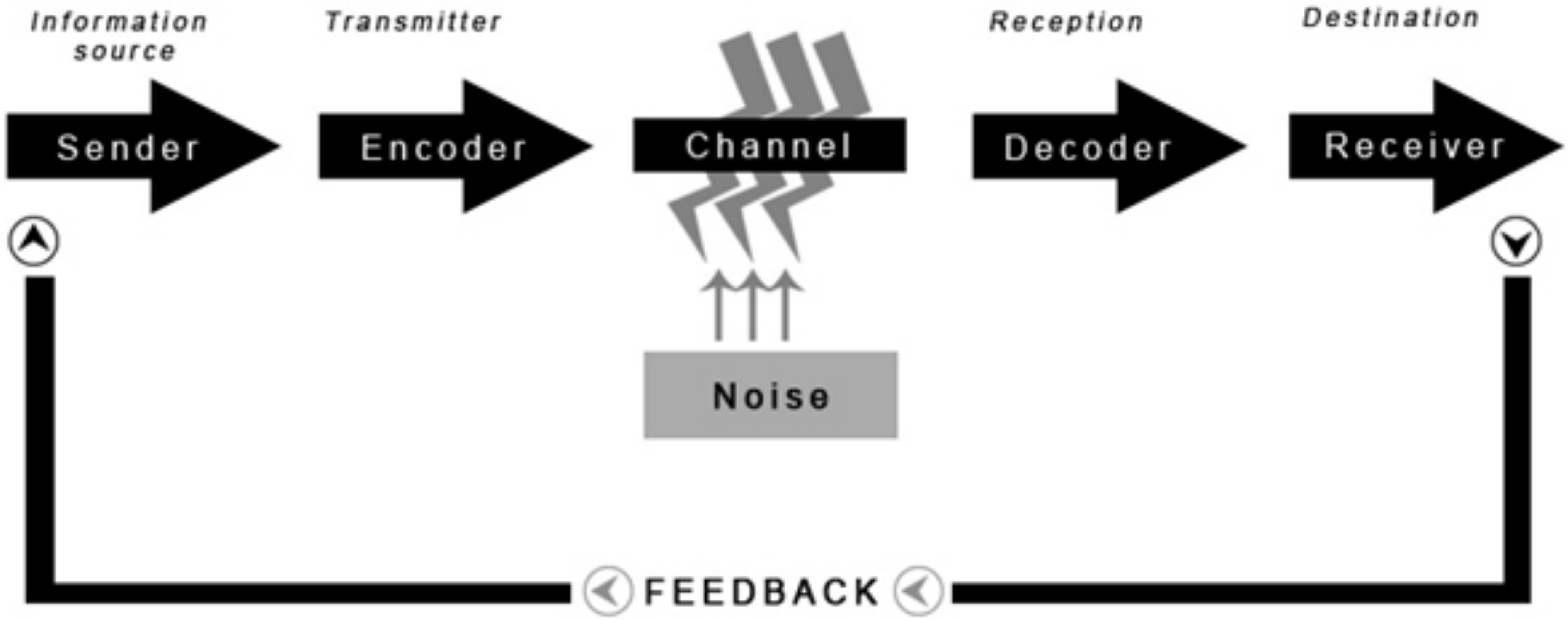
Lecture 7: *Information in spike trains*



SHANNON-WEAVER'S MODEL OF COMMUNICATION

Basics of probability theory

Product rule:

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$$= \sum_b P(a|b)P(b)$$

Bayes' rule:

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$
$$= \frac{P(b|a)P(a)}{\sum_{a'} P(b|a')P(a')}$$

Entropy as a measure of uncertainty:

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$$S(X) = -\sum_x p(x) \log_2(p(x))$$

Basics of information theory

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Additivity:

$$S(A,B) = S(A) + S(B)$$

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Chain rule:

$$S(A,B) = S(A) + S(B|A) = S(B) + S(A|B)$$

Recall: *basics of information theory*

Mutual information:

$$
\begin{aligned}
I(A;B) &= S(A) - S(A|B) \\
&= S(B) - S(B|A) \\
&= \sum_{a,b} P(a,b) \log_2 \left(\frac{P(a,b)}{P(a)P(b)} \right) \\
&= \sum_{a,b} P(a)P(b|a) \log_2 \left(\frac{P(b|a)}{P(b)} \right)
\end{aligned}
$$

Kullback-Liebler divergence (D_{KL}):

$$D_{\mathrm{KL}}(P, Q) = \sum_a P(a) \log_2 \frac{P(a)}{Q(a)}$$

Basics of information theory

Mutual information ≥ 0:

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Basics of information theory

Mutual information ≥ 0:

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$$D_{\mathrm{KL}}(P,Q) = \sum_a P(a)\log_2\frac{P(a)}{Q(a)}$$

Information as reduction in uncertainty:

$$I(B;A) = I(A;B)$$

$$I(A;B) = S(A) - S(A|B)$$

$$I(A;B) = S(A) - \langle S(A;b)\rangle_b$$

$$I(B;A) = S(B) - \langle S(B;a)\rangle_a$$

$$I(A;B) = \sum_{a,b} p(a,b)\log_2\left(\frac{p(a,b)}{p(a)p(b)}\right)$$

$$P(A,B) = P(A|B)P(B)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Entropy and information in a binary code:

$$H = -\sum_r P[r] \log_2 P[r]$$

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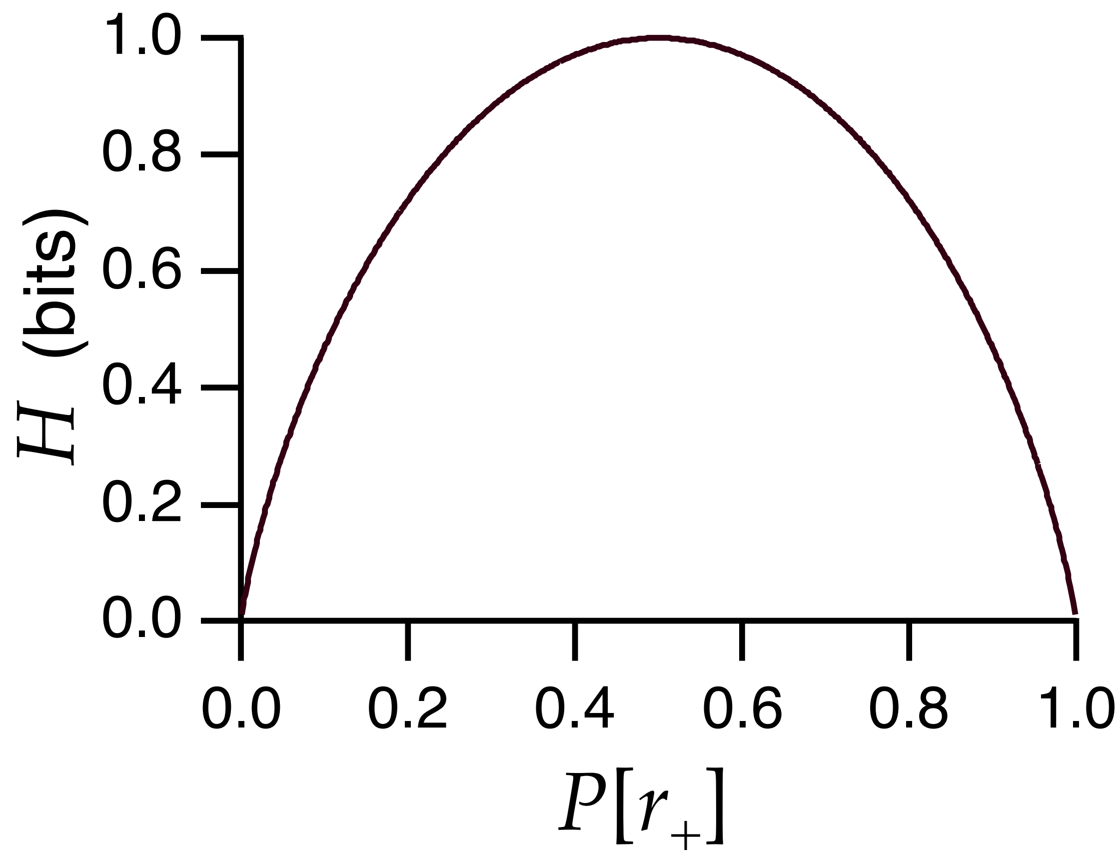
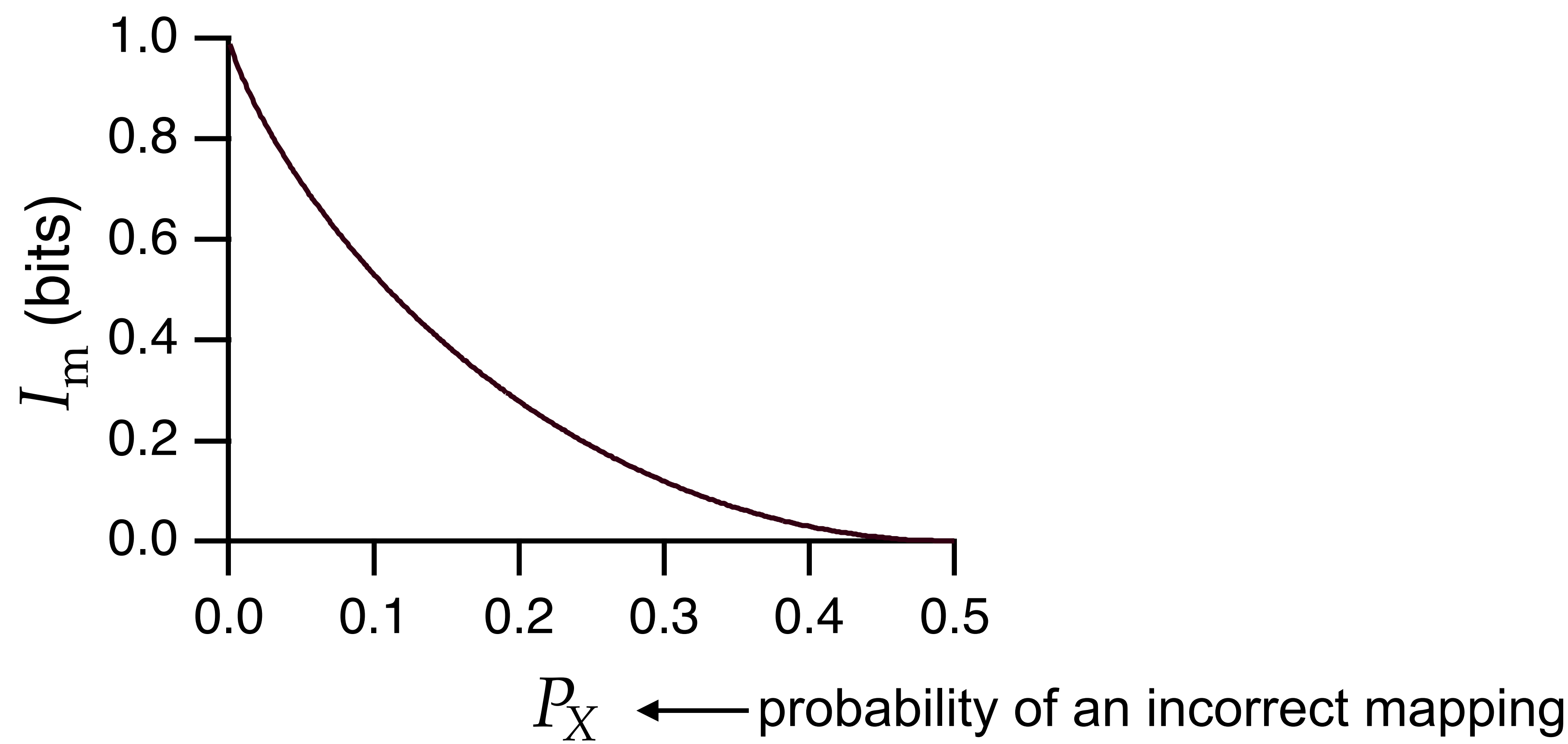
$$H = -(1 - P[r_+]) \log_2 (1 - P[r_+]) - P[r_+] \log_2 P[r_+]$$

Entropy in a binary code:



$$H = -(1 - P[r_+])\log_2(1 - P[r_+]) - P[r_+]\log_2 P[r_+]$$

Information in a binary code, binary stimulus:



$$I_m = 1 + (1 - P_X)\log_2(1 - P_X) + P_X \log_2 P_X$$

Entropy and information of a continuous variable:

$$
\begin{aligned}
H &= -\sum p[r]\Delta r \log_2(p[r]\Delta r) \\
&= -\sum p[r]\Delta r \log_2 p[r] - \log_2 \Delta r
\end{aligned}
$$

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$$\lim_{\Delta r \to 0} \left\{ H_{\mathrm{noise}} + \log_2 \Delta r \right\} = -\int ds \int dr\, p[s] p[r|s] \log_2 p[r|s]$$

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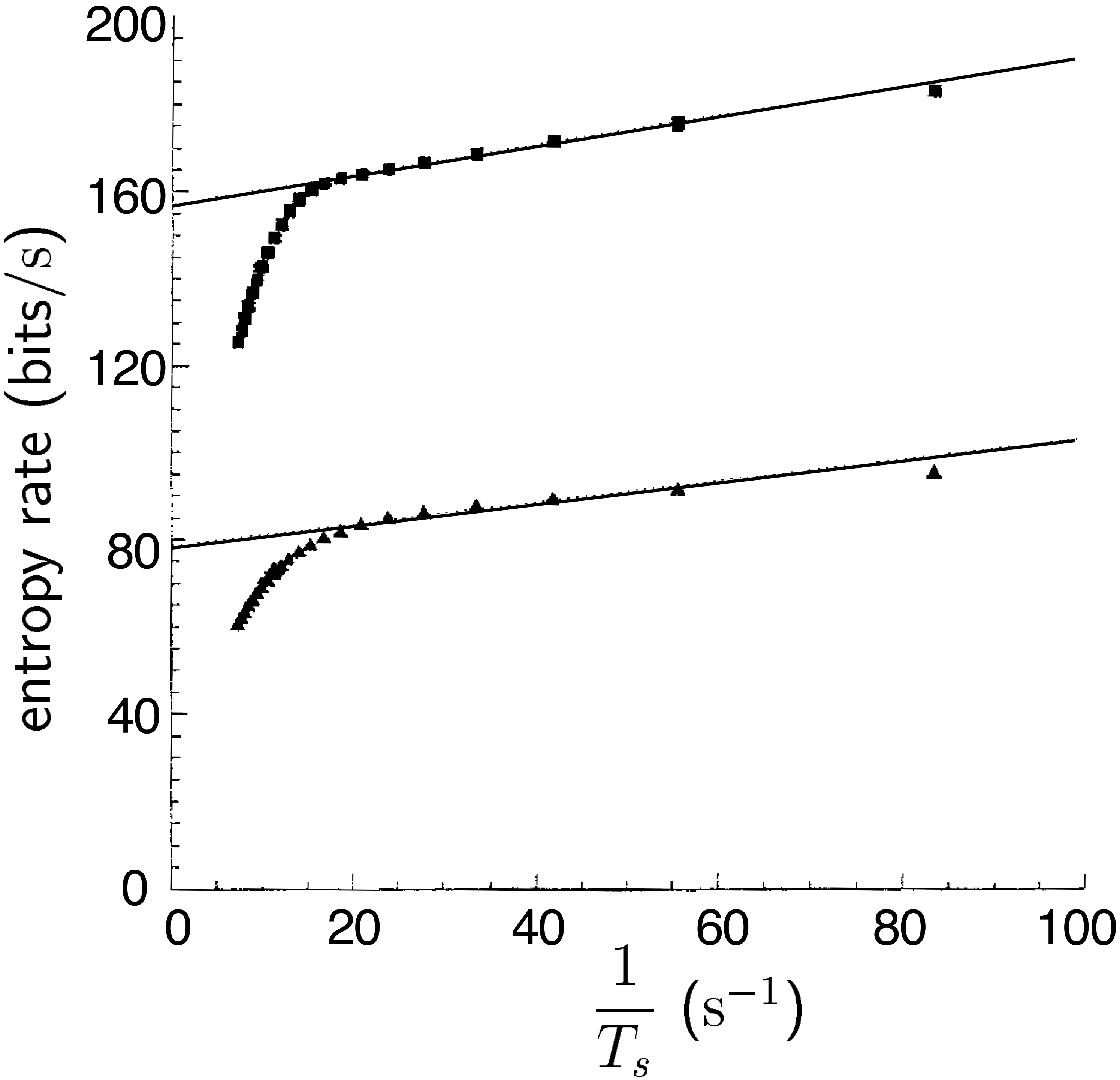
$$I_{\text{m}} = \int ds \int dr\, p[s] p[r|s] \log_2 \left(\frac{p[r|s]}{p[r]} \right)$$

Information in a spike train, enumerating states:

$$h = -\frac{1}{T_{\mathrm{s}}} \sum_{B} P[B] \log_2 P[B]$$

$$h_{\mathrm{noise}} = -\frac{\Delta t}{T} \sum_{t} \left(\frac{1}{T_s} \sum_{B} P[B(t)] \log_2 P[B(t)] \right)$$

Entropy estimation in the fly H1 neuron:



Information in single spikes:

$$I(1 \text{ spike; s}) = \frac{1}{T}\int_0^T dt\ \left(\frac{r(t)}{\bar{r}}\right)\log_2\left(\frac{r(t)}{\bar{r}}\right)$$

see Brenner et al., 2000

Information in single spikes:

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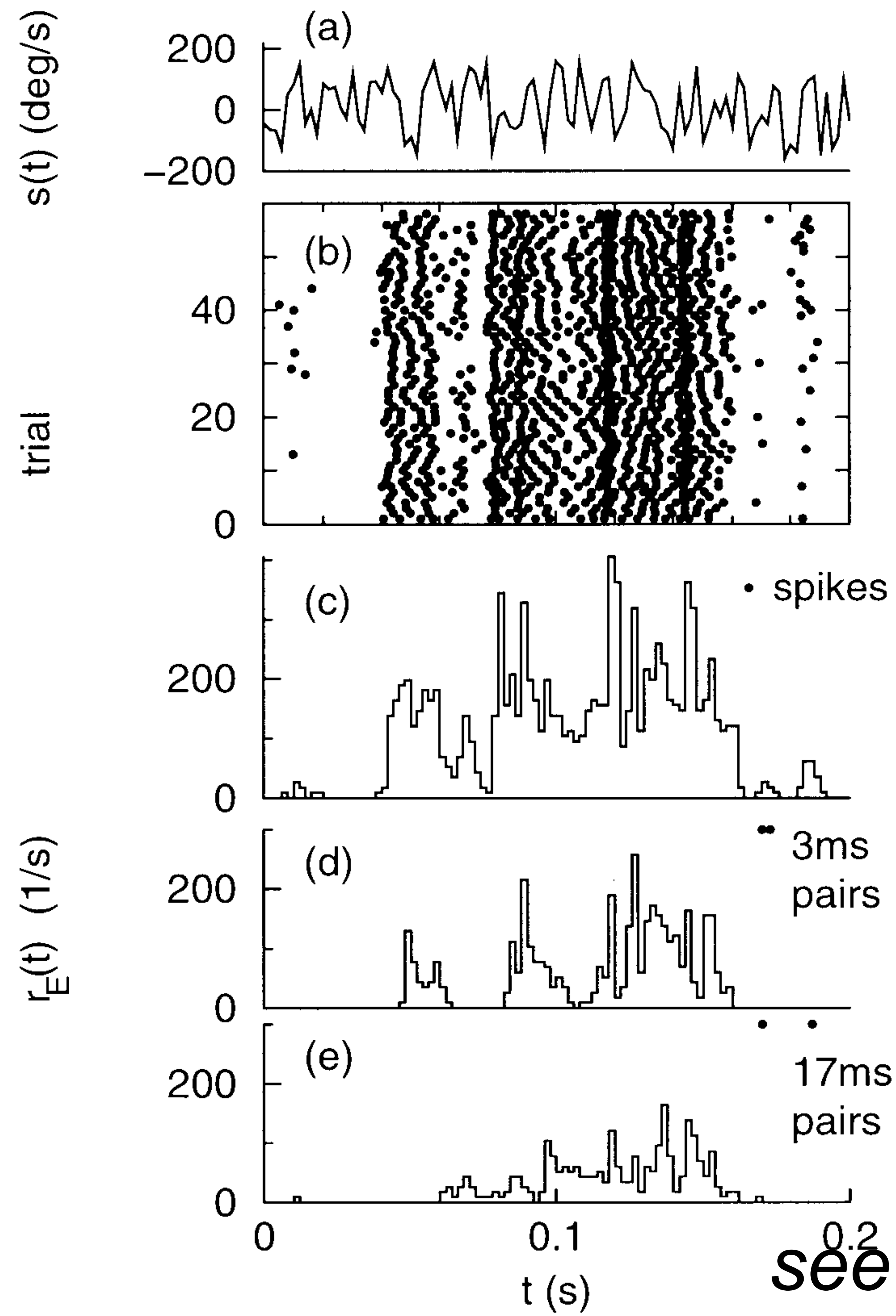
chalkboard interlude

see Brenner et al., 2000

Computing information from particular symbols:



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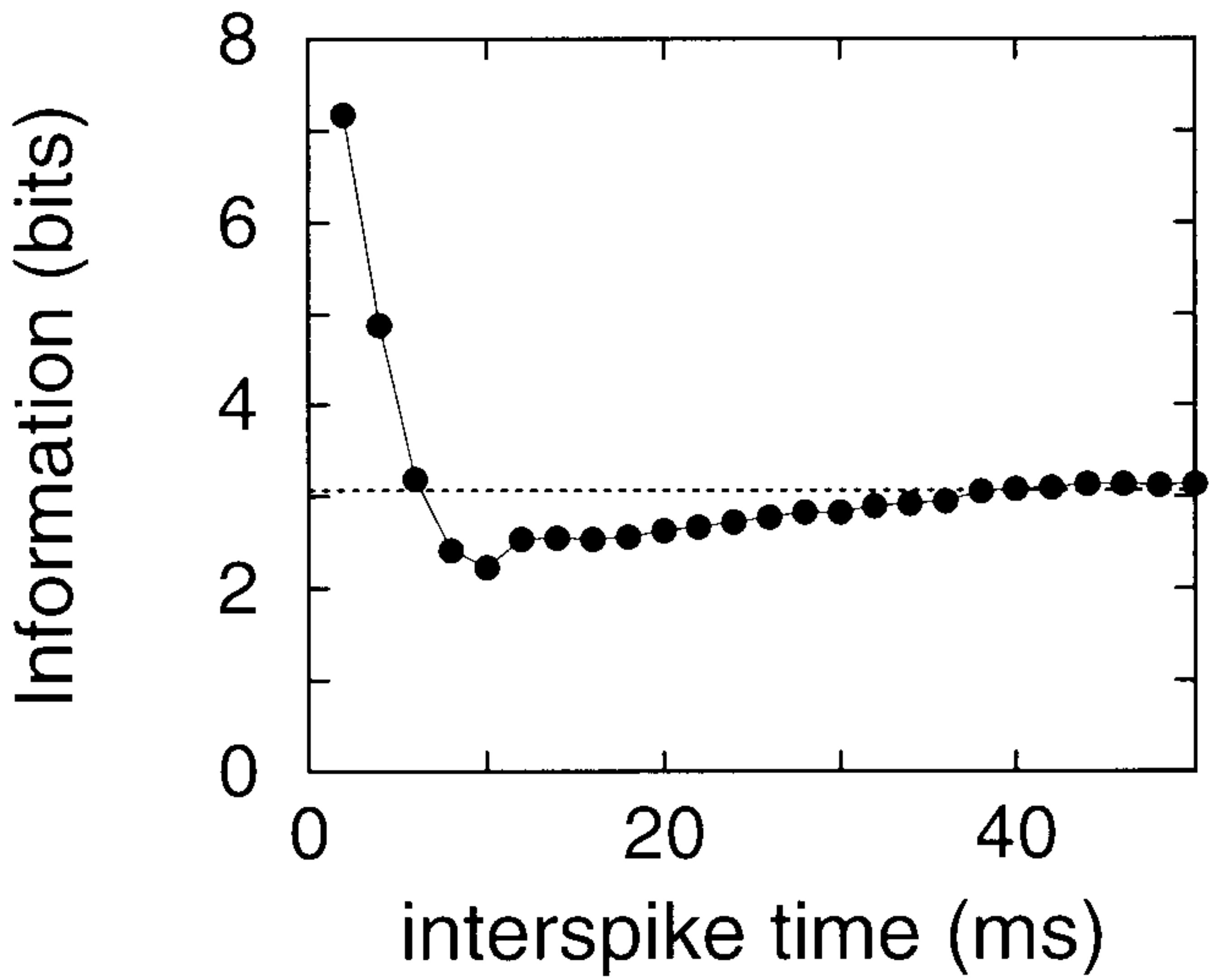
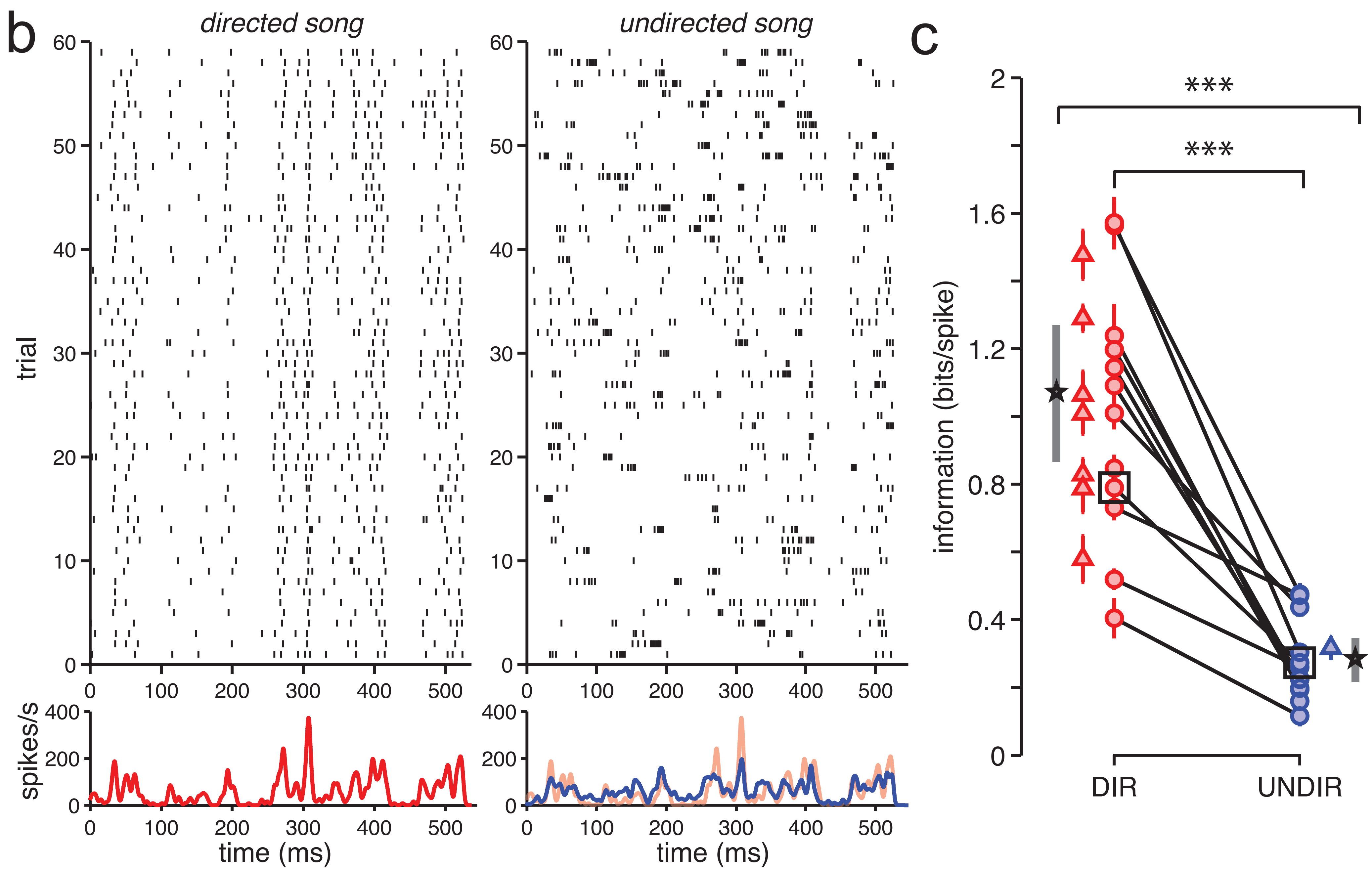
Computing the information in spike pairs:



Searching for the symbols in the neural code:



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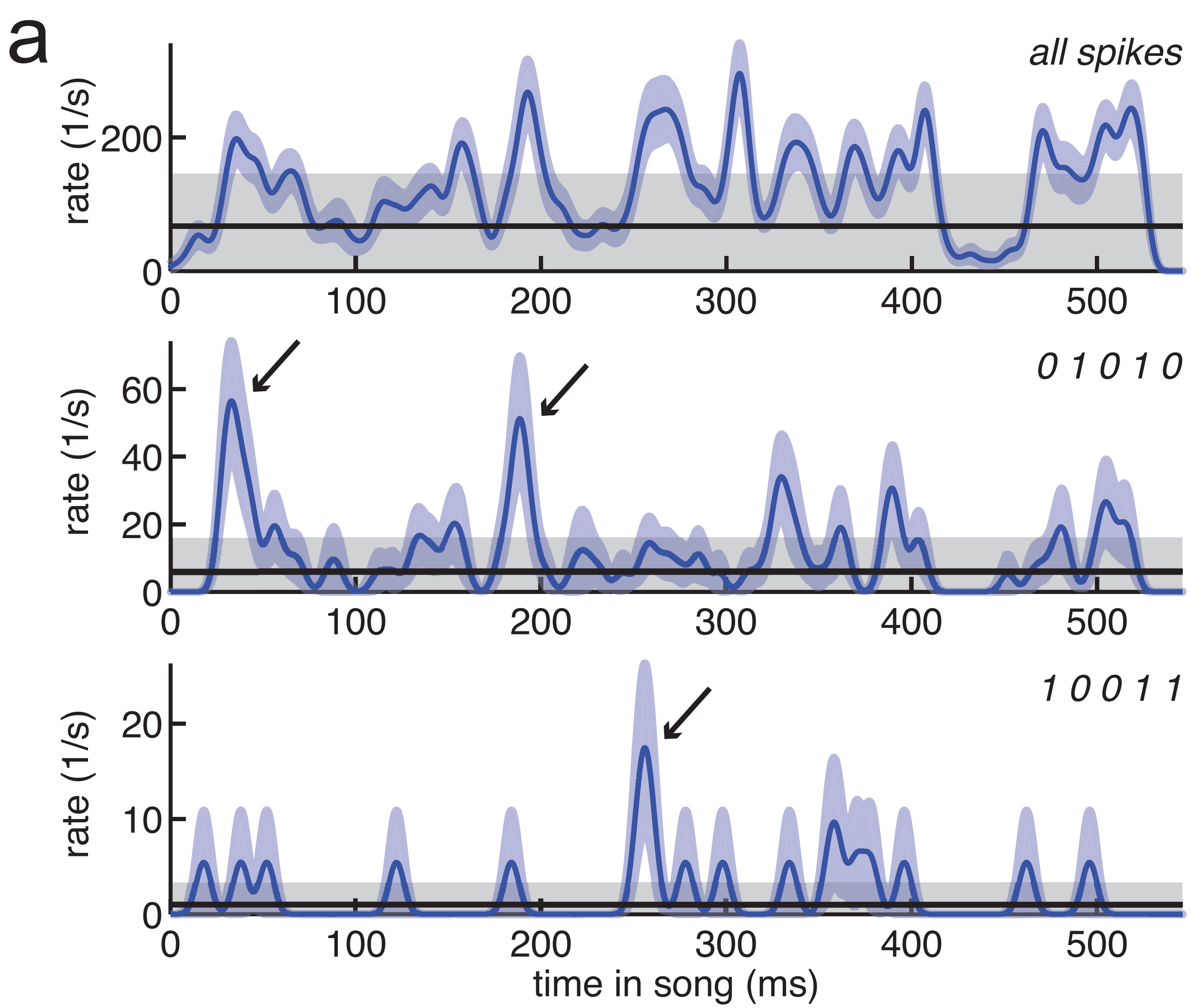

Searching for the symbols in the neural code: