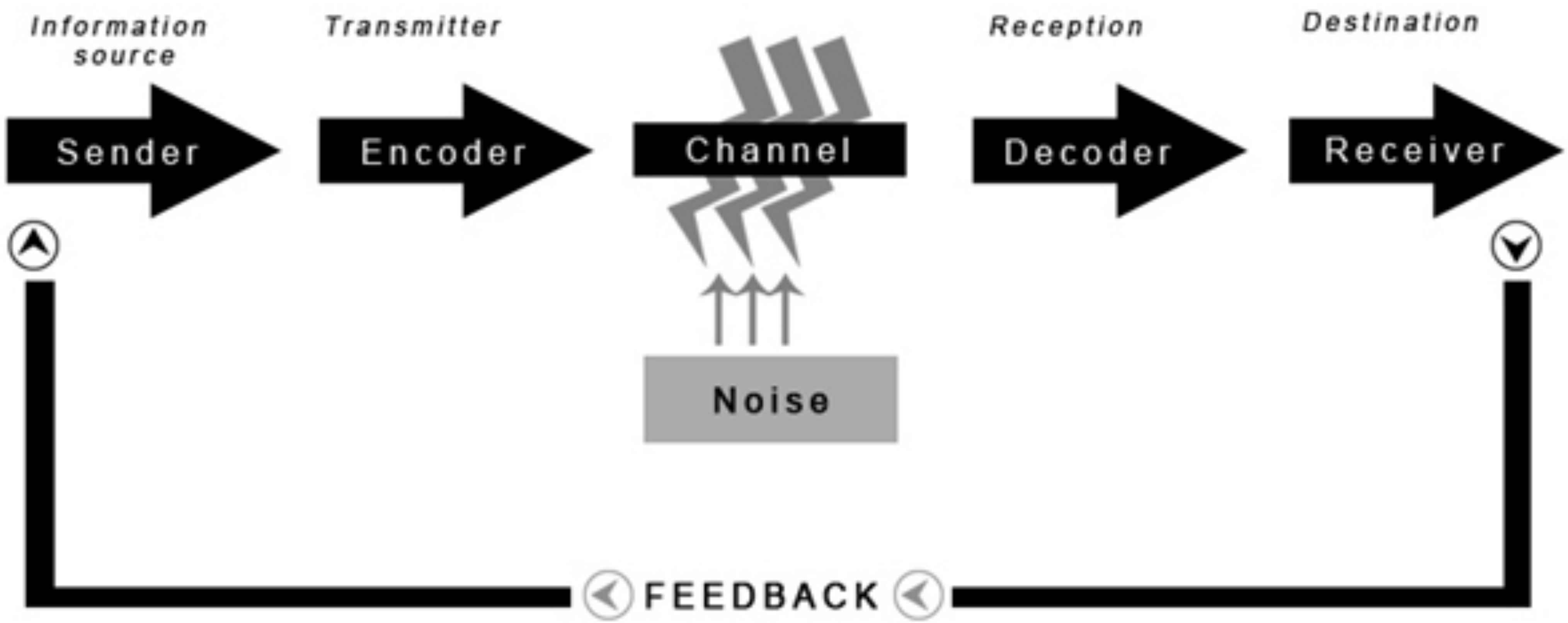
Lecture 10: *Decoding neural signals, part II*



Shannon-Weaver's Model of Communication

Review of previous results BEGIN

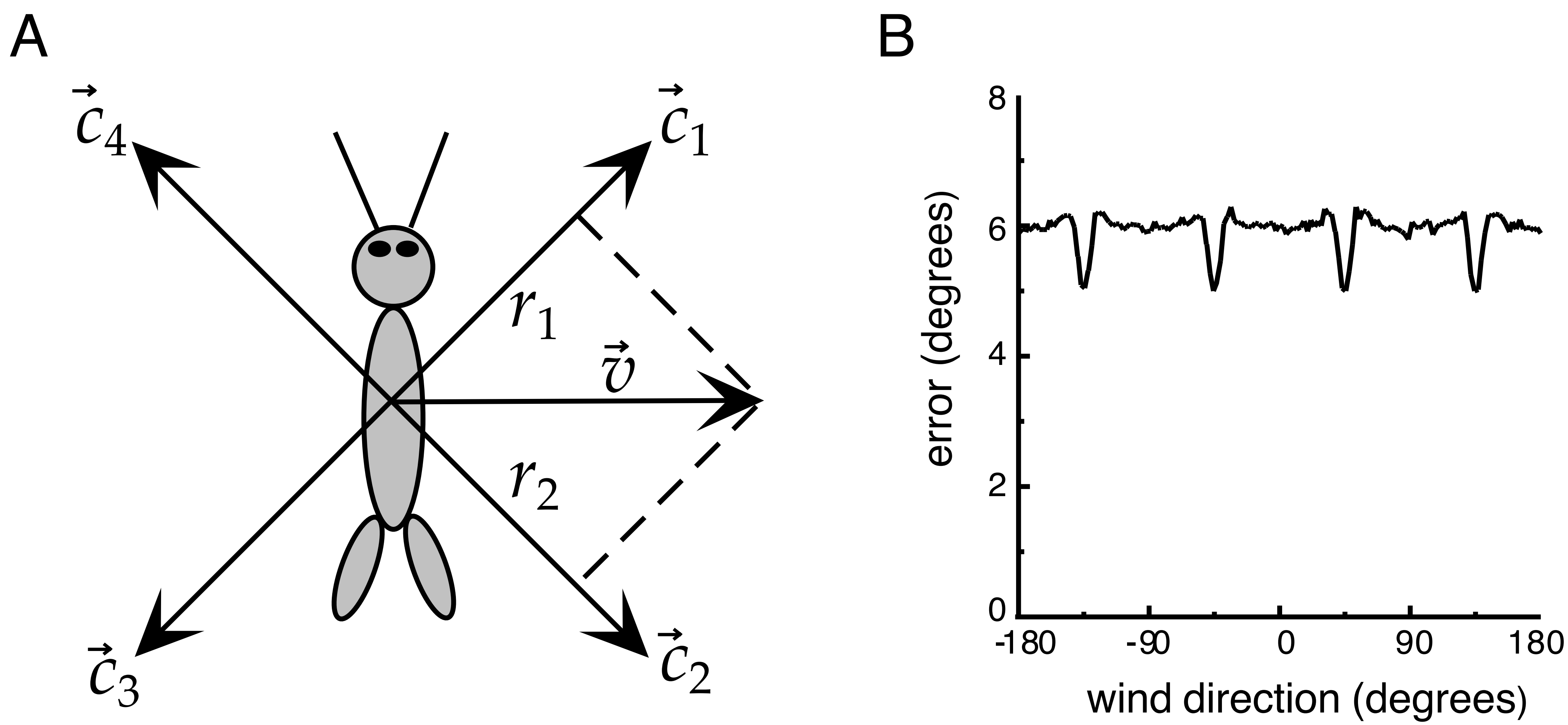
Optimal estimate of the stimulus, Bayesian version:

$$\int ds\, L(s, s_{\text{bayes}})\, p[s|\mathbf{r}]$$

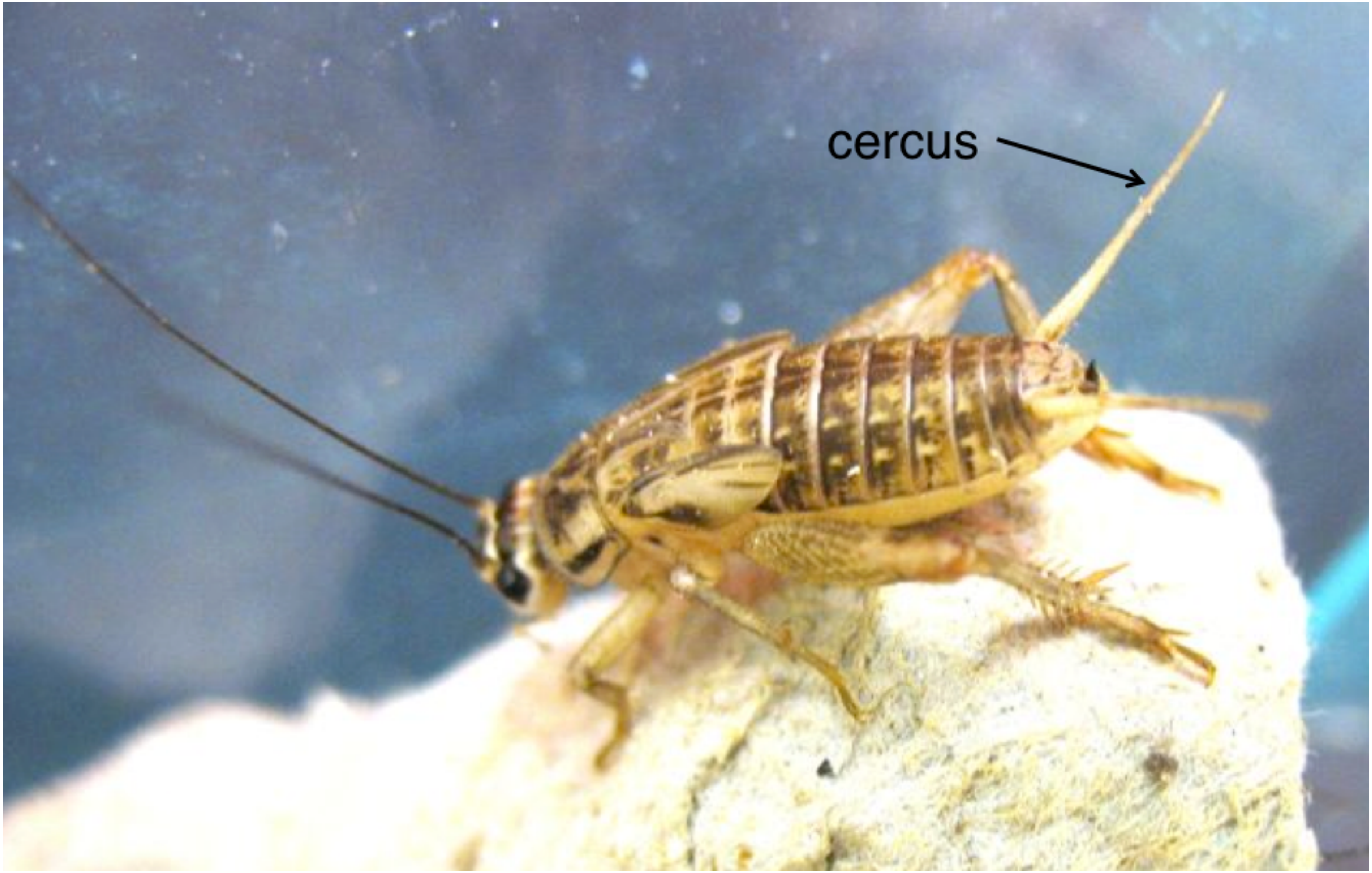
$$L(s, s_{\text{bayes}}) = (s - s_{\text{bayes}})^2 \qquad s_{\text{bayes}} = \int ds\, p[s|\mathbf{r}]s$$

$$p[s|\mathbf{r}] = \frac{p[\mathbf{r}|s]p[s]}{p[\mathbf{r}]}$$

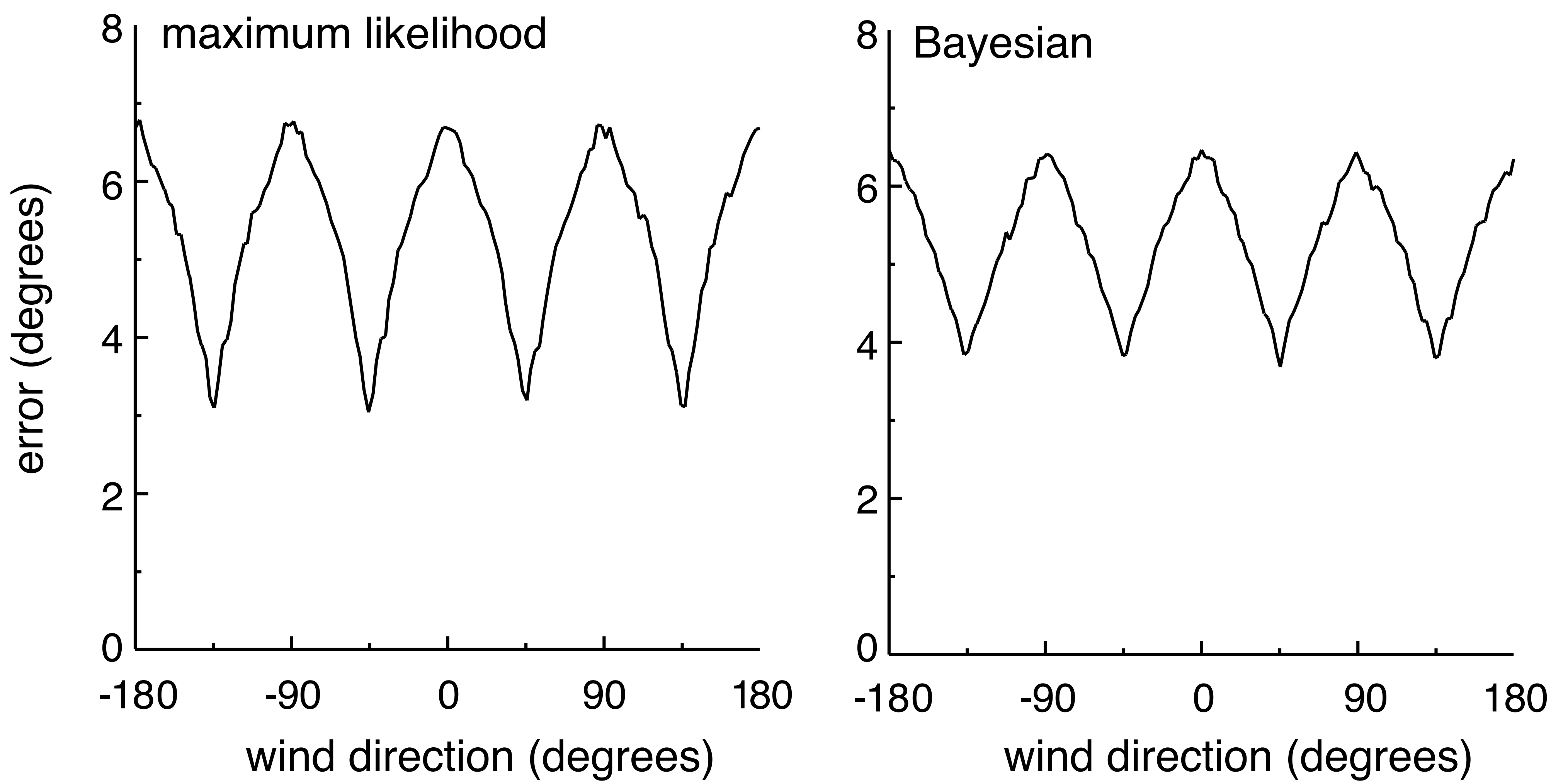
Estimating wind direction from the cricket cercal



Estimating wind direction from the cricket cercal



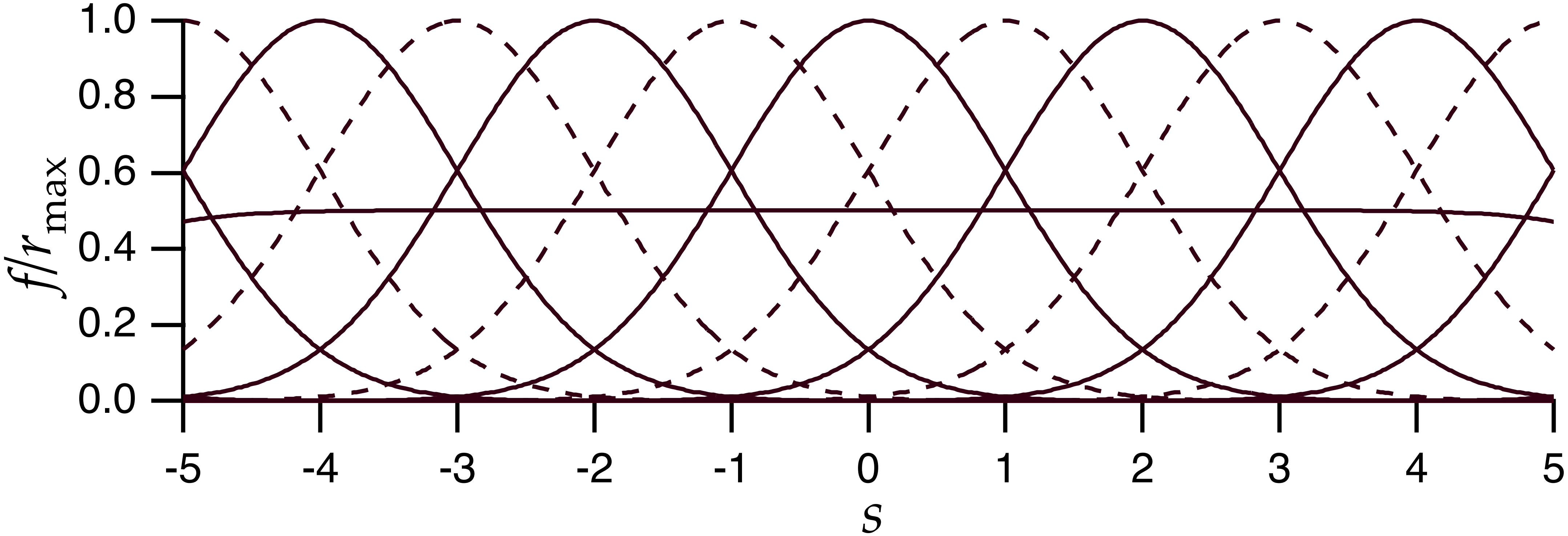
Comparing ML to Bayes for the cercal system:



Gaussian tuning curve for an arbitrary variable:

$$f_a(s) = r_{\max} \exp\left(-\frac{1}{2}\left(\frac{s - s_a}{\sigma_a}\right)^2\right)$$

A population of Gaussian tuned cells:

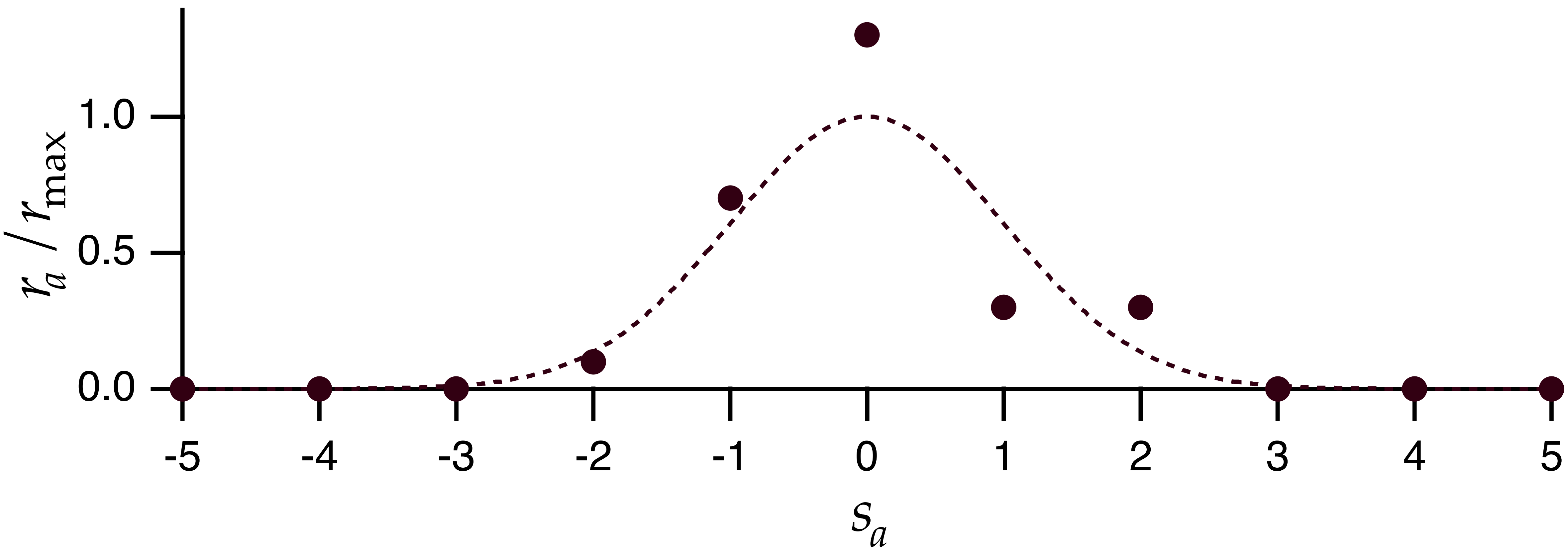


Conditional distribution for independent neurons:

$$P[r_a|s] = \frac{(f_a(s)T)^{r_a T}}{(r_a T)!} \exp(-f_a(s)T)$$

$$P[\mathbf{r}|s] = \prod_{a=1}^{N} \frac{(f_a(s)T)^{r_a T}}{(r_a T)!} \exp(-f_a(s)T)$$

Simulated response to s_a = 0:



Deriving the ML estimate of the stimulus:

$$P[r_a|s] = \frac{(f_a(s)T)^{r_a T}}{(r_a T)!} \exp(-f_a(s)T)$$

$$P[\mathbf{r}|s] = \prod_{a=1}^{N} \frac{(f_a(s)T)^{r_a T}}{(r_a T)!} \exp(-f_a(s)T)$$

$$\ln P[\mathbf{r}|s] = T \sum_{a=1}^{N} r_a \ln\big(f_a(s)\big) + \ldots$$

Deriving the ML estimate of the stimulus:

$$\ln P[\mathbf{r}|s] = T \sum_{a=1}^{N} r_a \ln\big(f_a(s)\big) + \ldots$$

$$\sum_{a=1}^{N} r_a \frac{f'_a(s_{\mathrm{ML}})}{f_a(s_{\mathrm{ML}})} = 0$$

$$s_{\mathrm{ML}} = \frac{\sum r_a s_a/\sigma_a^2}{\sum r_a/\sigma_a^2}$$

$$s_{\mathrm{ML}} = \frac{\sum r_a s_a}{\sum r_a}$$

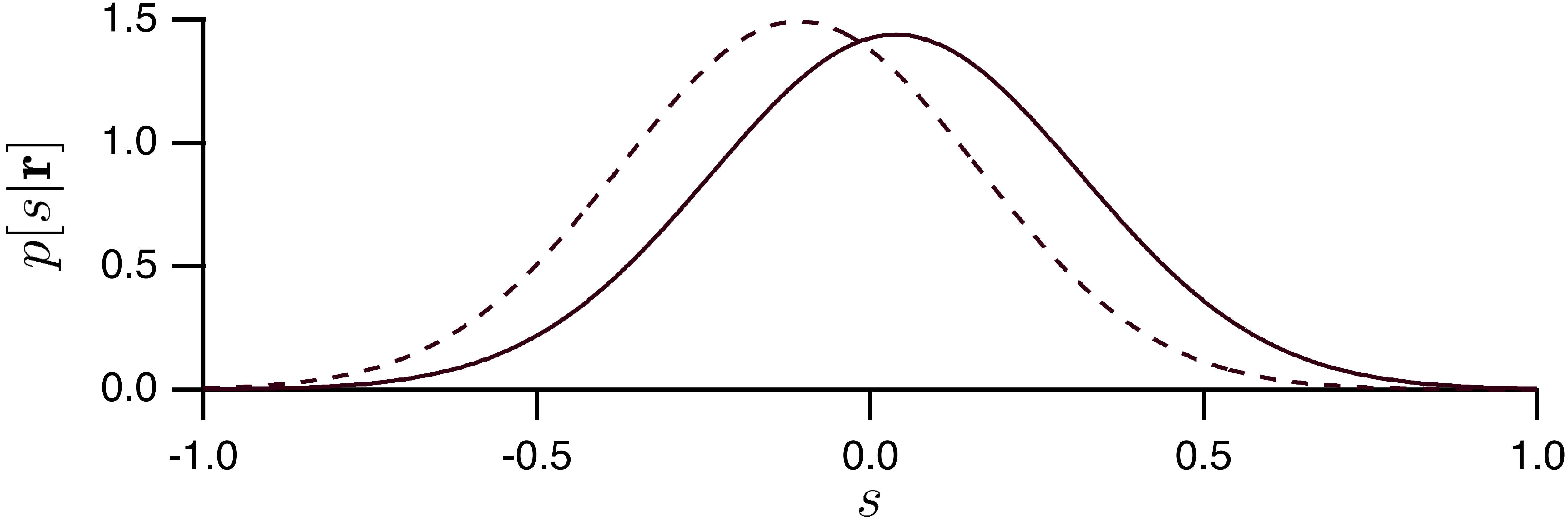
Constructing the MAP estimate of the stimulus:

$$\ln p[s|\mathbf{r}] = T \sum_{a=1}^{N} r_a \ln\big(f_a(s)\big) + \ln p[s] + \ldots$$

$$T \sum_{a=1}^{N} \frac{r_a f_a'(s_{\mathrm{MAP}})}{f_a(s_{\mathrm{MAP}})} + \frac{p'[s_{\mathrm{MAP}}]}{p[s_{\mathrm{MAP}}]} = 0$$

$$s_{\mathrm{MAP}} = \frac{T \sum r_a s_a / \sigma_a^2 + s_{\mathrm{prior}} / \sigma_{\mathrm{prior}}^2}{T \sum r_a / \sigma_a^2 + 1 / \sigma_{\mathrm{prior}}^2}$$

MAP allows us to take P(s) into account:



Bias and variance in our estimates:

$$b_{\text{est}}(s) = \langle s_{\text{est}} \rangle - s$$

$$\sigma^2_{\text{est}}(s) = \langle (s_{\text{est}} - \langle s_{\text{est}} \rangle)^2 \rangle$$

$$\left\langle (s_{\text{est}} - s)^2 \right\rangle = \left\langle (s_{\text{est}} - \langle s_{\text{est}} \rangle - b_{\text{est}}(s))^2 \right\rangle = \sigma^2_{\text{est}}(s) + b^2_{\text{est}}(s)$$

Review of previous results END

Likelihood ratios and the "score":

$$\frac{p[r|s+\Delta s]}{p[r|s]} \approx \frac{p[r|s] + \Delta s \partial p[r|s]/\partial s}{p[r|s]}$$

$$= 1 + \Delta s \frac{\partial \ln p[r|s]}{\partial s}$$

$$Z(r) = \frac{\partial \ln p[r|s]}{\partial s}$$

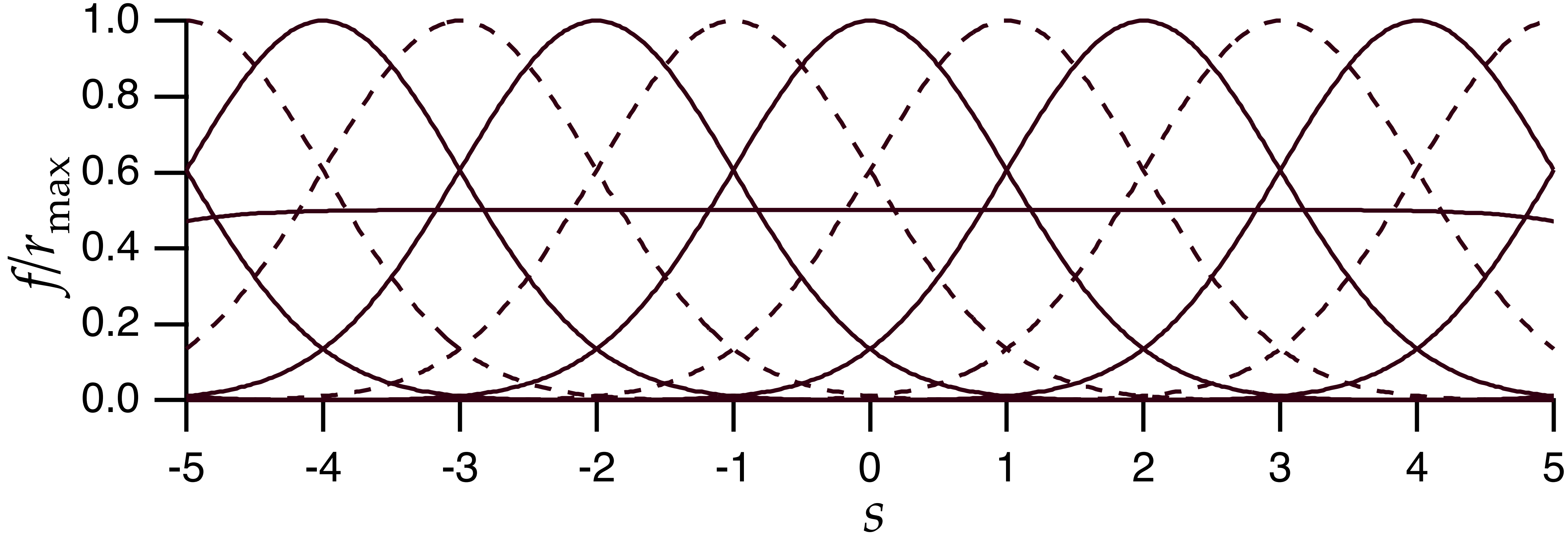
The Cramer-Rao bound on estimates of the stimulus:

$$\sigma^2_{\mathrm{est}}(s) \geq \frac{\left(1 + b'_{\mathrm{est}}(s)\right)^2}{I_{\mathrm{F}}(s)}$$

$$I_{\mathrm{F}}(s) = \left\langle -\frac{\partial^2 \ln p[\mathbf{r}|s]}{\partial s^2} \right\rangle = \int d\mathbf{r}\, p[\mathbf{r}|s] \left(-\frac{\partial^2 \ln p[\mathbf{r}|s]}{\partial s^2} \right)$$

$$I_{\mathrm{F}}(s) = \left\langle \left(\frac{\partial \ln p[\mathbf{r}|s]}{\partial s} \right)^2 \right\rangle = \int d\mathbf{r}\, p[\mathbf{r}|s] \left(\frac{\partial \ln p[\mathbf{r}|s]}{\partial s} \right)^2$$

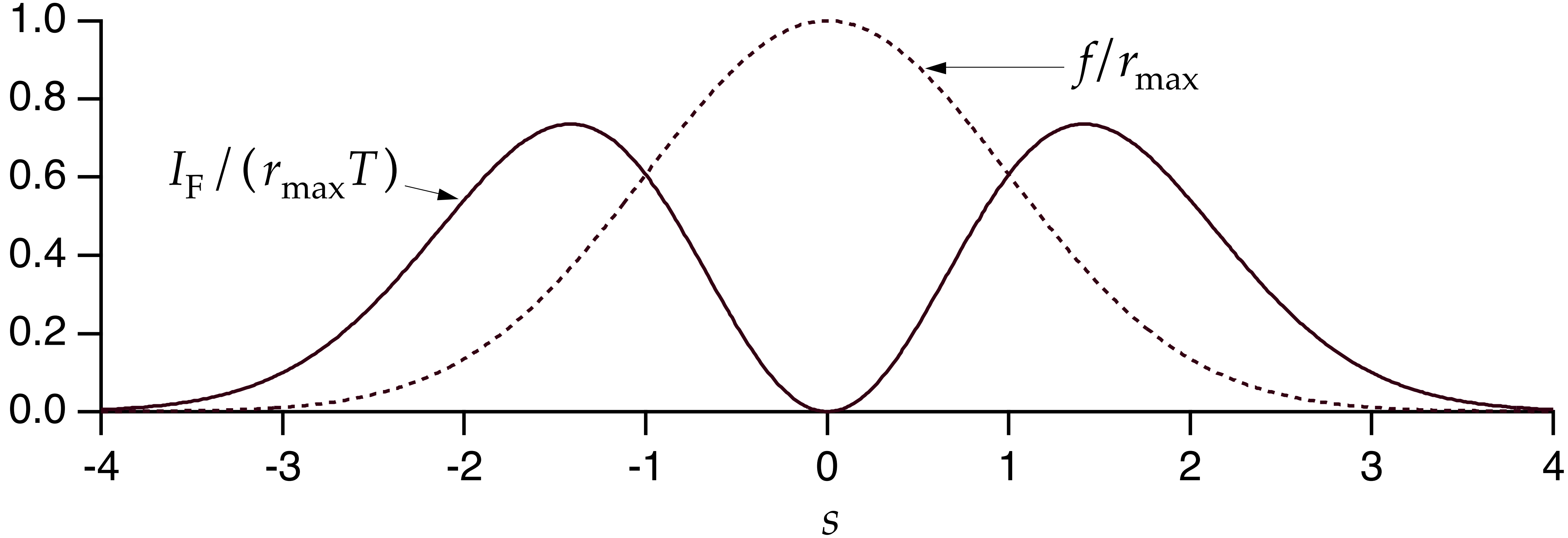
A population of Gaussian tuned cells:



The Fisher information for Poisson neurons:

$$I_{\mathrm{F}}(s) = T \sum_{a=1}^{N} \frac{\left(f_a'(s)\right)^2}{f_a(s)}$$

The Fisher information for a Gaussian tuning curve:



What's best? Narrow or wide tuning curves?

$$I_{\mathrm{F}}(s) = T \sum_{a=1}^{N} \frac{\left(f_a'(s)\right)^2}{f_a(s)}$$

$$I_{\mathrm{F}}(s) = T \sum_{a=1}^{N} \frac{r_{\max}(s-s_a)^2}{\sigma_r^4} \exp\left(-\frac{1}{2}\left(\frac{s-s_a}{\sigma_r}\right)^2\right)$$

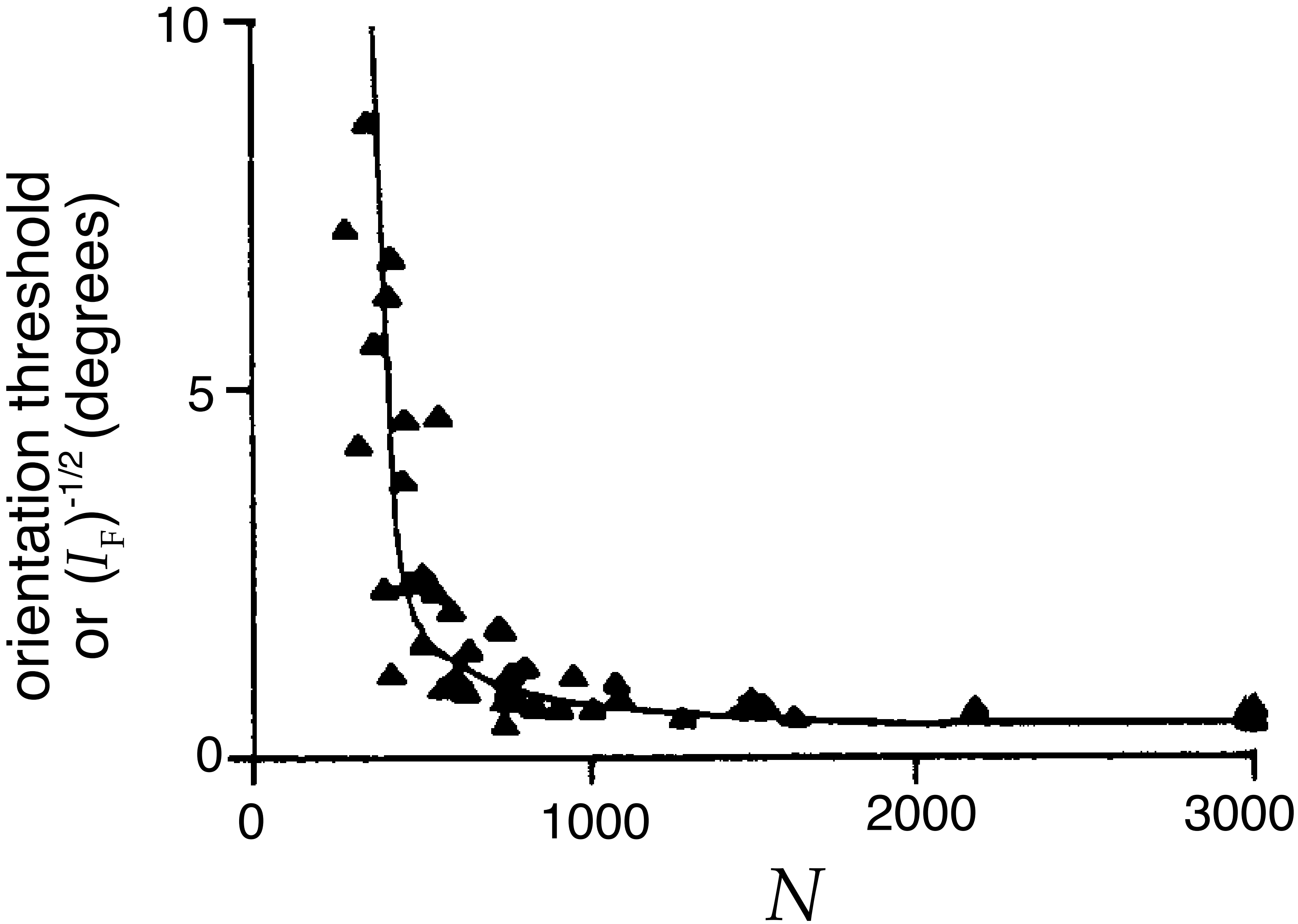
$$\begin{aligned} I_{\mathrm{F}}(s) &\approx \rho_s T \int_{-\infty}^{\infty} d\xi \, \frac{r_{\max}(s-\xi)^2}{\sigma_r^4} \exp\left(-\frac{1}{2}\left(\frac{s-\xi}{\sigma_r}\right)^2\right) \\ &= \frac{\sqrt{2\pi}\rho_s \sigma_r r_{\max} T}{\sigma_r^2} \,. \end{aligned}$$

...depends on the dimensionality of the stimulus!

$$\begin{aligned} I_{\mathrm{F}}(s) &\approx \rho_s T \int_{-\infty}^{\infty} d\xi \, \frac{r_{\max}(s-\xi)^2}{\sigma_r^4} \exp\left(-\frac{1}{2}\left(\frac{s-\xi}{\sigma_r}\right)^2\right) \\ &= \frac{\sqrt{2\pi}\rho_s \sigma_r r_{\max} T}{\sigma_r^2} \, . \end{aligned}$$

$$I_{\mathrm{F}} = \frac{(2\pi)^{D/2} D \rho_s \sigma_r^D r_{\max} T}{\sigma_r^2} = (2\pi)^{D/2} D \rho_s \sigma_r^{D-2} r_{\max} T$$

Comparing Fisher info to human subject performance:



Optimal discrimination:

$$d' = \Delta s \sqrt{I_{\mathrm{F}}(s)}$$

$$Z(r) = \frac{\partial \ln p[r|s]}{\partial s} \qquad Z = T \sum_{a=1}^{N} r_a \frac{f_a'(s)}{f_a(s)}$$