Adjustment Theory Terms

- **Frequency** (n_i): Number of times a measurement value appears.
- **Relative Frequency** (r_i): The ratio of a specific measurement frequency to the total number of measurements.
$$r_i = \frac{n_i}{n}$$
- **Relative Sum of Frequencies** (R_i): Cumulative sum of relative frequencies.
$$R_i = \sum_{j \leq i} r_j$$
- **Probability Density Function (PDF)**: Function that describes the relative likelihood of a random variable taking on a given value.
- **Cumulative Distribution Function (CDF)**: The probability that a random variable takes on a value less than or equal to x.
$$F(x) = P(X \leq x)$$
- **Expectation Value** ($E(X)$): The mean or expected value of a random variable.
$$E(X) = \sum x_i p_i$$
(discrete case)
$$E(X) = \int x f(x) dx$$
(continuous case)
- **Variance** (σ^2): Measure of spread around the mean.
$$\sigma^2 = E\{(X - E(X))^2\}$$
- **Standard Deviation** (σ): Square root of variance.
$$\sigma = \sqrt{\sigma^2}$$
- **Covariance** ($\mathrm{Cov}(X,Y)$): Measures the relationship between two random variables.
$$\mathrm{Cov}(X,Y) = E[(X - E(X))(Y - E(Y))]$$
- **Correlation Coefficient** (ρ): Normalized covariance measure.
$$\rho = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}$$

- **Law of Large Numbers**: States that the sample mean converges to the population mean as sample size increases.
- **Skewness**: Measures asymmetry of the probability distribution.
- **Kurtosis**: Measures the "tailedness" of the probability distribution.

- **Central Limit Theorem**: Theorem stating that the distribution of the sample mean approximates a normal distribution as the sample size increases.

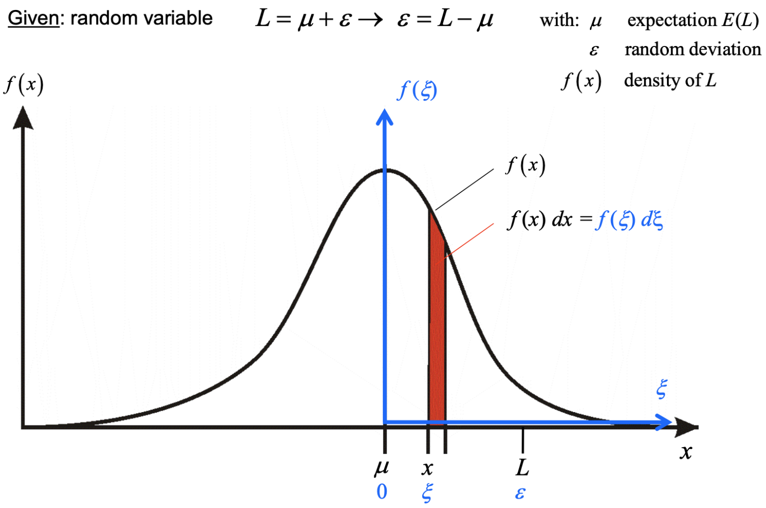

Figure 1: Central Limit Theorem

The Central Limit Theorem states that given a sufficiently large number of independent and identically distributed (i.i.d.) random variables with finite variance, their sum (or sample mean) follows a normal distribution:

$$X_n \sim N(\mu, \sigma^2/n)$$

where μ is the mean and σ^2 is the variance.

Shifting and Scaling of the Normal Distribution

A normal distribution is fully described by its expectation value μ and variance σ^2. The probability density function (PDF) of a normal distribution is given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Shifting the normal distribution to the mean value μ changes the function's center, but the shape remains the same. The spread of the function is controlled by σ^2. As the variance increases, the distribution flattens; as it decreases, it becomes more peaked.

Example: Application of CLT

Consider a sample of size $n = 30$ drawn from a uniform distribution $U(0,1)$. The mean of this sample will approximate a normal distribution:

$$\bar{X} \sim N\left(\frac{1}{2}, \frac{1}{12n}\right)$$

Thus, even though the original distribution is not normal, the sample mean follows an approximate normal distribution for sufficiently large n.

Suppose we take a random sample of size $n = 30$ from a uniform distribution $U(0, 1)$. According to the CLT, the sample mean follows an approximate normal distribution:

$$\bar{X} \sim N\left(\frac{1}{2}, \frac{1}{12n}\right).$$

Thus, even for non-normal distributions, the sample mean distribution will tend toward normality as n increases.

Essential Features of the Normal Distribution

- The normal density function is symmetric about its mean μ.
- The expectation value, median, and mode are equal to μ.
- The maximum density occurs at μ.
- The inflection points occur at $\mu \pm \sigma$.
- The probabilities of deviations within multiples of σ are:

$$\begin{aligned} P(\mu - \sigma \leq X \leq \mu + \sigma) &= 68.27\%, \\ P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) &= 95.45\%, \\ P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) &= 99.73\%. \end{aligned}$$

For confidence levels of 90%, 95%, and 99%, the corresponding intervals are:

$$\begin{aligned} P(\mu - 1.645\sigma \leq X \leq \mu + 1.645\sigma) &= 90\%, \\ P(\mu - 1.960\sigma \leq X \leq \mu + 1.960\sigma) &= 95\%, \\ P(\mu - 2.576\sigma \leq X \leq \mu + 2.576\sigma) &= 99\%. \end{aligned}$$

Theoretical and Practical Importance of the Normal Distribution

- The normal distribution plays a key role in statistics due to the Central Limit Theorem, which states that the sum of independent random variables tends to be normally distributed.
- Many natural and measurement processes follow approximately normal distributions, making it highly applicable in fields like geodesy, photogrammetry, and surveying.
- Statistical methods such as hypothesis testing, confidence intervals, and regression analysis rely on normal distribution assumptions.
- The probability values for standard deviations provide a basis for statistical decision-making.

Key statistical meassures in Adjustmeent Theory

Basic Notations from Probability and Statistics

Random variables represent outcomes of experiments.

Probability density functions (PDFs) define the likelihood of outcomes.

Glossary of Technical Terms

Summary of Key Concepts

Basic Notations from Probability and Statistics

- **Random variables** represent outcomes of experiments.
- **Probability Density Function (PDF)**: Function that describes the relative likelihood of a random variable taking on a given value.
- **Cumulative Distribution Function (CDF)**: The probability that a random variable takes on a value less than or equal to x.

Probability Density Function (PDF)

A probability density function (PDF) describes the likelihood of a continuous random variable taking on a particular value. It is defined such that:

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

where $f(x)$ satisfies:

- $f(x) \geq 0$ for all x.
- The total probability integrates to 1: $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Example: Consider a uniform distribution on the interval $[0, 1]$, where:

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Then, the probability that X falls between 0.2 and 0.5 is:

$$P(0.2 \leq X \leq 0.5) = \int_{0.2}^{0.5} 1\,dx = 0.5 - 0.2 = 0.3.$$

Cumulative Distribution Function (CDF)

The cumulative distribution function (CDF) gives the probability that a random variable X takes a value less than or equal to x:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt$$

Example: For a uniform distribution $U(0, 1)$:

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

Thus, the probability that $X \leq 0.5$ is:

$$P(X \leq 0.5) = F(0.5) = 0.5.$$

Frequency, Relative Frequency, and Relative Sum of Frequencies

- Frequencies describe occurrences in a dataset.
- Relative frequencies approximate probabilities for large samples.

Example: Tossing a fair die 100 times: Frequency of rolling a 3 might be 18.

$$r_3 = \frac{18}{100} = 0.18$$

Distributive Function

Used to compute probabilities for continuous and discrete variables. **Example:** For a uniform distribution over $[0, 1]$, the cumulative function is:

$$F(x) = x, \quad 0 \leq x \leq 1$$

- **Expectation Value** ($E(X)$): The mean or expected value of a random variable. (discrete case) (continuous case)
- **Variance** (σ^2): Measure of spread around the mean.

Expectation Value (Mean)

Defines the central tendency of a probability distribution. **Example:** A fair die with values $1, 2, 3, 4, 5, 6$ has an expected value:

$$E(X) = \sum_{i=1}^{6} x_i P(X = x_i) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6} = 3.5$$

Variance and Standard Deviation

Variance quantifies the spread of a distribution. **Example:** For a die:

$$\begin{aligned} \sigma^2 &= E(X^2) - [E(X)]^2 \\ &= \left(\sum_{i=1}^{6} x_i^2 P(X = x_i) \right) - 3.5^2 \\ &= (1 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 9 \cdot \frac{1}{6} + 16 \cdot \frac{1}{6} + 25 \cdot \frac{1}{6} + 36 \cdot \frac{1}{6}) - 12.25 \\ &= \frac{91}{6} - 12.25 = 2.92 \end{aligned}$$

Standard deviation:

$$\sigma = \sqrt{2.92} \approx 1.71$$