TTIC 31230, Fundamentals of Deep Learning

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Dropout

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Dropout can be viewed as an ensemble method.

To draw a model from the ensemble we randomly select a mask μ with

$$
\begin{cases}
\mu_i = 0 \text{ with probability } \alpha \\[2em]
\mu_i = 1 \text{ with probability } 1 - \alpha
\end{cases}
$$

Then we use the model $(\Phi, \ \mu)$ with weight layers defined by

$$
y_i = \text{Relu} \left(\sum_j W_{i,j} \ \mu_j x_j \right)
$$

Dropout Training

Repeat:

- Select a random dropout mask μ

- $\Phi \mathrel{-}= \nabla_\Phi \mathcal{L}(\Phi, \mu)$

Backpropagation must use the same mask μ used in the forward computation.

Test Time Scaling

At train time we have

$$y_i = \mathrm{Relu} \left(\sum_j W_{i,j} \, \mu_j x_j \right)$$

At test time we have

$$y_i = \mathrm{Relu} \left((1 - \alpha) \sum_j W_{i,j} \, x_j \right)$$

At test time we use the "average network".

Dropout for Least Squares Regression

Consider simple least square regression

$$\Phi^* = \underset{\Phi}{\text{argmin}} \ \ \text{E}_{(x,y)} \ E_\mu \left(y - \Phi \cdot (\mu \odot x)\right)^2$$

$$= \text{E}\left[(\mu \odot x)(\mu \odot x)^\top\right]^{-1} \text{E}\left[y(\mu \odot x)\right]$$

$$= \underset{\Phi}{\text{argmin}} \ \ \text{E}_{(x,y)}(y - (1 - \alpha)\Phi \cdot x)^2 + \sum_i \frac{1}{2}(\alpha - \alpha^2)\text{E}\left[x_i^2\right]\Phi_i^2$$

In this case dropout is equivalent to a form of L_2 regularization
— see Wager et al. (2013).

A Dropout Bound

$$KL(Q_{\alpha,\Phi},\ Q_{\alpha,0}) = E_{\mu\sim P_\alpha,\epsilon\sim\mathcal{N}(0,1)^d}\ \ln\frac{P_\alpha(\mu)e^{-\frac{1}{2}||\mu\odot\epsilon||^2}}{P_\alpha(\mu)e^{-\frac{1}{2}||\mu\odot(\Phi+\epsilon)||^2}}$$

$$= E_{\mu\sim P_\alpha}\ \frac{1}{2}||\mu\odot\Phi||^2$$

$$= \frac{1-\alpha}{2}||\Phi||^2$$

$$L(Q_{\alpha,\Phi}) \leq \frac{1}{1-\frac{1}{2\lambda}}\left(\hat{L}(Q_{\alpha,\Phi}) + \frac{\lambda L_{\max}}{N}\left(\frac{1-\alpha}{2}||\Phi||^2 + \ln\frac{1}{\delta}\right)\right)$$

END