2 Partial Derivatives for Distances

$$s_{ik} = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2} \tag{1}$$

x_k Unknown

$$\frac{\partial s_{ik}}{\partial x_k} = \frac{x_k - x_i}{s_{ik}} = \frac{\Delta x_{ik}}{s_{ik}} = \cos t_{ik}, \quad \texttt{ds\_dx\_to}(yi,\ xi,\ yk,\ xk) \tag{2}$$

y_k Unknown

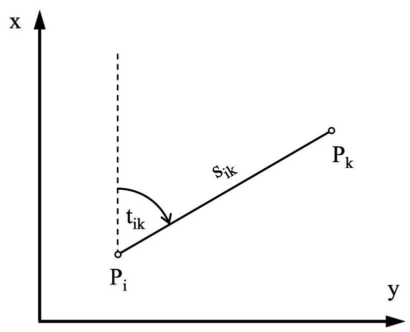
$$\frac{\partial s_{ik}}{\partial y_k} = \frac{y_k - y_i}{s_{ik}} = \frac{\Delta y_{ik}}{s_{ik}} = \sin t_{ik}, \quad \texttt{ds\_dy\_to}(yi,\ xi,\ yk,\ xk) \tag{3}$$

x_i Unknown

$$\frac{\partial s_{ik}}{\partial x_i} = -\frac{x_k - x_i}{s_{ik}} = -\frac{\Delta x_{ik}}{s_{ik}} = -\cos t_{ik}, \quad \texttt{ds\_dx\_from}(yi,\ xi,\ yk,\ xk) \tag{4}$$

y_i Unknown

$$\frac{\partial s_{ik}}{\partial y_i} = -\frac{y_k - y_i}{s_{ik}} = -\frac{\Delta y_{ik}}{s_{ik}} = -\sin t_{ik}, \quad \texttt{ds\_dy\_from}(yi,\ xi,\ yk,\ xk) \tag{5}$$



Notation information:

Index *i*: Station
Index *k*: Target

measured from ‚*i*' to ‚*k*'

Partial derivatives can be further simplified by using the equations for the grid bearing t_{ik} and distance s_{ik} .

Figure 1: Distance

3 Partial Derivatives for Directions

$$\alpha_{ikl} = \arctan\left(\frac{y_k - y_i}{x_k - x_i}\right) - \omega \tag{6}$$

w Direction Unknown

$$\frac{\partial r_{ik}}{\partial w_i} = -1 \tag{7}$$

x_k Unknown

$$\frac{\partial r_{ik}}{\partial x_k} = -\frac{y_k - y_i}{s_{ik}^2} = -\frac{\Delta y_{ik}}{s_{ik}^2} = -\frac{\sin t_{ik}}{s_{ik}}, \quad \texttt{dr\_dx\_to}(yi,\ xi,\ yk,\ xk) \tag{8}$$

y_k Unknown

$$\frac{\partial \alpha_{ikl}}{\partial y_k} = \frac{x_k - x_i}{s_{ik}^2} = \frac{\Delta x_{ik}}{s_{ik}^2} = \frac{\cos t_{ik}}{s_{ik}}, \quad \texttt{dr\_dy\_to}(yi,\ xi,\ yk,\ xk) \tag{9}$$

x_i Unknown

$$\frac{\partial r_{ik}}{\partial x_i} = \frac{\Delta y_{ik}}{s_{ik}^2}, \quad \texttt{dr\_dx\_from}(yi,\ xi,\ yk,\ xk) \tag{10}$$

y_i Unknown

$$\frac{\partial r_{ik}}{\partial y_i} = -\frac{\Delta x_{ik}}{s_{ik}^2}, \quad \texttt{dr\_dy\_from}(yi,\ xi,\ yk,\ xk) \tag{11}$$

4 Partial Derivatives for Angles

$$\alpha_{ikl} = \arctan\left(\frac{y(to) - y(from)}{x(to) - x(from)}\right) - \arctan\left(\frac{y(to) - y(from)}{x(to) - x(from)}\right) \tag{12}$$

$$\alpha_{ikl} = \arctan\left(\frac{y_k - y_i}{x_k - x_i}\right) - \arctan\left(\frac{y_l - y_i}{x_l - x_i}\right) \tag{13}$$

x_k Unknown

$$\frac{\partial \alpha_{ikl}}{\partial x_k} = -\frac{y_k - y_i}{s_{ik}^2} = -\frac{\Delta y_{ik}}{s_{ik}^2} = -\frac{\sin t_{ik}}{s_{ik}}, \quad \texttt{dr\_dx\_to}(yi,\ xi,\ yk,\ xk) \tag{14}$$

y_k Unknown

$$\frac{\partial \alpha_{ikl}}{\partial y_k} = \frac{x_k - x_i}{s_{ik}^2} = \frac{\Delta x_{ik}}{s_{ik}^2} = \frac{\cos t_{ik}}{s_{ik}}, \quad \texttt{dr\_dy\_to}(yi,\ xi,\ yk,\ xk) \tag{15}$$

x_l Unknown

$$\frac{\partial \alpha_{ikl}}{\partial x_l} = \frac{y_l - y_i}{s_{il}^2} = \frac{\Delta y_{il}}{s_{il}^2} = \frac{\sin t_{il}}{s_{il}}, \quad \texttt{dr\_dx\_from}(yi,\ xi,\ yk,\ xk) \tag{16}$$

y_l Unknown

$$\frac{\partial \alpha_{ikl}}{\partial y_l} = -\frac{x_l - x_i}{s_{il}^2} = -\frac{\Delta x_{il}}{s_{il}^2} = -\frac{\cos t_{il}}{s_{il}}, \quad \texttt{dr\_dy\_from}(yi,\ xi,\ yk,\ xk) \tag{17}$$

x_i Unknown

$$\frac{\partial \alpha_{ikl}}{\partial x_i} = \frac{\Delta y_{ik}}{s_{ik}^2} - \frac{\Delta y_{il}}{s_{il}^2} = \frac{\sin t_{ik}}{s_{ik}} - \frac{\sin t_{il}}{s_{il}}, \quad \texttt{dr\_dx\_from}(yi,\ xi,\ yk,\ xk) + \texttt{dr\_dx\_to}(yi,\ xi,\ yl,\ xl) \tag{18}$$

y_i Unknown

$$\frac{\partial \alpha_{ikl}}{\partial y_i} = -\frac{\Delta x_{ik}}{s_{ik}^2} + \frac{\Delta x_{il}}{s_{il}^2} = -\frac{\cos t_{ik}}{s_{ik}} + \frac{\cos t_{il}}{s_{il}}, \quad \texttt{dr\_dy\_from}(yi,\ xi,\ yk,\ xk) + \texttt{dr\_dy\_to}(yi,\ xi,\ yl,\ xl) \tag{19}$$

$$\alpha_{ikl} = \arctan\left(\frac{y_k - y_i}{x_k - x_i}\right) - \arctan\left(\frac{y_l - y_i}{x_l - x_i}\right) \tag{20}$$

Computation of Kartesian koordinates

$$\frac{\partial Y_P}{\partial d} = \sin t\ ; \quad \frac{\partial Y_P}{\partial t} = d \cdot \cos t$$

$$\begin{bmatrix} Y_P \\ X_P \end{bmatrix} = \begin{bmatrix} Y_S + d \cdot \sin t \\ X_S + d \cdot \cos t \end{bmatrix}$$

$$\frac{\partial X_P}{\partial d} = \cos t; \quad \frac{\partial X_P}{\partial t} = -d \cdot \sin t$$

Design matrix $\mathbf{F} = \mathbf{J}$:

$$\mathbf{J} = \begin{array}{cc} & \begin{array}{cc} d & t \end{array} \\ \begin{array}{c} Y_P \\ X_P \end{array} & \begin{bmatrix} \dfrac{\partial Y_P}{\partial d} & \dfrac{\partial Y_P}{\partial t} \\ \dfrac{\partial X_P}{\partial d} & \dfrac{\partial X_P}{\partial t} \end{bmatrix} \end{array} = \begin{bmatrix} \sin t & d \cdot \cos t \\ \cos t & -d \cdot \sin t \end{bmatrix}$$