# Homework 7

**Quantum Mechanics**
*November 2, 2022* C Seitz

**Problem 1.** *5.1*

**Solution**.
We are concerned here with the new ground state ket $|0\rangle$ in the presence of $H_1$ and the new ground state energy shift $\Delta_0$.

$$|0\rangle = \left|0^0\right\rangle + \sum_{k\neq 0} \left|k^0\right\rangle \frac{V_{k0}}{E_0^0 - E_k^0} + ...$$

$$\Delta_0 = V_{00} + \sum_{k\neq 0} \frac{|V_{k0}|^2}{E_0^0 - E_k^0} + ...$$

$$V_{nk} = b\left\langle n^0\right| x \left|k^0\right\rangle = b\sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{k}\delta_{n,k-1} + \sqrt{k+1}\delta_{n,k+1}\right)$$

The lowest nonvanishing order is then $V_{01}$. Therefore

$$\Delta_0 = -\frac{b^2\hbar}{2m\omega}\frac{1}{\hbar\omega} = -\frac{b^2}{2m\omega^2}$$

To solve it exactly, notice that the potential is of the form

$$V_1(x) = ax^2 + bx$$

The new potential shifts to the left by $b/2$, has a new minimum at $-b/2a$, and the gradient has changed:

$$V'(x) = 2ax \rightarrow V_1'(x) = 2ax + b$$

This change in the gradient will not change the energy differences w.r.t. the original problem (why?) so we have really just shifted the equilibrium point down by $-b/2m\omega^2$.

$$\Delta = -\frac{b}{2a} = -\frac{b}{2m\omega^2}$$

which is exactly what we got with perturbation theory.

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**Problem 2.** *5.2*

**Solution**. The perturbation Hamiltonian is

$$H_1 = \frac{Vx}{L}$$

$$\begin{aligned} V_{nk} = \left\langle n^0 \right| H_1 \left| k^0 \right\rangle &= \frac{V}{L} \left\langle n^0 \right| x \left| k^0 \right\rangle \\ &= \frac{V}{L} \int_0^L x \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{k\pi x}{L}\right) dx \end{aligned}$$

$$\Delta_0 = V_{00} + \sum_{k\neq 0} \frac{|V_{k0}|^2}{E_0^0 - E_k^0} + ...$$

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**Problem 3.** *5.5*

**Solution**. ■

**Problem 4.** *5.7*

**Solution**. ■

**Problem 5.** *5.12a*

**Solution**. ■

**Problem 6.** *5.24*

**Solution**. ■