TTIC 31230, Fundamentals of Deep Learning

David McAllester, Winter 2020

Gaussian VAEs

A General Autoencoder

$$y \quad z \quad \hat{y}$$

In generall we have either $P_\Phi(z)$ for z discrete or $\hat{p}_\Phi(z)$ for z continuous.

A General Autoencoder

$$y \quad z \quad \hat{y}$$

For the continuous case with $p_\Phi(z|y)$ and $\hat{p}_\Phi(z)$ both Gaussian we can assume without loss of generality that

$$\hat{p}_\Phi(z) = \mathcal{N}(0, I)$$

Gaussian VAEs: The Reparameterization Trick

$$-\ln p_\Phi(y) \;\leq\; E_{z\sim\hat{p}_\Phi(z|y)}\;-\ln\frac{p_\Phi(z)p_\Phi(y|z)}{\hat{p}_\Phi(z|y)}$$

$$=\;E_\epsilon\;-\ln\frac{p_\Phi(z)p_\Phi(y|z)}{\hat{p}_\Phi(z|y)}\quad z := f_\Phi(y,\epsilon)$$

ϵ is parameter-independent noise.

This supports SGD: $\nabla_\Phi\; E_{y,\epsilon}\;[\ldots] = E_{y,\epsilon}\;\nabla_\Phi\;[\ldots]$

Gaussian VAEs

$$\Phi^* = \underset{\Phi}{\mathrm{argmin}}\, E_{y,\epsilon} \; - \ln \frac{p_\Phi(z)p_\Phi(y|z)}{\hat{p}_\Phi(z|y)}$$

$$z \;=\; z_\Phi(y) + \sigma_\Phi(y) \odot \epsilon \quad \epsilon \sim \mathcal{N}(0, I)$$

$$\hat{p}_\Phi(z[i]|y) \;=\; \mathcal{N}(z_\Phi(y)[i], \sigma_\Phi(y)[i])$$

$$p_\Phi(z[i]) \;=\; \mathcal{N}(\mu_p, \sigma_p[i]) \;\; \mathrm{WLOG} = \mathcal{N}(0, 1)$$

$$p_\Phi(y|z) \;=\; \mathcal{N}(y_\Phi(z),\; \sigma^2 I)$$

END