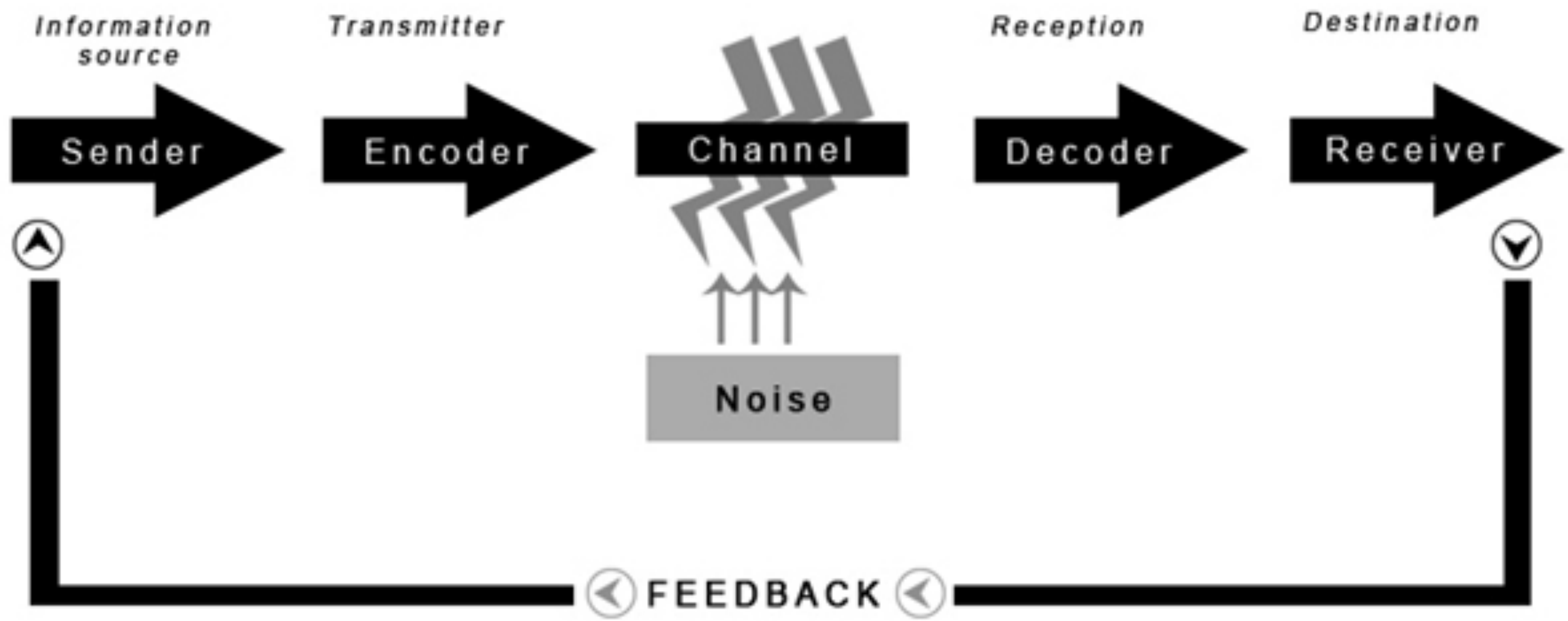
Lecture 9: *Decoding neural signals*



Shannon-Weaver's Model of Communication

A few definitions to start:

- $P[s]$, the probability of stimulus s being presented. This is often called the prior probability,
- $P[\mathbf{r}]$, the probability of response $\mathbf{r}$ being recorded,
- $P[\mathbf{r}, s]$, the probability of stimulus s being presented and response $\mathbf{r}$ being recorded,
- $P[\mathbf{r}|s]$, the conditional probability of evoking response $\mathbf{r}$ given that stimulus s was presented, and
- $P[s|\mathbf{r}]$, the conditional probability that stimulus s was presented given that the response $\mathbf{r}$ was recorded.

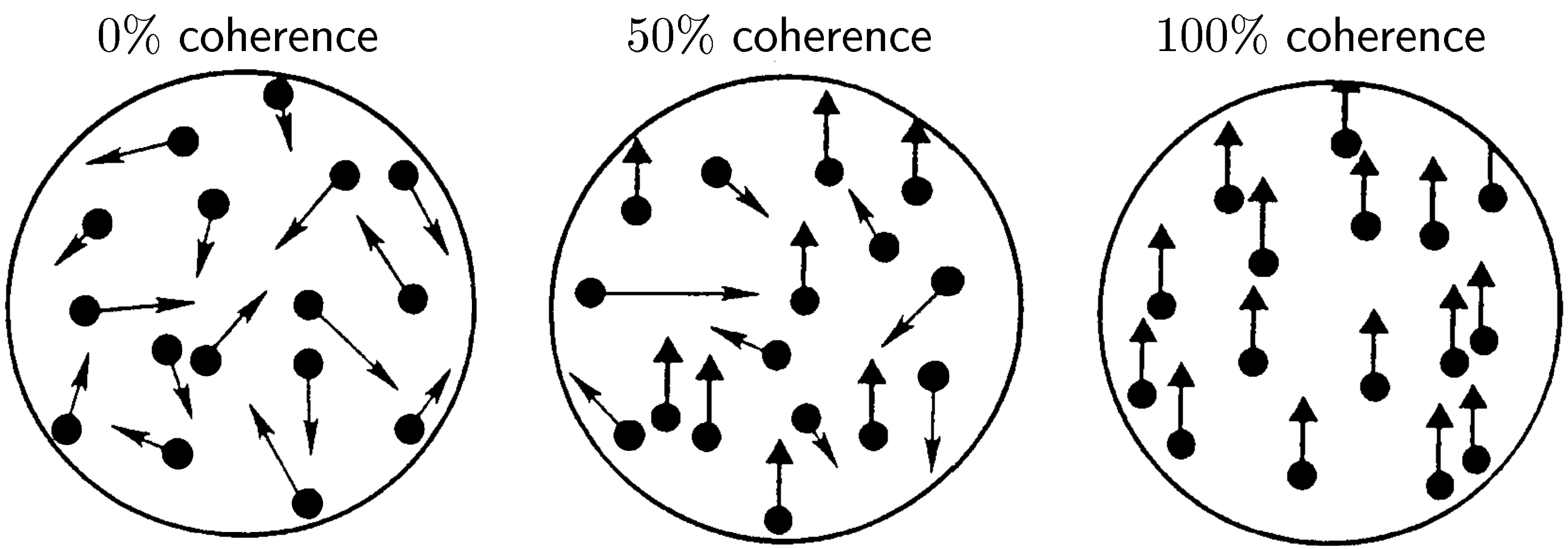
Bayes' Rule:

$$P[\mathbf{r}] = \sum_{s} P[\mathbf{r}|s]P[s] \text{ and similarly } P[s] = \sum_{\mathbf{r}} P[s|\mathbf{r}]P[\mathbf{r}]$$

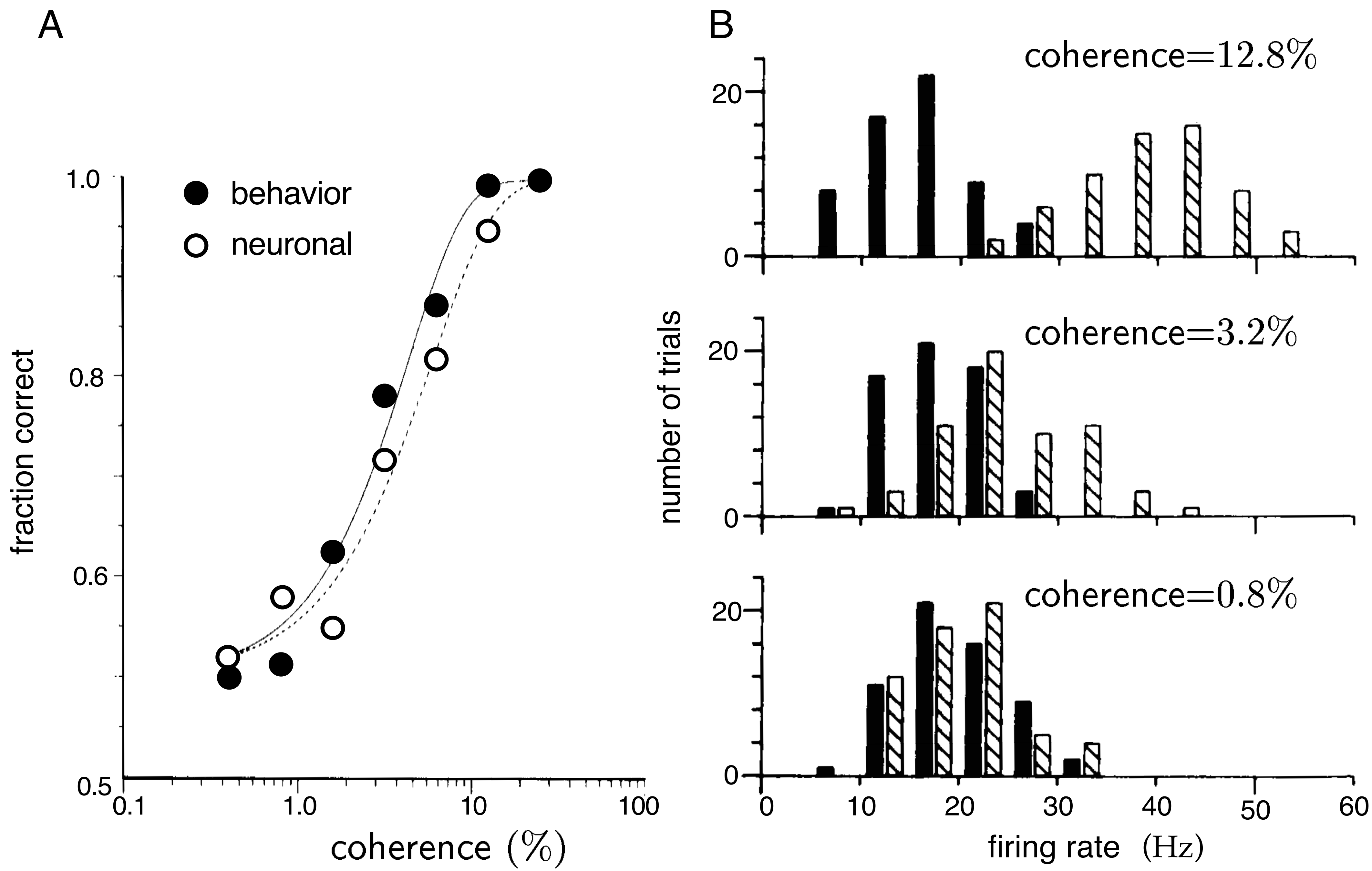
$$P[\mathbf{r}, s] = P[\mathbf{r}|s]P[s] = P[s|\mathbf{r}]P[\mathbf{r}]$$

$$P[s|\mathbf{r}] = \frac{P[\mathbf{r}|s]P[s]}{P[\mathbf{r}]}$$

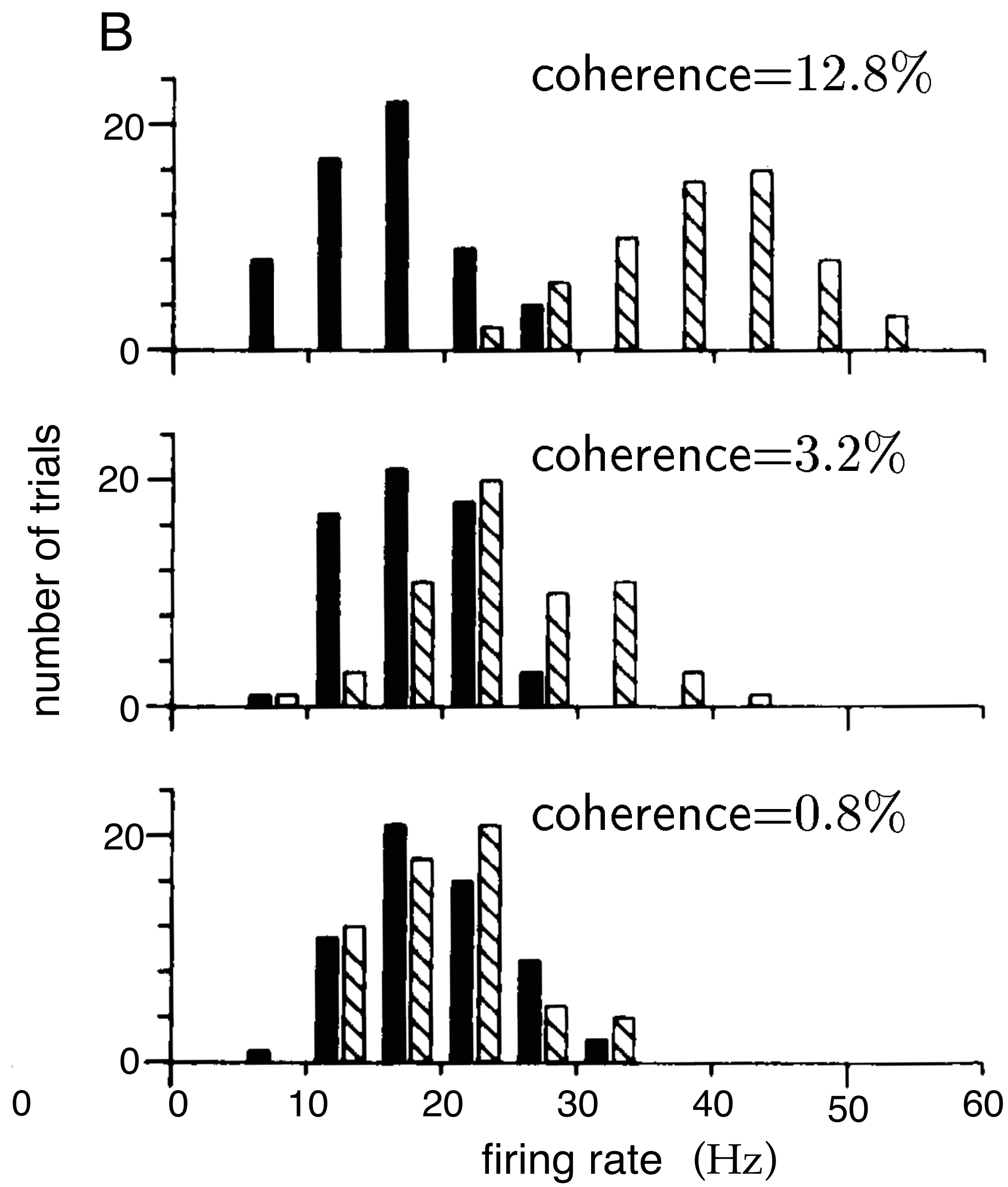
Stochastic stimulus, random moving dots:



Behavior on the random dot discrimination task:



Discriminability:



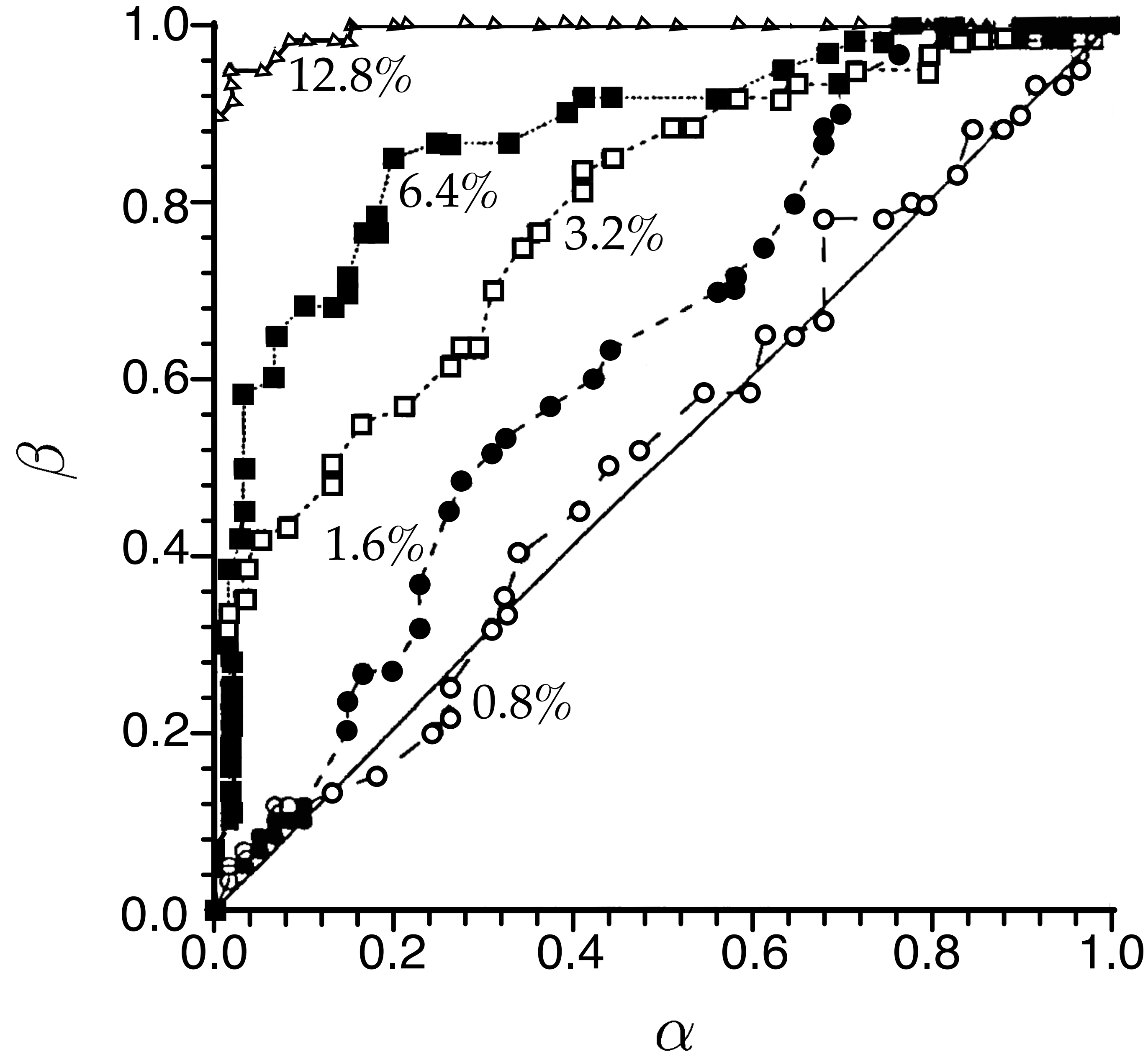
$$d' = \frac{\langle r \rangle_+ - \langle r \rangle_-}{\sigma_r}$$

Standard error measures in signal detection theory:

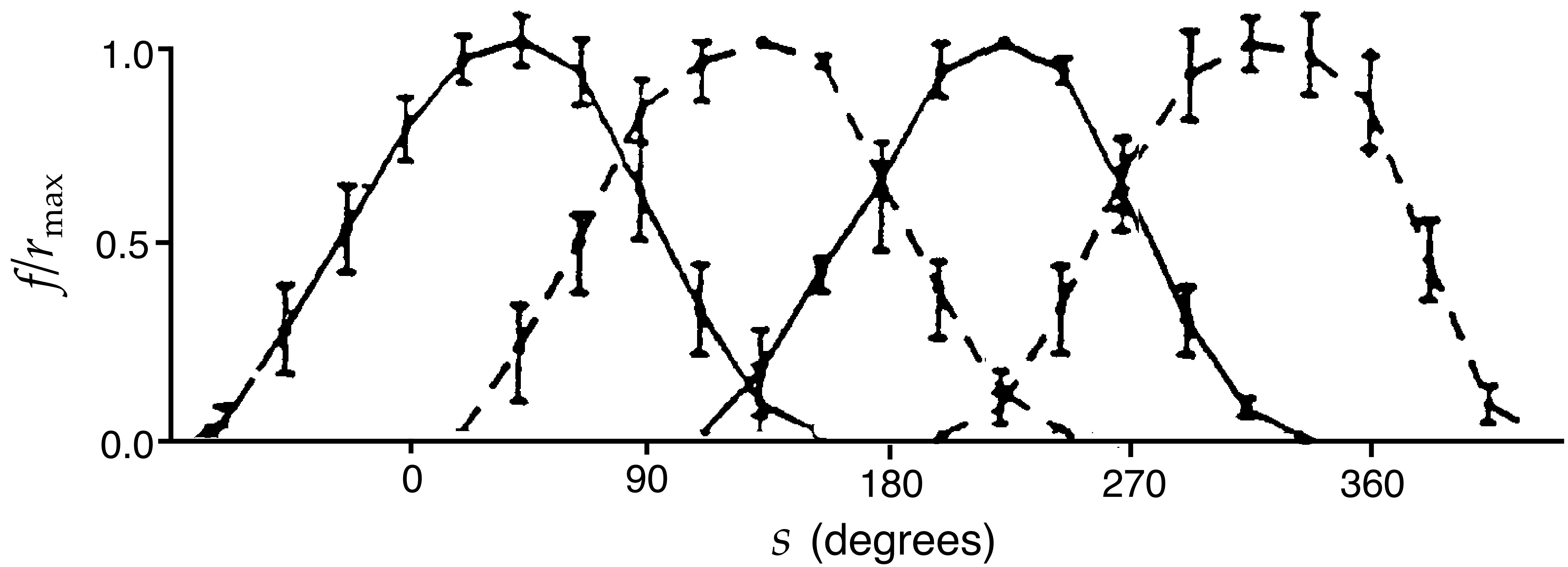
$$\begin{aligned} \alpha(z) &= P[r \geq z|-] && \text{is the size or false alarm rate of the test, and} \\ \beta(z) &= P[r \geq z|+] && \text{is the power or hit rate of the test .} \end{aligned}$$

	probability	
stimulus	correct	incorrect
$+$	β	$1-\beta$
$-$	$1-\alpha$	α

ROC curves for various coherence levels:



Population decoding:

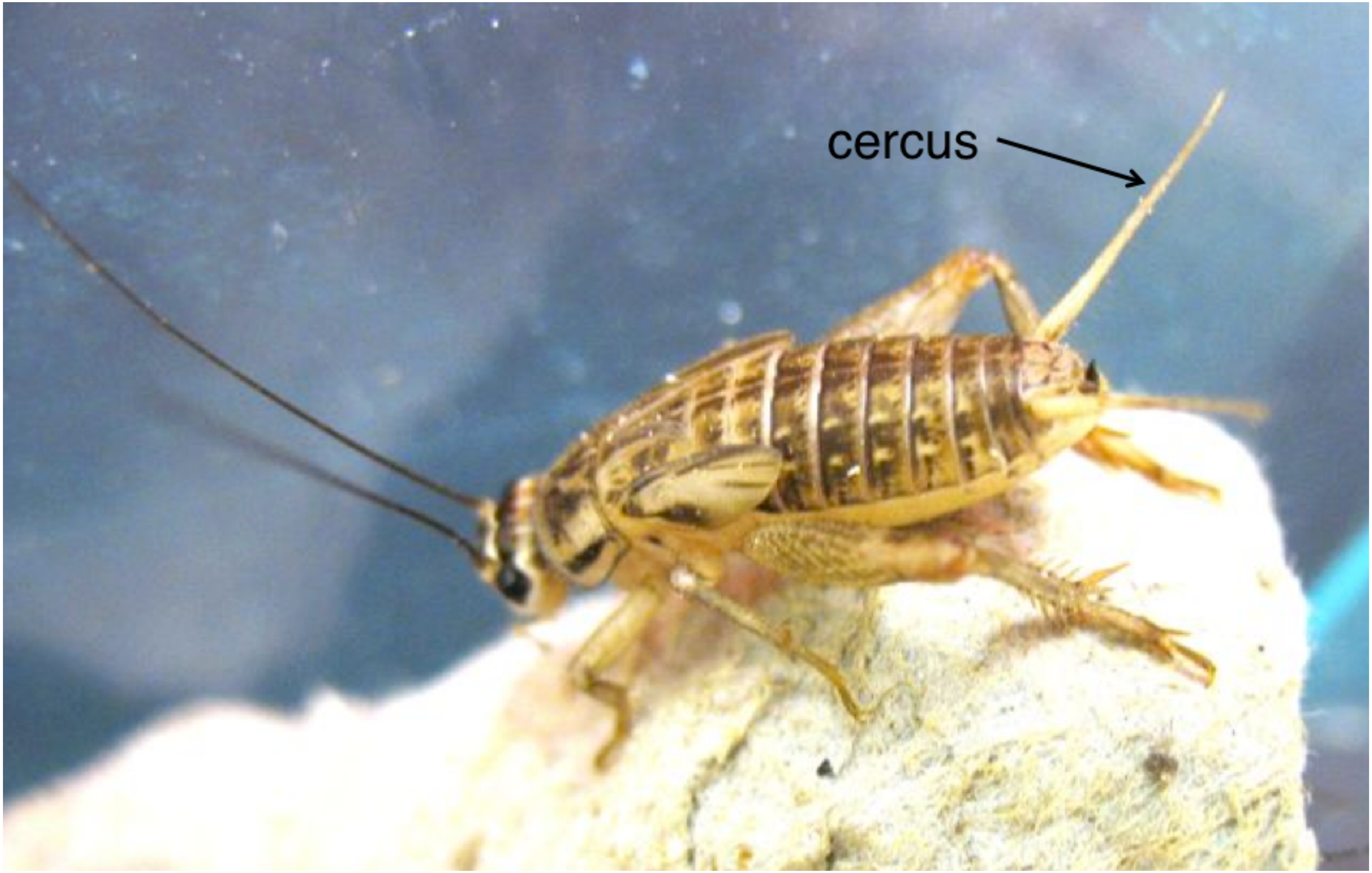


Cosine tuning curves:

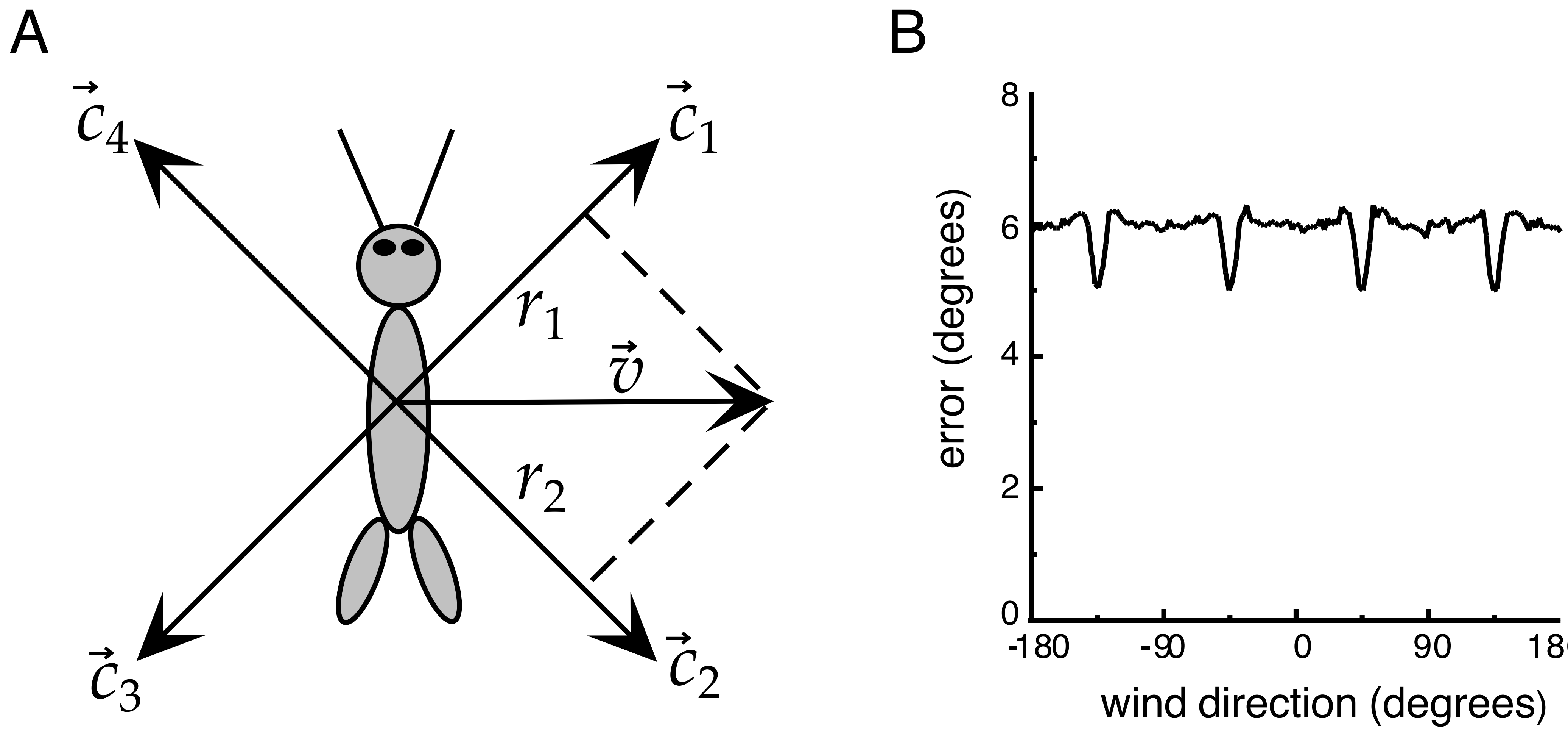
$$\left(\frac{f(s)}{r_{\max}}\right)_a = [(\cos(s - s_a)]_+$$

$$\left(\frac{f(s)}{r_{\max}}\right)_a = [\vec{v} \cdot \vec{c}_a]_+$$

Estimating wind direction from the cricket cercal system:



Estimating wind direction from the cricket cercal system:



Population vector estimate of direction:

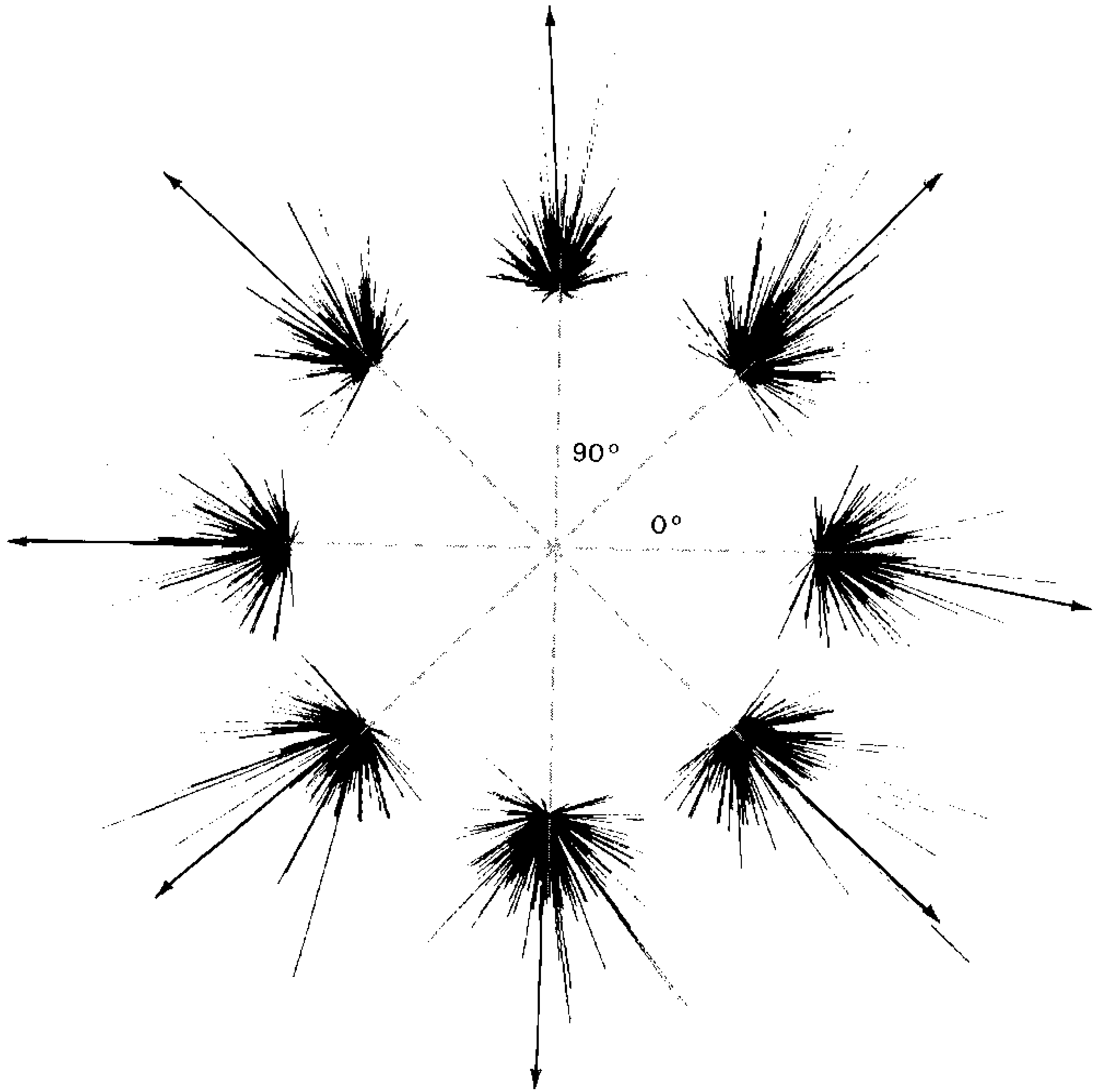
$$\vec{v}_{\text{pop}} = \sum_{a=1}^{4} \left(\frac{r}{r_{\max}} \right)_a \vec{c}_a$$

$$\left(\frac{\langle r \rangle - r_0}{r_{\max}} \right)_a = \left(\frac{f(s) - r_0}{r_{\max}} \right)_a = \vec{v} \cdot \vec{c}_a$$

$$\vec{v}_{\text{pop}} = \sum_{a=1}^{N} \left(\frac{r - r_0}{r_{\max}} \right)_a \vec{c}_a$$

$$\langle \vec{v}_{\text{pop}} \rangle = \sum_{a=1}^{N} (\vec{v} \cdot \vec{c}_a) \vec{c}_a$$

Population vectors in a center-out reach task (monkey):



Optimal estimate of the stimulus, Bayesian version:

$$p[s|\mathbf{r}] = \frac{p[\mathbf{r}|s]p[s]}{p[\mathbf{r}]}$$

$$\int ds\, L(s, s_{\text{bayes}})p[s|\mathbf{r}]$$

chalkboard interlude

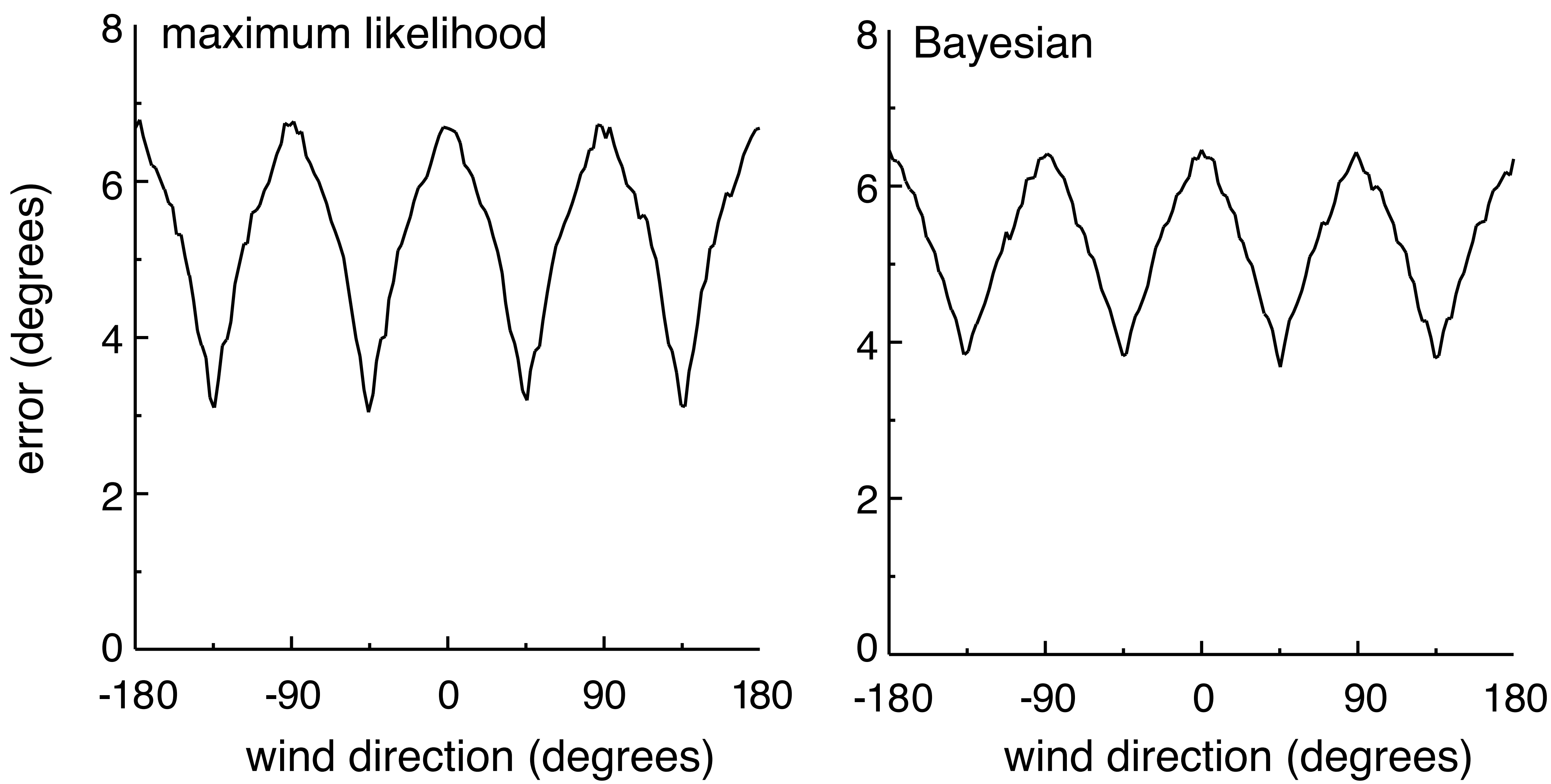
Optimal estimate of the stimulus, Bayesian version:

$$p[s|\mathbf{r}] = \frac{p[\mathbf{r}|s]p[s]}{p[\mathbf{r}]}$$

$$\int ds\, L(s, s_{\text{bayes}})p[s|\mathbf{r}]$$

$$s_{\text{bayes}} = \int ds\, p[s|\mathbf{r}]s$$

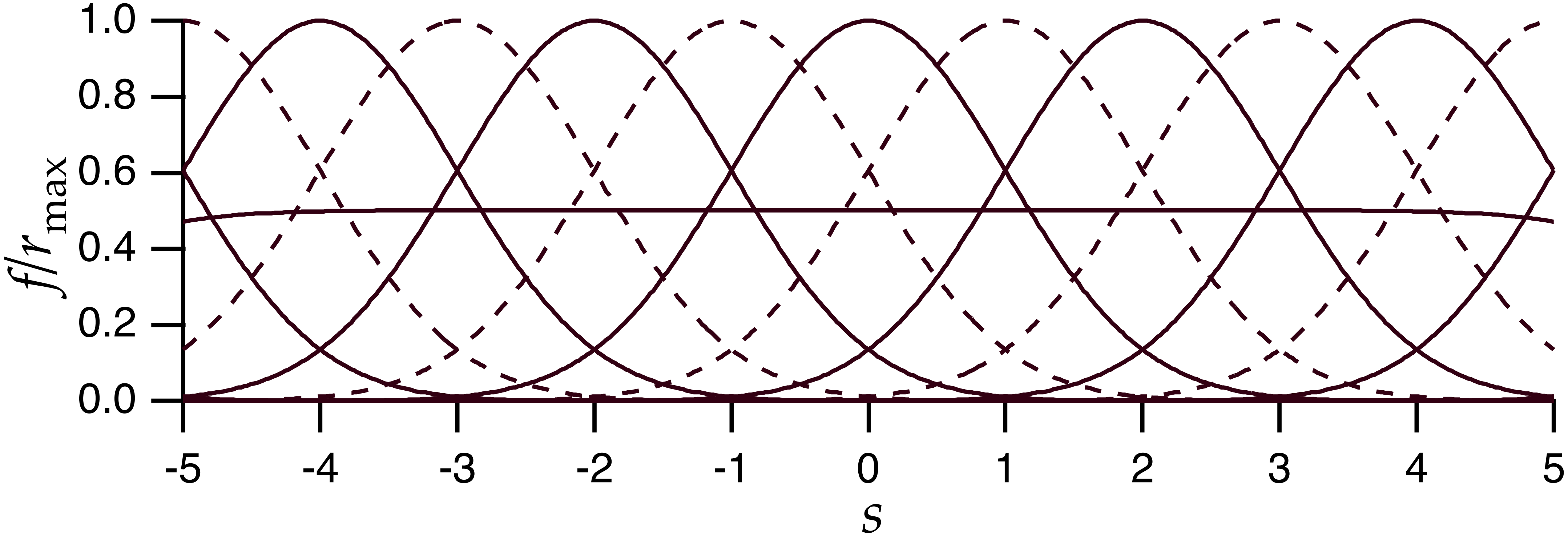
Comparing ML to Bayes for the cercal system:



Gaussian tuning curve for an arbitrary variable:

$$f_a(s) = r_{\max} \exp\left(-\frac{1}{2}\left(\frac{s - s_a}{\sigma_a}\right)^2\right)$$

A population of Gaussian tuned cells:

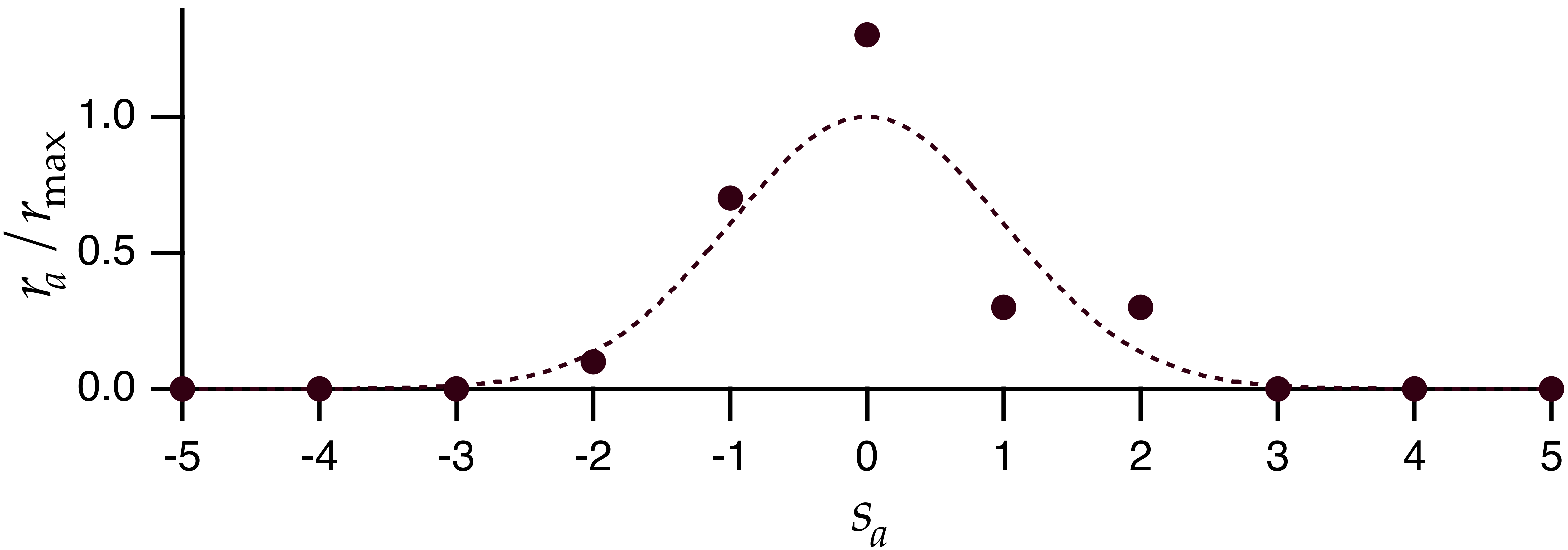


Conditional distribution for independent neurons:

$$P[r_a|s] = \frac{(f_a(s)T)^{r_a T}}{(r_a T)!} \exp(-f_a(s)T)$$

$$P[\mathbf{r}|s] = \prod_{a=1}^{N} \frac{(f_a(s)T)^{r_a T}}{(r_a T)!} \exp(-f_a(s)T)$$

Simulated response to s_a = 0:



Deriving the ML estimate of the stimulus:

$$P[r_a|s] = \frac{(f_a(s)T)^{r_a T}}{(r_a T)!}\exp(-f_a(s)T)$$

$$P[\mathbf{r}|s] = \prod_{a=1}^{N} \frac{(f_a(s)T)^{r_a T}}{(r_a T)!}\exp(-f_a(s)T)$$

$$\ln P[\mathbf{r}|s] = T\sum_{a=1}^{N} r_a \ln\big(f_a(s)\big) + \ldots$$

$$\sum_{a=1}^{N} r_a \frac{f_a'(s_{\mathrm{ML}})}{f_a(s_{\mathrm{ML}})} = 0 \qquad s_{\mathrm{ML}} = \frac{\sum r_a s_a/\sigma_a^2}{\sum r_a/\sigma_a^2} \quad s_{\mathrm{ML}} = \frac{\sum r_a s_a}{\sum r_a}$$

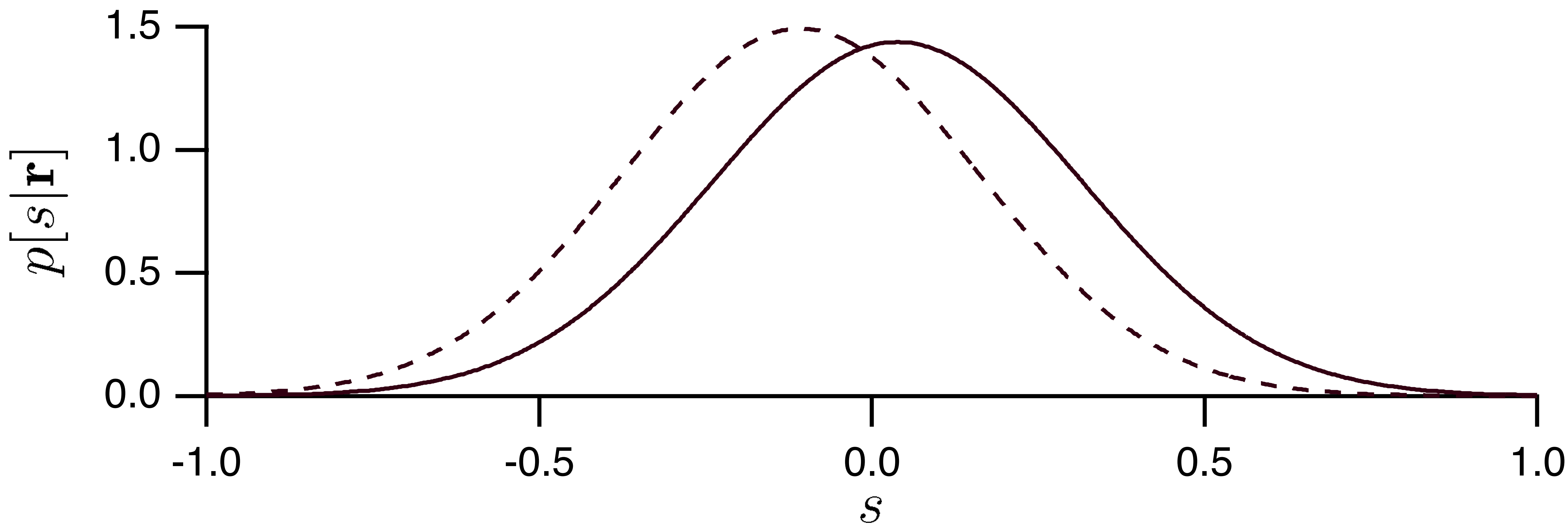
Constructing the MAP estimate of the stimulus:

$$\ln p[s|\mathbf{r}] = T\sum_{a=1}^{N} r_a \ln\big(f_a(s)\big) + \ln p[s] + \ldots$$

$$T\sum_{a=1}^{N} \frac{r_a f_a'(s_{\mathrm{MAP}})}{f_a(s_{\mathrm{MAP}})} + \frac{p'[s_{\mathrm{MAP}}]}{p[s_{\mathrm{MAP}}]} = 0$$

$$s_{\mathrm{MAP}} = \frac{T\sum r_a s_a/\sigma_a^2 + s_{\mathrm{prior}}/\sigma_{\mathrm{prior}}^2}{T\sum r_a/\sigma_a^2 + 1/\sigma_{\mathrm{prior}}^2}$$

MAP allows us to take P(s) into account:



Bias and variance in our estimates:

$$b_{\text{est}}(s) = \langle s_{\text{est}} \rangle - s$$

$$\sigma^2_{\text{est}}(s) = \langle (s_{\text{est}} - \langle s_{\text{est}} \rangle)^2 \rangle$$

$$\left\langle (s_{\text{est}} - s)^2 \right\rangle = \left\langle (s_{\text{est}} - \langle s_{\text{est}} \rangle - b_{\text{est}}(s))^2 \right\rangle = \sigma^2_{\text{est}}(s) + b^2_{\text{est}}(s)$$