

Eigenvectors and Eigenvalues

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1 Eigenvectors and Eigenvalues

For a $N \times N$ matrix we have a maximum of N eigenvectors and N eigenvalues. The definition of an eigenvector is that it satisfies the matrix equation

$$A\vec{e} = \lambda\vec{e}$$

Say we would like to find the \vec{e} and λ satisfy this equation for a particular A . For small N , we can find these eigenvectors and their corresponding eigenvalues relatively easily. For example, when $N = 2$

$$A\vec{e} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \lambda \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

This matrix equation represents the following system of equations

$$ae_1 + be_2 = \lambda e_1$$

$$ce_1 + de_2 = \lambda e_2$$

We have two equations and two unknowns so we can solve it easily. We want e_1 and e_2 in terms of a, b, c, d

$$ae_1 + be_2 = \lambda e_1$$
$$c \frac{be_2}{\lambda - a} + de_2 = \lambda e_2$$