Probabilistic graphical models

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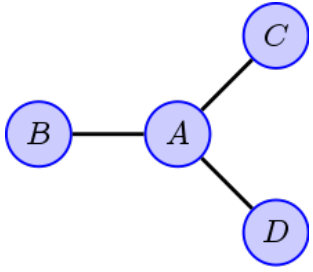
April 9, 2022

Outline

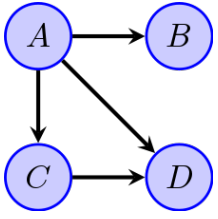
Probabilistic graphical models (PGMs)

Say we have a joint probability over gene expression $P(\boldsymbol{X})$
A PGM describes how $P(\boldsymbol{X})$ factors



Markov Random Fields (MRFs) e.g., Ising model

$$P(\boldsymbol{X};\Theta) = \frac{1}{Z}\prod_{i=1}^{N} P(\boldsymbol{X_i}, \mathcal{C}(X_i);\Theta_i)$$



Bayesian Network (BNs) - include causality

$$P(\boldsymbol{X}|\mathcal{G},\Theta) = \prod_{i=1}^{N} P(\boldsymbol{X_i}|\mathcal{C}(X_i),\Theta_i)$$

BNs as well as hybrid models have been used to examine gene expression

Markov random fields

$$P(\mathbf{x}) = \frac{\exp\left(-H(\mathbf{x})\right)}{\sum_i \exp\left(-H(\mathbf{x}_i)\right)}$$

Suppose the energy function can be written as a sum over cliques:

$$H(\mathbf{x}) = \sum_n \tilde{\psi}_n(c_n)$$

Let $\psi_n = \log \tilde{\psi}_n$, which means $P(\mathbf{x})$ factors according to

$$P(\mathbf{x}) = \frac{\prod_n \psi_n(c_n)}{\sum_i \prod_n \psi_n(c_n)}$$

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