Directions:
Please only put your final, well written solutions, in the space provided.
Give exact answers (simplified radicals or fractions).
If you use additional paper clearly label the question and upload pages after the question page.
Use complete sentences and explain your reason as much as possible.
There are **??** questions and **??** points total

Name: _____

1. Answer each question

   (a) [2 points] If the point $(5, -3)$ is on the graph of the function $y = f(x)$, where will that point land if it is transformed by $y = \frac{2}{3}f(x-5)+1$?

   (a) _____

   (b) [2 points] If the function $y = f(x)$ has a range of $(-1, 5]$, what will the range of the transformed function $y = 2f(x) - 3$ be?

   (b) _____

   (c) [2 points] Solve the equation $3x - 2 = -4x + 12$

   (c) _____

   (d) [4 points] What does it mean for an equation to represent a function of y in x?

2. To enroll in a phone plan, you shop around various vendors and find one you like. They offer the following data plan: The initial setup fee is \$100 and the first 5 gigabytes are priced at \$2 per gigabyte. Any additional data is priced at \$10 per gigabyte.

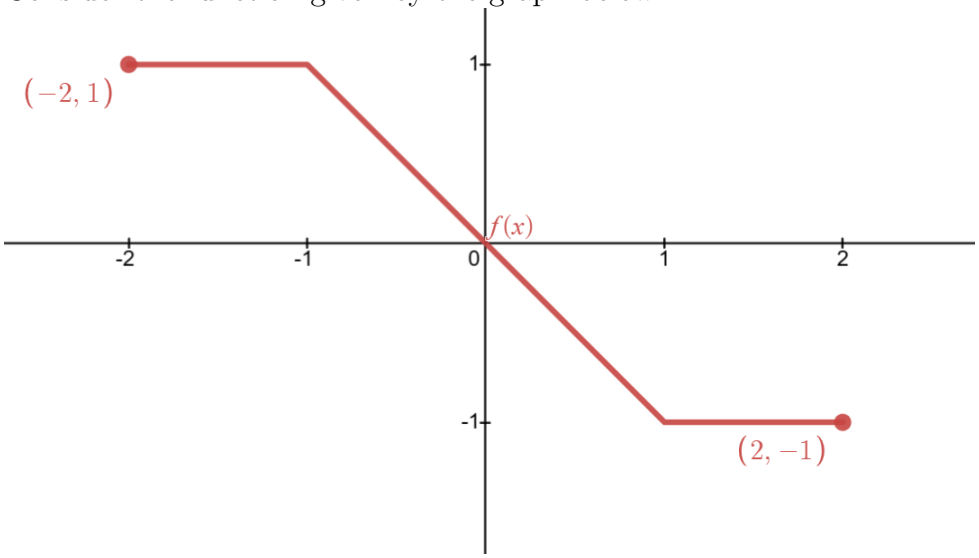
   (a) [4 points] What is the piecewise function that represents the cost for x gigabytes of data?

   (b) [4 points] Graph the function with an appropriate scale and indicate **all important points** such as intercepts, intersections, and cutoff values for the piecewise function. Be sure to label your axes and indicate clearly the scale.

   (c) [2 points] How much will it cost you to use 12 gigabytes of data?

   (c) _____

3. Consider the function given by the graph below:

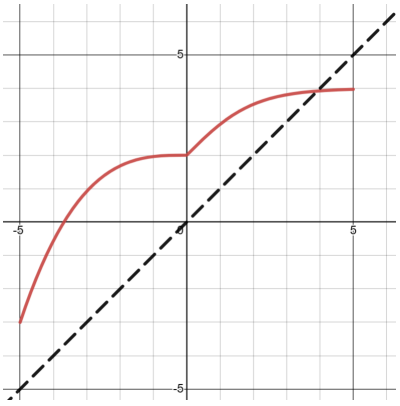


(a) [5 points] Sketch the graph of $y = 2f(x-1)$. Label at least 3 of the transformed points and clearly indicate your scale.

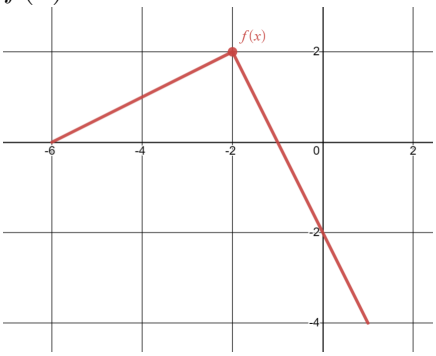
(b) [5 points] Sketch the graph of $y = -f\left(\frac{x}{2}\right) + 3$. Label at least 3 of the transformed points and clearly indicate your scale.

4. (a) [2 points] Please provide a clear definition for the statement: y is a function of x

(b) [4 points] Using the graph provided, sketch the inverse of the function shown in red.



(c) [4 points] Provide a short argument on whether or not the graph of the function $f(x)$ below has an inverse.



5. (a) [2 points] Suppose that we are considering two functions, $f(x)$ and $T(x)$. Provide both the algebraic and verbal definition for the following expression:

$$(f \circ T)(x)$$

(b) Let $f(x) = x^2 - 3$ and $T(x) = |x - 2| - 5$. Show all work and answer each question below.

i. [1 point] Evaluate $(f \circ T)(4)$.

i. _____

ii. [1 point] Evaluate $(T \circ f)(4)$.

ii. _____

iii. [2 points] What is $(f \circ T)(x)$?

iii. _____

iv. [2 points] What is $(T \circ f)(x)$?

iv. _____

v. [2 points] Explain your answer: Are T and f inverse functions of each other?

6. In a hospital, a patient is presenting with an unknown illness. In order to rule out different diagnoses, the patient's mouth and nose aare swabbed and a bacterial cultural is grown in a petri dish over the course of the next ten hours. The population of the bacteria on the dish is found to follow the following model:

$$P = f(t) = \frac{100t + 23}{5 + 7t}, 0 \leq t \leq 10$$

Where P is the population in thousands and t is the time in hours.

(a) [5 points] Determine the inverse of this function, $t = f^{-1}(P)$. Show all your steps.

(a) _____

(b) [3 points] Give both the numerical answer, rounded to the nearest tenth, and the practical meaning of $f^{-1}(12)$.

(b) _____

(c) [2 points] In order to properly identify the bacterial strain, there needs to be at least ten thousand bacteria present in the dish. How long will the lab need to wait until there are at least that many bacteria?

(c) _____

7. Answer each of the problems below. Show all work.

(a) [3 points] If $f(x) = x^2 + 2$ and $g(x) = \frac{1}{x-4}$, find and simplify $\left(\frac{f}{g}\right)(x)$.

(a) _____

(b) [3 points] Does the equation $100y - x = x^3$ represent y as an **invertible function** of x? Explain your answer.

(c) [3 points] If $h(x) = x^2 - 1$ and $q(x) = \frac{2}{x+1}$, find and simplify $(q \circ h)(x)$

(c) _____

(d) [1 point] Draw a mathematical dragon. All answers are acceptable :-)