

# Homework 2

Quantum Mechanics

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**Problem 1.** *Problem 1.12 from Sakurai*

**Solution.**

If we choose the representation such that  $|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $|2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$  then we can use the definition of the outer product to show that

$$H = a \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

The energy eigenvalues are then found by

$$\begin{aligned} \det(H - \lambda I) &= \det \begin{pmatrix} a - \lambda & a \\ a & -a - \lambda \end{pmatrix} \\ &= (a - \lambda)(-a - \lambda) - a^2 \\ &= \lambda^2 - 2a^2 = 0 \end{aligned}$$

therefore  $E_{\pm} = \pm\sqrt{2}a$ . The  $+$  eigenvector  $|\psi_1\rangle$  is given by the system

$$\begin{aligned} (\psi_1^1 + \psi_1^2) &= \sqrt{\frac{2}{a}}\psi_1^1 \\ (\psi_1^1 - \psi_1^2) &= \sqrt{\frac{2}{a}}\psi_1^2 \end{aligned}$$

The  $-$  eigenvector  $|\psi_2\rangle$  is given by the system

$$\begin{aligned} (\psi_2^1 + \psi_2^2) &= -\sqrt{\frac{2}{a}}\psi_2^1 \\ (\psi_2^1 - \psi_2^2) &= -\sqrt{\frac{2}{a}}\psi_2^2 \end{aligned}$$

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**Problem 2.** *Problem 1.13 from Sakurai*

**Solution.**

Writing  $H$  out in matrix form gives

$$\begin{aligned} H &= H_{11} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + H_{12} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + H_{22} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} H_{11} + H_{12} + H_{22} + 1 & H_{11} - H_{12} - H_{22} + 1 \\ H_{11} - H_{12} + H_{22} - 1 & H_{11} + H_{12} - H_{22} - 1 \end{pmatrix} \end{aligned}$$

$$\det(H - \lambda I) = \det \begin{pmatrix} H_{11} + H_{12} + H_{22} + 1 - \lambda & H_{11} - H_{12} - H_{22} + 1 \\ H_{11} - H_{12} + H_{22} - 1 & H_{11} + H_{12} - H_{22} - 1 - \lambda \end{pmatrix}$$

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**Problem 3.** *Problem 1.15 from Sakurai*

**Solution.** After the first measurement along  $+\hat{z}$ , all of our atoms are prepared in the  $|+\rangle$  state in the  $S_z$  basis. At the next apparatus oriented along  $\hat{n}$ , more atoms will be filtered out since  $|+\rangle$  is not an eigenket of the  $\mathbf{S} \cdot \hat{n}$  operator. Recall that  $|+\rangle_n$  is

$$|+\rangle_n = \cos \frac{\beta}{2} |+\rangle + \sin \frac{\beta}{2} |-\rangle$$

The probability the state  $|+\rangle$  survives is given by the inner product

$$\begin{aligned} |\langle + | + \rangle_n|^2 &= |\langle + | \cos \frac{\beta}{2} |+\rangle + \sin \frac{\beta}{2} |-\rangle|^2 \\ &= \cos^2 \frac{\beta}{2} \end{aligned}$$

After this, all atoms are in the  $|+\rangle_n$  state. We then filter the atoms one more time with an apparatus along  $-\hat{z}$ . The fraction that survive this one is given by

$$\begin{aligned}
 |\langle -|+\rangle_n|^2 &= |\langle -|\cos\frac{\beta}{2}|+\rangle + \langle -|\sin\frac{\beta}{2}|-\rangle|^2 \\
 &= \sin^2\frac{\beta}{2}
 \end{aligned}$$

Therefore the fraction output is  $\cos^2\frac{\beta}{2}\sin^2\frac{\beta}{2}$ . We can maximize this function by setting  $\beta = \pi/2$

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**Problem 4.** *Problem 1.16 from Sakurai*

**Solution.**

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**Problem 5.** *Problem 1.23 from Sakurai*

**Solution.**

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**Problem 6.** *Problem 1.24 from Sakurai*

**Solution.**

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