Homework 4

Quantum Mechanics
February 22, 2023 C Seitz

Problem 1. *Problem 2.65*

Solution. Let us call these states $|\alpha\rangle$ and $|\beta\rangle$:

$$|\alpha\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$
$$|\beta\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

If we choose a non-orthogonal basis, such as

$$|e_1\rangle = |0\rangle \quad |e_2\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

These states have the following representation in this new basis

$$\begin{aligned} |\alpha'\rangle &= (|e_1\rangle \langle e_1| + |e_2\rangle \langle e_2|) |\alpha\rangle \\ &= \frac{1}{\sqrt{2}} |e_1\rangle + |e_2\rangle \end{aligned}$$

$$\begin{aligned} |\beta'\rangle &= (|e_1\rangle \langle e_1| + |e_2\rangle \langle e_2|) |\beta\rangle \\ &= \frac{1}{\sqrt{2}} |e_1\rangle \end{aligned}$$

The norm is not preserved, because the change of basis matrix $|e_1\rangle \langle e_1| + |e_2\rangle \langle e_2|$ was not unitary. But it is clear that these states differ neither by a global or relative phase.

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Problem 2. *Problem 2.66*

Solution.

$$\begin{aligned}\langle\alpha| X_1 Z_2 |\alpha\rangle &= \frac{1}{2}(\langle 00| + \langle 11|)X_1 Z_2(|00\rangle + |11\rangle) \\ &= \frac{1}{2}(\langle 00| + \langle 11|)(|10\rangle - |01\rangle) = 0\end{aligned}$$

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Problem 3. *Problem 2.71*

Solution. ■

Problem 4. *Problem 2.72*

Solution. ■

Problem 5. *Problem 2.75*

Solution. ■

Problem 6. *Problem 2.79*

Solution. ■