Differentiation and Partial Differentiation Cheat Sheet

1. Basic Differentiation Rules

(a) Common Derivatives

- Power Rule: $d/dx\ (x^n) = n\ x^{(n-1)}$
- Constant Rule: $d/dx\ (C) = 0$
- Constant Multiple Rule: $d/dx\ [C\ f(x)] = C\ d/dx\ f(x)$
- Sum/Difference Rule: $d/dx\ [f(x) \pm g(x)] = f'(x) \pm g'(x)$

(b) Product and Quotient Rules

- Product Rule: $d/dx\ [f(x)\ g(x)] = f'(x)\ g(x) + f(x)\ g'(x)$
- Quotient Rule: $d/dx\ [f(x)/g(x)] = (f'(x)\ g(x) - f(x)\ g'(x))\ /\ g^2(x)$

(c) Chain Rule

- $d/dx\ f(g(x)) = f'(g(x)) * g'(x)$

(d) Common Function Derivatives

- $d/dx\ (e^x) = e^x$
- $d/dx\ (a^x) = a^x \ln a$
- $d/dx\ (\ln x) = 1/x$
- $d/dx\ (\sin x) = \cos x$
- $d/dx\ (\cos x) = -\sin x$
- $d/dx\ (\tan x) = \sec^2 x$

2. Higher-Order Derivatives

- Second Derivative: $f''(x) = d^2/dx^2\ f(x)$
- nth Derivative: $f^{(n)}(x) = d^n/dx^n\ f(x)$

3. Partial Differentiation

(a) Partial Derivatives

- d/dx f(x,y) = Differentiate with respect to x, treating y as constant.
- d/dy f(x,y) = Differentiate with respect to y, treating x as constant.

(b) Second-Order Partial Derivatives
- d^2 f/dx^2 = Second partial derivative with respect to x.
- d^2 f/dy^2 = Second partial derivative with respect to y.
- d^2 f/dxdy = d^2 f/dydx (Clairaut's Theorem if continuous)

(c) Gradient, Divergence, and Curl
- Gradient: Gradf = (df/dx, df/dy, df/dz)
- Divergence: Grad . F = dF1/dx + dF2/dy + dF3/dz
- Curl: Grad x F

4. Applications of Differentiation
- Tangent Line Equation: y - y0 = f'(x0)(x - x0)
- Optimization (Maxima & Minima):
  - f'(x) = 0 for critical points
  - f''(x) > 0 at a local minimum
  - f''(x) < 0 at a local maximum