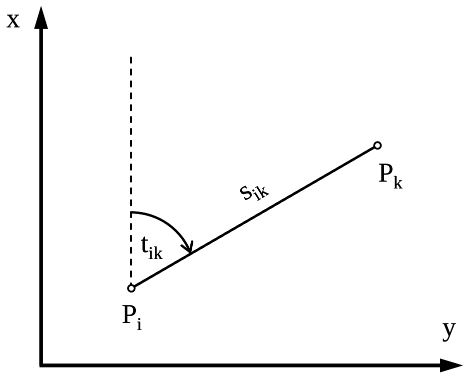
Partial derivatives of geodetic observation equations

Given: Observation Equations
Searched: Partial derivatives with respect to the unknowns for the linearization of the observation equations

1. Distances

$$s_{ik} = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2}$$



Notation information:

Index i: Station
Index k: Target

measured from ‚i' to ‚k'

Partial derivatives can be further simplified by using the equations for the grid bearing t_{ik} and distance s_{ik} .

Partial Derivatives:

$$\frac{\partial s_{ik}}{\partial x_k} = \frac{x_k - x_i}{s_{ik}} = \frac{\Delta x_{ik}}{s_{ik}} = \cos t_{ik} \qquad \frac{\partial s_{ik}}{\partial x_i} = -\frac{x_k - x_i}{s_{ik}} = \frac{-\Delta x}{s_{ik}} = -\cos t_{ik}$$

$$\frac{\partial s_{ik}}{\partial y_k} = \frac{y_k - y_i}{s_{ik}} = \frac{\Delta y_{ik}}{s_{ik}} = \sin t_{ik} \qquad \frac{\partial s_{ik}}{\partial y_i} = -\frac{y_k - y_i}{s_{ik}} = -\frac{\Delta y_{ik}}{s_{ik}} = -\sin t_{ik}$$

2. Directions

$$r_{ik} = \arctan\left(\frac{y_k - y_i}{x_k - x_i}\right) - \omega_i$$

Partial Derivatives:

$$\frac{\partial r_{ik}}{\partial y_k} = \frac{x_k - x_i}{s_{ik}^{\ 2}} = \frac{\Delta x_{ik}}{s_{ik}^{\ 2}} = \frac{\cos t_{ik}}{s_{ik}} \qquad \frac{\partial r_{ik}}{\partial y_i} = -\frac{x_k - x_i}{s_{ik}^{\ 2}} = -\frac{\Delta x_{ik}}{s_{ik}^{\ 2}} = -\frac{\cos t_{ik}}{s_{ik}}$$

$$\frac{\partial r_{ik}}{\partial x_k} = -\frac{y_k - y_i}{s_{ik}^{\ 2}} = -\frac{\Delta y_{ik}}{s_{ik}^{\ 2}} = -\frac{\sin t_{ik}}{s_{ik}} \qquad \frac{\partial r_{ik}}{\partial x_i} = \frac{y_k - y_i}{s_{ik}^{\ 2}} = \frac{\Delta y_{ik}}{s_{ik}^{\ 2}} = \frac{\sin t_{ik}}{s_{ik}} \qquad \frac{\partial r_{ik}}{\partial \omega_i} = -1$$

3. Angles

$$\alpha_{ikl} = \arctan\left(\frac{y_k - y_i}{x_k - x_i}\right) - \arctan\left(\frac{y_l - y_i}{x_l - x_i}\right)$$

Partial Derivatives:

$$\frac{\partial \alpha_{ikl}}{\partial y_k} = \frac{x_k - x_i}{s_{ik}^{\ 2}} = \frac{\Delta x_{ik}}{s_{ik}^{\ 2}} = \frac{\cos t_{ik}}{s_{ik}} \qquad \frac{\partial \alpha_{ikl}}{\partial y_l} = -\frac{x_l - x_i}{s_{il}^{\ 2}} = \frac{-\Delta x_{il}}{s_{il}^{\ 2}} = -\frac{\cos t_{il}}{s_{il}}$$

$$\frac{\partial \alpha_{ikl}}{\partial x_k} = -\frac{y_k - y_i}{s_{ik}^{\ 2}} = -\frac{\Delta y_{ik}}{s_{ik}^{\ 2}} = -\frac{\sin t_{ik}}{s_{ik}} \qquad \frac{\partial \alpha_{ikl}}{\partial x_l} = \frac{y_l - y_i}{s_{il}^{\ 2}} = \frac{\Delta y_{il}}{s_{il}^{\ 2}} = \frac{\sin t_{il}}{s_{il}}$$

$$\frac{\partial \alpha_{ikl}}{\partial y_i} = -\frac{\Delta x_{ik}}{s_{ik}^{\ 2}} + \frac{\Delta x_{il}}{s_{il}^{\ 2}} = -\frac{\cos t_{ik}}{s_{ik}} + \frac{\cos t_{il}}{s_{il}} \qquad \frac{\partial \alpha_{ikl}}{\partial x_i} = \frac{\Delta y_{ik}}{s_{ik}^{\ 2}} - \frac{\Delta y_{il}}{s_{il}^{\ 2}} = \frac{\sin t_{ik}}{s_{ik}} - \frac{\sin t_{il}}{s_{il}}$$