Chapter 6

Synchronous Computations

Synchronous Computations

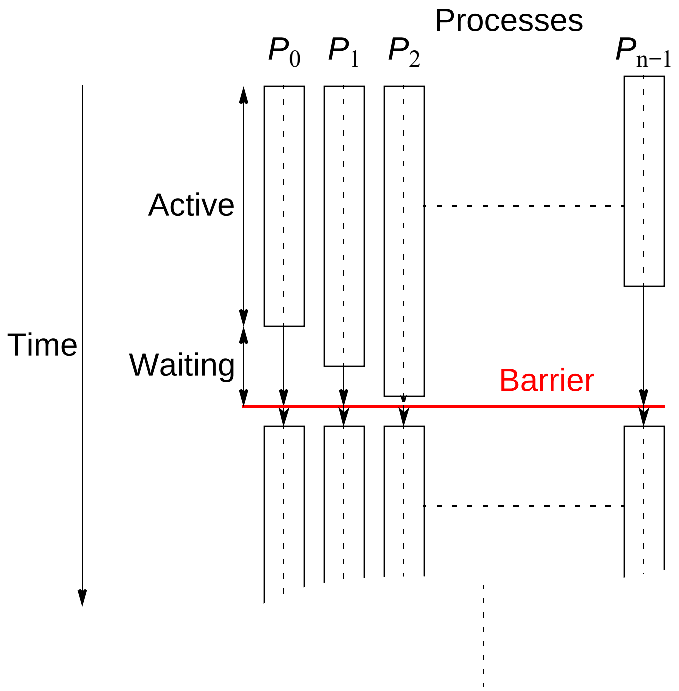
In a (fully) synchronous application, all the processes synchronized at regular points.

Barrier

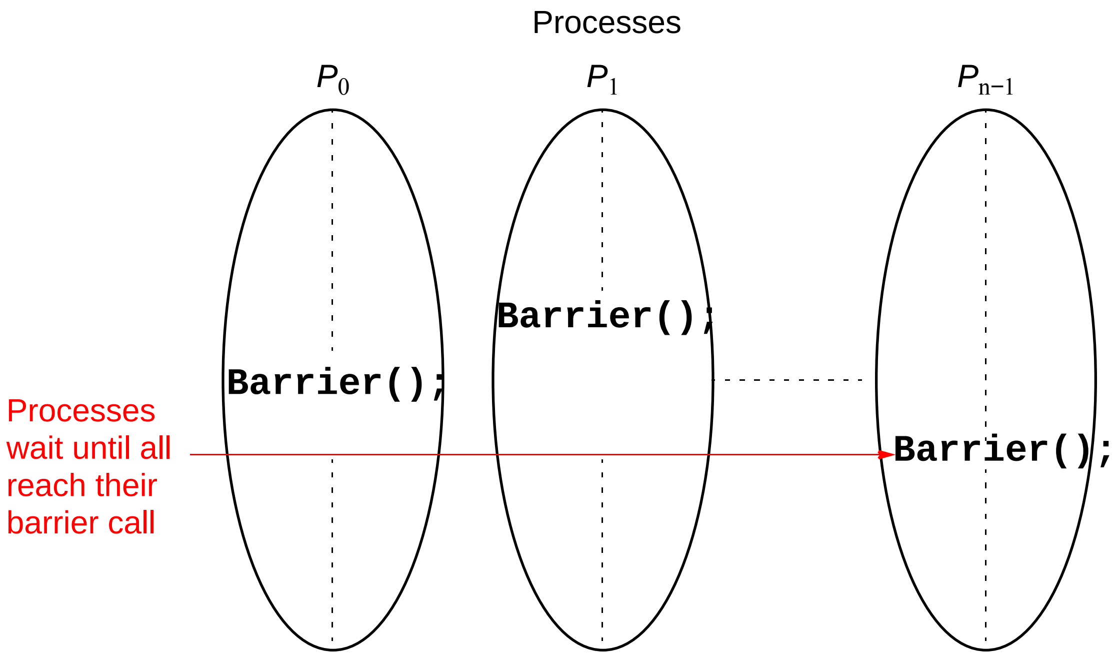
A basic mechanism for synchronizing processes - inserted at the point in each process where it must wait.

All processes can continue from this point when all the processes have reached it (or, in some implementations, when a stated number of processes have reached this point).

Processes reaching barrier at different times



In message-passing systems, barriers provided with library routines:



MPI

`MPI_Barrier()`

Barrier with a named communicator being the only parameter. Called by each process in the group, blocking until all members of the group have reached the barrier call and only returning then.

PVM

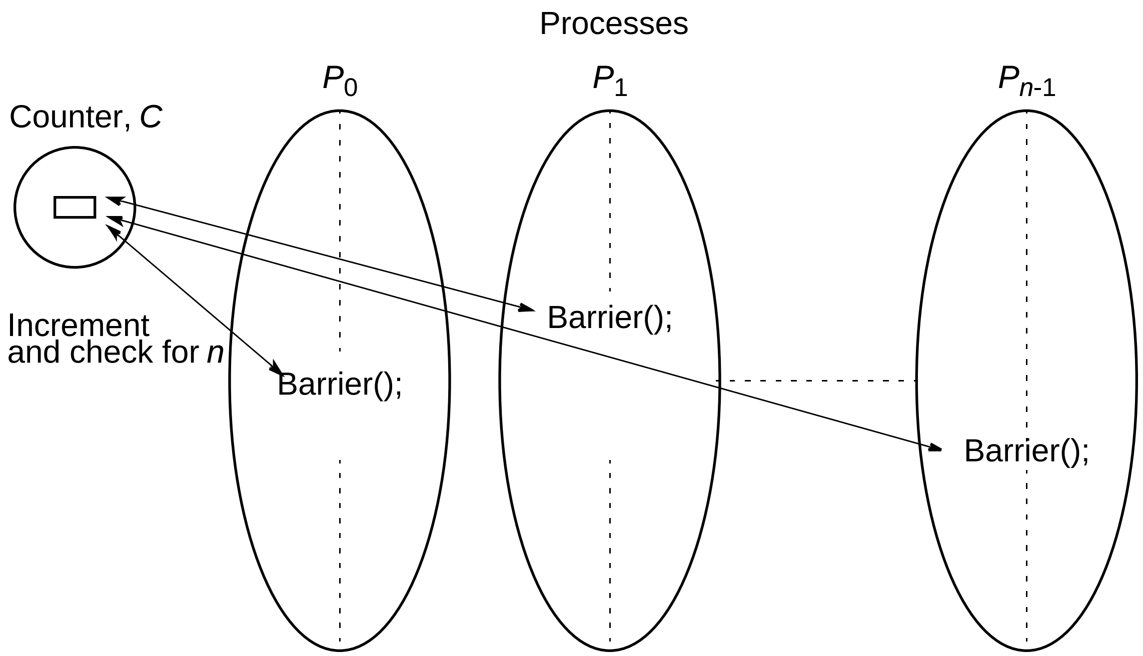
`pvm_barrier()`

similar barrier routine used with a named group of processes.

PVM has the unusual feature of specifying the number of processes that must reach the barrier to release the processes.

Barrier Implementation

Centralized counter implementation (*linear barrier*):



Counter-based barriers often have two phases:

- A process enters arrival phase and does not leave this phase until all processes have arrived in this phase.
- Then processes move to departure phase and are released.

Good implementations must take into account that a barrier might be used more than once in a process. Might be possible for a process to enter barrier for a second time before previous processes have left barrier for the first time. Two-phase handles this scenario.

Example code:

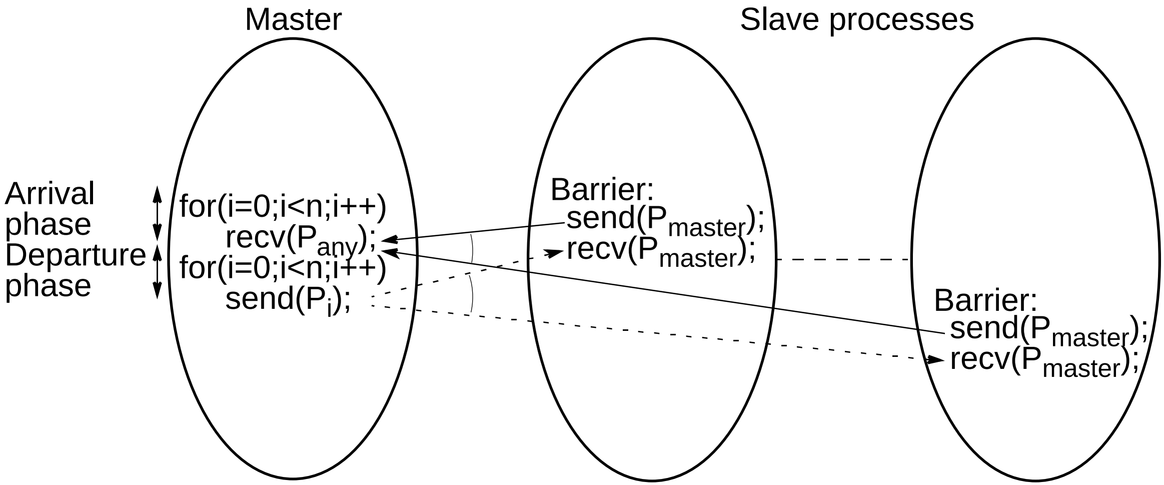
Master:

```
for (i = 0; i < n; i++)/*count slaves as they reach
barrier*/
   recv(P_any);
for (i = 0; i < n; i++)/* release slaves */
   send(P_i);
```

Slave processes:

```
send(P_master);
recv(P_master);
```

Barrier implementation in a message-passing system



Tree Implementation

More efficient. Suppose 8 processes, P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7:

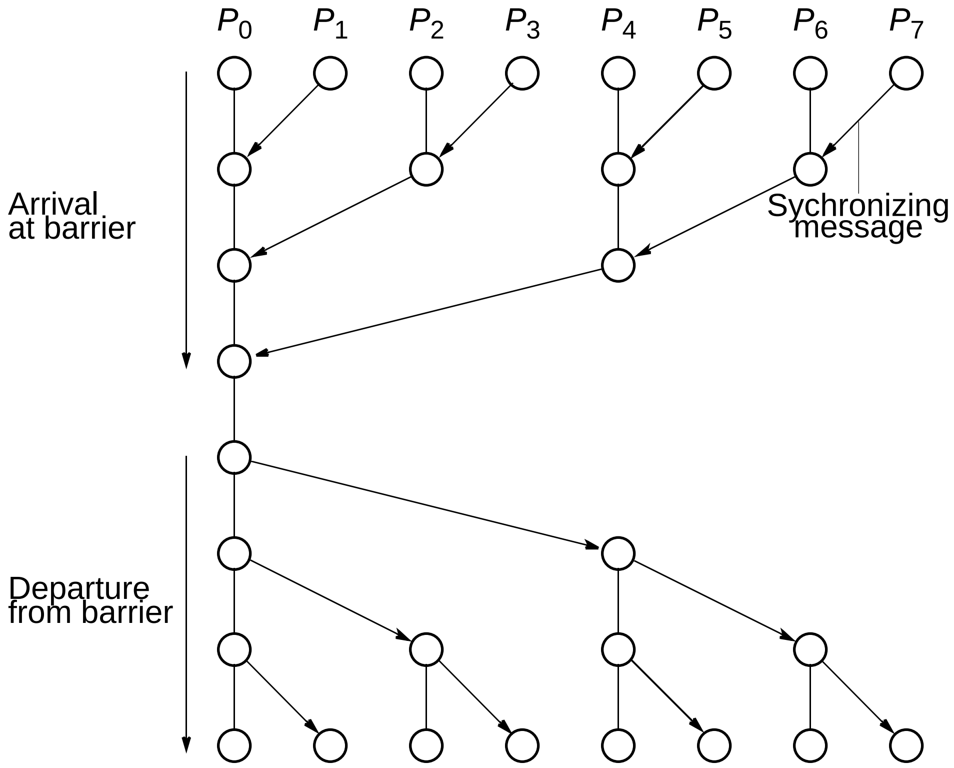
First stage: P_1 sends message to P_0; (when P_1 reaches its barrier)
P_3 sends message to P_2; (when P_3 reaches its barrier)
P_5 sends message to P_4; (when P_5 reaches its barrier)
P_7 sends message to P_6; (when P_7 reaches its barrier)

Second stage: P_2 sends message to P_0; (P_2 & P_3 reached their barrier)
P_6 sends message to P_4; (P_6 & P_7 reached their barrier)

Third stage: P_4 sends message to P_0; (P_4, P_5, P_6, & P_7 reached barrier)
P_0 terminates arrival phase;
(when P_0 reaches barrier & received message from P_4)

Release with a reverse tree construction.

Tree Barrier

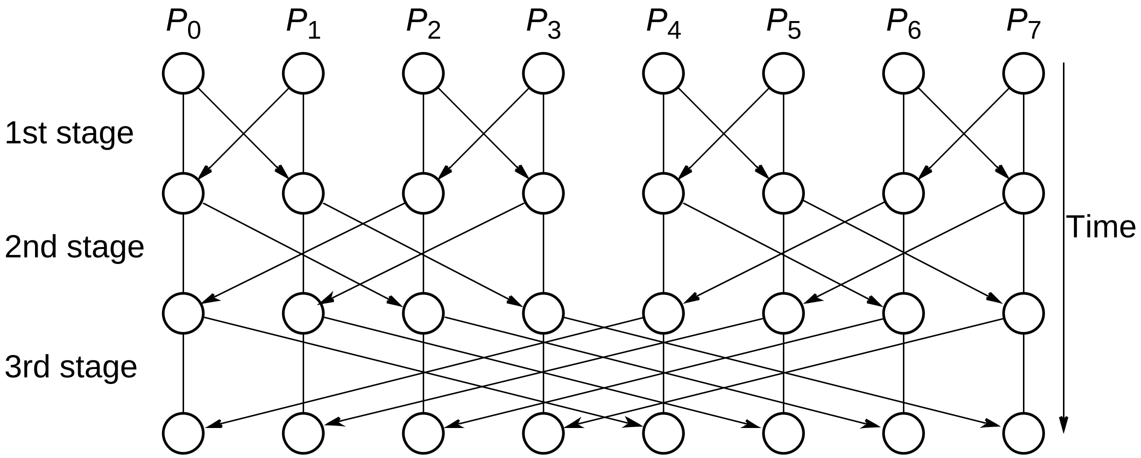


Butterfly Barrier

First stage $P_0 \leftrightarrow P_1, P_2 \leftrightarrow P_3, P_4 \leftrightarrow P_5, P_6 \leftrightarrow P_7$

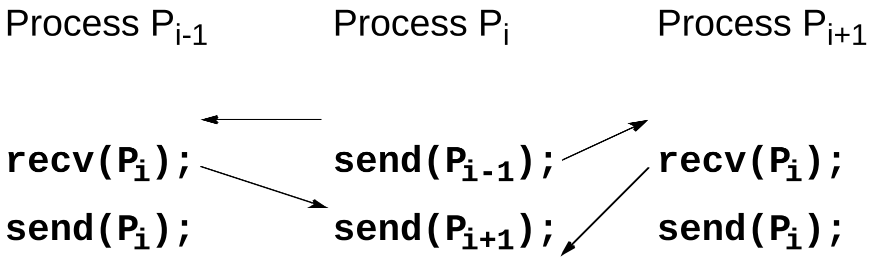
Second stage $P_0 \leftrightarrow P_2, P_1 \leftrightarrow P_3, P_4 \leftrightarrow P_6, P_5 \leftrightarrow P_7$

Third stage $P_0 \leftrightarrow P_4, P_1 \leftrightarrow P_5, P_2 \leftrightarrow P_6, P_3 \leftrightarrow P_7$



Local Synchronization

Suppose a process P_i needs to be synchronized and to exchange data with process P_{i-1} and process P_{i+1} before continuing:



```
Process Pi-1        Process Pi          Process Pi+1

recv(Pi);           send(Pi-1);         recv(Pi);
send(Pi);           send(Pi+1);         send(Pi);
```

Not a perfect three-process barrier because process P_{i-1} will only synchronize with P_i and continue as soon as P_i allows. Similarly, process P_{i+1} only synchronizes with P_i.

Deadlock

When a pair of processes each send and receive from each other, deadlock may occur.

Deadlock will occur if both processes perform the send, using synchronous routines first (or blocking routines without sufficient buffering). This is because neither will return; they will wait for matching receives that are never reached.

A Solution

Arrange for one process to receive first and then send and the other process to send first and then receive.

Example

Linear pipeline, deadlock can be avoided by arranging so the even-numbered processes perform their sends first and the odd-numbered processes perform their receives first.

Combined deadlock-free blocking `sendrecv()` routines

MPI provides **`MPI_Sendrecv()`** and **`MPI_Sendrecv_replace()`**.

Example

$$\begin{tikzcd}[column sep=small]
\text{Process } P_{i-1} & & \text{Process } P_i & & \text{Process } P_{i+1} \\
\texttt{sendrecv}(P_i); \arrow[rr, leftrightarrow] & & \texttt{sendrecv}(P_{i-1}); & & \\
& & \texttt{sendrecv}(P_{i+1}); \arrow[rr, leftrightarrow] & & \texttt{sendrecv}(P_i);
\end{tikzcd}$$

sendrev()s have 12 parameters!

Synchronized Computations

Can be classififed as:

- Fully synchronous

or

- Locally synchronous

In fully synchronous, all processes involved in the computation must be synchronized.

In locally synchronous, processes only need to synchronize with a set of logically nearby processes, not all processes involved in the computation

Fully Synchronized Computation Examples

Data Parallel Computations

Same operation performed on different data elements simultaneously; i.e., in parallel.

Particularly convenient because:

- Ease of programming (essentially only one program).
- Can scale easily to larger problem sizes.
- Many numeric and some non-numeric problems can be cast in a data parallel form.

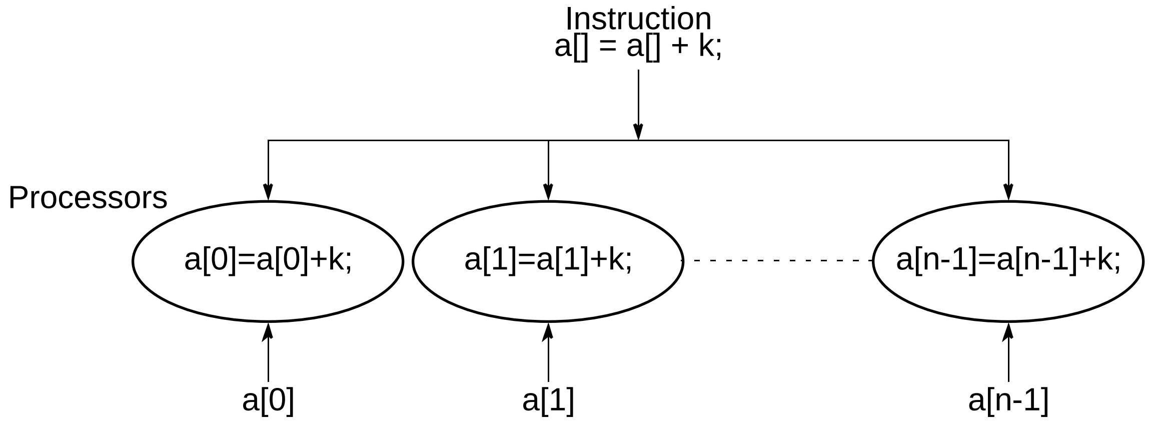
Example

To add the same constant to each element of an array:

```
for (i = 0; i < n; i++)
  a[i] = a[i] + k;
```

The statement **`a[i] = a[i] + k`** could be executed simultaneously by multiple processors, each using a different index **`i`** **($0 < i \leq n$)**.

Data Parallel Computation



Forall construct

Special “parallel” construct in parallel programming languages to specify data parallel operations

Example

```
forall (i = 0; i < n; i++) {
  body
}
```

states that *n* instances of the statements of the body can be executed simultaneously.

One value of the loop variable **`i`** is valid in each instance of the body, the first instance has **`i`** = 0, the next **`i`** = 1, and so on.

To add **k** to each element of an array, **a**, we can write

```
forall (i = 0; i < n; i++)
  a[i] = a[i] + k;
```

Data parallel technique applied to multiprocessors and multicomputers.

Example

To add **k** to the elements of an array:

```
i = myrank;
a[i] = a[i] + k;/* body */
barrier(mygroup);
```

where **`myrank`** is a process rank between 0 and $n - 1$.

Data Parallel Example - Prefix Sum Problem

Given a list of numbers, x_0, …, x_{n-1}, compute all the partial summations (i.e., $x_0 + x_1$; $x_0 + x_1 + x_2$; $x_0 + x_1 + x_2 + x_3$; …).

Can also be defined with associative operations other than addition. Widely studied. Practical applications in areas such as processor allocation, data compaction, sorting, and polynomial evaluation.

Data parallel method of adding all partial sums of 16 numbers

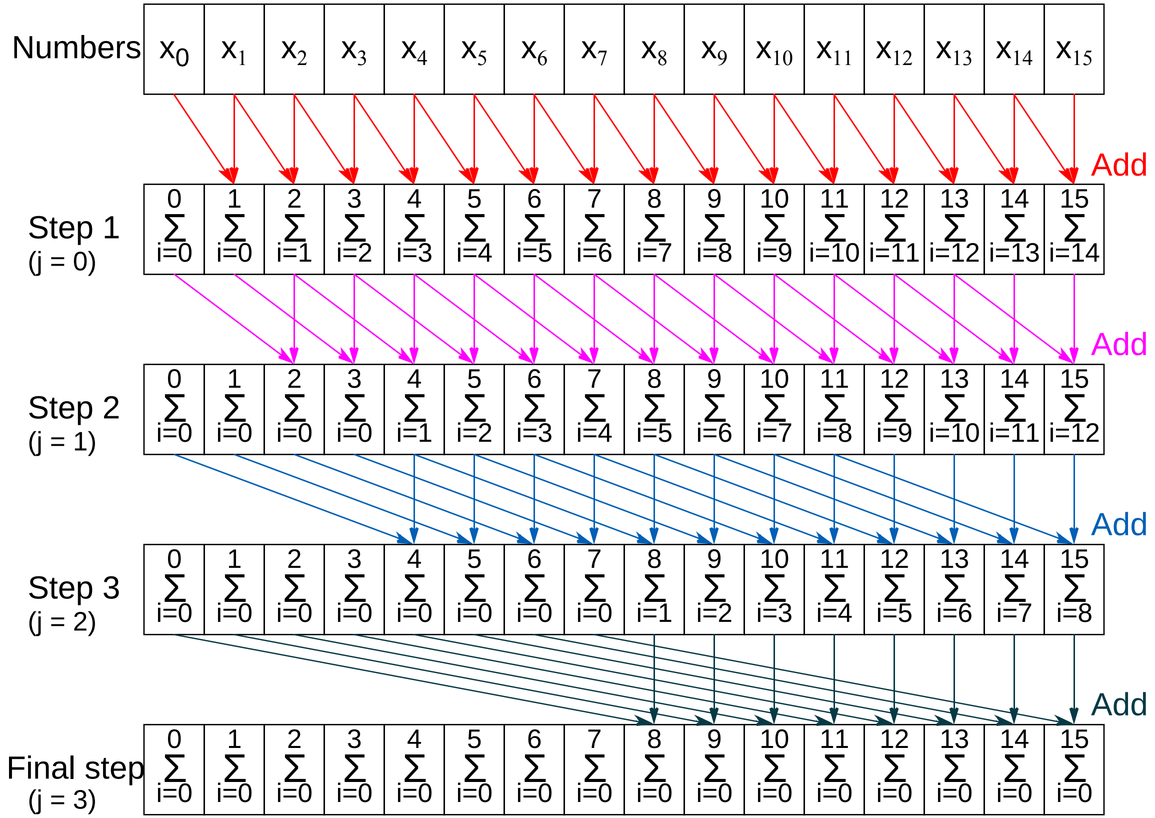
Sequential code

```
for (j = 0; j < log(n); j++) /* at each step, add*/
   for (i = 2^j; i < n; i++) /* to accumulating sum */
     x[i] = x[i] + x[i - 2^j];
```

Parallel code

```
for (j = 0; j < log(n); j++) /* at each step, add */
   forall (i = 0; i < n; i++)/*to sum */
      if (i >= 2^j) x[i] = x[i] + x[i - 2^j];
```

Data parallel prefix sum operation


Numbers
x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10} x_{11} x_{12} x_{13} x_{14} x_{15}
Add
Step 1 (j = 0)
$\sum_{i=0}^{0}$ $\sum_{i=0}^{1}$ $\sum_{i=1}^{2}$ $\sum_{i=2}^{3}$ $\sum_{i=3}^{4}$ $\sum_{i=4}^{5}$ $\sum_{i=5}^{6}$ $\sum_{i=6}^{7}$ $\sum_{i=7}^{8}$ $\sum_{i=8}^{9}$ $\sum_{i=9}^{10}$ $\sum_{i=10}^{11}$ $\sum_{i=11}^{12}$ $\sum_{i=12}^{13}$ $\sum_{i=13}^{14}$ $\sum_{i=14}^{15}$
Add
Step 2 (j = 1)
$\sum_{i=0}^{0}$ $\sum_{i=0}^{1}$ $\sum_{i=0}^{2}$ $\sum_{i=0}^{3}$ $\sum_{i=1}^{4}$ $\sum_{i=2}^{5}$ $\sum_{i=3}^{6}$ $\sum_{i=4}^{7}$ $\sum_{i=5}^{8}$ $\sum_{i=6}^{9}$ $\sum_{i=7}^{10}$ $\sum_{i=8}^{11}$ $\sum_{i=9}^{12}$ $\sum_{i=10}^{13}$ $\sum_{i=11}^{14}$ $\sum_{i=12}^{15}$
Add
Step 3 (j = 2)
$\sum_{i=0}^{0}$ $\sum_{i=0}^{1}$ $\sum_{i=0}^{2}$ $\sum_{i=0}^{3}$ $\sum_{i=0}^{4}$ $\sum_{i=0}^{5}$ $\sum_{i=0}^{6}$ $\sum_{i=0}^{7}$ $\sum_{i=1}^{8}$ $\sum_{i=2}^{9}$ $\sum_{i=3}^{10}$ $\sum_{i=4}^{11}$ $\sum_{i=5}^{12}$ $\sum_{i=6}^{13}$ $\sum_{i=7}^{14}$ $\sum_{i=8}^{15}$
Add
Final step (j = 3)
$\sum_{i=0}^{0}$ $\sum_{i=0}^{1}$ $\sum_{i=0}^{2}$ $\sum_{i=0}^{3}$ $\sum_{i=0}^{4}$ $\sum_{i=0}^{5}$ $\sum_{i=0}^{6}$ $\sum_{i=0}^{7}$ $\sum_{i=0}^{8}$ $\sum_{i=0}^{9}$ $\sum_{i=0}^{10}$ $\sum_{i=0}^{11}$ $\sum_{i=0}^{12}$ $\sum_{i=0}^{13}$ $\sum_{i=0}^{14}$ $\sum_{i=0}^{15}$

Synchronous Iteration
(Synchronous Parallelism)

Each iteration composed of several processes that start together at beginning of iteration. Next iteration cannot begin until all processes have finished previous iteration. Using **`forall`**:

```
for (j = 0; j < n; j++)/*for each synch. iteration */
   forall (i = 0; i < N; i++) {/*N procs each using*/
     body(i);                   /* specific value of i */
   }
```

or:

```
for (j = 0; j < n; j++) {/*for each synchr.iteration */
   i = myrank;          /*find value of i to be used */
   body(i);
   barrier(mygroup);
}
```

Another fully synchronous computation example

Solving a General System of Linear Equations by Iteration

Suppose the equations are of a general form with *n* equations and *n* unknowns

$$
\begin{array}{l}
a_{n-1,0}x_0 + a_{n-1,1}x_1 + a_{n-1,2}x_2 \ldots + a_{n-1,n-1}x_{n-1} = b_{n-1} \\
\quad\quad\quad\quad . \\
\quad\quad\quad\quad . \\
\quad\quad\quad\quad . \\
a_{2,0}x_0 + a_{2,1}x_1 + a_{2,2}x_2 \ldots + a_{2,n-1}x_{n-1} = b_2 \\
a_{1,0}x_0 + a_{1,1}x_1 + a_{1,2}x_2 \ldots + a_{1,n-1}x_{n-1} = b_1 \\
a_{0,0}x_0 + a_{0,1}x_1 + a_{0,2}x_2 \ldots + a_{0,n-1}x_{n-1} = b_0
\end{array}
$$

where the unknowns are $x_0, x_1, x_2, \ldots x_{n-1}$ $(0 \le i < n)$.

By rearranging the *i*th equation:

$$a_{i,0}x_0 + a_{i,1}x_1 + a_{i,2}x_2 \ldots \qquad + a_{i,n-1}x_{n-1} \quad = b_i$$

to

$x_i = (1/a_{i,i})[b_i - (a_{i,0}x_0 + a_{i,1}x_1 + a_{i,2}x_2 \ldots a_{i,i-1}x_{i-1} + a_{i,i+1}x_{i+1} \ldots + a_{i,n-1}x_{n-1})]$

or

$$x_i = \frac{1}{a_{i,i}}\left[b_i - \sum_{j \neq i} a_{i,j}x_j\right]$$

This equation gives x_i in terms of the other unknowns and can be be used as an iteration formula for each of the unknowns to obtain better approximations.

Jacobi Iteration

All values of *x* are updated together.

Can be proven that the Jacobi method will converge if the diagonal values of *a* have an absolute value greater than the sum of the absolute values of the other *a*'s on the row (the array of *a*'s is *diagonally dominant*) i.e. if

$$\sum_{j \neq i} |a_{i,j}| < |a_{i,i}|$$

This condition is a sufficient but not a necessary condition.

Termination

A simple, common approach. Compare values computed in one iteration to values obtained from the previous iteration.

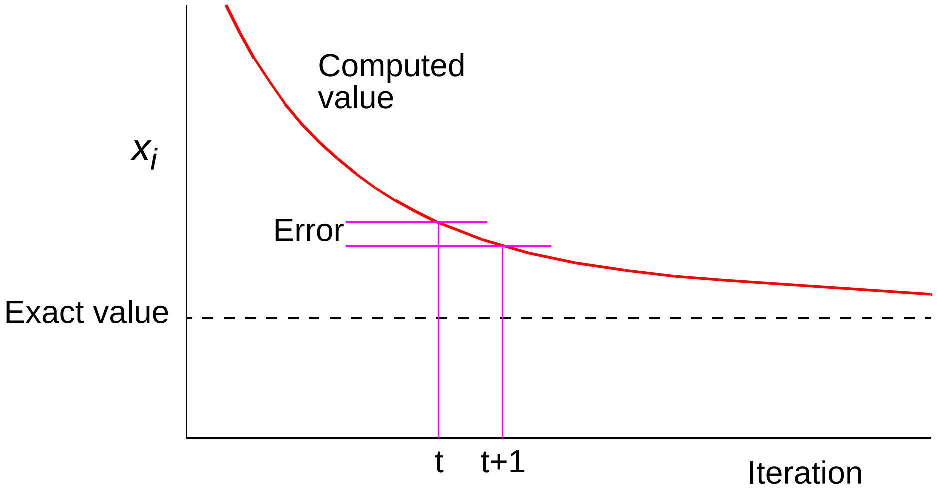
Terminate computation when all values are within given tolerance; i.e., when

$$\left| x_i^t - x_i^{t-1} \right| < \text{error tolerance}$$

for all *i*, where x_i^t is the value of x_i after the *t*th iteration and x_i^{t-1} is the value of x_i after the $(t - 1)$th iteration.

However, this does not guarantee the solution to that accuracy.

Convergence Rate



Parallel Code

Process P_i could be of the form

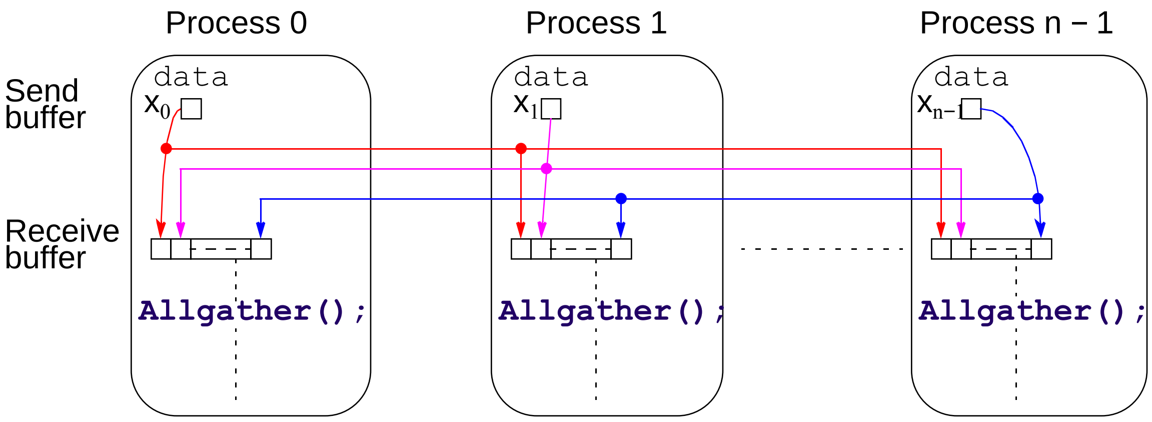
```
x[i] = b[i];                                 /*initialize unknown*/
for (iteration = 0; iteration < limit; iteration++) {
    sum = -a[i][i] * x[i];
    for (j = 0; j < n; j++)                  /* compute summation */
      sum = sum + a[i][j] * x[j];
    new_x[i] = (b[i] - sum) / a[i][i];/* compute unknown */
    allgather(&new_x[i]);                    /*bcast/rec values */
    global_barrier();                        /* wait for all procs */
}
```

`allgather()` sends the newly computed value of **`x[i]`** from process *i* to every other process and collects data broadcast from the other processes.

Introduce a new message-passing operation - Allgather.

Allgather

Broadcast and gather values in one composite construction.



Partitioning

Usually number of processors much fewer than number of data items to be processed. Partition the problem so that processors take on more than one data item.

block allocation – allocate groups of consecutive unknowns to processors in increasing order.

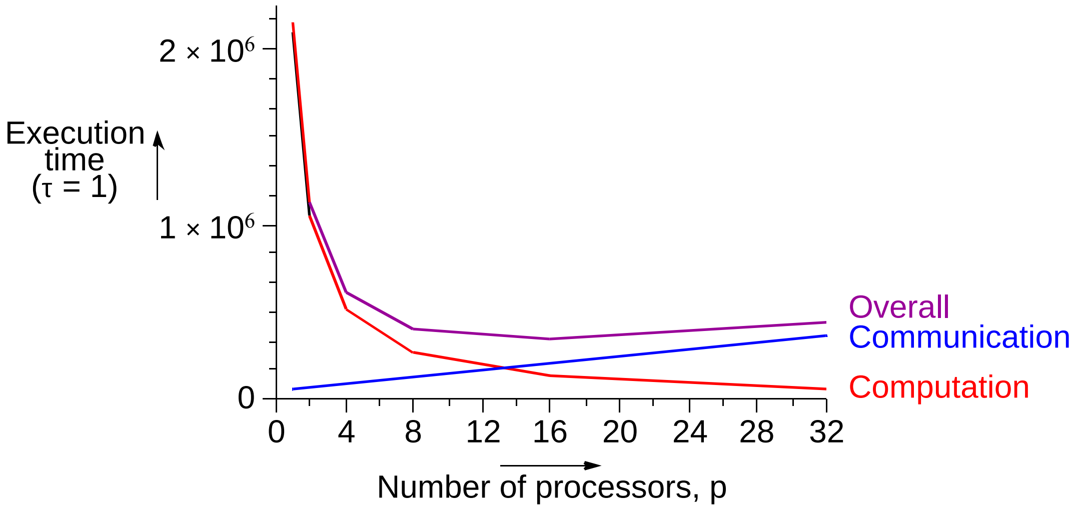
cyclic allocation – processors are allocated one unknown in order; i.e., processor P_0 is allocated x_0, x_p, x_{2p}, …, $x_{((n/p)-1)p}$, processor P_1 is allocated x_1, x_{p+1}, x_{2p+1}, …, $x_{((n/p)-1)p+1}$, and so on.

Cyclic allocation no particular advantage here (Indeed, may be disadvantageous because the indices of unknowns have to be computed in a more complex way).

Effects of computation and communication in Jacobi iteration

Consequences of different numbers of processors done in textbook.

Get:



Locally Synchronous Computation

Heat Distribution Problem

An area has known temperatures along each of its edges. Find the temperature distribution within.

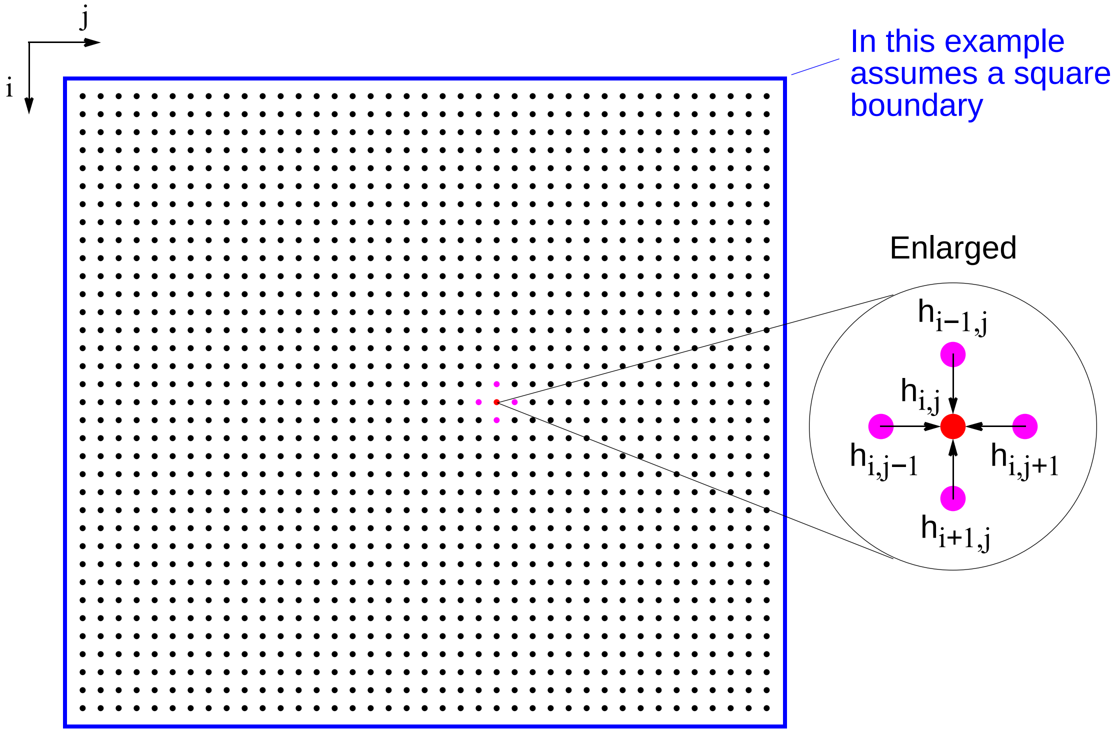
Divide area into fine mesh of points, $h_{i,j}$. Temperature at an inside point taken to be average of temperatures of four neighboring points. Convenient to describe edges by points.

Temperature of each point by iterating the equation:

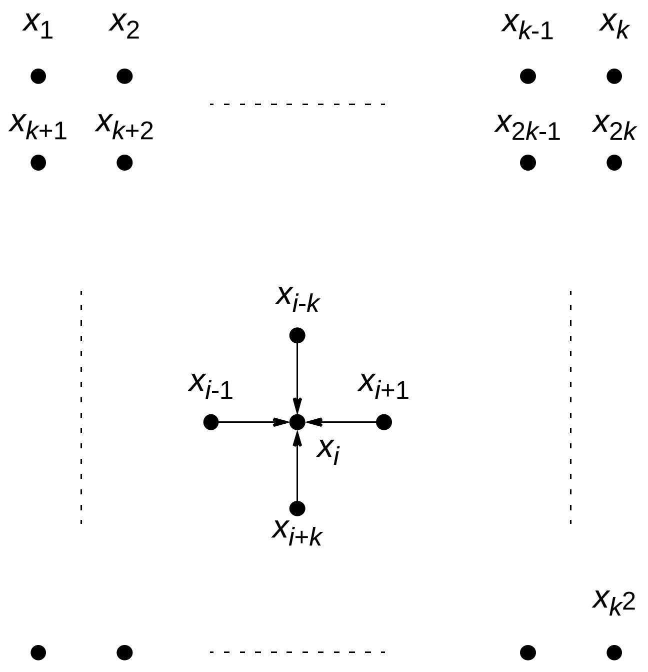
$$h_{i,j} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4}$$

($0 < i < n$, $0 < j < n$) for a fixed number of iterations or until the difference between iterations less than some very small amount.

Heat Distribution Problem



Natural ordering of heat distribution problem



Number points from 1 for convenience and include those representing the edges. Each point will then use the equation

$$x_i = \frac{x_{i-1} + x_{i+1} + x_{i-k} + x_{i+k}}{4}$$

Could be written as a linear equation containing the unknowns x_{i-k}, x_{i-1}, x_{i+1}, and x_{i+k}:

$$x_{i-k} + x_{i-1} - 4x_i + x_{i+1} + x_{i-k} = 0$$

Notice: solving a (sparse) system of linear equations.

Also solving Laplace’s equation.

Sequential Code

Using a fixed number of iterations

```
for (iteration = 0; iteration < limit; iteration++) {
 for (i = 1; i < n; i++)
  for (j = 1; j < n; j++)
   g[i][j] = 0.25*(h[i-1][j]+h[i+1][j]+h[i][j-
1]+h[i][j+1]);
 for (i = 1; i < n; i++)/* update points */
   for (j = 1; j < n; j++)
        h[i][j] = g[i][j];
}
```

To stop at some precision:

```
do {
 for (i = 1; i < n; i++)
  for (j = 1; j < n; j++)
   g[i][j] = 0.25*(h[i-1][j]+h[i+1][j]+h[i][j-1]+h[i][j+1]);

 for (i = 1; i < n; i++)/* update points */
   for (j = 1; j < n; j++)
     h[i][j] = g[i][j];

 continue = FALSE;      /* indicates whether to continue */
 for (i = 1; i < n; i++)/* check each pt for convergence */
   for (j = 1; j < n; j++)
     if (!converged(i,j) {/* point found not converged */
       continue = TRUE;
       break;
     }

} while (continue == TRUE);
```

Parallel Code

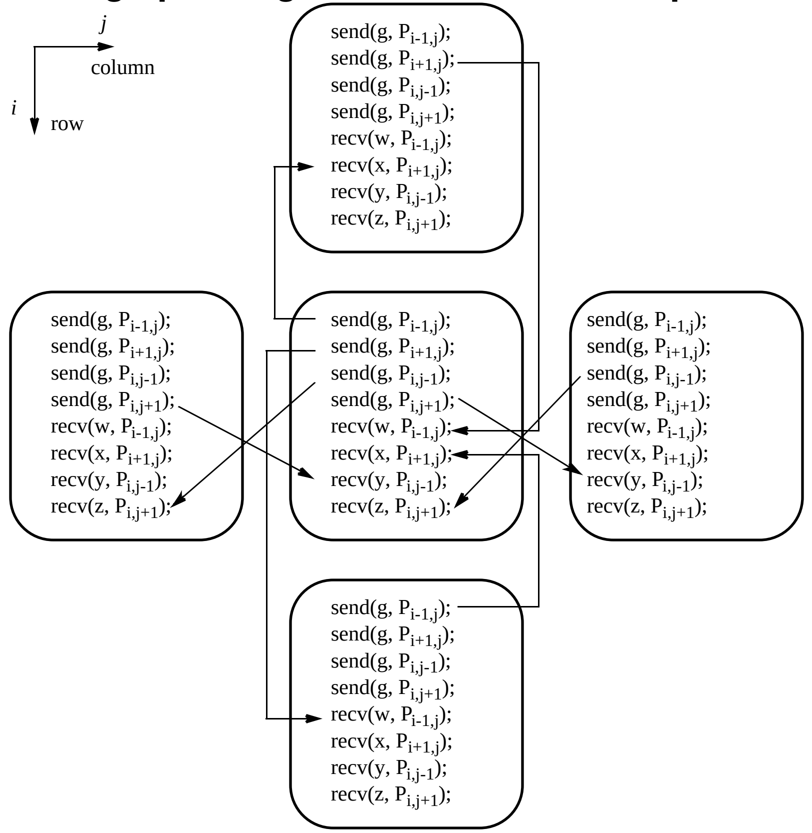
With fixed number of iterations, $P_{i,j}$ (except for the boundary points):

```
for (iteration = 0; iteration < limit; iteration++) {
   g = 0.25 * (w + x + y + z);
   send(&g, $P_{i-1,j}$);   /* non-blocking sends */
   send(&g, $P_{i+1,j}$);
   send(&g, $P_{i,j-1}$);
   send(&g, $P_{i,j+1}$);
   recv(&w, $P_{i-1,j}$);   /* synchronous receives */
   recv(&x, $P_{i+1,j}$);
   recv(&y, $P_{i,j-1}$);
   recv(&z, $P_{i,j+1}$);
}
```

Local barrier

Important to use **`send()`**s that do not block while waiting for the **`recv()`**s; otherwise the processes would deadlock, each waiting for a **`recv()`** before moving on - **`recv()`**s must be synchronous and wait for the **`send()`**s.

Message passing for heat distribution problem



Version where processes stop when they reach their required precision:

```
iteration = 0;
do {
   iteration++;
   g = 0.25 * (w + x + y + z);
   send(&g, P_{i-1,j});         /* locally blocking sends */
   send(&g, P_{i+1,j});
   send(&g, P_{i,j-1});
   send(&g, P_{i,j+1});
   recv(&w, P_{i-1,j});         /* locally blocking receives */
   recv(&x, P_{i+1,j});
   recv(&y, P_{i,j-1});
   recv(&z, P_{i,j+1});
} while((!converged(i, j)) || (iteration < limit));
send(&g, &i, &j, &iteration, P_{master});
```

To handle the processes operating at the edges:

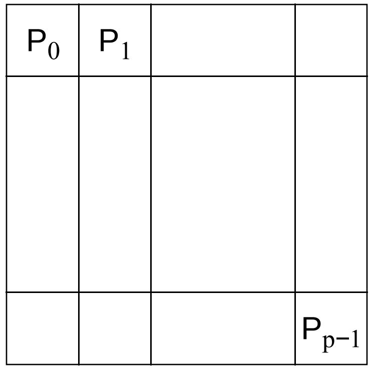
MPI has a construct to help here

```
if (last_row) w = bottom_value;
if (first_row) x = top_value;
if (first_column) y = left_value;
if (last_column) z = right_value;
iteration = 0;
do {
  iteration++;
  g = 0.25 * (w + x + y + z);
  if !(first_row) send(&g, P_{i-1,j});
  if !(last_row) send(&g, P_{i+1,j});
  if !(first_column) send(&g, P_{i,j-1});
  if !(last_column) send(&g, P_{i,j+1});
  if !(last_row) recv(&w, P_{i-1,j});
  if !(first_row) recv(&x, P_{i+1,j});
  if !(first_column) recv(&y, P_{i,j-1});
  if !(last_column) recv(&z, P_{i,j+1});
} while((!converged) || (iteration < limit));
send(&g, &i, &j, iteration, P_{master});
```

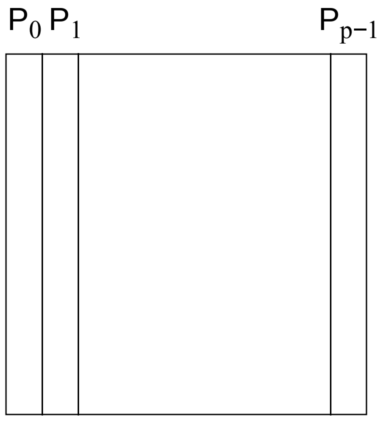
Partitioning

Normally allocate more than one point to each processor, because many more points than processors.

Points could be partitioned into square blocks or strips:



Blocks

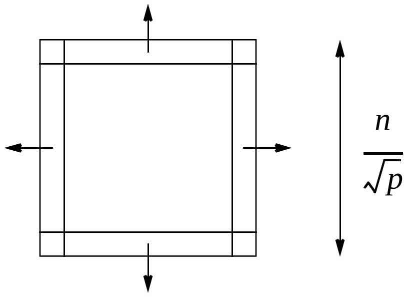


Strips (columns)

Block partition

Four edges where data points exchanged. Communication time given by

$$t_{\text{commsq}} = 8\left(t_{\text{startup}} + \frac{n}{\sqrt{p}}t_{\text{data}}\right)$$



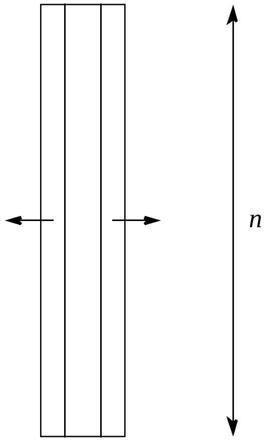
Square blocks

Communication consequences of partitioning

Strip partition

Two edges where data points are exchanged. Communication time is given by

$$t_{\text{commcol}} = 4(t_{\text{startup}} + nt_{\text{data}})$$



Strips

Optimum

In general, the strip partition is best for a large startup time, and a block partition is best for a small startup time.

With the previous equations, the block partition has a larger communication time than the strip partition if

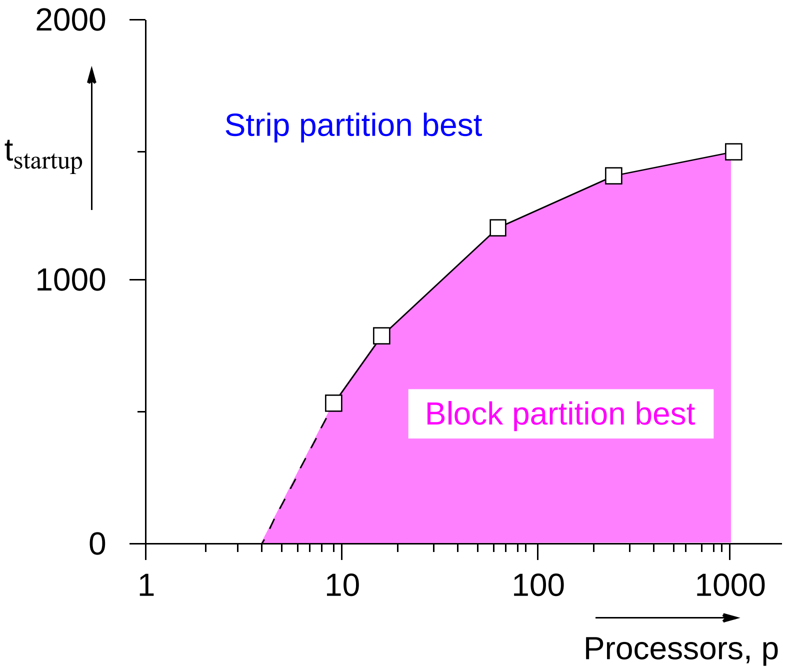
$$8\left(t_{\text{startup}} + \frac{n}{\sqrt{p}}t_{\text{data}}\right) > 4(t_{\text{startup}} + nt_{\text{data}})$$

or

$$t_{\text{startup}} > n\left(1 - \frac{2}{\sqrt{p}}\right)t_{\text{data}}$$

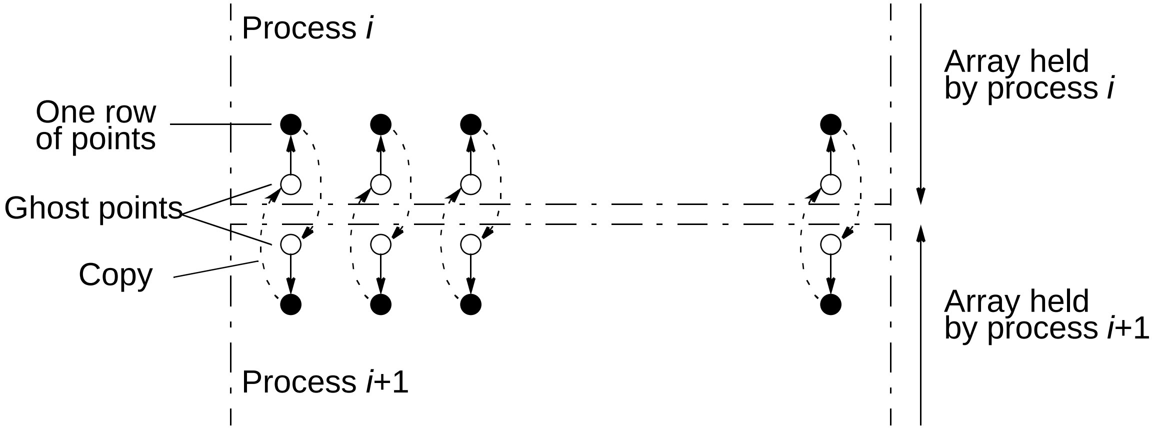
($p \geq 9$).

Startup times for block and strip partitions



Ghost Points

An additional row of points at each edgethat hold the values from the adjacent edge. Each array of points is increased to accommodate the ghost rows.



Safety and Deadlock

When all processes send their messages first and then receive all of their messages is “unsafe” because it relies upon buffering in the **`send()`**s. The amount of buffering is not specified in MPI.

If insufficient storage available, send routine may be delayed from returning until storage becomes available or until message can be sent without buffering.

Then, a locally blocking **`send()`** could behave as a synchronous **`send()`**, only returning when the matching **`recv()`** is executed. Since a matching **`recv()`** would never be executed if all the **`send()`**s are synchronous, deadlock would occur.

Making the code safe

Alternate the order of the **`send()`**s and **`recv()`**s in adjacent processes so that only one process performs the **`send()`**s first:

}

Then even synchronous **`send()`**s would not cause deadlock.

Good way you can test for safety is to replace message-passing routines in a program with synchronous versions.

MPI Safe Message Passing Routines

MPI offers several alternative methods for safe communication:

- Combined send and receive routines:

  `MPI_Sendrecv()`

  which is guaranteed not to deadlock

- Buffered send()s:

  `MPI_Bsend()`

  here the user provides explicit storage space

- Nonblocking routines:

  `MPI_Isend()` **and MPI_Irecv()**

which return immediately. Separate routine used to establish whether message has been received - **`MPI_Wait()`**, **`MPI_Waitall()`**, **`MPI_Waitany()`**, **`MPI_Test()`**, **`MPI_Testall()`** **or** **`MPI_Testany()`**.

Other fully synchronous problems

Cellular Automata

The problem space is divided into cells.

Each cell can be in one of a finite number of states.

Cells affected by their neighbors according to certain rules, and all cells are affected simultaneously in a “generation.”

Rules re-applied in subsequent generations so that cells evolve, or change state, from generation to generation.

Most famous cellular automata is the “Game of Life” devised by John Horton Conway, a Cambridge mathematician.

Also good assignment for graphical output.

The Game of Life

Board game - theoretically infinite two-dimensional array of cells. Each cell can hold one “organism” and has eight neighboring cells, including those diagonally adjacent. Initially, some cells occupied.

The following rules apply:

1. Every organism with two or three neighboring organisms survives for the next generation.
2. Every organism with four or more neighbors dies from overpopulation.
3. Every organism with one neighbor or none dies from isolation.
4. Each empty cell adjacent to exactly three occupied neighbors will give birth to an organism.

These rules were derived by Conway “after a long period of experimentation.”

Simple Fun Examples of Cellular Automata

“Sharks and Fishes”

An ocean could be modeled as a three-dimensional array of cells. Each cell can hold one fish or one shark (but not both).

Fish and sharks follow “rules.”

Fish

Might move around according to these rules:

1. If there is one empty adjacent cell, the fish moves to this cell.
2. If there is more than one empty adjacent cell, the fish moves to one cell chosen at random.
3. If there are no empty adjacent cells, the fish stays where it is.
4. If the fish moves and has reached its breeding age, it gives birth to a baby fish, which is left in the vacating cell.
5. Fish die after *x* generations.

Sharks

Might be governed by the following rules:

1. If one adjacent cell is occupied by a fish, the shark moves to this cell and eats the fish.
2. If more than one adjacent cell is occupied by a fish, the shark chooses one fish at random, moves to the cell occupied by the fish, and eats the fish.
3. If no fish in adjacent cells, the shark chooses an unoccupied adjacent cell to move to in a similar manner as fish move.
4. If the shark moves and has reached its breeding age, it gives birth to a baby shark, which is left in the vacating cell.
5. If a shark has not eaten for *y* generations, it dies.

Similar examples: “foxes and rabbits” -Behavior of rabbits to move around happily whereas behavior of foxes is to eat any rabbits they come across.

Sample Student Output

Serious Applications for Cellular Automata

Examples

- fluid/gas dynamics
- the movement of fluids and gases around objects
- diffusion of gases
- biological growth
- airflow across an airplane wing
- erosion/movement of sand at a beach or riverbank.