

TTIC 31230, Fundamentals of Deep Learning

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Loopy Belief Propagation (Loopy BP)

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We design an algorithm that is correct for tree graphs and use it on non-tree (loopy) graphs.

Belief Propagation on Trees

Belief Propagation is a message passing procedure (actually dynamic programming).

For each edge $\{i, j\}$ and possible value \tilde{y} for node i we define $Z_{j \rightarrow i}[c]$ to be the partition function for the subtree attached to i through j and with $\hat{y}[i]$ restricted to c .

The function $Z_{j \rightarrow i}$ on the possible values of node i is called the **message** from j to i .

The reverse direction message $Z_{i \rightarrow j}$ is defined similarly.

Dynamic Programming Computes the Messages

$$Z_{j \rightarrow i}[c] = \sum_{c'} e^{s_n[j, c'] + s_e[j, i, c', c]} \left(\prod_{k \in N(j), k \neq i} Z_{k \rightarrow j}[c'] \right)$$

Loopy BP

In a Loopy Graph we can initialize all messages $Z_{i \rightarrow j}[c] = 1$ and then repeating (until convergence) the updates

$$\tilde{Z}_{j \rightarrow i}[c] = \frac{1}{Z_{j \rightarrow i}} Z_{j \rightarrow i}[c] \quad Z_{j \rightarrow i} = \sum_c Z_{j \rightarrow i}[c]$$

$$Z_{j \rightarrow i}[c] = \sum_{c'} e^{s_n[j, c'] + s_e[j, i, c', c]} \left(\prod_{k \in N(j), k \neq i} \tilde{Z}_{k \rightarrow j}[c'] \right)$$

Computing Node Marginals from Messages

$$\begin{aligned} Z_i(c) &\doteq \sum_{\hat{y}: \hat{y}[i]=c} e^{s(\hat{y})} \\ &= e^{s_i[c]} \left(\prod_{j \in N(i)} Z_{j \rightarrow i}[c] \right) \\ \textcolor{red}{P_i(c)} &= Z_i(c)/Z, \quad Z = \sum_c Z_i(c) \end{aligned}$$

Computing Edge Marginals from Messages

$$\begin{aligned} Z_{i,j}(c, c') &\doteq \sum_{\hat{y}: \hat{y}[i]=c, \hat{y}[j]=c'} e^{s(\hat{y})} \\ &= e^{s_n[i,c]+s_n[j,c'] + s_e[i,j,c,c']} \\ &\quad \prod_{k \in N(i), k \neq j} Z_{k \rightarrow i}[c] \\ &\quad \prod_{k \in N(j), k \neq i} Z_{k \rightarrow j}[c'] \\ \textcolor{red}{P}_{i,j}(c, c') &= Z_{i,j}(c, c') / Z \quad Z = \sum_{c, c'} Z_{i,j}(c, c') \end{aligned}$$

END