Stochastic computation in recurrent networks of spiking neurons

Clayton Seitz

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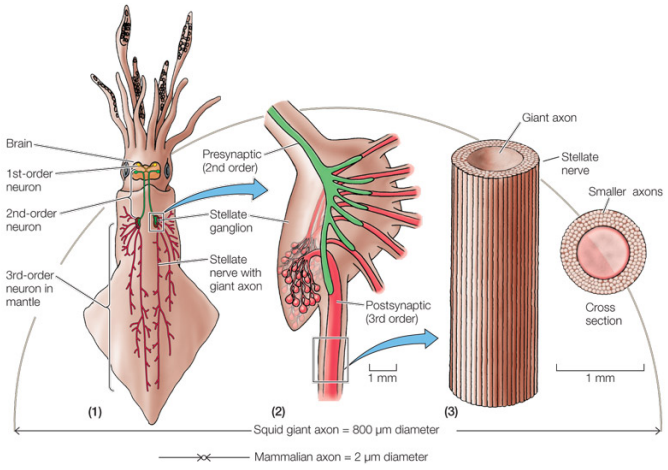
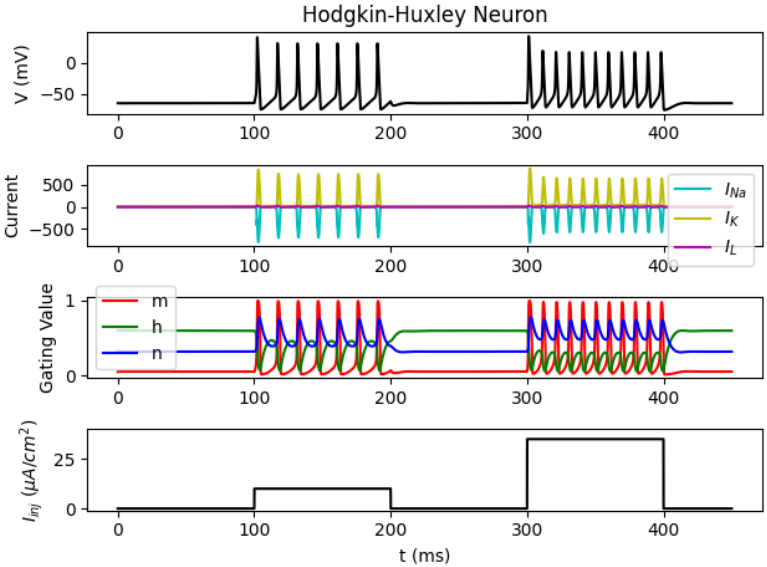
The squid giant axon

Hodkin and Huxley developed a mathematical model for nerve cell communication in 1952 using voltage data from the giant axon of a squid

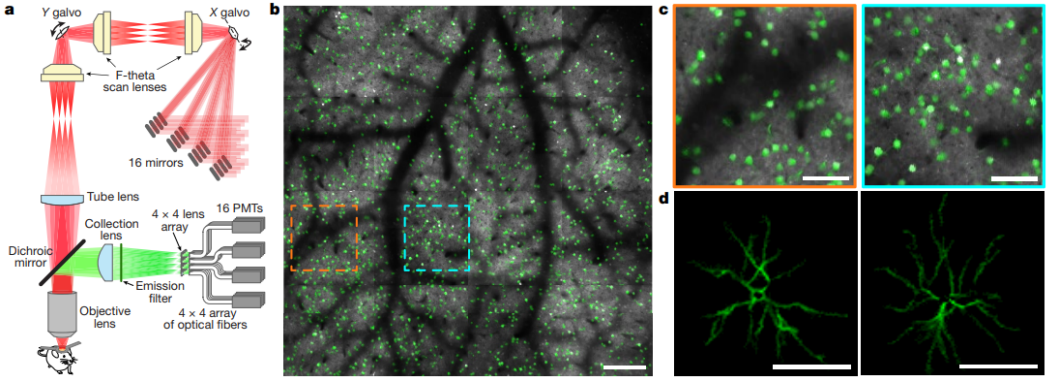


Na^+ and K^+ are the major charge carriers



Ca^{2+} sensors enable high-speed two-photon imaging

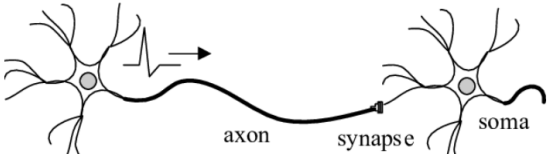
Animal models and experimental technologies have improved drastically

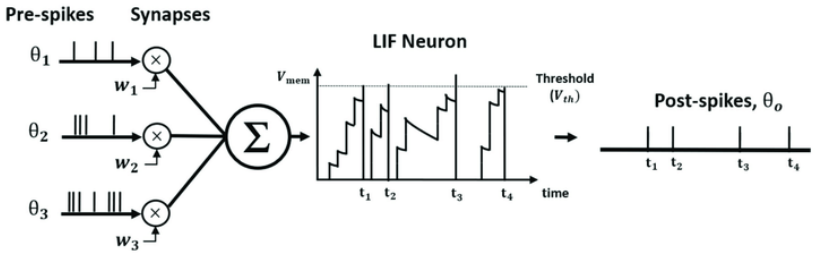


Scale bars: b, 250 um; c, d, 100 um

4mm^2 FOV at $\sim$ 8Hz

Spiking neural networks (SNN): integrate and fire models

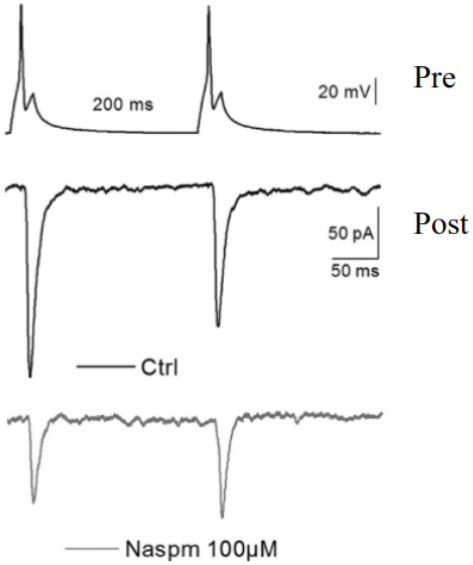




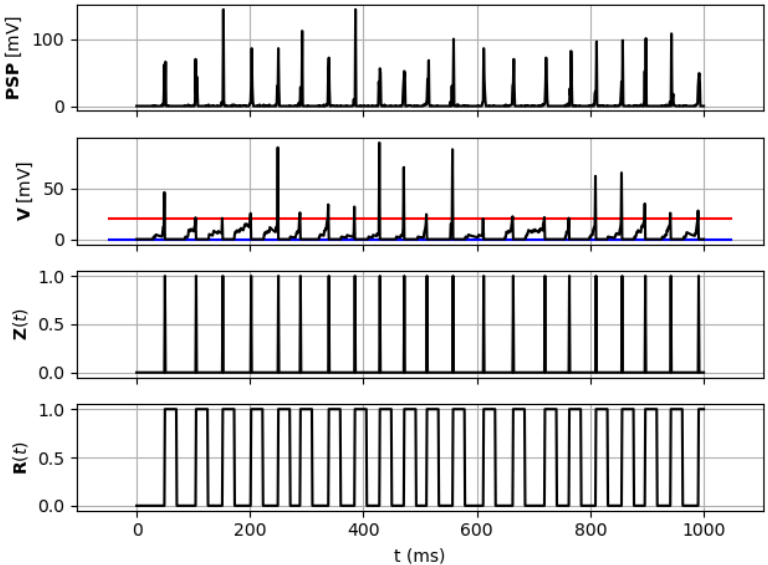
$$\tau \dot{V(t)} = -g_L V(t) + \sum_n w_n \theta_n(t)$$

Synaptic transmission

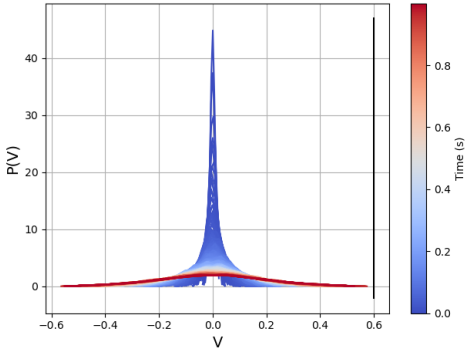
- Presynaptic action potentials contribute to post-synaptic currents
- The amplitude of post-synaptic currents change in time
- Amplitude changes thought to drive learning and memory



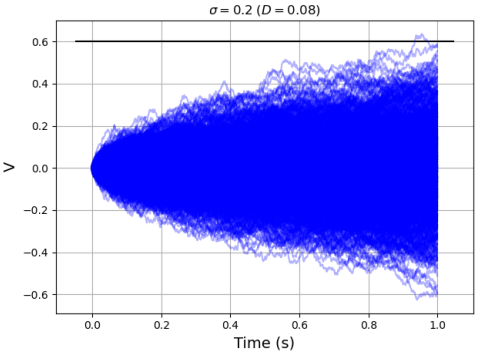
Synaptic current as a stochastic process



Fokker-Planck equation for Brownian motion



(a)

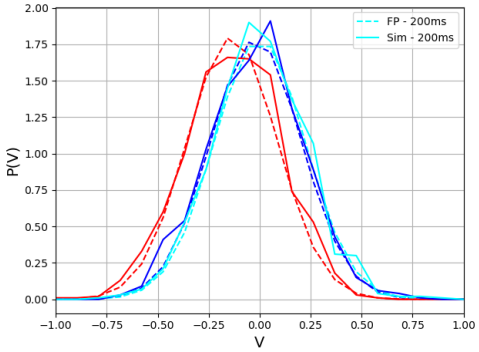


(b)

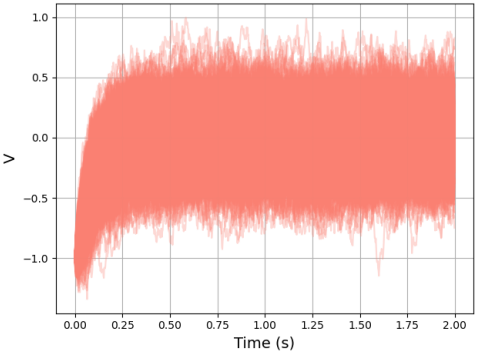
Predicting $I_n(t)$ is hard in complex networks. We instead solve for $P(V, t)$

$$\tau \frac{\partial P}{\partial t} = \sqrt{2D} \frac{\partial^2 P}{\partial V^2}$$

Fokker-Planck equation for Brownian motion



(c)



(d)

The Ornstein-Uhlenbeck (OU) process is a diffusion process with drift

$$\tau \frac{\partial P}{\partial t} = (\mu(t) - V)\frac{\partial P}{\partial V} + \sqrt{2D}\frac{\partial^2 P}{\partial V^2}$$