# Training a dynamical system by using multivariate information

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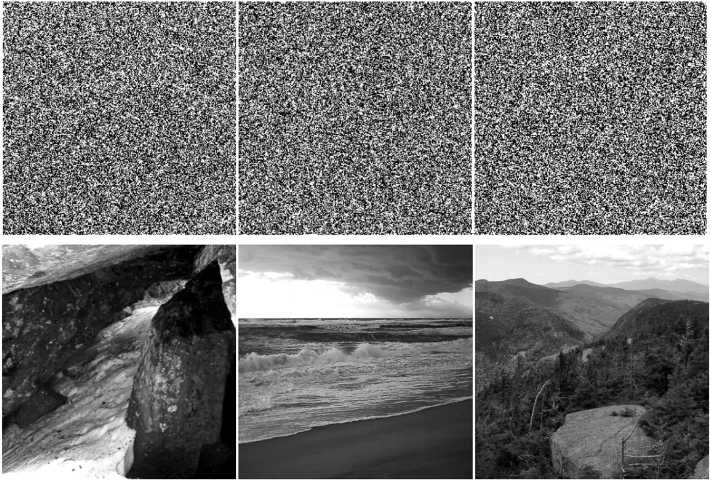
# Introduction

Neuroethology argues that neural networks evolve according to the stimuli to which they are exposed



# Channel coding for neural networks

Networks of neurons can be viewed as a communication channel
Except this communication channel *learns* the transformation $F$ based on the statistical structure of its input $X$. Visual cortex has learned an encoding for visual scenes (that perhaps maximizes information)

# Leaky integrate and fire neurons

A realistic LIF model might look like

$$\tau_m \frac{d\mathrm{V}[I]}{dt} = (\mathrm{V}[I] - E)\sum_j \mathrm{W}^0[I,j] + (\mathrm{V}[I] - E_{in})\sum_k \mathrm{W}^1[I,k])$$

Instead, we ignore changes in the voltage of the postsynaptic neuron due to subthreshold voltages of the presynaptic neuron and let matrices W learn the input-output voltage relationship

$$V[j,t{+}1] = \alpha V[j,t{+}1] + \sum_{i \neq j} W^0_{ij} z[i,t] + \sum_i W^1_{ij} x[i,t{+}1] - z[j,t] v_{th}$$

where $z = H(v - v_{th})$

# Estimating gradients

Say we have a model $\Phi = (W^0, W^1)$ and want to use gradient descent to train a network to have a target rate or a target branching parameter. The rate and its associated loss for a single unit is

$$r(t) = \frac{1}{\Delta t} \int_t^{t+\Delta t} d\tau \langle \rho(\tau) \rangle \qquad \mathcal{L} = \alpha(r - r_0)^2$$

We would like the standard update

$$\Delta W_{ij} = -\eta \frac{\partial \mathcal{L}}{\partial W_{ij}}$$

But it is intractable to compute $\frac{\partial \mathcal{L}}{\partial W_{ij}}$ since $\rho(t)$ depends on other neurons through space and time.

# Estimating gradients

Bellec et al. presented a solution to estimating $\frac{\partial \mathcal{L}}{\partial W_{ij}}$ for online learning, but it can just as well be used for supervised learning

$$\frac{\partial \mathcal{L}}{\partial W_{ij}} = \sum_t \frac{\partial \mathcal{L}}{\partial z[j,t]} \cdot \frac{\partial z[j,t]}{\partial W_{ij}}$$

where the gradient $\frac{\partial z[j,t]}{\partial W_{ij}}$ is computed locally.

# Multivariate information theory

# Adaptation of the transfer function

How do neuron transfer functions adapt to stimuli in an unsupervised manner?

# Adaptation defines an energy function over phase space

# Generalization bounds

What is the distance of a code defined by a particular energy function E

# The energy function defines a dynamical system

# The energy function is a generative model

# Application to natural image statistics