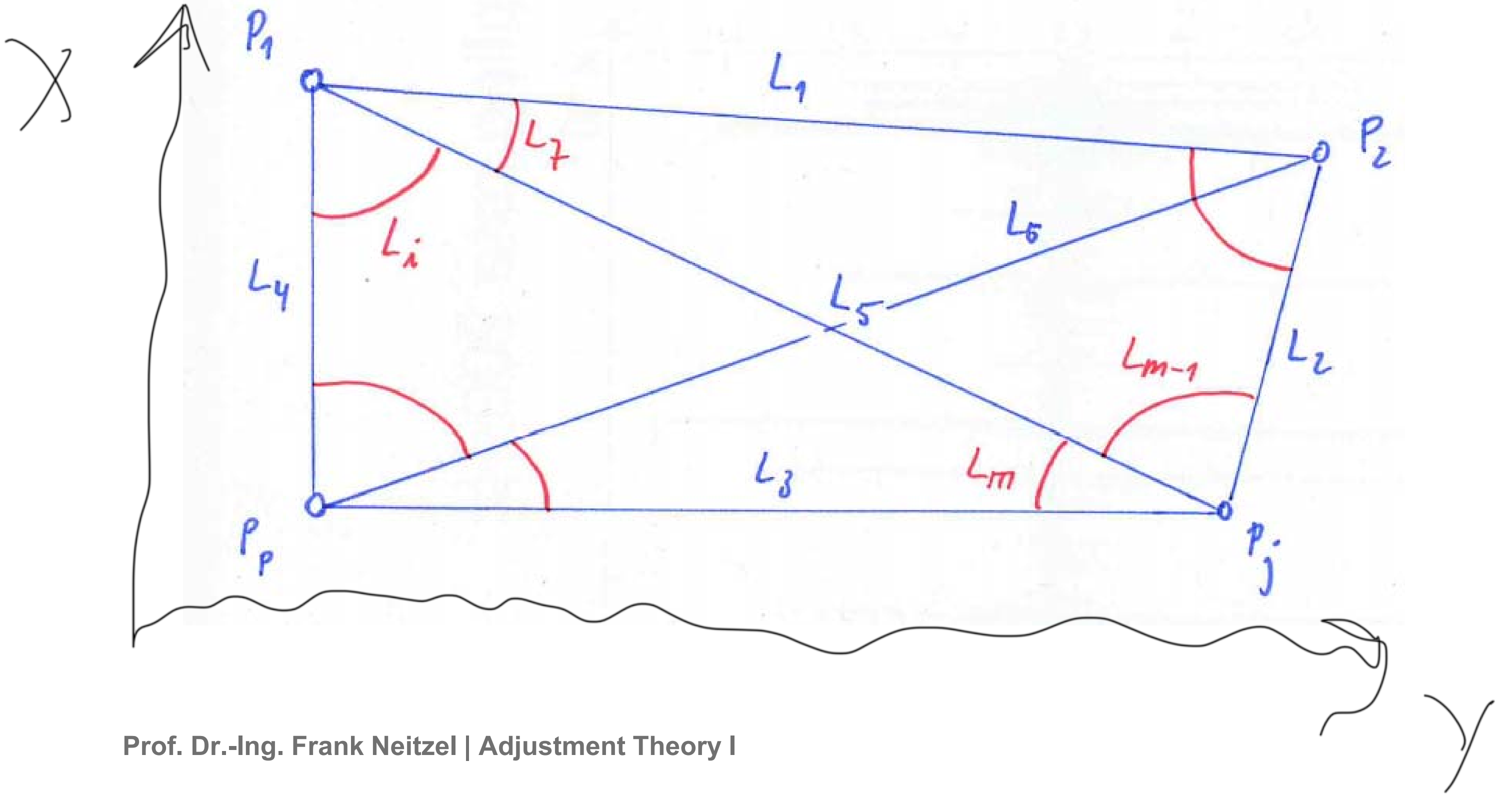
3.2 The m-dimensional random vector

3.2.1 Theoretical expectation and theoretical covariance matrix

Example: Measurement of directions and distances in a geodetic network



3.2.1 Theoretical expectation and theoretical covariance matrix

- Given: m random variables $L_1, L_2, \dots, L_m$

- Random vector

$$\mathbf{L}_{m\times 1} = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_m \end{bmatrix}$$

- Vector of expectations

$$\boldsymbol{\mu}_{m\times 1} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix} = E\{\mathbf{L}_{m\times 1}\} = \begin{bmatrix} E(L_1) \\ E(L_2) \\ \vdots \\ E(L_m) \end{bmatrix}$$

3.2.1 Theoretical expectation and theoretical covariance matrix

- Vector of random deviations

Expectation

$$\boldsymbol{\varepsilon}_{m\times1}=\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \vdots\\ \varepsilon_m\end{bmatrix}=\mathbf{L}_{m\times1}-\boldsymbol{\mu}_{m\times1}=\begin{bmatrix}L_1-\mu_1\\ L_2-\mu_2\\ \vdots\\ L_m-\mu_m\end{bmatrix}$$

Expectation of a expectation? = itself

- as in 2.4.1

$$E(\boldsymbol{\varepsilon}_{m\times1})=E\{\mathbf{L}_{m\times1}-\boldsymbol{\mu}_{L_{m\times1}}\}=\underbrace{E(\mathbf{L}_{m\times1})}_{\boldsymbol{\mu}_L}-\boldsymbol{\mu}_{L_{m\times1}}=\mathbf{0}$$

Covariance

$$E\{\boldsymbol{\varepsilon}_{L_{m\times1}}\cdot\boldsymbol{\varepsilon}^{\mathrm{T}}_{L_{1\times m}}\}=\begin{bmatrix}E(\varepsilon_1^2) & E(\varepsilon_1\cdot\varepsilon_2) & \cdots & E(\varepsilon_1\cdot\varepsilon_m)\\ E(\varepsilon_2\cdot\varepsilon_1) & E(\varepsilon_2^2) & \cdots & E(\varepsilon_2\cdot\varepsilon_m)\\ \vdots & \vdots & \ddots & \vdots\\ E(\varepsilon_m\cdot\varepsilon_1) & E(\varepsilon_m\cdot\varepsilon_2) & \cdots & E(\varepsilon_m\cdot\varepsilon_m)\end{bmatrix}=\boldsymbol{\Sigma}_{LL_{m\times m}}$$

Variance

3.2.1 Theoretical expectation and theoretical covariance matrix

- Theoretical VCM of L

$$\boldsymbol{\Sigma}_{LL_{m\times m}} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_m^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \rho_{12}\cdot\sigma_1\cdot\sigma_2 & \cdots & \sigma_{1m} \\ \rho_{21}\cdot\sigma_2\cdot\sigma_1 & \sigma_2^2 & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{m1}\cdot\sigma_m\cdot\sigma_1 & \rho_{m2}\cdot\sigma_m\cdot\sigma_2 & \cdots & \sigma_m^2 \end{bmatrix}$$

- Theoretical Variance of L_i

$\xi + 0$

$$E\left(\varepsilon_i^2\right) = \sigma_i^2$$

3.2.2 Empirical expectation and empirical covariance matrix

- Given: m random variables $\mathbf{L}_{m\times 1} = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_m \end{bmatrix}$
- $m \times n$-dimensional observation matrix

$$\mathbf{l}_{m\times n} = \begin{bmatrix} l_{11} & l_{21} & \cdots & l_{m1} \\ l_{12} & l_{22} & \cdots & l_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ l_{1n} & l_{2n} & \cdots & l_{mn} \end{bmatrix}$$

m different random variables were measured n times

- m-dimensional vector of mean values respective expectations

$$\bar{\mathbf{l}} = \begin{bmatrix} \bar{l}_1 \\ \bar{l}_2 \\ \vdots \\ \bar{l}_m \end{bmatrix} \qquad \text{resp.} \qquad \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix}$$

3.2.2 Empirical expectation and empirical covariance matrix

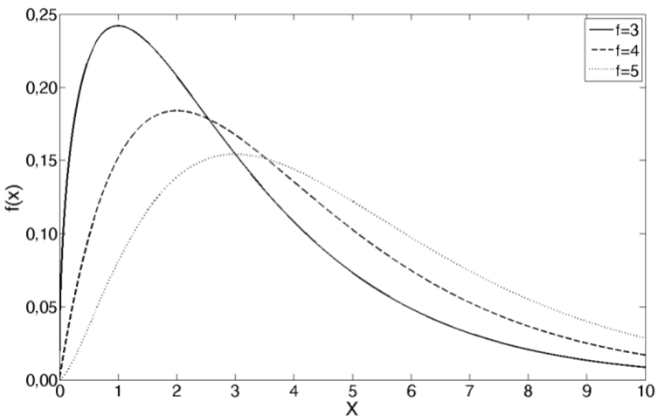
▶ m-dimensional VCM

$$\mathbf{S}_{ll_{m\times m}} = \begin{bmatrix} s_1^2 & s_{12} & \cdots & s_{1m} \\ s_{21} & s_2^2 & \cdots & s_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_m^2 \end{bmatrix}$$

▶ Correlation coefficient for the m-dimensional case

$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \cdot \sigma_j}$ respective $r_{ij} = \frac{s_{ij}}{s_i \cdot s_j}$ for $i, j = 1,2, \dots, m$ and $i \neq j$

→ All computations (e.g. residuals, empirical variances and covariances and correlation coefficients as in 2-dimensional case)



Adjustment Theory I

Chapter 3 - The Random Vector

Prof. Dr.-Ing Frank Neitzel | Institute of Geodesy and Geoinformation Science