Find an exact simplified solution to the equation in the interval $0 < x < 1$.

$$1 = 12\cos(x + 1) - 5$$

Find the solutions to the equation in the interval $0 \leq \beta \leq \pi$.

$$2\cos(2\beta) + 1 = 0$$

Simplify the trigonometric expression

$$\frac{2+\cot^2(x)}{\csc^2(x)}-1$$

Make the indicated trigonometric substitution in the given algebraic expression and simplify (see Example 7). Assume that $0 < \theta < \frac{\pi}{2}$

$$\frac{\sqrt{x^2 - 4}}{x} \quad , \quad x = 2\sec(\theta)$$

$$f(x) = x^2 - 3$$

$$T(x) = \sqrt{x+12} - 5$$

$$\begin{aligned}
(T \circ f)(x) &= T(f(x)) \\
(T \circ f)(x) &= T(x^2 - 3) \\
(T \circ f)(x) &= \sqrt{x^2 - 3 + 12} - 5 \\
(T \circ f)(x) &= \sqrt{x^2 + 9} - 5
\end{aligned}$$

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
b^2 &= c^2 - a^2 \\
b &= \pm\sqrt{c^2 - a^2}
\end{aligned}
$$

$$x(x^3 + 27) = x(x+3)(x^2 - 3x + 9)$$

$$\begin{aligned} 3x^{3/2} - 12x^{1/2} + 9x^{-1/2} &= 3x^{-1/2}\left(x^2 - 4x + 3\right) \\ &= 3x^{-1/2}(x-3)(x-1) \end{aligned}$$

$$f(x) = \sqrt[4]{x}$$
$$f(-x) = \sqrt[4]{-x}$$

$$x^4 - 10x^2 + 9 = 0$$

Replace the variable to reduce the overall degree of the problem. Here is the best choice for that:

$$x^2 = y$$

$$y^2 - 10y + 9 = 0$$

Once you solve for y, don't forget you need to actually solve for x, so go back to $x^2 = y$ and replace y with all the solutions you find. You may have up to 4 solutions for x.

$(-\infty,-2]$	$[-2,1-\sqrt{5}]$	$[1-\sqrt{5},2]$	$[2,1+\sqrt{5}]$	$[1+\sqrt{5},\infty)$
$\times$	$\times$	$\times$	Yo	$\times$

$$\begin{aligned} \frac{5x}{6} - \frac{\pi}{6} &= \frac{1}{6}(5x - \pi) \\ &= \frac{1}{6} \times 5\left(x - \frac{\pi}{5}\right) \\ &= \frac{5}{6}\left(x - \frac{\pi}{5}\right) \end{aligned}$$

$$\begin{aligned} \frac{3x}{4} - \frac{\pi}{6} &= \frac{9x}{12} - \frac{2\pi}{12} \\ &= \frac{1}{12}(9x - 2\pi) \\ &= \frac{1}{12} \times 9\left(x - \frac{2\pi}{9}\right) \\ &= \frac{3}{4}\left(x - \frac{2\pi}{9}\right) \end{aligned}$$

$$\begin{aligned} \log_5\left(\frac{\sqrt{5x^9}}{y}\right) &= \log_5\left(\frac{\sqrt{5}\cdot\sqrt{x^9}}{y}\right) \\ &= \log_5(\sqrt{5}) + \log_5\left(\sqrt{x^9}\right) - \log_5(y) \\ &= \log_5(5^{1/2}) + \log_5(x^{9/2}) - \log_5(y) \\ &= \tfrac{1}{2}\log_5(5) + \tfrac{9}{2}\log_5(x) - \log_5(y) \\ &= \tfrac{1}{2} + \tfrac{9}{2}\log_5(x) - \log_5(y) \end{aligned}$$

$$\text{Power Rule} \quad : \quad \log_B(A^n) = n\log_B(A)$$

$$\text{Product Rule} \quad : \quad \log_B(A \times C) = \log_B(A) + \log_B(C)$$

$$\text{Quotient Rule} \quad : \quad \log_B\left(\tfrac{A}{C}\right) = \log_B(A) - \log_B(C)$$

$$\begin{aligned} \log_{12}(9) + 2\log_{12}(4) &= \log_{12}(9) + \log_{12}(4^2) \\ \log_{12}(144) &= \log_{12}(12^2) = 2 \end{aligned}$$

$$\ln e^7 - \ln e^2 = 7\ln e - 2\ln e = 7 - 2$$

Recall the main definition of Logarithm:

$$\log_B x = y \leftrightarrow x = B^y$$

Then rearrange to get the Logarithm alone so you can use the definition:

$$\begin{aligned} y &= -\log_3(4x+7) + 2 \\ y - 2 &= -\log_3(4x+7) \\ -y + 2 &= \log_3(4x+7) \\ 3^{-y+2} &= 4x + 7 \\ 3^{-y+2} - 7 &= 4x \\ \frac{3^{-y+2} - 7}{4} &= x \\ f^{-1}(y) &= \frac{3^{-y+2} - 7}{4} \end{aligned}$$