What is white noise?

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1 Definition of white noise

I wanted to clarify what is meant when we say that a stochastic process is *white*. The term of course comes from its relationship to white light, which contains all frequencies. Technically, the power spectral density of white light or a white stochastic process is flat - it contains all frequencies at equal power. In the real world, a truly white power spectrum is an idealization, but it remains a useful concept in several fields.

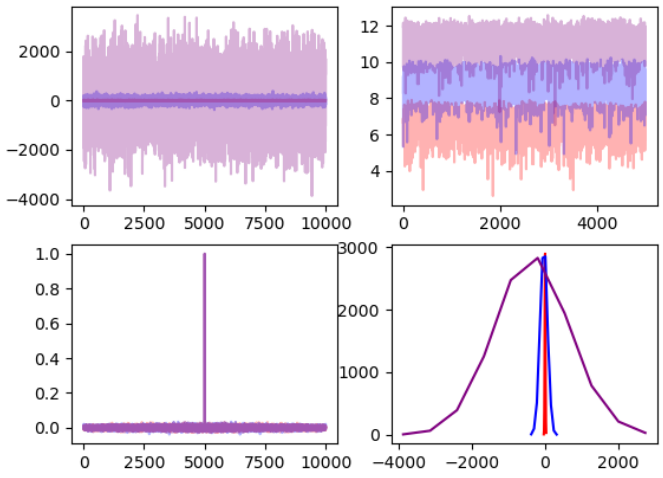

Figure 1: (UL) A stochastic process drawn from a normal distribution with variance 1,10,1000. (UR) Power spectral density of the three stochastic processes (LL) Autocorrelation function for the three processes. (LR) Empirical amplitude distribution for the three processes.

Recall that the Fourier transform of the autocorrelation function of a signal is equivalent to its power spectral density (PSD). That is, the autocorrelation

function and the PSD form a Fourier pair. We also know that Fourier transform of a delta function in the time domain is a flat function in the frequency domain (this has very important consequences in quantum mechanics when applied to position and momentum). Therefore, if we have a stochastic process with a flat PSD, then its autocorrelation function must be a delta function and it is called delta-correlated.

At this point I want to take note of the terminology. The term white noise refers to a property of the PSD and therefore the shape of the autocorrelation function. It refers to the "delta-correlated-ness" of the proces. It does not refer to say, the distribution the process is sampled from (in the case that the process has stationary statistics). For example, if the stochastic process is simply a sequence of samples from a Gaussian (normal) distribution, but the samples are drawn i.i.d, the PSD will be flat and the autocorrelation function will be a delta-function. The probability density of the amplitude and the temporal correlation structure are orthogonal concepts.