

# Homework 8

Quantum Mechanics

November 8, 2022

C SEITZ

## Problem 1. 5.27

Solution.

$$\frac{\langle \tilde{0} | H | \tilde{0} \rangle}{\langle \tilde{0} | \tilde{0} \rangle} \geq E_0$$

The denominator is easy to compute

$$2 \int_{-\infty}^0 e^{\beta x} dx = \frac{1}{\beta}$$

The numerator

$$\begin{aligned} \langle \tilde{0} | H | \tilde{0} \rangle &= \int_{-\infty}^{\infty} \psi^*(x) H \psi(x) dx \\ &= \int_{-\infty}^0 \psi^*(x) H \psi(x) dx + \int_0^{\infty} \psi^*(x) H \psi(x) dx \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^0 \psi^*(x) H \psi(x) dx &= \int_{-\infty}^0 -e^{\beta x} \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} e^{\beta x} + \frac{1}{2} m \omega^2 x^2 e^{2\beta x} dx \\ &= \int_{-\infty}^0 e^{2\beta x} \left( \frac{1}{2} m \omega^2 x^2 - \frac{\hbar^2 \beta^2}{2m} \right) dx \\ &= \left|_{-\infty}^0 \frac{1}{2} m \omega^2 \frac{e^{2\beta x} (1 - 2\beta x + 2\beta^2 x^2)}{4\beta^3} - e^{2\beta x} \frac{\hbar^2 \beta}{4m} \right. \\ &= \frac{1}{2} m \omega^2 \frac{1}{4\beta^3} - \frac{\hbar^2 \beta}{4m} \end{aligned}$$

$$\begin{aligned}
\int_0^\infty \psi^*(x) H \psi(x) dx &= \int_0^\infty -e^{-\beta x} \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} e^{-\beta x} + \frac{1}{2} m \omega^2 x^2 e^{-2\beta x} dx \\
&= \int_0^\infty e^{-2\beta x} \left( \frac{1}{2} m \omega^2 x^2 - \frac{\hbar^2 \beta^2}{2m} \right) dx \\
&= \left|_0^\infty \frac{1}{2} m \omega^2 \frac{e^{-2\beta x} (1 + 2\beta x + 2\beta^2 x^2)}{4\beta^3} - e^{-2\beta x} \frac{\hbar^2 \beta}{4m} \right. \\
&= \frac{1}{2} m \omega^2 \frac{1}{4\beta^3} - \frac{\hbar^2 \beta}{4m}
\end{aligned}$$

$$\bar{H} = \frac{\langle \tilde{0} | H | \tilde{0} \rangle}{\langle \tilde{0} | \tilde{0} \rangle} = \frac{m \omega^2}{4\beta^2} - \frac{\hbar^2 \beta^2}{2m}$$

$$\frac{d\bar{H}}{d\beta} = -\frac{m \omega^2}{4\beta} - \frac{\hbar^2 \beta}{m} = 0$$

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**Problem 2.** 5.29

**Solution.**

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**Problem 3.** 5.30

**Solution.**

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**Problem 4.** 5.32

**Solution.**

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**Problem 5.** 5.35

**Solution.**

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**Problem 6.** 5.36

**Solution.**

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