



Figure 1: Expectation values of position as a function of time for the infinite (left) and finite (right) square well

Project 2

Quantum Mechanics

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Part 1

We can work in a coordinate system centered on zero, and write

$$\begin{aligned}
 \langle x \rangle &= \sum_{a'} \sum_{a''} c_{a'}^* c_{a''} \langle a' | x | a'' \rangle \exp \left(\frac{-i(E_{a''} - E_{a'})t}{\hbar} \right) \\
 &= \frac{1}{2} \left(\langle 0 | x | 1 \rangle \exp \left(\frac{-i(E_1 - E_0)t}{\hbar} \right) + \langle 1 | x | 0 \rangle \exp \left(\frac{-i(E_0 - E_1)t}{\hbar} \right) \right) \\
 &= \beta \cos(\omega t)
 \end{aligned}$$

where $\beta = \langle 0 | x | 1 \rangle = \langle 1 | x | 0 \rangle$ (because x is Hermitian) and $\omega = (E_1 - E_0)/\hbar$. The angular frequency is higher for the finite square well because there is a large energy gap between the first excited state and the ground state (see the differential in the eigenvalue spectrum in Figure 1c).
