Stochastic computation in recurrent networks of spiking neurons

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Outline

A brief survey of deep learning architectures

- Perceptrons e.g. MLPs for classification of vectorized data
- Convolutional neural networks (CNNs) for image classification, segmentation
- Recurrent neural networks (RNNs) for temporal data
- Generative adversarial networks (GANs) and autoencoders e.g. VAEs for generative modeling
- ...

which are all trained offline on known samples from some (perhaps very complicated) population distribution

What *is* a deep network?

Frank Rosenblatt, using the McCulloch-Pitts neuron and the findings of Donald Hebb, went on to develop the first perceptron

Say there exists a set $\mathcal{X}$ of possible inputs and some set $\mathcal{Y}$ of possible outputs, and a parameter vector $\Phi \in \mathbb{R}^d$.

For $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ a deep network Φ computes a conditional probability $P_\Phi(y|x)$.

A general prescription for training a deep network

We assume a “population” probability distribution Pop on pairs (x, y).

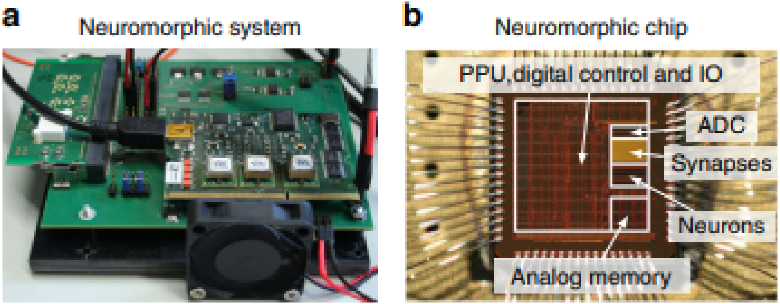
$$\Phi^* \quad = \quad \operatorname*{argmin}_{\Phi} \mathcal{L}(x, y, \Phi)$$

where $(x, y) \sim \mathrm{Pop}$. Learning (minimizing loss) occurs by some form of gradient descent e.g. stochastic gradient descent (SGD).

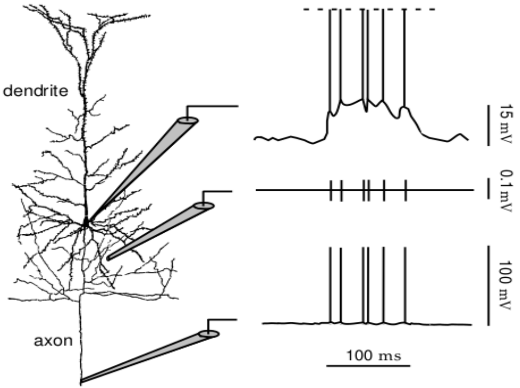
$$\Delta\Phi = -\eta \frac{\partial \mathcal{L}}{\partial \Phi}$$

Training large neural networks is computationally expensive

- The entire brain is estimated consume 10W of power
- Spiking networks (SNNs) perform computations in memory giving low-latency
- SNNs can in principle self-organize without backprop (unsupervised learning)



The third generation of neural networks: spiking nets

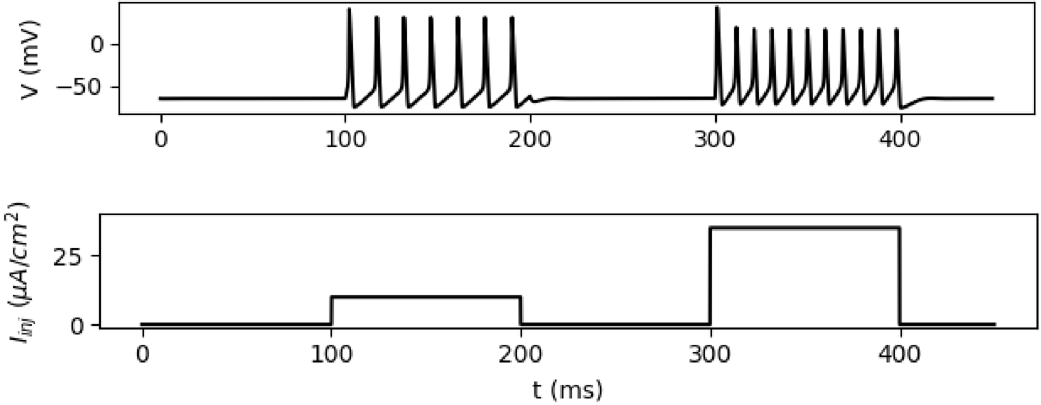


- $\sim$ 16 billion neurons in cortex
- A neuron receives on the order of 10^3 to 10^4 synaptic inputs
- Neurons communicate via action potentials in an all-or-nothing fashion

The third generation of neural networks: spiking nets

- Post-synaptic potentials (PSPs) allow pre-synaptic action potentials to change post-synaptic membrane potential
- PSPs can be positive or negative (excitatory or inhibitory)

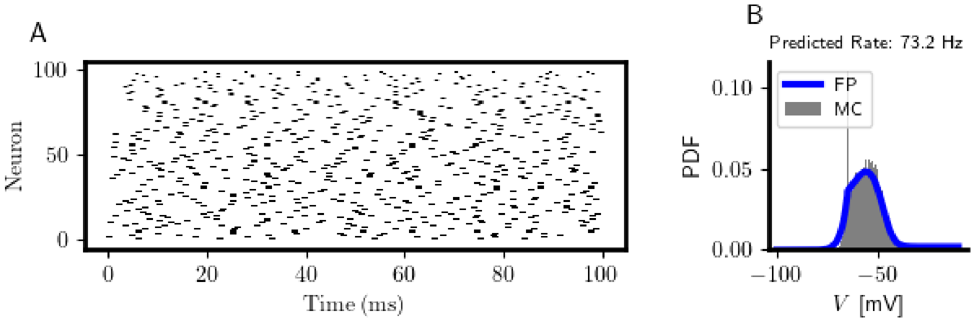
Integrate and fire (IF) neuron models



$$\tau \dot{V}(t) = g_\ell(E - V) + g_\ell \cdot \psi(V) + I(t)$$

Monte-Carlo simulation of uncoupled IF neurons

When $\psi(V) = g_\ell \Delta_T \exp\left(\frac{V - V_L}{\Delta_T}\right)$ we have the exponential integrate and fire model

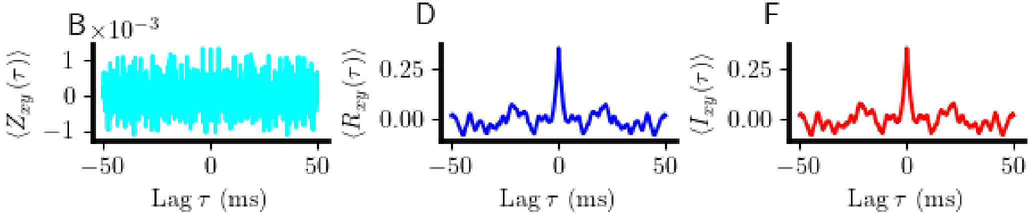


Langevin equations have a corresponding Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\sigma^2}{\tau}\frac{\partial^2 P}{\partial V^2} + \frac{\partial}{\partial V}\left(\frac{V - E + \psi}{\tau} P\right)$$

Synaptic coupling can induce correlations in spiking activity

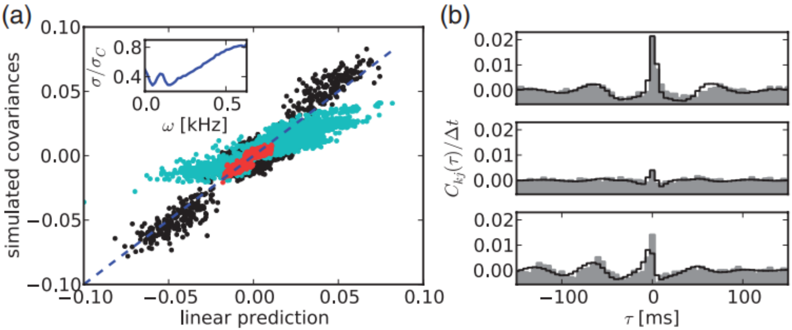
For special synaptic connectivity regimes dynamical variables can remain uncorrelated between neurons



Uncorrelated neural activity captures irregular spiking seen *in-vivo*

Predicting neuron correlations

The linear response of $r(t)$ allows us to also estimate the matrix of cross-correlations $C_{kj}(\tau)$ from the synaptic connectivity $\mathcal{C}$



This has important implications for brain-inspired machine learning