Detailed Balance

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1 Detailed Balance

This note will discuss the notion of *detailed balance* for Markov processes. Detailed balance is a property of a time-dependent probability density for which the net probability current is zero and in thus has a form that is independent of time. Under such conditions, we say that the density is stationary or at *equilibrium*. This concept has many important applications, for example in Markov Chain Monte Carlo (MCMC) algorithms, we design a Markov chain whose stationary distribution is a target distribution which we cannot sample from directly. Other examples come from thermodynamics and statistical mechanics, where detailed balance is synonymous with *reversibility* of a thermodynamic system.

We will start with the case where the phase space Ω of our system is discrete which implies that the density $P(\Omega)$ has finite support.