Homework 7

Quantum Mechanics
November 1, 2022 C Seitz

Problem 1. *5.1*

Solution.
We are concerned here with the new ground state ket $|0\rangle$ in the presence of H_1 and the new ground state energy shift Δ_0.

$$|0\rangle = \left|0^0\right\rangle + \sum_{k\neq 0} \left|k^0\right\rangle \frac{V_{k0}}{E_0^0 - E_k^0} + ...$$

$$\Delta_0 = V_{00} + \sum_{k\neq 0} \frac{|V_{k0}|^2}{E_0^0 - E_k^0} + ...$$

$$V_{nk} = b\left\langle n^0\right| x \left|k^0\right\rangle = b\sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{k}\delta_{n,k-1} + \sqrt{k+1}\delta_{n,k+1}\right)$$

The lowest nonvanishing order is then V_{01}. Therefore

$$\Delta_0 = -\frac{b^2\hbar}{2m\omega}\frac{1}{\hbar\omega} = -\frac{b^2}{2m\omega^2}$$

To solve it exactly, notice that the potential is of the form

$$V(x) = ax^2 + bx$$

This function shifts to the left by $b/2$ and has a new minimum at $-b/a$. Also, any multiplicative constant ϵ of the potential can just be absorbed into ω, such that $E_n = \hbar\omega\sqrt{\epsilon}(n + \frac{1}{2})$. Importantly, this implies that the energy differences for various n, remain the same. Therefore, we can conclude that

$$\Delta = -\frac{b}{2a} = -\frac{b}{2m\omega^2}$$

which is exactly what we got with second-order perturbation theory. ■

Problem 2. *5.2*

Solution. ■

Problem 3. *5.5*

Solution. ■

Problem 4. *5.7*

Solution. ■

Problem 5. *5.12a*

Solution. ■

Problem 6. *5.24*

Solution. ■