Homework 6

Quantum Mechanics
October 28th, 2022

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Problem 1. *Problem 3.12 from Sakurai*

Solution.

In general the ensemble average of an operator $[A]$ is defined as

$$[A] = \sum_i w_i \langle \alpha_i | A | \alpha_i \rangle$$

where $\sum_i w_i = 1$

$$\begin{aligned}
[\sigma_x] &= a \langle + | \sigma_x | + \rangle + (1-a) \langle -; y | \sigma_x | -; y \rangle \\
&= a \langle + | (|+\rangle \langle - | + |-\rangle \langle + |) | + \rangle + (1-a) \langle -; y | (|+\rangle \langle - | + |-\rangle \langle + |) | -; y \rangle \\
&= 0
\end{aligned}$$

$$\begin{aligned}
[\sigma_y] &= a \langle + | \sigma_y | + \rangle + (1-a) \langle -; y | \sigma_y | -; y \rangle \\
&= ai \langle + | (|+\rangle \langle - | - |-\rangle \langle + |) | + \rangle + i(1-a) \langle -; y | (|+\rangle \langle - | - |-\rangle \langle + |) | -; y \rangle \\
&= i(1-a) \langle -; y | \left(-\frac{i}{\sqrt{2}} |+\rangle - \frac{1}{\sqrt{2}} |-\rangle \right) \\
&= -i(1-a) \langle -; y | +; y \rangle = 0
\end{aligned}$$

$$\begin{aligned}
[\sigma_z] &= a \langle + | (|-\rangle \langle - | - |+\rangle \langle + |) | + \rangle + i(1-a) \langle -; y | (|-\rangle \langle - | - |+\rangle \langle + |) | -; y \rangle \\
&= -a + i(1-a) \langle -; y | \left(-\frac{i}{\sqrt{2}} |+\rangle - \frac{1}{\sqrt{2}} |-\rangle \right)
\end{aligned}$$

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Problem 2. *Problem 3.13 from Sakurai*

Solution. ■

Problem 3. *Problem 3.14 from Sakurai*

Solution. ■

Problem 4. *Problem 3.15 from Sakurai*

Solution. ■

Problem 5. *Problem 3.16 from Sakurai*

Solution. ■

Problem 6. *Problem 3.40 from Sakurai*

Solution. ■