$C_{2j} = a_{21} * b_{1j} + a_{22} * b_{2j} + \ldots + a_{2n} * b_{nj}$
….
$C_{nj} = a_{n1} * b_{1j} + a_{n2} * b_{2j} + \ldots + a_{nn} * b_{nj}$
$P_1 \ldots P_n$
In parallel
CR: working
ER: problem
CW: working
EW: working

PRAM
EREW with n^2 processors

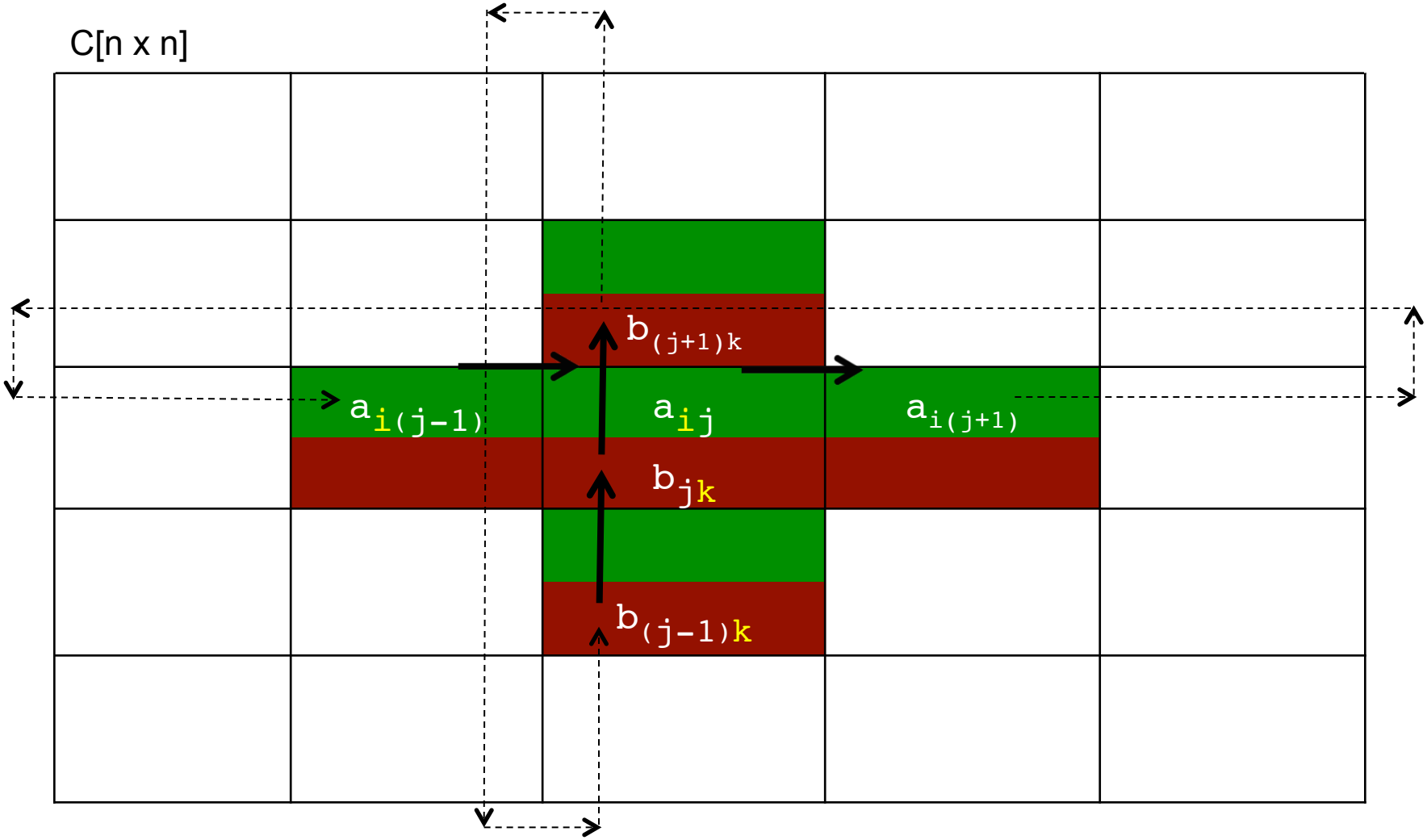


Why not use $n^3 > n^2$ processors with EREW?

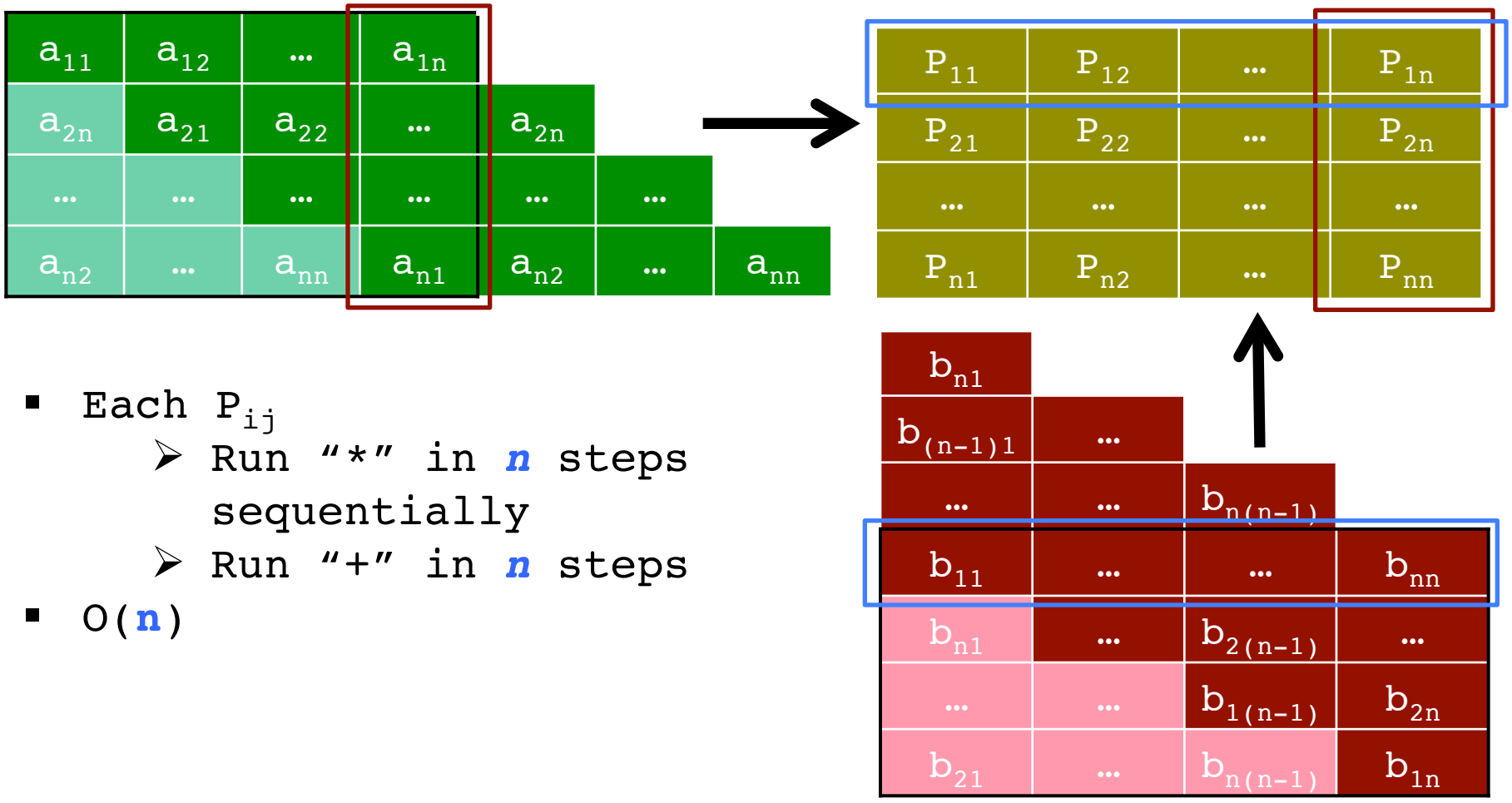
PRAM
EREW with n^2 processors



PRAM
EREW with n^2 processors

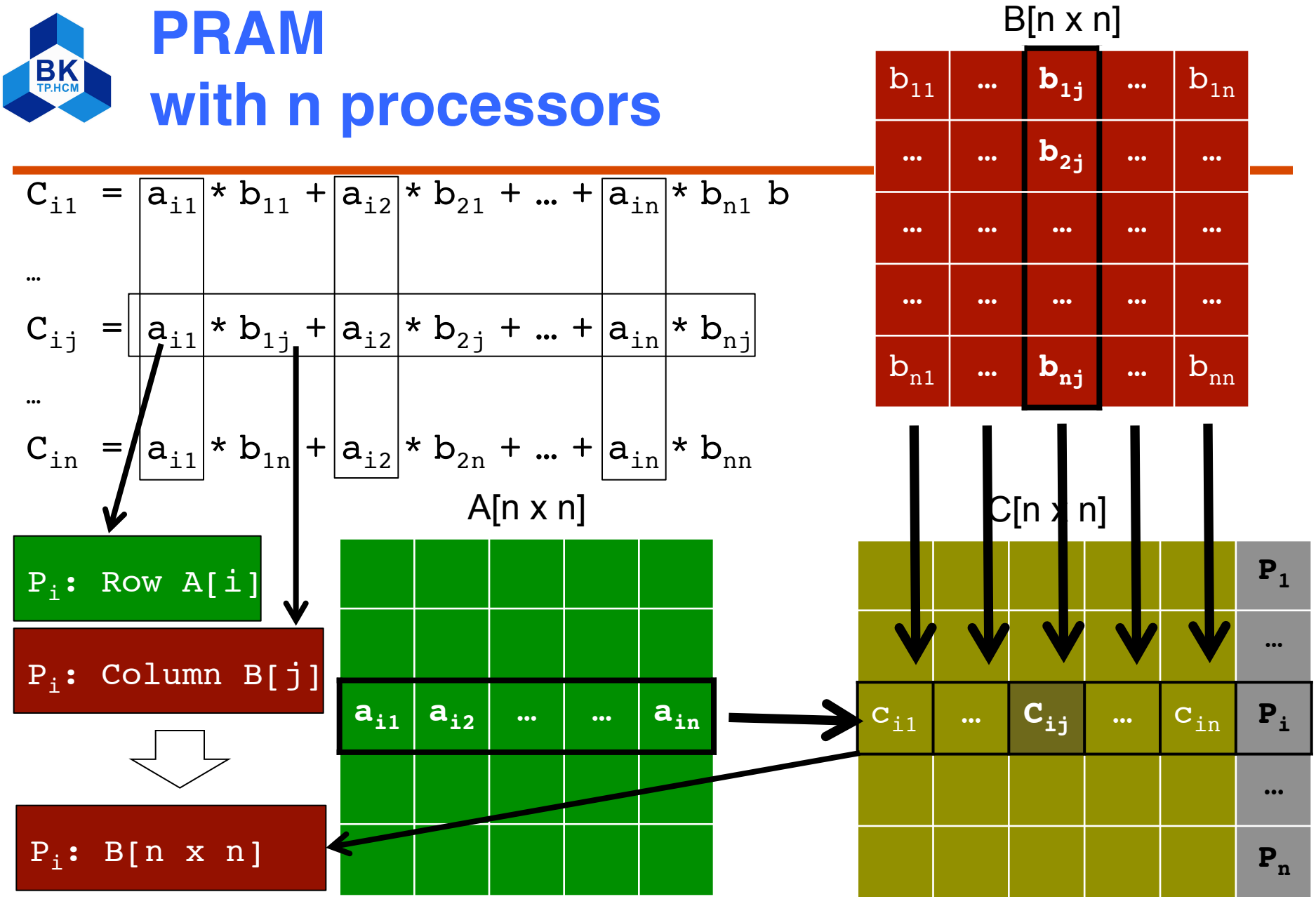


PRAM
EREW with n^2 processors



- Each P_{ij}
  - Run "*" in ***n*** steps sequentially
  - Run "+" in ***n*** steps
- O(***n***)

PRAM with n processors

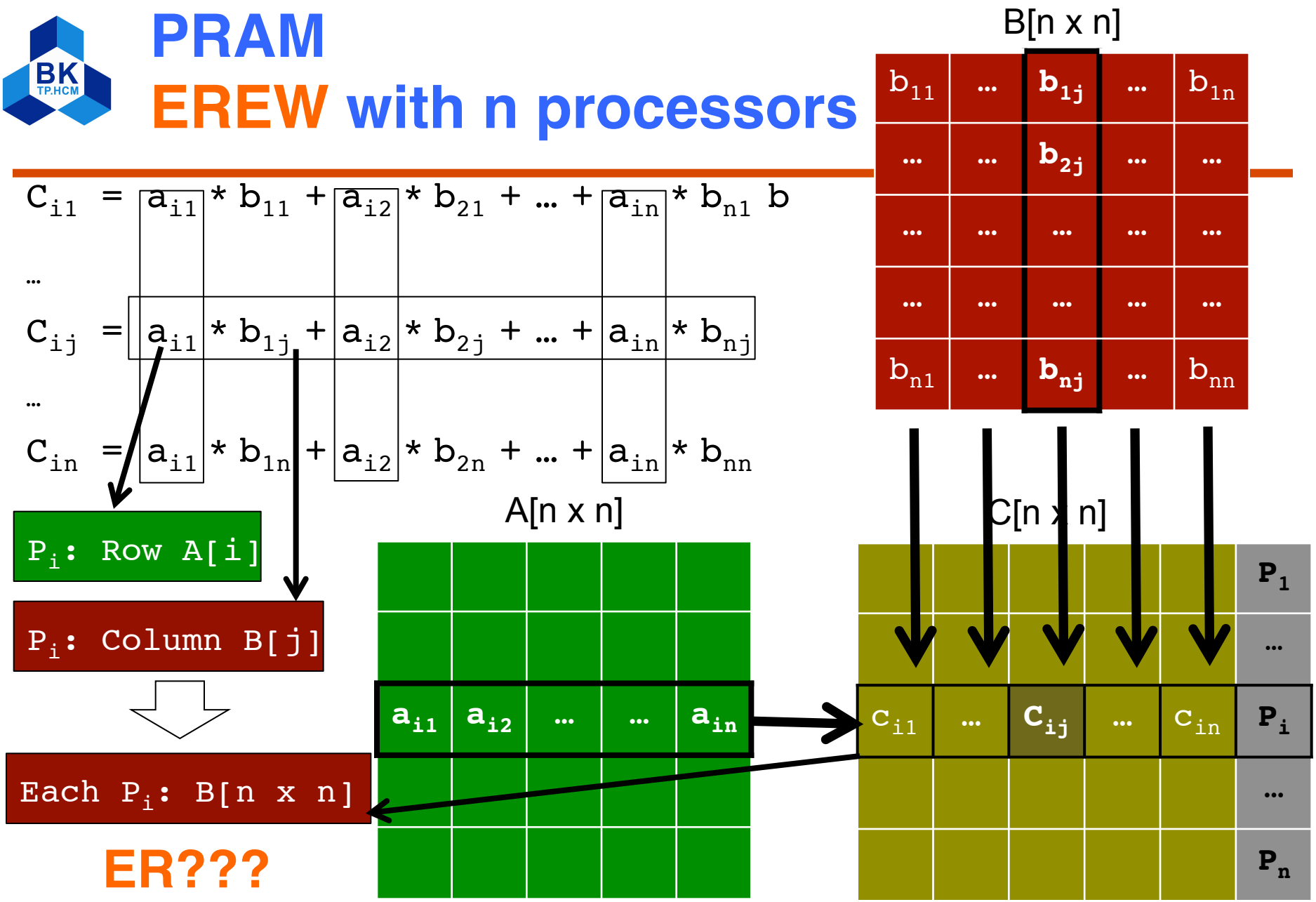


PRAM
CRCW with n processors

Each P_i:

- $C_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + \ldots + a_{in} * b_{nj}$ => O(**n**)
- Row C[i]: O(**n^2**)

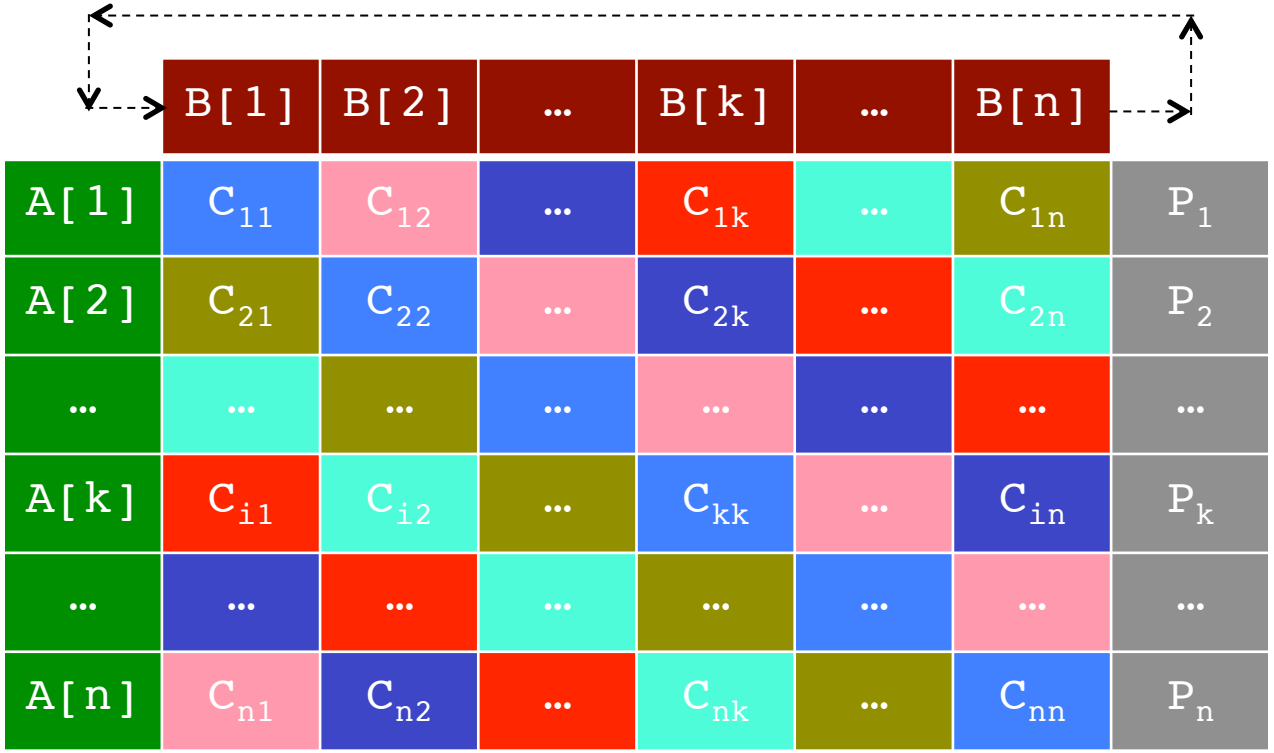
PRAM
EREW with n processors



PRAM
EREW with n processors

Each P_i:

- $C_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + \ldots + a_{in} * b_{nj} \Rightarrow O(n)$
- Row C[i]: $O(n^2)$



PRAM
EREW with k processors (k << n)

Each P_i:

- 1 step: $O(k*n)$ =>
- 1 phase (n/k steps): $O(k*n*n/k) \sim O(n^2)$
- n/k phases: $O(n^2*n/k) \sim O(n^3/k)$