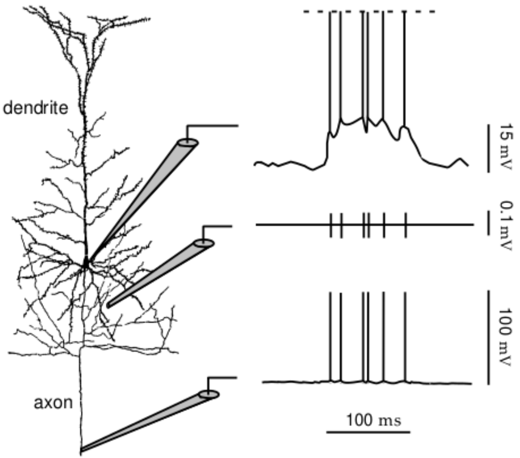
Stochastic computation in recurrent networks of spiking neurons

Clayton Seitz

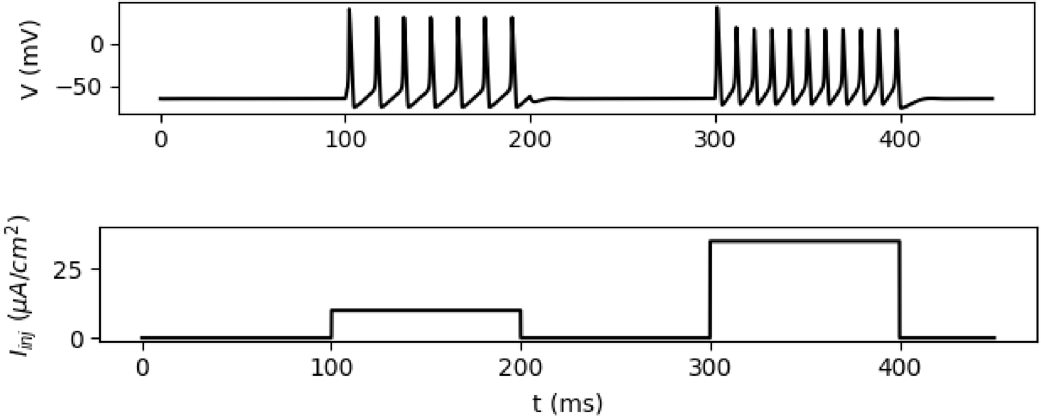
December 12, 2021

Introduction to networks of spiking neurons



- $\sim$ 16 billion neurons in cortex
- A neuron receives on the order of 10^3 to 10^4 synaptic inputs
- Neurons communicate via action potentials in an all-or-nothing fashion
- Post-synaptic potentials (PSPs) allow pre-synaptic action potentials to change post-synaptic membrane potential
- PSPs can be positive or negative (excitatory or inhibitory)

Integrate and fire (IF) neuron models

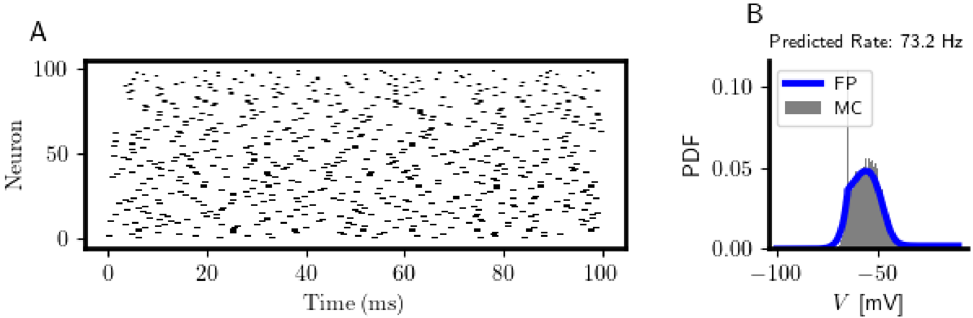


Monte-Carlo simulations of neurons often use a Langevin equation:

$$\tau \dot{V}(t) = g_\ell(E - V) + g_\ell \cdot \psi(V) + I(t)$$

Monte-Carlo simulation of an integrate and fire model

When $\psi(V) = g_\ell \Delta_T \exp\left(\frac{V - V_L}{\Delta_T}\right)$ we have the exponential integrate and fire model



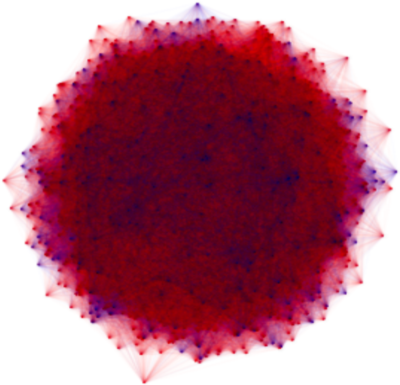
Langevin equations have a corresponding Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\sigma^2}{\tau}\frac{\partial^2 P}{\partial V^2} + \frac{\partial}{\partial V}\left(\frac{V - E + \psi}{\tau} P\right)$$

which we can occasionally solve numerically

Synaptic coupling can induce correlations in spiking activity

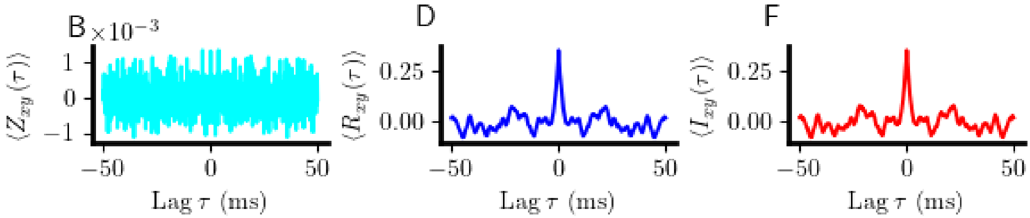
Figure: $N = 1000$ densely coupled excitatory-inhibitory neurons



- When neurons are coupled $I(t) = F(t) + R(t)$
- Non-trivial correlations then can arise from correlations in $F(t)$ and $R(t)$.
- Dynamics of $I(t)$ depend strongly on the connectivity matrix $\mathcal{C}$
- $\mathcal{C}$ itself is dynamic (synaptic plasticity)

Synaptic coupling can induce correlations in spiking activity

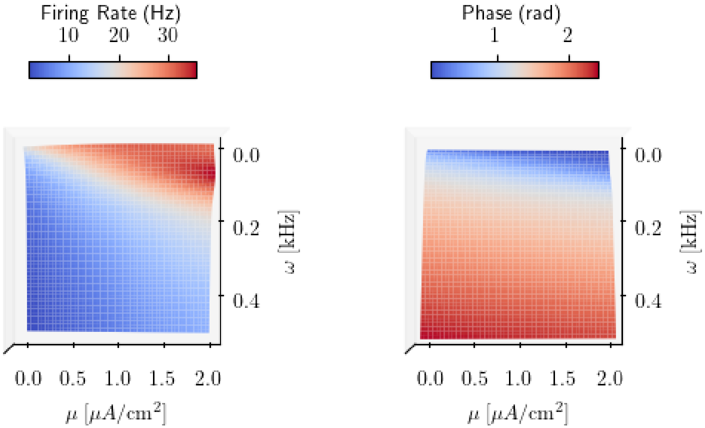
- For special $\mathcal{C}$, dynamical variables can remain uncorrelated between neurons



- Uncorrelated neural activity captures irregular spiking seen *in-vivo*
- However correlated activity is thought to be fundamental to the neural code

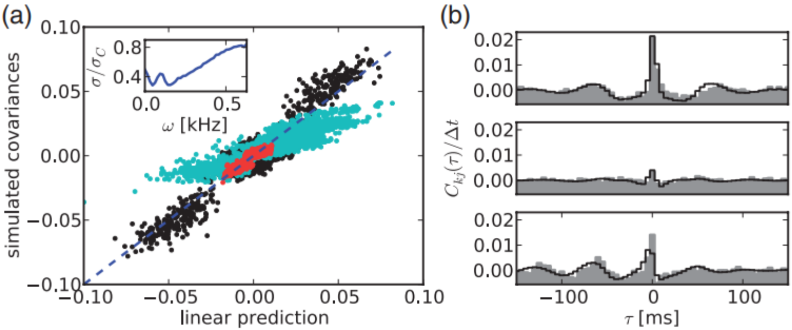
Linear response of the EIF model

- Neurons are often modeled as heterogeneous Poisson processes
- Heterogeneous Poisson processes have a time-dependent rate $r(t)$, which has a frequency response



Predicting neuron correlations

The linear response of $r(t)$ allows us to also estimate the matrix of cross-correlations $C_{kj}(\tau)$ from the synaptic connectivity $\mathcal{C}$



This has important implications for brain-inspired machine learning