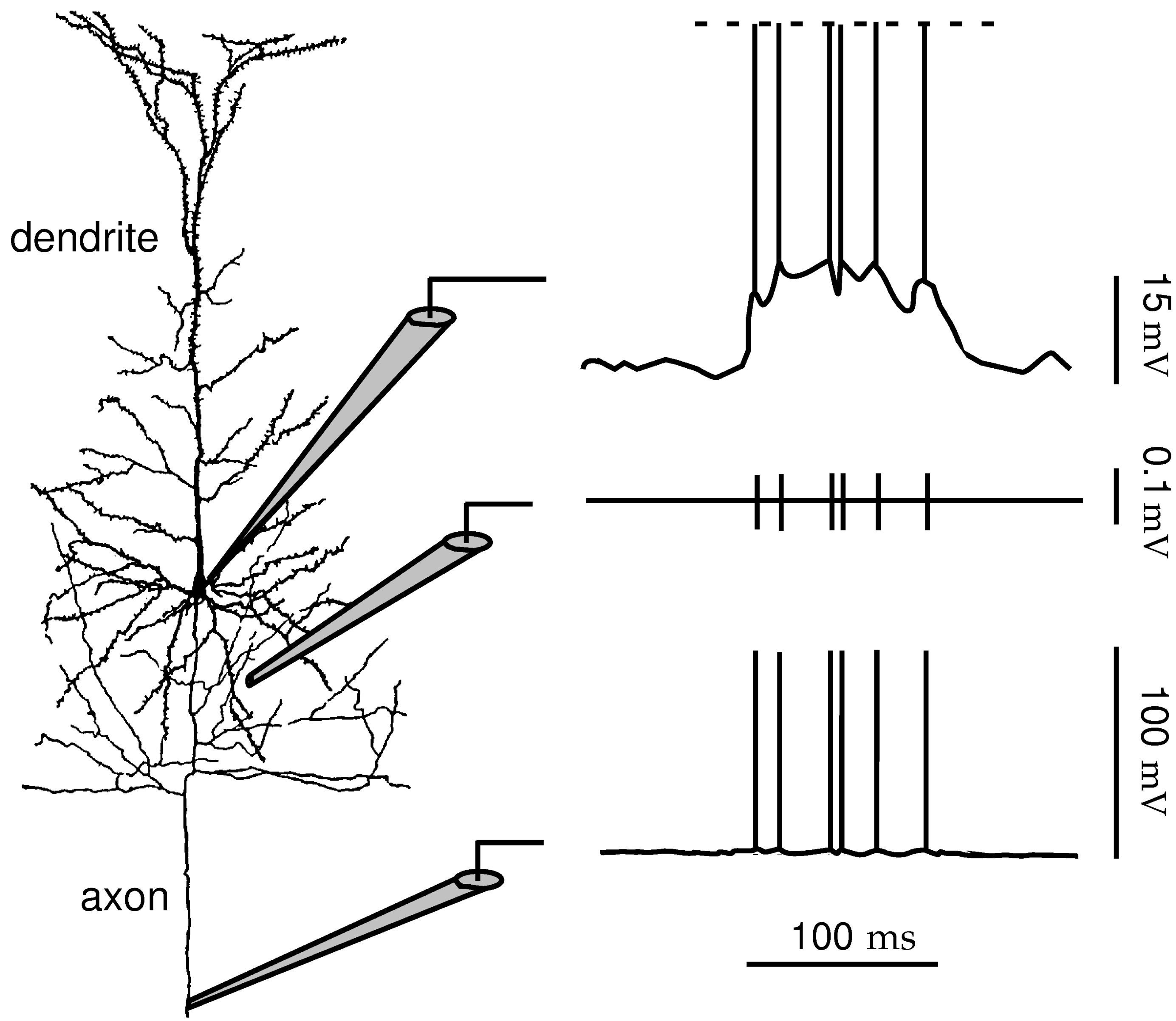
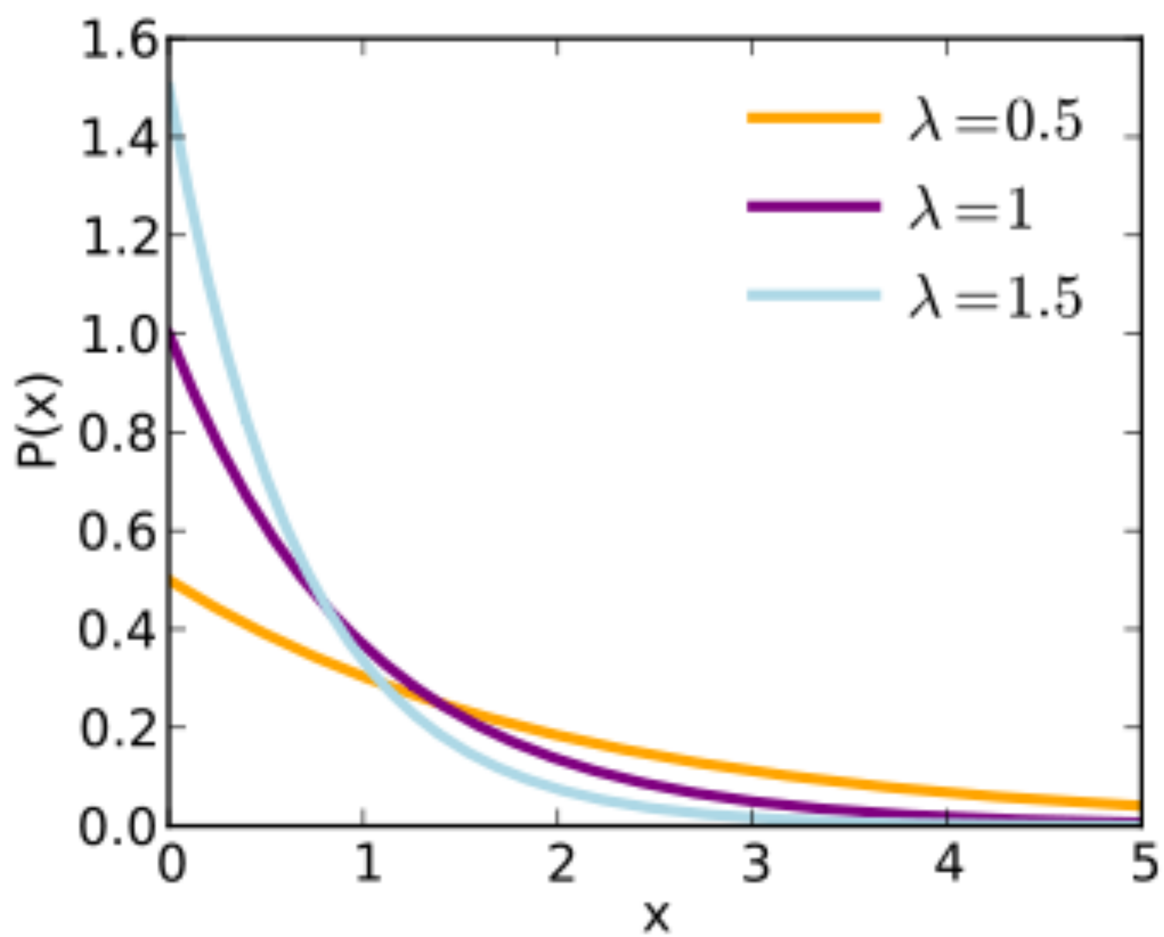
Lecture 4: Spike train statistics, part 2



$$\mathcal{S} = \{t_1, t_2, \ldots t_N\}$$

ISI distribution for a Poisson process:

$$P[\tau \leq t_{i+1} - t_i < \tau + \Delta t] = \mathrm{r}\Delta t \exp(-\mathrm{r}\tau)$$



Mean and variance of the ISI distribution:

$$\langle \tau \rangle = \int_0^\infty d\tau\, \tau \mathsf{r} \exp(-\mathsf{r}\tau) = \frac{1}{\mathsf{r}}$$

$$\sigma_\tau^2 = \int_0^\infty d\tau\, \tau^2 \mathsf{r} \exp(-\mathsf{r}\tau) - \langle \tau \rangle^2 = \frac{1}{\mathsf{r}^2}$$

chalkboard interlude

Coefficient of variation (CV):

$$C_V = \frac{\sigma_\tau}{\langle \tau \rangle}$$

$$\langle \tau \rangle = \int_0^\infty d\tau \, \tau \mathsf{r} \exp(-\mathsf{r}\tau) = \frac{1}{\mathsf{r}}$$

$$\sigma_\tau^2 = \int_0^\infty d\tau \, \tau^2 \mathsf{r} \exp(-\mathsf{r}\tau) - \langle \tau \rangle^2 = \frac{1}{\mathsf{r}^2}$$

Moment generating functions:

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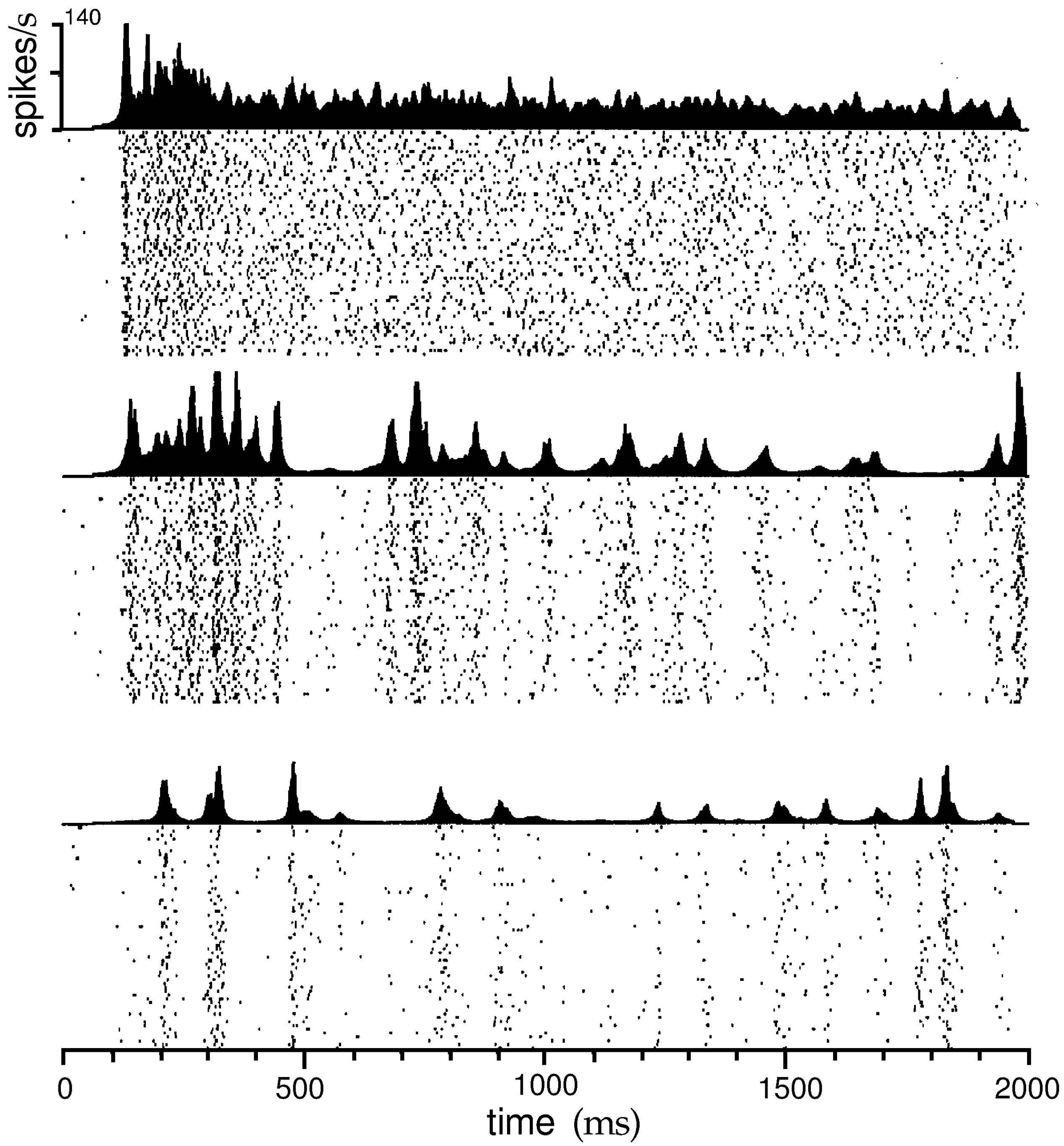
Inhomogeneous Poisson process:

$$p[t_1, t_2, \ldots, t_n] = \exp\left(-\int_0^T dt\, \mathsf{r}(t)\right) \prod_{i=1}^{n} \mathsf{r}(t_i)$$

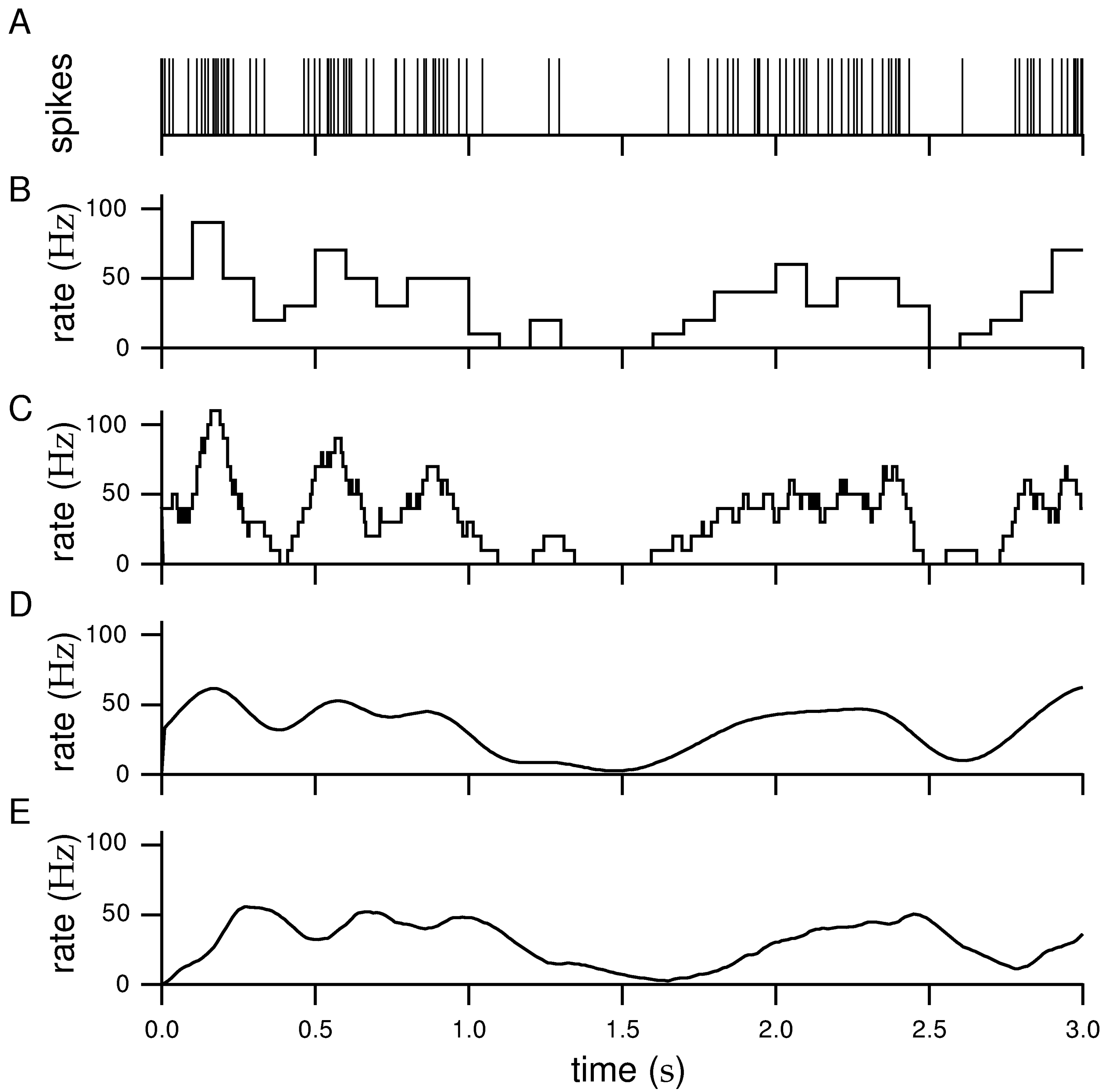
$$N_{\lambda(t)}[s, t) \sim \mathsf{Poiss}[\int_s^t d\xi\ \lambda(\xi)]$$

$$\frac{\mathcal{V}ar\left[N_{\lambda(t)}[s, t)\right]}{\mathcal{E}\left[N_{\lambda(t)}[s, t)\right]} = 1$$

Mean response functions and the PSTH:

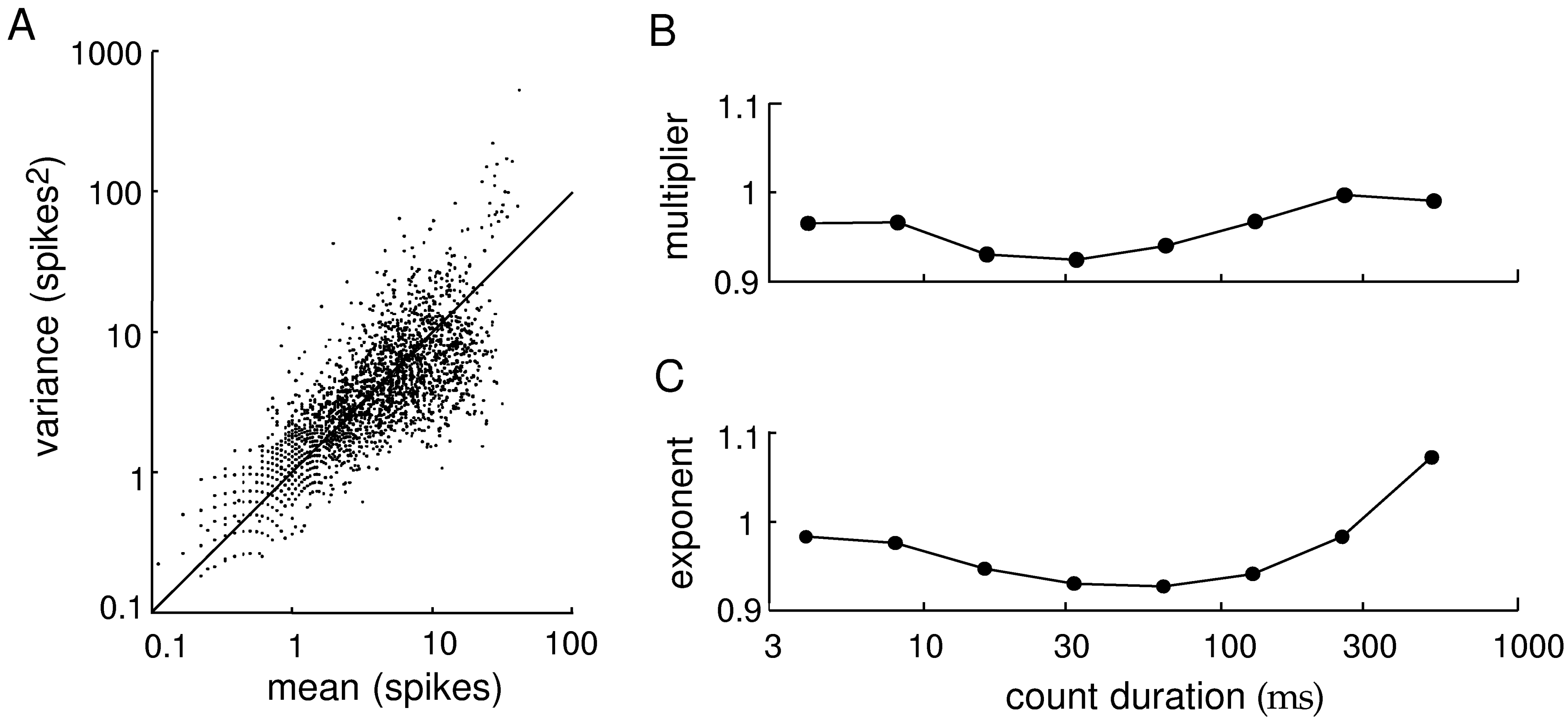


Estimating time-varying rates from data:

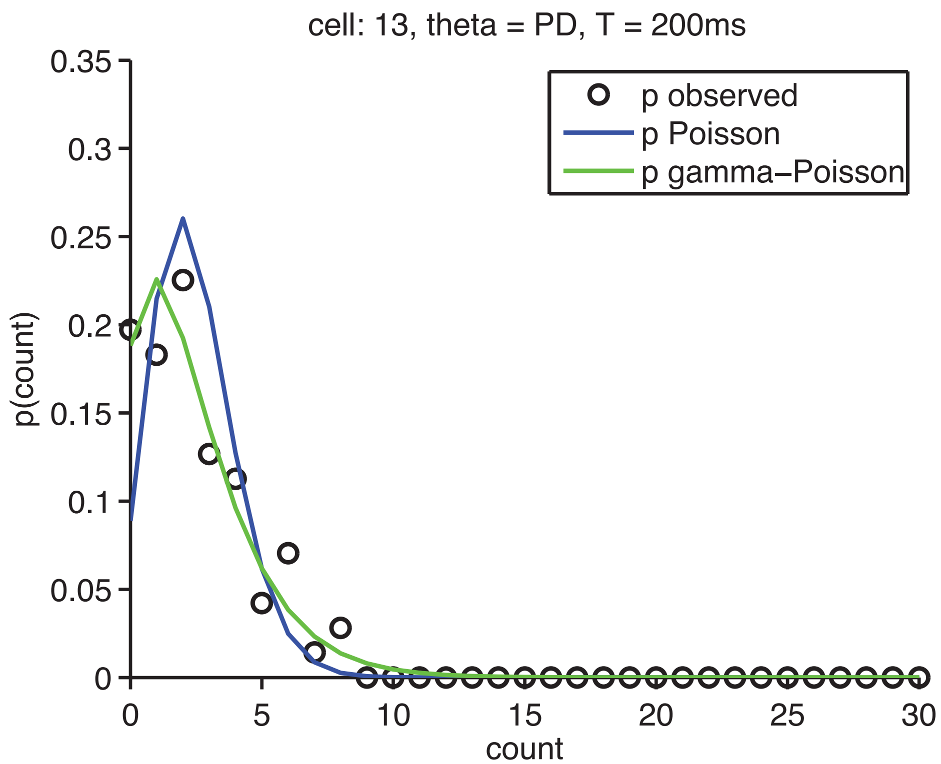


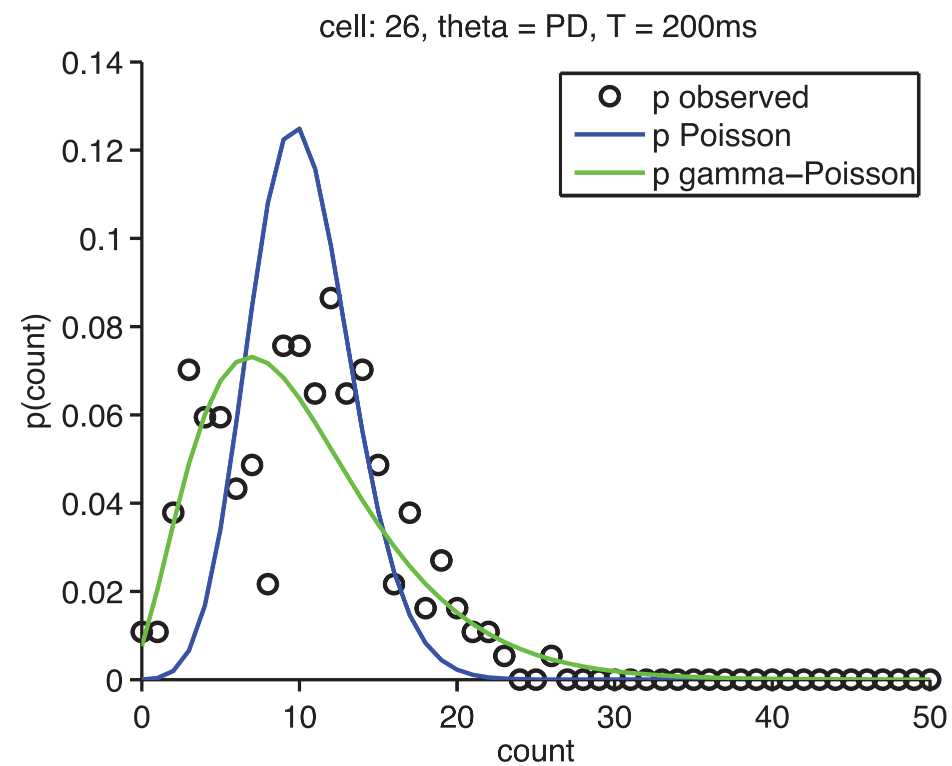
How well do these models fit?

$$\sigma_n^2 = A\langle n\rangle^B$$



In real data, spike counts often have variance >





Poisson:

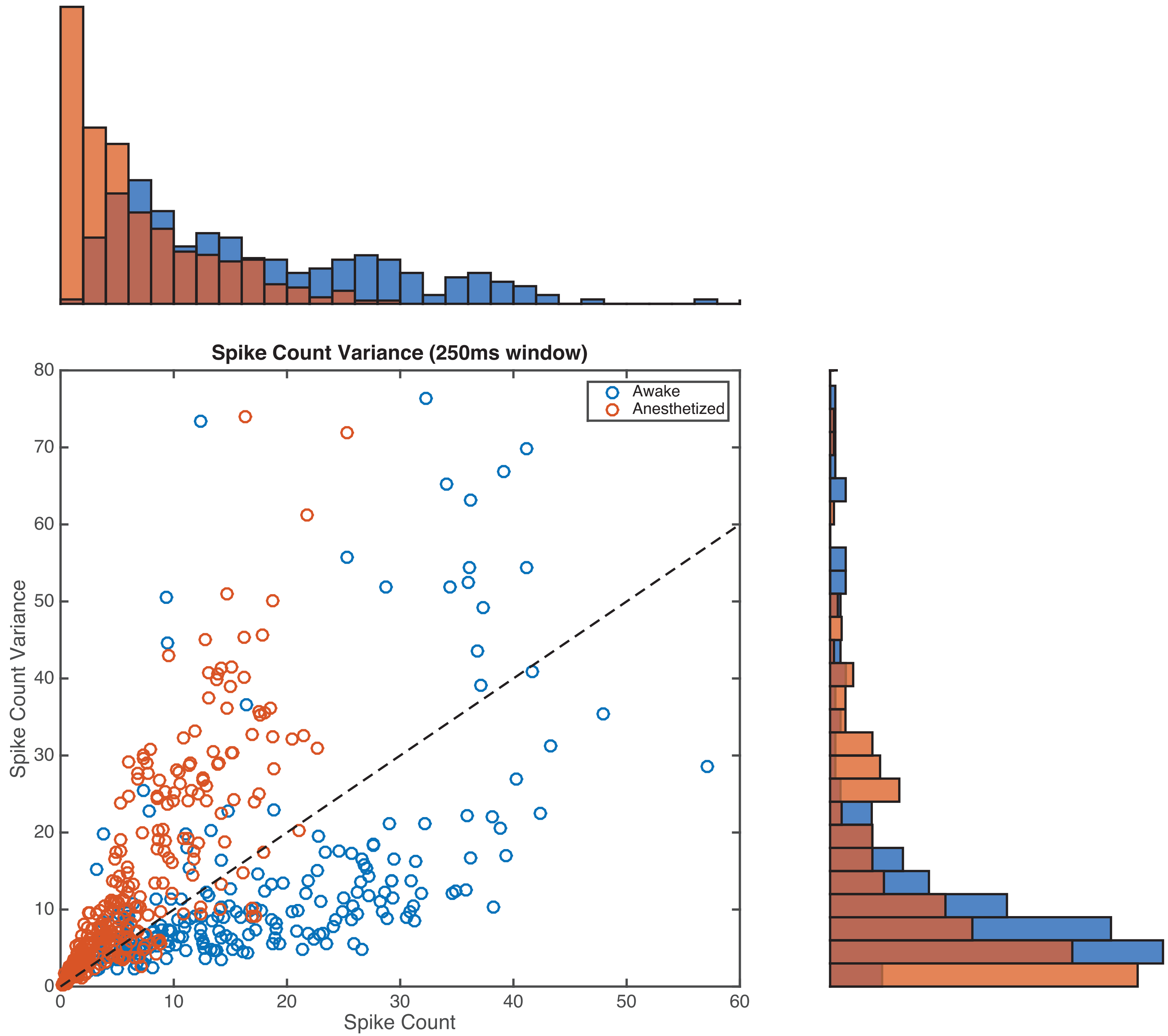
$$P(r_i|\theta) = \frac{\lambda_i(\theta)^k}{k!} e^{-\lambda_i(\theta)} \qquad \mathrm{var}(k) = \lambda$$

gamma-Poisson:

$$P(r_i|\theta) = \frac{\Gamma(\alpha + k)}{k!\Gamma(\alpha)} \frac{\beta^\alpha \lambda_i(\theta)^k}{(\lambda_i(\theta) + \beta)^{\alpha+k}} \qquad \mathrm{var}(k) = \lambda(1 + \sigma^2 \lambda)$$

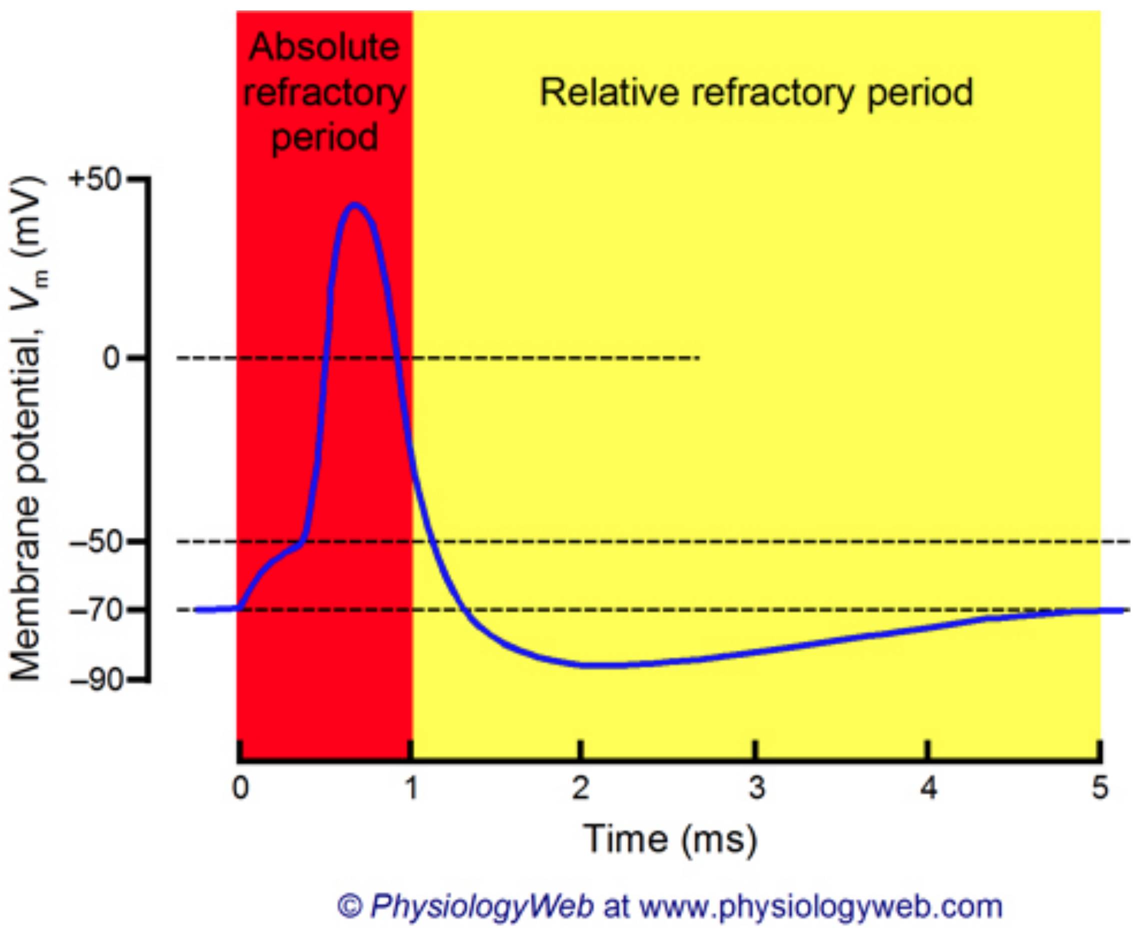
$$\alpha = \beta = 1/\sigma^2$$

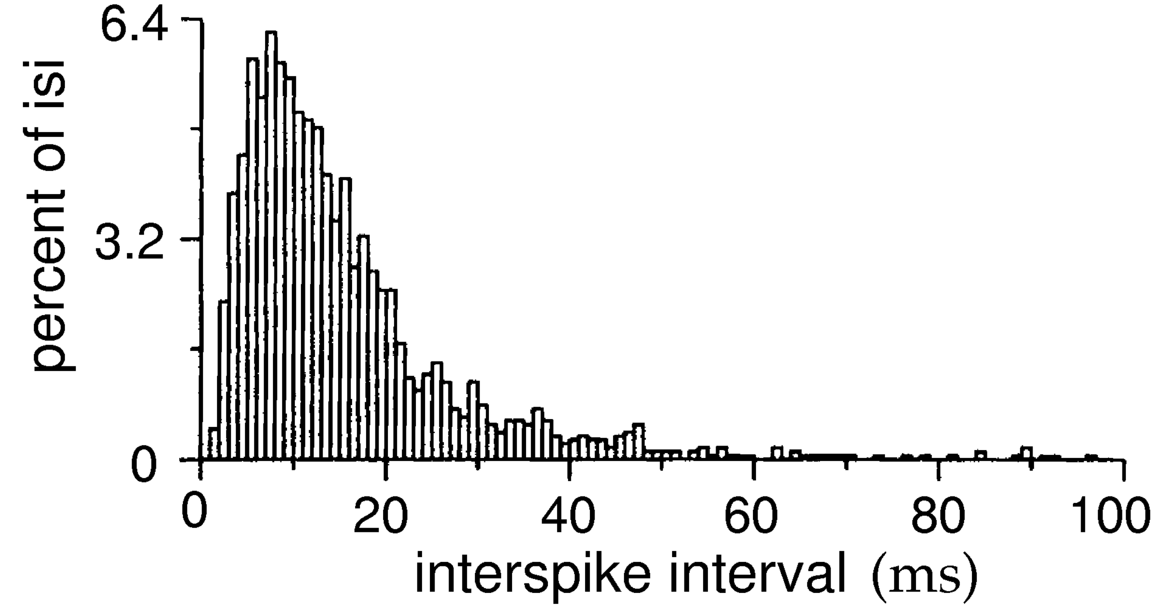
Fano Factor changes with brain state:



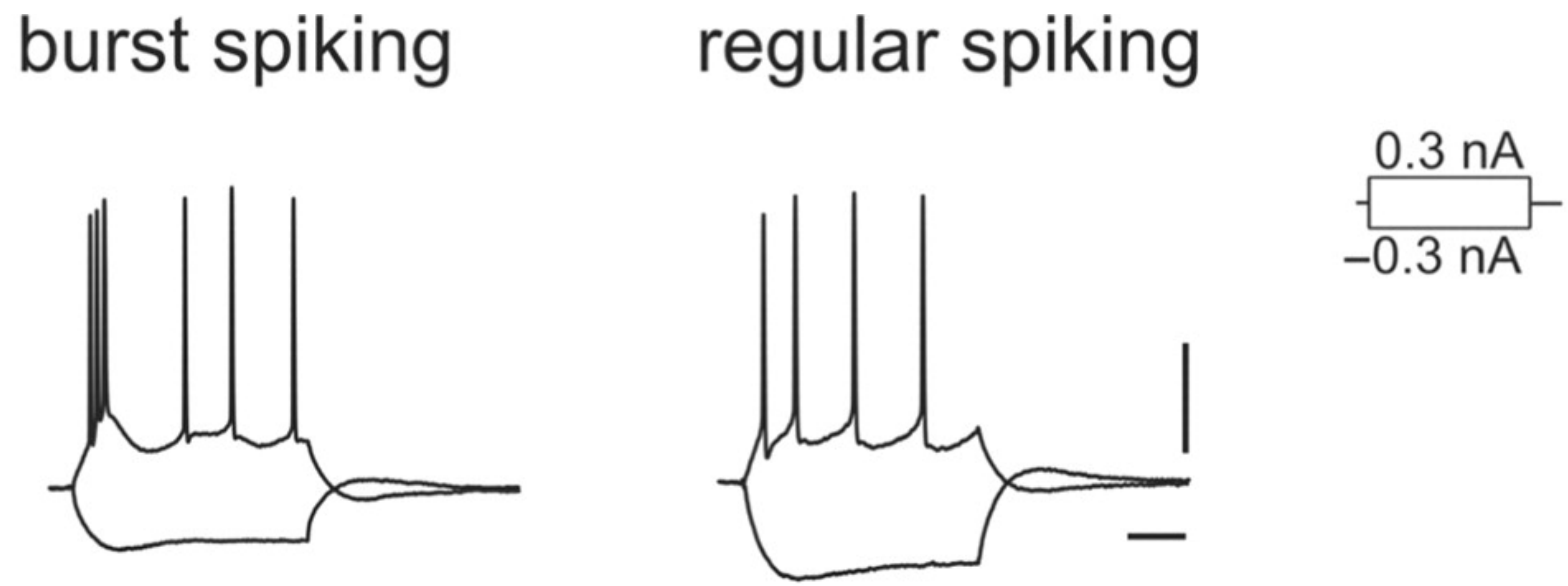
Joe Lombardo, Osborne lab data

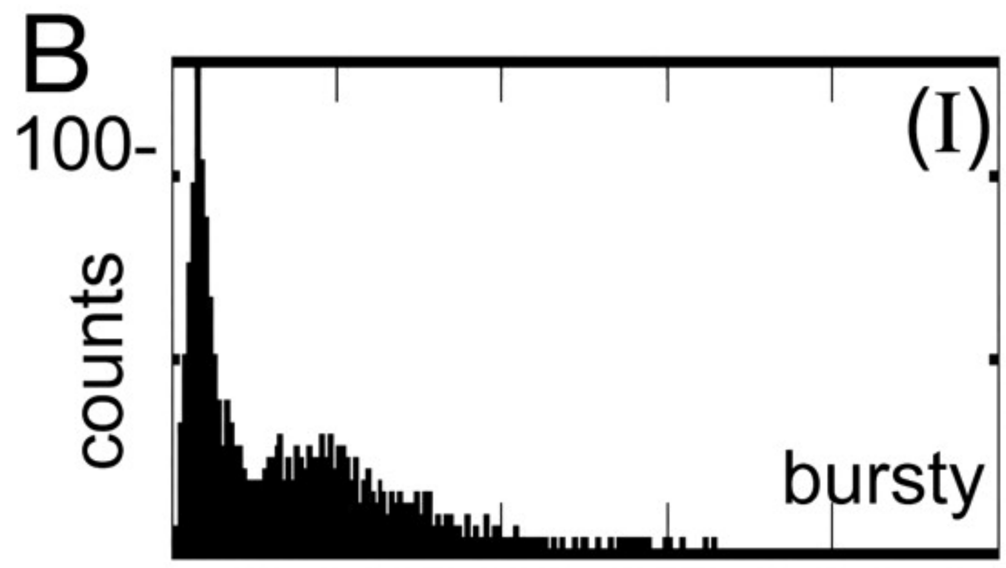
Problem: real neurons are refractory



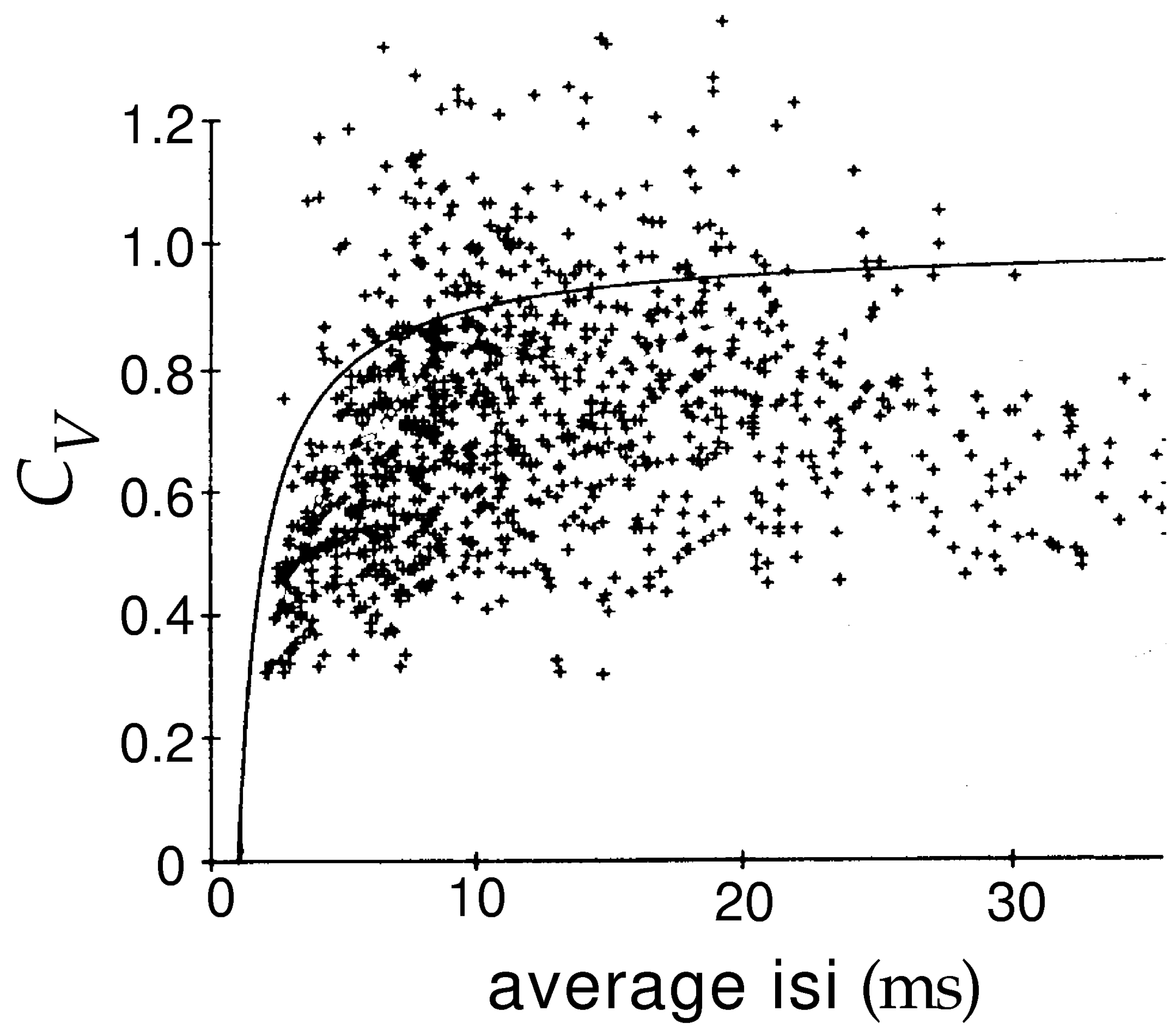


Problem: real neurons burst



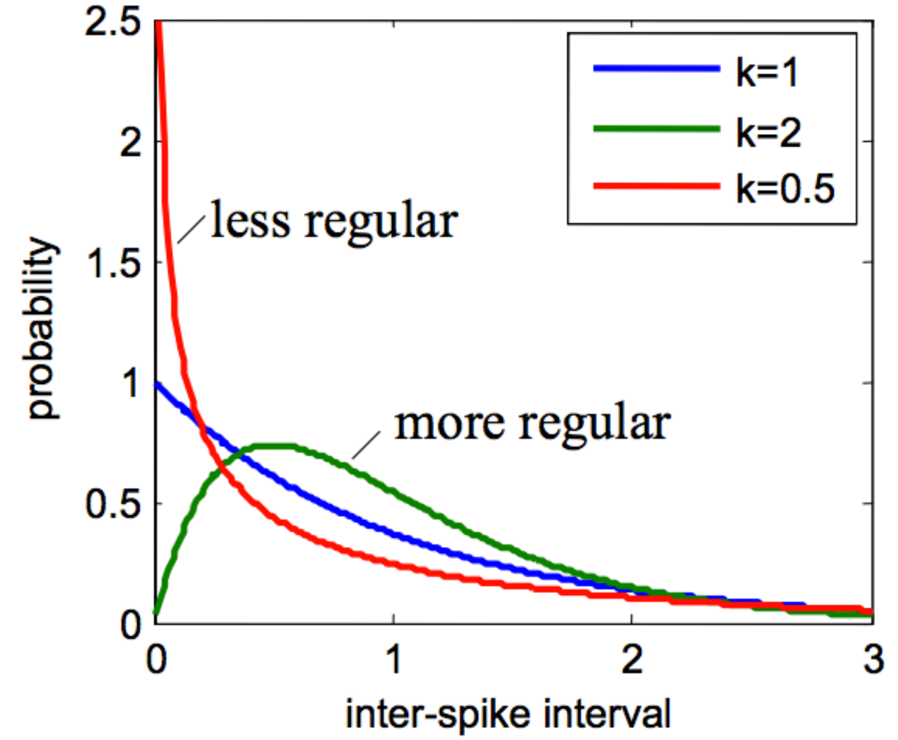


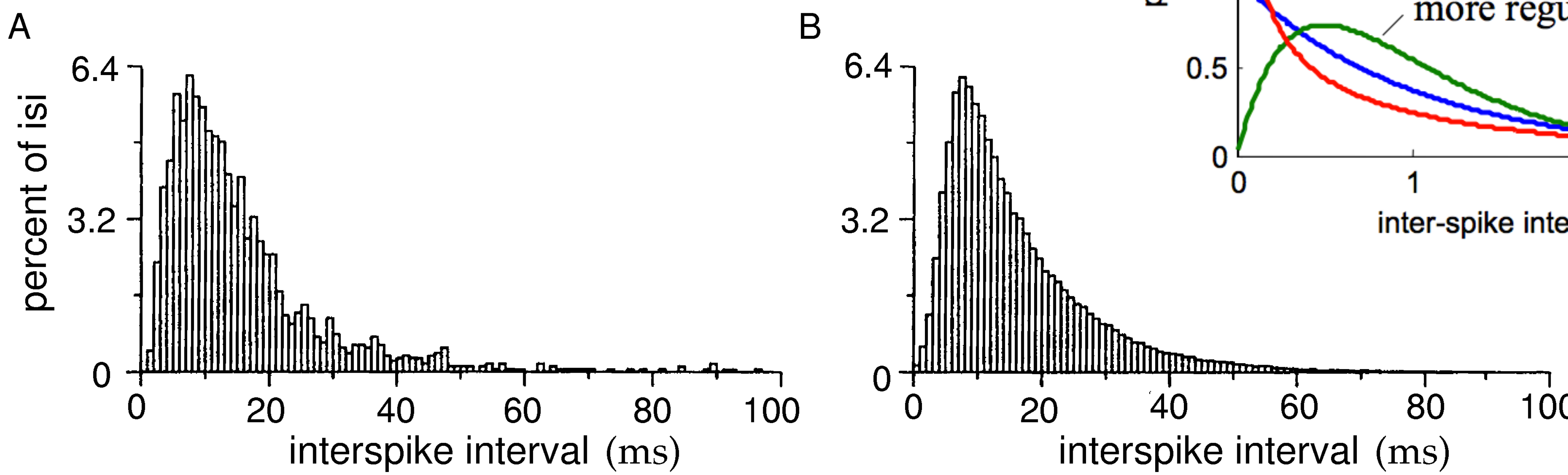
Poisson process with a refractory period:

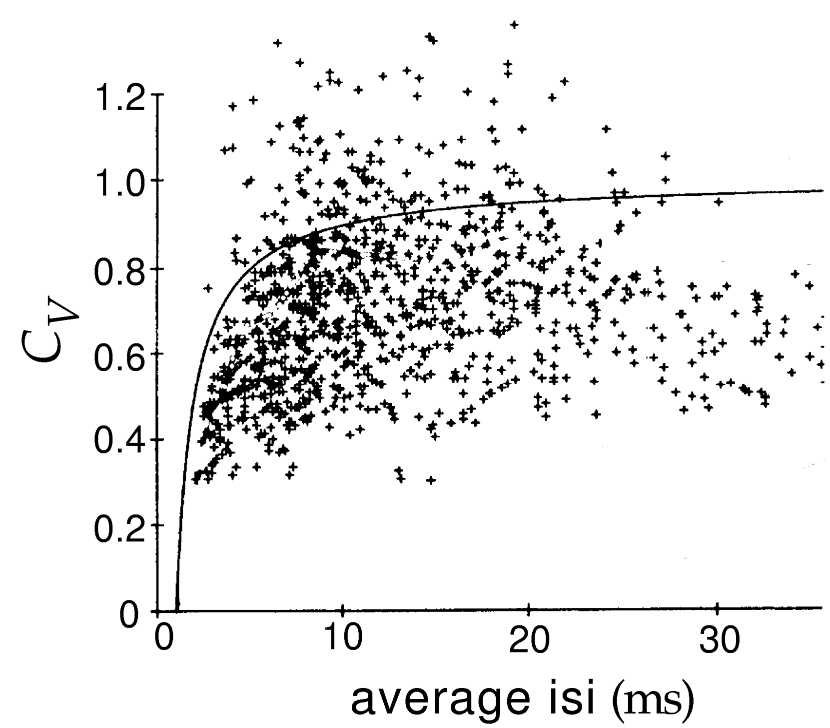


The Gamma distribution:

$$p[\tau] = \frac{r(r\tau)^k \exp(-r\tau)}{k!}$$

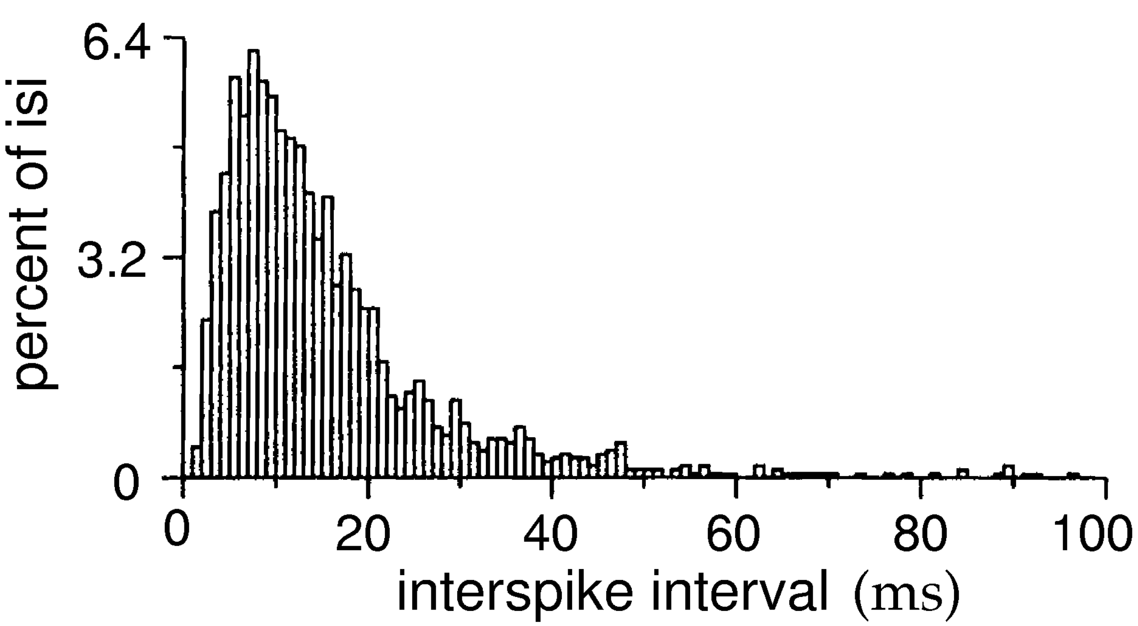




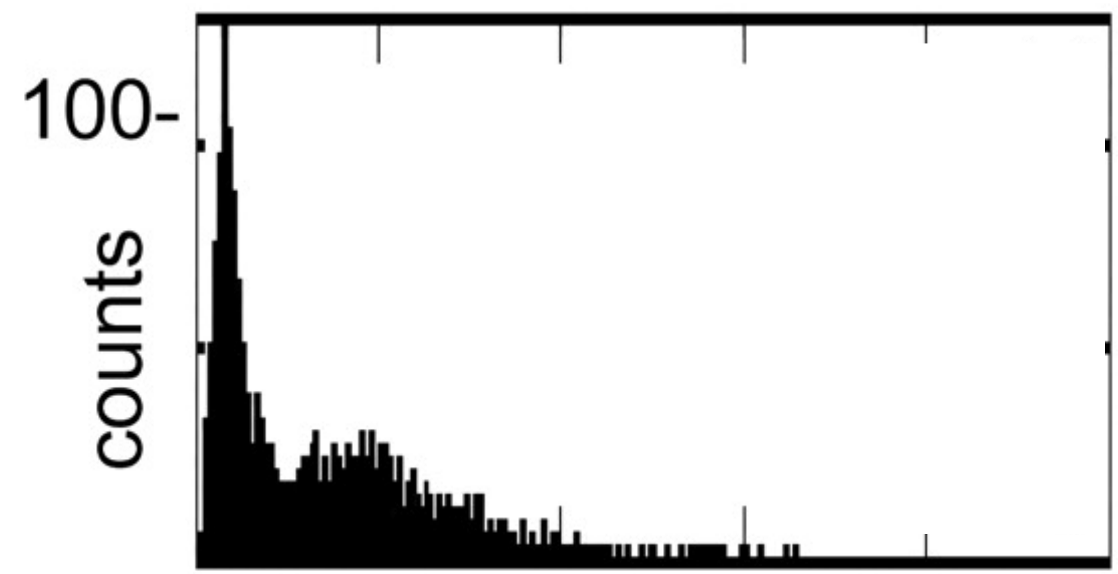


Real neuronal spiking displays history dependence:

Refractory:



Bursty:



The doubly stochastic Poisson process:

Some parameter of the process (sometimes the rate) is a random variable itself.
Sometimes used to model bursting (Cox process)

example: the randomly scaled IHPP:

$$\lambda(t) = s \cdot \rho(t) \qquad \text{with } \rho(t) \text{ fixed and } s \sim \mathsf{Gamma}(\alpha, \beta)$$

Conway's Wizard Puzzle

Last night I sat behind two wizards, Azemelius and Bartholomew, on a bus. I heard this conversation:

Azemelius: I have a positive integer number of children, whose ages are positive integers. The product of their ages is my own age, and the sum of their ages is the number on this bus.

Bartholomew (looking at the number of the bus): Perhaps if you told me your age and how many children you had, I could work out their ages?

Azemelius: No, you could not.

Bartholomew: Aha! At last I know how old you are! (Bartholomew had been trying to find Azemelius's age for a long time.)

What is the number of the bus?