

# A brief introduction to graphical models

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# Outline

Introduction to graphical models

Graphical models of gene expression

Graphical models in image processing

References

# The logic of generative modeling

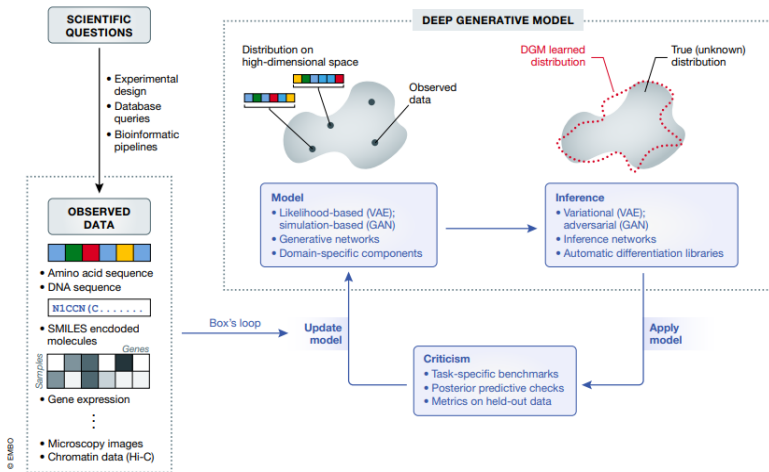
Say we have a set of variables  $x = (x_1, x_2, \dots, x_n)$  which might have some statistical dependence

The variable  $x$  might be an amino acid sequence, gene expression data, microscopy image, etc.

- ▶ Often we are handed a batch of empirical samples  $\{x_i\}_{i=1}^N$
- ▶ We want to know the generating distribution  $p(x)$

In supervised **generative learning**, we try to explicitly learn the joint distribution  $p(x) = \prod_{i=1}^{N-1} p(x_i | x_{i+1:N}) p(x_N)$ , which is generally more difficult than discriminative learning.

# Generative modeling with feedback



# Perks of generative modeling

- ▶ Fitting complete multivariate distributions  $p(\mathbf{x})$  goes beyond correlation-based or clustering approaches
- ▶ Correlations cannot discover partial correlation in the context of other neighbors
- ▶ Fitting  $p(\mathbf{x})$  permits sampling based inference

# Why generative modeling is difficult

When describing a distribution over multiple variables, we may not know the proper normalization  $Z$ . That is,

$$p(x) = \frac{1}{Z} \tilde{p}(x)$$

This **very important** situation arises in several contexts:

1. In **Bayesian inference** where  $p(x_1|x_2) = p(x_2|x_1)p(x_1)/p(x_2)$  is intractable due to  $Z = p(x_2) = \int p(x_2|x_1)p(x_1)dx_1$ . This integral can be very difficult or impossible to compute.
2. In models from statistical physics, e.g. the Ising model, we only know  $\tilde{p}(x) = e^{-H(x)}$  where  $H(x)$  is the Hamiltonian

In **image processing**, graphical models are a standard technique for:

- ▶ Image denoising
- ▶ Image interpolation (resolution enhancement)
- ▶ Semantic image segmentation

In biology, graphical models are used widely in **omics** i.e.

- ▶ Genomics
- ▶ Transcriptomics
- ▶ Proteomics

# Bayesian image reconstruction

Say a fluorophore emits photons at a rate  $\lambda_n$ . This is the best we can do according to QM

For a CMOS array with quantum efficiency  $\gamma$  [ $e^-/p$ ] we have

$$I_n = \gamma g_n P_n(\lambda_n) + G_n(\mu_n; \sigma_n^2) + \beta$$

where  $\mu_n$  [ADU] is the detector offset and  $g_n$  [ADU/ $e^-$ ] is the gain.

All we know is  $\lambda_n$ , so both the true signal  $I_n$  and the detected signal  $\hat{I}_n$  are stochastic processes.

$$P_\lambda(I_n, \hat{I}_n) = \frac{1}{Z} \frac{\exp(-\lambda_n) \lambda_n^p}{p!} \exp\left(-\frac{(D - g_n p - \mu_n)^2}{\sigma_n^2}\right)$$



# Bayesian image reconstruction

Marginalizing over  $p$  gives the noise model as a function of the rate  $\lambda_n$

$$P_{\lambda}(I_n) = \frac{1}{Z} \sum_p \frac{\exp(-\lambda_n) \lambda_n^p}{p!} \exp\left(-\frac{(D - g_n p - \mu_n)^2}{\sigma_n^2}\right)$$

# References I