TTIC 31230, Fundamentals of Deep Learning

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Deep Graphical Models

aka, Energy Based Models

Distributions on Exponentially Large Sets

$$\Phi^* = \operatorname*{argmin}_{\Phi} E_{(x,y)\sim\mathrm{Pop}} \; -\ln \; P(y|x)$$

$$\Phi^* = \operatorname*{argmin}_{\Phi} E_{y\sim\mathrm{Pop}} \; -\ln \; P(y)$$

The structured case: $y \in \mathcal{Y}$ where $\mathcal{Y}$ is discrete but iteration over $\hat{y} \in \mathcal{Y}$ is infeasible.

Semantic Segmentation

We want to assign each pixel to one of Y semantic classes.

For example "person", "car", "building", "sky" or "other".

Constructing a Graph

We construct a graph whose nodes are the pixels and where there is an edges between each pixel and its four nearest neighboring pixels.

Labeling the Nodes of a Graph

$\hat{y}$ assigns a semantic class $\hat{y}[n]$ to each node (pixel) n.

We assign a score to $\hat{y}$ by assigning a score to each node and each edge of the graph.

$$s(\hat{y}) = \underbrace{\sum_{n \in \mathrm{Nodes}} s^N[n, \hat{y}[n]]}_{\text{Node Scores}} + \underbrace{\sum_{\langle n,m \rangle \in \mathrm{Edges}} s^E[\langle n, m\rangle, \hat{y}[n], \hat{y}[m]]}_{\text{Edge Scores}}$$

Using Deep Networks

For input x we use a network to compute the score tensors.

$$s^N[N, Y] = f^N_\Phi(x)$$

$$s^E[E, Y, Y] = f^E_\Phi(x)$$

Exponential Softmax

$$\text{for } \hat{y} \quad s(\hat{y}) \;=\; \sum_n \; s^N[n, \hat{y}[n]] + \sum_{\langle n,\, m\rangle \in \text{Edges}} \; s^E[\langle n, m\rangle, \hat{y}[n],\; \hat{y}[m]]$$

$$\text{for } \hat{y} \;\; P_s(\hat{y}) \;=\; \text{softmax}_{\hat{y}} \;\; s(\hat{y}) \quad \text{all possible } \hat{y}$$

$$\mathcal{L} \;=\; -\ln P_s(y) \qquad \text{gold label (training label) } y$$

Exponential Softmax is Typically Intractable

$\hat{y}$ assigns a label $\hat{y}[n]$ to each node n.

$s(\hat{y})$ is defined by a sum over node and edge tensor scores.

$P_s(\hat{y})$ is defined by an exponential softmax over $s(\hat{y})$.

Computing Z in general is #P hard (there is an easy direct reduction from SAT).

Compactly Representing Scores on Exponentially Many Labels

The tensor $s^N[N, Y]$ holds NY scores.

The tensor $s^E[E, Y, Y]$ holds EY^2 scores where e ranges over edges $\langle n, m \rangle \in \text{Edges}$.

Back-Propagation Through Exponential Softmax

$$\begin{aligned} s^N[I,Y] &= f^N_\Phi(x) \\ s^E[E,Y,Y] &= f^E_\Phi(x) \end{aligned}$$

$$s(\hat{y}) = \sum_n s^N[n, \hat{y}[n]] + \sum_{\langle n,m\rangle \in \mathrm{Edges}} s^E[\langle n,m\rangle, \hat{y}[n],\ \hat{y}[m]]$$

$$P_s(\hat{y}) = \operatorname*{softmax}_{\hat{y}}\ s(\hat{y}) \quad \text{all possible } \hat{y}$$

$$\mathcal{L} = -\ln P_s(y) \quad \text{gold label } y$$

We want the gradients $s^N.\mathrm{grad}[N,Y]$ and $s^E.\mathrm{grad}[E,Y,Y]$.

END