

Homework 7

Quantum Mechanics

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Problem 1. 5.1

Solution.

We are concerned here with the new ground state ket $|0\rangle$ in the presence of H_1 and the new ground state energy shift Δ_0 .

$$|0\rangle = |0^0\rangle + \sum_{k \neq 0} |k^0\rangle \frac{V_{k0}}{E_0^0 - E_k^0} + \dots$$

$$\Delta_0 = V_{00} + \sum_{k \neq 0} \frac{|V_{k0}|^2}{E_0^0 - E_k^0} + \dots$$

$$V_{nk} = b \langle n^0 | x | k^0 \rangle = b \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{k} \delta_{n,k-1} + \sqrt{k+1} \delta_{n,k+1} \right)$$

The lowest nonvanishing order is then V_{01} . Therefore

$$\Delta_0 = -\frac{b^2 \hbar}{2m\omega} \frac{1}{\hbar\omega} = -\frac{b^2}{2m\omega^2}$$

To solve it exactly, notice that the potential is of the form

$$V(x) = ax^2 + bx$$

This function shifts to the left by $b/2$ and has a new minimum at $-b/a$. Also, any multiplicative constant ϵ of the potential can just be absorbed into ω , such that $E_n = \hbar\omega\sqrt{\epsilon}(n + \frac{1}{2})$. Therefore, we can conclude that

$$\Delta = -\frac{b}{2a} = -\frac{b}{2m\omega^2}$$

which is exactly what we got with second-order perturbation theory. ■

Problem 2. *5.2*

Solution. ■

Problem 3. *5.5*

Solution. ■

Problem 4. *5.7*

Solution. ■

Problem 5. *5.12a*

Solution. ■

Problem 6. *5.24*

Solution. ■