Bayesian inference and memory in recurrent networks of spiking neurons

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Outline

1. A short note on deep learning
2. Deep generative models
3. Biologically inspired neural networks
4. Synaptic connectivity as an internal model

A brief survey of deep learning architectures

- Perceptrons e.g. MLPs for classification of vectorized data
- Convolutional neural networks (CNNs) for image classification, segmentation
- Recurrent neural networks (RNNs) for temporal data
- Generative adversarial networks (GANs) and autoencoders e.g. VAEs for generative modeling
- ...

which are all trained offline on known samples from some (perhaps very complicated) population distribution

Review of Bayesian inference

Recall Bayes theorem from fundamental probability theory

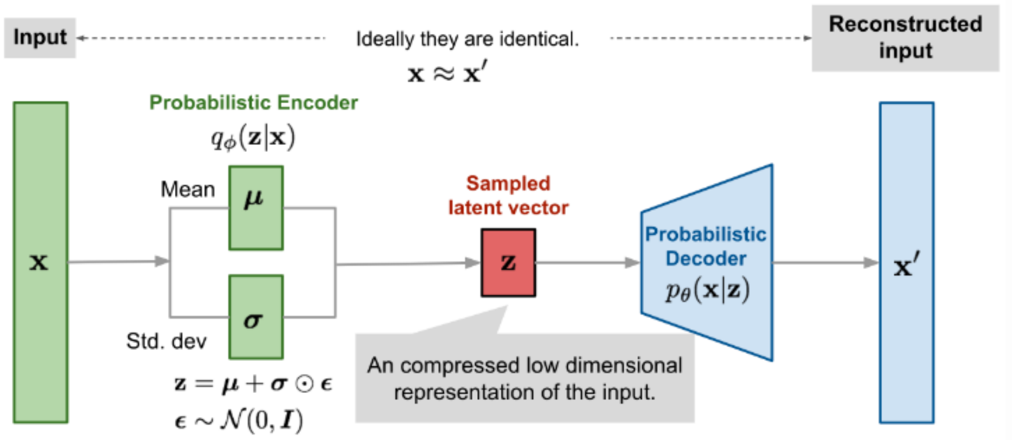
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{\int P(B|A)P(A)dA}$$

$P(A|B)$ is called the posterior, $P(B|A)$ the likelihood, $P(A)$ the prior, and $P(B)$ the evidence

$$P(B) = \int P(B|A)P(A)dA$$

The prior and likelihood are often known explicitly while calculating the evidence is often intractable. Monte-Carlo Markov Chain (MCMC) methods and Variational Inference offer solutions

Deep generative models: variational autoencoders (VAEs)



In essence, we *approximate* the true posterior $P(Z|X)$ with a neural network

Deep generative models: variational autoencoders (VAE)

When training a VAE we're concerned with the following problem:

$$\min_{\phi} \; \mathbb{E}_{x \sim Pop, z \sim P_\phi(z|x)} \left[\ln \frac{P_\phi(z|x)}{P(z)} - \ln P_\phi(x|z) \right] .$$

We can model $P_\phi(z|x)$ with an encoder and $P_\phi(x|z)$ with a decoder as follows:

$$P_\phi(z|x) = \mathcal{N}\left(\mu_{\phi,z}(x), \Sigma_{\phi,z}(x)\right)$$

$$P_\phi(x|z) = \mathcal{N}\left(\mu_{\phi,x}(z), \sigma^2 I\right) ,$$

where $\mu_{\phi,z}, \Sigma_{\phi,z}, \mu_{\phi,x}$ are neural networks, and $\Sigma_{\phi,z}(x)$ is diagonal.

Let $P(z)$ (the prior over z) to be $\mathcal{N}(0, I)$.

Using Monte-Carlo Markov Chain (MCMC) to sample the posterior

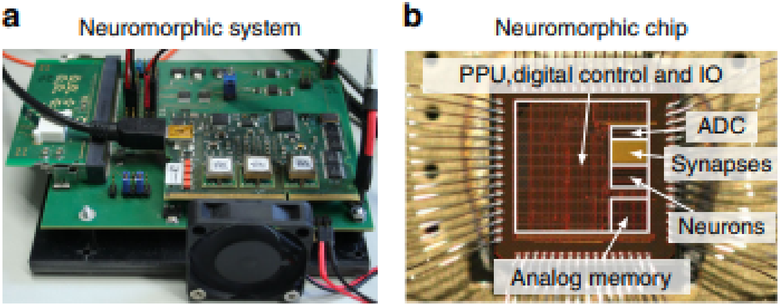
Monte Carlo methods estimate distributions by repeated sampling

If calculating $P(B)$ is intractable and we require samples from the posterior $P(A|B)$ we can use MCMC

A prominent hypothesis in neuroscience is that neurons use

SNNs can run on emerging dedicated hardware

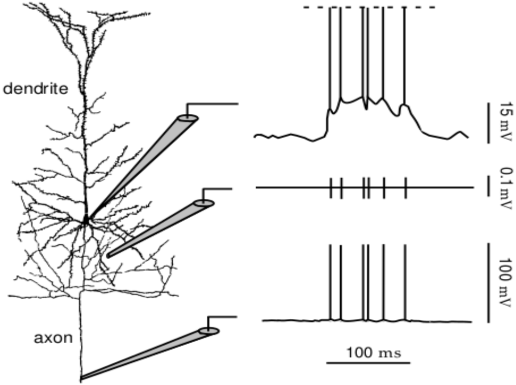
- The entire brain is estimated consume 10W of power
- Spiking networks (SNNs) perform computations in memory giving low-latency
- SNNs can in principle self-organize without backprop (unsupervised learning)



SNNs can encode information in the phase of neural responses

Thus a coding of analog variables by firing rates seems quite dubious in the context of fast cortical computations

The third generation of neural networks: spiking nets

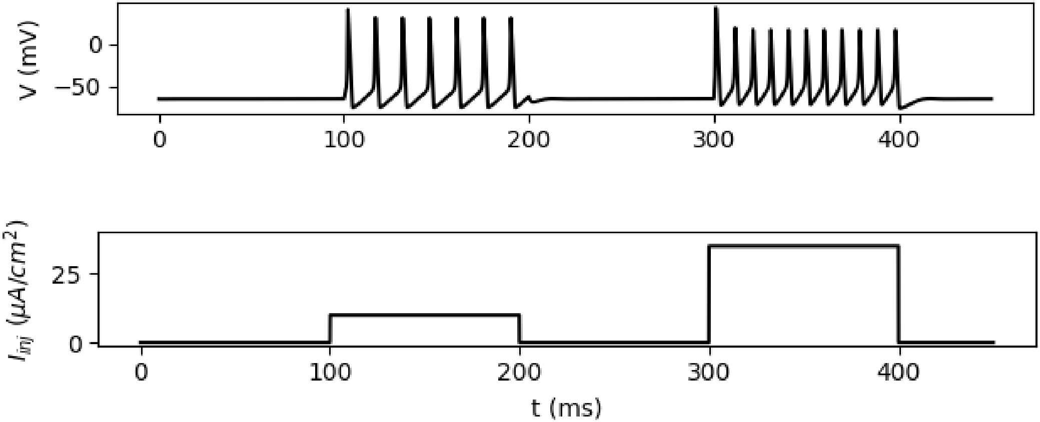


- $\sim$ 16 billion neurons in cortex
- A neuron receives on the order of 10^3 to 10^4 synaptic inputs
- Neurons communicate via action potentials in an all-or-nothing fashion

The third generation of neural networks: spiking nets

- Post-synaptic potentials (PSPs) allow pre-synaptic action potentials to change post-synaptic membrane potential
- PSPs can be positive or negative (excitatory or inhibitory)

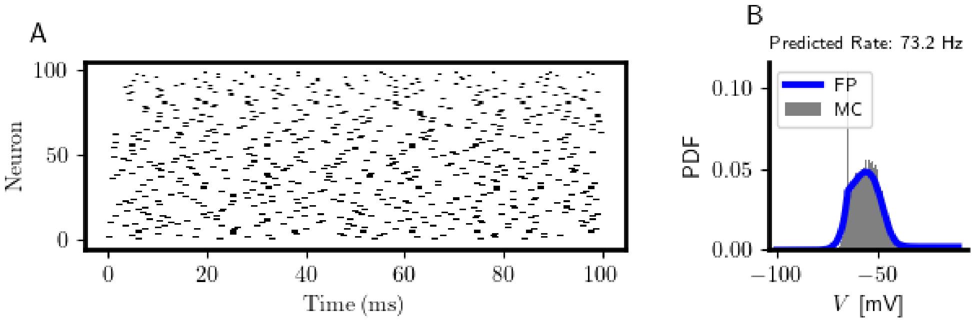
Integrate and fire (IF) neuron models



$$\tau \dot{V}(t) = g_\ell(E - V) + g_\ell \cdot \psi(V) + I(t)$$

Monte-Carlo simulation of uncoupled IF neurons

When $\psi(V) = g_\ell \Delta_T \exp\left(\frac{V - V_L}{\Delta_T}\right)$ we have the exponential integrate and fire model

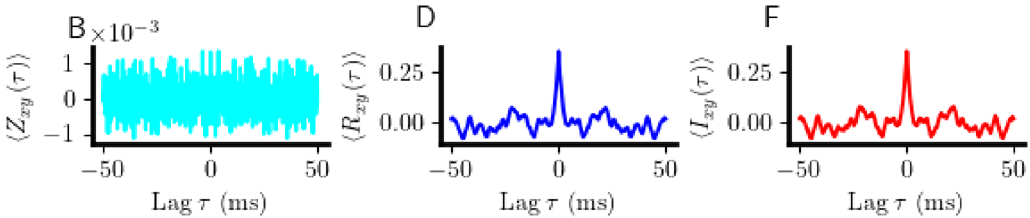


Langevin equations have a corresponding Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\sigma^2}{\tau}\frac{\partial^2 P}{\partial V^2} + \frac{\partial}{\partial V}\left(\frac{V - E + \psi}{\tau} P\right)$$

Synaptic coupling can induce correlations in spiking activity

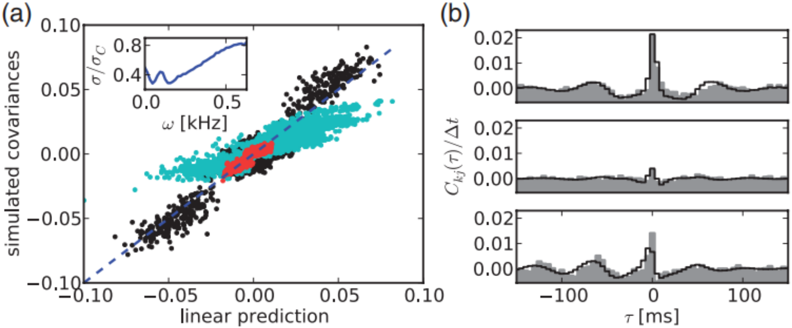
For special synaptic connectivity regimes dynamical variables can remain uncorrelated between neurons



Uncorrelated neural activity captures irregular spiking seen *in-vivo*

Predicting neuron correlations

The linear response of $r(t)$ allows us to also estimate the matrix of cross-correlations $C_{kj}(\tau)$ from the synaptic connectivity $\mathcal{C}$



This has important implications for brain-inspired machine learning