TTIC 31230, Fundamentals of Deep Learning

David McAllester, Winter 2020

Representing Functions with Deep Circuits

Circuit Complexity Theory

Representing Functions by Circuits

We can define functions of Boolean variables (the corners of $[0,1]^d$) with Boolean circuits or linear threshold circuits.

We can define functions of $[0,1]^d$ with feed-forward real-valued networks (Deep models).

Circuit Complexity Theory

Building on work of Ajtai, Sipser and others, Hastad proved (1987) that any bounded-depth Boolean circuit computing the parity function must have exponential size.

Matus Telgarsky recently gave some formal conditions under which shallow networks provably require exponentially more parameters than deeper networks (COLT 2016).

The Kaurnaugh Model of DNNs

The Karnaugh map, also known as the K-map, is a method to simplify boolean algebra expressions.

$$\begin{aligned} F(A,B,C,D) &= AC' + AB' + BCD' + AD' \\ &= (A+B)(A+C)(B'+C'+D')(A+D') \end{aligned}$$

The Kaurnaugh Model of DNNs

Many very different circuits compute the same function.

A Kaurnaugh Person Detector

Wheel or Face

Hand or Flower Hand or Flower

Leg or Tree Leg or Tree

The set of locally minimal models (circuits) could be vast (exponential) without damaging performance.

Is a Boolean circuit a distributed representation?

END