# Neural dynamics of vision

## A computational perspective

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Dedicated to Calvin and Hobbes.

# Contents

# Preface

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## Another sample section

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## Structure of book

Each unit will focus on <SOMETHING>.

## About the companion website

The website[1] for this file contains:

- A link to (freely downlodable) latest version of this document.
- Link to download LaTeX source for this document.
- Miscellaneous material (e.g. suggested readings etc).

## Acknowledgements

- A special word of thanks goes to Professor Don Knuth[2] (for TEX) and Leslie Lamport[3] (for LATEX).
- I'll also like to thank Gummi[4] developers and LaTeXila[5] development team for their awesome LATEX editors.
- I'm deeply indebted my parents, colleagues and friends for their support and encouragement.

Amber Jain
http://amberj.devio.us/

[1]https://github.com/amberj/latex-book-template
[2]http://www-cs-faculty.stanford.edu/~uno/
[3]http://www.lamport.org/
[4]http://gummi.midnightcoding.org/
[5]http://projects.gnome.org/latexila/

# 1

# The Neural Code

*"This is a quote and I don't know who said this."*
– Author's name, *Source of this quote*

## 1.1 Section heading

# 2

# Microscopy and Image Analysis

*"This is a quote and I don't know who said this."*
– Author's name, *Source of this quote*

## 2.1 Section heading

# 3

# Error Correcting Codes

> *"This is a quote and I don't know who said this."*
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> – Author's name, *Source of this quote*

## 3.1 Section heading

# 4

# Natural Image Statistics and Gabor Analysis

*"This is a quote and I don't know who said this."*
– Author's name, *Source of this quote*

## 4.1 Section heading

# 5

# Semantic Coding

*"This is a quote and I don't know who said this."*
– Author's name, *Source of this quote*

## 5.1 Section heading

# 6

# Information and Coding Theory

*"We may have knowledge of the past but cannot control it; we may control the future but have no knowledge of it"*

– Claude Shannon

## 6.1 Introduction

Information theory is a framework first introduced by Claude Shannon's seminal paper *A mathematical theory of communication* published in 1948. At it's core, information theory makes the intuitive concept of *information* mathematically rigorous and forms the foundation of many modern communication systems. Neural circuits in the visual system are an especially interesting example of such a communication system. Therefore, in this section, the information theoretic concepts necessary for studying neural circuits are introduced.

## 6.2 Entropy

The concept of entropy is not exclusive to information theory; rather, it is used widely in disciplines such as physics and mathematical statistics. In fact, entropy was originally defined in statistical physics when Ludwig Boltzmann gave a statistical description of a thermodynamic system of particles. Since this is arguably the more intuitive path as opposed an entirely mathematical description, I will follow a similar line of reasoning in the following paragraphs.

In every application, the entropy $\mathbf{H}$ is a measure of uncertainty or how much information is contained in a random variable $x$. In information theory, the entropy is a property of a probability distribution of a random variable

$P(x)$ where $x$ can take on continuous or discrete values. For the discrete case, we can express the entropy in bits

$$\mathbf{H} = -\sum_{x \in S} P(x) \log P(x) \tag{6.1}$$

where the set $S$ spans the entire space of possible discrete values of $x$. Notice that $\mathbf{H} \geq 0$ since $P(x) \leq 1$ and therefore $\log P(x) \leq 0$ for all $x$. We might guess that the $P(x)$ with maximum entropy is the uniform distribution and to prove that we need to introduce a famous inequality.

### 6.2.1 Jensen's Inequality

Jensen's inequality is a statement about convexity. Consider a binary variable $x$ that takes the value 0 with probability $\alpha$ and value 1 with probability $1 - \alpha$.

$$x = \begin{cases} 0 & \alpha \\ 1 & 1 - \alpha \end{cases}$$

A function $f$ of the variable $x$ is said to be *convex* if the following inequality holds

$$\alpha f(x) + (1 - \alpha) f(y) \leq f(\alpha x + (1 - \alpha) y)$$

which when generalized for an arbitrary random variable $x$ forms Jensen's inequality

$$\mathbf{E}[f(x)] \leq f(\mathbf{E}[x]) \tag{6.2}$$

and if we flip the inequality we call the function *concave*.

$$\begin{aligned} \mathbf{H} &= -\sum_{x \in S} P(x) \log P(x) \\ &= \sum_{x \in S} \frac{1}{N} \log N = \log N \end{aligned}$$

We have now shown that the upper bound on the entropy for a random variable with $N$ possible values is $\log N$.

### 6.2.2 Kraft's Inequality

Kraft's inequality is a constraint on prefix-free codes. A code is prefix free if and only-if the following statement is true

$$\sum_i 2^{-l_i} \leq 1 \tag{6.3}$$

for code lengths $l_i$.

### 6.2.3 Example 1: Applying Jensen's Inequality

Let's consider a function $f : \mathbb{R} \rightarrow \mathbb{R}$. Using Jensen's inequality, we can prove that $f = x^2$ or $f = x \log x$ are convex functions. Let's begin by applying it to $x^2$ for a general normalized probability distribution $p(x)$.

$$\begin{aligned} \int p(x) f(x) dx &= \int x^2 p(x) dx \\ &= x^2 - 2 \int x dx \\ &= 0 \leq x^2 \ \forall x \end{aligned}$$

We have a similar proof for $f(x) = x \log x$

$$\begin{aligned} \int p(x) f(x) dx &= \int x \log x \ p(x) dx \\ &= x \log x - \int \frac{d}{dx} x \log x \ dx \\ &= 0 \leq \mu \log \mu \end{aligned}$$

where $\mu = \mathbf{E}[x] \geq 0$ since $f$ is only defined on $[0, \infty]$.

### 6.2.4 Example 2: Proving Cauchy-Schwarz

A common form of the Cauchy-Schwarz inequality states that for two vectors $u$ and $v$, we have

$$u \cdot v \leq \|u\| \, \|v\|$$