Problem Set 2

Information and Coding Theory

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Problem 0.1. *Find tight upper and lower bounds on two extremely biased coins where the first coin is distributed according to*

$$P = \begin{cases} 0 & \epsilon \\ 1 & 1-\epsilon \end{cases}$$

and the second is distributed according to

$$Q = \begin{cases} 0 & 2\epsilon \\ 1 & 1-2\epsilon \end{cases}$$

Solution. I will assume that distinguishing the two coins means that, given a sequence of n flips, we can say whether it is coin P or coin Q 90 percent of the time. Setting this threshold gives

$$D(P||Q) \leq \frac{1}{2\ln 2}||P-Q||_1^2 = \frac{1}{2\ln 2}\left(\frac{8}{5}\right)^2$$

To start, we write out the KL-Divergence between the distributions P and Q for a sequence of n coin tosses

$$\begin{aligned} D(P(x_1 \dots x_n)||Q(x_1 \dots x_n)) &= nD(P||Q) \\ &= n\left[P(0)\log\frac{1}{Q(0)} + P(1)\log\frac{1}{Q(1)}\right] \\ &= n\left[\epsilon\log\frac{1}{2\epsilon} + (1-\epsilon)\log\frac{1}{1-2\epsilon}\right] \end{aligned}$$

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