Chapter 2

Abstract Machine Models

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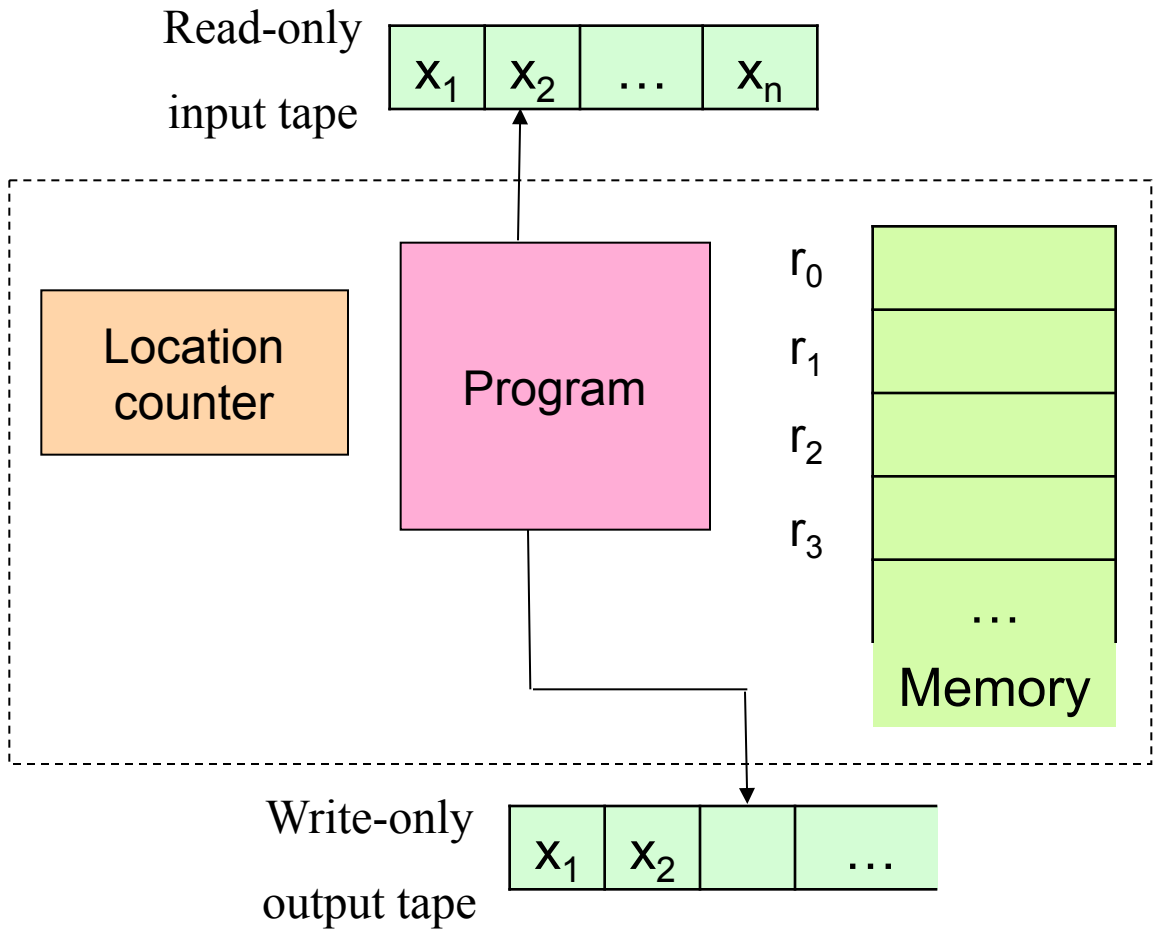
Abstract Machine Models

- An abstract machine model is mainly used in the design and analysis of parallel algorithms without worry about the details of physical machines.
- Three abstract machine models:
  - PRAM
  - BSP
  - Phase Parallel

RAM (1)

- RAM (Random Access Machine)



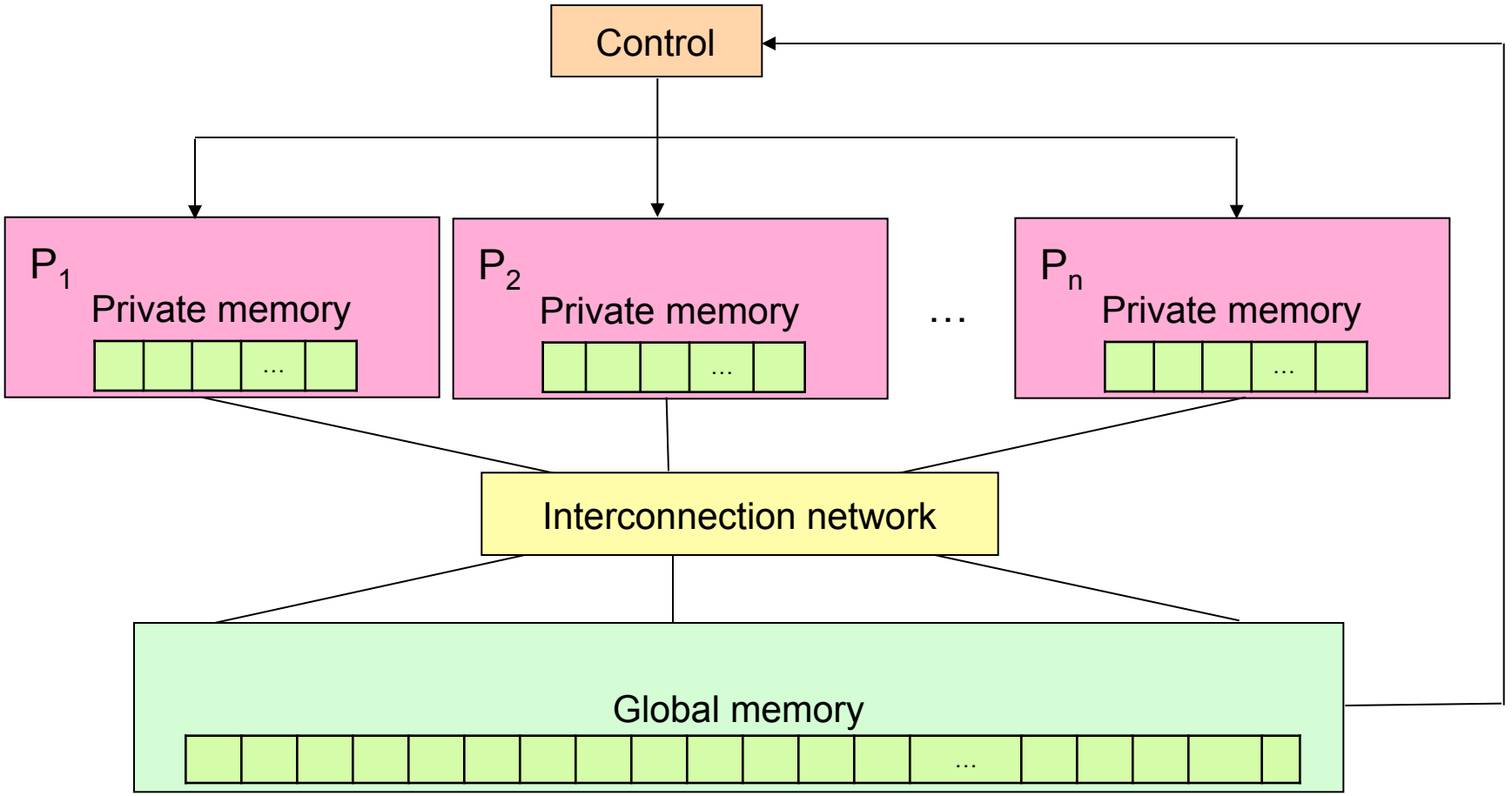
RAM (2)

RAM model of serial computers

- Memory is a sequence of words, each capable of containing an integer
- Each memory access takes one unit of time
- Basic operations (add, multiply, compare) take one unit time
- Instructions are not modifiable
- Read-only input tape, write-only output tape

PRAM (1)

Parallel Random Access Machine (Introduced by Fortune and Wyllie, 1978)



PRAM (2)

- A control unit
- An unbounded set of processors, each with its own private memory and an unique index
- Input stored in global memory or a single *active* processing element
- Step: (1) read a value from a single private/global memory location
  (2) perform a RAM operation
  (3) write into a single private/global memory location
- During a computation step: a processor may activate another processor
- All active, enable processors must execute ***the same instruction*** (albeit on different memory location)???
- Computation terminates when the last processor halts

PRAM (3)

PRAM composed of:

- P processors, each with its own unmodifiable program
- A single shared memory composed of a sequence of words, each capable of containing an arbitrary integer
- a read-only input tape
- a write-only output tape

PRAM model is a synchronous (SIMD)/ asynchronous (MIMD), shared address space parallel computer

- Processors share a common clock but may execute different instructions in each cycle

PRAM(4)

- Definition:

  The **cost** of a PRAM computation is the product of the parallel time complexity and the number of processors used.

Ex: a PRAM algorithm that has time complexity O(log ***p***) using ***p*** processors has cost O(***p*** log ***p***)

Time Complexity Problem

- Time complexity of a PRAM algorithm is often expressed in the big-*O* notation
- Machine size *n* is usually small in existing parallel computers
- Ex:
  - Three PRAM algorithms A, B and C have time complexities if 7n, (n log n)/4, n log log n.
  - Big-*O* notation: A(O(n)) < C(O(n log log n)) < B(O(n log n))
  - Machines with no more than 1024 processors:

    log n ≤ log 1024 = 10 and log log n ≤ log log 1024 < 4

    and thus: B < C < A

Conflicts Resolution Schemes (1)

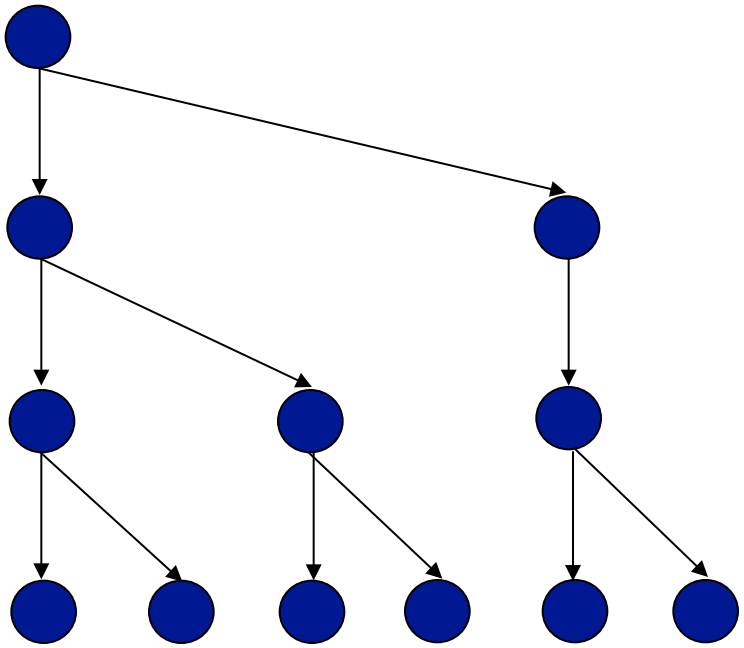
- PRAM execution can result in simultaneous access to the same location in shared memory.
  - Exclusive Read (ER)
    - » No two processors can simultaneously read the same memory location.
  - Exclusive Write (EW)
    - » No two processors can simultaneously write to the same memory location.
  - Concurrent Read (CR)
    - » Processors can simultaneously read the same memory location.
  - Concurrent Write (CW)
    - » Processors can simultaneously write to the same memory location, using some conflict resolution scheme.

Conflicts Resolution Schemes (2)

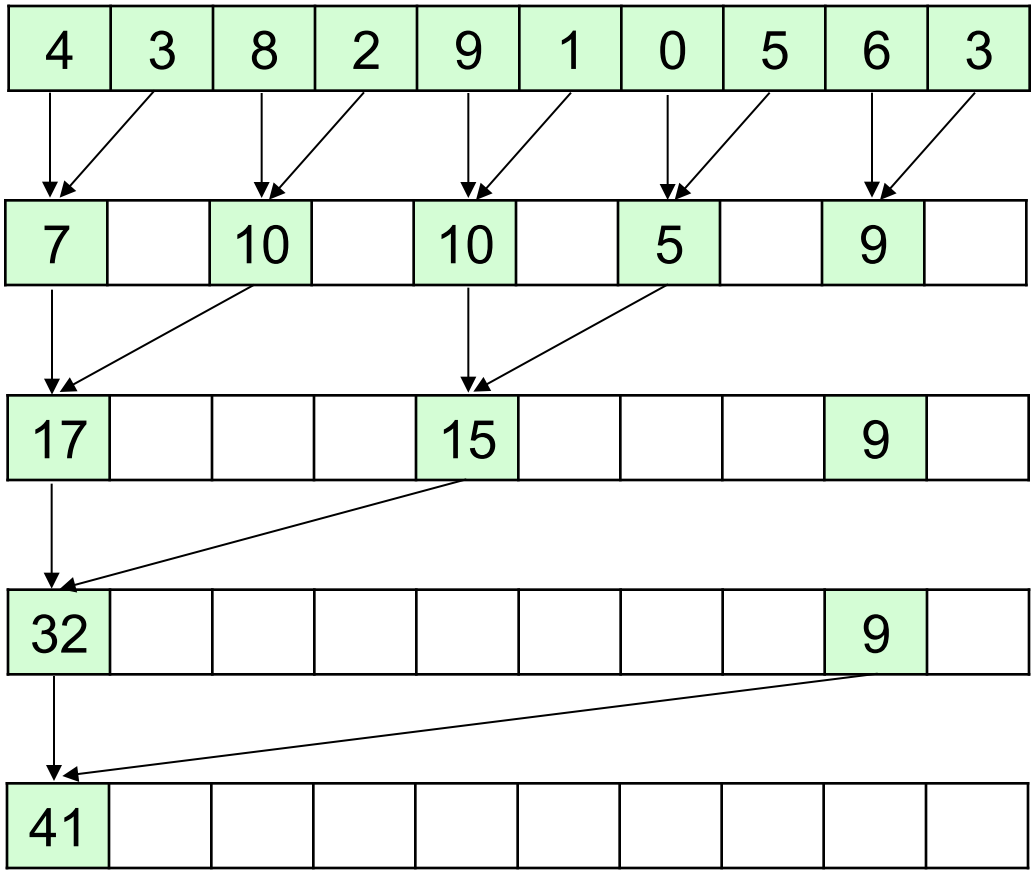
- Common/Identical CRCW
  - All processors writing to the same memory location must be writing the same value.
  - The software must ensure that different values are not attempted to be written.
- Arbitrary CRCW
  - Different values may be written to the same memory location, and an arbitrary one succeeds.
- Priority CRCW
  - An index is associated with the processors and when more than one processor write occurs, the lowest-numbered processor succeeds.
  - The hardware must resolve any conflicts

PRAM Algorithm

- Begin with a single active processor active
- Two phases:
  - A sufficient number of processors are activated
  - These activated processors perform the computation in parallel
- $\lceil \log p \rceil$ activation steps: p processors to become active
- The number of active processors can be double by executing a single instruction



Parallel Reduction (1)



Parallel Reduction (2)

(EREW PRAM Algorithm in Figure2-7, page 32, book [1])

Ex: SUM(EREW)

Initial condition: List of $n \geq 1$ elements stored in $A[0..(n-1)]$

Final condition: Sum of elements stored in $A[0]$

Global variables: n, $A[0..(n-1)]$, j

begin

spawn (P_0, P_1,…, $P_{\lfloor n/2 \rfloor -1}$)

for all P_i where $0 \leq i \leq \lfloor n/2 \rfloor -1$ do

for $j \leftarrow 0$ to $\lceil \log n \rceil - 1$ do

if i modulo $2^j = 0$ and $2i+2^j < n$ the

$A[2i] \leftarrow A[2i] + A[2i+2^j]$

endif

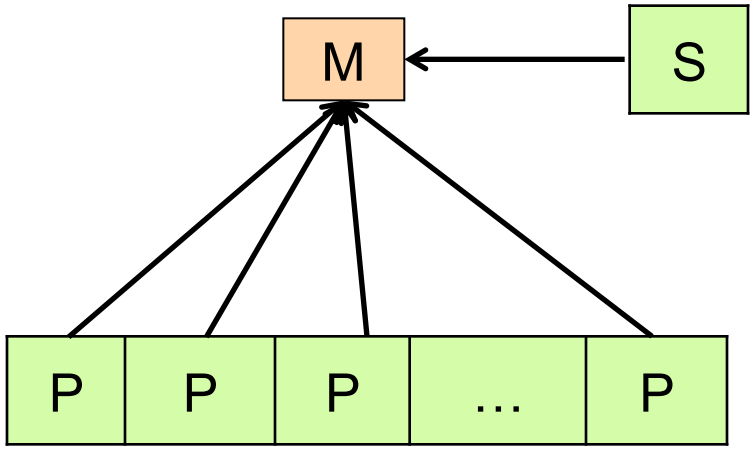
endfor

endfor

end

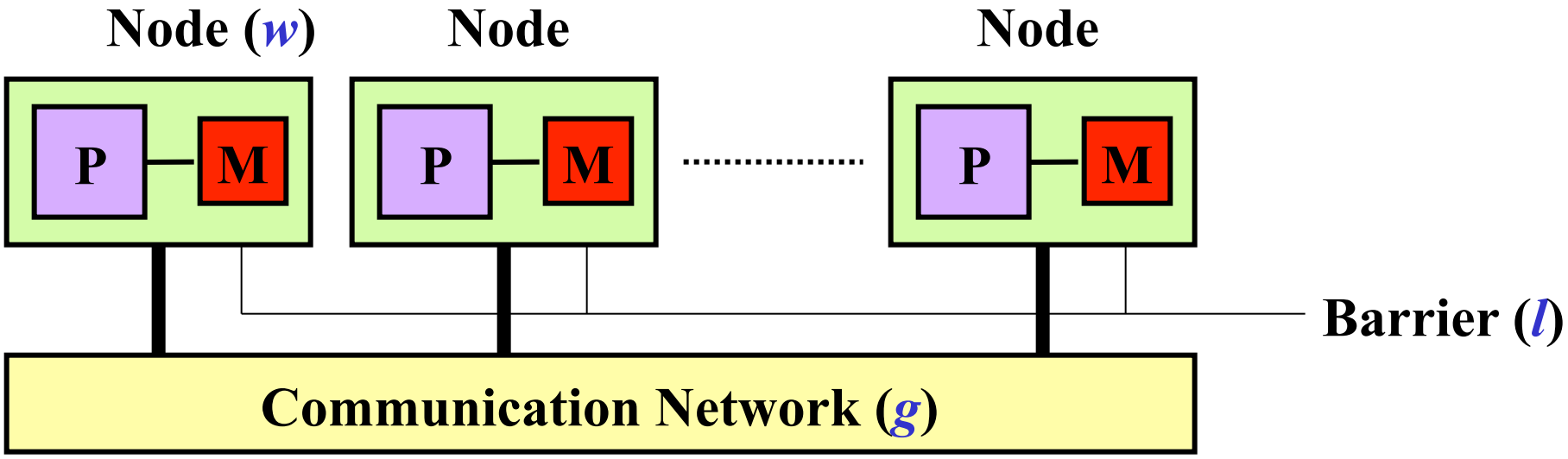
Broadcasting on a PRAM

- "Broadcast" can be done on CREW PRAM in O(1) steps:
  - Broadcaster sends value to shared memory
  - Processors read from shared memory
- Requires log*P* steps on EREW PRAM



BSP – Bulk Synchronous Parallel

- BSP Model
  - Proposed by Leslie Valiant of Harvard University
  - Developed by W.F.McColl of Oxford University



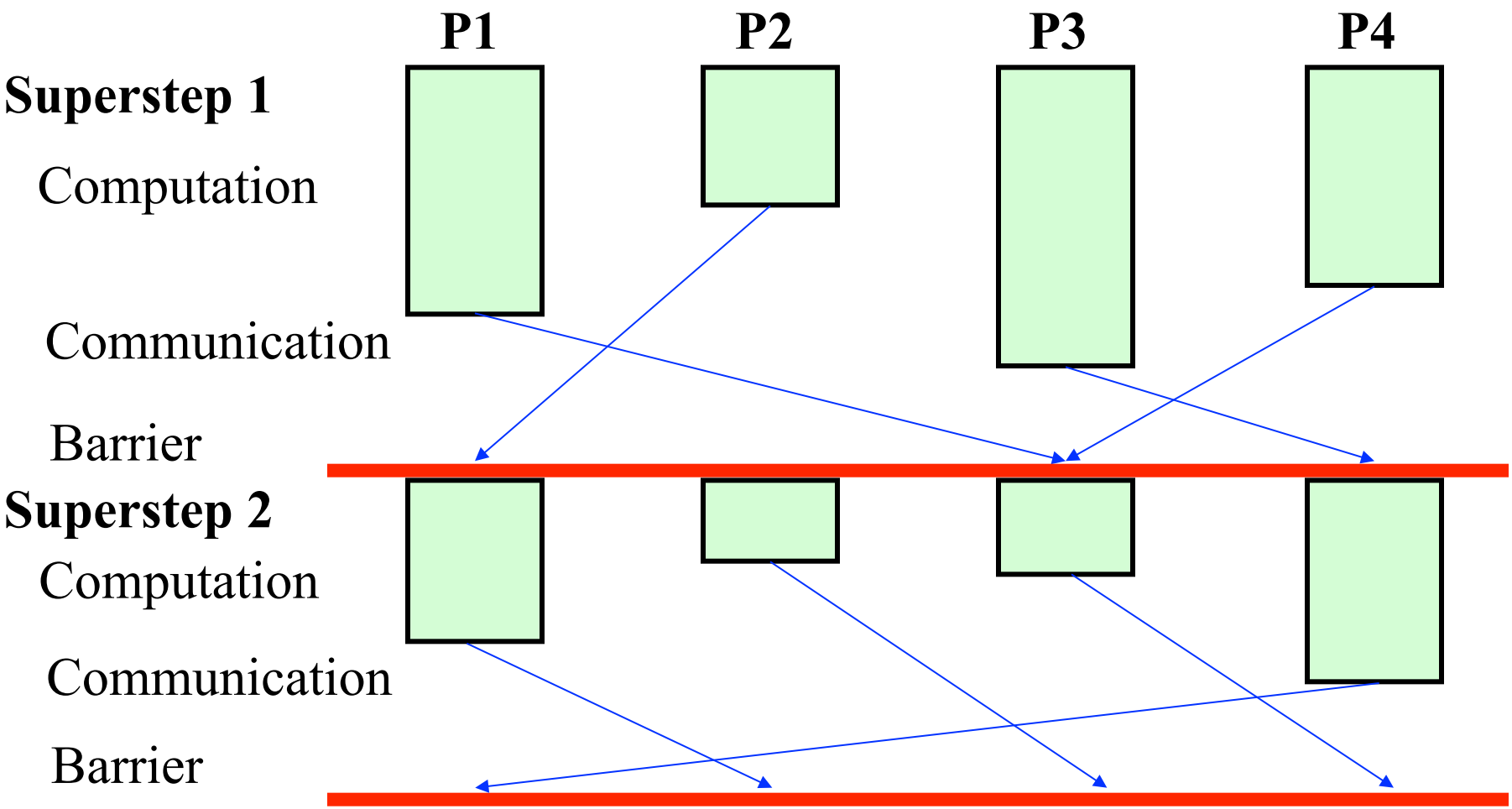
BSP Model

- A set of n nodes (processor/memory pairs)
- Communication Network
  - Point-to-point, message passing (or shared variable)
- Barrier synchronizing facility
  - All or subset
- Distributed memory architecture

BSP Programs

- A BSP program:
  - *n* processes, each residing on a node
  - Executing a strict sequence of *supersteps*
  - In each superstep, a process executes:
    - » Computation operations: *w* cycles
    - » Communication: *gh* cycles
    - » Barrier synchronization: *l* cycles

A Figure of BSP Programs



Three Parameters

- The basic time unit is a cycle (or time step)
- ***w*** parameter
  - Maximum computation time within each superstep
  - Computation operation takes at most ***w*** cycles.
- ***g*** parameter
  - Number of cycles for communication of unit message when all processors are involved in communication - network bandwidth
  - (total number of local operations performed by all processors in one second) / (total number of words delivered by the communication network in one second)
  - ***h*** relation coefficient ~ number of incoming or outgoing messages for a superstep
  - Communication operation takes ***gh*** cycles.
- ***l*** parameter
  - Barrier synchronization takes ***l*** cycles.

Time Complexity of BSP Algorithms

- Execution time of a superstep:
  - Sequence of the computation, the communication, and the synchronization operations: $w + gh + l$
  - Overlapping the computation, the communication, and the synchronization operations: $\max\{w, gh, l\}$

Phase Parallel

- Proposed by Kai Hwang & Zhiwei Xu
- Similar to the BSP:
  - A parallel program: sequence of phases
  - Next phase cannot begin until all operations in the current phase have finished
  - Three types of phases:
    - » Parallelism phase: the overhead work involved in process management, such as process creation and grouping for parallel processing
    - » Computation phase: local computation (data are available)
    - » Interaction phase: communication, synchronization or aggregation (e.g., reduction and scan)
- Different computation phases may execute different workloads at different speed.

Ref

- BSP: http://www.computingreviews.com/hottopic/hottopic_essay.cfm?htname=BSP