# Homework 2

**Quantum Mechanics**

*August 29th, 2022* Clayton Seitz

**Problem 1.** *Problem 1.12 from Sakurai*

**Solution**.

If we choose the representation such that $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $|2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$ then we can use the definition of the outer product to show that

$$H = a\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$$

The energy eigenvalues are then found by

$$\begin{aligned}\det(H-\lambda I) &= \det\begin{pmatrix}a-\lambda & a\\ a & -a-\lambda\end{pmatrix}\\ &= (a-\lambda)(-a-\lambda)-a^2\\ &= \lambda^2-2a^2=0\end{aligned}$$

therefore $E_{\pm} = \pm\sqrt{2}a$. The $+$ eigenvector $|\psi_1\rangle$ is given by the system

$$\begin{aligned}(\psi_1^1+\psi_1^2) &= \sqrt{\frac{2}{a}}\psi_1^1\\ (\psi_1^1-\psi_1^2) &= \sqrt{\frac{2}{a}}\psi_1^2\end{aligned}$$

The $-$ eigenvector $|\psi_2\rangle$ is given by the system

$$\begin{aligned}(\psi_2^1+\psi_2^2) &= -\sqrt{\frac{2}{a}}\psi_2^1\\ (\psi_2^1-\psi_2^2) &= -\sqrt{\frac{2}{a}}\psi_2^2\end{aligned}$$

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**Problem 2.** *Problem 1.13 from Sakurai*

**Solution**.

Writing $H$ out in matrix form gives

$$\begin{aligned} H &= H_{11}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + H_{12}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + H_{22}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} H_{11} + H_{12} + H_{22} + 1 & H_{11} - H_{12} - H_{22} + 1 \\ H_{11} - H_{12} + H_{22} - 1 & H_{11} + H_{12} - H_{22} - 1 \end{pmatrix} \end{aligned}$$

$$\det(H - \lambda I) = \det\begin{pmatrix} H_{11} + H_{12} + H_{22} + 1 - \lambda & H_{11} - H_{12} - H_{22} + 1 \\ H_{11} - H_{12} + H_{22} - 1 & H_{11} + H_{12} - H_{22} - 1 - \lambda \end{pmatrix}$$

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**Problem 3.** *Problem 1.15 from Sakurai*

**Solution**.

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**Problem 4.** *Problem 1.16 from Sakurai*

**Solution**.

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**Problem 5.** *Problem 1.23 from Sakurai*

**Solution**.

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**Problem 6.** *Problem 1.24 from Sakurai*

**Solution**.

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