Statistical inference and memory in recurrent networks of spiking neurons

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Outline

The Hopfield Network

Proc. Natl. Acad. Sci. USA
Vol. 79, pp. 2554–2558, April 1982
Biophysics

Neural networks and physical systems with emergent collective computational abilities

(associative memory/parallel processing/categorization/content-addressable memory/fail-soft devices)

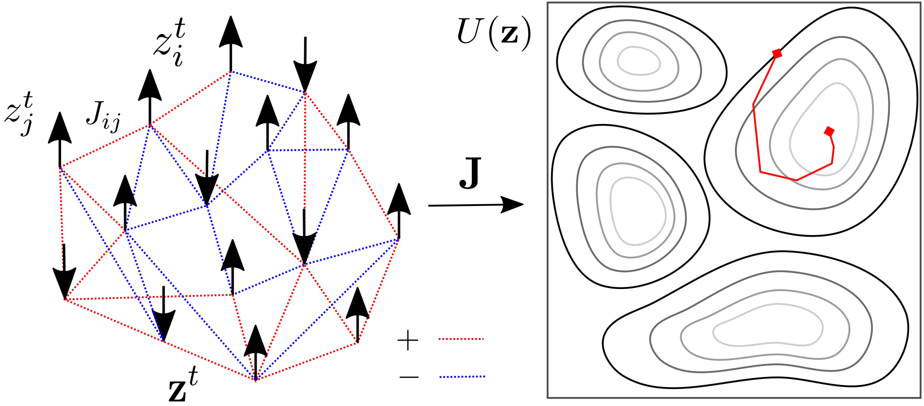
J. J. HOPFIELD

Division of Chemistry and Biology, California Institute of Technology, Pasadena, California 91125; and Bell Laboratories, Murray Hill, New Jersey 07974

Contributed by John J. Hopfield, January 15, 1982

$>$ 23,000 citations in 2021

The Hopfield Network



Recurrent artificial neural network which resembles a spin glass

The network stores binary patterns ξ as attractor states

Serves as content-addressable or associative memory

The Hopfield Network

The Boltzmann Machine

The stochastic counterpart of the Hopfield Network

Review of Bayesian inference

Recall Bayes theorem from fundamental probability theory

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{\int P(B|A)P(A)dA}$$

$P(A|B)$ is called the posterior, $P(B|A)$ the likelihood, $P(A)$ the prior, and $P(B)$ the evidence

$$P(B) = \int P(B|A)P(A)dA$$

Calculating this integral is often intractable. Monte-Carlo Markov Chain (MCMC) methods and variational methods offer solutions

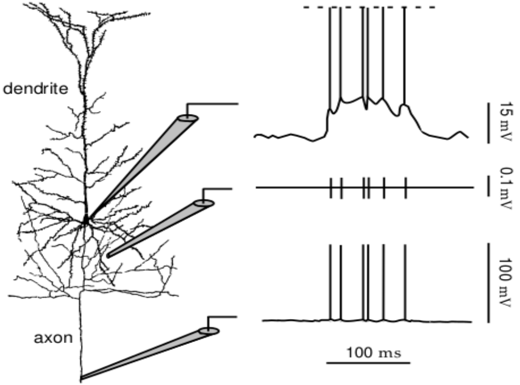
Monte-Carlo Markov Chain (MCMC) to sample the posterior

Monte Carlo methods estimate distributions by repeated sampling

If calculating $P(B)$ is intractable and we require samples from the posterior $P(A|B)$ we can use MCMC

A prominent hypothesis in neuroscience is that neurons use

The third generation of neural networks: spiking nets

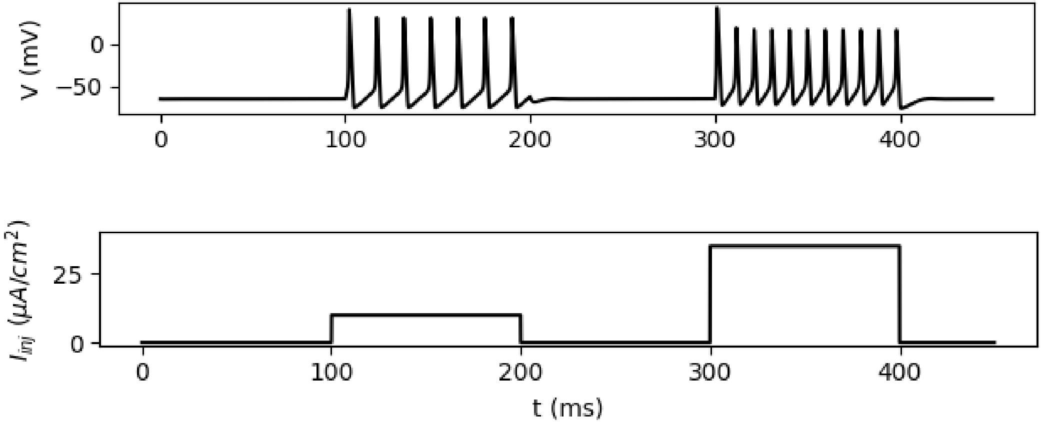


- $\sim$ 16 billion neurons in cortex
- A neuron receives on the order of 10^3 to 10^4 synaptic inputs
- Neurons communicate via action potentials in an all-or-nothing fashion

The third generation of neural networks: spiking nets

- Post-synaptic potentials (PSPs) allow pre-synaptic action potentials to change post-synaptic membrane potential
- PSPs can be positive or negative (excitatory or inhibitory)

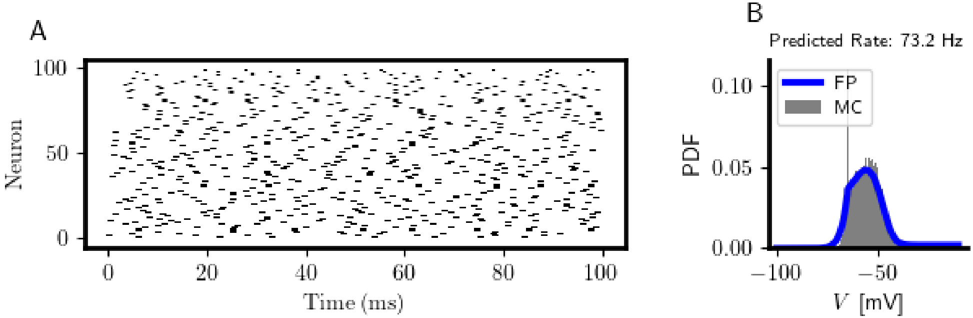
Integrate and fire (IF) neuron models



$$\tau \dot{V}(t) = g_\ell(E - V) + g_\ell \cdot \psi(V) + I(t)$$

Monte-Carlo simulation of uncoupled IF neurons

When $\psi(V) = g_\ell \Delta_T \exp\left(\frac{V - V_L}{\Delta_T}\right)$ we have the exponential integrate and fire model

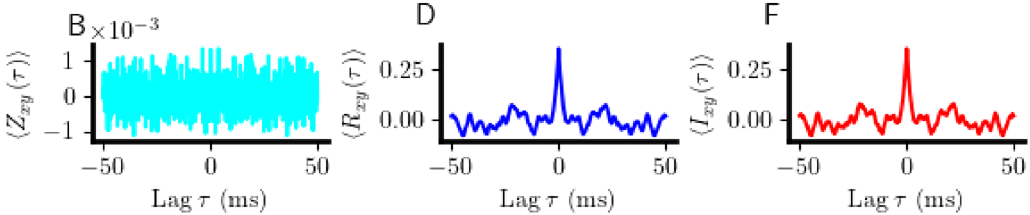


Langevin equations have a corresponding Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\sigma^2}{\tau}\frac{\partial^2 P}{\partial V^2} + \frac{\partial}{\partial V}\left(\frac{V - E + \psi}{\tau} P\right)$$

Synaptic coupling can induce correlations in spiking activity

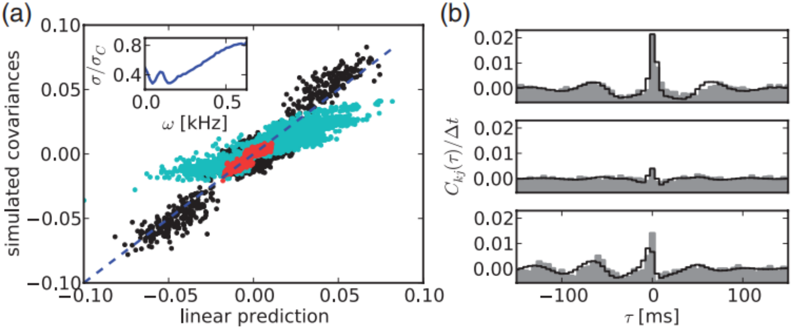
For special synaptic connectivity regimes dynamical variables can remain uncorrelated between neurons



Uncorrelated neural activity captures irregular spiking seen *in-vivo*

Predicting neuron correlations

The linear response of $r(t)$ allows us to also estimate the matrix of cross-correlations $C_{kj}(\tau)$ from the synaptic connectivity $\mathcal{C}$



This has important implications for brain-inspired machine learning