Bayesian image reconstruction

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Outline

References

Photon statistics of CMOS cameras

- Imaging noise consists of shot noise, thermal noise, and readout noise
- Shot noise is Poisson, thermal noise and readout noise are Gaussian

For a CMOS pixel n, the true signal S_n $[\mathrm{ADU}]$ is a Poisson process with rate parameter λ_n

$$S_n = \gamma g_n P_n(\lambda_n)$$

where $\gamma \; [e^{-}/p]$ is the quantum efficiency and $g_n \; [\mathrm{ADU}/e^{-}]$ is the pixel's gain

$$P(S_n) = \frac{\exp\left(-\lambda_n\right)\lambda_n^p}{p!}$$

But what is the distribution over the corrupted signal $P(\hat{S}_n)$?

Photon statistics of CMOS cameras

To find $P(\hat{S}_n)$, we first evaluate the joint density $P(S_n, \hat{S}_n)$

$$\begin{aligned} P(S_n, \hat{S}_n) &= P(\hat{S}_n | S_n = s) P(S_n = s) \\ &= \frac{1}{Z} \exp\left(-\frac{(\hat{S}_n - g_n s - \mu_n)^2}{\sigma_n^2}\right) \frac{\exp\left(-\lambda_n\right) \lambda_n^s}{s!} \end{aligned}$$

Marginalizing over S_n gives the desired distribution over $\hat{S}_n$

$$P(\hat{S}_n) = \frac{1}{Z} \sum_{s=0}^{\infty} \frac{\exp\left(-\lambda_n\right) \lambda_n^s}{s!} \exp\left(-\frac{(\hat{S}_n - g_n s - \mu_n)^2}{\sigma_n^2}\right)$$

Bayesian parameter inference for CMOS photon statistics

The parameters in our model $\theta = (\lambda_n, g_n, \mu_n, \sigma_n^2)$ are unknown apriori

$$P(\theta|\hat{S}_n) \propto P(\hat{S}_n|\theta)P(\theta)$$

We can just computed the likelihood $P(\hat{S}_n|\theta)$ on the last slide. Samples from the posterior can be found for example by MCMC or we could use MAP estimation

Either of these approaches only make sense for stationary statistics, which means the physical locations and photophysics of the sample remain unchanged in time

For example photostable fluorophores like quantum dots would be a good choice

Consequences for single molecule localization

The point spread function

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