

Chapter 6

Processor Organization

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Outline

❑ Criteria:

- Diameter, bisection width, etc.

❑ Processor Organizations:

- Mesh, binary tree, hypertree, pyramid, butterfly, hypercube, shuffle-exchange



Criteria

❑ Diameter

- The largest distance between two nodes
- Lower diameter is better

❑ Bisection width

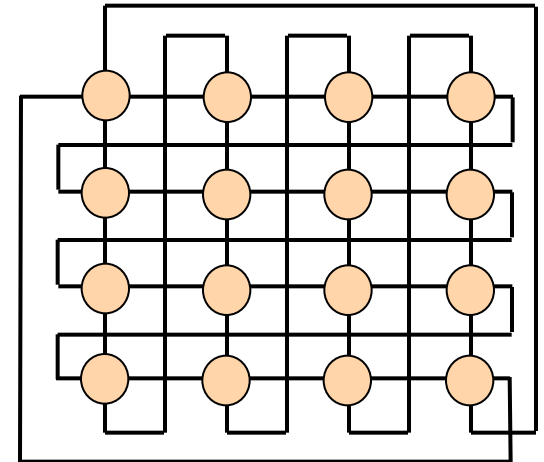
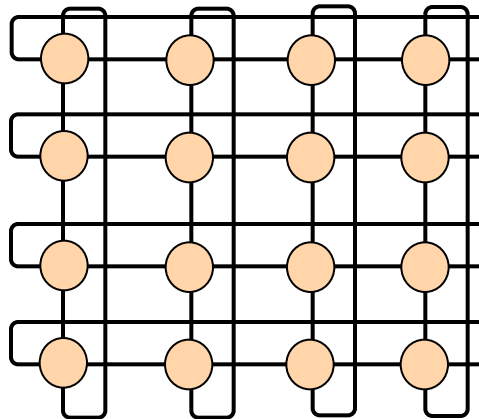
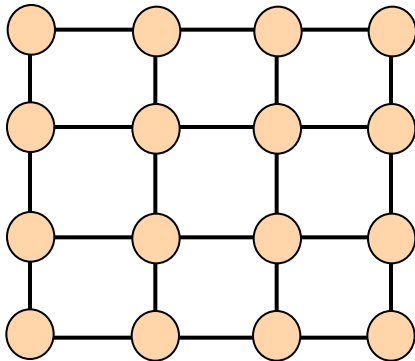
The minimum number of edges that must be removed in order to divide the network into two halves (within one)

❑ Number of edges per node

❑ Maximum edge length

Mesh (1)

- ❑ Q-dimensional lattice
- ❑ Communication is allowed only between neighboring nodes. Interior nodes communicate with $2q$ other nodes.





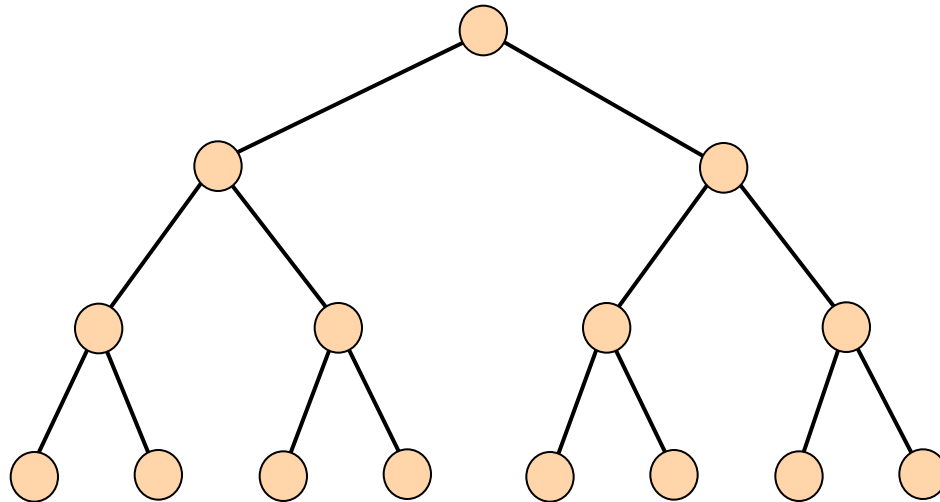
Mesh (2)

- Q-dimensional mesh with k^q nodes
 - Diameter: $q(k-1)$
 - Bisection width: k^{q-1}
 - The maximum number of edges per node: $2q$
 - The maximum edge length is a constant

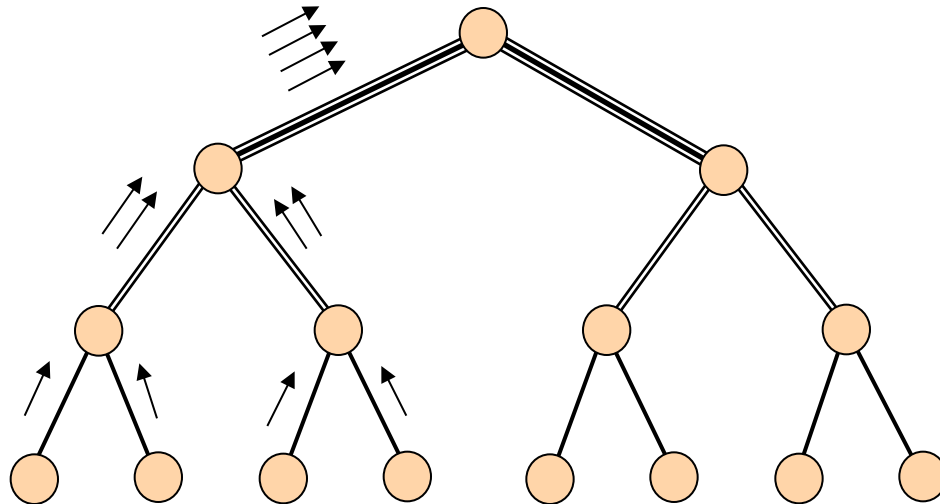


Binary Tree

- Depth $k-1$: 2^k-1 nodes
- Diameter: $2(k-1)$
- Bisection width: 1
- Length of the longest edge: increasing

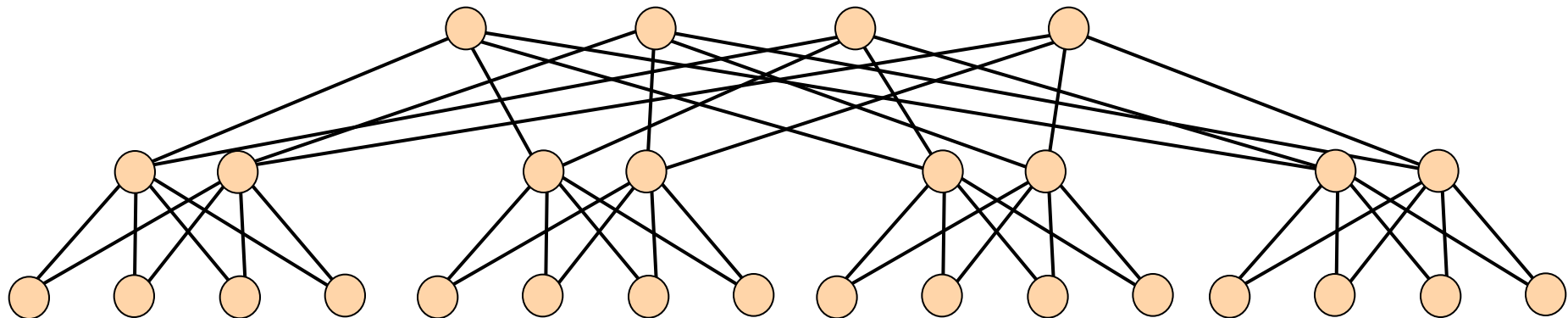
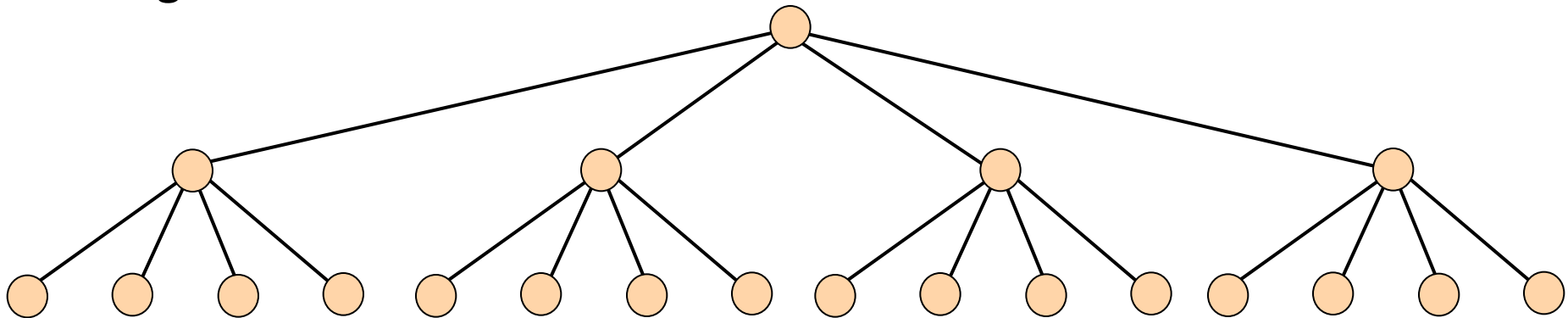


- Bandwidth problem on binary tree



Hypertree (1)

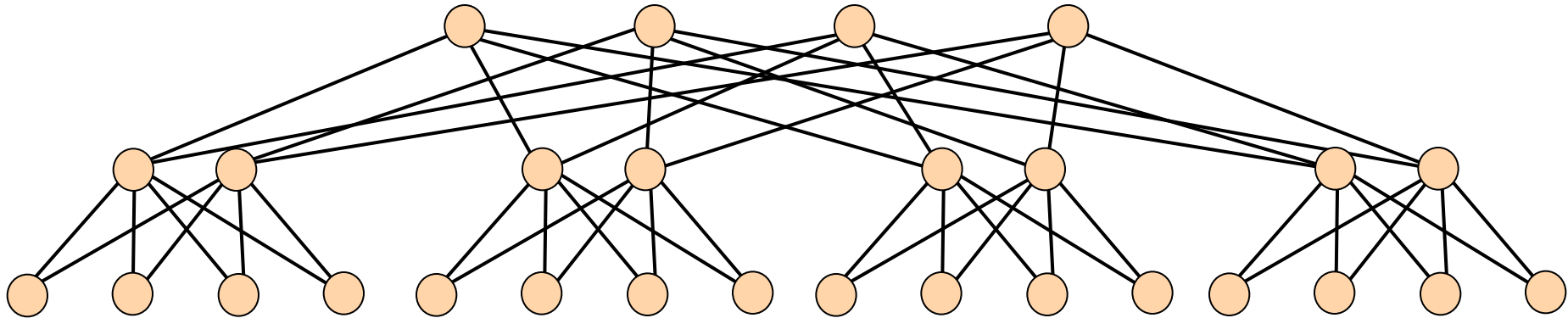
- Hypertree of degree k and depth d : a complete k -ary tree of height d .



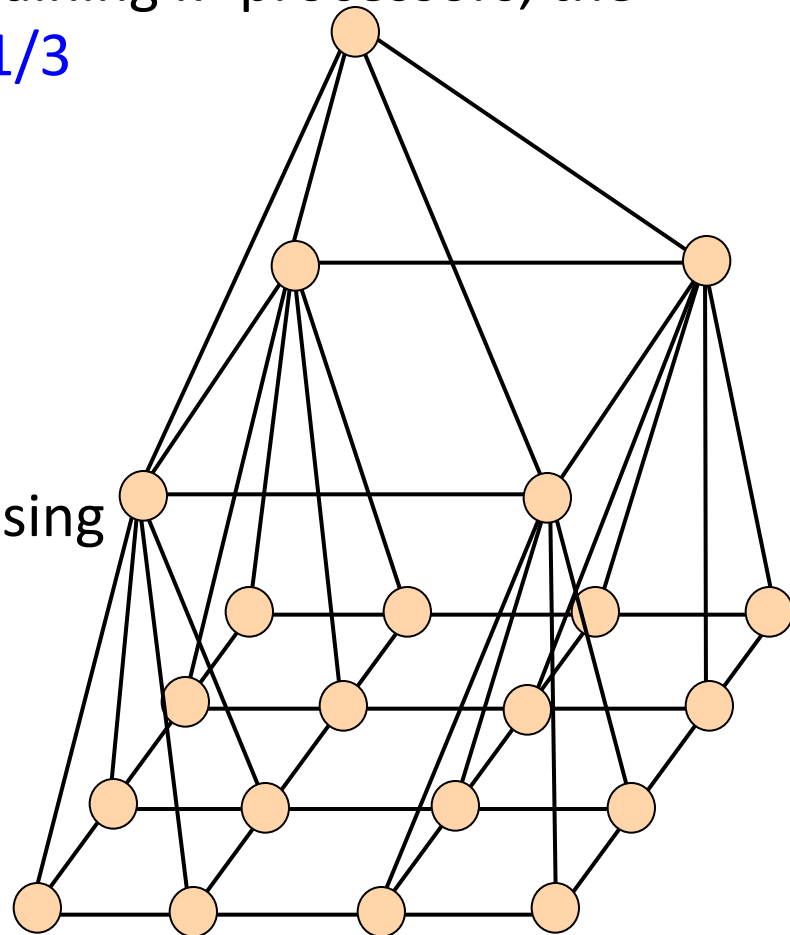


Hypertree (2)

- A 4-ary hypertree with depth d has 4^d leaves and $2^d(2^{d+1}-1)$ nodes in all
 - Diameter: $2d$
 - Bisection width: 2^{d+1}
 - The number of edges per node ≤ 6
 - Length of the longest edge: increasing



- ❑ Size k^2 : base a 2D mesh network containing k^2 processors, the total number of processors = $(\frac{4}{3})k^2 - \frac{1}{3}$
- ❑ A pyramid of size k^2 :
 - Diameter: $2\log k$
 - Bisection width: $2k$
 - Maximum of links per node: 9
 - Length of the longest edge: increasing

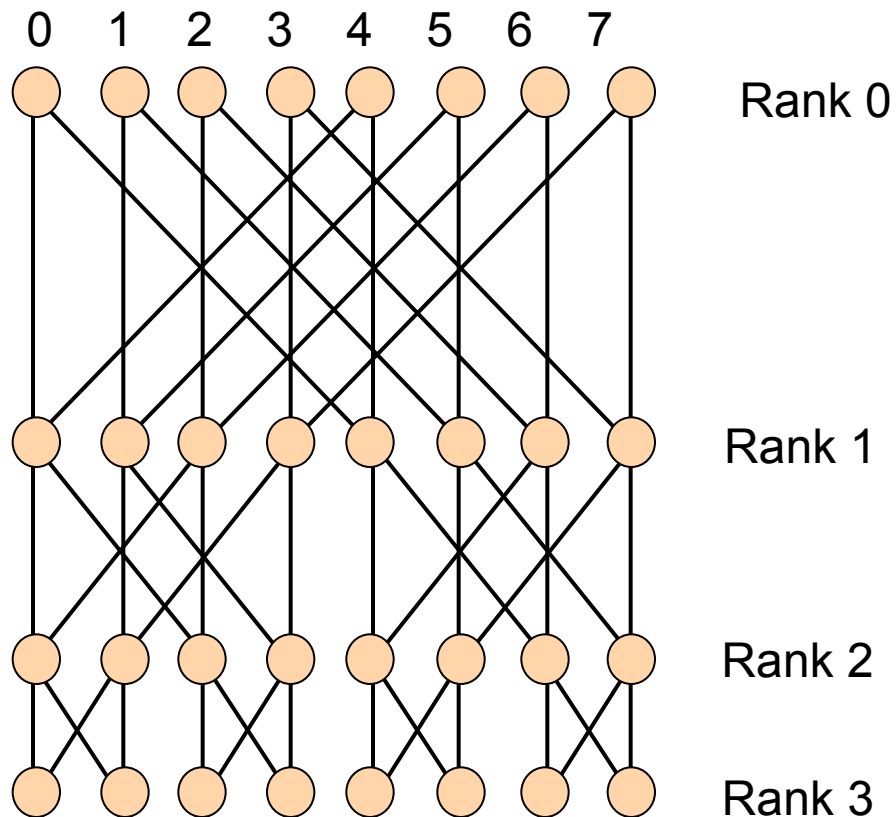




Butterfly (1)

- ❑ $(k+1)2^k$ nodes divided into $k+1$ rows (rank), each contains $n=2^k$ nodes.
- ❑ Ranks are labeled 0 through k
- ❑ $\text{Node}(i,j)$: j -th node on the i -th rank
- ❑ $\text{Node}(i,j)$ is connected to two nodes on rank $i-1$: $\text{node}(i-1,j)$ and $\text{node}(i-1,m)$, where m is the integer found by inverting the i -th most significant bit in the binary representation of j
- ❑ If $\text{node}(i,j)$ is connected to $\text{node}(i-1,m)$, then $\text{node}(i,m)$ is connected to $\text{node}(i-1,j)$
- ❑ Diameter= $2k$
- ❑ Bisection width= 2^k
- ❑ Length of the longest edge: increasing

Butterfly (2)



Node(1,5): $i=1, j=5$

$j = 5 = \mathbf{101}$ (binary)

↓ $i=1$

$\mathbf{001} = 1$

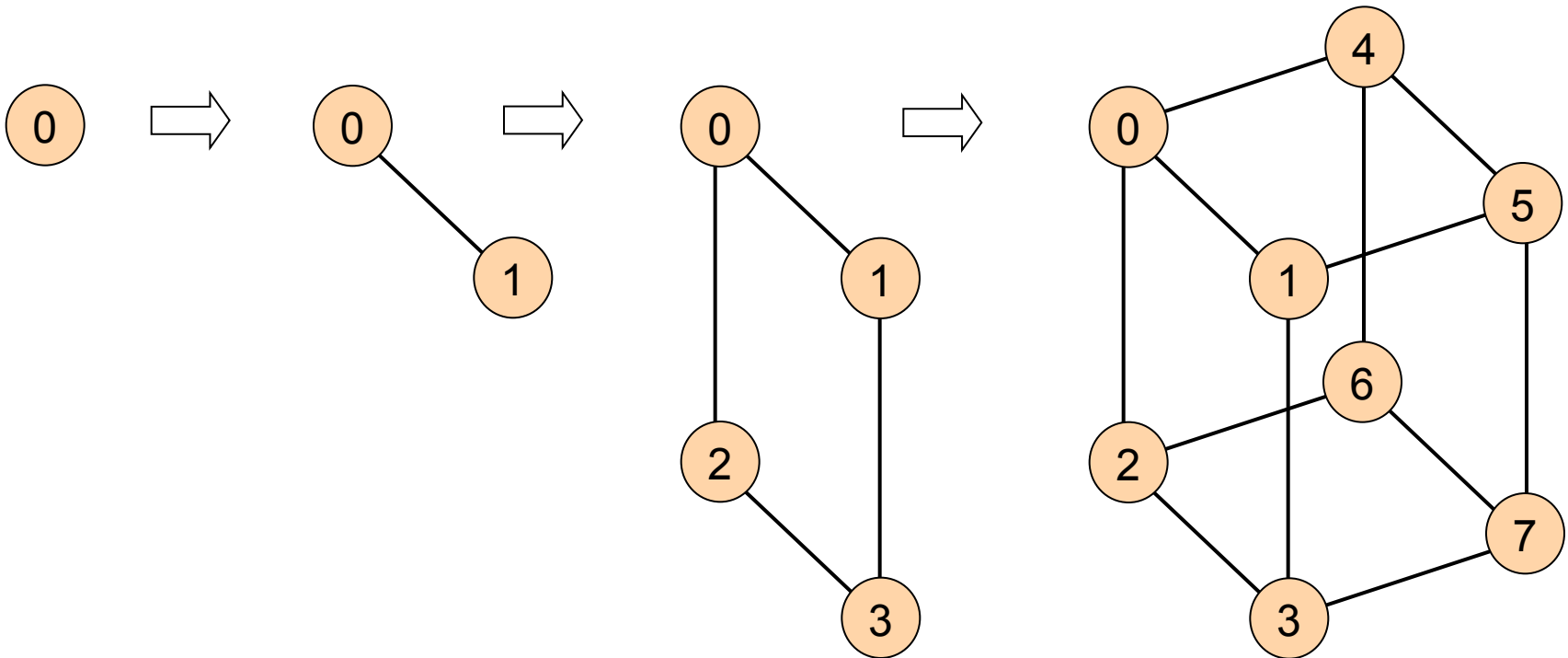
Node(1,5) is connected to
node(0,1)



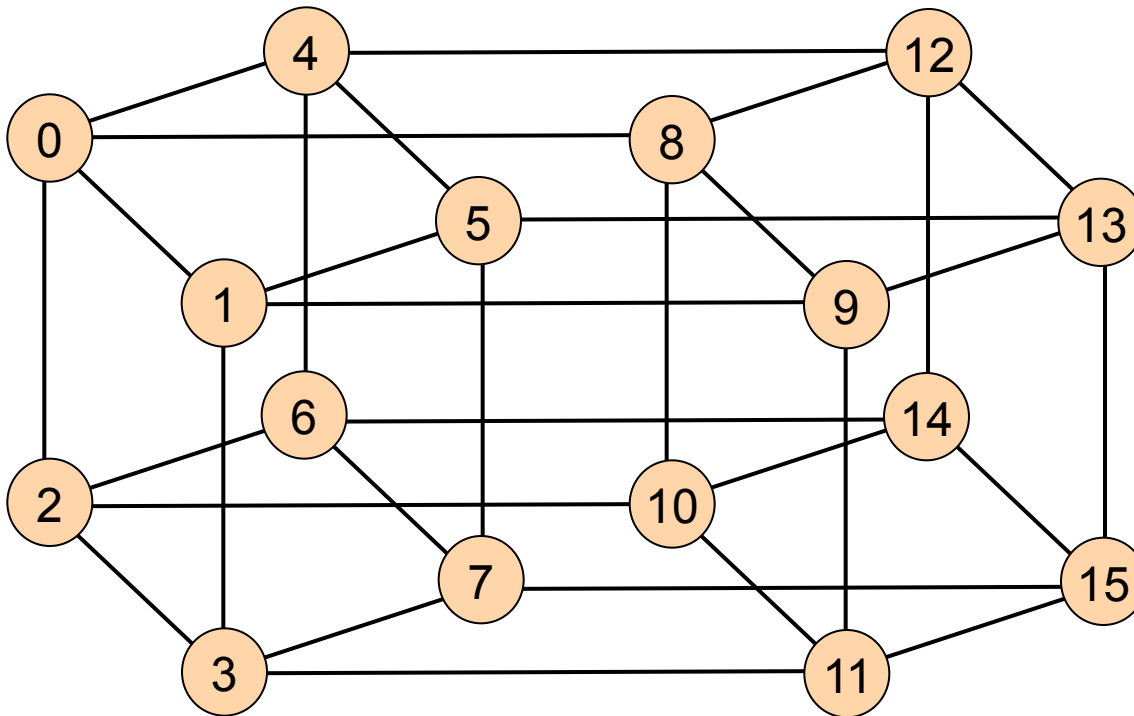
Hypercube (1)

- ❑ 2^k nodes form a k -dimensional hypercube
- ❑ Nodes are labeled $0, 1, 2, \dots, 2^k-1$
- ❑ Two nodes are adjacent if their labels differ in exactly one bit position
- ❑ Diameter= k
- ❑ Bisection width= 2^{k-1}
- ❑ Number of edges per node is k
- ❑ Length of the longest edge: increasing

Hypercube (2)



Hypercube (3)



□ 5 = **0101**

□ 1 = **0001**

□ 4 = **0100**

□ 13 = **1101**



Others

- ❑ Torus

 - <http://clusterdesign.org/torus/>

 - <http://www.fujitsu.com/global/about/tech/k/whatis/network/>

- ❑ Cube-Connected cycles

- ❑ Shuffle-Exchange

- ❑ De Bruijn