1. BS model. PDE $\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0.$

Q: $dS_t = rS_t\,dt + \sigma S_t\,dW_t^Q$

2. $\mathcal{L}\_PDE = \left(\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2 V_{SS}+rSV_S-rV\right)^2$

3. boundary (terminal condition)

$V(T,S)=\max(S-K,0)$

$\rightarrow \mathcal{L}\_terminal = (V(T,S)-\max(S-K,0))^2.$

4. PINN. → approximates risk-neutral EV.

$V(t,S)=E^Q[e^{-r(T-t)}\Phi(S_T)].$

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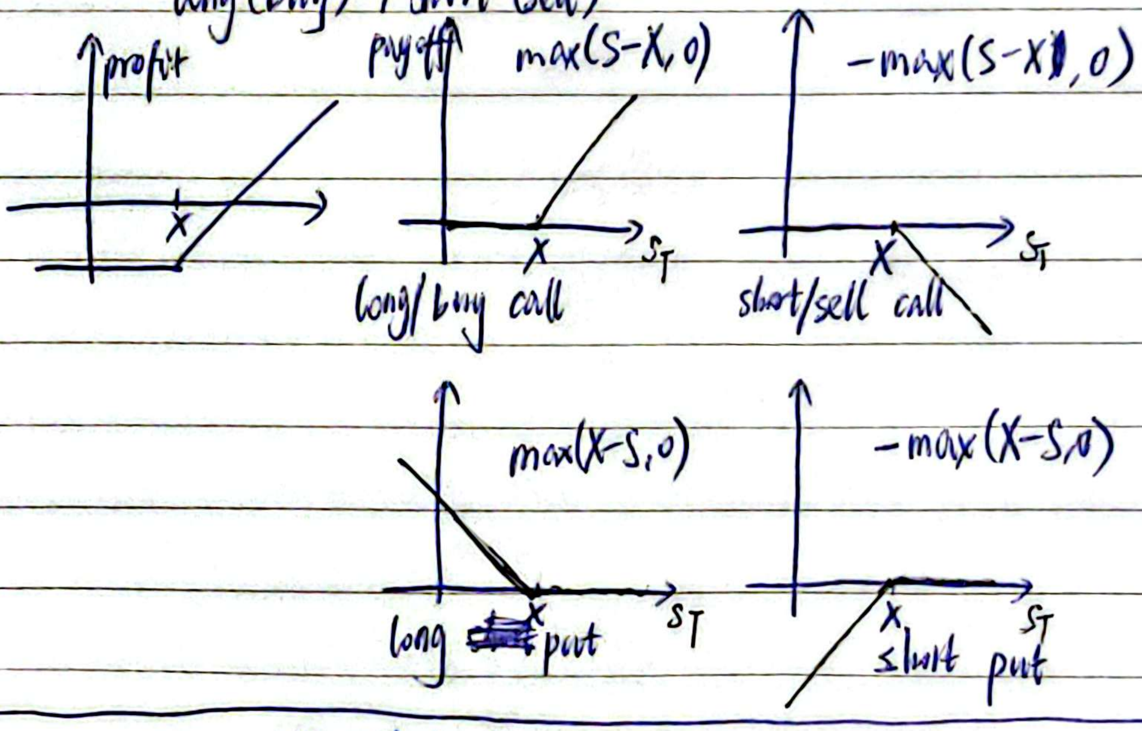
options: call / put.

long (buy) / short (sell)



Martingale: X_t. Stochastic process.

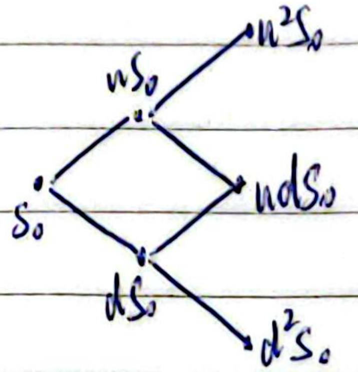
① $E[|X_t|]<\infty$

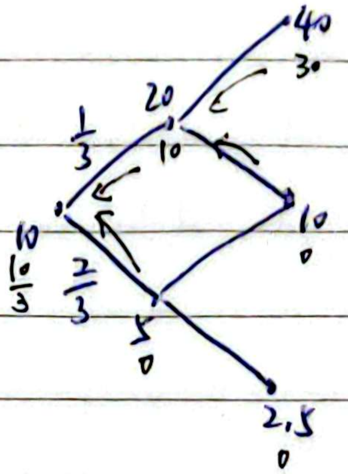
② filtration of market information

③ $E[X_{t+1}|\mathcal{F}_t]=X_t.$

Risk Neutral Pricing $\xrightarrow[\text{Version}]{\text{Discrete}}$ Risk Neutral Binomial Tree (Trinomial) ...

risk neutral: (Q)
- no arbitrage.
- expected growth rate. $\rightarrow r_f$
- derivatives prices $= \sum_{t=1}^{n} PV(CF_t)$





$X=10.$

assuming $e^{r\Delta t}=1$

t: 0 1 2

$$p = \frac{e^{r\Delta t}-d}{u-d} = \frac{1-\frac{1}{2}}{2-\frac{1}{2}} = \frac{1}{3}. \qquad u=e^{\sigma\sqrt{\Delta t}} \quad d=e^{-\sigma\sqrt{\Delta t}}$$

options price: $V_0 = e^{-r\Delta t}(qV_u + (1-q)V_d) \rightarrow$ discrete

$V(t) = E^Q[e^{-r(T-t)}\underbrace{\Phi(S_T)}_{\text{payoff at maturity}}]. \rightarrow$ continuous

Martingale. (the discounted price of an asset must be a martingale)

$$E^Q[S_{t+\Delta t}|S_t] = e^{r\Delta t}S_t$$

$$\Rightarrow p\cdot uS_t + (1-p)dS_t = e^{r\Delta t}S_t.$$

$$\Rightarrow pu + (1-p)d = e^{r\Delta t}$$

$$\Rightarrow p(u-d) = e^{r\Delta t} - d.$$

$$\Rightarrow p = \frac{e^{r\Delta t}-d}{u-d} \quad (0<p<1)$$