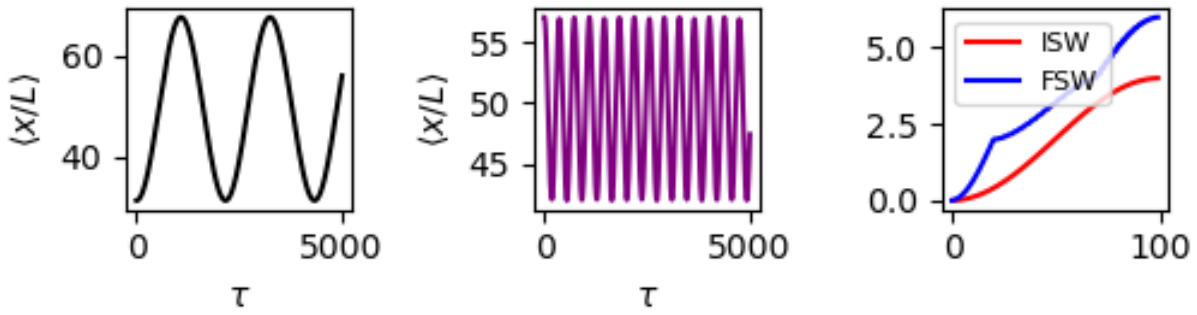

Figure 1: Expectation values of position as a function of time for the infinite (left) and finite (right) square well

Project 2

Quantum Mechanics

December 7, 2022

C Seitz

Part 1

We can work in a coordinate system centered on zero, and write

$$\begin{aligned}\langle x\rangle &= \sum_{a'}\sum_{a''} c^*_{a'}c_{a''}\langle a'|\,x\,|a''\rangle \exp\left(\frac{-i(E_{a''}-E_{a'})t}{\hbar}\right)\\ &= \frac{1}{2}\left(\langle 0|\,x\,|1\rangle \exp\left(\frac{-i(E_1-E_0)t}{\hbar}\right)+\langle 1|\,x\,|0\rangle \exp\left(\frac{-i(E_0-E_1)t}{\hbar}\right)\right)\\ &= \beta\cos(\omega t)\end{aligned}$$

where $\beta = \langle 0|\,x\,|1\rangle = \langle 1|\,x\,|0\rangle$ (because x is Hermitian) and $\omega = (E_1 - E_0)/\hbar$. The angular frequency is higher for the finite square well because there is a large energy gap between the first excited state and the ground state (see the differential in the eigenvalue spectrum in Figure 1c).

Part 2

We are using the time-dependent Hamiltonian

$$H(x,t) = H_0(x) + \lambda(1 - e^{-t/\tau})V(x)$$

where $V(x)$ is the finite square well potential. We are assuming that $\tau \to \infty$ so the potentially turns on exactly at $t = 0$, giving a constant perturbation. Notice that V is going to be needed in energy basis, so we will need to transform V using the unitary operator (the $|i\rangle$ basis to the $|\epsilon_n\rangle$ basis).

(H) We are after $P_n(t) = |c_n^{(1)}(t)|^2$ for $n = 1, 2, 3$. In the text, we are given

$$\begin{aligned} c_n^{(1)}(t) &= -\frac{i}{\hbar} V_{ni} \int_0^t e^{i\omega_{ni}t} dt \\ &= \frac{V_{ni}}{E_n - E_i}(1 - e^{i\omega_{ni}t}) \end{aligned}$$

where $V_{ni} = \langle n| V |i\rangle$.

$$|c_n^{(1)}(t)|^2 = \frac{4|V_{ni}|^2}{|E_n - E_i|^2} \sin^2\left(\frac{(E_n - E_i)t}{2\hbar}\right)$$