MATH 112 Class Notes
for Asynchronous Sections

Department of Mathematics
The University of Arizona
2024-2025 Academic Year

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Syllabus Highlights

Welcome to Math 112! The following information highlights some of the most important parts of the full syllabus. Students are still expected to know the contents of the full syllabus, which is posted in D2L.

Required Materials

- ALEKS access
- Graphing Calculator

In-Person Class Meetings

The in-person class meetings are an important and mandatory part of the course. These in-person class meetings allow students time to interact with their instructor and classmates, ask questions, share ideas, and collaborate on solving problems. In-class activities may include group work, quizzes, class discussions, student presentations, and collecting written assignments.

Online Work

The book and online homework are hosted on http://aleks.com (ALEKS). Enrollment in the ALEKS Math 112 course should be completed by the end of the first day of classes. Failure to enroll by the third day of classes may result in administrative drop from the course.

Exams

There will be three midterm exams and a cumulative final exam.

Course Grade Break Down

- Homework/Quizzes: 150 points
- Midterm 1: 100 points
- Midterm 2: 100 points
- Midterm 3: 100 points
- Final Exam 150 points

Getting Help

- Office hours and appointments via email
- Tutoring at ThinkTank

Actions That May Result in an Administrative Drop

- Failing to sign up for ALEKS
- Failing to come to class the first day
- Missing more than 3 classes
- Missing more than 5 assignments

Quick Reference Guide

Math Department Information

Math Dept Home	https://www.math.arizona.edu
Math 112 Homepage	https://math112.math.arizona.edu
Math Common Final Exam Schedule	https://www.math.arizona.edu/academics/courses/finals

University Information

U of A Home	https://www.arizona.edu/
COVID-19 Information	https://covid19.arizona.edu
Final Exam Schedule	https://registrar.arizona.edu/finals
Important Dates and Deadlines	https://registrar.arizona.edu/courses/dates-deadlines
U of A Computing Homepage	https://it.arizona.edu
24/7 IT Computing Support Center	https://it.arizona.edu/get-support
UAccess	https://uaccess.arizona.edu/
D2L	https://d2l.arizona.edu/d2l/loginh/

Important University Policies

Code of Academic Integrity
https://deanofstudents.arizona.edu/policies/code-academic-integrity

Student Code of Conduct
https://deanofstudents.arizona.edu/student-rights-responsibilities/student-code-conduct

Services for Students

University Tutoring Services	https://thinktank.arizona.edu
Disability Resource Center	https://drc.arizona.edu
Campus Health	https://health.arizona.edu
Counseling & Psychological Services	https://health.arizona.edu/counseling-psych-services

Course Expectations

Mathematics is often challenging. That is a part of what makes it feel so satisfying when you finally solve the problem or understand the concept that you have been struggling with. Mathematics is more about creating and deep thinking than memorizing procedures. It may take several different approaches and many mistakes before a problem is solved. Mathematics at its best is a collaborative endeavor. You will know that you have succeeded in solving a problem when you can explain your solution to others and they can understand and verify your results, and you will learn an enormous amount by listening to others, asking questions, and sharing ideas.

To be most successful in this course, work to meet the following expectations. Think of these as Habits of Highly Successful Scholars.

1. Stay focused on mathematics during class. Put away all distractions.

2. Be prepared for class. This means you should review your notes and work on assigned problems before class. Bring your notes, calculator, and note-taking materials to class.

3. In between your Safe Zone and Panic Zone is your Learning Edge. Always work at your Learning Edge. This includes working to catch up, if you fall behind; and working on extension questions, if you are ahead of the class.

4. Give your best effort at doing mathematics. This includes being persistent even if you feel frustrated at times. It also includes making mistakes and learning from them. If you are not making mistakes, you are stuck in the Safe Zone, and are not at your Learning Edge.

5. Ask questions – of yourself, your classmates, and your instructor. Be brave. Be curious. Only accept mathematics that makes sense to you.

6. Actively and respectfully listen to your classmates and your instructor.

7. Communicate about mathematics positively and meaningfully, to your group to ro the class. Be mindful of how much you are speaking versus listening and whether everyone's ideas have been heard.

8. Be respectful of everyone in the classroom community. Contribute to a positive, supportive learning environment.

9. Be reflective about yourself as a learner and as a mathematician. This may include recognizing and challenging your own beliefs and feelings about mathematics, and perhaps changing your perception of what it means to do mathematics. Recognize how you learn best and put a plan into action.

Strategies for Checking Answers

No mathematical work is complete until it has been checked and verified. As in science, mathematical results should never be taken on faith or on the say so of some authority; only accept mathematics that makes sense to you. This includes checking and verifying your own work.

There are many ways to check your answer to a problem. These are some strategies you can try:

1. Check to see whether your answer makes sense. For example, if you are solving a problem to find the speed of a car driving on the highway, 150 mph is not a reasonable solution.
2. Make sure that the units in an application problem work out. For example, if you calculate the time needed for a journey by taking the speed times the distance, the units would give (miles per hour) × (miles), or miles^2 per hour. This does not make sense. Instead, if you calculate the time needed for a journey as distance divided by the speed, the units give (miles) ÷ (miles per hour), which simplifies to hours. This does make sense! (Note that working with the units gives you clues about how to correctly combine the given quantities!).
3. Take the answer you found and substitute it back into the original equation. When you are asked to solve an equation, this is the simplest and most straight-forward way to check your results.
4. When you are solving a word problem, check to see whether your solution yields the desired outcome. For example, if you are solving for the dimensions of a rectangle that give a certain area, use the length and width you found in your solution to calculate the area, and see if you get what you expect.
5. Consider a reasonable domain for the solution. For example, if the problem states that a person is given one 100 mg dosage of a drug, and you need to calculate the amount of the drug left in the person's system after a certain amount of time, then your answer must be greater than or equal to 0 mg, and less than or equal to 100 mg. So, if you get an answer of 120 mg or -5 mg, you know you've done something incorrectly.
6. Solve the problem using a different technique or algorithm. For example, if you solved a quadratic equation by factoring, you could check it by using the quadratic formula.
7. Use a different approach to solve the problem. For example, if you solved a problem strictly algebraically, you can try graphing on your calculator to check your solution. (This can work the other way around as well!)
8. Build a concrete example of the more abstract question. For example, if you are asked how the graph of $y = 2f(x)$ is transformed from the graph of $y = f(x)$, you can pick a specific function, say $y = \sqrt{x}$, and take a look at the graph of $y = \sqrt{x}$ compared to the graph of $y = 2\sqrt{x}$.

Note: Sometimes students try to check by re-doing the problem in the same way they approached it the first time. If you made a mistake the first time, it is very likely to make the same mistake the second time. (This is human nature.) This is not recommended as a way to check your work (though it may help in a pinch on an exam).

Quadratic Formula Program for Graphing Calculators

Quadratic Formula - TI 82, TI-83, TI-84, TI-84+

Introduction

This program solves equations of the form $Ax^2 + Bx + C = 0$ by using the quadratic formula.

The Program

:Prompt A,B,C	{Prompt is in PRGM under I/O}
:(B^2-4*A*C)→D	{The arrow is STO}
:If D<0	{If is in PRGM under CTL} {< is in TEST }
:Then	{Then is in PRGM under CTL}
:Disp "NO REAL SOLUTIONS"	{Disp is in PRGM under I/O} {Words within "" are typed using ALPHA}
:Goto 1	{Goto is in PRGM under CTL}
:Else	{Else is in PRGM under CTL}
:((-B+√(D))/(2*A))→E	{The √ is square root} {The - is the negative sign}
:((-B-√(D))/(2*A))→F	{The second - is the subtraction sign}
:Disp "SOLUTIONS",E,F	{Disp is in PRGM under I/O} {Words within "" are typed using ALPHA}
:Lbl 1	{Lbl is in PRGM under CTL}

Running the program

You will be asked to enter values for *A*, *B*, and *C* according to the quadratic formula. To test your program, try the following:

A=2, *B*=3, *C*=4. Your answer should be NO REAL SOLUTIONS.

A=5, *B*=4, *C*=-2. Your answer should be .348331477355, -1.14833147735.

Quadratic Formula - TI 85, TI-86

Introduction

This program solves equations of the form $Ax^2 + Bx + C = 0$ by using the quadratic formula.

The Program

:Prompt A,B,C	{Prompt is in PRGM under I/O}
:(B^2-4*A*C)→D	{The arrow is STO}
:If D<0	{If is in PRGM under CTL} {< is in TEST }
:Then	{Then is in PRGM under CTL}
:Disp "NO REAL SOLUTIONS"	{Disp is in PRGM under I/O} {Words within "" are typed using ALPHA}
:Goto P	{Goto is in PRGM under CTL}
:Else	{Else is in PRGM under CTL}
:((-B+ √ (D))/(2A))→E	{The√ is square root} {The - is the negative sign}
:((-B- √ (D))/(2A))→F	{The second - is the subtraction sign}
:Disp "SOLUTIONS",E,F	{Disp is in PRGM under I/O} {Words within "" are typed using ALPHA}
:Lbl P	{Lbl is in PRGM under CTL}

Running the program

You will be asked to enter values for *A*, *B*, and *C* according to the quadratic formula. To test your program, try the following:

A=2, *B*=3, *C*=4. Your answer should be NO REAL SOLUTIONS.

A=5, *B*=4, *C*=-2. Your answer should be .348331477355, -1.14833147735.

Quadratic Formula – Casio models

Introduction

This program solves equations of the form $Ax^2 + Bx + C = 0$ by using the quadratic formula.

The Program

'QUADRATIC'	{This will be the name of the program}
"A"?→A	{" is in ALPHA} {? is in PRGM} {→ is on the , button}
"B"?→B	
"C"?→C	
(B^2-4×A×C) →D	{× is the times sign}
D<0 ⇒ Goto 1	{⇒, Goto are in PRGM under JMP} {< is in PRGM under REL} {0 is a zero}
((-B+ √ D) ÷(2A))→E	{The √ is the square root symbol} {The - is the negative sign}
((-B- √ D) ÷(2A))→F	{The second - is a subtraction sign}
"SOLUTIONS"	
E◢	{◢ is in PRGM, do not hit EXE}
F◢	
Goto 2	
Lbl 1	{Lbl is in PRGM under JMP}
"NO REAL SOLUTIONS"	
Lbl 2	

Running the program

You will be asked to enter values for *A*, *B*, and *C* according to the quadratic formula. To test your program, try the following:

A=2, *B*=3, *C*=4. Your answer should be NO REAL SOLUTIONS.

A=5, *B*=4, *C*=-2. Your answer should be .348331477355, -1.14833147735.

UNIT 1 – Functions, Graphs, and Linear Functions

Section: Functions

Objectives

- Understand the definition of function.
- Understand and properly use the phrase "---___ is a function of ___."
- Given a table of values, an equation in two variables, or a graph, determine whether it represents a function, and explain your reasoning.
- Identify the domain of a function given by an expression.
- Use function notation correctly.
- Evaluate a function at a value in its domain.
- Evaluate a function at an algebraic expression.

Review Material

- Interval notation
- Linear inequalities

Introductory Example

In a certain city, sales tax is 9%. Write an expression to express the total cost of purchasing an item with a selling price of x dollars, after tax is added.

Class Notes and Examples

- What are the things you need to know about functions?

Example 1

As you drive at a fairly constant speed of approximately 60 mph, from Tucson to Bisbee (94 miles), you pass through Benson, which is 40 miles from Tucson. Sketch a graph of your distance from Tucson against time. Indicate on your graph the time when you reach Benson. What are the input, output, domain, and range of the function you have graphed?

Example 2

The following table shows market data that a sunglass manufacturer has gathered on a particular pair of sunglasses:

Purchase price	\$46	\$40	\$37	\$30	\$27
Consumer demand (quantity purchased)	16,000	40,000	52,000	80,000	92,000

Does this table represent the demand (# of units) as a function of the price? Why or why not? Does it represent the price as a function of the demand (# of units)? Why or why not?

Example 3

The following table shows a sampling of test scores from a midterm exam

Student	1	2	3	4	5
Midterm Grade	85%	90%	100%	100%	70%

Does this table represent midterm grades as a function of students? Why or why not?
Does it represent students as a function of midterm grades? Why or why not?

More Practice

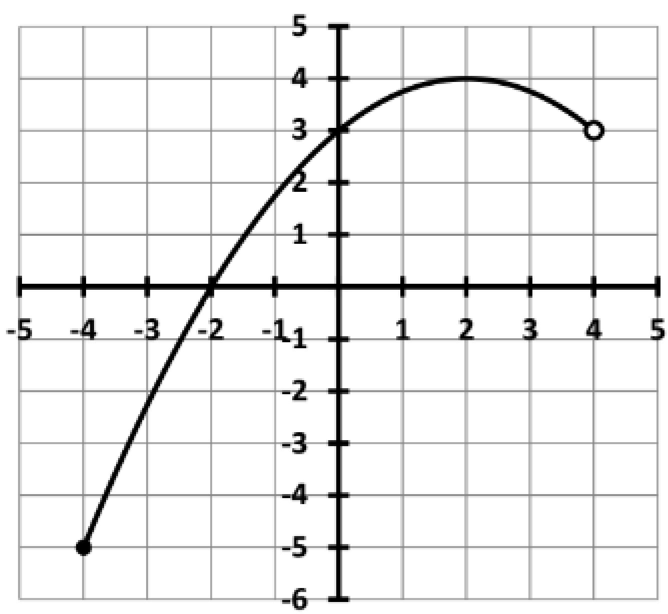
1. For each of the following, determine the domain of the function, and evaluate $g(2)$ and $g(a+3)$.

   (A) $g(x) = 3x^2 - 5x$

   (B) $g(x) = \frac{2x-1}{x+3}$

(C) $g(x) = \sqrt{2 - 5x} - 1$

2. Determine the domain and range of the following function.



3. For the function $f(x) = 3x^2 - 5x$, find $\frac{f(h+4)-f(4)}{h}$ and simplify completely.

4. Which of the following equations represent y as a function of x? How do you know?

$3 + xy^2 = 0$
$x^2 + y^2 - 4 = 0$
$x^2 + 5y^4 = -2$
$4 + y - 3x = 2$
$5x + y^2 = 10$
$3y^3 + 2x = 7$
$4x^2 = 9 + y$

Additional Comments or Examples

Section: Graphs of Functions

Objectives

- Find the intercepts of a function algebraically and graphically.
- Given the graph of a function, identify the domain and range of the function.
- Given the graph of a function, identify open intervals where the graph is increasing/decreasing/constant/positive/negative.
- Sketch a complete graph of a function which includes all important features.
- Using your knowledge of domain and range, as well as context provided, find an appropriate viewing window for a function on a graphing calculator.
- Use a calculator to answer questions about a function in a given context.

Review Material

- Interval notation
- Solving linear and quadratic equations

Introductory Example

A business has determined a mathematical model based on market data for its profit, P (in dollars) as a function of the number of items sold, x. The model is given by the function $P(x) = -0.1x^2 + 150x - 14000$. Graph this function in an appropriate viewing window.

Class Notes and Examples

- What are the things you need to know about graphs of functions?

Example 1

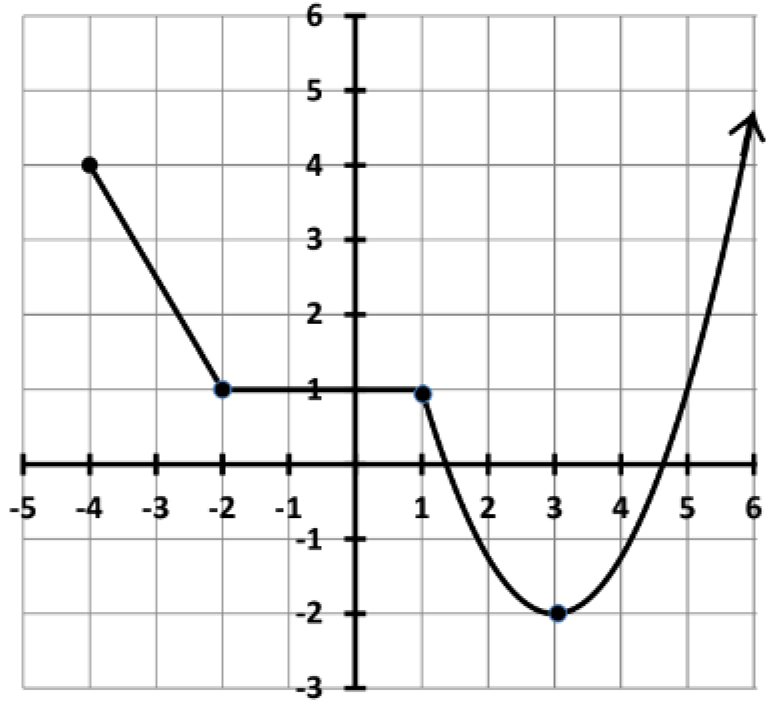
A single 1000 mg dose of a drug is administered to a patient orally. The steady-state concentration of the drug in the patient's bloodstream, measured in micrograms per milliliter, is given approximately by the function $C(t) = 247e^{-0.1t} - 173e^{-2.8t}$ where t is time in hours since the drug was administered. The drug is effective only if the concentration remains between 100 and 400 mcg/mL. Graph the function $C(t)$ in an appropriate window. Use the graph to determine the time frame during which the drug remains effective.

Example 2

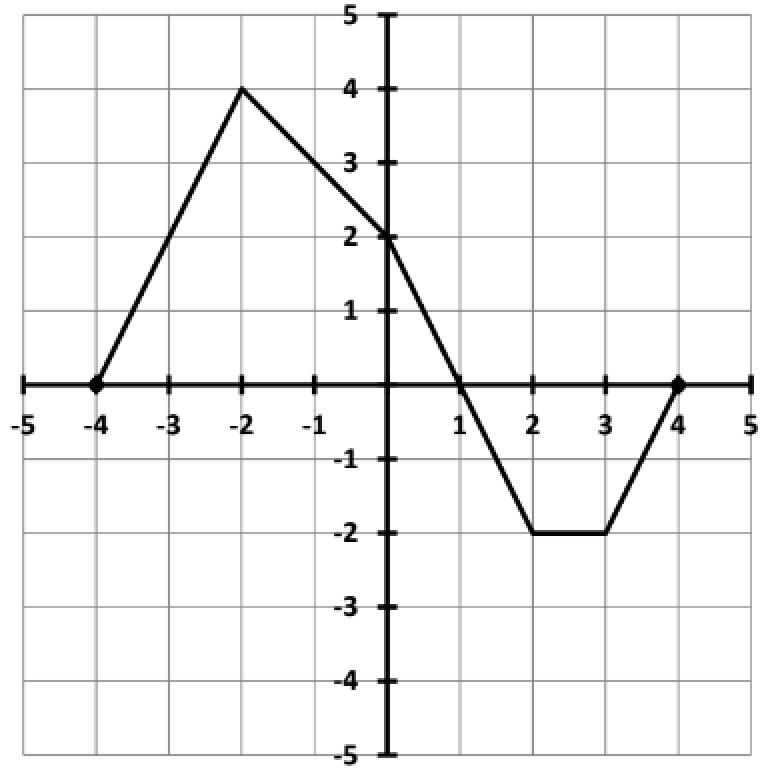
The cost, *C* (in thousands of dollars), of removing *x* percent of a certain pollutant from a lake is given by the formula $C(x) = \frac{18x}{100-x}$. Graph this function in an appropriate window. What is a reasonable domain for this function?

More Practice

1. Determine the domain and range for the function graphed below. Then identify the open intervals on which the function below is increasing, decreasing, and constant.



2. Identify the open intervals where the function graphed below is increasing, decreasing, and constant. Then determine the intervals where the function is positive and negative. Finally, list all of the intercepts.



3. Determine the domain, range, and intercepts of each function. Determine the intervals on which the function is increasing/decreasing/constant, and on which the function is positive/negative.

(A) $g(x) = \sqrt{2x-5} - 1$

(B) $T(x) = x^3 - 4x$

Additional Comments or Examples

Section: Linear Functions

Objectives

- Identify a function given by an expression as a linear function, and identify the slope and *y*-intercept.
- Determine the slope of a line, given two points on the line.
- Sketch the graph of a linear function, given a point and the slope, or given the equation of the line.
- Create linear functions, given two points on the graph of the function.
- Solve application problems involving linear functions.

Review Material

- Solving linear equations
- Finding the domain of a function in a practical context

Introductory Example

A company produces a pair of skates for $43.53 and sells each pair for $89.95. The fixed costs (costs incurred regardless of the number of skates produced) are $742.72.

- Write the total cost as a function of the number of pairs of skates produced.
- Write the revenue as a function of the number of pairs of skates produced/sold.
- Graph these two functions on the same set of axes. Indicate your viewing window. Think about an appropriate domain for this context.

Class Notes and Examples

- What are the things you need to know about linear functions?

Example 1

A business bought a machine for $25,000. After 4 years, the machine is valued at $10,000. Assume the value of the machine depreciates linearly. Express the value of the machine as a function of time in years. Graph this function in an appropriate window.

Example 2
Recall the following table from a previous example. It shows market data that a sunglass manufacturer has gathered on a particular pair of sunglasses:

Purchase price	$46	$40	$37	$30	$27
Consumer demand (quantity purchased)	16,000	40,000	52,000	80,000	92,000

Find a formula for the demand (in thousands of units) as a function of the price. Graph this function in an appropriate window.

Example 3

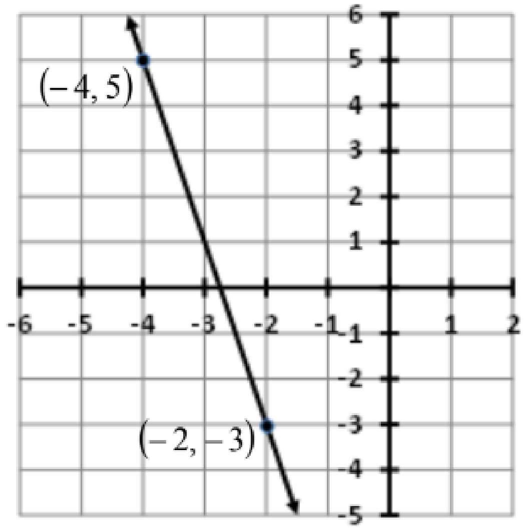
On most state highways, the fine for speeding depends on the speed of the car. In a certain state, the fine for driving 70 mph is \$25, and the fine for driving 85 mph is \$100. Assuming that this relationship is linear, write an equation for the fine as a function of the speed of the car.

More Practice

1. Determine an equation of the linear function passing through the points $(-4,\ 5)$ and $(2,\ 1)$.

2. Determine the intercepts for the linear function: $f(x) = -\frac{1}{2}x + 5$.

3. Determine an equation in slope-intercept form for the linear function graphed below. Identify the intercepts.



4. Determine an equation for the line with slope 0 passing through the point $(7,\ 3)$.

5. Determine an equation for the line with undefined slope passing through the point $(7,\ 3)$.

Additional Comments or Examples

Section: Piecewise Linear Functions

Objectives

- Evaluate a piecewise-defined function at a value in its domain.
- Graph a piecewise-defined function (with only linear pieces).
- Solve application problems involving piecewise-defined functions.

Review Material

- Solving linear equations
- Finding intercepts of functions
- Determining the equation of a line

Introductory Example

A cell phone carrier charges \$50 per month per line, with 2 Gb of data included. Users who exceed 2 Gb of data are charged \$15 per Gb over 2 Gb. Express the total monthly cost for one phone line as a function of the number of Gb of data used. Graph this function in an appropriate window.

Class Notes and Examples

- What are the things you need to know about piecewise-defined functions?

Example 1

A company is planning to order polo shirts pre-printed with their company logo from an online supplier. The cost to create the template for the logo is \$50. The cost per shirt is tiered, according to the number of shirts ordered. The cost per shirt is \$20 for up to 30 shirts. For orders of more than 30 shirts, the charge per shirt is \$15. Write a function to represent the total cost of ordering x shirts. Graph this function. If the total budget is \$1000, how many shirts can be ordered? What if the total budget is \$550?

Example 2

The following data illustrate the way federal income tax brackets are set up. This is a simplified version of the actual tax tables, for taxable incomes of single individuals up to $37,950 in the 2016 tax year.

- A taxable income between $0-$9,325 is charged 10% in tax.
- A taxable income between $9,326-$37,950 is charged 10% of the first $9,325, and 15% of the amount over $9,325.

Write a piecewise-linear function to represent the income tax for a single taxpayer as a function of taxable income, for taxable incomes up to $37,950. Graph this function.

More Practice

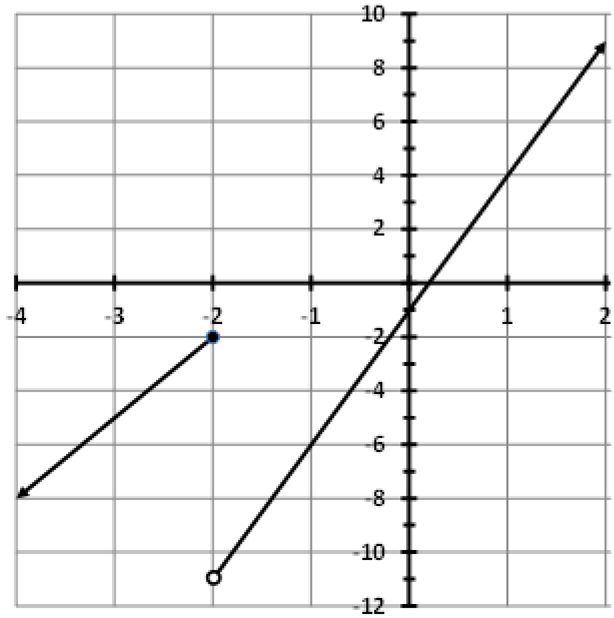
1. Let $f(x) = \begin{cases} -x - 1 & \text{if } x \leq -2 \\ 2x + 3 & \text{if } x > -2 \end{cases}$

(A) Determine $f(-3)$, $f(1)$, and $f(5)$.

(B) Graph the function.

(C) Determine the intercepts.

2. Determine a rule for the function graphed below.



Additional Comments or Examples

Section: Transformations of Functions

Objectives

- Identify by sight the graphs of basic functions, such as the constant function, the identity function, the square function, the cube function, the absolute value function, the square root function, and the cube root function.
- Given a function represented by an equation, write an equation that translates the graph of the function up/down, left/right.
- Given a function represented by an equation, write an equation that reflects the graph of the function across the x-axis and/or y-axis.
- Given a function represented by an equation, write an equation that expands or compresses the graph of the function horizontally and/or vertically.
- Given an equation that represents a transformation of a basic function, identify the transformation(s) involved, and graph the new function.

Review Material

- Domain and range
- Graphs of basic functions

Introductory Example

Recall this example from earlier in the semester: You begin a trip at 8:00 am. As you drive at a fairly constant speed of approximately 60 mph, from Tucson to Bisbee (94 miles), you pass through Benson, which is 40 miles from Tucson. Suppose your friend takes the same trip, but starts 1 hour later. Express your distance from Tucson as a function of time since 8:00 am. Graph both your distance and your friend's distance from Tucson on the same set of axes, where time is measured in hours since 8:00 am.

Class Notes and Examples

Exploratory Activity:

We transform the graph of a known, or "base" function, by adding/subtracting, multiplying/dividing a constant in various ways to the function. Let's explore the relationship between the graphs of a base function and related functions that are transformations of that function, by using our graphing calculators.

	Domain & Range	Graph (Note: max., min., intercepts)	Relationship between graph and graph of base function
Base function: $y = f(x) = \sqrt{x}$			
$y = f(x) + 2 = \sqrt{x} + 2$			
$y = f(x) - 2 = \sqrt{x} - 2$			
$y = f(x+2) = \sqrt{x+2}$			
$y = f(x-2) = \sqrt{x-2}$			
$y = -f(x) = -\sqrt{x}$			
$y = f(-x) = \sqrt{-x}$			

	Domain & Range	Graph	Relationship between graph and graph of base function
Base function: $y = f(x) = (x^3 - x)$			
$y = 2f(x) = 2(x^3 - x)$			
$y = \frac{1}{2}f(x) = \frac{1}{2}(x^3 - x)$			
$y = f(2x) = (2x)^3 - (2x)$			
$y = f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^3 - \left(\frac{1}{2}x\right)$			

If you start with the function $f(x)$, what transformation are described by each of the following, and how do we do those transformations graphically?

- $f(x) + C$, if C is positive? negative?

- $f(x + C)$, if C is positive? negative?

- $-f(x)$

- $f(-x)$

- $Cf(x)$, if $C > 1$? if $0 < C < 1$?

- $f(Cx)$, if $C > 1$? if $0 < C < 1$?

Example 1

A manufacturer has determined the cost and revenue functions for producing a certain product are given by $C(x) = 24x + 5200$ and $R(x) = 66x$. Graph these functions in an appropriate window.

(A) What would be the effect on the graph of the cost function if the fixed costs increased by \$800? What would the new cost function be?

(B) What is the original profit function? Predict what effect would the change in part (A) have on the profit function? Why does this make sense? Check your prediction graphically.

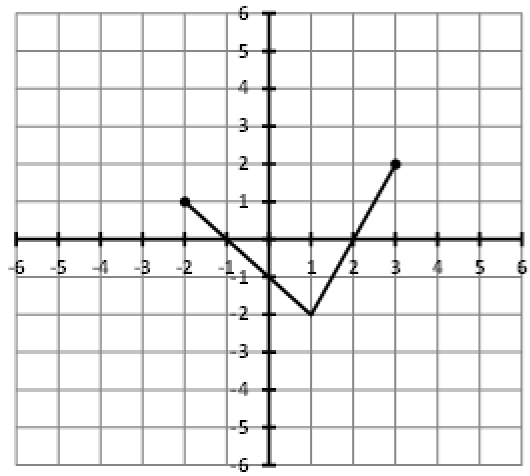
(C) What would be the effect on the graph of the revenue function if the price per unit was increased by 10%? What would the new revenue function be?

Example 2

Shelby invests $100 in an account bearing 2% interest compounded annually. The amount in the account at time *t* in years is given by $A(t) = 100(1.02)^t$. Graph this function, and identify the intercepts. How would you alter the function $A(t)$ if the initial amount of money is doubled? How would the graph be changed?

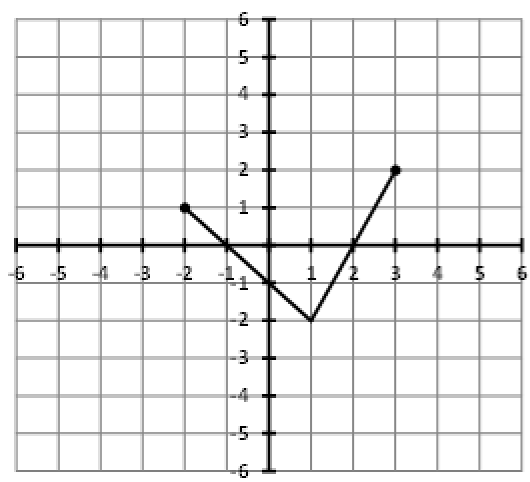
More Practice

1. Given the graph of $y = f(x)$ below, determine the following.



(A) What are the domain and range of $f(x)$?

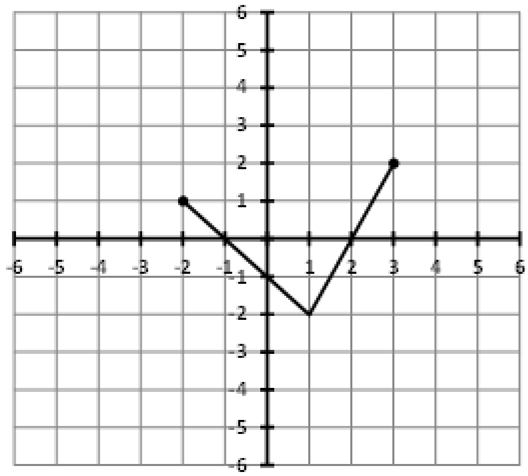
(B) Sketch the graph of $y = f(x) - 2$ on the graph below.



(C) What are the domain and range of $y = f(x) - 2$?

2. Given the graph of $y = f(x)$ below, determine the following.

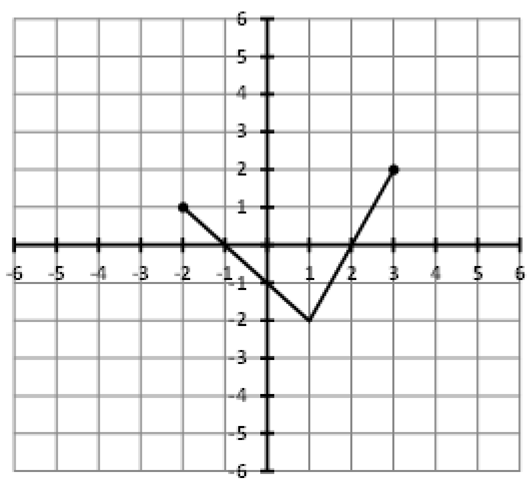
(A) Sketch the graph of $y = f(x + 3)$ on the graph below.



(B) What are the domain and range of $y = f(x + 3)$?

3. Given the graph of $y = f(x)$ below, determine the following.

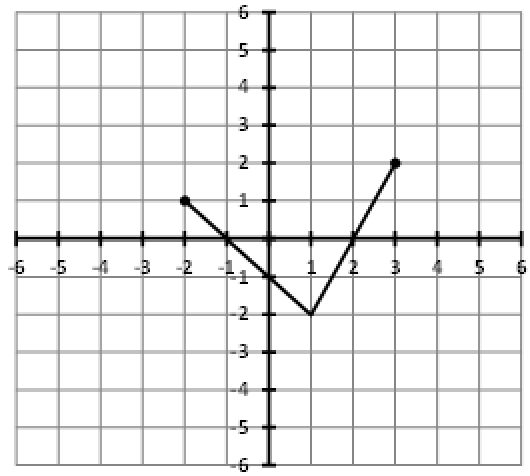
(A) Sketch the graph of $y = -f(x) + 1$ on the graph below.



(B) What are the domain and range of $y = -f(x) + 1$?

4. Given the graph of $y = f(x)$ below, determine the following.

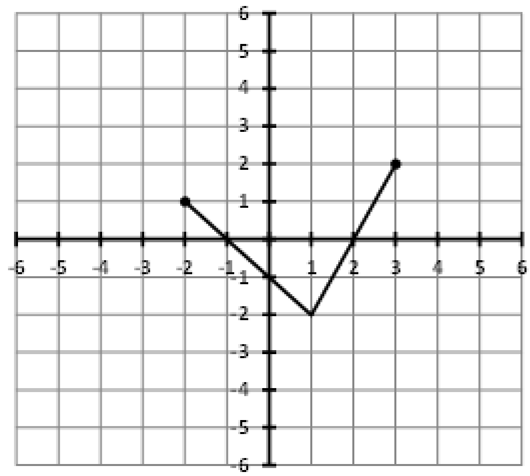
(A) Sketch the graph of $y = f(-x)$ on the graph below.



(B) What are the domain and range of $y = f(-x)$?

5. Given the graph of $y = f(x)$ below, determine the following.

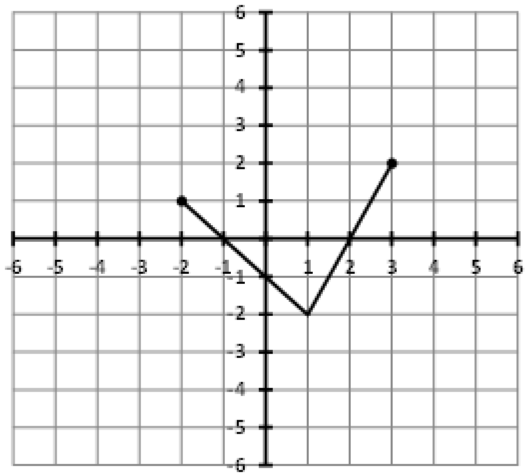
(A) Sketch the graph of $y = 2f(x)$ on the graph below.



(B) What are the domain and range of $y = 2f(x)$?

6. Given the graph of $y = f(x)$ below, determine the following.

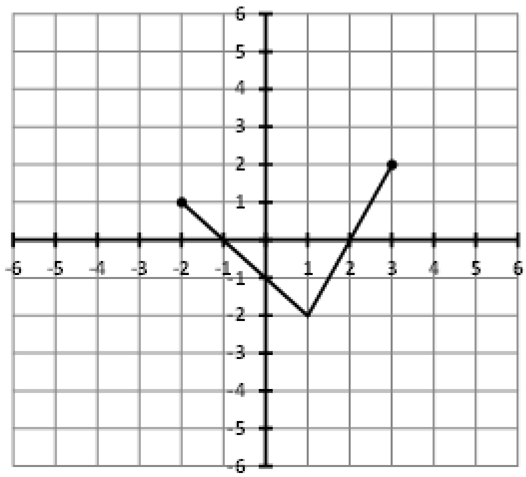
(A) Sketch the graph of $y = \frac{1}{2}f(x-1)$ on the graph below.



(B) What are the domain and range of $y = \frac{1}{2}f(x-1)$?

7. Given the graph of $y = f(x)$ below, determine the following.

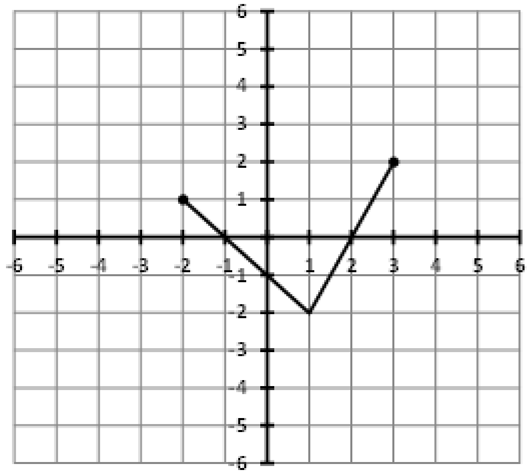
(A) Sketch the graph of $y = f(2x)$ on the graph below.



(B) What are the domain and range of $y = f(2x)$?

8. Given the graph of $y = f(x)$ below, determine the following:

(A) Sketch the graph of $y = f\left(\frac{1}{2}x\right)$ on the graph below.



(B) What are the domain and range of $y = f\left(\frac{1}{2}x\right)$?

9. For each transformation of one of our basic graphs, list the base function and describe the transformations in words. Be sure to describe the transformations in the appropriate order. Identify the domain and range of the transformed function. Verify by graphing.

(A) $y = (x-1)^3 - 4$

(B) $y = -(x-2)^2 + 3$

(C) $y = 5\sqrt{x+7}$

(D) $y = |-5x| + 2$

10. Suppose the point (-3, 5) is on the graph of $y = g(x)$. What point must be on the graph of $y = 2g(-x) - 9$?

11. Suppose the point (1, 4) is on the graph of $y = f(x)$. What point must be on the graph of $y = -f(2x) + 3$?

Additional Comments or Examples

UNIT 2 – Combined, Inverse, and Polynomial Functions

Section: Combining Functions

Objectives

- Find the sum, difference, product, or quotient of functions represented graphically, algebraically, or in table form.
- Find the domain of the sum, difference, product, or quotient of functions.
- Compose two functions represented graphically, algebraically, or in table form.
- Given a compound function, identify two functions whose composition yields the given function.
- Find the domain of a composition of two functions.

Review Material

- Finding domain of functions
- Interval notation and the intersection of intervals
- Function notation

Introductory Example

Recall this example from earlier in the semester. A company produces a pair of skates for \$43.53 and sells each pair for \$89.95. The fixed costs (costs incurred regardless of the number of skates produced) are \$742.72. We wrote the total cost function, $C(x)$, and the revenue function, $R(x)$. Now let's find the difference of these functions, $(R-C)(x) = R(x) - C(x)$, and graph this new function on the same set of axes as $C(x)$ and$R(x)$. What does $(R-C)(x)$ represent?

Class Notes and Examples

- What are the things you need to know about combining functions?

Example 1

The local jazz society puts on a series of weekly concerts during the fall. When concert tickets are priced at \$15, the average attendance is 400 people. The society has been researching different ticket prices, and they've found that each \$2 increase in price will generally result in 25 fewer people attending. Write an expression to represent the attendance, $A(x)$, as a function of the ticket price, x. Use this function and the price function, $p(x) = x$, to find the product, $(pA)(x) = p(x) \cdot A(x)$, and graph this function in an appropriate window. What does this product represent?

Example 2

The sales tax that applies at a particular store is 9%. Write an expression to represent the total price paid, $P(x)$, for an item priced at *x* dollars, including the sales tax. The store is having a big sale, and the price of every item is discounted by \$5. Write a second expression to represent the sale price, $S(x)$ for an item originally priced at *x* dollars (before tax). Find the composition of the functions, $(P \circ S)(x) = P\big(S(x)\big)$. What does this composition represent?

More Practice

1. Use the functions $f(x) = 3x - 7$ and $g(x) = \sqrt{x+1}$ to determine a simplified formula for each of the following. Indicate the domain of each function as well.

(A) $\left(\frac{f}{g}\right)(x)$

(B) $(f \circ g)(x)$

(C) $(g \circ f)(x)$

2. Using the same functions as above, determine $(f \circ g)(3)$ in two different ways. First, by evaluating the function found in part (1B), and second, by evaluating $f\big(g(3)\big)$ where you evaluate $g(3)$ and then evaluate $f(the\ result)$. In which situations is it easier to use one method over the other?

3. If $h(x) = \sqrt{x^2 + 1}$, find two functions $f(x)$ and $g(x)$ so that $(f \circ g)(x) = h(x)$. Can you find a second pair of functions that will work?

4. If $h(x) = \frac{2x}{\sqrt{5-2x}}$, find two functions $f(x)$ and $g(x)$ so that $(f \circ g)(x) = h(x)$. Can you find a second pair of functions that will work?

5. The following tables show the unemployment rate and the crime rate in a particular city.

t, time in years since 2010	U, unemployment rate
0	0.02
1	0.023
2	0.03
3	0.032

U, unemployment rate	C, crime rate (percentage in decimal form of people affected by violent or property crimes)
0.01	0.015
0.02	0.021
0.03	0.028
0.04	0.031
0.05	0.037

Use these tables to compute $(C \circ U)(2)$ and give a practical interpretation. What is the domain of $(C \circ U)$?

Additional Comments or Examples

Section: Inverse Functions

Objectives

- Given a function represented by a graph, an equation, a table of values, or a verbal description, determine whether the function is one-to-one (i.e. has an inverse function).
- Verify algebraically that two functions are inverse functions by finding the composition of the two functions.
- Given the graph of a one-to-one function, identify the domain and range of the inverse function, and sketch its graph.
- Find the inverse function of a one-to-one function represented by a graph, an equation, or a table of values.

Review Material

- Solving linear equations
- Composition of functions

Introductory Example

Recall this example from earlier in the semester: We found that the demand for a certain pair of sunglasses could be expressed as a function of the purchase price using the equation $q = f(p) = -4000p + 200{,}000$. Use this equation to express p as a function of q. What does this new function tell us?

Class Notes and Examples

- What are the things you need to know about inverse functions?

Example 1

Verbal descriptions of several functions are given below. Explain what the inverse of the function described tells you, and determine whether this inverse qualifies as a function.

(A) $H(d)$ represents the height above ground of a ball thrown horizontally off a balcony when the ball is d meters away.

(B) $T(p)$ represents the sales tax paid on an item that sells for p dollars.

(C) $C(t)$ represents the average cost of a gallon of unleaded gasoline in Tucson t days after September 1.

(D) $R(n)$ represents the revenue generated by selling n cups of coffee at a football game.

(E) $V(h)$ represents the volume of water in a rain barrel when the water is h inches deep.

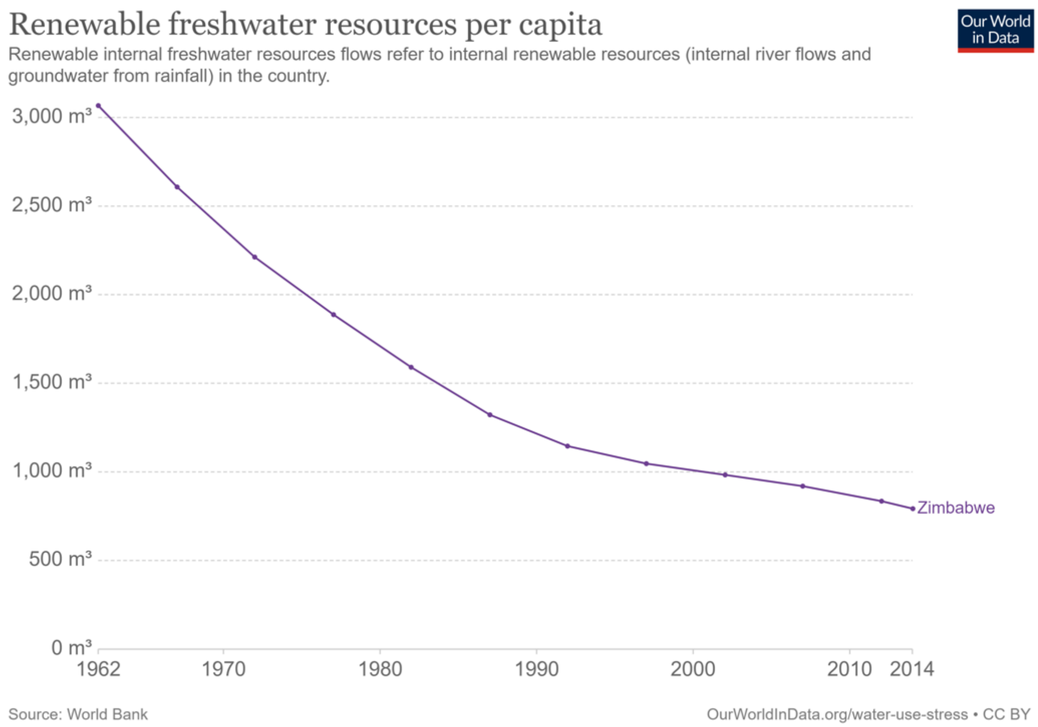
(F) $P(x)$ represents the profit function found from the revenue function $R(x) = x\left(\frac{100{,}000 - x}{2000}\right)$ and the cost function $C(x) = 1200 + 10x$, where x is the number of units sold.

Example 2

Suppose a cost-benefit model is given by $C = f(x) = \frac{6.6x}{100-x}$, where C is the cost, in thousands of dollars, of removing x percent of a given pollutant. Determine a formula for the inverse function and give a practical interpretation.

More Practice

1. The graph below shows the amount of renewable freshwater (in cubic meters) per person in Zimbabwe as a function of time over several decades. Call this function $W = f(t)$, where t is the year. Use this graph to estimate $f^{-1}(2000)$ and give a practical interpretation.



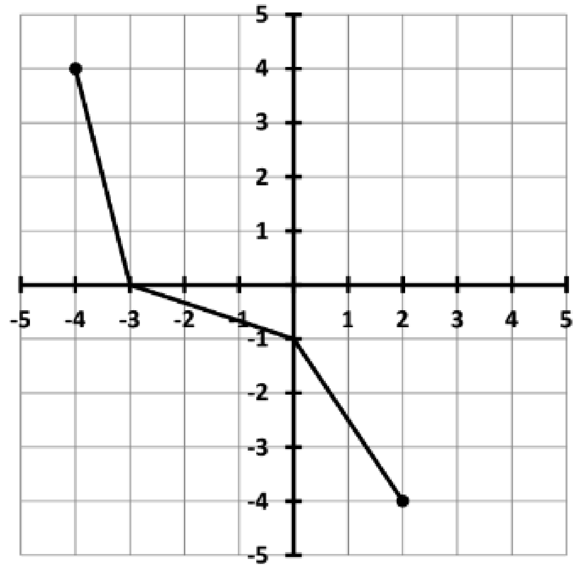
2. Determine the inverse function and determine the value of $f^{-1}(2)$ for each of the functions below.

(A) $f(x) = 7x^3 + 1$

(B) The function *f* described by the set of ordered pairs below.

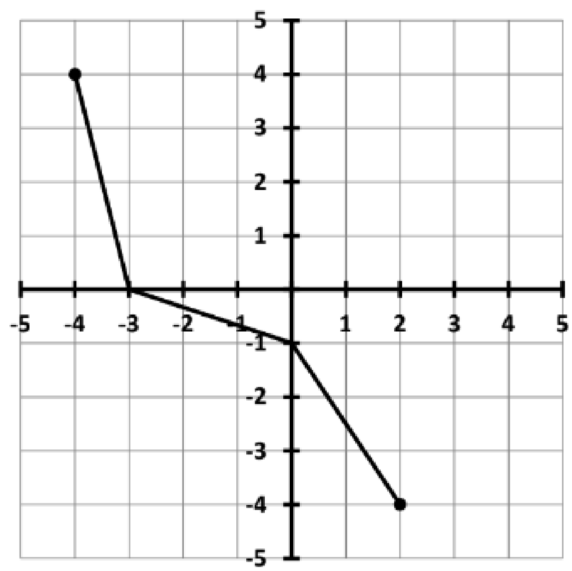
n	3	-4	1	2
$f(n)$	2	6	-1	0

3. Consider the graph of the function $y = f(x)$.



(A) What are the domain and range of the function?

(B) Sketch a graph of the inverse function $y = f^{-1}(x)$.



(C) What are the domain and range of the inverse function?

Additional Comments or Examples

Section: Quadratic Functions and Applications

Objectives

- Given a function represented algebraically, determine whether the function is quadratic.
- Write a quadratic function in different forms, such as factored form or vertex form.
- Identify and understand the components of a quadratic function in standard form, and identify the impact of these components on the graph of the function.
- Identify the vertex or maximum/minimum of a quadratic function using either the standard form of the function or the vertex formula.
- Find the equation of a quadratic function, given the vertex and a point, or two zeros and a point.
- Set up a quadratic function to model certain "real life" situations, and interpret results in "real life" terms. Examples include projectile motion models, area models, and demand/cost/revenue/profit models.

Review Material

- Solving quadratic equations
- Quadratic Formula program in calculator

Introductory Example

Recall this problem from earlier in the semester: A business has created a mathematical model based on market data for its profit, P (in dollars) as a function of the number of items sold, x. The model is given by the function $P(x) = -0.1x^2 + 150x - 14000$. This is an example of a quadratic function. How many units must be produced and sold in order to maximize profit? What is the maximum profit?

Class Notes and Examples

- What are the things you need to know about quadratic functions?

Example 1

A stone is thrown upward. Its height in meters t seconds after release is given by $h(t) = -4.9t^2 + 49t + 277.4$.

(A) How many seconds does it take the stone to reach its maximum height? What is the maximum height the stone reaches?

(B) When does the stone strike the ground?

Example 2

A concert venue holds a maximum of 1000 people. With ticket prices at \$30, the average attendance is 650 people. It is predicted that for each dollar the ticket price is lowered, approximately 25 more people attend. Create a function to represent the revenue generated from ticket sales, and use this to find the maximum possible revenue.

Example 3

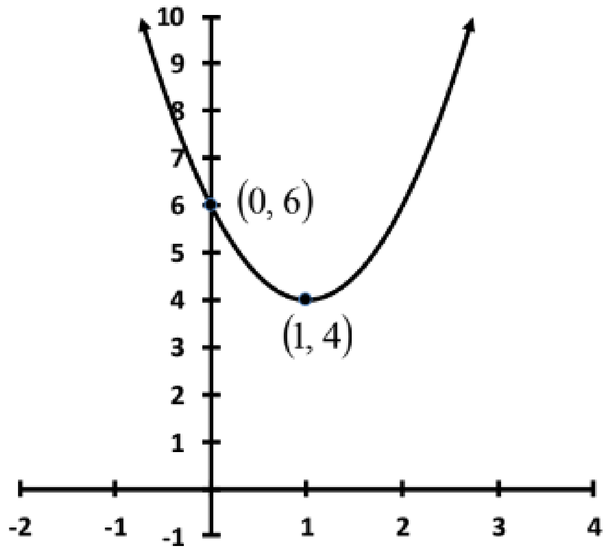
Suppose a sunglass manufacturer determines the demand function for a certain line of sunglasses is given by $p = 50 - \frac{1}{4000}x$, where *p* is the price per pair and *x* is the number of pairs sold. The fixed cost of producing a line of sunglasses is \$25,000 and each pair of sunglasses costs \$3 to make. How many pairs of sunglasses should be produced and sold in order to maximize profit?

Example 4

A sports team has 1500 feet of fencing to enclose a field. What is the maximum area that can be enclosed? What are the dimensions (length and width) of the enclosed field?

More Practice

1. Determine a formula for the parabola graphed below.



2. Determine a formula for the parabola that goes through the points $(-5,\ 0)$, $(3,\ 0)$, and $(4,\ -12)$.

3. Solve for x: $x^2 - 6x - 1 = 0$.

Additional Comments or Examples

Section: Polynomial Functions

Objectives

- Identify the general form for a polynomial function.
- Determine the end behavior of polynomials from the leading term property.
- Determine the zeros of a polynomial function.
- Sketch the graph of a polynomial function.
- Use the end behavior of a polynomial to determine the possible degree of the polynomial and sign of the leading coefficient.
- Determine a possible equation of a polynomial function given its graph.

Review Material

- Factoring
- Solving quadratic equations

Introductory Example

Based on actuarial life tables, the average number of years of life remaining for a female of age x (for ages up to 100) can be given approximately by the function $N(x) = 0.0000004x^4 - 0.00004x^3 - 0.002x^2 - 1.0191x + 81.955$. Graph this function in an appropriate window.
[Data source: http://www.lifeexpectancycalculators.com/actuarial-life-tables.html]

Class Notes and Examples

Exploratory Activity

Let's take a look at some polynomial functions, and explore the relationship between the graphs of the functions and their algebraic forms, by using our graphing calculators.

	Degree and Leading Term	Graph (Note: intercepts, max., min.)	Zeros and Factored Form
$y = x^3$			
$y = x^4$			
$y = x^5$			
$y = x^6$			
$y = x^3 + 7x^2 + 6x$			
$y = x^4 - 5x^2 + 4$			
$y = \frac{1}{2}x^2(x+3)(x-1)(x-4)$			

Class Notes and Examples

- What are the things you need to know about polynomial functions?

General Form:

Degree:

Leading Term/Leading Coefficient:

Long-term Behavior of Polynomial Functions:

	Even Degree	Odd Degree
Positive Leading Coefficient		
Negative Leading Coefficient		

Zeros:

Example 1

A 12” x 8” rectangular piece of cardboard is folded into an open-top box. The folding is made possible by cutting identical squares out of the four corners of the cardboard. Determine a formula for the volume of this box, and find the maximum possible volume. [Hint: a drawing or a physical model may prove extremely valuable.]

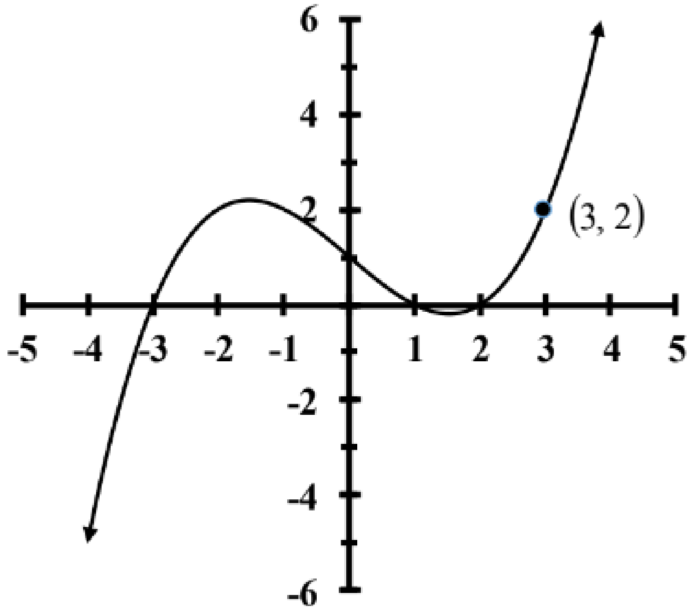
Example 2

For a particular product, it is determined that the cost function and revenue function are given by $C(x) = 0.00003x^3 + 7x + 1500$ and $R(x) = -0.01x^2 + 40x$, where x is the number of units produced and sold. Find a function to represent the profit as a function of x, and determine the number of units that should be produced and sold in order to maximize profit.

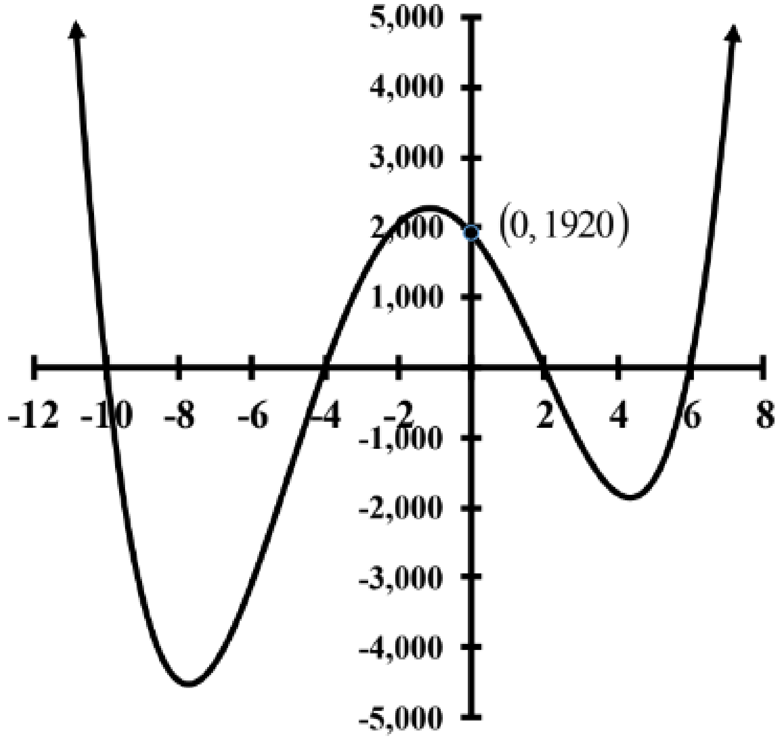
More Practice

1. Determine a possible equation for the polynomial functions graphed below. Verify by graphing on your calculator

(A)



(B)



5,000
4,000
3,000
2,000
1,000
-1,000
-2,000
-3,000
-4,000
-5,000
-12
-10
-8
-6
-4
-2
2
4
6
8
(0, 1920)

2. Determine the degree, leading coefficient, end behavior, and zeros for the following polynomial functions. Sketch a rough prediction of each graph by hand. Verify by graphing on your calculator.

(A) $M(x) = x^3 - x^2 - 6x$

(B) $R(x) = 0.5(x+3)(x+1)(x-2)(x-5)$

Additional Comments or Examples

UNIT 3 – Rational, Exponential and Logarithmic Functions

Section: Rational Functions

Objectives

- Identify whether a function represented by an equation is a rational function.
- Determine the domain, range, and intercepts of a rational function represented by an equation or a graph.
- Given a rational function represented by an equation, determine the equation of any vertical or horizontal asymptote for the function.
- Use the horizontal asymptote of a rational function to determine the long-term behavior of the function, over vice versa.
- Solve application problems involving rational functions.

Review Material

- Finding domain, range, intercepts
- Solving equations involving rational expressions
- Solving linear and quadratic equations

Introductory Example

A clothes dryer is purchased for \$750, and electricity to run it costs approximately \$95 per year. Write a function to express the total cost of operating the dryer for x years. Then create a function to express the **average** cost per year of operating the dryer. Graph this function in an appropriate window. What is the long-term behavior of this function?

Class Notes and Examples

- What are the things you need to know about rational functions?

Example 1

Recall this example from earlier in the semester. The cost, *C* (in thousands of dollars), of removing *x* percent of a certain pollutant from a lake is given by the formula $C(x) = \frac{18x}{100-x}$.

(A) Graph this function.

(B) If the city has $30,000 to clean up the lake, what percentage of pollutants can be removed?

(C) How can you visualize this on the graph?

Example 2

A company that manufactures bicycles has determined that a new employee can assemble $M(d)$ bicycles per day after d days of on-the-job training, where $M(d) = \frac{100d^2}{3d^2+10}$.

(A) After how many days of training would an employee be able to assemble 25 bicycles per day?

(B) What does the horizontal asymptote tell you about this problem?

More Practice

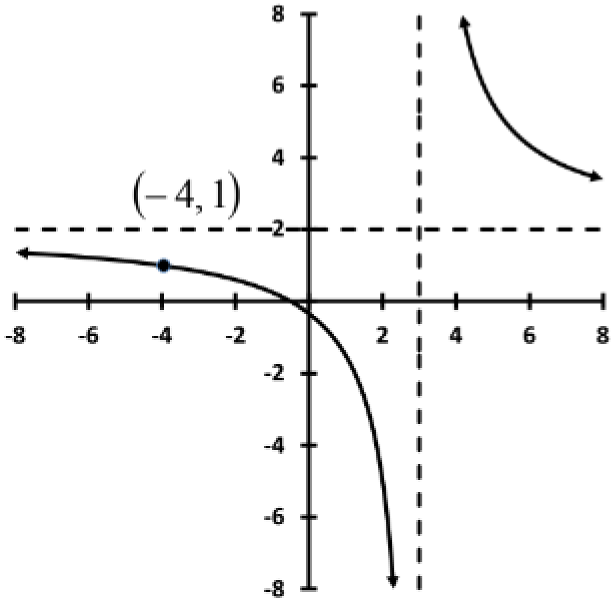
1. Determine the domain, zeros, and equations for all asymptotes for each function. Verify by graphing. Use the TABLE feature of your calculator to assist you.

   (A) $p(x) = \frac{3x-2}{x+1}$

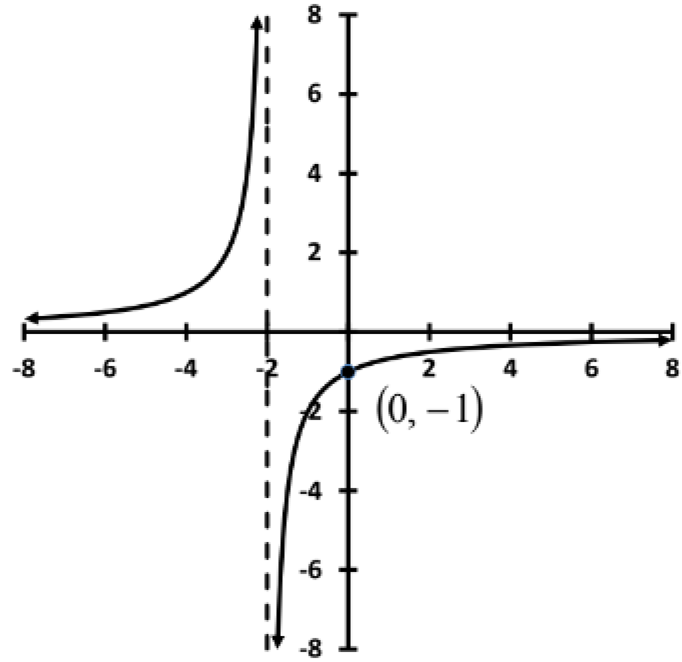
(B) $R(x) = \frac{2x+1}{x^2-16}$

2. Determine a formula for the rational functions graphed below.

(A)



(B)



3. How many different functions can you create that have a vertical asymptote at $x = 1$?

Additional Comments or Examples

Section: Exponential Functions

Objectives

- Identify exponential functions.
- Understand and identify the characteristics of exponential functions of the form $y = C \cdot b^x$, including the domain, range, intercept, asymptote, end behavior, and shape of graphs.
- Determine the formula for an exponential function given its graph.
- Solve compound interest and exponential growth/decay application problems.

Review Material

- Properties of exponents
- Solving systems of equations using substitution

Introductory Example

Nathan has \$100 to open a savings account. He has found an account that offers 2.5% interest compounded annually. Write a function to represent the balance in the account as a function of time in years, assuming that the initial deposit and all subsequent interest is kept in the account. Graph this function in an appropriate window.

Class Notes and Examples

- What are the things you need to know about exponential functions?

What is the general form of an exponential function?

What is the domain of an exponential function? The range?

What is/are the intercept(s)? The asymptote(s)?

What is the formula for compound interest if interest is compounded annually? *n* times per year? continuously?

Example 1

A person plans to invest $5,000 into a money market account. Determine the interest rate that is needed for the money to grow to $45,000 in 30 years if the interest is compounded quarterly. Round the interest rate to the nearest 0.01%.

Example 2

Bacteria from raw eggs has come in contact with some other ingredients that you are putting into a pasta salad. Initially there were 500 bacteria present; one hour later there were 4000 bacteria in the salad. The population of bacteria in the pasta salad can be modeled by an exponential function of the form $P(t) = C \cdot b^t$, where t is measured in hours. Determine the function, and use the graph to determine the number of hours it takes for the population to double in size.

Example 3

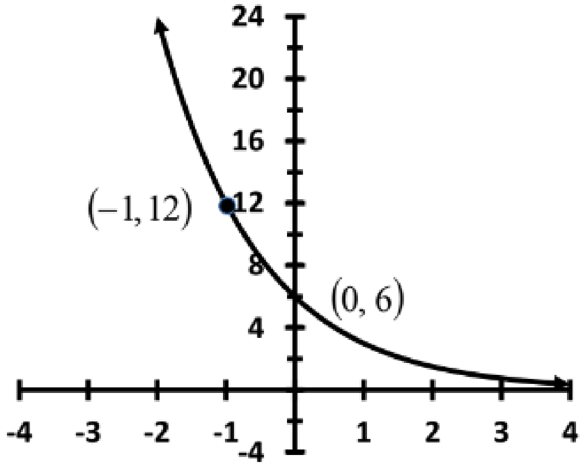
The amount of radioactive material in a 200-gram sample of bismuth can be modeled by the function $A(t) = 200e^{-0.1386t}$, where *t* is time in days. What is the half-life of this radioactive material?

More Practice

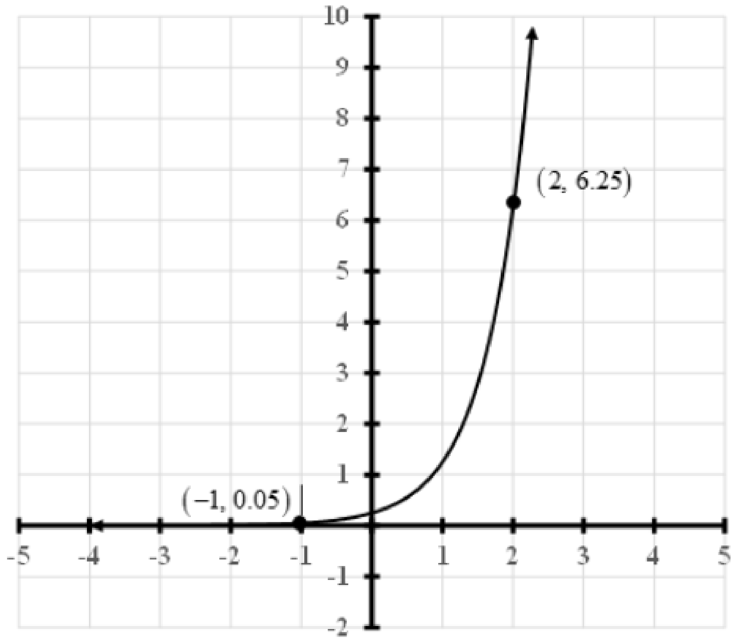
1. Determine a function of the form $y = C \cdot b^x$ that passes through the points $(1,\ 12)$ and $(3,\ 192)$.

2. Determine a function of the form $y = C \cdot b^x$ for each of the exponential functions graphed.

(A)



(B)



3. The function $y = -2e^{x+3}$ was created using transformations of the function $y = e^{x}$. Determine the transformations that were performed. Give the domain, horizontal asymptote, range, and *y*-intercept. Graph the given function.

Transformation(s): ____

Domain: __________________

Horizontal Asymptote: ____________________

Range: ____________________

y-intercept: ________________

Graph:

Additional Comments or Examples

Section: Logarithmic Functions

Objectives

- Apply the definition of a logarithm to convert an equation in logarithmic form into exponential form.
- Given an equation in exponential form, rewrite the equation in logarithmic form.
- Properly identify and use the notation for natural (base e) and common (base 10) logarithm.
- Evaluate natural and common logarithms on a calculator.
- Know and identify the shape and basic features of the graphs of log functions.
- Determine the domain, range, intercepts of logarithmic functions.

Review Material

- Inverse functions

Introductory Example

Suppose \$100 is invested in an account earning 5% interest compounded annually. How long will it take the amount in the account to grow to \$200?

Class Notes and Examples

- What are the things you need to know about logarithmic functions?

What is a logarithm?

What is the notation for logarithms?

What is the exponential equivalent of a logarithm?

How do you evaluate simple logarithmic expressions?

What is the common logarithmic function? The natural logarithmic function?

What is the domain of a logarithmic function? The range?

What does the graph look like in general?

What is the Change of Base Formula?

Example 1

Suppose $100 is invested in an account earning 5% interest compounded annually. How long will it take for the amount in the account to grow to $200? How about to D dollars?

Example 2

A population of 1000 bacteria is on the kitchen counter. The amount of time (*T*), in hours, that it takes the population to reach *x* bacteria is given by $T = f(x) = 16 \cdot \ln\left(\frac{x}{1000}\right)$.

(A) How long will it take for the population to double?

(B) How many bacteria will there be in 3 hours?

(C) Determine a formula for the inverse function, $f^{-1}(T)$.

(D) What does the inverse function tell you?

More Practice

1. Evaluate each of the following.

(A) $\log_3(81)$

(B) $\log_2\left(\frac{1}{8}\right)$

(C) $\log_{16}(4)$

2. For the function $f(x) = \log(3x + 2)$, determine the domain, vertical asymptote, range, and intercept(s). Graph the given function.

Domain: ___________________

Vertical Asymptote: _______________________

Range: ____________________

x-intercept: _________________

y-intercept: _________________

Graph:

3. Rewrite $\log_5(3) = M$ in exponential form.

4. Rewrite $7^C = 12$ in logarithmic form.

5. Solve for the exact value of x: $4(3)^x = 20$

Additional Comments or Examples

Section: Properties of Logarithms

Objectives

- Apply the properties of logarithms to expand expressions involving the logarithm of a product/quotient/power into a sum/difference of logarithms.
- Apply the properties of logarithms to condense a sum/difference of logarithms into a single logarithm.

Review Material

- Definition of logarithm
- Evaluating logarithms on calculator

Class Notes and Examples

Exploratory Activity

Let's evaluate some logarithms using our calculators, and try to discover some of the more advanced properties of logarithms.

Evaluate:

$\log_5(25) =$ $\log_5(125) =$ $\log_5(25 \cdot 125) =$

$\log_2\left(\frac{1}{8}\right) =$ $\log_2(4) =$ $\log_2\left(\frac{1}{8} \cdot 4\right) =$

$\log_{\mathrm{b}}(b^5) =$ $\log_{\mathrm{b}}(b^2) =$ $\log_{\mathrm{b}}(b^5 \cdot b^2) =$

In general, $\log_{\mathrm{b}}(x \cdot y) =$ ______________________________

$\log_2(32) =$ $\log_2(4) =$ $\log_2\left(\frac{32}{4}\right) =$

$\log_3(9) =$ $\log_3(81) =$ $\log_3\left(\frac{9}{81}\right) =$

$\log_{\mathrm{b}}(b^5) =$ $\log_{\mathrm{b}}(b^2) =$ $\log_{\mathrm{b}}\left(\frac{b^5}{b^2}\right) =$

In general, $\log_{\mathrm{b}}\left(\frac{x}{y}\right) =$ ______________________________

$\log_2(4^5) =$ $5 \cdot \log_2(4) =$

$\log(100^{-2}) =$ $-2 \cdot \log(100) =$

$\ln(e^7) =$ $7 \cdot \ln(e) =$

In general, $\log_b(x^p) =$ ______________________________

Two special values to remember: $\log_b b =$ ______ $\log_b 1 =$ _________

- What are the things you need to know about properties of logarithms?

What is the property for the logarithm of a product?

What is the property for the logarithm of a quotient?

What is the property for the logarithm of a power?

What are the inverse properties for logarithms and exponentials?

Example 1
Use properties of logarithms to rewrite each of the following as a single logarithm.

(A) $\ln(6x) + \frac{1}{2}\ln(x) - \ln(2x)$

(B) $\log(5z) - \log(x) - 3\log(3y) + \log(t)$

(C) $2\log_2(x) + \log_2(y) - 4\log_2(P) - \frac{1}{3}\log_2(Q) + \log_2(z)$

Example 2
Use properties of logarithms to expand each expression as much as possible.

(A) $\ln(10xe^{3x})$

(B) $\log\left(\frac{2x^4}{y\sqrt{z}}\right)$

(C) $\log_5\left(\sqrt{5a}\right)$

More Practice

1. Use the natural logarithm and a property of logarithms to solve $4(3)^x = 20.$

   Note: As we saw in the last section, you could also solve this equation using a logarithm with base 3. The point of this problem is to learn a second method. Why? For one, this new method will not require the change of base formula at the end if we want an approximate answer. Secondly, some problems on Midterm 3 will have instructions like this one that force you to use a logarithm with a particular base.

2. Using a property of logarithms, solve $\log(-x-2)+\log(1-x)=1$.

3. Using a property of logarithms, solve $\log_3(3x + 17) - \log_3(x + 1) = 2$.

Additional Comments or Examples

Section: Exponential and Logarithmic Equations and Applications

Objectives

- Solve exponential equations.
- Solve log equations.
- Solve exponential and logarithmic equations in the context of applications, including compound interest, doubling time, half-life, logistic models.
- Set up and solve application problems involving exponential growth and decay.

Review Material

Solving linear and quadratic equations

Introductory Example

Recall this example from earlier this semester: A cup of coffee that is initially $125^{\circ}F$ is placed in a room kept at a constant $72^{\circ}F$. The temperature of the coffee, T, as a function of time is given by $T(x) = 72 + 53(0.8)^x$, where x is measured in minutes. How long will it take the coffee to cool to $100^{\circ}F$?

Class Notes and Examples

- What are the things you need to know about solving exponential and logarithmic equations?

What is the general strategy for solving exponential equations?

What is the general strategy for solving log equations?

Why is it necessary to check the solutions to a logarithmic equation?

Example 1

The world population has been growing roughly exponentially for the past 30 years. In 1987, the world population was approximately 5 billion. In 1998, the world population was approximately 6 billion.

(A) Find an exponential equation of the form $y = Ce^{kt}$ which models the world population with t representing the number of years since 1987. Use at least 6 decimal places for the value of k.

(B) What does this model predict the population was in 2018? Round to the nearest 0.01 billion.

(C) According to this model, when did the population reach 7 billion?

(D) When is the population predicted to reach 8 billion?

(E) What is the doubling time for the world population?

Example 2

Suppose Matt wants to have \$10,000 saved in 9 years. How much should he invest today at 3.4% interest compounded continuously in order to reach his goal?

Create your own extension questions and answers. What if…

Example 3

A sample of bismuth initially contains 200 g of radioactive material. After 7 days, 37.9% of the radioactive bismuth remains.

(A) Find an exponential equation which models the amount of radioactive bismuth in the sample as a function of time.

(B) According to your model, when will 20% of the radioactive bismuth remain?

(C) When will the amount of radioactive bismuth remaining be 10 g?

(D) What is the half-life for this material?

(E) How much radioactive bismuth will be there after 14 days?

(F) What percentage of the radioactive bismuth will remain after 28 days?

Example 4

The doubling time for an investment is given by the equation $T = \frac{\ln(2)}{\ln(1+r)}$, where r is the interest rate in decimal form (e.g., 3% = 0.03). At what interest rate would you need to invest in order to double your investment in 10 years?

Example 5

Due to oxygen supply, available food, and other variables, the fish population, $P(t)$, in a particular lake grows according to the function:

$$P(t) = \frac{18{,}000}{1+24e^{-t/4}} \qquad (t \geq 0).$$

where t is time in months since the fish were introduced into the lake.

(A) How many fish were first introduced into the lake?

(B) Graph the function and identify the upper bound of the fish population.

(C) Use algebra to find out when will the population reach 10,000.

(D) How many fish will be in the lake at the end of 2 years?

More Practice

1. Solve the equation: $\log(z) + \log(z - 15) = 2$

2. Solve the equation: $1 = \frac{2}{1+199e^{-0.12t}}$

3. Solve the equation: $140\left(\frac{1}{2}\right)^{t/4} = 350$

4. The population, $P(t)$, of the United States (in millions) can be approximated by the model $P(t) = 227e^{0.0093t}$ where t is the number of years since 1980.

(A) Find the formula for the inverse function.

(B) Evaluate $P^{-1}(265)$. Round your answer to two decimal places. Explain what this means in practical terms.

Additional Comments or Examples