# **Chapter 2**

**PRAM: Matrix** 

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### **Matrix addition**

	A[n	xn]				B[n	xn]				C[n	xn]	
a <sub>11</sub>	a <sub>12</sub>		a <sub>ln</sub>		b <sub>11</sub>	b <sub>12</sub>	•••	b <sub>1n</sub>		C <sub>11</sub>	C <sub>12</sub>		C <sub>ln</sub>
a <sub>21</sub>	a <sub>22</sub>		a <sub>2n</sub>		b <sub>21</sub>	b <sub>22</sub>		b <sub>2n</sub>	_	C <sub>21</sub>	C <sub>22</sub>	•••	C <sub>2n</sub>
				+					_			•••	
a <sub>n1</sub>	a <sub>n2</sub>		$a_{nn}$		b <sub>n1</sub>	b <sub>n2</sub>		b <sub>nn</sub>		C <sub>n1</sub>	C <sub>n2</sub>	•••	C <sub>nn</sub>

### C[nxn]

$c_{11} = a_{11} + b_{11}$	$c_{12} = a_{12} + b_{12}$	 $c_{1n} = a_{1n} + b_{1n}$
$c_{21} = a_{21} + b_{21}$	$c_{22} = a_{22} + b_{22}$	 $c_{2n} = a_{2n} + b_{2n}$
•••		 
$c_{n1} = a_{n1} + b_{n1}$	$c_{n2} = a_{n2} + b_{n2}$	 $c_{nn} = a_{nn} + b_{nn}$



# Matrix addition: PRAM with nxn processors

#### C[nxn]

$c_{11} = a_{11} + b_{11}$	$c_{12} = a_{12} + b_{12}$	•••	$c_{1n} = a_{1n} + b_{1n}$
P <sub>11</sub>	P <sub>12</sub>		P <sub>1n</sub>
$c_{21} = a_{21} + b_{21}$	$c_{22} = a_{22} + b_{22}$	•••	$c_{2n} = a_{2n} + b_{2n}$
P <sub>21</sub>	P <sub>22</sub>		P <sub>2n</sub>
•••	•••	•••	•••
$c_{n1} = a_{n1} + b_{n1}$	$c_{n2} = a_{n2} + b_{n2}$	•••	$c_{nn} = a_{nn} + b_{nn}$
P <sub>n1</sub>	P <sub>n2</sub>		$\mathbf{P}_{ ext{nn}}$

> All (nxn) processors run '+' in parallel: O(1) =>

### P<sub>ij</sub> processor:

o Read:  $a_{ij}$  &  $b_{ij}$ 

Write: C<sub>ii</sub>

#### No overlapping data

CRCW: O(1)

EREW: O(1)



# Matrix addition: PRAM with n processors

➤ All n processors run '+' in parallel in n steps: O(n) ??? =>

#### C[nxn]

Step 1	Step 2	•••	Step n
$c_{11} = a_{11} + b_{11}$	$c_{12} = a_{12} + b_{12}$	•••	$c_{1n} = a_{1n} + b_{1n}$
$\mathbf{P}_1$	$\mathbf{P}_1$	•••	<b>P</b> <sub>1</sub>
$c_{21} = a_{21} + b_{21}$	$c_{22} = a_{22} + b_{22}$	•••	$c_{2n} = a_{2n} + b_{2n}$
$\mathbf{P}_2$	$P_2$		P <sub>2</sub>
•••	•••	•••	•••
			•••
$c_{n1} = a_{n1} + b_{n1}$	$c_{n2} = a_{n2} + b_{n2}$	•••	$c_{nn} = a_{nn} + b_{nn}$
P <sub>n</sub>	P <sub>n</sub>	•••	P <sub>n</sub>

P<sub>ij</sub> processor:

o Read:  $a_{ij} \& b_{ij}$ 

o Write: Cii

No overlapping data

o CRCW: O(n)

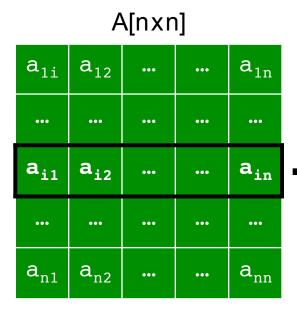
o EREW: O(n)

Your algorithm with k processors (k << n)?

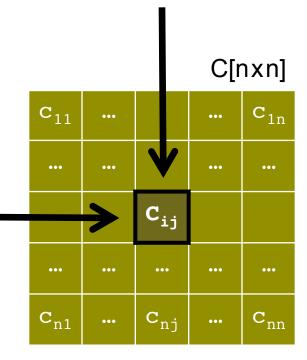


# **Matrix multiplication**

$$C_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + ... + a_{in} * b_{nj}$$



#### B[nx n] $b_{11}$ $\mathbf{b_{1j}}$ $b_{1n}$ $b_{2j}$ ••• ••• ••• ••• ••• ••• ••• ••• ••• ••• $b_{nj}$ $b_{n1}$ $b_{nn}$ •••





- $\mathbf{C}_{ij} = \mathbf{a}_{i1} * \mathbf{b}_{1j} + \mathbf{a}_{i2} * \mathbf{b}_{2j} + ... + \mathbf{a}_{in} * \mathbf{b}_{nj}$
- Vector: A[i<sup>th</sup> row] x B[j<sup>th</sup> column]

Number of operations = 
$$n * opt('*') + (n-1) * opt('+')$$
  
n processors ~  $O(1) + O(log(n))$ 

$$\sim O(\log(n))$$

### C[nxn]

C <sub>11</sub>	•••		•••	C <sub>1n</sub>
•••	•••		•••	•••
		C <sub>ij</sub>		
•••		•••		•••
C <sub>n1</sub>		<b>c</b> <sub>nj</sub>		<b>c</b> <sub>nn</sub>



# PRAM Matrix multiplication

```
C<sub>ij</sub> using n processors: O(log(n))
C[nxn]: (n*n)C<sub>ij</sub>

n processors: O(n<sup>2</sup> * log(n))

n<sup>3</sup> processors: O(log(n))

n<sup>2</sup> processors???
```

### $C[n \times n]$

C <sub>11</sub>	•••		•••	C <sub>1n</sub>
•••	•••		•••	•••
		C <sub>ij</sub>		
				•••
C <sub>n1</sub>		C <sub>nj</sub>		C <sub>nn</sub>

# PRAM

- C<sub>ij</sub> using n processors: O(log(n))

  C[nxn]: (n\*n)C<sub>ij</sub>

  n processors: O(n<sup>2</sup> \* log(n))

  n<sup>3</sup> processors: O(log(n))

  n<sup>2</sup> processors???
- => Difference between CRCW & EREW???

### C[nxn]

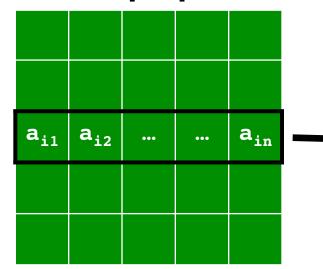
<b>c</b> <sub>11</sub>	•••		•••	C <sub>1n</sub>
•••	•••		•••	•••
		C <sub>ij</sub>		
				•••
C <sub>n1</sub>		C <sub>nj</sub>		<b>c</b> <sub>nn</sub>



### Row

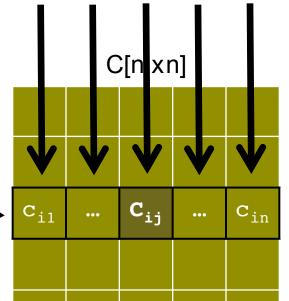
$$C_{i1} = \begin{bmatrix} a_{i1} * b_{11} + a_{i2} * b_{21} + ... + a_{in} * b_{n1} \\ C_{i2} = a_{i1} * b_{12} + a_{i2} * b_{22} + ... + a_{in} * b_{n2} \\ ... \\ C_{in} = \begin{bmatrix} a_{i1} * b_{1n} + a_{i2} * b_{2n} + ... + a_{in} * b_{nn} \end{bmatrix}$$

### A[nxn]



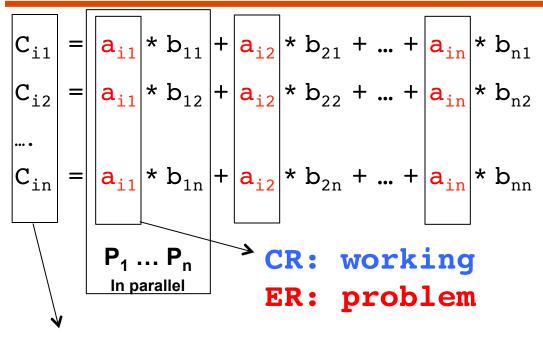
#### B[nxn]

b <sub>11</sub>	•••	b <sub>1j</sub>	•••	b <sub>1n</sub>
•••	•••	b <sub>2j</sub>	•••	•••
•••	•••	•••	•••	•••
•••	•••	•••	•••	•••
b <sub>n1</sub>	•••	b <sub>nj</sub>	•••	b <sub>nn</sub>





# **Concurrent Read: in row**



CW: working

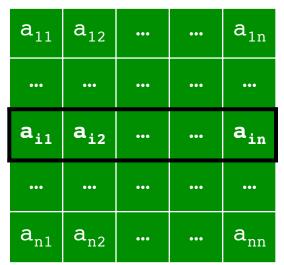
EW: working



# Column

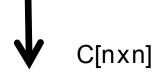
$$C_{1j} = a_{11} * b_{1j} + a_{12} * b_{2j} + ... + a_{1n} * b_{nj}$$
 $C_{2j} = a_{21} * b_{1j} + a_{22} * b_{2j} + ... + a_{2n} * b_{nj}$ 
...
 $C_{nj} = a_{n1} * b_{1j} + a_{n2} * b_{2j} + ... + a_{nn} * b_{nj}$ 

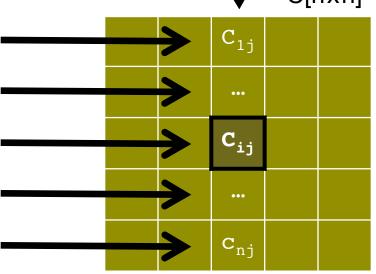
### A[nxn]



#### B[nxn]

b <sub>11</sub>	•••	b <sub>1j</sub>	•••	b <sub>1n</sub>
•••	•••	b <sub>2j</sub>	•••	•••
•••	•••	•••	•••	•••
•••	•••	•••	•••	•••
b <sub>n1</sub>	•••	b <sub>nj</sub>	•••	b <sub>nn</sub>







### Column

$$C_{1j} = \begin{bmatrix} a_{11} * b_{1j} + a_{12} * b_{2j} + ... + a_{1n} * b_{nj} \\ C_{2j} = a_{21} * b_{1j} + a_{22} * b_{2j} + ... + a_{2n} * b_{nj} \\ ... \\ C_{nj} = \begin{bmatrix} a_{n1} * b_{1j} + a_{n2} * b_{2j} + ... + a_{nn} * b_{nj} \\ \end{bmatrix}$$

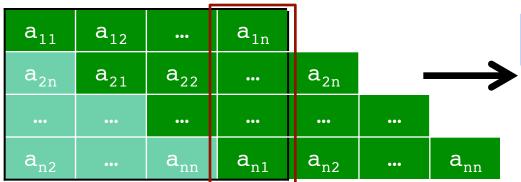
$$P_{1} ... P_{n}$$
In parallel

$$CR: working$$
ER: problem

CW: working

EW: working

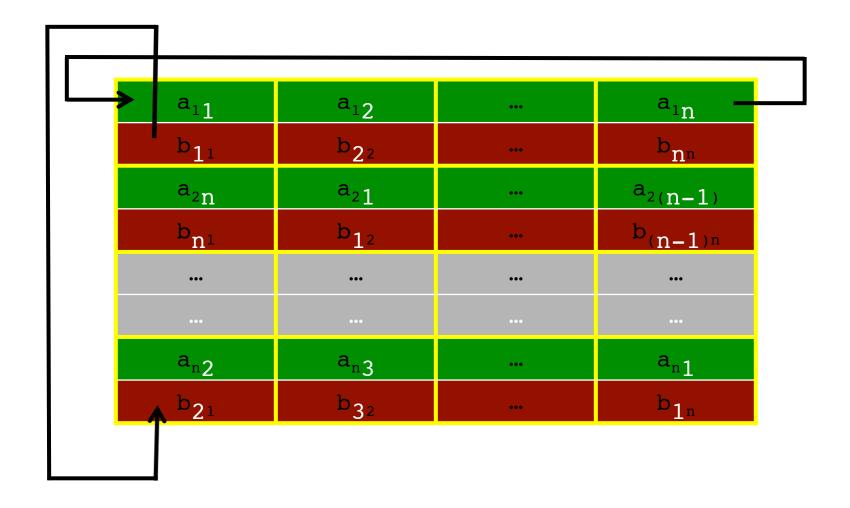




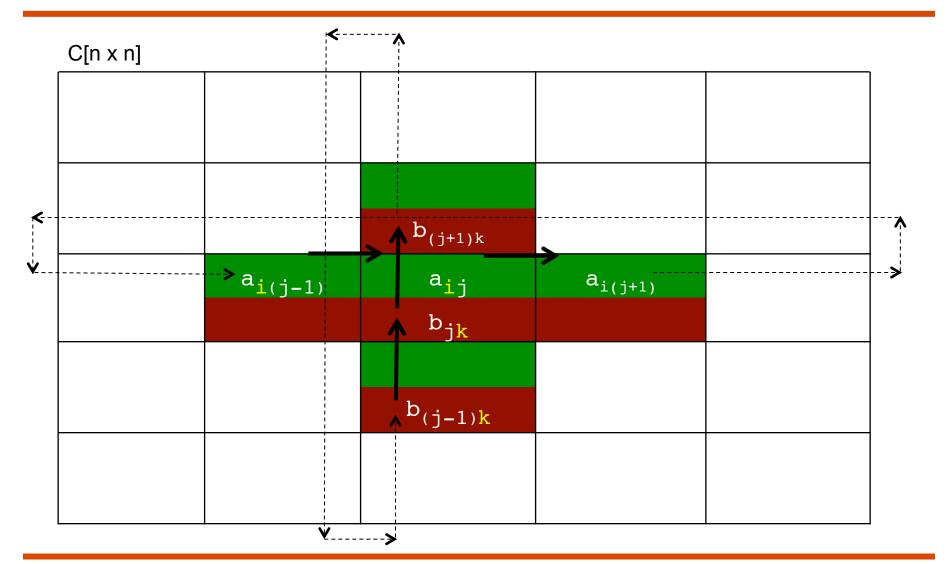
Why not use  $n^3 > n^2$  processors with EREW?

b <sub>n1</sub>		<b>^</b>	
b <sub>(n-1)1</sub>		ı	
	•••	$b_{n(n-1)}$	
b <sub>11</sub>			b <sub>nn</sub>
b <sub>n1</sub>		b <sub>2(n-1)</sub>	•••
		b <sub>1(n-1)</sub>	b <sub>2n</sub>
b <sub>21</sub>	•••	$b_{n(n-1)}$	b <sub>1n</sub>











a <sub>11</sub>	<b>a</b> <sub>12</sub>		a <sub>1n</sub>				
a <sub>2n</sub>	a <sub>21</sub>	a <sub>22</sub>		a <sub>2n</sub>	•		
	•••						
a <sub>n2</sub>	•••	a <sub>nn</sub>	$a_{n1}$	a <sub>n2</sub>	•••	a <sub>nn</sub>	

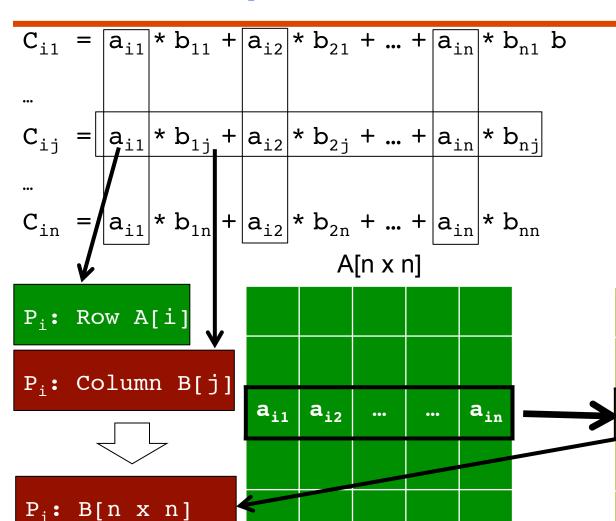
P <sub>11</sub>	P <sub>12</sub>		P <sub>1n</sub>
P <sub>21</sub>	P <sub>22</sub>		P <sub>2n</sub>
		•••	•••
P <sub>n1</sub>	P <sub>n2</sub>	•••	P <sub>nn</sub>

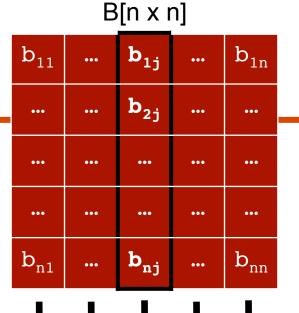
- Each P<sub>ij</sub>
  - > Run "\*" in n steps sequentially
  - $\triangleright$  Run "+" in n steps
- O(n)

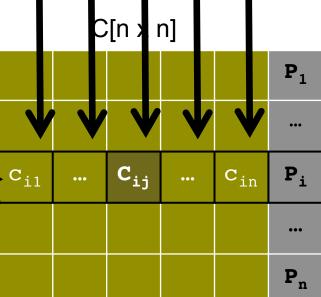
- nl	<b>−</b> n2	•••	- nn
b <sub>n1</sub>		<b>1</b>	
b <sub>(n-1)1</sub>	•••		_
	•••	$b_{n(n-1)}$	
b <sub>11</sub>		•••	b <sub>nn</sub>
b <sub>n1</sub>		b <sub>2(n-1)</sub>	
		b <sub>1(n-1)</sub>	b <sub>2n</sub>
b <sub>21</sub>		$b_{n(n-1)}$	b <sub>1n</sub>



# PRAM with n processors









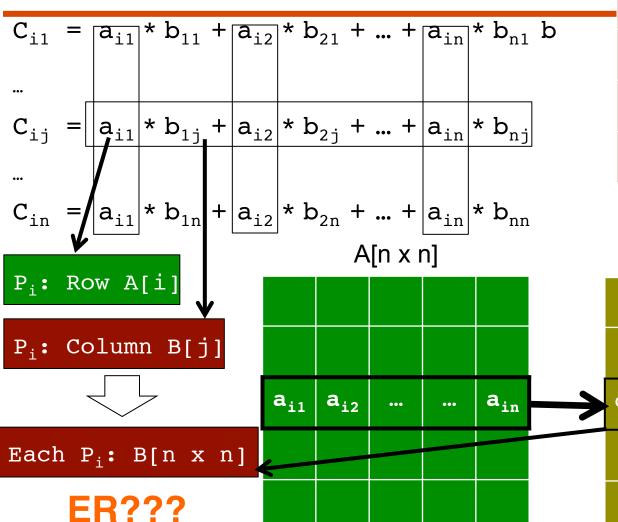
# PRAM CRCW with n processors

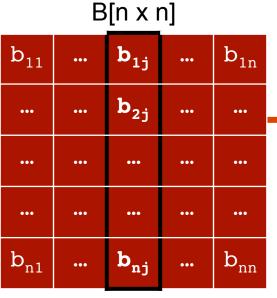
### Each P<sub>i</sub>:

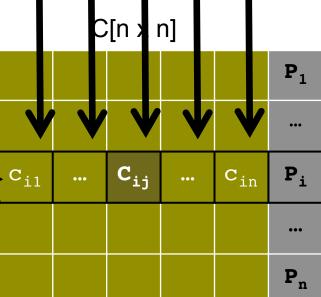
- $C_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + ... + a_{in} * b_{nj} => O(n)$
- Row C[i]:  $O(n^2)$



# PRAM EREW with n processors









# PRAM EREW with n processors

### Each P<sub>i</sub>:

•  $C_{ij} = a_{i1} * b_{1j} + a_{i2} * b_{2j} + ... + a_{in} * b_{nj} => O(n)$ 

• Row C[i]:  $O(n^2)$ 

<b>/</b>							<b>-</b>	
Ÿ <b>&gt;</b>	B[1]	B[2]	•••	B[k]	•••	B[n]	<b>&gt;</b>	
A[1]	C <sub>11</sub>	C <sub>12</sub>	•••	C <sub>1k</sub>		C <sub>1n</sub>	$P_1$	
A[2]	C <sub>21</sub>	C <sub>22</sub>		C <sub>2k</sub>		C <sub>2n</sub>	$P_2$	
							•••	
A[k]	C <sub>i1</sub>	C <sub>i2</sub>		$C_{\mathrm{kk}}$		C <sub>in</sub>	$P_k$	
							•••	
A[n]	C <sub>n1</sub>	C <sub>n2</sub>		C <sub>nk</sub>		C <sub>nn</sub>	P <sub>n</sub>	



# PRAM EREW with k processors (k << n)

### Each P<sub>i</sub>:

- 1 step: O(k\*n) =>
- 1 phase (n/k steps):  $O(k*n*n/k) \sim O(n^2)$
- n/k phases:  $O(n^2*n/k) \sim O(n^3/k)$

Step 1		Step n/k							
	_	B[1]	•••	B[k]		B[n-k]	<b></b>	B[n]	
Phase_1	A[1]	C <sub>11</sub>		$\mathtt{C_{1k}}$		C <sub>1(n-k)</sub>		C <sub>1n</sub>	P <sub>1</sub>
			k x k						
	A[k]	$C_{\mathrm{k1}}$		$C_{\mathrm{kk}}$		C <sub>k(n-k)</sub>		$C_{k(n-k)}$	$P_{k}$
Phase n/k									
	A[n-k]	C <sub>(n-k)1</sub>		$C_{(n-k)k}$		$C_{(n-k)(n-k)}$		$C_{(n-k)n}$	<b>P</b> <sub>1</sub>
	A[n]	C <sub>n1</sub>		C <sub>nk</sub>		$C_{n(n-k)}$		C <sub>nn</sub>	$P_k$