Directions:
Please only put your final, well written solutions, in the space provided.
Give exact answers (simplified radicals or fractions).
If you use additional paper clearly label the question and upload pages after the question page.
Use complete sentences and explain your reason as much as possible.
There are 4 questions and 40 points total

Name: _____

1. For this question, we will be considering the function

$$f(x) = -\frac{2}{5}(x+1)^2 + 3$$

(a) [1 point] What is the vertex of the parabola formed by $f(x)$?

(a) _____

(b) [1 point] What is the range of $f(x)$?

(b) _____

(c) [1 point] What is the vertical intercept of $f(x)$?

(c) _____

(d) [2 points] What is/are the horizontal intercept(s) of $f(x)$?

(d) _____

(e) [1 point] On what interval(s) is $f(x)$ negative?

(e) _____

(f) [1 point] On what interval(s) is $f(x)$ increasing?

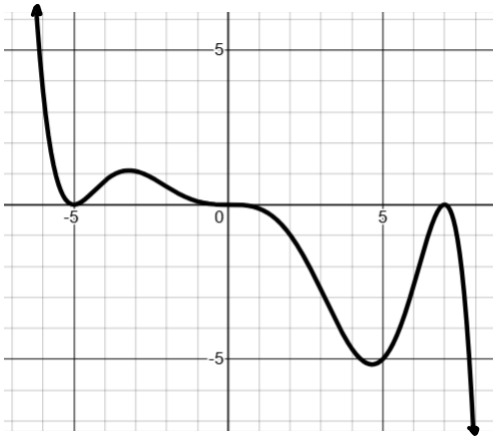
(f) _____

(g) [3 points] Sketch the graph of $f(x)$ carefully label the vertex, all intercepts, and have a clear scale on your horizontal and vertical axis.

2. (a) [3 points] Given the function $h(x) = \frac{1}{4}(x+5)(x-3)(x-9)$, for what values of x is the function $h(x)$ positive? Demonstrate/explain your reasoning and working out. Give your answer in interval notation.

(a) _____

(b) Given the graph below, answer each question



i. [1 point] What is a reasonable degree for this polynomial?

i. _____

ii. [1 point] Is the leading coefficient of this polynomial *positive* or *negative*?

ii. _____

iii. [1 point] What are the *factors* of this polynomial?

iii. _____

iv. [1 point] Using only a + or − for the leading coefficient, write the factored form for the polynomial.

iv. _____

(c) [3 points] Sketch the graph of $g(x) = x(x-4)^2(x+2)$. Label all intercepts and emphasize the shape of the polynomail near the horizontal intercepts. The vertical values of any minimums or maximums *do not need to be to scale*. I want to see the correct *shape* and end behavior. Make sure your sketch is fully connected and indicate end behaviors with arrows.

3. For this question, we will be using the function

$$R(x) = \frac{-(x+4)(x-2)(x-3)}{x^3+27}$$

(a) [1 point] What is the domain of $R(x)$?

(a) _____

(b) [1 point] What are (if any) the *vertical asymptotes* of $R(x)$?

(b) _____

(c) [1 point] What is (if any) the *end behavior* of $R(x)$? (If it has horizontal asymptote, state the equation, if it does not state there is no H.A.)

(c) _____

(d) [1 point] What is the vertical intercept of $R(x)$?

(d) _____

(e) [1 point] What are the horizontal intercepts of $R(x)$?

(e) _____

(f) [5 points] Sketch the graph of $R(x)$. Include and label all intercepts, vertical asymptote(s) and horizontal asymptote. Sketch asymptotes as a dotted or dashed line and/or in a different color if you prefer.The vertical scale does not need to be perfect, but it should be clear where the end behavior is and the behavior of the function near any vertical asymptotes.

4. (a) [3 points] Carefully sketch the graph of

$$f(x) = \begin{cases} -1 & -1 \le x < 0 \\ 1 & 0 \le x < 1 \end{cases}$$

Use the graph you created to answer the remaining questions. Read the notation *carefully* and if you have any questions ask on Teams.

(b) [1 point] What is $f(0)$?

(b) _____

(c) [1 point] What is $\lim_{x \to 0^-} f(x)$?

(c) _____

(d) [1 point] What is $\lim_{x \to 0^+} f(x)$?

(d) _____

(e) [1 point] What is $\lim_{x \to 0} f(x)$?

(e) _____

(f) [1 point] What is $f(0.5)$?

(f) _____

(g) [1 point] What is $\lim_{x \to 0.5} f(x)$?

(g) _____

(h) [1 point] What is $\lim_{x \to -0.5} f(x)$?

(h) _____