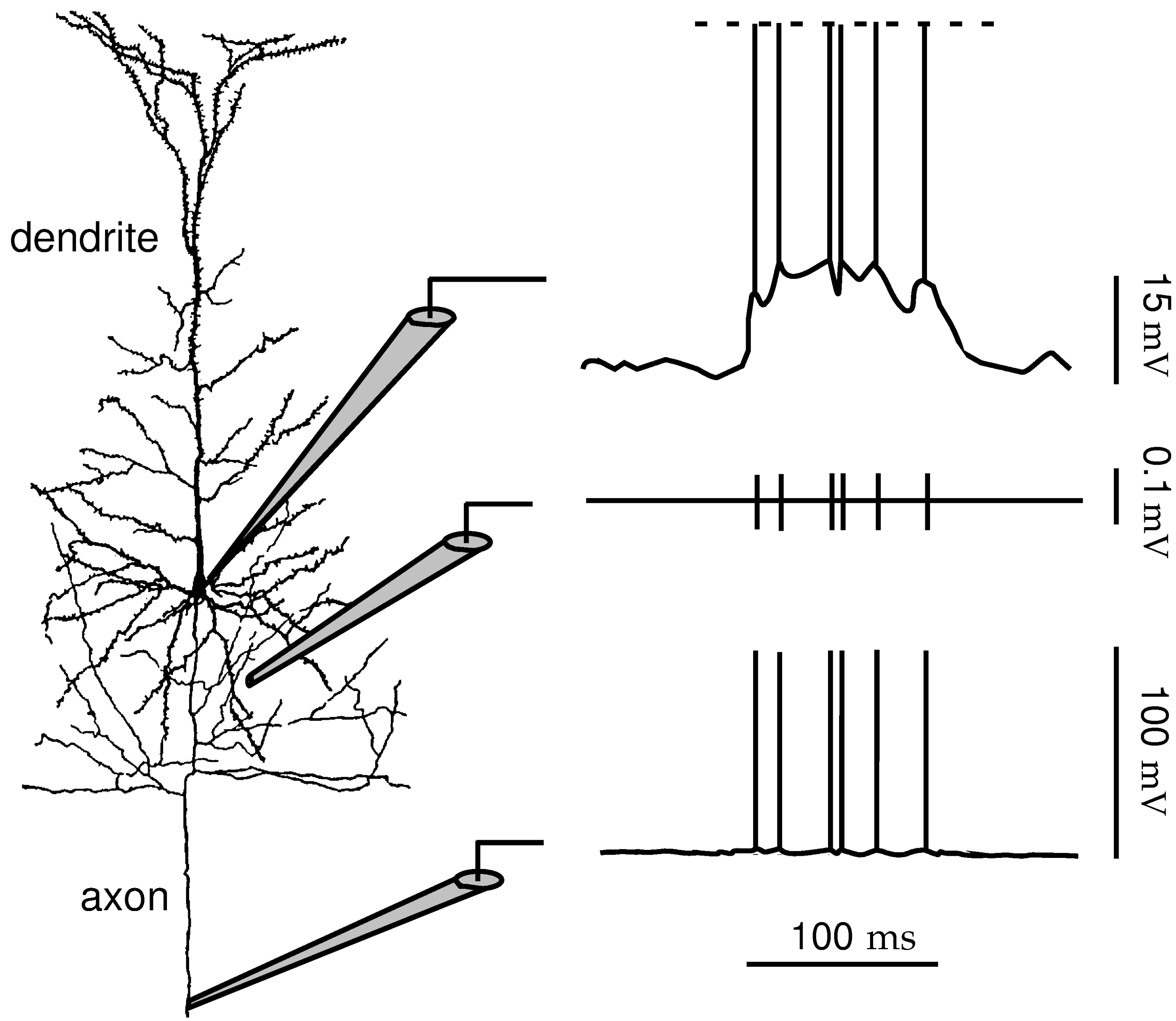
Lecture 6: *Stimulus encoding by single neurons part 2: STC and MID and GLMs*



$$\mathcal{S} = \{t_1, t_2, \ldots t_N\}$$

The STA as the optimal kernel (for white noise

$$E = \frac{1}{T}\int_0^T dt\ (\mathrm{r}_{\mathrm{est}}(t) - \mathrm{r}(t))^2$$

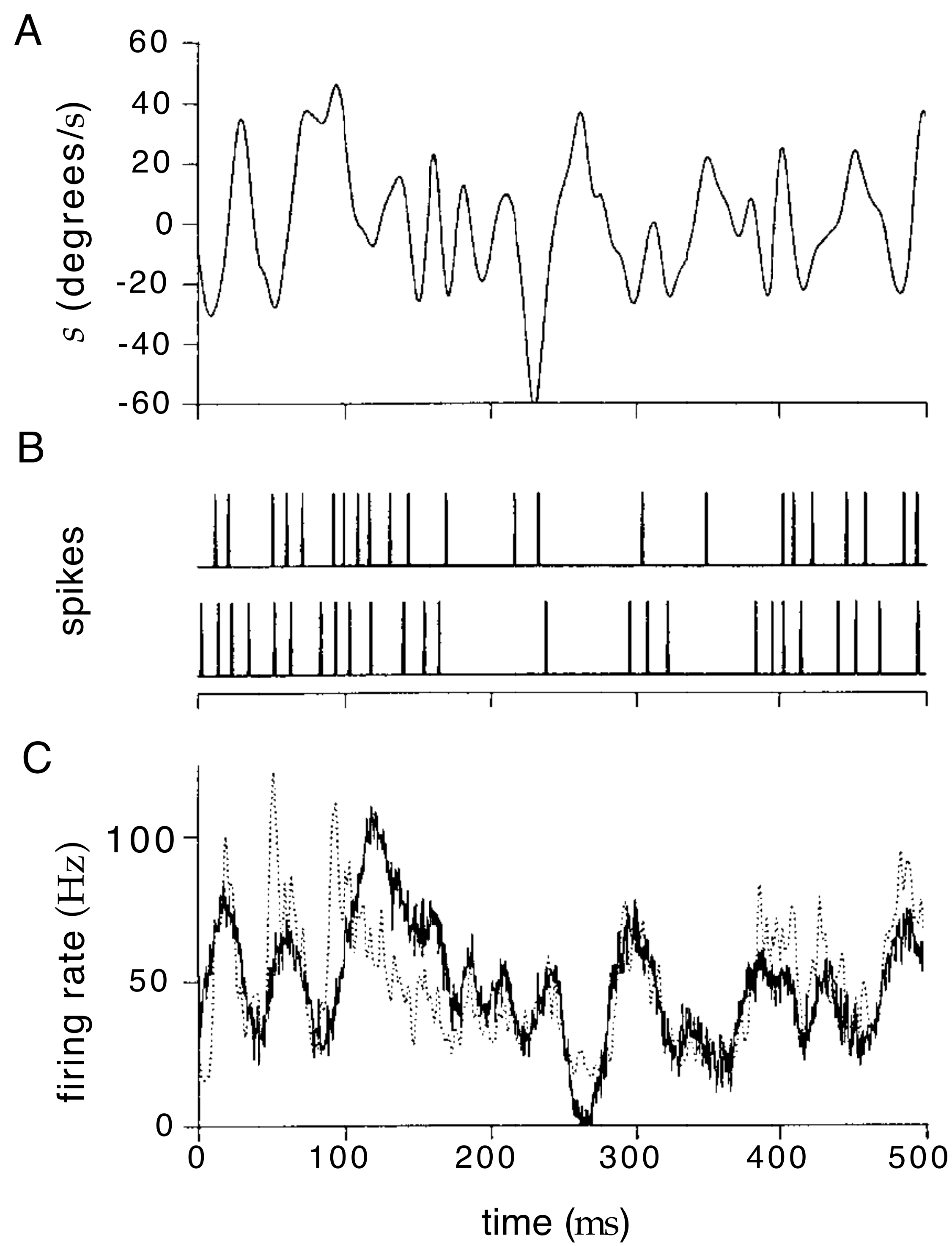
$$\int_0^\infty d\tau'\, Q_{ss}(\tau - \tau')D(\tau') = Q_{\mathrm{r}s}(-\tau)$$

$$Q_{ss}(\tau) = \sigma_s^2 \delta(\tau)$$

$$\sigma_s^2 \int_0^\infty d\tau'\, \delta(\tau - \tau')D(\tau') = \sigma_s^2 D(\tau)$$

$$D(\tau) = \frac{Q_{\mathrm{r}s}(-\tau)}{\sigma_s^2} = \frac{\langle r\rangle C(\tau)}{\sigma_s^2}$$

Example from the fly H1 neuron:

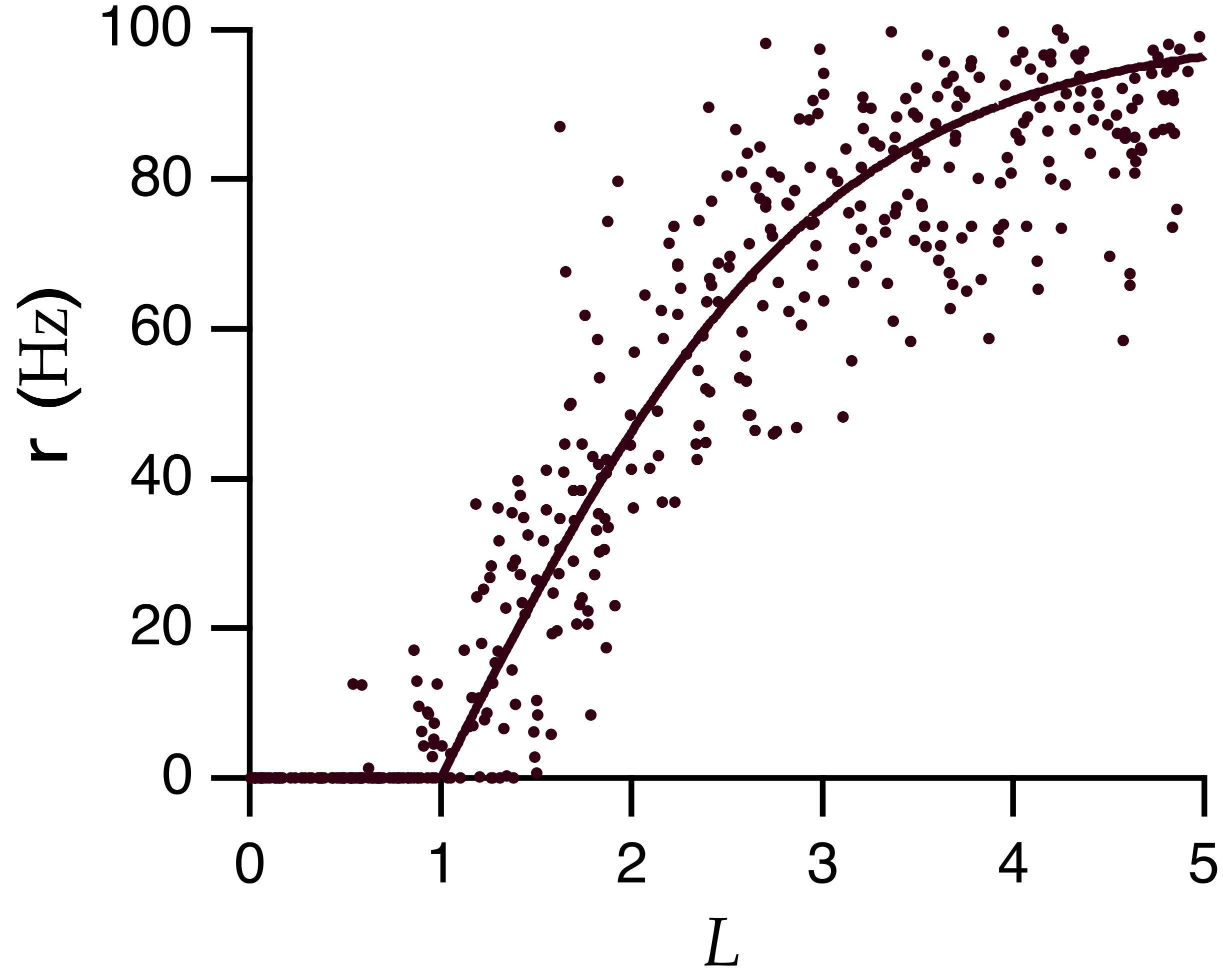


What's missing? Including static nonlinearities:

$$L(t) = \int_0^\infty d\tau D(\tau) s(t-\tau)$$

$$\mathsf{r}_{\text{est}}(t) = \mathsf{r}_0 + F(L(t))$$

A graphical method for finding F(L):



Reverse-correlation example: V1 simple cell

$$C(x, y, \tau) = \frac{1}{\langle n \rangle} \left\langle \sum_{i=1}^{n} s(x, y, t_i - \tau) \right\rangle$$

$$Q_{\mathrm{r}s}(x, y, \tau) = \frac{1}{T} \int_0^T dt\, \mathrm{r}(t) s(x, y, t + \tau)$$

$$C(x, y, \tau) = \frac{Q_{\mathrm{r}s}(x, y, -\tau)}{\langle r \rangle}$$

$$L(t) = \int_0^\infty d\tau \int dx dy\, D(x, y, \tau) s(x, y, t - \tau)$$

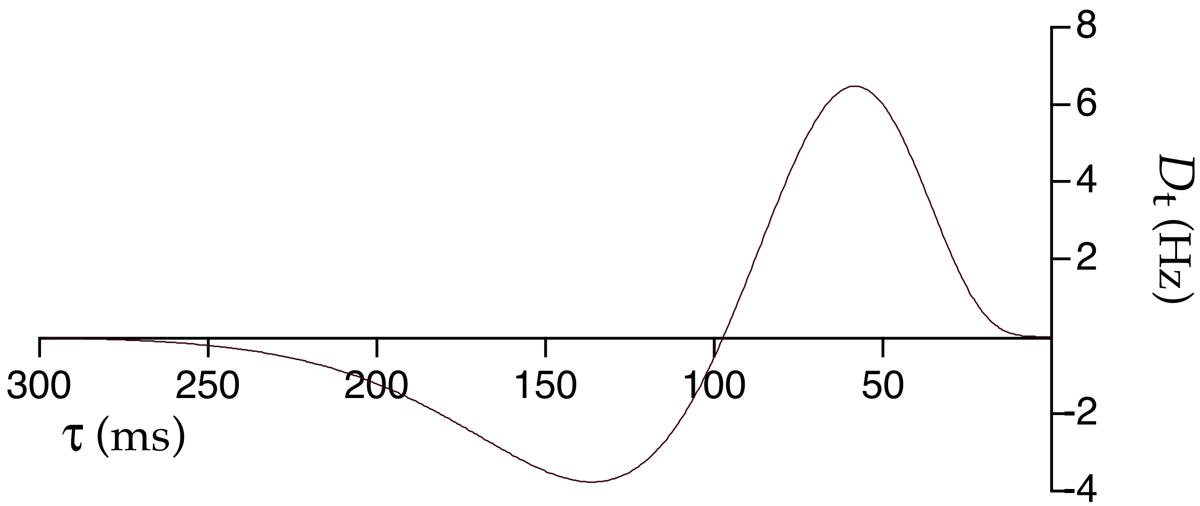
$$D(x, y, \tau) = \frac{Q_{\mathrm{r}s}(x, y, -\tau)}{\sigma_s^2} = \frac{\langle r \rangle C(x, y, \tau)}{\sigma_s^2}$$

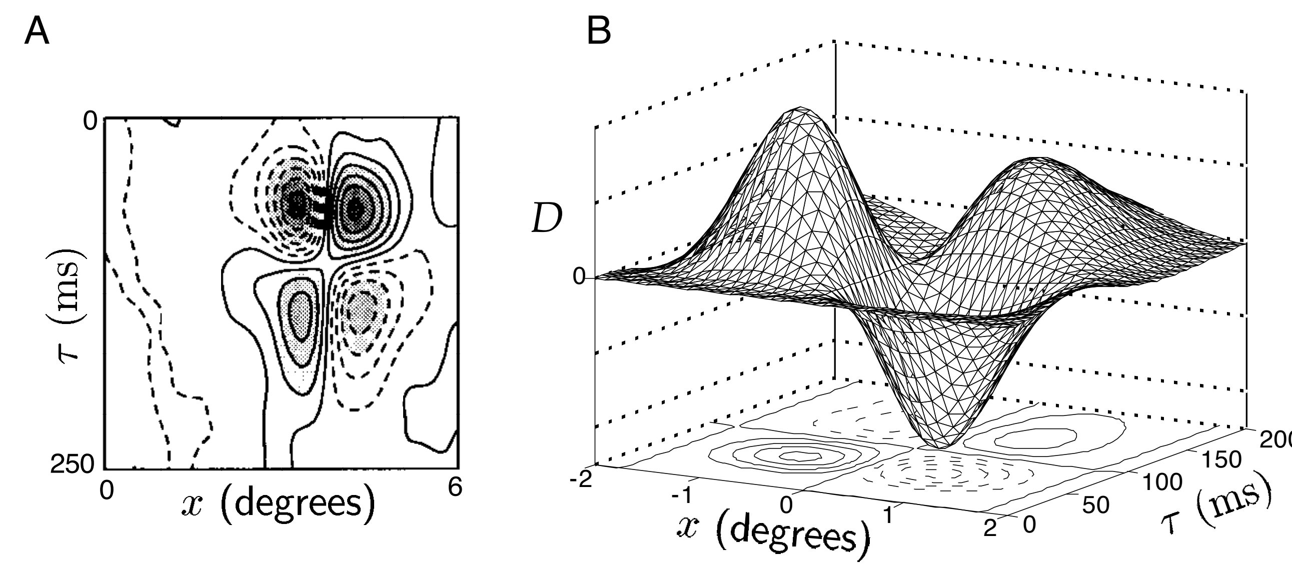
Space-time separable linear filter:

$$D(x, y, \tau) = D_s(x, y) D_t(\tau)$$

$$D_s(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) \cos(kx - \phi)$$

$$D_t(\tau) = \alpha \exp(-\alpha\tau) \left(\frac{(\alpha\tau)^5}{5!} - \frac{(\alpha\tau)^7}{7!}\right)$$





Simple cell nonlinearity:

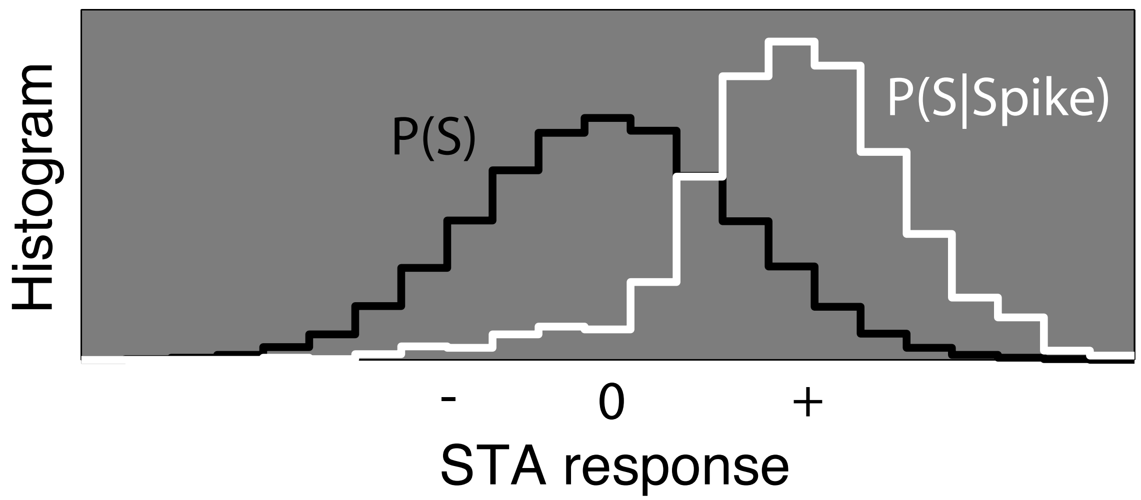
$$F(L) = \frac{G[L]_+^2}{A_{1/2}^2 + G[L]_+^2}$$

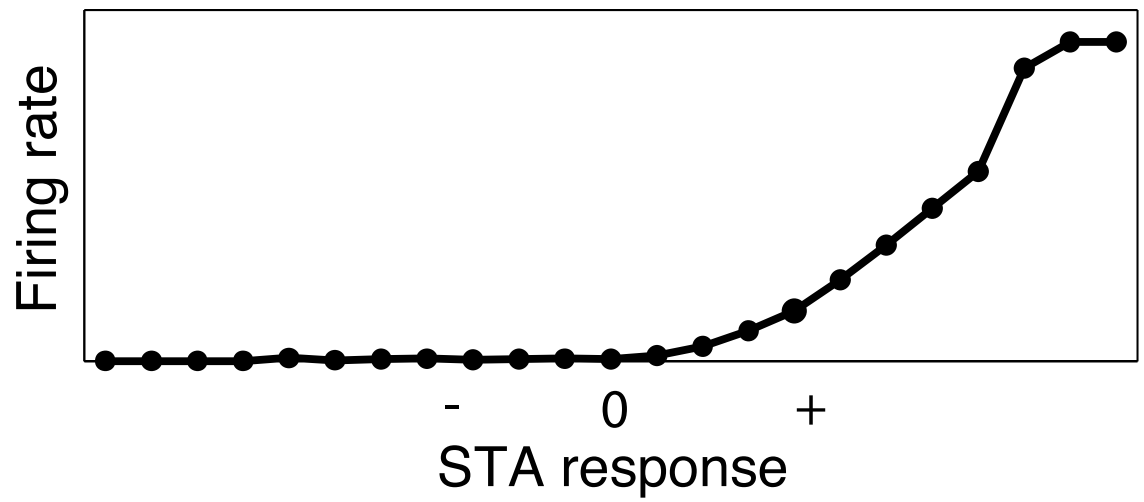
???

The Bayesian approach for F(L):

$$\mathcal{P}(\text{spike}|\vec{s}) = ?$$

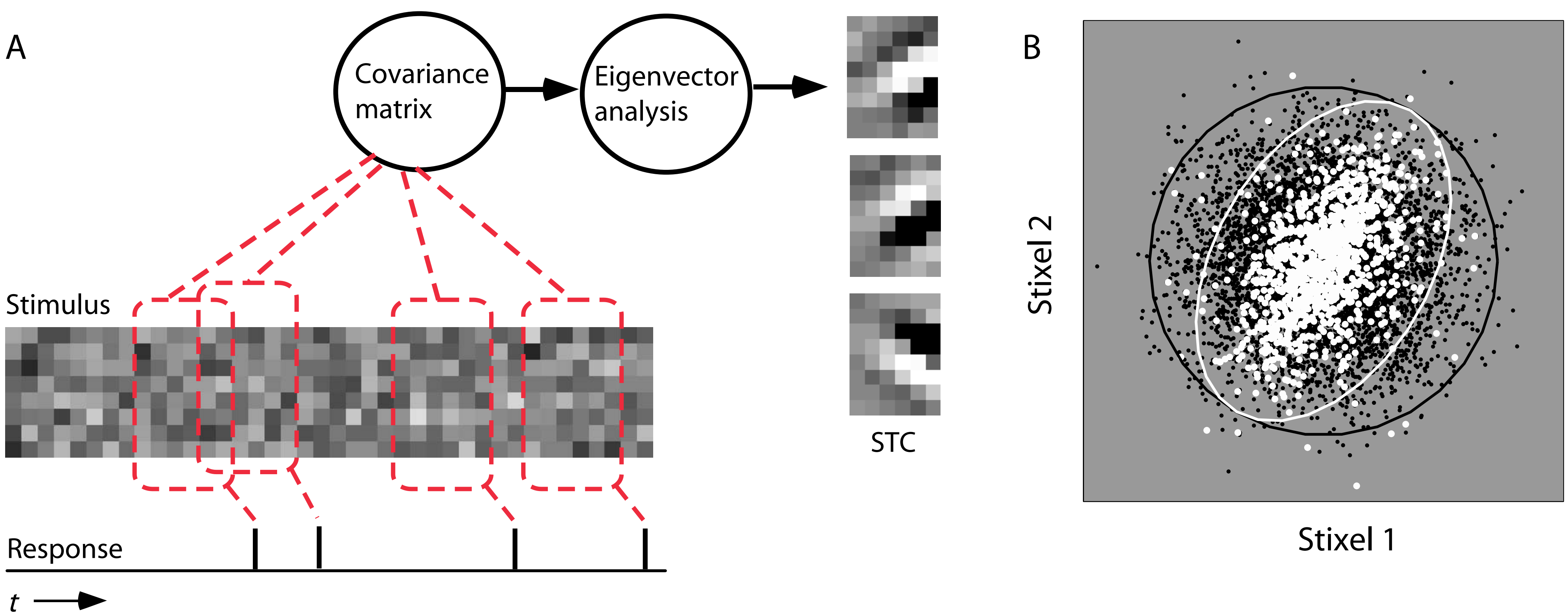
$$\mathcal{P}(\text{spike}|\vec{s}) = \frac{\mathcal{P}(\text{spike})\mathcal{P}(\vec{s}|\text{spike})}{\mathcal{P}(\vec{s})}$$





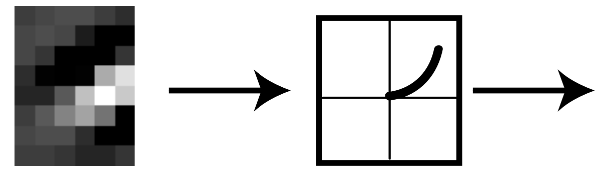
Spike-triggered covariance (STC):

$$cov = \frac{1}{N-1}\sum_{n=1}^{N}\left[\vec{s}\,(t_n)_{-C(\tau)}\right]\left[\vec{s}\,(t_n)_{-C(\tau)}\right]^T$$

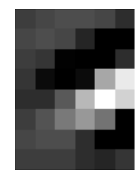


How many filters? Assessing significance:

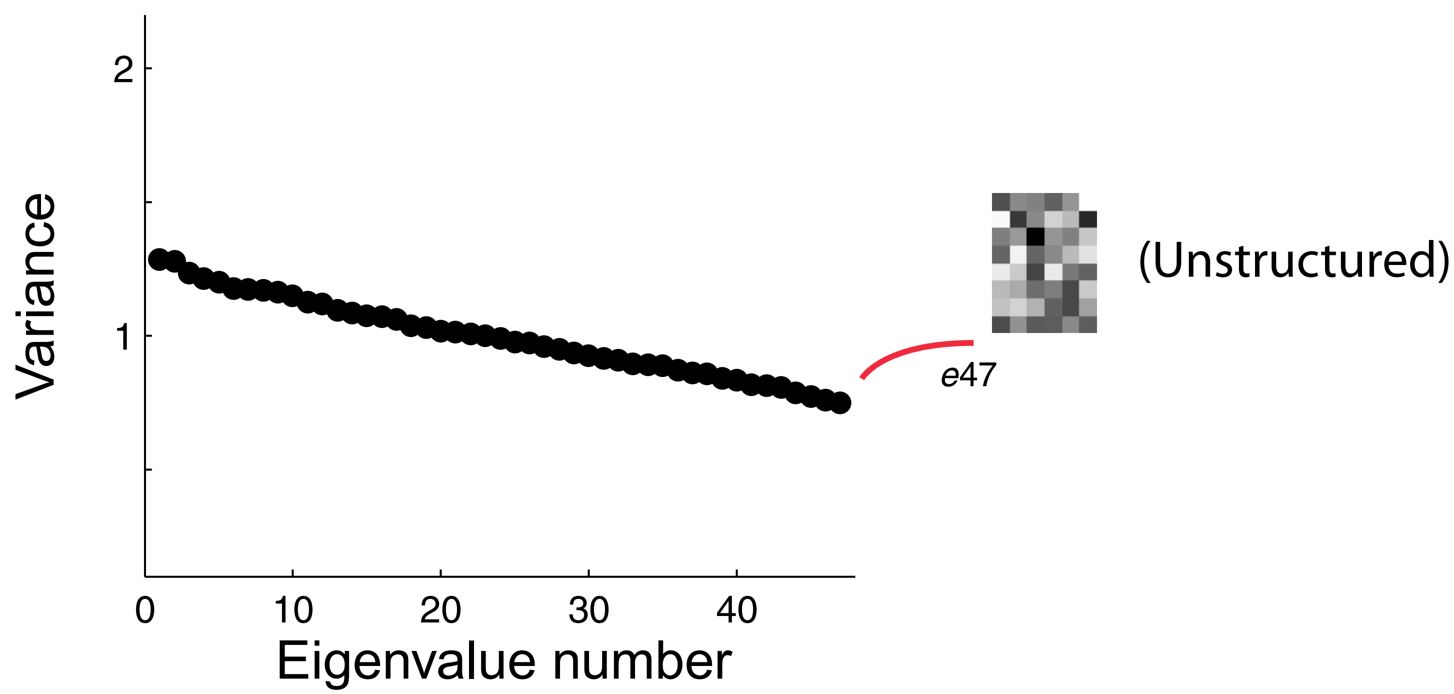
Model neuron:



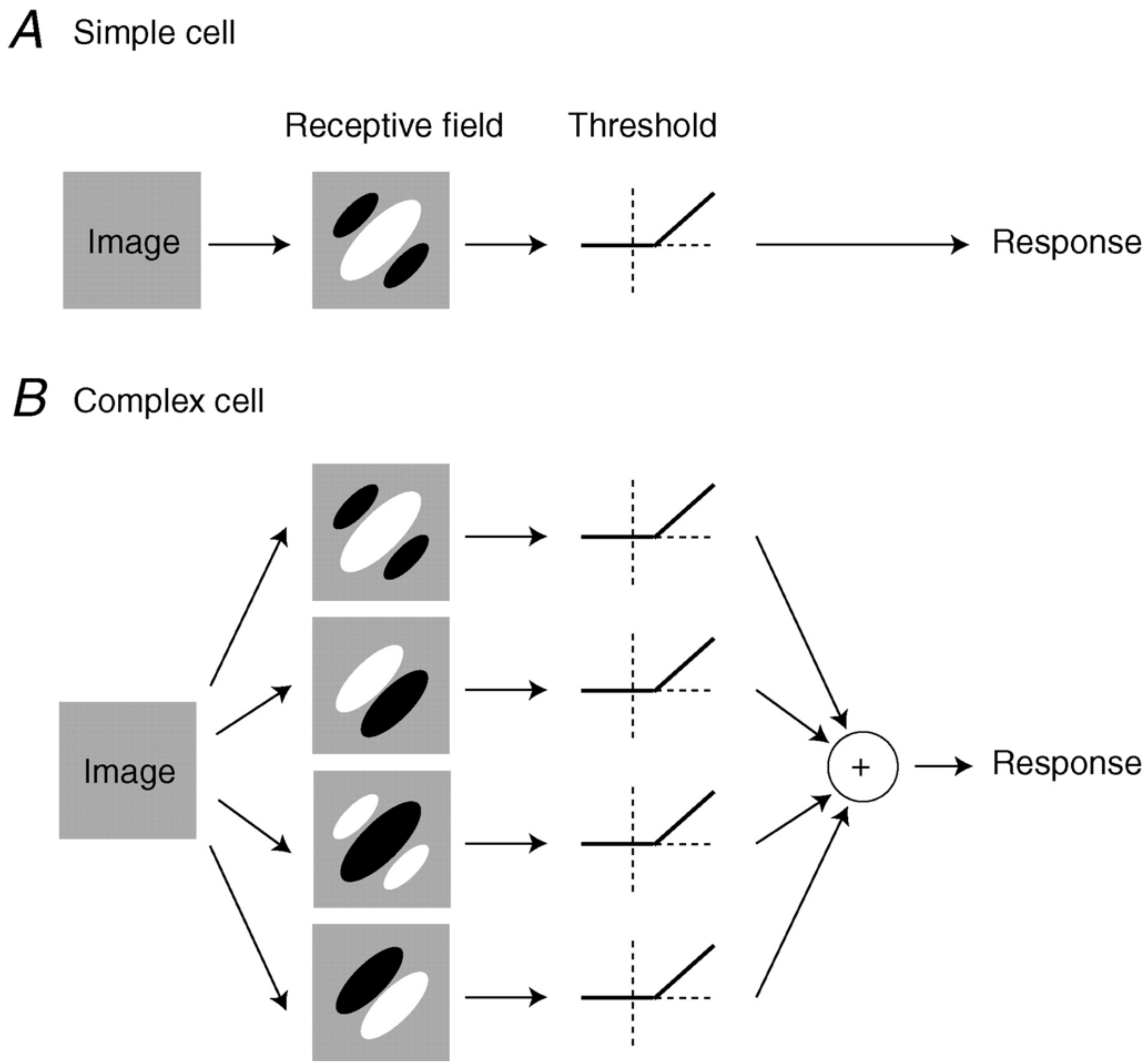
STA analysis:



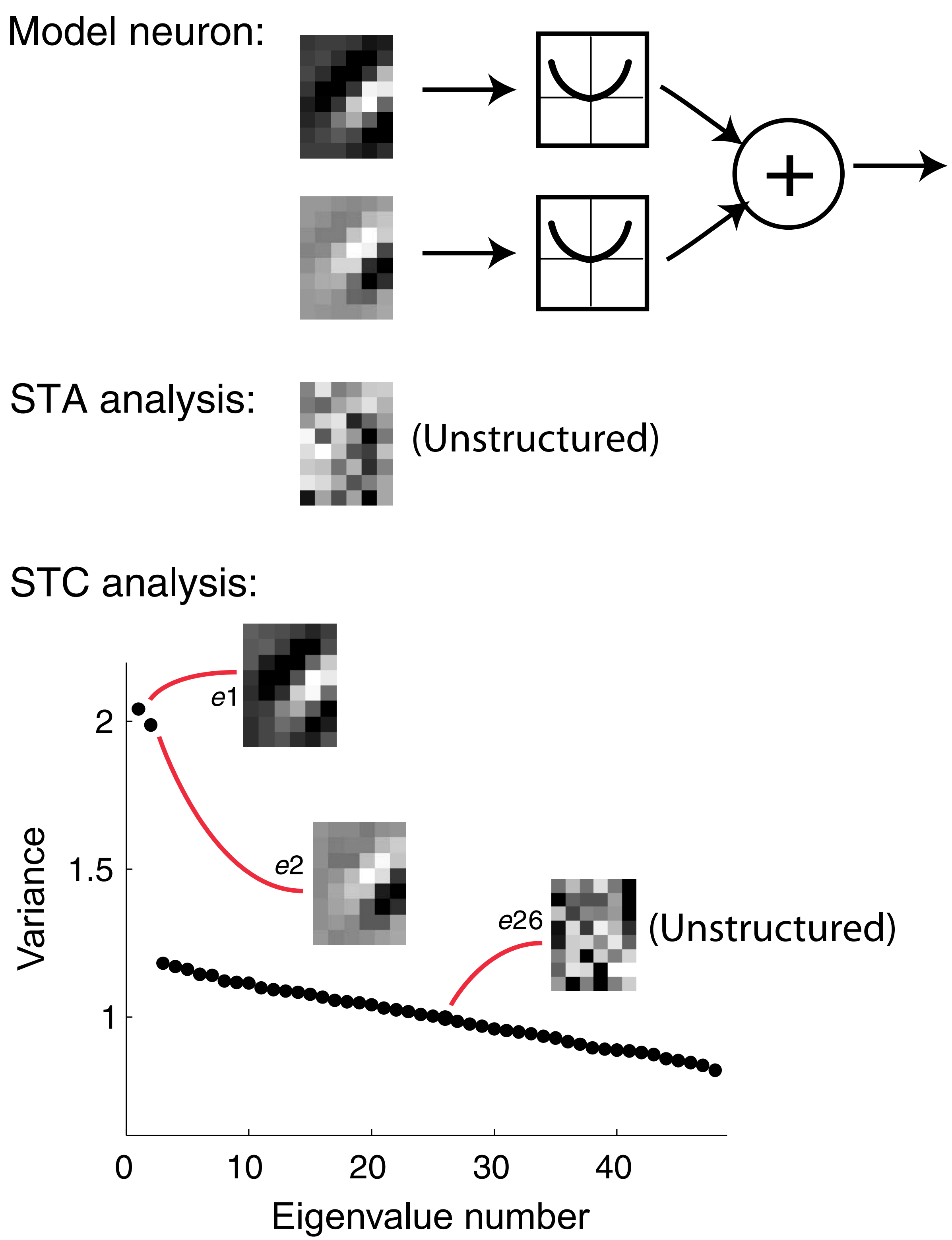
STC analysis:



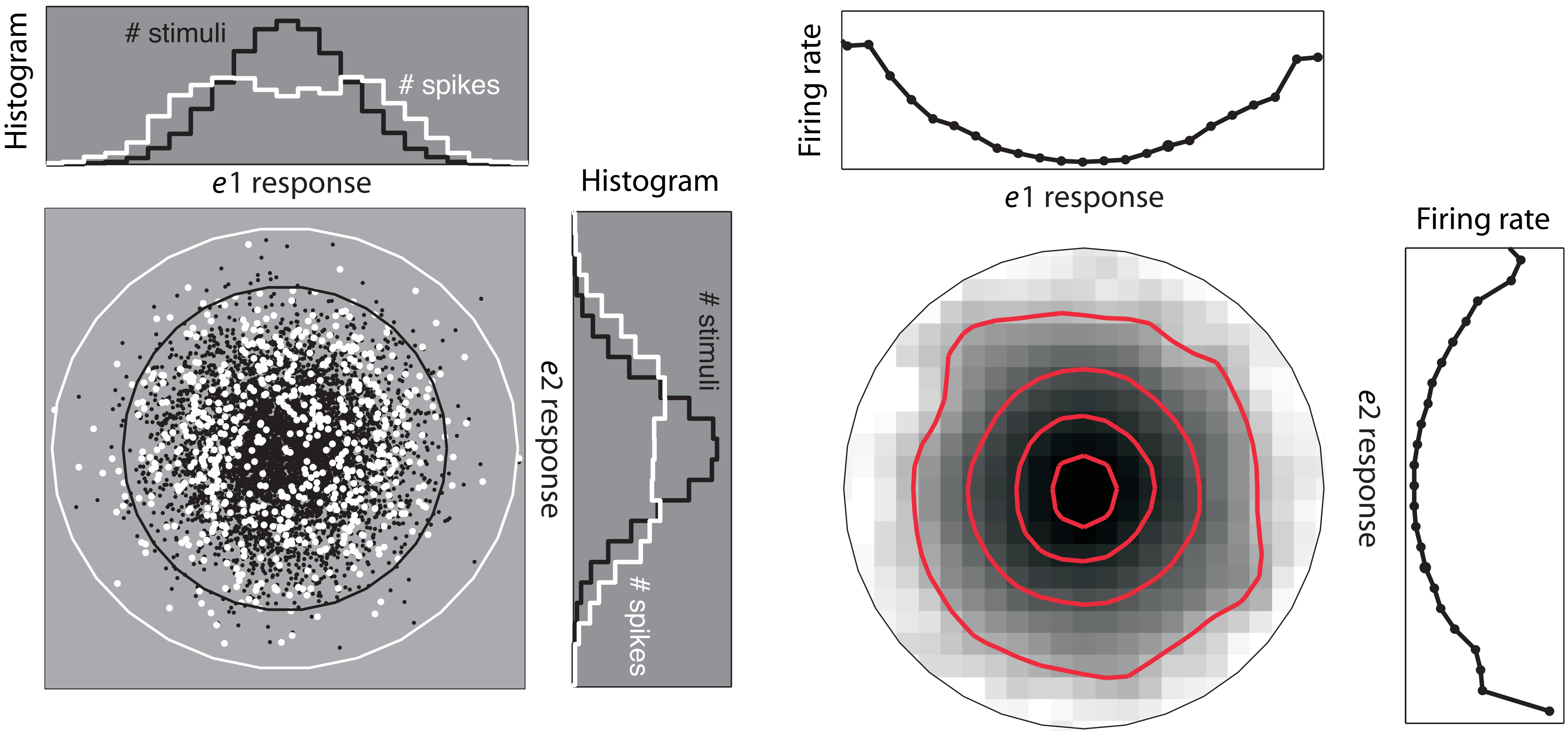
Example: a V1 complex cell



How many filters? Assessing significance:



'Curse of dimensionality' and parametric nonlinearities:

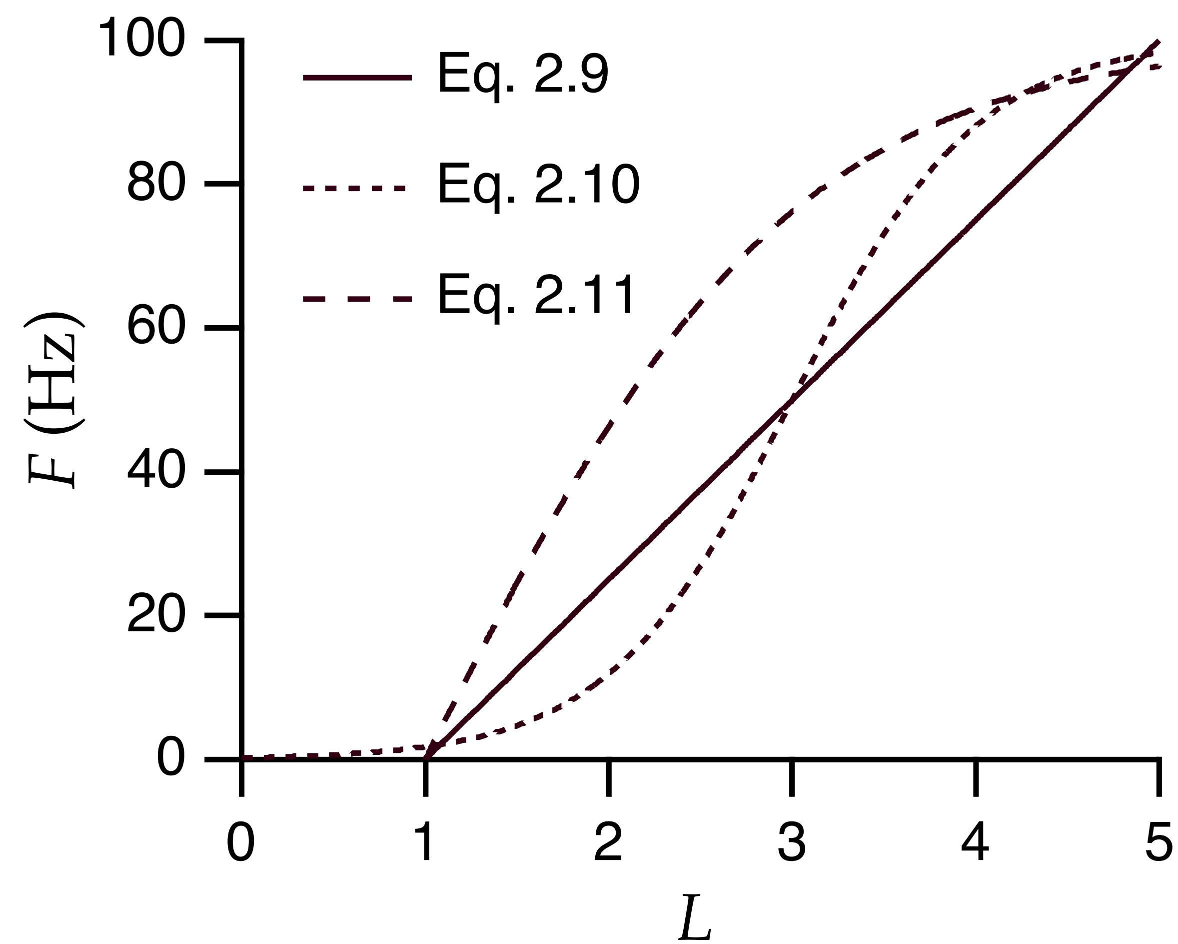


Some commonly used nonlinearities:

$$F(L) = G[L - L_0]_+$$

$$F(L) = \frac{\mathrm{r}_{\max}}{1 + \exp\left(g_1(L_{1/2} - L)\right)}$$

$$F(L) = \mathrm{r}_{\max}[\tanh\left(g_2(L - L_0)\right)]_+$$

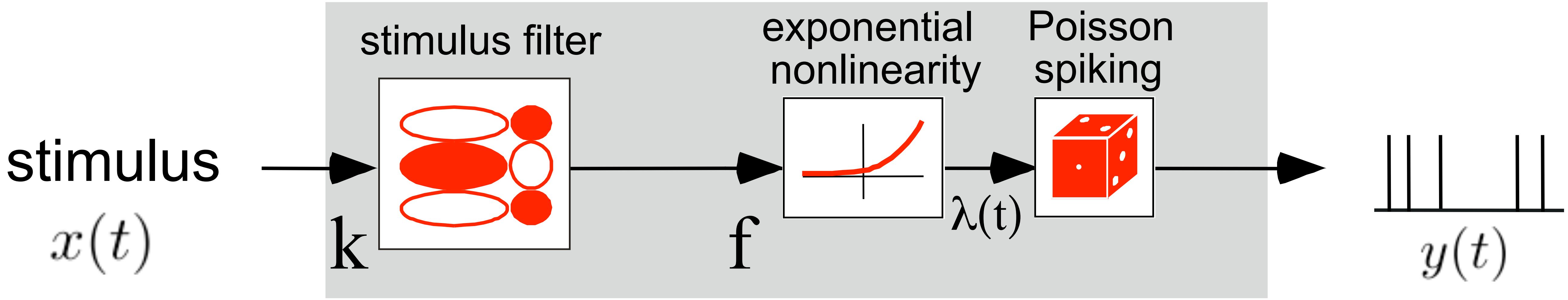


Practical issues when when fitting LNP models:

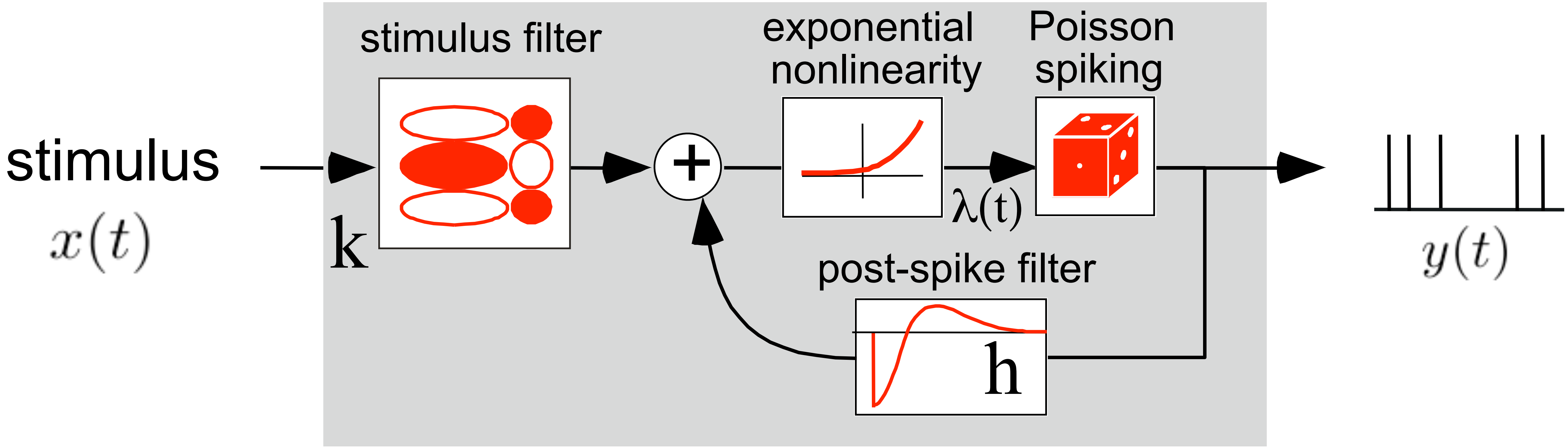
Even if one is careful to design an experiment and data analysis methodology that leads to accurate and artifact-free estimates, a spike-triggered analysis can still fail if the model assumptions are wrong. Two examples of failure of the LNP model are as follows: (1) there is no low-dimensional subspace in which the neural response may be described or (2) the neural response has a strong dependence on spike history (e.g., refractoriness, bursting, adaptation) that cannot be described by an inhomogeneous Poisson process. STA/STC analysis of data simulated using more realistic spike generation models, such as Hodgkin–Huxley (Agüera y Arcas & Fairhall, 2003; Agüera y Arcas, Fairhall, & Bialek, 2001, 2003; Pillow & Simoncelli, 2006) and integrate-and-fire (Pillow & Simoncelli, 2003), produces biased estimates and artifactual filters. Although the STA/STC filters might in some cases still provide a reasonable description of a neuron's response, it is important to recognize that the LNP model provides only a crude approximation of the neural response (see Interpretation issues section).

from Schwartz et al., 2006

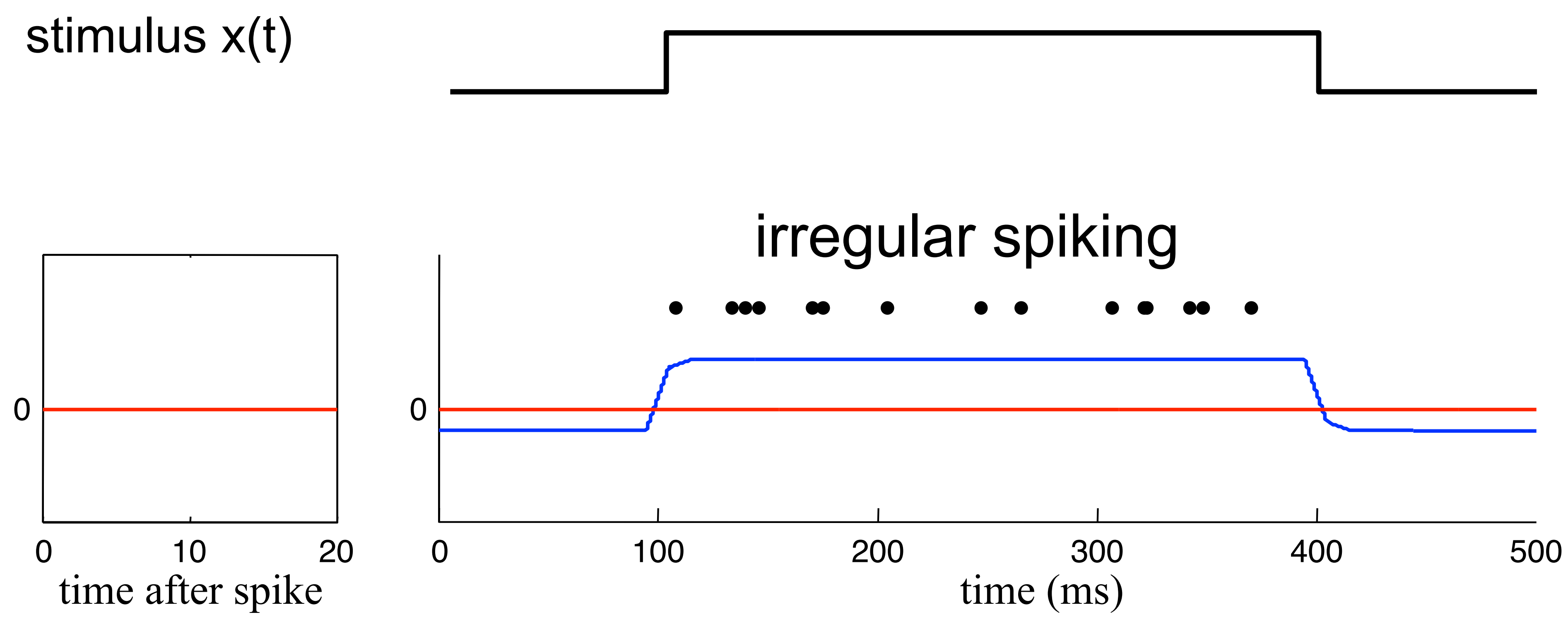
Introduction to Generalized Linear Models (GLMs):



Adding spike history dependence:

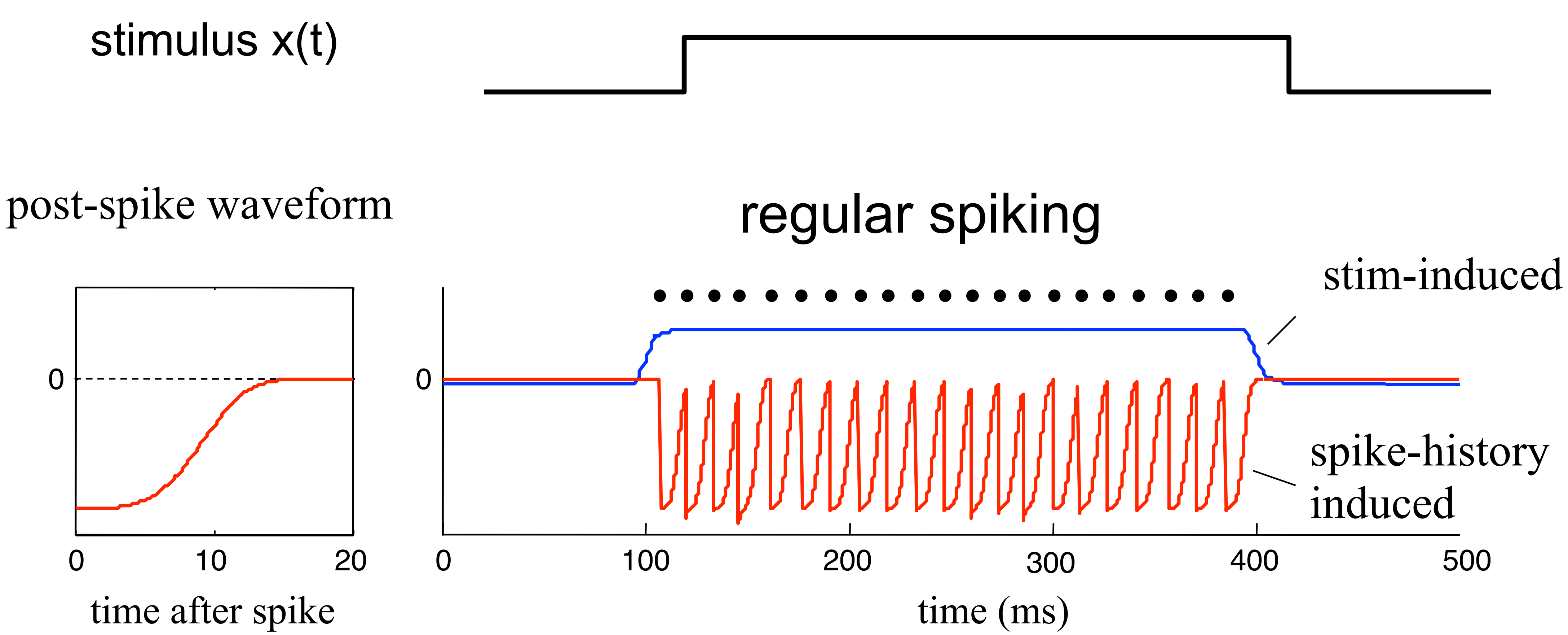


Different post-spike filters create different dynamics:



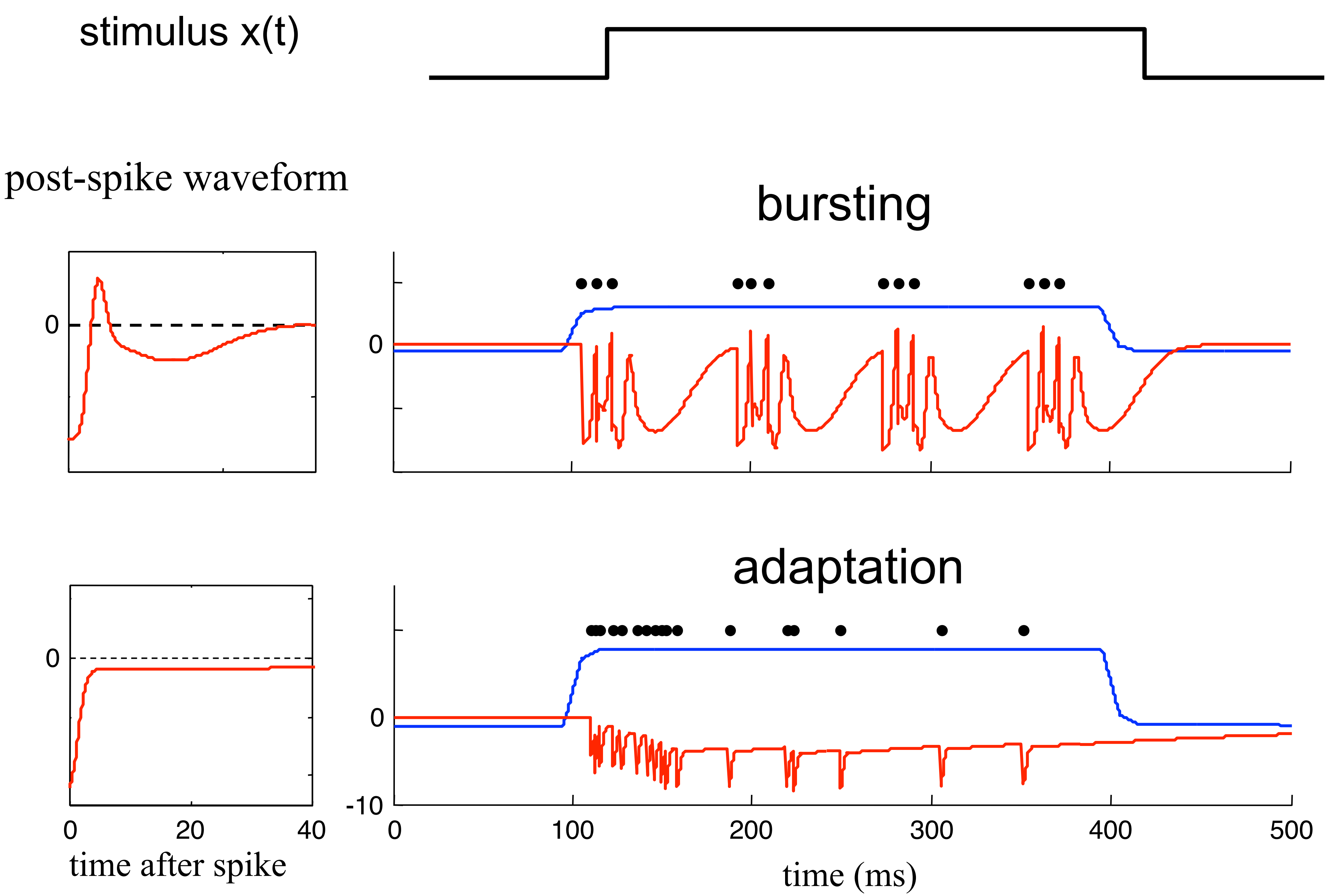
from J. Pillow

Regular spiking:



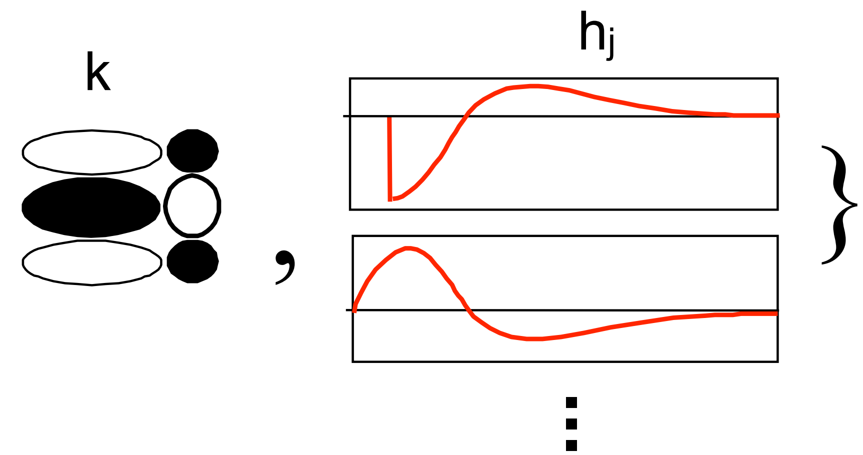
from J. Pillow

Bursting, adaptation:



from J. Pillow

Fitting GLMs to neural data:

Model parameters: $\theta = \{$ 

$\}$

conditional intensity
(spike rate)

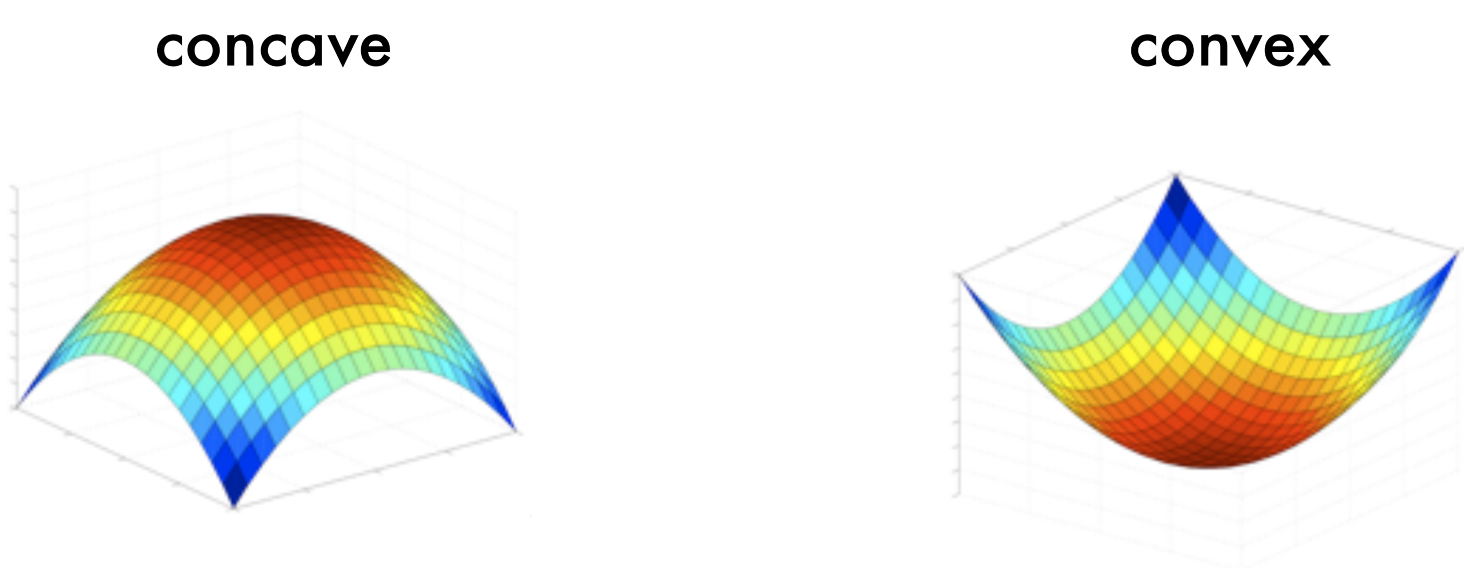
$$\lambda(t) = f(k \cdot x(t) + \sum h_j \cdot y_j^{hist}(t))$$

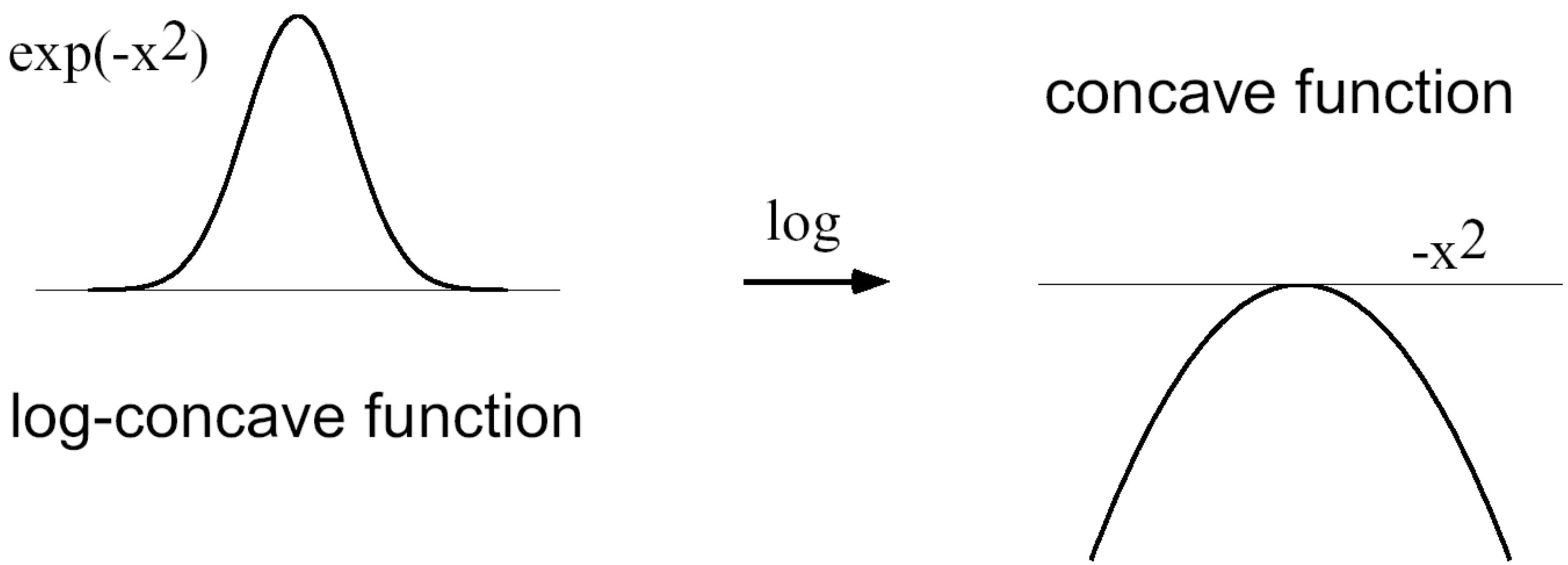
stimulus vector   spike history vector

single bin: $$P(y \text{ spikes}|\lambda(t)) = \frac{1}{y!}\lambda(t)^y e^{-\lambda(t)}$$

from J. Pillow

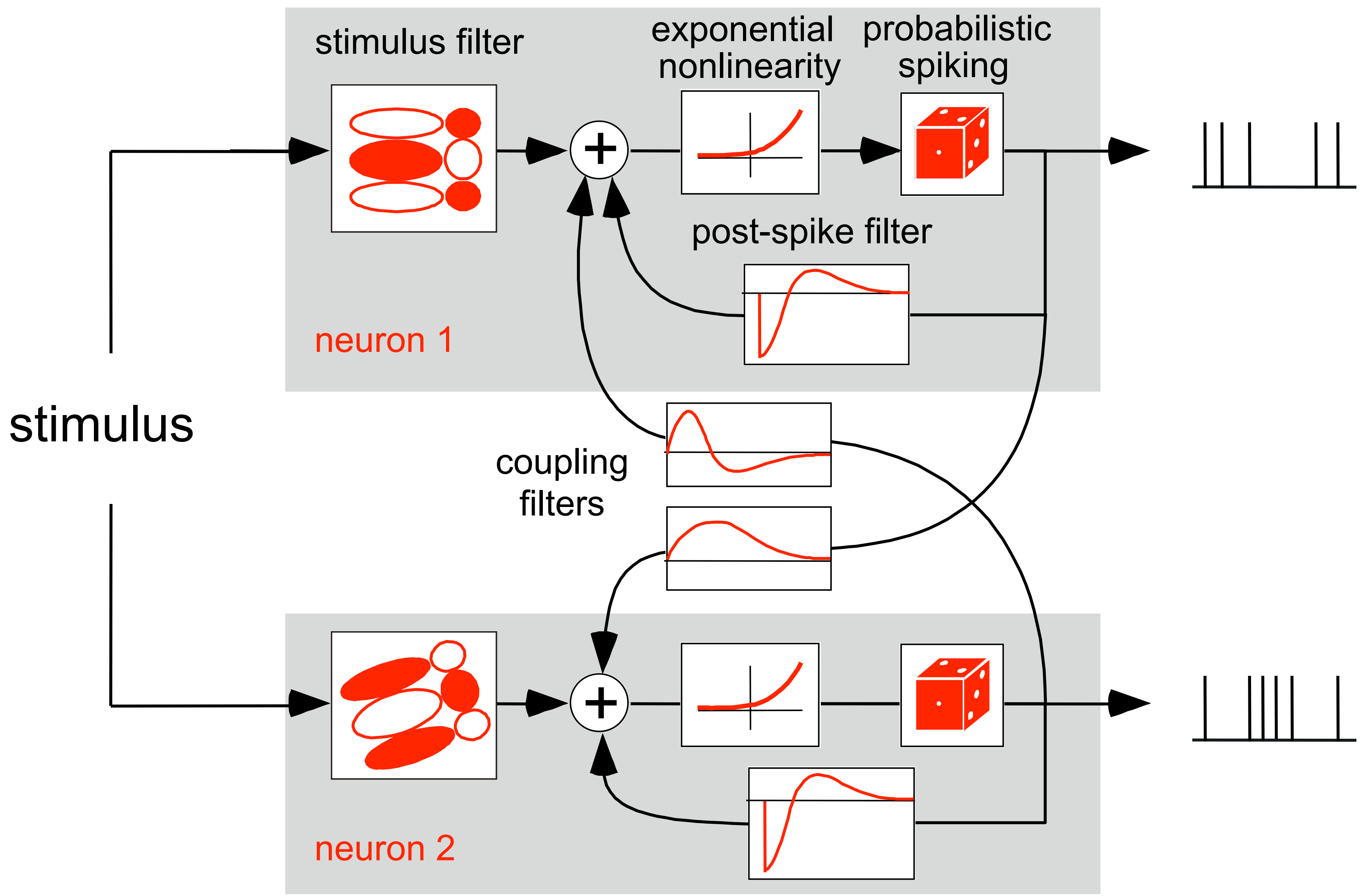
Log-concavity and maximization:



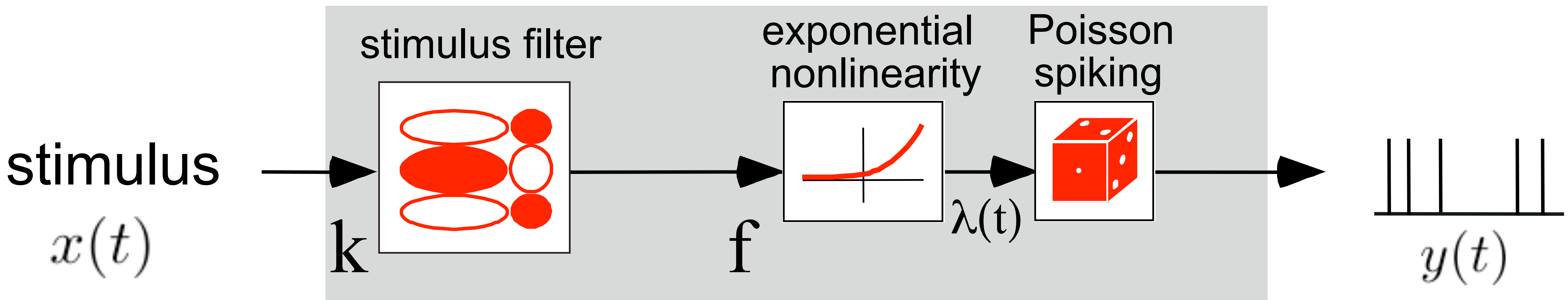


Th'm: the GLM log-likelihood is concave in its parameters for any stimulus and spike data if the nonlinearity is convex and log-concave.   see Paninski 2004

Preview of what's to come: coupling neurons



Basic LNP model:



“Maximally informative” dimensions:

$$I_{\mathrm{spike}} = \int d\mathbf{s} P(\mathbf{s}|\mathrm{spike}) \log_2 \left[\frac{P(\mathbf{s}|\mathrm{spike})}{P(\mathbf{s})} \right]$$

$$I(\mathbf{v}) = \int dx P_{\mathbf{v}}(x|\mathrm{spike}) \log_2 \left[\frac{P_{\mathbf{v}}(x|\mathrm{spike})}{P_{\mathbf{v}}(x)} \right]$$

chalkboard interlude

Entropy as a measure of uncertainty:

$$\begin{aligned} \text{uncertainty} &= \log(n) \\ &= \log(1/p) \\ &= -\log(p) \end{aligned}$$

$$u_i = -\log(p_i)$$

$$\langle u_i \rangle = -\sum_i p_i \log(p_i)$$

$$S(X) = -\sum_x p(x) \log_2(p(x))$$

Information as reduction in uncertainty:

$$I(X;Y) = S(X) - \langle S(X|Y) \rangle_y$$

$$I(X;Y) = S(Y) - \langle S(Y|X) \rangle_x$$

$$I(X;Y) = \sum_{x,y} P(X,Y) \log_2 \left(\frac{P(X,Y)}{P(X)P(Y)} \right)$$

$$P(X,Y) = P(X|Y)P(Y)$$

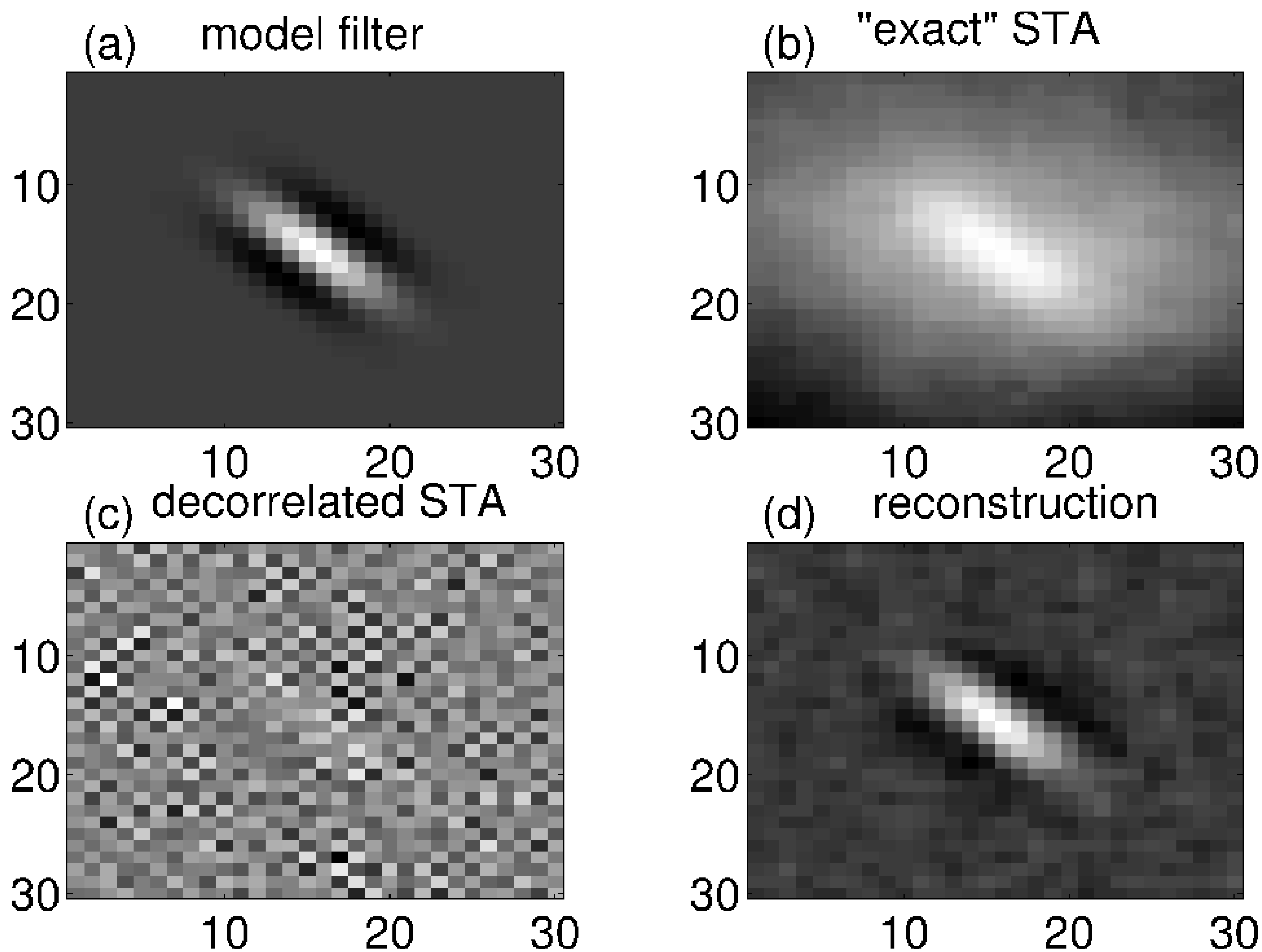
$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

“Maximally informative” dimensions:

$$I_{\text{spike}} = \int d\mathbf{s} P(\mathbf{s}|\text{spike}) \log_2 \left[\frac{P(\mathbf{s}|\text{spike})}{P(\mathbf{s})} \right]$$

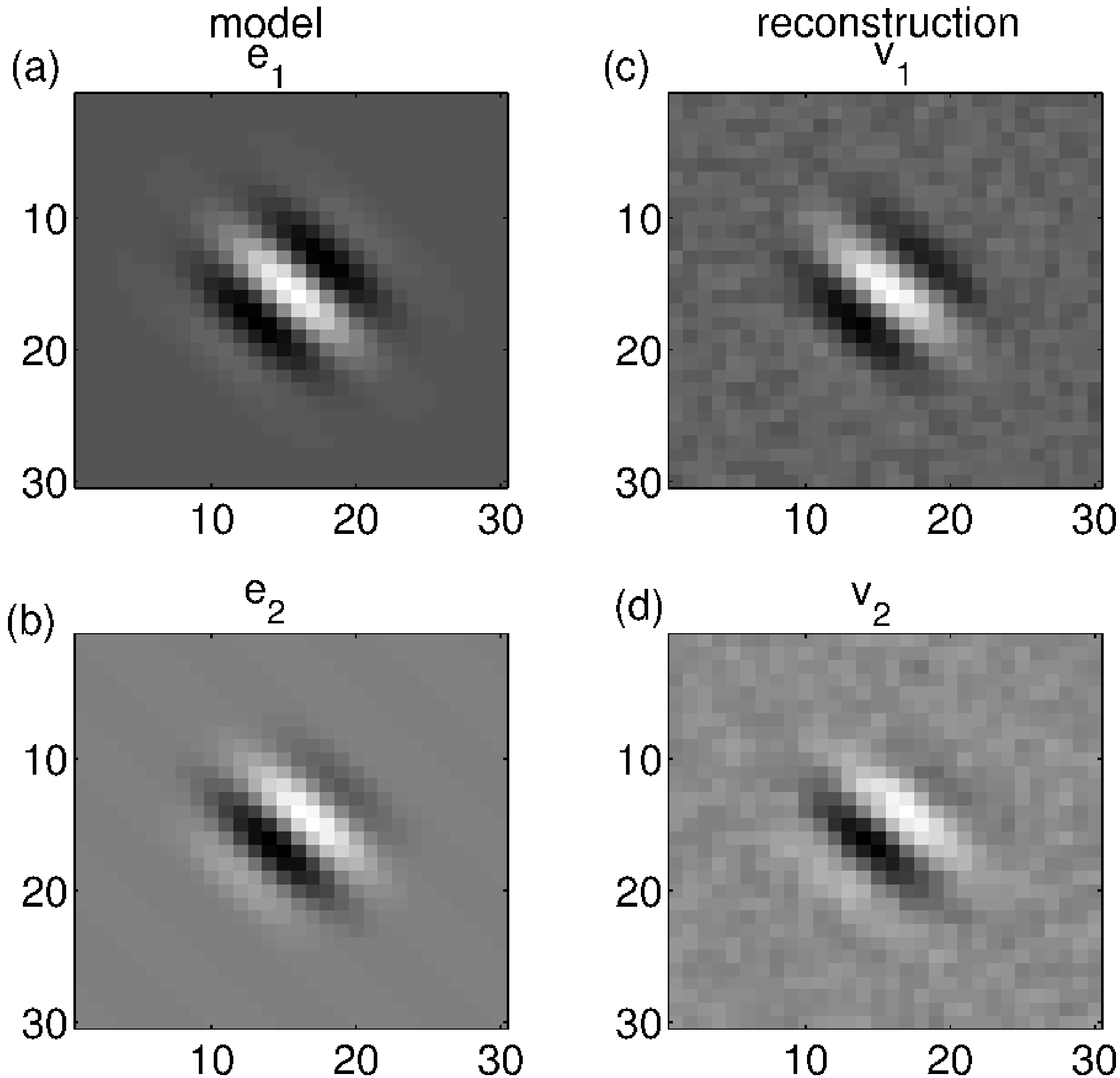
$$I(\mathbf{v}) = \int dx P_{\mathbf{v}}(x|\text{spike}) \log_2 \left[\frac{P_{\mathbf{v}}(x|\text{spike})}{P_{\mathbf{v}}(x)} \right]$$

“Maximally informative” dimensions:



Sharpee, Rust, Bialek 2004

…works for a complex cell, too:



Sharpee, Rust, Bialek 2004