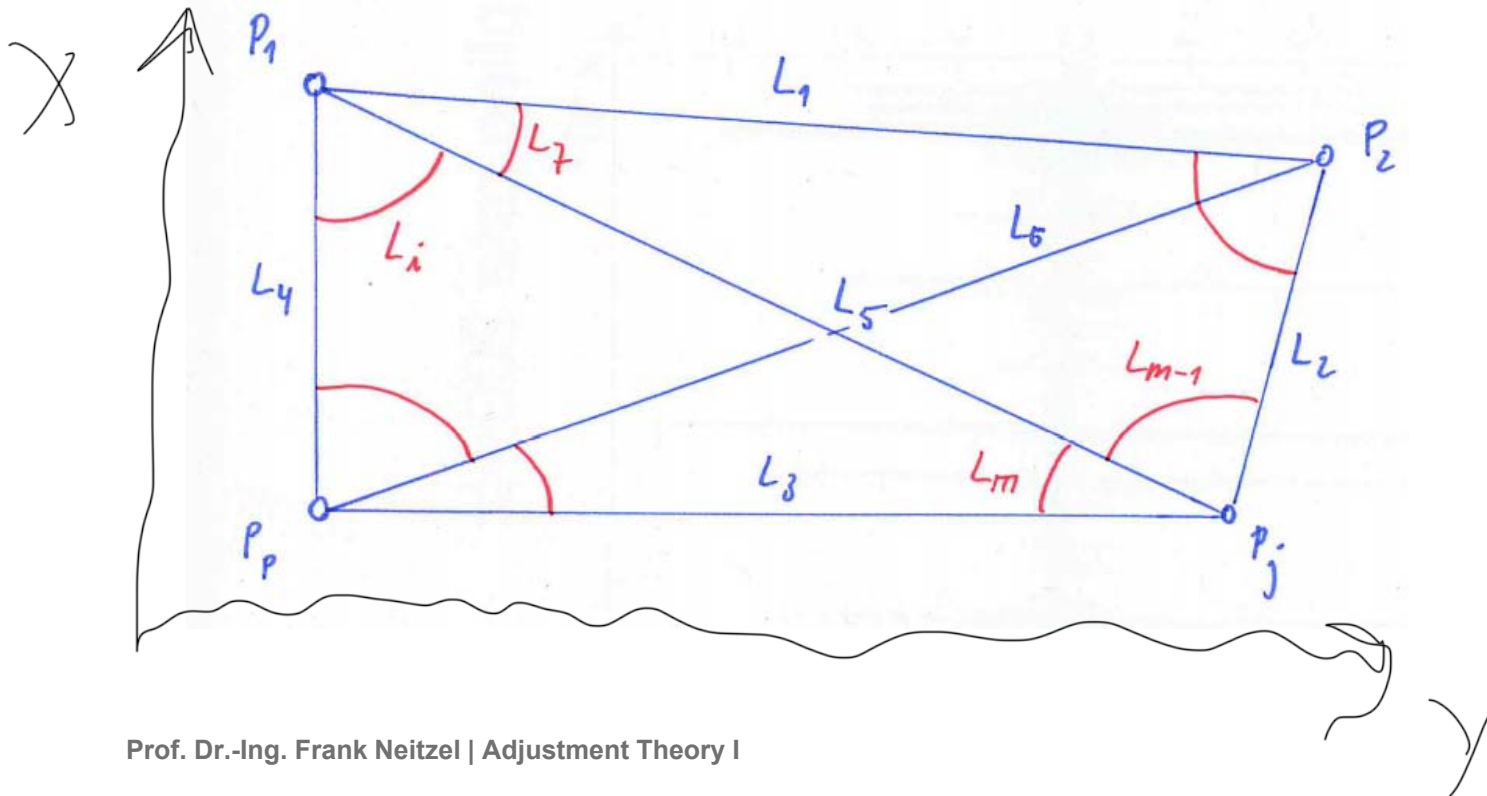


3.2 The m -dimensional random vector

3.2.1 Theoretical expectation and theoretical covariance matrix

Example: Measurement of directions and distances in a geodetic network



3.2.1 Theoretical expectation and theoretical covariance matrix

► Given: m random variables L_1, L_2, \dots, L_m

► Random vector

$$\mathbf{L}_{m \times 1} = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_m \end{bmatrix}$$

► Vector of expectations

$$\boldsymbol{\mu}_{m \times 1} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix} = E\{\mathbf{L}_{m \times 1}\} = \begin{bmatrix} E(L_1) \\ E(L_2) \\ \vdots \\ E(L_m) \end{bmatrix}$$

3.2.1 Theoretical expectation and theoretical covariance matrix

► Vector of random deviations

$$\boldsymbol{\varepsilon}_{m \times 1} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix} = \mathbf{L}_{m \times 1} - \boldsymbol{\mu}_{m \times 1} = \begin{bmatrix} L_1 - \mu_1 \\ L_2 - \mu_2 \\ \vdots \\ L_m - \mu_m \end{bmatrix}$$

Expectation

► as in 2.4.1

Expectation of a expectation = let self

$$E(\boldsymbol{\varepsilon}_{m \times 1}) = E\{\mathbf{L}_{m \times 1} - \boldsymbol{\mu}_{L_{m \times 1}}\} = \underbrace{E(\mathbf{L}_{m \times 1})}_{\boldsymbol{\mu}_L} - \boldsymbol{\mu}_{L_{m \times 1}} = \mathbf{0}$$

$$E\{\boldsymbol{\varepsilon}_{L_{m \times 1}} \cdot \boldsymbol{\varepsilon}_{L_{1 \times m}}^T\} = \begin{bmatrix} E(\varepsilon_1^2) & E(\varepsilon_1 \cdot \varepsilon_2) & \cdots & E(\varepsilon_1 \cdot \varepsilon_m) \\ E(\varepsilon_2 \cdot \varepsilon_1) & E(\varepsilon_2^2) & \cdots & E(\varepsilon_2 \cdot \varepsilon_m) \\ \vdots & \vdots & \ddots & \vdots \\ E(\varepsilon_m \cdot \varepsilon_1) & E(\varepsilon_m \cdot \varepsilon_2) & \cdots & E(\varepsilon_m \cdot \varepsilon_m) \end{bmatrix} = \boldsymbol{\Sigma}_{LL_{m \times m}}$$

Covariance

Variance

3.2.1 Theoretical expectation and theoretical covariance matrix

► Theoretical VCM of L

$$\Sigma_{LL_{m \times m}} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_m^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \rho_{12} \cdot \sigma_1 \cdot \sigma_2 & \cdots & \sigma_{1m} \\ \rho_{21} \cdot \sigma_2 \cdot \sigma_1 & \sigma_2^2 & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{m1} \cdot \sigma_m \cdot \sigma_1 & \rho_{m2} \cdot \sigma_m \cdot \sigma_2 & \cdots & \sigma_m^2 \end{bmatrix}$$

► Theoretical Variance of L_i

ε_i

$\varepsilon_i \longrightarrow E(\varepsilon_i^2) = \sigma_i^2$

3.2.2 Empirical expectation and empirical covariance matrix

► Given: m random variables $\mathbf{L}_{m \times 1} = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_m \end{bmatrix}$

► $m \times n$ -dimensional observation matrix

$$\mathbf{l}_{m \times n} = \begin{bmatrix} l_{11} & l_{21} & \cdots & l_{m1} \\ l_{12} & l_{22} & \cdots & l_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ l_{1n} & l_{2n} & \cdots & l_{mn} \end{bmatrix}$$

m different random variables were measured n times

► m -dimensional vector of mean values respective expectations

$$\bar{\mathbf{l}} = \begin{bmatrix} \bar{l}_1 \\ \bar{l}_2 \\ \vdots \\ \bar{l}_m \end{bmatrix} \quad \text{resp.} \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix}$$

3.2.2 Empirical expectation and empirical covariance matrix

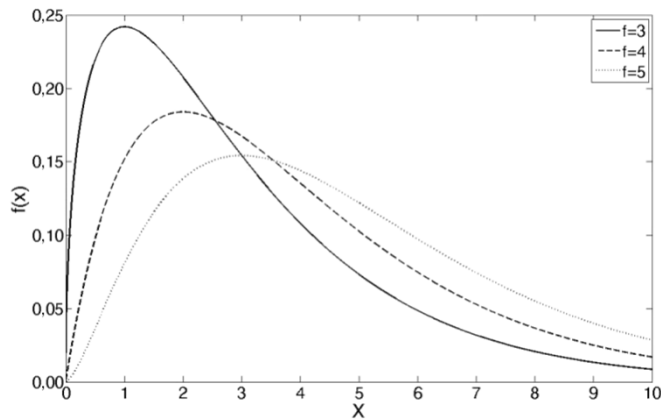
► m -dimensional VCM

$$\mathbf{S}_{ll_{m \times m}} = \begin{bmatrix} s_1^2 & s_{12} & \cdots & s_{1m} \\ s_{21} & s_2^2 & \cdots & s_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_m^2 \end{bmatrix}$$

► Correlation coefficient for the m -dimensional case

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \cdot \sigma_j} \quad \text{respective} \quad r_{ij} = \frac{s_{ij}}{s_i \cdot s_j} \quad \text{for } i, j = 1, 2, \dots, m \text{ and } i \neq j$$

→ All computations (e.g. residuals, empirical variances and covariances and correlation coefficients as in 2-dimensional case)



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Adjustment Theory I

Chapter 3 - The Random Vector