The Nature of Measurement: Part 1, The Inexactness of Measurement - Counting vs. Measuring

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Surveyors and mappers measure and portray distances, angles, directions, areas, heights, slopes, volumes and coordinates and various features on or near the surface of the earth. The surveyor's role in society is to execute earth-related measurements and interpret the data collected. The role of the mapper is to plot, portray, display, or store the data in digital format. Thus, "surveying and mapping" encompasses all of the technology and processes, as well as the decisions and analyses associated with earth-related measurements and their portrayal.

Measurement is not an exclusive function of surveyors and mappers. Everyone from physical and behavioral scientists to postal clerks and homemakers gather and use measured data on a daily basis. A person may simply want to know human body weight, waistline girth, air temperature, time of day, volume of recipe ingredients, or other variables that affect our comfort or daily habits and needs. Certain professions or trades may involve reading dials or scales through much of each work day.

This short introduction to measurement science is intended as a review for those already expert in the subject, as a training aid for surveying and mapping personnel who use and process measurement related data, and as a primer for others who use and interpret the data. This is the first in a series of articles, each covering a different aspect of measurement.

Measurements are Inexact

Even with the most sophisticated equipment, a measurement is only an estimate of the true size of a quantity. Exactness simply does not exist in the physical world. "Truth" is always elusive when it comes to measurement. This is because the instruments, as well as the people using them are imperfect, because the environment in which the instruments and people operate influences the process, and because the behavior of people, instruments, and the environment cannot be fully predicted.

How to approximate the degree of inexactness of a reading (or "observation") is a fundamental concern of any professional surveyor. It should also be of concern to others who use measured data, since any misunderstanding could lead a person or group of people to wrong conclusions. Examples in later installments will illustrate that the consequences could be unhealthy, costly or disastrous. If measured data is to be interpreted and used correctly, the concept of inexactness must never be denied.

Two Kinds of Numerical Information

Understanding the difference between numbers as measurements and numbers as counts may be the key to eliminating confusion over why measurements of the same quantity often (perhaps usually) disagree, and why readings should never be accepted at "face value."

Counting involves whole numbers only. There are no decimals involved in counting objects, unless the units lend themselves to divisibility by multiples of 10 (such as when dollars and cents are the counted objects). In any case, the last digit expressed in a count is an exact number, without variation. Furthermore, there can be no continuation of the number beyond the last digit expressed. It ends both in practice and in theory with that digit. For instance,

$147.92 (or 14,792 pennies) could never be 147.916 or 147.9224 or even 147.920 unless some division of the penny had been devised and counted. Accountants, bankers, roll-takers, census-takers and "bean counters" of all kinds are involved in counting objects. We have all learned to count.

When the numbers being considered represent measurements, we enter int**o** a whole new world of concepts and abstractions. Some of the concepts may be quite mysterious to many people, even to some regular users of measured data. In contrast to counts, measurements always involve numbers that are inexact. In theory, such numbers continue to infinity. A measurement expressed as 147.92 feet might be 147.92247 or 147.91832, or any other string of numbers that would round to 147.92. We do not know what lies behind that last digit expressed, and even the last digit could be off a little. There is always a mystery and uncertainty to a measured number. Since measurers can never resolve the value exactly, and the number must be expressed in finite terms, no one really knows the exact value.

I believe that most people were not taught the difference between numbers as counts and numbers as measurements. If measurement was taught properly in our schools, people might learn not to expect exactness in measured data, they would be better able to interpret such data, and perhaps it would also help many people to learn not to expect perfection in anything. The imperfect world of measurement is a mirror of the imperfect world in which we live.

Measurement Is An Abstract Science

Each of us seems to have an individual "level of abstraction" that is a "comfort zone" between that which is understood and that which is too "deep" to process or consider. Many people, even with the ability to comprehend, try to avoid abstract concepts, even at the most elementary level. This nature of measurement may be too abstract for many people. Such people are often afraid of indefinite and vague ideas or concepts. If they also were not taught the difference between measurements and counts in school, they may try to force exactness into situations. This is foolhardy. Anyone who uses measurements must accept the inexactness and work with it.

Perhaps the term "bean counters" is often used in a derogatory way because counting of any kind is a simple act. Everything is either "black or white," either "right" or "wrong." There is no judgment or estimation needed, no abstract theory, no special training or skill required. Children can do it before they start developing abstract thinking. Measurement analysis, on the other hand is abstract, requiring judgment, application of statistics and other analytical procedures, and accepting uncertainty as a reality.

The basic difference between a number representing a count and one representing a measurement is that the count *can be exact*, whereas no measurement can be exact. A count is a *discrete* number, whereas a measurement is a *continuous* number.

If a person has spent a lifetime thinking numbers were all the same, it may take a while for all of this to become clear.

Making The Distinction: Examples

In case the distinction between numbers as counts and numbers as measurements is not yet clear, perhaps some examples will help.

Guessing the number of jelly beans in a jar is a measurement, since it is an estimate based on some judgment. One person may merely guess the number by looking at the jar. Another might make a quick count or estimate of the beans occupying a cross-section of the jar and, how high the beans appear to be stacked and then compute a volume of beans. Still another person may weigh 10 beans, weigh the full jar of beans, weigh an identical empty jar, and with a little arithmetic called extrapolation, compute a number. The second person may come closer to the true number than the person making a "wild guess," and the third person may come even closer. The second and third persons have refined the method of measurement. All three are making measurements. However, when the beans are actually counted, there is no estimating.

When census takers go door-to-door, counting the people in each household, then add these numbers, they have a count of the population. However, when they take samples of populations, then apply various assumptions as to population densities in an area and arrive at a total, they are estimating. This is a measurement of the population.

In surveying, the final number used for the measured value may be comprised of both counts and estimates. The surveyor counts the number of tape lengths, for example, but estimates the final reading on the tape. The surveyor may count the repetitions of an angle using the "method of repetition." Ultimately, the number used is a measurement because of inexactness in estimating readings, aligning marks with taping pins, aligning micrometer lines, centering the theodolite over the ground point with a plumb bob or optical plummet, and several other phenomena that must be estimated.

Basic Concepts Must Be Accepted

That all measurements are inexact must be accepted before any of the theories and deeper levels of abstraction related to measurement science can be understood. Without appreciating this, a person has no hope of using measurement data properly.

After accepting and understanding the basic concept of inexactness, the next stage is to understand the difference between measurements and counts. This is perhaps a higher level of abstraction and may be more difficult for some people to integrate into their thinking.

I maintain that we have not been taught the distinction very well in our elementary schools and beyond. When something is widely misunderstood for years, it may be difficult to change the ways of thinking.

The public, apparently handicapped by their schooling or inability to deal with basic abstract concepts, usually thinks of numbers as counts. Indeed, a symptom of this is often experienced by surveyors when clients and users of our work fail to understand why two surveyors cannot agree on the measurement of the same quantity.

An attorney who attended one of my seminars came to me later and said "I think I finally understand it now—a measurement shown by a surveyor on a plat is just a professional opinion." I said, "You've got it!" You see, the value used by any particular measurer is just an opinion, an estimate, or a perception by that individual. It is affected by bias of many kinds.

This serves to introduce the subject for the next part of this series. A count cannot contain an error. Any difference between two discrete counts of the same thing is caused by a mistake. Only measurements can have errors in them. It is primarily these errors that are responsible for discrepancies between measurements of the same thing, and the root cause of the inexactness we have been discussing. Measurements are susceptible to both errors and mistakes. Counts are susceptible only to mistakes. This will all be explained further in the second part of this series. n

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The Nature of Measurement: Part II, Mistakes and Errors

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In the first part of this series, a distinction was drawn between numbers as counts versus numbers as measurements. It was explained that counts can be exact whereas measurements are always inexact. We ended with the thought that counts are susceptible to mistakes, whereas measurements are susceptible to both mistakes and errors.

As with counts and measurements, I maintain that most of us have also been conditioned to think that error and mistake are the same things. Our education, the media, literature and even dictionaries mostly use the terms as synonyms. As a result, most of the public, and perhaps even some in the sciences and engineering, may have trouble analyzing numbers which derive from measurements. The difference between a mistake and an error is important to understand if you are to analyze measured data properly. If we accept the proposition that counts are susceptible to *mistakes* only, and measurements are susceptible to both *mistakes* and *errors* (as we define the terms), analysis of numerical data must follow different procedures, depending on the nature of the data (whether count or measurement.)

Cause and Prevention of Mistakes

A mistake is a “blunder”, a “goof”, a “slip-up”. Mistakes occur in both counting and measuring, and in a lot of other things. Mistakes in measurement can be traced to carelessness, inattention, improper training, bad habits, lack of innate ability, poor judgment, adverse measuring or observing conditions, and various negative attitudes, emotions and perceptions that plague humans.

Mistakes can be caused by making a decision without sufficient information or evidence. People often try to use their intellect, experience or precedent to decide something, without having gathered relevant evidence concerning the specific matter under investigation. Land surveyors, for example, often make decisions about the location of property corners without sufficient evidence. This failure falls under the category of poor judgment or bad habits.

We make mistakes through transposing numbers, striking wrong keys on calculators, misreading scales, etc. In everyday affairs, people make mistakes in a similar manner, perhaps mis-dialing a phone number or using the wrong date on a letter.

I feel that there are few true “accidents”. Most should be classified as mistakes, and someone should take responsibility for them. The fender benders I observe on a section of curved road near where I live on frosty mornings are caused by lack of proper training, bad habits, negative emotions and attitudes such as impatience and selfishness, or just plain carelessness and lack of common sense. Human inability to confront and control individual oversights and poor judgments lurk behind most "accidents" or mistakes.

Many mistakes are easily recognized and realized. Others may have such small effects that they go unnoticed. An example in measurement would be transposing the last two digits in a large number like 1,834.65 feet. Others, based more on judgment or acting without complete information, may not have immediate, or consistent, consequences. Driving too fast for conditions may not always result in disaster, for example. Voting for the crook instead of the honest person or buying a faulty product may not have immediate consequences. The mistake is realized later, as “hindsight” reveals it.

Mistakes will never be completely eliminated from measurements, but they can be reduced in most cases by developing the measurer in a way that he or she learns to be more careful, attentive, conscientious and proud of the work being done. Through proper training and development of good work habits, development and maintenance of positive attitudes, and understanding the theory and practice involved with the variable being measured, mistakes can be controlled and practically eliminated. We will always need to confront them, however, because human imperfections make them inevitable.

Errors and Their Sources

When properly defined, “error” pertains only to measurements—that is, to estimating anything where exactness is not possible. It does not apply to counts, where exactness is possible. Errors are unavoidable even for the most thoroughly trained and motivated measurer. They occur to some extent in virtually every measurement because of imperfections of instruments and people, as well as influences of the natural environment. There are basically two types of errors in measurements: systematic and random. I will explain them later. First, let's examine the sources of errors. There are four of them:

Natural errors. Measurements are usually made in an environment that is essentially uncontrollable (outdoors). Effects on instruments and processes from such factors as temperature, atmospheric pressure, atmospheric refraction, humidity, solar and other heat, wind, gravity, and earth's curvature must be measured, and readings must be corrected for these variables if accurate results are to be expected.

Instrumental errors. All measurements employ instruments, from the simple plumb line to the most sophisticated electronic apparatus. Some error is always present in the measurements due to imperfection in manufacture, adjustment or basic characteristics of the instrument. Even when "in adjustment" there is error, since the adjusting process usually must involve human manipulation and judgment with no perfection in the procedure used.

Personal errors. Since humans are directly involved with all measurements, and since humans are imperfect, errors are inevitable in measurements. Automation and electronics have reduced personal errors in measurements, but not eliminated them. People still perform centering and alignment judgments, for example, even when readings are digital.

Calculation errors. Unless sufficient digits are recorded and carried through all computation steps, and unless conversion factors and constants contain sufficient digits, round-off errors occur. Significant figures in measurements directly affect the significant figures in computed results. Significant figures and round-off errors, and the broader subject of precision, are subjects for other parts in this series.

Systematic Errors

Systematic errors are those that generally obey or follow some mathematical or physical law. The cause of such errors can usually be traced to instrument maladjustment, lack of calibration, or the environment. If they are discovered, they can be quantified through instrument tests or calibration and through understanding the various effects of nature. Since they are discoverable, they essentially can be corrected. Systematic errors occur when the cross-hairs of a surveying instrument get out of adjustment, or when a surveyor's steel tape or a tailor's cloth tape become stretched. They occur when the tires on a vehicle are not of a diameter which would yield a true distance consistent with what the odometer reads. They occur in electronic distance measurements because the measurements are affected by changes in the temperature and atmospheric pressure. They occur because of manufacturing errors in graduations of any type of scale.

Random Errors

Random errors are unavoidable. They follow random patterns, or the laws of “chance”. They have unknown signs; thus, they are expressed as “plus or minus”. The magnitude of such an error is unknown, but it can be estimated. These errors are caused by human and instrument imperfections as well as uncertainties in determining the effects of the environment on measurements.

Personal errors are nearly all random in nature. People cannot perceive anything with exactness. In surveying, this refers to the alignment of cross-hairs on targets, centering of instruments over ground points, reading rods and scales, centering level bubbles, etc. Random errors are small misjudgments, not mistakes. They are what happens when a person tries to “do it right”, but misses the mark by a small amount due to imperfections in the system (which includes himself).

Furthermore, all instruments, even so-called automatic systems with precise digital readings, have certain imperfections in all their components, be it the optics, electronics, or mechanical features. Unpredictable changes in adjustment of instruments, or loose fitting parts can cause random errors.

There will inevitably be uncertainties in determining all variables affecting instruments that are used in a natural environment. In fact, some influences are essentially unmeasurable, such as constantly varying wind speed and changing radiant heat from the sun. Calculation errors (round-off errors) have a random effect on calculated results. In a long series of calculations, such an error “propagates” and affects both intermediate and final results. Error propagation will be discussed in another part of this series.

Understanding the nature of random errors helps to understand why systematic errors are never really fully corrected, since the observation of the physical phenomena causing the error, or the aligning and calibration of instruments in itself contains personal, random errors. Thus, measurements have “uncertainties” or random errors which remain unquantifiable. Random errors are dealt with by controlling or managing them. It is a quality control process. They cannot be corrected or eliminated, only minimized and controlled.

Mistakes vs. Random Errors

Many people, even professionals who use measurements regularly have some difficulty differentiating between mistakes and random errors when applying the concepts in practice. The difference deserves a little more explanation here, since the misconceptions are so widespread.

Mistakes occur because of negligence, while random errors occur due to imperfection. Here we will define negligence as either deliberate or wilful deviation from accepted practices or adopted standards (whether what is accepted or adopted is known to the individual or not), or an occurrence caused by carelessness, acting with insufficient or faulty information, etc.

Mistakes are either deliberate, as in fraud or tampering with data, or they occur because someone is unwilling to study, learn and employ correct procedures, to control emotions, to keep in practice, to focus on the task at hand, or to think. By contrast, random errors occur naturally, even when the individual is attempting to perform the procedure correctly.

It is true that there is a "gray area" between random errors and mistakes. If a person is too hasty in some mechanical measuring procedure, for example, the large random errors that occur start to look a lot like mistakes. Since it is poor judgment to hurry in such an instance, the results have a lot of scatter, and some might say that they contain small mistakes. Whether they are small mistakes or large random errors does not matter as much as knowing how to deal with the data.

Mostly, however, there is a clear distinction between a mistake and a random error. If a person observed 1,874.56 feet from a scale and recorded it as 1,874.65 feet, that is a mistake. Let us say, however, that one person observed the reading as 1,874.56 feet and another person observed it as 1,874.65 feet, and neither person made a mistake in either observing or recording the numbers as they appeared to that person. If the range of 0.09 feet is acceptable according to standards and specifications, we can say that the difference between the readings was caused by random errors.

We have all learned that a miscount or any mistake is generally frowned upon, and reflects upon us in a negative way. People sometimes get punished for mistakes, and rewarded for consistently avoiding them. This is as it should be. However, it has also seemingly taught us to think of everything as either "right" or "wrong." We have grown to feel that someone must be blamed for the "wrongs." We cannot apply this process to inexact, perceived things. Two people can conscientiously measure the same quantity and get different values, even after applying careful measurement analysis and correcting for systematic errors. What is wonderful about the science of measurement is that both these people can be equally "right." Isn't that a mirror of life itself? Two people can perceive a situation differently and neither is considered "wrong" (unless, of course, one of them has not corrected initial perceptions for bias or prejudice.) The key to "getting it right," in measurement or anything else, is to recognize, then either avoid or remove, both mistakes and errors.

A point we will develop further in this series is that, truly, "to err is human." But that does not apply in the same sense to mistakes. Although it is "human" to do both, a person should never rationalize mistakes away by citing this old quotation. To do so is a "cop-out" if it applies to mistakes. It avoids taking responsibility and indicates an unwillingness to be held accountable. However, there is no need for condemnation, criticism, blame or apology where random errors are concerned (assuming they are controlled and managed).

To cope with numerical data of any kind, it is extremely important to learn the difference between a measurement and a count, between a mistake and an error, and between a systematic and a random error.

The Nature of Measurement: Part III: Dealing With Errors

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The last part in this series defined "error" and made a differentiation between mistake and error, then separated errors between "systematic" and "random." This part of the series further explains errors and how to deal with them.

Dealing With Systematic Errors

Once discovered and quantified, systematic errors can be essentially compensated or corrected by either observing (measuring) in such a way that the error is canceled in the mechanical process of measuring or by calculating a correction and adding this to the observation (reading).

The first requirement is to recognize and accept the possible existence of errors. Next, identify the various sources that might be affecting a reading systematically. Then, the analyst must determine what the "system" is. That is, what is the relationship between the changes in readings and the size of the quantity, as affected by a particular systematic error? Is it a constant, unchanging with the size of the quantity? Is it linear, changing in direct proportion to the change in the variable causing the error, and/or in proportion to the size of the quantity being measured? Or, does it follow some other mathematical relationship? Is there some physics involved?

As an example, we discovered many years ago that a material will stretch when tension is applied to it. The thicker the material, the less it will stretch, per unit of length, given any particular tension. We have tested the surveyor's steel tape and determined that the expansion caused by tension is fairly linear for this ductile material. Thus, we can use a formula for the systematic error that considers the tension applied, the length over which it is applied, the cross-sectional area of the tape, and the elasticity of steel. The stretch obeys known mathematical and physical laws.

All instruments must be calibrated; that is, compared with a standard, under known (measurable) conditions. For example, we calibrate a tape by comparing its length with a standard known to be "accurate" (another subject in this series), under known tension, temperature (since the tapes expand and contract with changes in temperature), and "support condition" (whether laying fully supported or supported only between the points of reading).

The general formula for systematic errors is $T = R + C$, where T = the "true" value, R = the reading, and C = the total of all observable corrections. On the cover of my book, entitled *Surveying Measurements and their Analysis*, and on my seminar brochures for Surveyor's Educational Seminars, is the caption, "The truth is equal to one's initial observations plus the corrections discovered through added experience or knowledge." You see, error theory can be applied to affairs outside the world of measurement. A person often is far from possessing the truth of a perceived matter until that person is willing to get rid of biases or prejudices affecting such observation. As has been said, measurement theory is somewhat of a mirror of many things in life. Understanding these concepts helps a person to deal with life and its pitfalls.

Examples of Systematic Errors

To illustrate systematic errors, I'll use my digital, battery-operated bathroom scale as an example. One might think it is exact, since it is electronic (a common fallacy, even among some surveyors). To check the scale, I placed 25-pound barbell plates on it, one at a time. Before I did this, I noted that it read -1.1 pounds (actually 998.9) with nothing on it. Then, when adding the barbell plates one at a time, I observed readings of 24.2, 49.5, 74.9, and 100.1 lbs.

The -1.1 pound indexing error represents what is called a constant error. I must add 1.1 pounds to anything I weigh, regardless of its mass. I can use it to correct the readings for the barbell plates. The readings, corrected for this constant error are 25.3, 50.6, 76.0 and 101.2.

At this point the observer must begin to use what we call "measurement analysis" in surveying. The data must be interpreted so that it can be applied to our benefit. If I divide 50.6 by two, I get 25.3, the same number I observed for the one plate. If I divide 76.0 by three, I get 25.33, almost exactly the 25.3 for one or two plates. And, 101.2 divided by four is also 25.3. If the plates were manufactured accurately at 25.0 pounds each (a faith assumption I chose to make here), then I have a "systematic error" of +0.3 pounds per each 25 pounds, or +1.2 for each 100 lbs. It is "systematic" because it follows a "system." It is predictable. It obeys mathematical laws. There is a linear relationship between the weight and the error.

To apply this new knowledge to my body weight, I would immediately add 1.1 pounds to what I observe, then subtract 0.012 pounds times the reading. An algebraic equation is True Weight = Observed Weight + Constant Correction + Variable Correction. In my case for one weighing, TW = 144.5 +1.1 -0.012(144.5) = 143.9 lbs. (to the nearest tenth).

It is observed that, to apply a correction, I simply change the sign of the error. That is, since the constant error was negative, the correction for it is positive. Likewise, since the variable error was positive (the reading was high for a "known" weight), the correction to compensate for it is negative in sign.

Example One

As an illustration of the mistakes that can be made in neglecting systematic errors such as these, I will recall an experience in accompanying my wife to the doctor a few times several years ago. They weighed her each time on the same scale. The nurse recorded the weight for the doctor to use in his professional decisions. I distinctly remember that my wife was wearing light clothing one warm day and heavier clothes on a cooler day. I knew that both weights were inaccurate, as they did not account for the clothing at all, and that the discrepancy between the two weight readings was also inaccurate because the difference in bulk of the clothing was ignored. Yet, this $60-per-hour physician was making judgments about my wife's health and progress, partly based on this data! He was making comments and recommendations affected by just a couple of pounds, and the systematic errors exceeded this amount. By the way, my wife needed to gain a little, not lose weight. She'd want me to say that.

Example Two

I used to do a lot of measuring of road race courses, when I was running marathons and other distance races. To check the accuracy of a course, I would calibrate my automobile odometer, using the Interstate Highway mile markers. On a long trip, I would glance at the odometer the instant I crossed one of those little green mile posts and estimate the odometer reading to hundredths of a mile. After 10 or 20 miles, a pattern would become apparent, much like what happened with the barbell plate weighing. One such odometer calibration yielded readings of 1.02, 2.02, 4.06, 5.08, 7.11, 10.16, 12.20, 15.25, 18.29, and 20.32. A little bit of analysis here will reveal that every time I had an accurate mile (applying faith again to the accuracy of the mile posts and intervals between), the reading averaged 1.016 miles. Thus, in the future, in checking a 10- mile race course, marked by the race director, I would "lay out" readings of 1.02, 2.03, 3.05, 4.06, 5.08,….., 10.16 miles. In 1973, I found that the Athens Marathon (that's in Ohio, not Greece) was about 0.75 miles short of the regulation distance of 26.21875 miles, and in 1974, it was approximately 1.5 miles long! I used to be unpopular with both race directors and the runners. Those runners in 1973 thought they had all set "PRs" (personal records), since the course was so short. They wanted to deny my findings. They were on a "high" from their apparent success, and the race director wanted me to shut up and "just come and run" like everybody else (his exact words, in fact). But, in 1974, some of the runners were willing to listen to me. The race director really hated me after what I publicized about the long course.

Several years later, a professional runner was denied a $50,000 bonus when he missed breaking the world- best time for a marathon by less than five seconds. I am convinced to this day that the course was long, considering the methods used at the time for the measuring and subsequent calibration/certification of the course. The man was probably cheated out of a bonus because of misunderstandings about measurement science.

Consequences of Systematic Errors

The example about my wife's weight when visiting the doctor, the checking and calibration of road race courses and other such examples help to illustrate how misunderstandings about measurement can hurt somebody. The results can cause undeserved celebrations, undeserved disappointments, unnecessary cost, lost rewards, lost time and inconvenience.

In surveying, when systematic errors occur and are not corrected or compensated, surveyors often have unnecessarily large discrepancies between their measurements and those of others, and even between duplications of their own measurements. This causes embarrassment for the profession and confusion for the users of the data. Systematic errors that go uncorrected make us look silly and incompetent.

Analysis of Random Errors

Random errors follow statistical behavioral laws, such as probability and compensation, which dictate a tendency of plus and minus errors to cancel, predict that large errors occur less frequently than small ones, and say that an error of any particular size is as likely to be positive as negative. A characteristic theoretical pattern of error distribution occurs upon analysis of a large number of repeated measurements of a quantity. Thus, the mathematics disciplines associated with random errors are statistics and probability. It is not geometry, although geometry may affect the results. For example, I have determined, through statistical analysis, that the standard deviation in reading a one-second optical theodolite is consistently between 0.6 and 1.0 seconds. The value varies with skill of the observer, quality and cleanliness of the optics of the instrument, lighting and probably other factors. Statistical analysis of random errors is the subject of another in this series on measurement.

Two Types of Measurements

Measurements can be direct or indirect. A direct measurement is one where the reading observed represents the quantity measured, without a need to add, take averages or use geometric formulas to compute the value desired. An indirect measurement requires calculation. After a little experience, surveyors begin to realize that few, if any, measurements are truly direct. For one reason, averages are usually taken to arrive at a final reading, or readings are often combined in some other way, such as when tape lengths are added together. Also, many values are indirectly determined from other measured values, such as areas or unknown sides of a triangle. But, the most important reason why measurements are generally indirect is that calculation must be performed to correct for systematic errors. "What you see" is not what you have. It is the indirect nature of measurements that forces the need to often apply some rather sophisticated mathematical procedures to analysis of errors and thus determine a "best value" to represent the size of the quantity. Some of these refinements will be discussed later in this series.

Learning the Difference Is Important

Learning the difference between a measurement and a count, between a mistake and an error, and between a systematic and a random error is extremely important in coping with numerical data of any kind. Similarly, the difference between precision and accuracy must be understood. These terms and their meanings are the subject of the next article. Gradually, the full meaning of this very enlightening science will be revealed.

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The Nature of Measurement: Part IV-Precision and Accuracy

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In this series we have discussed the difference between numbers as measurements and numbers as counts, the difference between errors and mistakes, and the various types and categories of measurement errors. Counts, we learned, can be exact whereas measurements cannot. Mistakes and errors are also different concepts, and occur for fundamentally different reasons, despite the fact that we have been conditioned to think of them as synonymous. Mistakes should be seen as misjudgments caused by carelessness and similar causes. Errors, on the other hand, arise from imperfections in instruments and the perceptions of people estimating readings and other variables, as well as fluctuations in the environmental conditions surrounding the measuring operations. Using the concepts as defined in measurement science, we found that errors cannot occur in counts. Errors can occur only in measurements. Mistakes can occur in both counts and measurements. We also defined two types of errors: systematic and random. Systematic errors obey mathematical and/or physical laws, and thus are predictable, correctable or avoidable. Random errors, by contrast, were explained as being unavoidable, because of imperfections in measurement systems (people, instruments, and nature). Such errors can be controlled, minimized, investigated and estimated, but never eliminated. They follow statistical laws of probability, and in this sense do have some predictability when studies are made to evaluate them.

With this as background, we can now look at two other very important concepts in measurement: precision and accuracy. Confusing these two concepts can lead to incorrect analysis of both measurement data and computed results, and either incorrectly written standards or mis-application of theoretically correct standards. Precision and accuracy relate directly to random and systematic errors, respectively.

Definitions of Terms

As with mistakes and errors, precision and accuracy have unfortunately been treated as synonyms among the general populace. However, in the world of measurement (and, in reality, everywhere else, whether it is recognized or not) there is an important difference between these concepts, regardless of words used to describe them.

Precision is in one sense related to the care and refinement of the measuring process, including instrument quality, attentiveness of the observer, stability of the environment in which the measurements are executed and overall design of the survey. In another sense, it is the degree of numerical agreement among measurements of the same quantity, or the repeatability of the readings. In still a third sense, it is simply the number of decimal places expressed in a measured quantity. All three meanings are consistent, since the care and refinement of the method directly affects the agreement among repeated measurements. This agreement reveals the decimal place, which has meaning. Precision relates to the method of measurement and the expression of the measured value.

Accuracy has only one meaning: conformity with the truth. The "truth" in measurement science is defined by one or more of the following:

• An adopted physical or other standard (on distance, weight, volume, time, etc.)

• Geometric laws

• A system decided as correct by some recognized authority

Conformity with the truth is decided by the stability and precision in defining these standards, laws and systems, by calibration of instruments, avoiding or removing mistakes, and by detecting and removing systematic errors caused by the environment or instrument adjustments.

It can be realized that conformity with a standard, and thus accuracy, can change as the definition of the standard changes. The rod, for example, certainly varied when it was defined by the toe-to-heel length of the first 16 men coming out of church. As modern definitions of our official unit (the meter) become more refined, the standard, and thus accuracy as related to the standard, becomes more stable. However, there will be variations caused by historical differences in definitions.

Geometric laws are the exception as to the unknown nature of the "true value." For example, the sum of the angles of a closed figure is (n-2)180º. Accuracy of the sum of the angles can thus be checked. But, the accuracy of any particular angle cannot.

Besides adopted legal standards and geometric truth, surveyors or government authorities sometimes also choose to select something as "true," such as a system (datum) of monuments of high "order of accuracy," and make subsequent measurements fit such "control." But, as with "situation ethics" in the real world, or other arbitrary philosophies purportedly embodying truth, using such systems to decide accuracy works only until a "better" standard comes along. Rejection of the NAD 27 datum in favor of the NAD 83 datum, then almost immediately rejecting this in favor of the High Accuracy Reference Network (HARN) adjusted control are examples of what happens when standards are set that have inherent errors in them.

Theoretically the true value exists, but it is elusive because of errors and variations in the standards and systems used to determine "truth." If stable standards and systems of control are more accurate than the ability of the surveyor to measure, accuracy reduces to the control of errors and mistakes in individual measurements.

Precision Further Explained

Precision is usually viewed in a relative or comparative sense. Precision and random errors are directly related. Precision increases when random errors decrease, and the reverse is also true. Statistical measures such as scatter, range and standard deviation are smaller when precision is higher because of the better quality control of random errors.

Precision can be observed in a limited sense simply by duplicating a measurement. One surveyor might observe 1,248.54 feet on one occasion and 1,248.59 feet another time. The 0.05-foot discrepancy is an indication of the precision. If another surveyor measures the distance as 1,249.32 feet and repeats it as 1,249.34 feet, the second surveyor has better precision than the first surveyor because the latter's two readings agree better. Of course, to better study precision of one measuring procedure, or to compare two different procedures, more than one repetition of each procedure is desirable. When the number of repetitions, using a specific method are sufficiently high, a statistical measure of precision (called standard deviation) can be computed. How to do this, and the value of this precision index in determining and comparing precision, will be the subject of a future article.

Precision can be determined, also in a limited way, by observing the number of digits expressed in a measurement. A level instrument operator may be able to observe a reading on a conventional level rod to the thousandths of a foot at a sighting distance of 20 or 30 feet, to the hundredths at 50-to-150 feet, and to the tenths for distances of 300-to-400 feet. Rod readings of 3.423 feet, 3.42 feet, and 3.4 feet are different in precision. The limitation of using this means to evaluate precision is that observers and computational personnel do not always express numbers to the correct precision. For example a reading using a method yielding hundredths might be recorded as 23.4 when it actually should be 23.40, and the opposite also occurs. Sometimes computational personnel simply record whatever comes out of the calculator, according to the arbitrary setting of the "FIX" on the calculator, without evaluating the precision of the numbers used in the computational steps. All of this relates to significant figures, which will be discussed in a later article.

Another way to evaluate precision, again in a limited sense, is to simply consider the instrumentation. A surveyor using a 1" theodolite is using a more precise instrument than one using a 1' vernier transit. However, the total methodology, including the geometric aspects of the survey, must always be considered when evaluating precision, not just the "least count" of the instrument. Suppose, for example, the transit surveyor used geodetic targets on tripods and optical plummets for centering over the ground stations, and had very long lines in the traverse. And, suppose the surveyor with the precise theodolite employed range poles for sightings and plumb bobs for centering the instrument, and had short sight distances. The overall method of the transit surveyor might be considered more precise, despite the fact that the reading circle is less precise than that of the theodolite.

There are many variables affecting precision in the field: geometric aspects of the survey; distances; repetitions made; slope angles; several variables related to the refinement of the instrument; skill of the observer; general care taken in centering and aligning; and general quality control over other aspects of the observations. In fact, everything affecting random errors affects precision, since reducing random errors improves it. The concept of precision and what affects it is more complex than what many realize. Complete evaluation of it requires a thorough understanding of the errors and, preferably, conducting controlled statistical tests of the measurement method.

Accuracy Further Explained

Suppose two surveyors measured a distance. One determined it to be 1,248.725 feet and the other surveyor reports it as 1,248.852 feet. Which is right? This question is answered when we identify which surveyor paid more attention to detection and removal of systematic errors and mistakes. Suppose both surveyors used electronic distance instruments. The first surveyor calibrated the instrument using a National Geodetic Survey calibration baseline and found the constant and PPM corrections; performed tests to determine accurate values for the reflector constants; checked and adjusted optical plummets of the instruments; and carefully observed the temperature and atmospheric pressure, keying these values into the instrument so as to compensate for these natural errors. The second surveyor did none of these calibration and systematic error tests or compensation procedures, and had left last seasons's temperature and pressure values in the instrument. Several applicable systematic errors would occur in the second surveyor's measurements from the sources indicated here, and possibly others, that, when combined, could easily account for the approximate 0.13-foot discrepancy.

Remember, the true value is always equal to the reading plus the sum of the applicable corrections. There are no exceptions to this. No surveyor is exempt from the need to correct for systematic errors in the quest for accuracy. Just as a person cannot find truth in any situation until all biases and prejudices have been compensated or removed from his or her thinking, all of the facts are known, and actions are taken consistent with the facts, accuracy in measured data is impossible without detecting and removing systematic errors, and knowing the error sources in the first place.

Comparing and Contrasting Precision and Accuracy

A measurement can be precise but inaccurate, as well as accurate but imprecise. For example, if a measurement was made with much care using a highly refined instrument, repeated readings of the same quantity would agree closely and thus precision would exist. But if the instrument contained one or more undetected, uncorrected systematic errors, the results would be inaccurate. In contrast, it is possible that the mean of several repeated measurements of this same quantity, using a less refined (but calibrated) method, could be closer to the true value and thus this procedure would yield more accurate results even though there was less agreement among the readings.

An example that explains the difference between precision and accuracy better than any other in surveying has to do with error of closure in traversing. Many surveyors seem to think that error of closure checks the accuracy of the work. Wrong! Error of closure primarily checks the precision, not the accuracy. It checks accuracy only in that it can find blunders. But, since it cannot detect systematic errors in the distances, it cannot fully check accuracy. I have always been dismayed (nothing new for me) that we still have so many written and adopted surveying measurement standards that dictate using error of closure as a means to check accuracy. When surveyors speak of a "1 part in 5,000 survey," I have found that many are only talking about the actual error of closure, not predicted relative errors, which consider the design of the procedures and control of systematic errors. You will get the same error of closure in the field whether systematic errors in distances were corrected or not!

Another surveying example relates to leveling. Let us say that an enterprising leveling crew discovers they can make better time by using a very long level rod, lengthening back-sight distances, as they level up a slope. They set a bench mark on top of a hill, after dozens of turns in the level line. Then they level back down the hill and "close" on the initial bench mark, and observe a nearly perfect closure. So, they conclude that their work was "good" and they go have a beer to celebrate their "results." Job well done? Look again. Since they had longer backsight distances going up the hill (made possible by use of the long rod), they had cumulative systematic errors from at least four sources:

- Any line-of-sight (collimation) error in the instrument, however small

- Earth's curvature and atmospheric re- fraction

- Any systematic error in the graduations (length) of the level rod

- Rod being slightly out of plumb each time

But, you say "aren't these errors compensated?" Yes, when the leveling crew went back down the hill the accumulated errors going up the hill were taken out of the circuit, resulting in apparently good results. But the error is still in the bench mark on top of the hill.

We must realize that precision is only a measure of control of random errors, nothing more. You cannot achieve accuracy in measurements merely by controlling random errors, either in the field or in the computational process. The most sophisticated instruments or least squares adjustment software, complete with error ellipses and pretty graphics, will do relatively little to achieve accuracy in your final values.

It is also possible to have accuracy without precision. For example, my automobile odometer is less precise than a surveying steel tape because the reading to tenths (with estimates to hundredths) of a mile is not as precise as the taped readings, even when the various random errors in taping are considered. However, if I calibrate the odometer and apply correction factors to establish a long distance, I am probably more accurate than the surveyor who applied no correction or calibration factors to a taped measurement. This statement becomes more valid as the distance becomes longer, because eventually the inaccuracy that comes from lack of correcting for systematic errors in the taping will exceed the imprecision in estimating the odometer reading to only hundredths of a mile at the two end points.

Closing Remarks

The caption and logo in my textbook, *Surveying Measurements and their Analysis* reads, "The truth is equal to one's initial observations plus the corrections discovered through added experience or knowledge." It is intended to apply to measurements, and to anything else where truth is being sought through observations. It is universal in its applications. Let us be reminded that truth is never found until biases are discovered and appropriate corrections applied. Measurements or opinions are but perceptions, as they stand alone. They approach truth only when compared with an acceptable standard.

The basic problem with precision, whether it pertains to measurement or opinion, is that it does not check anything outside of its own closed system. Isn't it rather foolish to allow repeatability to provide confidence regarding observations, or to check anything only against itself or something or someone with similar biases? Learning to make comparisons and weigh things against standards is probably one of the hardest things for anybody to learn and to do consistently. Perhaps this is why precision is so often confused with accuracy when it comes to evaluating measurement quality in surveying.

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The Nature of Measurement: Part V-On Property Corners and Measurement Science

Dr. Ben Buckner, LS, PE

In the first four parts of the Nature of Measurement series, we explained the basic science of measurement. With this as background, it is appropriate to postpone further discussion of measurement theory, and place some of the basic concepts into the context of land boundary surveying.

Most specialties within the broad profession of surveying and mapping, such as geodetic, photogrammetric, topographic, engineering and construction surveying, as well as cartography and land information systems, are concerned primarily with measurements and portrayal of three-dimensional, earth-related data. Thus, the science of measurement as it affects specifications and standards on the accuracy of data sources, data gathering and data portrayal, is the primary body of science employed in these specialties. In land boundary surveying, however, we have an additional concern. This is accuracy in position of property corners, which depends on something besides measurement accuracy.

Accuracy of a Property Corner

The "something" is evidence of where the corner was located by the original surveyor. Starting as far back as the Land Act of 1800 regarding public land surveys, and much earlier in boundary cases for other land systems, accuracy of a corner position relates only to how closely it agrees with the position where the original surveyor placed it. This means, for example, the location where an iron pipe monument came to rest after the last whack of the original surveyor's sledge hammer, or where a stone rested on the day of the original survey. The accurate position is not where that surveyor mathematically or otherwise intended it to be, the precise coordinate position resulting from a weighted least-squares adjustment of the original (or subsequent) data, the position of the monument if disturbed, or the position of a new monument set by a later surveyor who may have ignored some original evidence. Where it existed in the field is controlling over dimensions and other citations in the description. "Truth" is only found in where the corner was monumented originally. Measurements, even monuments and the original record, are just evidence. None of these, by themselves, is final truth or proof.

The original surveyor has no problem determining truth. He or she is the "alpha and the omega" on this. That surveyor has a responsibility to obey the intent of the conveyance, and to preserve the position of the corner monuments by several means (including accurate measurements). But, as time passes and property owners acquiesce in the set lines, that surveyor is forgiven for mistakes, errors, ambiguities in interpreting intent, making measurements, setting monuments and preparing descriptions. The retracement surveyor inherits the problem of determining what the original surveyor left as evidence for the corner being investigated. Retracement surveyors have a simple role in life. The task may not be simple, but the role is.

Corner vs. Monument

In order to better understand the points being made here, it is important to differentiate between "corner" and "monument." A corner is a point where a property line changes direction, or the point of intersection of two or more boundary lines. A monument is a physical marker, marking or natural feature identifying the location of a property corner, found in place, set or otherwise marked to preserve and perpetuate the position of the described and surveyed corner. A monument marks, or identifies, and helps perpetuate the location of a corner. A monument is vulnerable to disturbance and is never permanent. The corner is permanent and unalterable, unless altered by deed or other legal means. A monument, or evidence of one, may or may not exist at a corner. One may never have been set. Or, all evidence of the original may have been destroyed. Yet the corner, being described in a legal description and made part of a deed, does exist. Records, verbal and other physical evidence are supposed to be sought and used.

At the risk of confusing the issues, it needs to be said that a boundary retracement surveyor is primarily looking for corners, not monuments. Monuments are just part of the evidence. The final conclusion is measurements relate to monuments; analysis of evidence relates to corners.

Two Bodies of Knowledge

From all this, we must conclude that some body of knowledge other than measurement science, must apply to locating property corners. Indeed, there is! This second body of knowledge embraces law, primarily case law on boundaries and judicial rules of evidence, as well as history and other disciplines. The practice of this body of knowledge is detective work. The person practicing it is properly delving into forensic science more than measurement science. Let no one deny it—this other body of knowledge is as complex and interesting as that related to measurement error theory.

The "art" of surveying is not just a free-wheeling "I think I'll put it here today because I am licensed" act. It is making judgments. Property surveyors need complete knowledge of both the measurement science and the law on boundaries in order to begin to make anything called judgments. I believe it is not just two bodies of knowledge, but two distinct ways of thinking.

There are two different analytical processes, each probably using a different area of the brain. This dual way of thinking can become confusing, even for seasoned surveyors. Most people seem to be able to think one way or the other. Crossing over into another way is difficult. In my opinion, truly professional surveyors know how to think both ways simultaneously, and can apply both bodies of knowledge to analyses.

We have two components of "truth" applicable to locating land boundary corners. One is measurement "truth"—accuracy of dimensions between physical features described; the other is evidentiary "truth"—accuracy of conclusions about the corner position, based on the evidence.

The Mulford Effect

Possibly the most quoted book on land surveying this century is A. C. Mulford's *Boundaries and Landmarks*. And, probably the most famous quote is from his preface, where he declares:

It is far more important to have faulty measurements on the place where the line exists, than an accurate measurement where the line does not exist at all.

I don't think Mulford intended for surveyors to disregard measurement accuracy. Yet, I have heard many surveyors quote this, get "puffed up" about their ability to put the corner back where the original surveyor placed it, and scoff at the idea of correcting for systematic errors or doing any kind of measurement analysis other than proportionate measurement. My own perspective on the subject, and what I would like to think Mulford would say now if he knew how many surveyors have misused his earlier statement, is:

It is important to first locate the corner from analysis of all relevant evidence bearing on its original position, applying common law rules and principles and, after the corner is thus located and monumented, to perform accurate measurements between the monuments, to analyze the measurement uncertainty, and to make appropriate and theoretically correct statements about this uncertainty.

In this statement, use of measurements in the first phase of restoring the corner is implicit, even though not expressly stated. If measurements are cited in a description or on a plat, they are part of the evidence. Where monuments are "called for," the case law dictates that measurements are secondary or informative, but they must be considered nevertheless. Therefore, analysis of their precision and accuracy becomes involved in the process of analyzing the evidence. Furthermore, when all other evidence of the corner is lost, measurements rise to the status of "controlling." Thus, the importance of accuracy and error control, both in the original measurements and in retraced measurements, cannot be denied.

Professional surveyors cannot ignore measurement accuracy and analysis of measurement uncertainty for three reasons. The first is explained in the previous paragraph. From a practical and legal standpoint, measurements are part of the evidence. The second is a little more philosophical. Measurements embody the very meaning of surveying. Ignoring measurement accuracy and analysis is tantamount to a doctor ignoring medicine or a lawyer ignoring rules of evidence. Third, accuracy in measurement helps preserve the evidence for future generations. This may be the most important reason, since it affects both the public and the profession. It leaves the survey in better shape than before, to everybody's benefit. It is simply the professional and the "right" thing to do.

Measurements and Monuments

Distances and directions must be made with respect to something. Throughout land boundary surveying history, dimensions have referred to physical monuments cited in descriptions or on plats. Physical monuments are a convenience for surveyors and land owners, to enable them to visually perceive the property boundaries.

Coordinates represent measurements with respect to an origin that is often outside the local survey area. Coordinates, whether geodetic, geocentric, local geodetic horizon or state plane, represent mathematical reference ties of property monuments to the mathematical ellipsoid.

But, none of the monuments, even those presumably set to represent the corners, are the corners themselves, nor are the coordinate reference ties. As a collection of physical objects and measured data, they are but evidence of the corners.

A Word on Positional Tolerance

The controversy and sometimes emotional reactions to ideas such as "positional tolerance" are grounded in misunderstandings of the need to consider both the science of measurements and the art of evaluation of evidence. The controversy is also caused by some basic misunderstanding about the difference between a corner and a monument, and on the historical and cultural importance placed on physical monuments. Some of it is just fear of having to deal with anything different.

If all we've said thus far has been digested, positional tolerance can be put in its place. Positional tolerance is not intended to relate to certainty of corner positions in regard to proper evaluation of the evidence. It should be viewed as referring only to the accuracy and level of certainty of measurements as related to the positions of the monuments purportedly marking the corners relative to each other, to a specified control monument, or to a geodetic or other datum. Positional tolerance is just another way of expressing uncertainties in measurement, for the sake of preserving evidence of the relative positions of interdependent corners in a land survey, and for expressing the measurement integrity of the survey.

Any surveyors who would certify to a positional tolerance of a corner are being foolish, unless they have somehow learned to quantify judgment. "As per evaluation of the evidence" cannot be placed into the context of error ellipses or statistical level of certainty. Personally, I think we need to keep the two certifications or declarations separate, and also that there are probably ways other than positional tolerance to describe measurement quality (the subject of another discussion, perhaps).

Toward True Professional Surveying

Besides the cases of complete incompetence, where some surveyors fail to use either adequate evidence or proper measurement analysis, I think there are three common failings being made in retracements. In each, the surveyor is getting it partly right, but neglects something. The first is the practice of using measurement and calculations almost exclusively to determine or establish "correct" positions. This is the failing that Chief Justice Thomas M. Cooley addressed in his famous statements over 100 years ago. It is the same failing that many "total station/COCO jockeys" still have today. The second failing is to disregard measurements as an aid to help gain a preponderance of evidence or a "best fit" solution in difficult situations. The third is to do a fair job on the evidence, perhaps even integrating measurement analysis to help gain a preponderance, but to do shoddy work in correcting for systematic errors and controlling random errors in the new measurements, and/or even just leaving the old, inaccurate measurements in the deed or on the plat.

We must learn to combine measurement science with rules of evidence and forensic science, in order to both find original corner positions and preserve them for future generations. Frankly, I think this combination of knowledge and analytical skills is what makes this branch of surveying so interesting, unique and special. When a land surveyor develops both areas of expertise, and has the ability to switch automatically from one to the other, that surveyor is a true professional. Such a surveyor towers above land surveyors who neglect some of the analysis and evidence, is more well rounded than geodetic surveyors who are prone to look only at the positional accuracy, and is certainly broader and more analytical than most lawyers who haven't gotten past the difference between a measurement and a count. Land surveying—what a wonderfully complex and interesting profession!

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The Nature of Measurement: Part 7—Significant Figures in Measurements

Dr. Ben Buckner, LS, PE

How to express a measurement is a complex matter. The number of digits observed and recorded is only the initial perception and expression of the quantity. What happens after that, as the data are corrected, adjusted and subjected to geometric manipulations and unit conversions, affects the precision of the measurement and, therefore, the expression of that measurement.

The number of digits recorded or reported should be related only to the precision of the observation and the geometric strength of the survey. Perceived significance, or that resulting from arbitrariness, may be quite different from actual significance. A displayed or recorded measurement viewed as 456.874 feet may be significant to other than the six digits displayed. A precision index, such as standard deviation, 95% error or a good estimate can override the convention that the number is precise to plus or minus half the last decimal place shown. If an analysis of the procedures showed an uncertainty of plus or minus 0.058 feet, the value has four significant figures and would be correctly shown as 456.9 feet. However, to avoid round-off errors in the future, it should be shown as 456.87 feet (Rule 4 below).

If the decimal places shown in a measurement are a result of proper analysis of the precision of the observation and subsequent attention to computational rules on significant figures and round-off errors, then the number has been correctly represented. If it is merely what the scales of the instruments read, what the computer or calculator screen displayed or based on wild estimates, it has probably been incorrectly represented.

The Meaning of Significant Figures

The number of digits expressed in a measurement is a statement of someone's perception of the significant figures of that quantity. Significant figures have often been defined in simple terms such as "the number of digits that have meaning." Some authors define them as "those that are read directly, plus the first doubtful one." However, such definitions are too simple to explain the full meaning of this abstract and sometimes confusing concept. For this reason, I define the term as follows:

The number of significant figures in a measurement is the number of figures that have meaning as estimated from the precision with which the quantity was observed and the data was reduced, said meaning being limited by the judgment of the person evaluating the precision of the observation and the computations.

The method and the analysis of its precision are what gives "meaning" or "significance" to the digits. When a user accepts this, he or she realizes that one cannot decide significant figures just by examining the digits or decimal places shown. Figures do not attain significance merely because they have been displayed. The above definition also says that different observers can conceivably have different opinions as to the "meaning," as perceived from an analysis of the precision. Thus, significance is at the mercy of the varying judgment of different observers and is not absolute. This definition also goes beyond a consideration of just the "raw readings" because the precision of a measured quantity is affected by many variables, most of them usually having greater effects on precision than the "least count" or refinement of instrument scale displays. For this reason, everyone working with the data must employ measurement theory, including random error analysis, error propagation and rules concerning significant figures in computations; otherwise measurements will probably be misrepresented.

Rule 1: *The recorder of a measurement should report the value to the appropriate number of figures as reflected by the measuring and computational precision, showing one extra figure to avoid future round-off errors.*

Whenever a person records a number in a field book, on a computation form, on a plat or elsewhere, the number of digits expressed is important because they affect the interpretation of the precision of the measurement, the time required to manipulate the number during future calculations and the precision of calculations using the number. The precision of any computed quantity obviously starts with the precision of the measurements used in the calculations. A user of the number often may have no means to evaluate the precision other than through the digits expressed by someone else. To an expert in measurement, a distance expressed as 125.8 feet does not mean the same thing as 125.80 feet. The non-professional or the uninformed surveyor or engineer may not see or care about the difference, but the difference is very real, and misunderstandings about it can affect the interpretation and use of measured data. For the remainder of this discussion, it must be assumed that numbers have been assigned an appropriate number of significant figures.

Rule 2: *In adding or subtracting, the number of significant figures in the sum or difference is determined by the fewest decimal places in the numbers involved.*

The sum 12.574 + 8,894.3 - 0.25860 = 8,906.615 as added directly with a calculator whose "FIX" is set to three decimal places but is properly rounded to 8,906.6 because the number with the "fewest decimal places" is the one to tenths. Some call this the "left-most decimal place" rule. If I measured three distances in segments, adding them to get the total, it is logical that the precision of the sum is determined by the one with the lowest precision. It would be a waste of time and effort to have some to higher precision than the least precise one.

Rule 3: *In multiplication or division, the number of significant figures in the product or quotient is the same as the fewest number of significant figures in the measured values used, conversion factors or constants having no influence on the significant figures in results.*

The computation of 34,780.07 x 56.999 ÷ 21.33 = 92,940.89123, but the correct answer is 92,940. This is because the 21.33 had only four significant figures. That the calculator was arbitrarily set to "FIX 5" has no bearing on the precision of the result. When conversion factors, functions or constants are involved, only the measured numbers dictate the significant figures. A length of 1,345.87 inches, converted to feet, is 112.156 feet because the conversion 12 inches/foot is an exact number. There are six significant figures in the measured value before and after conversion. Precision to hundredths of an inch is comparable to precision to thousandths of a foot. When the units are changed the decimal places often change in the converted number.

Rule 4: *To avoid round-off errors in calculations:*

• Use at least one digit more in conversion factors or constants than is in the measured value with the least number of significant figures

• Carry one extra digit in computed quantities to avoid round-off errors in calculating other quantities

• Record values appropriately as in Rule 1

In the example demonstrating multiplication and division, the answer was 92,940 (to four significant figures). Using this rule, I would record and retain 92,941, which rounds the calculated answer to five figures, not four.

Note: hereafter, substitute Pi for p and Delta for D

The following example dramatically illustrates the effect of round-off error and the importance of this rule. The area of a fraction of a circle is determined using pR2D÷360º where R = 348.56 feet, and D = 40º. I have assumed that the 40º is exact. If I did otherwise, we would be forced to deal with error propagation involving two variables, which is beyond the scope of what I am trying to illustrate here. As we have contrived the example, the round-off error will be affected by the significant figures used in the radius and the chosen value of p. Using five significant figures in the radius as expressed, the following results, using various popular values of p, are:

p = 3.14, Area = 42,387.932345 (rounds to 42,388 — large error)

p = 3.1416, Area = 42,409.5312913 (rounds to 42,410 — small error)

p = 3.14159, Area = 42,409.3962979 (rounds to 42,409 — no error)

p = 3.141592654, Area = 42,409.43212 (rounds to 42,409 — no error)

Note that it does not matter how many figures are used in p beyond six digits. The answer, to five places, will always be the same, and thus round-off error is avoided by applying the "one extra figure" rule. Anyone who uses approximations or those misleading tables of conversion factors often found in professional publications, textbooks, and other references will probably have round-off errors.

Because p, trigonometric functions and other such numbers are usually derived to more than sufficient precision in modern electronic calculators or computers, this rule may often be unimportant. But anytime such values are not generated within the programmed systems to sufficient precision, round-off errors can occur. A conversion using 2.47 acres per hectare is good only to three significant figures as is a feet-to-meters conversion using 3.28 feet per meter. The computed answer changes in the decimal place to the right of the last digit of the constant used. If this place is not at least one digit beyond the analyzed precision warranted by the data, round-off error occurs. However, if the conversion factor is an exact number, such as 12 inches per foot, the answer retains the significant figures of the measurements, as explained in Rule 3.

Dealing With Zeroes

Zeroes have always been a source of confusion to anyone using measured data. It is best to think of zeroes in terms of how they affect the significant figures of computed quantities. The number 0.000465 has the same effect as 0.0465 or 46.5. Assuming the originator of such numbers analyzed their precision properly, there are only three significant figures in each because only these figures would affect any answer involving multiplication or division with this number. Zeroes at the beginning (either to left or right of the decimal point) are not significant. However, 125.8 has different significant figures than 125.80. Normally, a zero at the end is just as likely as the other nine random possibilities. However, there are exceptions when we know otherwise. In the example used for Rule 3, the correctly rounded number of 92,940 had four significant figures, not five. It can get more confusing when there are two or more zeroes. If the moon is 186,000 miles away, is that significant to units, tens, hundreds, or thousands? Most of the zeros are probably just there to give us the size of the number. The most likely value is either three or four significant figures. Knowing more about the measuring method would help resolve it.

Significant Figures an Indicator of Precision

As mentioned previously, a measured number is understood to be precise to plus or minus half the last decimal place shown unless some other precision is stated. For example, a distance to thousandths of a foot is understood to be plus or minus 0.0005 feet. A bearing shown to seconds is understood to be precise to plus or minus 0.5 seconds. An area shown to hundredths of an acre is understood to be precise to plus or minus 0.005 acres. All of these statements assume that the rules on significant figures were observed by those manipulating the data and that no "extra" digits were shown.

Let us put each of the above common expressions to a test. Suppose a surveyor used a total station and a prism pole to measure a distance and expressed it as 2,004.782 feet. At this distance, I think we could easily show that errors caused by instrument and target centering, failing to keep the prism pole plumb, observing temperature and pressure, observing the vertical angle used to convert to horizontal distance and various instrumental and other errors, would preclude having precision to plus or minus 0.0005 feet. The precision is probably plus or minus a few hundredths. A more honest expression of the distance is probably 2,004.8 feet, or 2,004.78 in view of the "one extra digit" rule.

A bearing is generally computed using an angle with respect to some fixed line. That angle contains random errors owing to the type of target; pointings to both targets; instrument reading precision; target and instrument centering; bubble centering and several geometric factors including sensitivity of the plate bubble, size of the vertical angle, sight distances to both targets and even the size of the angle itself. For sight distances under 100 feet, the surveyor is lucky to have plus or minus 1-minute precision in the angles. Expressing bearings to the nearest second hardly seems appropriate.

An area expressed to 0.01 acres implies a precision of plus or minus about 218 square feet (0.005 acres). If the tract happened to be square, it can be shown (using error propagation) that this implies the errors in the sides are about 0.74 feet each. That is only 1 part in 280 for distance precision! Something is clearly wrong here. If one assumed the precision in the distances to be more like 1 part in 2800, the area should at least be shown to thousandths of an acre. Using the "one extra digit" rule to avoid round-off errors in the future, it should probably be expressed to ten-thousandths. If the precise area was 1.0042 acres, showing 1.00 acres on the plat is a misrepresentation of the precision. The area error here is about 183 square feet. This not only cheats someone of an area the size of an average room in a house (worth perhaps several hundred or even thousands of dollars), but if a future surveyor took the 1.00 acre expression and made computations to divide the tract in half, the dividing line would be about 0.44 feet off of its correct position.

"More or Less" Not Enough

As a related point concerning area or any measured quantity, the commonly used term "more or less" is an unfortunate abomination of measurement science. Any plus or minus cited or implied ought to be based on measurement analysis, precision and significant figures. It should be quantified accordingly, not left hanging as if the surveyor were hiding from something or so unsure of his measurements that he had to end the statement with an embarrassing "I don't know what I have here—your guess is as good as mine." The whole thing is rooted in ignorance of the nature of measurement.

Unwarranted Extra Digits

Lack of attention to significant figures comes in three forms, and there are consequences for all three types of oversight. The first misuse is when unwarranted digits are reported in measured values. This falsifies the precision, misleads the user and usually results in false precision expressed in future numbers calculated. Unsuspecting users of the data may think they are precise to the extra decimal places shown, based on methods better than those actually used. Expressing an angle to seconds when it may only be good to minutes is an example of this misuse. Digitizing or scaling coordinates from maps and showing any numbers to the right of the decimal place is another example. The consequences of taking the measured number out of the context of the methods employed to derive it could be serious. This oversight does not contribute to round-off error, but is a misrepresentation of results. Although fraud may be too strong of a word (in law, fraud usually carries with it the intent to deceive and gain advantage), reporting excess figures (beyond the "one extra") is misrepresentation.

Insufficient Significant Figures

The second problem is when insufficient significant figures are shown somewhere in the measuring or computing process, owing either to faulty initial recording of data or subsequent expressions of values computed. Showing acreage to too low of a precision is an example. The person using the data often has no way to retrieve the true precision resulting from the field procedures and thus precision of any calculations made with the faulty expressions of the numbers is affected. Sophisticated computer programs having high internal computational precision and fancy mathematical procedures cannot create precision in imprecise data. They can only retain the inherent precision of the data. If values have been reported to insufficient precision, the loss of precision in overall results can exceed that caused by the three error sources associated with the observations (nature, instruments, people). In other words, thousands of dollars spent on instruments, computer hardware and software, personnel training and field work could easily be wasted if someone neglects the importance of round-off errors in calculations. Remember, "garbage in, garbage out."

Accumulated Round-off Errors

The third problem occurs when the computational personnel violate rules regarding round-off errors or otherwise misuse reported data. This problem is different from merely reporting numbers incorrectly, as above. This problem represents the deterioration of precision as a result of accumulated round-off errors. What may have been good data originally can evolve to numerical "garbage." The usual faults here are not using sufficient digits in conversion factors and failing to follow the "one extra digit" rule throughout the computational procedures.

Violation of the rules of expression and computation of measured values to be published in legal documents, relied on by others as the beginning place to derive other quantities, and often used as a basis of determining the precise monetary value of land, should be considered a violation of minimum acceptable standards for professional surveyors. As experts in measurement, surveyors should report measured quantities to the appropriate precision and follow the basic rules of computation to avoid unnecessary round-off errors in derived measurements. Of all of the errors associated with measurement, round-off error is the only one that can be avoided.

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The Nature of Measurement: Part 8—Basic Statistical Analysis of Random Errors

Dr. Ben Buckner, LS, PE

Random error was introduced and defined in Part 2 of this series ("Mistakes and Errors," *Professional Surveyor,* April 1997, pp. 19-22). The nature of these errors was more completely explained in Part 3 ("Dealing with Errors," *Professional Surveyor,* May/June 1997, pp. 66-68), where I stated that random errors follow statistical behavioral laws such as the laws of probability and compensation. In Part 6 ("Level of Certainty," *Professional Surveyor,* November/December 1997, pp. 30-32), I mentioned that statisticians and researchers use controlled experiments to determine repeatability and probability of certain outcomes or occurrence of certain selected events. In that article standard deviation was introduced and mentioned as a tool for testing precision. Level of certainty was explained, in part, by stating that the standard deviation is at the 68.3 percent level of certainty. With this as review, I will further explain this statistical tool and how to use it to test precision by using repetitions of a measurement.

Definition of Standard Deviation

The standard deviation is the value on the X axis of the probability curve that occurs at the points of inflection of the curve. To those who wish to apply this tool to practical matters in surveying, it is an estimate of the precision in the variable being tested, or the expected random error in this variable. The standard deviation is sometimes called "standard deviation of a single value," meaning that it is the expected precision of one measurement using the procedure. To arrive at a reliable value for such a single observation, a test must be done, using several repetitions of a measurement, simulating the field conditions to be used later.The late Sol A. Bauer, a former surveyor from Cleveland, Ohio, and ACSM President in 1949, performed extensive testing of his field procedures over a 15-year period starting in the early 1930s. His results were published in *Surveying and Mapping,* Volume VII, No. 3 (1947). In a later article in Volume XI, No. 2 (1951), Mr. Bauer discussed the need for more study of errors by the Property Surveys Division of ACSM. His pioneering studies set a good example and illustrate what is possible, especially considering that he didn't have the advantage of the computer for data reduction. Sol Bauer is a rare exception.

Lack of Interest in Error Testing

Overall interest in the investigation of errors seems to be relatively low among surveying professionals. It has probably been limited to analyses by federal government surveyors for high order control surveying, by engineers employed by instrument manufacturers and by a handful of people who have taken an academic interest in this important aspect of surveying. It was my experience when I was a college professor that research money was not available for such testing. Such limited testing has left the profession with incomplete and often inaccurate data on the precision of even the most common surveying methods. It is probable that most surveyors simply use assumptions, "defaults," closing errors, "least counts" or "professional judgment" to estimate errors, if they are estimated or confronted at all.

A surveyor does not have very good understanding of or control over errors unless they are investigated using some sort of controlled experiments. Assumptions, or manufacturers' statements as to precision, can be considerably far from reality, and the geometry and atmospheric conditions of the survey affect the errors much more than many realize. This has been proven to me many times, in practice, in my own research and while teaching seminars on measurement analysis in which group opinions ("judgments") were often solicited, then compared with results of statistical testing and geometric analyses.

Errors that are close to reality are the only thing that makes least squares adjustments valid and justifiable because the weights for adjustments are proportional to the inverse of the squares of the random errors. Furthermore, when propagating errors to determine error estimates for any indirect measurement, or to estimate positional errors, having even one poor estimate in the several errors used in the propagation causes the results to be "garbage." I shudder when I think of the thousands of people using sophisticated software for analysis of errors who have not carefully considered the numerical estimates of the errors being plugged into those computers (or used as defaults)—who then consider the results, with their error ellipses and data printouts, as being worth any more than the print paper itself.

Almost any measuring procedure can be tested for precision. Often individual error sources can be isolated and tested for their contribution to the total error occurring in the procedure. This breakdown has several advantages, which will be cited in the last section herein.

Basic Measurement Error Testing

Precision tests should be made to isolate random error sources and eliminate or compensate for systematic errors that might affect the data. For example, horizontal angle errors due to horizontal axis tilt caused by bubble misleveling are far different over flat terrain compared with hilly terrain. Long distances have the effect of reducing directional errors compared with short distances, when considering target or instrument centering errors as the source. Most such effects can be quantified by using geometric analysis. Ideally, errors should be investigated under several types of conditions. Even changing the observer can change precision. Any experiment must be carefully devised to control the error sources and know what the results mean.

If I am considering precision of a horizontal angle, I might start by breaking down the angle into the variables that contribute to the total random error. I count at least 15 such variables, including 1) reading, 2) pointing, 3) target centering, 4) instrument centering, 5) bubble centering, 6) bubble sensitivity, 7) distance to backsight, 8) distance to foresight, 9) vertical angle to backsight, 10) vertical angle to foresight, 11) measuring program (repetition versus directional method), 12) number of repetitions, 13) sighting conditions (sun, haze, heat waves, ground stability and so forth), 14) criteria used for rejecting any readings and 15) size of the horizontal angle itself. Note that all three error sources are represented here—1) instrumental, 2) personal and 3) natural.

How many readers may have thought that the most significant, or perhaps the only, error source was the reading precision of the instrument? Of the 15 variables above, reading precision would probably rank among the bottom three in importance for most situations, yet it is most often used to denote the precision of the angles. How these 15 variables could be tested might be the subject of a graduate thesis. How they combine into a final angular error is done by error propagation, which will be addressed in the next article in this series. Let us now look at the reading error.

Making A Statistical Test for Precision

Data gathering• A 1-second optical theodolite was tested by carefully focusing the eyepiece, rotating the mirror to gain optimum lighting of the circle, then rotating the micrometer knob to align the coincidence marks and estimating each reading to 0.1." The micrometer knob was then moved slightly to "destroy" the reading, and the coincidence marks realigned to get a second independent reading. This procedure was repeated until 25 readings were recorded. The readings, just as they were taken, are recorded in the first column of Table 1.

Each reading is assumed to contain only my personal, random error in aligning the coincidence marks, combined with a less important error of making an estimate to the tenth of a second on the scale. These two errors define the "reading error." Although personal in nature, being a measure of the care and skill of the observer, it is affected by the quality and cleanliness of the instrument optics, the lighting, the micrometer focus and possibly changing environmental factors. Thus, I have introduced several more variables that could be further varied and tested. Space limitations do not permit discussion of how these other variables might be isolated. It will suffice to say that they will cause slightly different results between identical tests, even with the same instrument and observer. The differences are much less significant, however, than the difference between the results of a valid test and "least counts" or numbers based on incomplete information or assumptions.

Calculation of standard deviation Standard deviation is calculated from the equation:

s = SQRT(S*vi*2/*n*-1) (1)

where s = standard deviation, vi = the "ith" (or any) residual and n = the number of observations in the set. The

S symbol means "sum of." A "residual" is the difference between an observation and the arithmetic mean, expressed as:

vi = xi - x (2)

where xi = the value of the particular observation, *x* = the arithmetic mean and

x = (S*xi*/*n*) (3)

The solution of the standard deviation is next illustrated, with intermediate steps shown in Table 1 and final calculations shown below the table.

Table 1.

Theodolite Micrometer Readings

Calculation of Standard Deviation of Theodolite Readings

i	Reading	vi	vi2
1	29.0	0.064	0.004
2	28.2	-0.736	0.542
3	28.3	-0.636	0.404
4	27.8	-1.136	1.290

5 30.1 1.164 1.355

6 27.4 -1.536 2.359

7 29.0 0.064 0.004

8 27.8 -1.136 1.290

9 29.2 0.264 0.070

10 30.1 1.164 1.355

11 29.2 0.264 0.070

12 28.9 -0.036 0.001

13 28.5 -0.436 0.190

14 28.7 -0.236 0.056

15 28.2 -0.736 0.542

16 28.5 -0.436 0.190

17 29.8 0.864 0.746

18 29.6 0.664 0.441

19 29.1 0.164 0.027

20 29.7 0.764 0.584

21 27.5 -1.436 2.062

22 28.2 -0.736 0.542

23 31.0 2.064 4.260

24 28.8 -0.136 0.018

25 30.8 1.864 3.474

= 0.955 sec. (5)

The surveyor does not need to do the calculations longhand, as above. Most scientific calculators, computer spreadsheets and statistical software have routines requiring only data input and a few keystrokes to execute them. The complete solution is shown here to teach understanding of the equation, which is important to understand its usefulness. Some observations in this regard are as follows.

Explanation of the Results

Note that if the sample were twice as large, both the numerator and denominator in the equation for s would also be essentially doubled. Thus, s is relatively unaffected by sample size, assuming that "n" is large enough. We could have done 50, 100 or more repetitions, and if all were done with the same skill and care, we could expect s to be essentially the same as calculated above. This is valuable insight, as it teaches that we can assume the error to be approximately the same in the future, given similar procedures (instrument, observer, conditions and so forth). My tests for this instrument have always yielded s between about 0.7 and 0.9 seconds. The above result was actually a little high, which indicates that I am probably getting out of practice taking readings.

Of course, s does not stabilize until a certain minimum number of repetitions are made. That number is generally around 15 or 20 when there are measurable deviations. If deviations are not easily detected, the number of observations must be much higher. Paraphrasing B. Austin Barry in *Engineering Measurements*, crudeness of method hides discrepancies, whereas refinement of method amplifies discrepancies. A crude method would be difficult to quantify as to precision using the standard deviation test.

A study of the equation also reveals that given any particular size of sample (denominator constant), a larger standard deviation results when the numerator is larger. This obviously happens when the residuals, or deviations from the mean, are larger. Larger deviations mean higher random errors or less agreement among duplicate readings, which, by definition, is lower precision. All of this illustrates how and why the standard deviation is a precision index.

Relating Error to Level of Certainty

Note that Equation 2 for a residual looks a lot like the equation for systematic error, except that the definition of error has true value in it rather than mean. The significance of this is that the standard deviation is an estimate of precision, not accuracy, because we are investigating the scatter of readings among themselves, each being compared with the mean and not the unknown true or exact value.

Relating all of the above to level of certainty (the third dimension to a measurement), recall that the standard deviation is at the 68.3 percent certainty level. From my test, I can declare that I can make one reading (note that I did not say measure one angle!) with this instrument and be 68.3 percent confident that it is precise to ±0.955 seconds. Recall also from Part 6 that if I want more certainty, I must widen the range (for example, the 2s error is about 95 percent certain). Some commonly accepted multipliers were given in Part 6. A more complete table is provided in my *Surveying Measurements and their Analysis* text and other books on the subject. Readers are invited to test the theory using the sample test. For example, 17 of the 25 readings will fall within ± s with respect to the mean. That is 68 percent, as the theory predicts. Multiplying s by 1.645 yields 1.57. Adding and subtracting this from 28.936 yields a range of 27.37 to 30.50. There are two values outside of this range (30.8 and 31.0), leaving 92 percent within the range. This is as close to the theoretical 90 percent as is possible in this set of 25 readings. The results follow the theory, as expected.

Much more could be said here about the data, the equations and the analyses of the testing procedures, but space does not permit doing so. For example, had we plotted a histogram (bar graph) of this data, or the normal probability curve, or even constructed a simple frequency distribution table, we would have seen the symmetrical shape of the data. This shape would reveal the attributes of random errors mentioned in earlier parts in this series—equal likelihood of an error of any size being positive as negative, small errors more likely than large errors and so forth. Readers are invited to study the text mentioned above for more thorough explanations and visual study of the graphics associated with statistical analysis of data.

In this test, we isolated one error source in measuring horizontal angles with a particular theodolite. Because of the repeatability of the results, we can rely on the standard deviation discovered to predict reading precision in future observations. This is helpful in several ways. One is simply to be able to cite a close estimate of range of error, with a level of certainty attached. If we can perform controlled tests to isolate other error sources (a "systems analysis"), we can observe the relative magnitudes of the various contributing sources. This gives us control over errors. For example, with such information we can take measures to reduce the contributions from some sources, have data for making wise decisions on modifying procedures to achieve specified standards with minimum effort and expense and so forth. Another related use of such knowledge is for derivation of specifications and devising theoretically sound measurement standards. As mentioned earlier, we also need close estimates of the random errors to perform weighted least squares adjustments. The first step to analysis of anything is knowledge. If we do not know our errors, we cannot control, manage or even discuss them wisely.

Error Testing Is Complex

Most of the applications are not done with individual variable test results viewed in isolation. All known error sources must be investigated individually, then combined mathematically, using error propagation. If we consider the errors stemming from the other 14 variables listed earlier, we see that we have an even more complex problem if we want to analyze the precision of just one angle measurement at one point. The mathematical procedures of error propagation for combining them into a final estimate of the error will be the subject of the next article in this series.

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The Nature of Measurement: Part 9 - The Concept of Random Error Propagation

Dr. Ben Buckner, LS, PE

Error theory is covered briefly in my review seminars. One multiple choice question taken from the NCEES "Typical Questions" booklet asks that if the standard error in measuring one tape length with a 100-ft. tape is ±0.02 ft., what is the standard error in measuring 1,296.88 ft. with this tape? The four answer choices are ±0.05, ±0.07, ±0.13 and ±0.26 ft. A compilation of responses from the many seminar groups, representing more than 1,000 attendees, reveals that about 92 percent choose ±0.26 as the answer, about 4 percent choose ±0.07, and the remaining 4 percent choose one of the other answers or do not respond. The correct answer is ±0.07, and it is calculated by taking 0.02 times the square root of 13 (the number of tape lengths). This is called "error in a series" and is an example of error propagation in addition.

The popular answer of ±0.26 results from treating the error in a tape length as a systematic error and indicates a failure to recognize what the ± (plus or minus) sign means, the significance of the "standard error" term or the difference between a systematic and a random error. In short, the vast majority of the people aspiring to become licensed surveyors do not understand random errors and their propagation, even on the most elementary level.

The Present "State of the Art"

One would think that licensed surveyors might do much better, but this is not the case. Problems such as the above have been used as part of group discussion at my professional development seminars on analysis of surveying measurements. More than 90 percent of the licensed surveyors attending these seminars fail to answer such questions correctly. When it comes to more complex error propagation problems, fewer than 5 percent analyze them correctly.

Occasionally a few surveyors become defensive, rationalizing that "we never need to do that in practice." Many seem to conclude that error propagation is unimportant, impractical and just a lot of theoretical nonsense. If that were the case, we might as well forget discussing concepts such as positional tolerance, positional error, designing measurement specifications and developing theoretically sound measurement standards because error propagation is a significant part of these concepts.

A thorough coverage of error propagation would require several hours of explanation or several chapters in a textbook. It can easily comprise two or three semester hours of college course work, preferably preceded by mathematics courses in calculus and statistics. Space does not permit doing more than introducing the concept here, illustrating how it works and trying to demonstrate the importance of using error propagation in the overall analysis of errors.

Definition

The *Glossary of Mapping Sciences* (ACSM, ASPRS, ASCE, 1994) defines propagation of error as "the effect of error in a quantity on a function of that quantity." This brief definition leaves open the possibility that error propagation deals with both systematic and random errors and therefore is a bit misleading. For example, the effect of a systematic error of 0.02 per 100-ft. tape length would indeed have an effect of 0.26 ft. over a distance of about 1,300 ft. I believe that error propagation applies to random errors, not systematic errors. The reasons why this is proper will be revealed later. I define the concept as follows:

Propagation of errors (or error propagation) is the mathematical process used to estimate the expected random error in a computed or indirectly measured quantity, caused by one or more identified and estimated random errors in one or more identified variables that influence the precision of the quantity.

A problem that contains only one random error and one variable is simple. In the above problem, any contributing error was lumped into the "per tape length" statement, with no breakdown into individual sources. This makes the problem very simple and renders it only an academic exercise to illustrate how error in a series works.

Real error propagation situations are not so simple. There are three considerations, each contributing to the complexity of the problem: 1) the number of variables involved, 2) the number of random error sources contributing to the error in each variable and 3) the mathematical complexity of the equation(s) (the functions) used to compute the desired quantity.

For instance, if we considered random errors in determining the temperature and tension on the tape and separately considered reading and end-marking errors, we would have four variables in the taping problem. Then we might consider that the temperature error has random errors of calibrating the thermometer, as well as reading it. The same is true for the tension handle. If the quantity being computed is area, not distance, it is a much more complex equation than if it were merely the error in a distance.

General Equation for Error Propagation

If y = f(x1, x2, x3, … xn); that is, "y" is a function of several variables called x1, x2, x3, and so forth, the general equation for error propagation, in somewhat simplified form, is:

Ey = [Ex12 + Ex22 + Ex32 + … + Exn2]1/2 (1)

where Ex12, Ex22, Ex32 through Exn2, are the random errors contributed by variable 1, 2, 3 through n, respectively, and Ey is the computed random error in "y" from these several contributing variables or error sources.

The mathematics above is simple. Identifying the variables and their error sources, then evaluating the resulting Ex1, Ex2, Ex3, … Exn is what creates an analysis problem. How to identify and analyze the variables involved and the random error sources contributing to the error in each variable goes beyond the scope of this article. Also, we cannot present all of the statistical procedures (beyond what was covered in Part 8 ["Basic Statistical Analysis of Random Errors," *Professional Surveyor,* April 1998, pp.56-58]) for arriving at a sound numeric value for each contributing error. These numbers must be carefully determined, or else the result is only "garbage." However, we can explain the mathematical procedures in evaluating each Ex once the variables, their error sources and good estimates of the individual contributing errors have been identified and determined.

One way to evaluate each Ex is through the use of differential calculus. Each Ex can be evaluated separately as follows:

Ex1 = (jy/jx1) ex1, E

x2 = (jy/jx2) ex2 and so forth (2)

In words, this says take the partial derivative of "y" with respect to each variable (x1, x2 and so forth), and multiply the result by the error in the respective variable.

Example Using Calculus

Suppose A = LW. This is the area of a rectangle, where A = area, L = length and W = width. The calculus gives us "W" as the partial derivative of "A" with respect to "L," and "L" as the partial derivative of "A" with respect to "W." Substituting these derivatives into Equation (2) yields:

EAL = W eL, EAW = L eW

where EAL is the error in the area resulting from error in "L" and EAW is the error in the area resulting from error in "W." Substituting these into Equation (1) results in:

EA = [(W eL)2 + (L eW)2]1/2

This is the formula for propagation of random errors in a product, often listed among other simple propagation formulas in textbooks.

Say that L = 300.00 ±0.04 feet, W = 200.00 ±0.03 feet. Substituting these values into the last formula yields EAL = ±8 sq. ft. and EAW = ±9 sq. ft., from which EA = ±12 sq. ft. as the propagated error in the area.

Note that the propagation equations for error in addition, error in a series and error in a mean, commonly listed in textbooks and other references, can be derived from calculus and the general formula (Equation 1) as easily as what was done here for the error in a product.

Rationale of Error Propagation

Error propagation is, in a sense, just adding the contributing errors. However, unlike systematic errors, random errors cannot be added algebraically. Systematic errors accumulate, whereas random errors tend to compensate. The sign of a systematic error remains the same; therefore, the error must accumulate. However, the sign of a random error, as likely to be positive as negative, causes the compensatory effect. Taking the square root of the sum of the squares of the errors yields an answer reflecting this effect.

In the above example, because the errors can be plus or minus and lie anywhere within the estimated range, the resulting error will be less than the absolute maximum possible but greater than zero. The probability of the total error accumulating to the absolute maximum of 17 is as improbable as the error exactly canceling to zero. The best estimate is somewhere between the two, which is dictated by the law of compensation. The square root of the sum of the squares of the individual contributing errors yields the best estimate of the propagated error in the function being determined.

The random errors in the equations can be at any level of certainty, as long as the level is consistent. That is, they can be standard deviations, 90 percent errors, 95 percent errors, error estimates and so forth. The error in the quantity being computed will be at the same level of certainty as that of the error estimates used to compute it.

Error Propagation Without Calculus

The calculus approach obviously does not work if a person does not know calculus. Another approach is available. I call it the "rate-of-change method." This approach is not inconsistent with the calculus approach. In fact, the results of both are virtually the same because differential calculus is just a way of determining rates of change.

Understanding this approach requires some simple reasoning. In the above example, the effect of an error in "L" on "A" is logically reasoned to be the difference between two values for "A," one computed with and the other without the error estimate in "L" added to "L." Similar reasoning applies for the error in "W." In this example, A = 60,000 sq. ft., error free. If the error of 0.04 ft. in "L" is added to "L," the area would be 60,008 sq. ft., which is 8 sq. ft. larger than the error-free 60,000. Had we considered the error to be negative, the erroneous area would have been 59,992 sq. ft., which is 8 sq. ft. smaller than the error-free result. Note that because we anticipate that we will be squaring the error effects, it does not matter whether we make them plus or minus. In magnitude, (-8)2 is the same as (+8).2 If we consider the effect of the error in "W," we calculate the area (holding "W" fixed), changing "W" by its error. The result is either 60,009 sq. ft. or 59,991 sq. ft., which when subtracted from the nominal 60,000 sq. ft., results in ±9 sq. ft. error in the area. It should be obvious that there is no difference in the results, with or without calculus. The propagated error in the area using either approach is the square root of the sum of 82 and 92, which is ±12 sq. ft.

Using the rate-of-change method, the procedure is always the same: compute the quantity, error-free, then re-compute it by adding (or subtracting) the error in each variable to (or from) the error-free quantity. Subtract each of these values from the error-free quantity and then take the square root of the sum of their squares of these differences.

This method may be easier to conceptualize than the calculus method. Also, it is probably no more cumbersome to use, especially if the calculations are computerized. A few simple programmed steps will yield the answers.

Graphical Analysis

Some problems, such as the area problem and others that can be sketched on a horizontal or vertical plane, lend themselves easily to graphical analysis. This approach bypasses the need for calculus and affords another way to literally see how the process works in a geometric sense. The figure below shows the 200 x 300 ft. rectangle with the effect of the errors in the two dimensions (positive errors assumed).

The cross-hatched areas are the area errors from the two error sources. It is easily seen that the area created by the error in "L" is EAL = 0.04 x 200 = 8 sq. ft. and the error caused by the error in "W" is EAW = 0.03 x 300 = 9 sq. ft. Remember that random errors are added using the square root of the sum of the squares of the contributing errors. Applying this reality to the graphical analysis, we have the same ±12 sq. ft. as the expected error in the area, with no calculus or computations of areas and erroneous areas, as with the rate-of-change method.

Dealing with Complex Problems

The more variables contributing to the errors and the more error sources in each variable, the longer the equation is for the square root of the sum of the squares. Although most problems do not lend themselves to a graphical analysis, taking partial derivatives and using the general formula or applying the rate-of-change approach works in any case. The complexity of the analysis is somewhat proportional to the complexity of the equation for the variable whose magnitude and error estimate are being determined. Let us look at a couple of examples.

Hour Angle Formula. In the hour angle method for astronomic azimuth, there are three variables: 1) latitude of the observation station, 2) declination of the celestial body and 3) hour angle. However, the equation itself has six trigonometric functions, two products, one subtraction and one quotient, and each of the variables has more than one contributing error source. The error in the latitude, if scaled from a map, would consist of the propagated error resulting from map inaccuracy, identification of the point on the ground, plotting the point on the map and the scaling error.

The error in the hour angle consists of the error in the longitude (which would have the same error sources as the latitude) plus the error in the timing. The timing error is affected by the accuracy of the time source, the starting/stopping precision in using a stopwatch and a small error in the DUT1 correction.

After all of this is considered and computed, the result is the estimated error in one solar azimuth at one point in time. To get the estimated error in the azimuth of a referenced line, we would need to consider the errors in the horizontal angles, as well as the number of repetitions used to arrive at a mean azimuth of the line. The angle errors would be affected by most of the usual sources, such as reading, pointing, ground target centering, bubble (effect on horizontal axis) centering and so forth. These errors are thoroughly analyzed and discussed in my manual, *Astronomic and Grid Azimuth*.

Coordinate Geometry Problems. Problems such as area by coordinates or DMDs involve equations for latitude and departure, which involve the length and directions of lines. To determine the error in such an area, one would need to consider all errors in distances and angles, perform an error propagation for each departure and latitude and then perform another error propagation for the area using the area formula itself. For more on application of error propagation to complex problems the reader is referred to my text, *Surveying Measurements and their Analysis*, several chapters of which are devoted to the subject.

Regardless of the complexity of the situation, the procedures are the same—identify the mathematical function to compute the desired quantity, identify the variables and error sources, do some rather thorough analysis or consult with experts to get some good values for the individual error estimates and then use either calculus or the rate-of-change method to propagate the errors.

Application

Error propagation is just one of the many aspects of analyzing errors. I used only a few examples here to illustrate, but the list is endless. Have you ever wondered what the error of closure ought to be in a given traverse, taking into consideration all of its unique geometric considerations, instruments used and overall field method?

Have you ever wanted a good estimate of the uncertainty in an elevation difference determined trigonometrically, or in a distance determined by a coordinate inverse or the law of cosines, or the positional error of a point with respect to some fixed "control point"?

How about deciding how many repetitions of a horizontal angle are needed to achieve a specified precision with a particular instrument or deciding an overall field method from among more than one choice? In all cases, the answer is found by applying the procedures outlined in this article on error propagation.

Errors are part of measurement reality. Knowing what you have in a computed quantity is part of the professional responsibility of any surveyor. Knowing how to use error propagation is an important part of the specialized body of knowledge called "surveying."

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The Nature of Measurement: Part 10 - Achieving Accuracy in Distances

Dr. Ben Buckner, LS, PE

Many typical instrumental and natural errors in distance measurements are often misunderstood or overlooked. Being systematic in nature, such errors cannot be found by "closing" surveys because relative error of closure checks precision, not accuracy. If these errors are not removed, distances will be inaccurate.

Some of these errors are purely systematic. That is, they obey known mathematical or physical laws and generally change in proportion to the change in the condition causing the error. Others are constant and do not change with some variable condition. Still other errors are systematic in nature but random in effect. Such errors become randomly distributed in the survey. The first type of error will be simply called systematic, the second will be called constant and the third will be called quasi-systematic.

The errors to be discussed are the most common, but the list is by no means complete. I have not attempted to provide a thorough explanation of the testing and calibration procedures. The goal is to encourage achieving the highest practical accuracy in measurements.

Modern land surveying standards are starting to require distances or relative positions accurate to a few hundredths of a foot (depending on "class" of land and scope of survey). For example, Kentucky's new standards, which began to be enforced July 1, 1998, require distances to be accurate to 0.05 feet + 100 PPM (parts per million) in urban and suburban surveys. Washington's standard is more stringent, allowing a positional tolerance of 0.07 feet + 50 PPM for urban surveys. Oregon's standard requires positional accuracy of 0.10 feet or 1:10,000, whichever is greater. To achieve any particular positional accuracy, distances must be more accurate than the positional error allowed because direction error also contributes to the positional error.

To control systematic errors in electronically measured distances, the constant and scale error of the instrument must be determined by proper use of a 4-point base line, temperature and pressure errors must be controlled, reflector constants must be known and applied, tribrach optical plummets and prism pole circular bubbles must be kept adjusted and slope measurements must be properly reduced to horizontal. In many cases (for example, urban surveys), use of tripod mounted reflectors may be necessary in place of prism poles.

Natural Errors in EDM

As a "rule of thumb," an error of 1ºC in the air temperature causes about 1 PPM error in a light wave measurement. Similarly, an error of 0.1 inches in pressure affects the measured distance by 1 PPM, as does an elevation difference of 100 feet. Such errors are negligible for topographic surveys but can be significant for boundary surveys.

For example, suppose the field crews do not change the PPM variable in the total station during times when there are wide swings of temperature. If the temperature goes from 5ºC to 24ºC (41ºF to 75ºF) on a spring day, and the PPM correction was not changed from morning to afternoon, the field crew would have a discrepancy of 0.06 feet in a 3,000-foot distance at the two different points in time.

Atmospheric pressure can also change several tenths of an inch over a few hours when weather is unstable. A weather radio broadcasts the pressure at the weather observation station, which may be miles away from the job site, and the weather may be significantly different there, causing another discrepancy. Considering that 100 feet in elevation difference causes 0.1 inches pressure difference, surveyors working in hilly or mountainous areas have an additional problem of constantly watching the pressure as they change elevation.

The biggest oversight concerning pressure occurs when the field crew uses the cited pressure directly from the weather broadcast. This is not an error, but a mistake because the pressure cited is "sea level" rather than local. Because the pressure drops with altitude, approximately 0.1 inches per 100 feet of altitude above sea level must be subtracted from the sea level pressure. The 1,900-foot contour passes through my back yard. If I were surveying in this subdivision, I would probably start with the broadcast pressure, then subtract 0.1 times 19, or 1.9 inches from that for keying into the total station. Neglecting this difference would cause 19 PPM error in the distances. Thus, a distance of 3,000 feet would have an error from this source of 0.06 feet, the same amount caused by the 19ºC error. If such differences are cumulative, not compensatory, we have a total error of 0.12 feet as a result of overlooking two commonly misunderstood aspects of temperature and pressure.

Instrument Errors in EDM

Without getting into complicated electronic aspects, let us simply say that two errors are associated with the EDM measurement: a constant error and a scale (PPM) error. An electronic distance instrument can be easily calibrated to discover these corrections. When done, the errors in future measurements from these sources can be controlled within a few millimeters (a hundredth of a foot or so). However, the test must be carefully and properly made. As a general specification, I try to have not more than one millimeter error built into the test from any one source. That way, once several error sources become combined, I probably have an accurate calibration within a few millimeters.

The NGS (formerly Coast and Geodetic Survey) installed hundreds of EDM base lines throughout the United States during the last 25 years. Although many of the stations have been disturbed, many have been maintained and some have been remeasured. Generally the intervals between points are accurate to a few tenths of a millimeter. It isn't important to understand how these accuracies were accomplished. Suffice to say that the published intervals represent "truth" for the surveyor. The base lines consist of four marked monuments, all in a straight line over uniformly sloping terrain. Typically, they are called the "0," "150," "430" and "1,400" stations, the numbers representing the distance from the "0" point. The accurate values, both horizontal and "mark-to-mark" (slope), are published on the description sheets.

EDM Calibration

Those who remember how to calibrate a steel tape will recall that a tape is not magically its optimum at some textbook values for temperature and tension. The same holds for EDM. To have accurate distances with a tape, the surveyor must calibrate it—that is, stretch it across a base line of known and highly accurate distance and observe the reading on the tape markings. An EDM must be checked in a similar way. In the case of a tape, we must consider the temperature and tension. In the case of the EDM, we must consider the temperature and pressure. With a tape, we simply record the calibration temperature and tension. These become the "standardization" conditions. With a total station, we can render the effect of the natural errors negligible by keying the readings into the instrument. When calibrating an EDM in this way, the instrument test will result in a scale and constant error that is truly an instrument error, unaffected by temperature and pressure.

To avoid having an effect on distances of more than one millimeter, we can apply the above "rules of thumb" on temperature and pressure. Most EDM base lines have a full length of about 1,400 meters. Keeping the error from temperature under 1 millimeter for this distance translates to an allowable error of about 0.7ºC (1.3ºF) and 0.07 inches of Mercury. The second requirement when calibrating an EDM is to have tribrach optical plummets adjusted to within about 1 millimeter. The reflectors must be placed on tripods with adjusted tribrachs. A good calibration would be impossible using a hand-held prism pole.

Only one reflector should be used for an EDM calibration, and the reflector constant must be known and keyed into the instrument. Doing so compensates for this constant. Thus, the test yields only the instrument constant. If the reflector constant is set to "0" in the instrument, the test results in a "system constant" and is only good for use of the instrument with that particular reflector.

The minimum a surveyor can measure and resolve both the EDM constant and scale errors is three intervals (that is, measure the 150, the 430, and the 1,400 meter lengths). A better set of data, yielding better results, requires six intervals. For the typical 4-point line, these six lengths are 150, 280, 430, 970, 1,250 and 1,400 meters. These measurements are accomplished by an efficient program of moving the instrument and reflector, minimizing the set-ups. All the while, the temperature and pressure should be monitored. It is best to do the calibration on a day when these variables are predicted to be fairly constant and then work efficiently to complete all measurements.

It is advisable to make about 6 to 10 repetitions of each distance, using the mean of the set. You can use either the horizontal or slope distances, as long as you are consistent. A total station that automatically calculates the horizontal distance is best because there is no need to measure tripod heights. For older instruments that do not calculate horizontal distance automatically, setting the reflector at the same height as the EDM and using the slope distances is convenient. After the data is collected, a least squares fit (linear regression) is done to determine the constant and scale corrections. These numbers are used to correct future distances, the theory being the same as when using the calibration error of a tape to correct measured distances.

The main difference between tape and EDM corrections is the scale correction in the latter. The existence of the scale error creates the need for four points in the base line, the measuring of six intervals instead of just one and the linear regression calculations. The most common oversight of many surveyors is just to go out and check one interval along a base line. This is insufficient because it does not isolate the scale error from the constant error. If the constant error was small, the EDM readings might check the published base line lengths for short distances but will mysteriously begin to deviate as the distances become longer.

As an example, a total station was tested. The constant correction was 0.4 millimeters (essentially negligible), and the scale correction was 21.5 PPM. A field reading of 3,000.00 feet would have a "true value" equal to the reading plus the sum of the corrections, the basic equation of $T = R + C$ being applied. The "R" is the observed EDM reading. The "C" is the sum of the two corrections. The constant correction (after making the units conversions) is

0.001 feet, and the scale correction amounts to 0.0645 feet, for a total correction of 0.066 feet. Thus, to the nearest hundredth, the accurate value T = 3,000.07 feet.

Other Systematic Errors

Optical plummets are more precise than plumb bobs but can be less accurate. Unless deflected, plumb lines by definition always point along the vertical and thus are "true." Optical plummets can easily become unadjusted. They can be checked and adjusted to within a millimeter or two. If the adjustment of the tribrach is uncertain, use of a plumb bob may be advisable.

Prism pole bubbles are another error source. Field crews should be taught to check the adjustment of rod bubbles frequently, using any one of several simple tests. Remember, these are precision instruments and as important for ultimate accuracy as the EDM instrument itself.

Reflector constants can vary a few millimeters from that cited by the manufacturer. The value can be determined to one or two millimeters by a simple test field.

Most land surveying distances are horizontal, not slope or geodetic. Unless the horizontal distance is computed in the total station, care must be taken that instrument and reflector heights are equalized so that slope distances are accurate. When attempting to project distances to a plane surface, other geometric errors exist because of the reality that the earth is round, not flat. When using state plane coordinates, for example, the scale and elevation factors must be carefully determined and applied.

It is a shame that some field crews probably check the oil level and tire pressure of the survey vehicle more often than they make accuracy checks of their surveying instruments. Modern survey standards cannot be achieved unless attention is paid to the errors outlined in this article. Alone, any one of the sources may be small and often insignificant. If enough errors are overlooked, accuracy may be far from what is desirable, and minimum accuracy standards may not be achieved, regardless of the precision indicated by traverse closure.

For more details of the various instrument test procedures, readers should contact the author.

***The Nature of Measurement*: Part XI - Suggested Measurement Standards for Property Surveys**

Dr. Ben Buckner, LS, PE

The following suggested standards evolved during 1996 and 1997. They were written after studying several sets of state standards, considering the various proposed ACSM/ALTA standards publicized during this period and drawing from my own knowledge of measurement science. These standards are similar to those prepared by a committee of the Northeast Chapter of the Tennessee Association of Professional Land Surveyors and suggested to the Tennessee state licensing board to replace the current standards.

I believe that these standards, if adopted by licensing boards and professional surveying associations, will overcome some misunderstanding about measurement accuracy and ease concern over positional tolerance. The standards are theoretically sound and easily attainable with modern instrumentation. The standards do not dictate methods. They are not specifications. The surveyor must read and understand them, then derive specifications designed to yield the accuracies dictated by the standards. Of course, the user must have a professional level knowledge of the science of measurement to interpret and apply the standards properly.

The standards are presented in a familiar "legal" format that can be easily adopted. I would caution any association or board against too much modification of any suggested standards. I have seen drastic changes made to well-considered suggestions where the surgeons simply lacked understanding and didn't bother to ask the author what was meant. The result is usually something impossible to apply. If something is not understood, the clarification will probably be found somewhere in Parts 1 through 10 of this series, which form the background of the standards.

Material in italics below is explanatory and is not part of the standards.

The Standards

A. Calibration of Instruments & Error Control

I. Basic Definitions

a. **Precision** is relative agreement among duplicate measurements of the same quantity or the repeatability of a measurement. Precision is a function of random errors and their control in measurements. Relative error of closure of a survey checks the precision. Precision is also sometimes used to denote the significant figures expressed in a measured number.

b. **Accuracy** is the degree of conformity of a final measured value, with respect to the true value as defined by accepted, legal standards on weights and measures, to geometric laws, or to accepted datums. Distance accuracy relates to standards established by the United States government defining the meter. Angle accuracy relates to geometric divisions of the circle and to geometric laws on closed figures. Accuracy is achieved by compensating or correcting for systematic errors and avoiding or removing mistakes from measurements and, to a lesser extent, by controlling random errors.

II. Instrument Calibration and Adjustment

Distance measuring instruments, retroprisms, and associated instrumentation shall by calibrated using calibration base lines and other methods to assure they are operating within acceptable tolerances and to determine instrument constants for the purpose of correcting measured data. Angle measuring and leveling instruments shall be checked periodically for maladjustment of axes and general functioning. Optical plumbing devices, such as the optical plummets in tribrachs and prism pole bubbles shall likewise by periodically checked for proper adjustment and adjusted if necessary.

III. Compensation for Systematic Errors

The surveyor shall, to the extent necessary to achieve the standards herein, compensate or correct for systematic errors associated with the instruments, natural phenomena, calibration and maladjustment of instruments, and other sources.

IV. Control and Management of Random Errors

The Surveyor shall control random errors in measurements by using due care andand specified procedures designed to achieve the required results. The surveyor shall use appropriate error propagation and other measurement design theory for selection of instruments, field procedures, geometric layouts, and computational procedures to control and adjust random errors in order to achieve the required relative positional tolerances specified herein.

V. Avoiding and Eliminating Mistakes

The surveyor shall take measurements to avoid, detect, and remove mistakes (blunders) from field measurements and calculations. Mistakes shall not be adjusted, mathematically, but removed prior to adjustment of closing errors.

B. Accuracy Standards For Property Line Surveys

I. Basic Definitions

a. Definitions Related to Property Line Evidence

1. A **corner** is a point where a property line changes direction, or the point of intersection of two or more boundary lines. A monument or evidence of a monument may or may not exist at a corner.
2. A **monument**, as used in property line surveys, is a physical marker, marking, or natural feature identifying the location of a property corner, found in place, or set or otherwise marked to preserve and perpetuate the position of the described and surveyed corner.

b. Definitions Pertinent to Positional Tolerance and Errors

1. **Reference Point**. A reference point may be a monument at the corner of a recorded subdivision or a prominent corner of a metes and bounds survey (generally being the point of beginning, points of commencement, or other such point). Generally, one reference point will be selected for any particular survey.

Author's Note: *Positional tolerance must relate to something identifiable, be it a physical monument or a geodetic reference system. Stating accuracy without "respect to what" is incomplete. A monumented "reference Point" is used here. This reference could also be a geodetic control monument. It could even be an ellipsoid-based coordinate system, with no particular physical monument mentioned, but I was afraid too much misunderstanding would spring from this suggestion. If all standards* *required referencing property surveys to such systems, then both the title identity and the coordinate reference would be accomplished with the reference point monument. In my opinion, requiring ties to systems such as state plane coordinates is overdue, with the densification and accuracy of control now realized by the high-accuracy reference networks. However, that is a separate issue.*

2. **Relative Positional Tolerance**. Relative positional tolerance or accuracy is the maximum acceptable error in position of a corner monument with respect to the monument identifying the selected reference point in a surveyed tract. Relative positional tolerance is not related to differences between measured and recorded distances or to uncertainty of the reference point or its identifying monument with respect to geodetic, cadastral, or other references. It is a standard for the accuracy of survey measurements made directly or indirectly between physical monuments. Where one or both of the monuments are set during the survey, the tolerance refers to the final monument position, not to temporary points set prior to final monumentation. In all cases, monuments shall have distinct markings to which measurements refer.
3. **Relative Positional Error**. Relative positional error or uncertainty is the computed error or uncertainty of one physical point (i.e., monumented corner) with respect to another. In these standards, it is specifically the uncertainty of any corner monument with respect to the selected reference point monument. It is computed using either analysis of component distance and direction errors between points or from a correctly weighted least squares analysis. Computed positional errors shall be less than the allowable tolerances.

Author's Note: *Positional tolerance or accuracy is what is expected. This is the "standard". Positional error is what the surveyor estimates, using calculations involving random errors, with error propagation, to calculate a defensible positional error. This error would then be compared with the standard. In practice, this is a very important step. Surveyors must know error theory, how to test instruments and methods for statistically determined random errors and how to perform the error propagation and other procedures to evaluate the survey.*

II. Application

a. Application Relative to Accuracy and Monuments

1. The following standards on relative positional tolerance and calculated positional error apply to accuracy, not to precision of measurements.
2. These standards apply only to accuracy of measurements between monuments marking analyzed corner positions, not to the best location of the corner position as analyzed from the evidence with embodies procedures involving case law related to retracing boundaries. Thus, a corner shall be properly set from evidence first, then monumented, in order to apply these standards. The surveyor shall use instruments and methods designed to yield the minimum standards.

b. Application Relative to Survey Classes

1. The relative positional accuracy appropriate for a particular property survey shall be based on the potential use of the land, as related to its capability or suitability for development, not to existing use.
2. Survey Classes

1. **Class 1:** Developed land and land which has a high potential to be developed, either because of its physical attributes or because of its location or possible future demand. This class typically includes what is generally considered urban and suburban land, but can also include some rural land.

2. **Class 2:** Land which is unlikely to be developed because of its physical attributes or remote location. This class includes mountains, marshes, and other areas unlikely to be intensely developed.

   Author's Note: *Because land uses change over time, particularly in regard to "developable land", there is value in anticipating this and making the accuracy of the surveys match what would be desirable in the future. The dimension of time is important in any planning scheme. Suggesting placing surveys into only two classes will, of course, go against tradition, but I hope reasonable surveyors will see the merit of two classes. To put this into perspective, with the use of modern instruments such as total stations, there is probably no significant difference between what surveyors are achieving in "Class 1" and "Class 2" areas anyway. In fact, the "Class 2" areas may even be surveyed more accurately. This is because they are usually rural, with longer lines, as opposed to the shorter sight lines typical of urban surveys, which works in favor of increasing relative accuracy. For this reason there is actually merit in having no differentiation between "classes" based on land. However, I figured some would never accept this, so two classes are suggested.*

3. Required Relative Positional Tolerance (Accuracy).
1. **Class1:** ± (0.07 feet + 1:20,000) or ±(20 mm. + 50 ppm)
2. **Class2:** ± (0.26 feet + 1:50,000) or ±(80 mm. + 200 ppm)

   Author's Note: *The fixed distance with a PPM should be familiar to most surveyors. The exact tolerance is thus variable and depends on the distance between the reference point and the monument being considered. Note that "error of closure" is not in the standards as related to accuracy because it relates to precision. Unless systematic errors have been compensated or corrected to the extent dictated by the standards, the desired accuracy will not result. Unless surveyors reject error of closure concepts in relation to accuracy, the professional will always suffer from its embarassing misunderstanding of basic measurement science.*

About the Author

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***The Nature of Measurement*: Part 12 - Universal Truths Found in Measurement Science**

Dr. Ben Buckner, LS, PE, CP

After years of reflecting on measurement, I have discovered some axioms or maxims imbedded in its essence. An axiom is an accepted principle, proverbial truth or maxim. A maxim is a general truth, moral reflection or rule of conduct. Whatever word chosen to describe them, there are universal truths at the heart of surveying, and they are applicable to any situation at any time.

Being preoccupied with measurement science has contributed significantly to my understanding of life's events and provided a flow of insight regarding choices and reactions to various situations involving human interaction. Studying measurement science has helped me learn more about life and relating to the world than have many books on human behavior. The philosophies and applications outlined in such books generally suffer from the limited experience and perspective of their writers. However, measurement science concepts apply universally, are self evident and leave little room for debate. When something can be seen as an axiom or maxim, we waste our time with opinion and intellectual reasoning about it. The *Holy Bible*, particularly books such as Proverbs, is the only written source that has been more significant for me than the science of measurement for gaining insight to guide my behavior and my reaction to the behavior of others.

This last article in the series on the nature of measurement explains how, with ample study and reflection on the nature of measurement, one can see truths that have little to do directly with surveying or measurements, but which can be applied in just about any situation being evaluated or considered. It is my desire that others gain as much as I have from this body of knowledge. I hope the following will help you along your journey.

Axiom #1: We are responsible for our mistakes.

In measurement, we learn that errors and mistakes are not the same. Errors are unavoidable. Mistakes are generally caused by carelessness, inattention, lack of information, poor training, negative and uncontrolled emotions and so forth, and thus are avoidable. Unless mistakes are either avoided or corrected promptly, there will be inevitable consequences. The consequences are usually cost, inconvenience, damage or pain for someone, whether dealing with measurements or something else. Thousands of lawsuits undoubtedly occur daily because people have been damaged by mistakes.

The very nature of mistakes requires us to take responsibility for them. Understanding this should help anyone see that rationalizing any mistake away is unacceptable. It teaches further that the best way to deal with mistakes is to avoid them by being careful and attentive, gaining the right education or training, staying current as to the knowledge required to execute tasks or practice our chosen profession, keeping sober and in control of our thoughts and movements and maintaining a positive and professional attitude. This aspect of life, learned by application of elementary measurement science, is universal. Taking responsibility for mistakes is a sign of a professional attitude and a mature and emotionally healthy individual or society.

Axiom #2: Truth is approached whenever we take our initial observations and modify them using corrections gained by knowledge of where we were in error initially.

In measurement science, we simply say:

$$\text{True Value} = \text{Reading} + \text{Sum of Corrections}$$

OR

$$(T = R + C_T)$$

When dealing with measurements, we are confronted with the reality that accuracy cannot exist unless the systematic errors are sought, found, evaluated and corrected or compensated. Human perceptions and readings of measurements in surveying are just part of a larger universe, which we can call observations. Without corrections, observations of any kind, and of any phenomena, are merely biased perceptions that may be far from the truth. In any event involving human interaction, our perception is inevitably biased or confused by emotions or preconceived ideas, manipulation of our emotions by propaganda or drama, deliberate or sloppy reporting of facts, lack of research into the facts, lack of relevant evidence, or lack of background education that would provide the knowledge of the theory and principles and the ability to think analytically. Knowledge has a natural way of helping remove bias. Truth has a way of setting us free from prejudices and misconceptions that block the view of accuracy.

As for human perceptions, our ego and pride often get in the way, and we readily and often unquestioningly decide and declare that the initial perception gained from the observation is accurate. The way of a fool always seems right to him. We often do not want to be "confused with facts" and resent it when more complete evidence surfaces that dictates a mind change or alteration of the proposed solution to the problem. One's reaction, when confronted with the facts (systematic errors) can include anger, defensiveness, vindictive behavior or denial. Often, the result is rigid adherence to the biased perception adopted and owned by the individual, with probable upset of some relationship.

If I understand and accept the aspect of measurement science embraced by this axiom, I will always question my own and others' perceptions, knowing they are undoubtedly biased. This assumes, of course, that my quest in life is to ascertain the truth. If it is to gain something personally, find the easy way or try to look good in the eyes of others (play "politics"), then no amount of knowledge in this regard will help me get closer to the truth. If truth is the goal, I am not afraid to alter my initial observation. I know, from this universal truth of the way things evolve, that it is normal, inevitable and necessary to often change my initial perceptions.

Axiom #3: Our perceptions are inexact, whether of measured quantities or other subjective phenomena.

This axiom relates to random errors, which lurk beneath the surface and frustrate any arrival at truth through observation. Therefore, regarding the equation cited under Axiom #2, I must modify the theory with the reality that I only approach, rather than find, the true value.

Some biases are unknown or difficult to detect. If we knew what they were, they would be systematic and therefore removable. All of our judgments, no matter how carefully considered, are flawed. This may come as a shock to some people, but this condition cannot be denied. Once we fully accept this, we begin to see that the judgments of others might be as good as our own, or at least we begin to consider that possibility. In discussions or debates of all types, we can get more outside of our own little world and weigh everyone else's comments. This universal truth can help to develop humility, objectivity and an attitude of cooperation and compromise.

When this axiom is fully understood and placed into operation, we can cease to expect perfection from employees, the guy driving the car in front of us, our spouse, ourselves or anyone. We will eventually tend to blame less and seek solutions or resolutions instead. As we see the scatter in numerical values of surveyed data, it should constantly remind us of the wide range of well-considered opinions regarding subjective data and to entertain the possibility that any one of these observations could be as close to the truth as any other. The fact that one of the observations happens to be our own, or that many may agree on an issue, doesn't create truth.

Axiom #4: Agreement of the majority is insufficient to declare that the best solution has been decided.

This axiom stems from the concepts of precision and accuracy. Reflecting on the two concepts helps us understand why it is important to distinguish between them and that they apply to everything, not just measurements. As we learn in surveying, precision relates little to accuracy. For instance, making several repetitions of a measurement, or closing a survey on itself, finds a few possible mistakes and helps evaluate precision. However, it does not detect systematic errors and therefore does not adequately check accuracy.

Earlier in this series I defined accuracy in measurements as adherence to defined standards on weights and measures, geometric laws or accepted datums. In affairs outside of measurement, accuracy isn't so easy to define. I will simply say that it is that which adheres to accepted legal, ethical or moral standards. It is what wisdom suggests would benefit others and not selected factions or groups having motives based in fear and other forms of self interest. Accuracy, as a measure of quality or what is "right," checks observations against something outside of the self or a small group, the something representing an absolute standard and ideally being free of bias.

Unfortunately, the truth is always elusive. Precision is much easier to observe. It requires only checking something against itself (usually our own work) or against similarly biased observations rather than a higher, outside standard. Making comparisons and weighing things against standards outside of ourselves is probably one of the hardest things for anybody to learn and to do consistently. Perhaps this is why precision is so often confused with accuracy when it comes to evaluating measurement quality in surveying.

In a court of law, the collective strength of evidence gained by the testimony of several witnesses is sufficient to gain a preponderance of evidence or prove an issue beyond a reasonable doubt only if each testimony is given independently, without collaboration, and if the witnesses are unprejudiced in their testimony. If each person gives "the truth and nothing but the truth," then a relatively few witnesses can be effective in proving a point. Truth will be obvious when the testimony is unbiased and unswayed by fear, temptation, greed and so forth. But, if witnesses successfully conspire to testify untruthfully but with a consistent story, justice will not be served. We have precision without accuracy.

A wise person seeks many counselors or consultants when trying to clarify an important issue or decide a course of action. Unfortunately, if all of the consultants are similarly biased, the counsel may not lead to the best decisions. If many people consulted have fears, prejudices, biases or self interests (often the same ones as the seeker), the collective advice may result in actions that are illegal, unethical, immoral and thus not "right." This can happen even if there was no deliberate attempt to manipulate everyone to agree according to some self interest. It occurs simply because the person is not wise enough to consult people who might disagree with his or her preconceived notion or pet project. Most of us are guilty of seeking support only from people who we anticipate are going to agree with us and avoiding contact with those who we suspect will not agree. Thus, we surround ourselves with many like-minded people and, being fed by the confidence we get from this selected and biased group, begin to feel in time that we must be "right." Hopefully, the insight gained from this axiom will help us know the best sources of counsel.

As an example of the above failure, I have observed countless times, in professional meetings and matters, how issues are pushed by people who feel they must be right because so many people seemingly agree with them. The "good old boy" organizations that surveyors call professional associations are often guilty of this. Members who often have genuine vision and goals based on serving the public and strengthening the profession are often ignored, ostracized and discouraged to the point of dropping out of the associations as the "status-quo" people band together into an unmovable and unchangeable core wallowing in a sea of fear and bias.

Another example of how repeatability has affected the strength of our profession is how the content of the NCEES examinations for land surveyors has been decided over the last 25 years. The tasks surveyors perform, based on return of thousands of questionnaires, have been used to decide exam content. If the majority did not use or apply a procedure, the subject matter was either omitted or greatly de-emphasized on the exam. That it might better protect the public if surveyors were expected to have certain knowledge and apply certain technical procedures has not been a factor in deciding the exam content. The knowledge base of the majority of the practitioners and the level of their practice has been the deciding factor. The system has emphasized the collective opinion of the majority, thus focusing on precision rather than what might be best.

There are countless examples of how precision without accuracy changed the course of history. Adolf Hitler's vast following for many years is one. The Watergate scandal during President Nixon's term of office is another. One of those involved in that scandal said that (paraphrased) someone needed to stop and ask "is this right," and they may have been able to stop what happened. This man recognized, too late perhaps, that they were a closed group whose collective opinion was not centered on truth.

We should always look for repeatability or agreement among wise people whose motives are unselfish or not biased in any particular political direction. Repeatability, when sought in this way, validates. Nonetheless, just as closing a survey on itself does not ensure accuracy, a large number of people agreeing on something does not in itself make it right.

Another lesson we can learn from all of this is that to disagree with the majority may be the right thing. Some non-conformists often have a point.

Axiom #5: Imperfection lies within the finite dimensions of humankind, whereas perfection is found at the infinite dimension.

We approach zero error or ultimate precision as we approach infinity. This is illustrated by the relationship between the error in a single value and the error in the mean of a large number of repetitions of a quantity. The error in the mean of a set of observations in a measuring procedure is equal to the standard deviation for the procedure divided by the square root of the number of observations in the set.

Perfection is found at infinity because the square root of infinity is still infinity and anything divided by infinity is zero. No matter how many times I repeat an angle, I cannot resolve it to zero error because I must operate with a finite number of observations.

Can we mortals find perfection in anything? This axiom, based on measurement science and mathematics, teaches us that we cannot. I have not yet met a person or group on this planet who has absolutely accurate and consistently correct judgments on all matters. The error in any individual or collective judgment or decision made by humans is short of perfection, no matter how long we ponder the situation or how many experts we consult.

The value of seeking many counselors, as discussed in the previous section, is driven home with this axiom. It teaches that if we seek the opinions and judgments of many, we will probably make wiser decisions. Although we cannot ever know what is "right," we can do better by seeking more opinions. Of course, the opinions, if used (they can be rejected), must be based on objective thought and evaluation of the evidence; otherwise "wild values" contaminate the sample. Wise people rarely make important decisions about anything having significant potential consequences without consulting various reliable sources. A wise man listens to counsel. Plans succeed with many advisors because the error in the judgment is reduced by the square root of the number of unbiased experts consulted. Likewise, plans are more apt to fail when made alone. The square root of one is always one.

Axiom #6: Absolute certainty is impossible in the real world. Certainty is delusion, whereas uncertainty is reality.

Remember the bell curve? The standard deviation, occurring at the points of inflexion, is the 68.3 percent level of certainty. To gain more certainty, we must widen the range of error. To have 100 percent certainty in anything measured, the range of error or the tolerance must be infinitely large. Statistical analysis and this curve teach us this.

From the above, we realize that 100 percent certainty is not possible in the real world where human judgment or perception is involved. This is true regarding both measurement science and personal decisions.

The truly open-minded person is uncertain. This is a conclusion we must reach after considering the above. As we ponder all of this, we begin to realize that certainty inevitably leads to rigidity and closed-mindedness, where new evidence is difficult to consider or accept, with consequent unwillingness to modify perceptions to further approach truth. Indeed, a fool is one who ignores facts, is not open to evidence or new knowledge, has his mind "made up," makes decisions with insufficient information or simply leans on his limited understanding.

Well-balanced people seem to know instinctively that they do not have the luxury of an infinitely large tolerance and so have some reservations or doubt about any decision. It is healthy to have self confidence when judgments are well considered, but some measure of caution and doubt is healthy too. I can proceed in a matter as an act of faith, but faith, by definition, is not susceptible to such analysis.

I remember that when I was a young Army lieutenant in training, the training officer would ask (right in my face) "Are you sure, Lieutenant?" I was trained to reply "Yes, Sir!" I was lying, of course. But I couldn't answer, "I am 99 percent sure." It wouldn't have been acceptable, and it would have been at least 90 percent certain that I would be doing push-ups had I replied with anything other than an absolute "Yes, Sir!"

We have been conditioned to expect the "Yes, Sirs" from our peers, our political leaders, our children, our spouses, our employees and so forth. The certification on your survey plat is a statement of 100 percent certainty. The law will use it against you later, declaring, "You said it was so."

Somehow, we have had it all wrong. We make a serious mistake when we expect certainty from ourselves or others. We should begin admitting uncertainty. It is more honest. This may go contrary to the way we are taught by parents, employers, politicians and peers. Nonetheless, it is true because we operate within finite dimensions.

Living with uncertainty may frighten some people. However, when the alternatives and the consequences of closed-mindedness and rigidity are considered, uncertainty becomes therapeutic. It is the route to better solutions and improved personal and other relationships, so it is bound to ultimately make a person feel better than always being certain.

Summary

Hopefully, this last in the series on the nature of measurement will give readers a deeper appreciation of our profession and the ability to tap into a source of insight and wisdom perhaps not recognized before. When you probe into the nature of measurement, the theory, concepts and equations begin to be seen as a mirror of life itself.

This exploration focused on the science of measurement. Before leaving here, I must say that there is more insight to be gained from a similar consideration of that other aspect of our professional thinking—that of evaluation of evidence and the role of the land surveyor in retracement. However, this aspect was completely omitted here for lack of space and the desire to focus on one theme. Maybe some other time we can tell "the rest of the story."

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