Neural dynamics of vision

A computational perspective

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Dedicated to Calvin and Hobbes.

Contents

Preface

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Structure of book

Each unit will focus on <SOMETHING>.

About the companion website

The website[1] for this file contains:

- A link to (freely downlodable) latest version of this document.
- Link to download LaTeX source for this document.
- Miscellaneous material (e.g. suggested readings etc).

Acknowledgements

- A special word of thanks goes to Professor Don Knuth[2] (for TEX) and Leslie Lamport[3] (for LATEX).
- I'll also like to thank Gummi[4] developers and LaTeXila[5] development team for their awesome LATEX editors.
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Amber Jain
`http://amberj.devio.us/`

[1] `https://github.com/amberj/latex-book-template`
[2] `http://www-cs-faculty.stanford.edu/~uno/`
[3] `http://www.lamport.org/`
[4] `http://gummi.midnightcoding.org/`
[5] `http://projects.gnome.org/latexila/`

1

Experimental facts of life

"This is a quote and I don't know who said this."
– Author's name, *Source of this quote*

1.1 Section heading

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Fundamental information and coding theory

"This is a quote and I don't know who said this."
– Author's name, *Source of this quote*

2.1 Section heading

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Microscopy and image analysis for imaging neural ensembles

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Error correcting codes in neural networks

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– Author's name, *Source of this quote*

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Natural image statistics and Gabor analysis

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Natural image representation in the visual cortex

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Semantic coding

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– Author's name, *Source of this quote*

7.1 Section heading

8

Information and Coding Theory

> *"We may have knowledge of the past but cannot control it; we may control the future but have no knowledge of it"*
>
> – Claude Shannon

8.1 Introduction

Information theory is a framework first introduced by Claude Shannon's seminal paper *A mathematical theory of communication* published in 1948. At it's core, information theory makes the intuitive concept of *information* mathematically rigorous and forms the foundation of many modern communication systems. Neural circuits in the visual system are an especially interesting example of such a communication system. Therefore, in this section, the information theoretic concepts necessary for studying neural circuits are introduced.

8.2 Entropy

The concept of entropy is not exclusive to information theory; rather, it is used widely in disciplines such as physics and mathematical statistics. In fact, entropy was originally defined in statistical physics when Ludwig Boltzmann gave a statistical description of a thermodynamic system of particles. Since this is arguably the more intuitive path as opposed an entirely mathematical description, I will follow a similar line of reasoning in the following paragraphs.

In every application, the entropy $\mathbf{H}$ is a measure of uncertainty or how much information is contained in a random variable x. In information theory, the entropy is a property of a probability distribution of a random variable

$P(x)$ where x can take on continuous or discrete values. For the discrete case, we can express the entropy in bits

$$\mathbf{H} = -\sum_{x \in S} P(x) \log P(x) \tag{8.1}$$

where the set S spans the entire space of possible discrete values of x. Notice that $\mathbf{H} \geq 0$ since $P(x) \leq 1$ and therefore $\log P(x) \leq 0$ for all x. We might guess that the $P(x)$ with maximum entropy is the uniform distribution and to prove that we need to introduce a famous inequality.

8.2.1 Jensen's Inequality

Jensen's inequality is a statement about convexity. Consider a binary variable x that takes the value 0 with probability α and value 1 with probability $1-\alpha$.

$$x = \begin{cases} 0 & \alpha \\ 1 & 1-\alpha \end{cases}$$

A function f of the variable x is said to be *convex* if the following inequality holds

$$\alpha f(x) + (1-\alpha) f(y) \leq f(\alpha x + (1-\alpha) y)$$

which when generalized for an arbitrary random variable x forms Jensen's inequality

$$\mathbf{E}[f(x)] \leq f(\mathbf{E}[x]) \tag{8.2}$$

and if we flip the inequality we call the function *concave*.

$$\begin{aligned} \mathbf{H} &= -\sum_{x \in S} P(x) \log P(x) \\ &= \sum_{x \in S} \frac{1}{N} \log N = \log N \end{aligned}$$

We have now shown that the upper bound on the entropy for a random variable with N possible values is $\log N$.

8.2.2 Kraft's Inequality

Kraft's inequality is a constraint on prefix-free codes. A code is prefix free if and only-if the following statement is true

$$\sum_i 2^{-l_i} \leq 1 \tag{8.3}$$

for code lengths l_i.

8.2.3 Example 1: Applying Jensen's Inequality

Let's consider a function $f : \mathbb{R} \to \mathbb{R}$. Using Jensen's inequality, we can prove that $f = x^2$ or $f = x \log x$ are convex functions. Let's begin by applying it to x^2 for a general normalized probability distribution $p(x)$.

$$\begin{aligned}
\int p(x)f(x)dx &= \int x^2 p(x)dx \\
&= x^2 - 2\int x dx \\
&= 0 \leq x^2 \ \forall x
\end{aligned}$$

We have a similar proof for $f(x) = x \log x$

$$\begin{aligned}
\int p(x)f(x)dx &= \int x \log x \ p(x)dx \\
&= x \log x - \int \frac{d}{dx} x \log x \ dx \\
&= 0 \leq \mu \log \mu
\end{aligned}$$

where $\mu = \mathbf{E}[x] \geq 0$ since f is only defined on $[0, \infty]$.

8.2.4 Example 2: Proving Cauchy-Schwarz

A common form of the Cauchy-Schwarz inequality states that for two vectors u and v, we have

$$u \cdot v \leq \|u\| \, \|v\|$$