7.3 Adjustment of Horizontal Surveys: Triangulation

Partial derivatives:

we know $(\arctan x)' = \frac{1}{1+x^2}$

$$\frac{\partial r_{ik}}{\partial y_k} = \frac{\partial\left(\arctan\frac{y_k-y_i}{x_k-x_i} - \omega_i\right)}{\partial y_k} = \frac{1}{1+\left(\frac{y_k-y_i}{x_k-x_i}\right)^2} \cdot \frac{1}{x_k-x_i}$$

$$= \frac{1}{1+\frac{(y_k-y_i)^2}{(x_k-x_i)^2}} \cdot \frac{1}{x_k-x_i} = \frac{1}{\frac{(x_k-x_i)^2}{(x_k-x_i)^2}+\frac{(y_k-y_i)^2}{(x_k-x_i)^2}} \cdot \frac{1}{x_k-x_i}$$

$$\frac{1}{\frac{x}{y}} = \frac{y}{x}$$

$$= \frac{(x_k-x_i)^2}{\underbrace{(x_k-x_i)^2+(y_k-y_i)^2}_{\text{distance squared}}} \cdot \frac{1}{x_k-x_i} = \frac{x_k-x_i}{s_{ik}^2} = \frac{\Delta x_{ik}}{s_{ik}^2}$$

7.3 Adjustment of Horizontal Surveys: Triangulation

$$\left.\frac{\partial r_{ik}}{\partial y_i}, \frac{\partial r_{ik}}{\partial x_k}, \frac{\partial r_{ik}}{\partial x_i}\right\}$$ See handout! $$\frac{\partial r_{ik}}{\partial \omega_i} = -1$$

2. Directions $$r_{ik} = \arctan\left(\frac{y_k - y_i}{x_k - x_i}\right) - \omega_i$$

Partial Derivatives:

$$\frac{\partial r_{ik}}{\partial y_k} = \frac{x_k - x_i}{s_{ik}^{2}} = \frac{\Delta x_{ik}}{s_{ik}^{2}} = \frac{\cos t_{ik}}{s_{ik}} \qquad \frac{\partial r_{ik}}{\partial y_i} = -\frac{x_k - x_i}{s_{ik}^{2}} = -\frac{\Delta x_{ik}}{s_{ik}^{2}} = -\frac{\cos t_{ik}}{s_{ik}}$$

$$\frac{\partial r_{ik}}{\partial x_k} = -\frac{y_k - y_i}{s_{ik}^{2}} = -\frac{\Delta y_{ik}}{s_{ik}^{2}} = -\frac{\sin t_{ik}}{s_{ik}} \qquad \frac{\partial r_{ik}}{\partial x_i} = \frac{y_k - y_i}{s_{ik}^{2}} = \frac{\Delta y_{ik}}{s_{ik}^{2}} = \frac{\sin t_{ik}}{s_{ik}} \qquad \frac{\partial r_{ik}}{\partial \omega_i} = -1$$

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Weights in triangulation networks

- Precision for horizontal directions given in

$$\sigma_{r_i} = \sqrt{b_0^2 + \left(\frac{b_1}{d_i} \cdot \rho\right)^2}$$

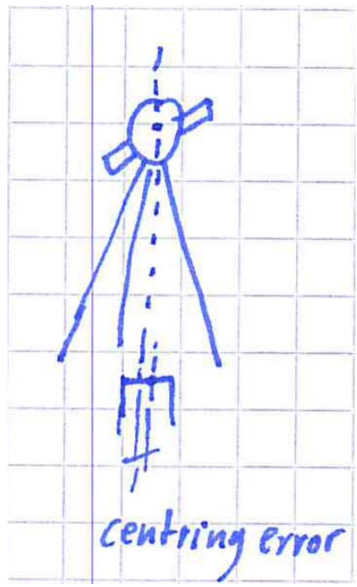
angular

$\rho = \frac{200}{\pi}$ if computation in gon = [illegible]

b_0: constant part of precision
→ usually given by manufacturer or from experience

b_1: centring error → often chosen as 1 mm

d_i: length of sight distance



$\frac{b_i}{2 d_i \pi} = \frac{\alpha}{400} \quad | \cdot 2\pi$

$\frac{b_i}{d_i} = \frac{\alpha \cdot \cancel{2}\pi}{\cancel{400}\,200} \quad | \rho = \frac{200}{\pi}$

$\frac{b_i}{d_i} = \frac{\alpha}{\rho} \Rightarrow \boxed{\frac{b_i}{d_i} \cdot \rho = \alpha}$

radian gon

the shorter the dist the larger the error

7.3 Adjustment of Horizontal Surveys: Triangulation

- Typical values for standard deviation σ_{r_i}:

  with $b_0 = 0.3 \text{ mgon}, b_1 = 1 \text{ mm}$

and $d_i = 100 \text{ m}$	→	$\sigma_{r_i} = 0.70 \text{ mgon}$
50 m	→	$\sigma_{r_i} = 1.31 \text{ mgon}$
10 m	→	$\sigma_{r_i} = 6.37 \text{ mgon}$
3 m	→	$\sigma_{r_i} = 21.22 \text{ mgon}$

- Variance matrix of the observations

$$\boldsymbol{\Sigma}_{LL} = \begin{bmatrix} \sigma_{r_1}^2 & & & 0 \\ & \sigma_{r_2}^2 & & \\ & & \ddots & \\ 0 & & & \sigma_{r_n}^2 \end{bmatrix}$$

- With reference variance σ_0^2
- Cofactor matrix of observations: $\mathbf{Q}_{LL} = \frac{1}{\sigma_0^2} \boldsymbol{\Sigma}_{LL}$
- Weight matrix of observations: $\mathbf{P} = \mathbf{Q}_{LL}^{-1}$

Example

The observed directions of the triangulation network depicted in Figure 1 are listed in Table 2. The points 1, 2, 4, 5 and 6 are control points (error free) and their 2D coordinates are given in Table 1. Calculate the adjusted coordinates of point 3 using least squares adjustment.

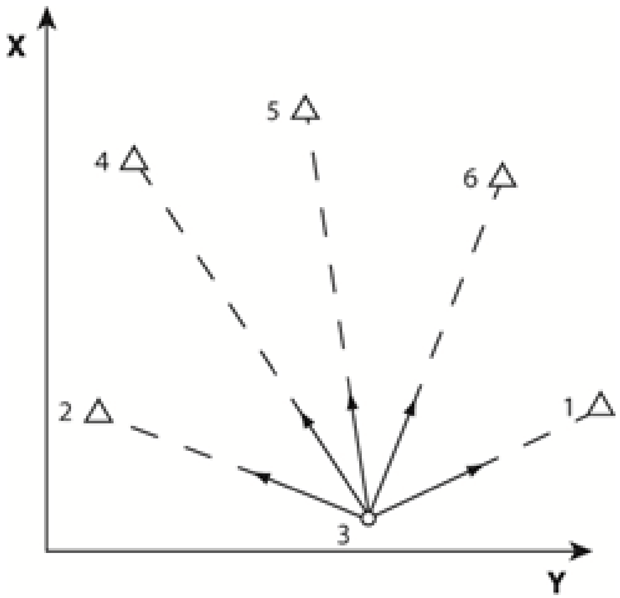


Figure 1: Triangulation network

Table 1: 2D coordinates of control points

Point No.	y [m]	x [m]
1	682.415	321.052
2	203.526	310.527
4	251.992	506.222
5	420.028	522.646
6	594.553	501.494

Approximate values for the coordinates of point 3:

y_3^0 : 493.7 ; x_3^0: 242.9

(graphical coordinates from a map)

7.3 Adjustment of Horizontal Surveys: Triangulation

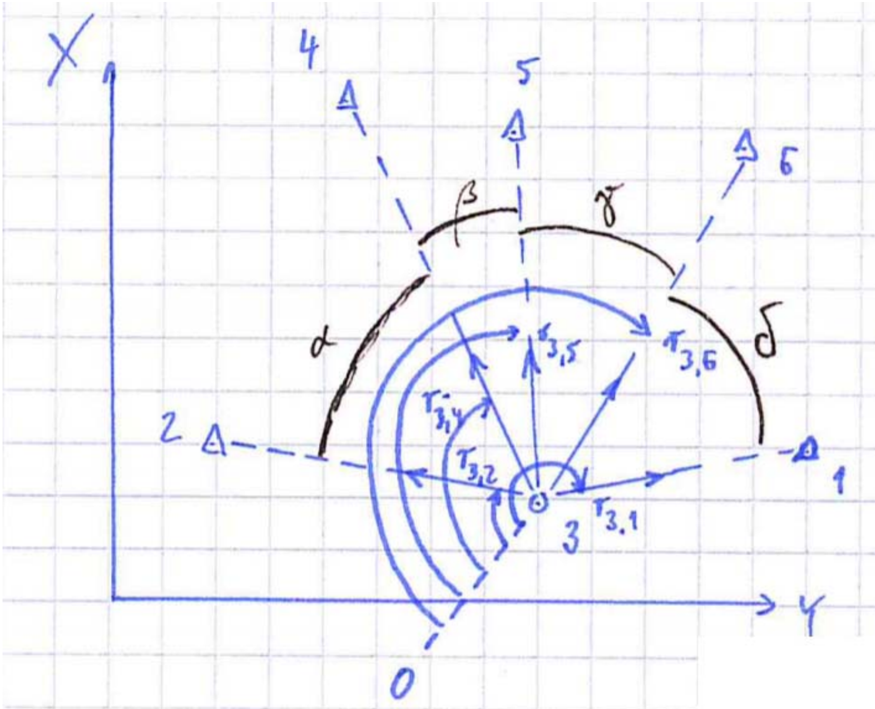
Stay
at home
pass exam

Table 2: Observed directions

Instrument station	Foresight station	Direction [gon]
3	1	206.9094
	2	46.5027
	4	84.6449
	5	115.5251
	6	155.5891

- The observed directions are uncorr. and have been obtained with a precision of 1 mgon
- Set up an appropriate functional model as well as the observation equations
- Set up the stochastic model
- Choose appropriate values for the break-off conditions ε and δ and justify your decision
- Solve the normal equation system and determine the 2D coordinates of point 3 as well as their standard deviations
- Calculate the residuals and the adjusted observations as well as their standard deviations

7.3 Adjustment of Horizontal Surveys: Triangulation



Instrument station	Foresight station	Direction [gon]
3	1	206.9094
	2	46.5027
	4	84.6449
	5	115.5251
	6	155.5891

7.3 Adjustment of Horizontal Surveys: Triangulation

General considerations:

- What are our unknowns?

  → Coordinates for point 3, orientation

  → We introduce y_3, x_3, ω_3

- What are our observations?

  → Directions

  → $r_{31}, r_{32}, r_{34}, r_{35}, r_{36}$

  Resection?

- Observations reduced into projection?

  → not necessary

- What are our fixed values?

  → $y_1, x_1; y_2, x_2; y_4, x_4; y_5, x_5; y_6, x_6$

- Redundancy?

  → $r = n - u$ → $r = 5 - 3$ → $r = 2$

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Functional model:

$$r_{31} = \arctan\frac{y_1 - y_3}{x_1 - x_3} - \omega_3$$

$$r_{32} = \arctan\frac{y_2 - y_3}{x_2 - x_3} - \omega_3$$

$$r_{34} = \arctan\frac{y_4 - y_3}{x_4 - x_3} - \omega_3$$

$$r_{35} = \arctan\frac{y_5 - y_3}{x_5 - x_3} - \omega_3$$

$$r_{36} = \arctan\frac{y_6 - y_3}{x_6 - x_3} - \omega_3$$

observation

unknown

fixed values

7.3 Adjustment of Horizontal Surveys: Triangulation

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Observation equations:

v = residuals

$$r_{31} + v_{r_{31}} = \arctan \ldots$$

$$r_{32} + v_{r_{32}} = \arctan \ldots$$

$$\vdots$$

$$r_{36} + v_{r_{36}} = \arctan \ldots$$

Please note: Perform computation in [rad]!

gon

Observation vector:

$$\mathbf{L} = \begin{bmatrix} 206.9094 \\ 46.5027 \\ 84.6449 \\ 115.5251 \\ 155.5891 \end{bmatrix} \cdot \frac{1}{\rho}$$

= conversion into radians

with $\rho = \frac{200 \text{ gon}}{\pi}$

Stochastic model of the observations:

$$\sigma_i = 1\ \mathrm{mgon} = 0.001\ \mathrm{gon} = \frac{0.001}{\rho}\,\mathrm{rad}$$

for $i = 1, \dots, n$

$$\rightarrow \boldsymbol{\Sigma}_{LL} = \begin{bmatrix} \left(\frac{0.001}{\rho}\right)^2 & & & \\ & \left(\frac{0.001}{\rho}\right)^2 & & \\ & & \ddots & \\ & & & \left(\frac{0.001}{\rho}\right)^2 \end{bmatrix}$$

with $\sigma_0 = 1\ \mathrm{mgon} = \frac{0.001}{\rho}\ \mathrm{rad}$

$$\mathbf{Q}_{LL} = \frac{1}{\sigma_0^2} \cdot \boldsymbol{\Sigma}_{LL} \;\rightarrow\; \mathbf{Q}_{LL} = \mathbf{I} \;\rightarrow\; \mathbf{P} = \mathbf{I}$$

7.3 Adjustment of Horizontal Surveys: Triangulation

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Vector of adjusted unknowns:

$$\hat{\mathbf{X}} = \begin{bmatrix} \hat{x}_3 \\ \hat{y}_3 \\ \hat{\omega}_3 \end{bmatrix}$$

Nonlinear functional model

→ Solution from iterative computation with linearised functional model

→ Introduction of appropriate values x_3^0, y_3^0, ω_3^0

→ ω_3^0 is a linear term, we can choose arbitrary values, e.g. $\omega_3^0 = 0$

why?

Vector of starting values:

$$\mathbf{X}^0 = \begin{bmatrix} x_3^0 \\ y_3^0 \\ \omega_3^0 \end{bmatrix}$$

Vector of adjusted reduced unknowns:

$$\hat{\mathbf{x}} = \hat{\mathbf{X}} - \mathbf{X}^0 = \begin{bmatrix} d\hat{x}_3 \\ d\hat{y}_3 \\ d\hat{\omega}_3 \end{bmatrix} = \begin{bmatrix} \hat{x}_3 - x_3^0 \\ \hat{y}_3 - y_3^0 \\ \hat{\omega}_3 - \omega_3^0 \end{bmatrix}$$

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Vector of reduced observations:

$l = L - \Phi(x^0)$

$$\mathbf{l} = \underbrace{\begin{bmatrix} 206.9094 \cdot \frac{1}{\rho} \\ 46.5027 \cdot \frac{1}{\rho} \\ 84.6449 \cdot \frac{1}{\rho} \\ 115.5251 \cdot \frac{1}{\rho} \\ 155.5891 \cdot \frac{1}{\rho} \end{bmatrix}}_{L} - \underbrace{\begin{bmatrix} \left(\arctan\frac{y_1-y_3^0}{x_1-x_3^0} - \omega_3^0\right) \\ \left(\arctan\frac{y_2-y_3^0}{x_2-x_3^0} - \omega_3^0\right) \\ \left(\arctan\frac{y_4-y_3^0}{x_4-x_3^0} - \omega_3^0\right) \\ \left(\arctan\frac{y_5-y_3^0}{x_5-x_3^0} - \omega_3^0\right) \\ \left(\arctan\frac{y_6-y_3^0}{x_6-x_3^0} - \omega_3^0\right) \end{bmatrix}}_{\Phi(x^0)}$$

Quadrants!

7.3 Adjustment of Horizontal Surveys: Triangulation

Jacobian matrix:

$$
\mathbf{J} = \begin{array}{c} \\ r_{31} \\ r_{32} \\ r_{34} \\ r_{35} \\ r_{36} \end{array}
\begin{array}{c}
\begin{array}{ccc} x_3^0 & y_3^0 & \omega_3^0 \end{array} \\
\begin{bmatrix}
\frac{\partial r_{31}}{\partial x_3^0} & \frac{\partial r_{31}}{\partial y_3^0} & \frac{\partial r_{31}}{\partial \omega_3^0} \\
\frac{\partial r_{32}}{\partial x_3^0} & \vdots & -1 \\
\vdots & \vdots & -1 \\
\vdots & \vdots & -1 \\
\frac{\partial r_{36}}{\partial x_3^0} & \frac{\partial r_{36}}{\partial y_3^0} & -1
\end{bmatrix}
\end{array}
$$

Partial derivatives → see handout!

Design matrix:

$$\mathbf{A} = \mathbf{J}$$

Normal equations:

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\hat{\mathbf{x}} = \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{l}$$

Solution of normal equations:

$$\hat{\mathbf{x}} = \underbrace{\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)}_{\mathbf{N}}^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{l}$$

Adjusted unknowns:

$$\hat{\mathbf{X}} = \mathbf{X}^{0} + \hat{\mathbf{x}}$$

Residuals:

$$\mathbf{v} = \mathbf{A}\hat{\mathbf{x}} - \mathbf{l}$$

Adjusted observations:

$$\hat{\mathbf{L}} = \mathbf{L} + \mathbf{v}$$

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Check:

$$\max |\hat{x}_i| < \varepsilon$$

and

$$\max \left|\hat{L}_i - \Phi_i(\widehat{\mathbf{X}})\right| < \delta$$

no

Use $\widehat{\mathbf{X}}$ as new $\mathbf{X}^0$

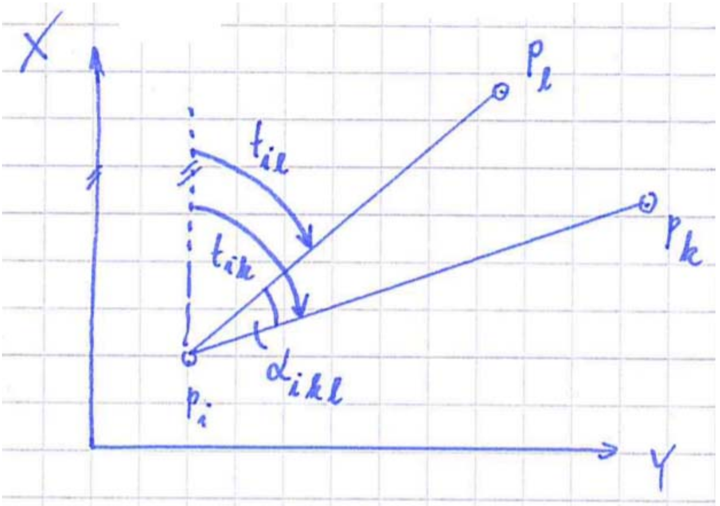
yes

$\boxed{\widehat{\mathbf{X}} \text{ is final result}}$

$s_0, \mathbf{\Sigma}_{\hat{X}\hat{X}}, \ldots$

Conversion of final results from [rad] into [gon]

Triangulation with angles as observations



Functional model:

$$\alpha_{ikl} = t_{ik} - t_{il}$$

$$\boxed{\alpha_{ikl} = \arctan\frac{y_k - y_i}{x_k - x_i} - \arctan\frac{y_l - y_i}{x_l - x_i}}$$

Partial derivatives → See handout!

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3. Angles

$$\alpha_{ikl} = \arctan\left(\frac{y_k - y_i}{x_k - x_i}\right) - \arctan\left(\frac{y_l - y_i}{x_l - x_i}\right)$$

Partial Derivatives:

$$\frac{\partial \alpha_{ikl}}{\partial y_k} = \frac{x_k - x_i}{s_{ik}^{\;2}} = \frac{\Delta x_{ik}}{s_{ik}^{\;2}} = \frac{\cos t_{ik}}{s_{ik}} \qquad \frac{\partial \alpha_{ikl}}{\partial y_l} = -\frac{x_l - x_i}{s_{il}^{\;2}} = \frac{-\Delta x_{il}}{s_{il}^{\;2}} = -\frac{\cos t_{il}}{s_{il}}$$

$$\frac{\partial \alpha_{ikl}}{\partial x_k} = -\frac{y_k - y_i}{s_{ik}^{\;2}} = -\frac{\Delta y_{ik}}{s_{ik}^{\;2}} = -\frac{\sin t_{ik}}{s_{ik}} \qquad \frac{\partial \alpha_{ikl}}{\partial x_l} = \frac{y_l - y_i}{s_{il}^{\;2}} = \frac{\Delta y_{il}}{s_{il}^{\;2}} = \frac{\sin t_{il}}{s_{il}}$$

$$\frac{\partial \alpha_{ikl}}{\partial y_i} = -\frac{\Delta x_{ik}}{s_{ik}^{\;2}} + \frac{\Delta x_{il}}{s_{il}^{\;2}} = -\frac{\cos t_{ik}}{s_{ik}} + \frac{\cos t_{il}}{s_{il}} \qquad \frac{\partial \alpha_{ikl}}{\partial x_i} = \frac{\Delta y_{ik}}{s_{ik}^{\;2}} - \frac{\Delta y_{il}}{s_{il}^{\;2}} = \frac{\sin t_{ik}}{s_{ik}} - \frac{\sin t_{il}}{s_{il}}$$

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Attention: We measure directions, not angles!

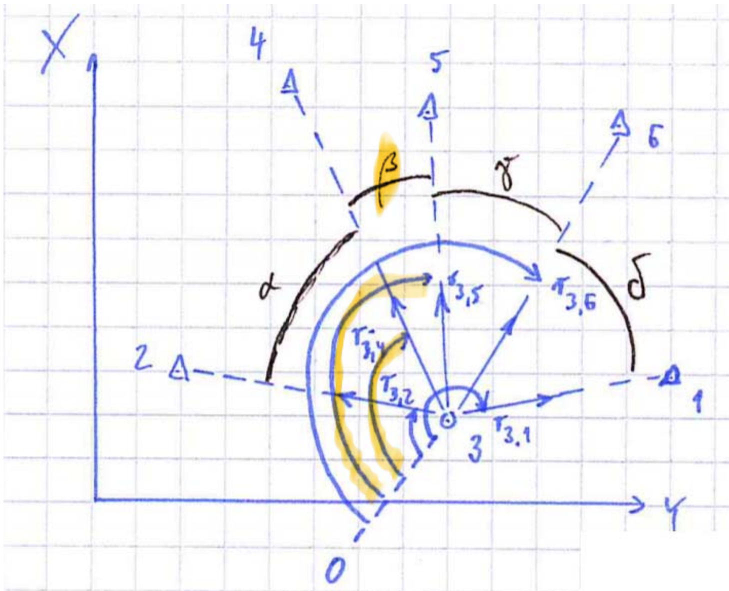
How can we compute angles?

→ Differences of directions!

In our example:

these are correlated

$$\left.\begin{aligned} \alpha &= r_{34} - r_{32} \\ \beta &= r_{35} - r_{34} \\ \gamma &= r_{36} - r_{35} \\ \delta &= r_{31} - r_{36} \end{aligned}\right\}$$ Reduction to interval $[0, \ldots, 400[$ gon



→ Derived observations!

→ VCM for these derived observations? From VC propagation!

Σ Matrix for α β γ δ

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Example

The Gauss-Krueger coordinates of the control points, which can be regarded as fixed (error free) values are listed in Table 1. The measurements of the combined horizontal network depicted in Figure 1 are listed in Table 2. Calculate the adjusted Gauss-Krueger coordinates of point 1 and 15 using least squares adjustment.

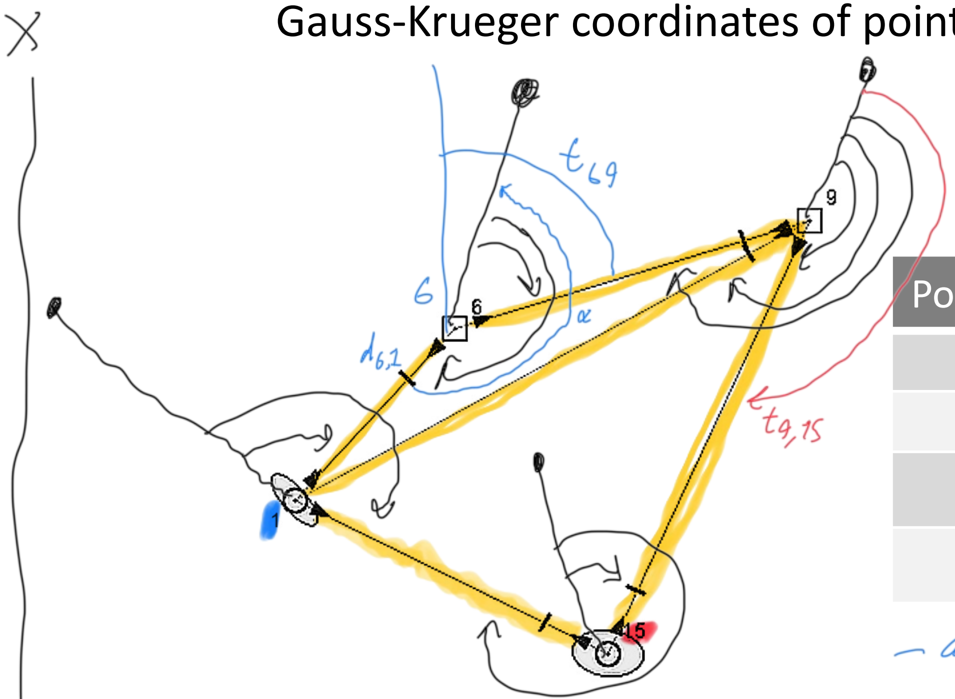


Figure 1: Combined horizontal network

– azimuth $t_{6,9}$ from given coord

– α from given measurements

Table 1: Gauss-Krueger coordinates for control points

Point No.	Easting [m]	Northing [m]	Remarks
6	53 17 651.428	49 68 940.373	Fixed point
9	53 24 162.853	49 70 922.160	Fixed point
1	to be computed, see blackboard		Initial values
15			Initial values

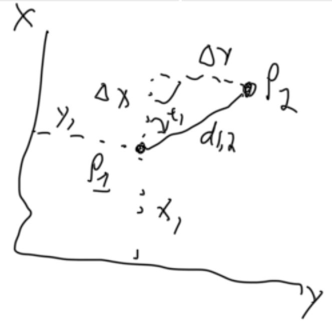
– azimuth $t_{6,1} = t_{6,9} + \alpha$

– $d_{6,1}$ from

– $y_1 = y_6 + d_{6,1} \cdot \sin t_{6,1}$

$x_1 = x_6 + d_{6,1} \cdot \cos t_{6,1}$

for point 1



$\Delta y = d_{1,2} \cdot \sin t_{1,2}$

$\Delta x = d_{1,2} \cdot \cos t_{1,2}$

$y_2 = y_1 + \Delta y \Rightarrow y_2 = y_1 + d_{1,2} \cdot \sin t_{1,2}$

$x_2 = x_2 + \Delta x \Rightarrow$

for point 15

7.4 Adjustment of Horizontal Surveys: Combined Network

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- azemith $t_{9,6}$ from coord
  or from $t_{6,9}+200$ gon
- β from measur
- azimuth $t_{9,15}=t_{9,6}-\beta$
- $d_{9,15}$ from measure
- $Y_{15}=Y_9+d_{9,15}\cdot\sin t_{9,15}$
  $X_{15}=X_9+d_{9,15}\cdot\cos t_{9,15}$

Table 2: Observed distances and directions

From	To	Horizontal directions [gon]	Horizontal distances [m]
1	6	148.0875	
	15	228.9044	
6	1	248.0883	4307.851
	9	81.1917	
9	15	207.9027	
	1	248.4428	10759.852
	6	261.1921	6806.332
15	1	358.9060	6399.069
	9	57.9014	8751.757

- The distances measurements have been performed with a precision of 10 cm and are already reduced into the Gauss-Krueger projection
- The observation of directions has been performed with a precision of 1 mgon
- All measurements (distances and directions) are uncorrelated

7.4 Adjustment of Horizontal Surveys: Combined Network

- Compute appropriate initial values for the coordinates of points 1 and 15
- Set up an appropriate functional model as well as the observation equations
- Set up the stochastic model
- Choose appropriate values for the break-off conditions ε and δ and justify your decision
- Solve the normal equation system and determine the Gauss-Krueger coordinates of point 1 and 15 as well as their standard deviations
- Calculate the residuals and the adjusted observations as well as their standard deviations

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General considerations:

- What are our unknowns?

→ Coordinates of points 1 and 15, orientation unknowns

→ We introduce $y_1, x_1;\ y_{15}, x_{15}; \omega_1, \omega_6, \omega_9, \omega_{15}$

- What are our observations?

→ Distances and directions

→ $s_{6,1}, s_{9,1}, s_{9,6}, s_{15,1}, s_{15,9}$

→ $r_{1,6}, r_{1,15}, r_{6,1}, r_{6,9}, r_{9,15}, r_{9,1}, r_{9,6}, r_{15,1}, r_{15,9}$

7.4 Adjustment of Horizontal Surveys: Combined Network

- Observations reduced into projection?

  → Distances: Yes!

  → Directions: Not necessary, conformal coordinates

- What are our fixed values?

  → $y_6, x_6; y_9, x_9$

- Redundancy?

  → $r = n - u$ → $r = 14 - 8$ → $r = 6$

Functional model:

$$s_{6,1} = \sqrt{(y_1 - y_6)^2 + (x_1 - x_6)^2}$$

$$\vdots$$

$$s_{15,9} = \sqrt{(y_9 - y_{15})^2 + (x_9 - x_{15})^2}$$

$$r_{1,6} = \arctan\frac{y_6 - y_1}{x_6 - x_1} - \omega_1$$

$$\vdots$$

$$r_{15,9} = \arctan\frac{y_9 - y_{15}}{x_9 - x_{15}} - \omega_{15}$$

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Observation equations:

$$s_{6,1} + v_{s_{6,1}} = \sqrt{}$$

$$\vdots$$

$$s_{15,9} + v_{s_{15,9}} = \sqrt{}$$

$$r_{1,6} + v_{r_{1,6}} = \arctan —$$

$$\vdots$$

$$r_{15,9} + v_{r_{15,9}} = \arctan —$$

Please note: Perform computations in [rad]!

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Observation vector:

$$\mathbf{L} = \begin{bmatrix} s_{6,1} \\ s_{9,1} \\ s_{9,6} \\ s_{15,1} \\ s_{15,9} \\ r_{1,6} \\ r_{1,15} \\ r_{6,1} \\ r_{6,9} \\ r_{9,15} \\ r_{9,1} \\ r_{9,6} \\ r_{15,1} \\ r_{15,9} \end{bmatrix}$$

Values from Table 2

$r_{1,6}, \ldots, r_{15,9}$: $\cdot \frac{1}{\rho}$ with $\rho = \frac{200\,\text{gon}}{\pi}$

Stochastic model of the observations:

- Distances: $\sigma_{s_i} = 10 \text{ cm} = 0.10 \text{ m}$
- Directions: $\sigma_{r_i} = 1 \text{ mgon} = 0.001 \text{ gon} = \frac{0.001}{\rho} \text{ rad}$

$$
\boldsymbol{\Sigma}_{LL} = \begin{array}{c|c}
 & \begin{array}{ccccccc} s_{6,1} & s_{9,1} & \cdots & s_{15,9} & r_{1,6} & \cdots & r_{15,9} \end{array} \\
\begin{array}{c} s_{6,1} \\ s_{9,1} \\ \vdots \\ s_{15,9} \\ \\ \vdots \\ \\ r_{15,9} \end{array} &
\left[\begin{array}{cccc|ccc}
0.10^2 & & & & & & \\
 & 0.10^2 & & & & 0 & \\
 & & \ddots & & & & \\
 & & & 0.10^2 & & & \\
\hline
 & & & & \left(\frac{0.001}{\rho}\right)^2 & & \\
 & 0 & & & & \ddots & \\
 & & & & & & \left(\frac{0.001}{\rho}\right)^2
\end{array}\right]
\end{array}
$$

- Attention:

  Sequence of variances must coincide with the sequence of the observations in vector $\mathbf{L}$

$$\mathbf{Q}_{LL} = \frac{1}{\sigma_0^2}\mathbf{\Sigma}_{LL}$$

with $\sigma_0^2 = 1$

$$\mathbf{Q}_{LL} = \mathbf{\Sigma}_{LL}$$

$$\mathbf{P} = \mathbf{Q}_{LL}^{-1}$$

Vector of adjusted unknowns:

$$\hat{\mathbf{X}} = [\hat{x}_1 \quad \hat{y}_1 \quad \hat{x}_{15} \quad \hat{y}_{15} \quad \hat{\omega}_1 \quad \hat{\omega}_6 \quad \hat{\omega}_9 \quad \hat{\omega}_{15}]^{\mathrm{T}}$$

→ Nonlinear functional model

→ Solution from iterative computing with linearised functional model

→ Introduction of approximate values $x_1^0, y_1^0; x_{15}^0, y_{15}^0; \omega_1^0, \omega_6^0, \omega_9^0, \omega_{15}^0$

→ $\omega_{1,\dots,15}$: linear terms

Vector of starting values:

$$\mathbf{X}^0 = [x_1^0 \quad y_1^0 \quad x_{15}^0 \quad y_{15}^0 \quad \omega_1^0 \quad \omega_6^0 \quad \omega_9^0 \quad \omega_{15}^0]^{\mathrm{T}}$$

Vector of adjusted reduced unknowns:

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 - x_1^0 \\ \hat{y}_1 - y_1^0 \\ \hat{x}_{15} - x_{15}^0 \\ \hat{y}_{15} - y_{15}^0 \\ \widehat{\omega}_1 - \omega_1^0 \\ \widehat{\omega}_6 - \omega_6^0 \\ \widehat{\omega}_9 - \omega_9^0 \\ \widehat{\omega}_{15} - \omega_{15}^0 \end{bmatrix}$$

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Vector of reduced observations:

$$\mathbf{l} = \begin{bmatrix} s_{6,1} - s_{6,1}^0 \\ \vdots \\ s_{15,9} - s_{15,9}^0 \\ r_{1,6} - r_{1,6}^0 \\ \vdots \\ r_{15,9} - r_{15,9}^0 \end{bmatrix} = \begin{bmatrix} 4307.851 - \sqrt{(x_1^0 - x_6)^2 + (y_1^0 - y_6)^2} \\ \vdots \\ 8751.757 - \sqrt{(x_9 - x_{15}^0)^2 + (y_9 - y_{15}^0)^2} \\ 148.0875 \cdot \frac{1}{\rho} - \left(\arctan \frac{y_6 - y_1^0}{x_6 - x_1^0} - \omega_1^0\right) \\ \vdots \\ 57.9014 \cdot \frac{1}{\rho} - \left(\arctan \frac{y_9 - y_{15}^0}{x_9 - x_{15}^0} - \omega_{15}^0\right) \end{bmatrix} \left.\vphantom{\begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix}}\right\} [\text{rad}]$$

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Jacobian matrix:

Attention: Sequence must coincide with sequence of unknowns in vector $\hat{\mathbf{x}}$

Attention: Sequence must coincide with sequence of observations in vector $\mathbf{L}$

$$
\mathbf{J} = \begin{array}{c} \\ s_{6,1} \\ s_{9,1} \\ \vdots \\ r_{1,6} \\ \vdots \\ r_{15,9} \end{array}
\begin{array}{c}
\begin{array}{cccccccc} x_1 & y_1 & & x_{15} & y_{15} & \omega_1 & \omega_6 & \omega_9 \quad \omega_{15} \end{array} \\
\begin{bmatrix}
\frac{\partial s_{6,1}}{\partial x_1} & \frac{\partial s_{6,1}}{\partial y_1} & \cdots & & \\
\frac{\partial s_{9,1}}{\partial x_1} & \frac{\partial s_{9,1}}{\partial y_1} & \cdots & & \\
 & & & & \\
\frac{\partial r_{1,6}}{\partial x_1} & \frac{\partial r_{1,6}}{\partial y_1} & & \ddots & \\
\vdots & \vdots & & & \\
\frac{\partial r_{15,9}}{\partial x_1} & \frac{\partial r_{15,9}}{\partial y_1} & \cdots & & \frac{\partial r_{15,9}}{\partial \omega_{15}}
\end{bmatrix}
\end{array}
$$

Partial derivatives → see handout!

Design matrix:

$$\mathbf{A} = \mathbf{J}$$

Normal equations:

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\hat{\mathbf{x}} = \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{l}$$

Solution of normal equations:

$$\hat{\mathbf{x}} = \big(\underbrace{\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}}_{\mathbf{N}}\big)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{l}$$

Adjusted unknowns:

$$\hat{\mathbf{X}} = \mathbf{X}^{0} + \hat{\mathbf{x}}$$

Residuals:

$$\mathbf{v} = \mathbf{A}\hat{\mathbf{x}} - \mathbf{l}$$

Adjusted observations:

$$\hat{\mathbf{L}} = \mathbf{L} + \mathbf{v}$$

7.4 Adjustment of Horizontal Surveys: Combined Network

Check:

$$\max |\hat{x}_i| < \varepsilon$$

and

$$\max \left|\hat{L}_i - \Phi_i(\widehat{\mathbf{X}})\right| < \delta$$

no

Use $\widehat{\mathbf{X}}$ as new $\mathbf{X}^0$

yes

$\widehat{\mathbf{X}}$ is final result

Conversion of final results from [rad] into [gon]

Empirical reference standard deviation:

$$s_0 = \sqrt{\frac{\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}}{n-u}}$$

VCM of adjusted unknowns:

$$\mathbf{\Sigma}_{\hat{X}\hat{X}} = s_0^2 \cdot \mathbf{Q}_{\hat{X}\hat{X}} \qquad \text{with} \qquad \mathbf{Q}_{\hat{X}\hat{X}} = \mathbf{N}^{-1}$$

Standard deviation of unknowns:

Computed from diagonal elements (square root) of $\mathbf{\Sigma}_{\hat{X}\hat{X}}$

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Remark:

Nowadays geodetic networks with measurement of

- Distances
- Directions
- GNSS baselines

Course „Transformation of Geodetic Networks“
3rd semester