Homework 3

Quantum Mechanics
Sept 15th, 2022

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Problem 1. *Problem 2.1 from Sakurai*

Solution. The Heisenberg equation of motion reads

$$\frac{dA}{dt} = \frac{1}{i\hbar}[A, H]$$

For the spin precession problem, we have the Hamiltonian

$$H = -\left(\frac{eB}{mc}\right) S_z = \omega S_z$$

For $A = S_x, S_y, S_z$, the time evolution is given by

$$\frac{dS_x}{dt} = \frac{\omega}{i\hbar}[S_x, S_z] = -\omega S_y$$
$$\frac{dS_y}{dt} = \frac{\omega}{i\hbar}[S_y, S_z] = \omega S_x$$
$$\frac{dS_z}{dt} = \frac{\omega}{i\hbar}[S_z, S_z] = 0$$

The above system has a straightforward solution:

$$S_x(t) = \cos(\omega t)$$
$$S_y(t) = \sin(\omega t)$$
$$S_z(t) = S_z(0)$$

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Problem 2. *Problem 2.3 from Sakurai*

Solution. We are given that $\vec{B} = B\hat{z}$ and that we are in the eigenstate $|\psi(0)\rangle = |\boldsymbol{S}\cdot\hat{\boldsymbol{n}}\rangle_+$, which reads

$$
\begin{aligned}
|\psi(0)\rangle &= \psi_+ |+\rangle + \psi_- |-\rangle \\
&= \cos\frac{\beta}{2} |+\rangle + \sin\frac{\beta}{2} |-\rangle
\end{aligned}
$$

where we have set $\alpha = 0$ since the ket is in the x-z plane. This state will evolve according to a Hamiltonian

$$
H = -\left(\frac{eB}{m_e c}\right) S_z = \omega S_z
$$

Clearly the eigenkets of the Hamiltonian are the eigenkets of S_z

$$
\begin{aligned}
|\psi(t)\rangle &= \psi_+(0)\exp\left(\frac{-iE_+t}{\hbar}\right)|+\rangle + \psi_-(0)\exp\left(\frac{-iE_-t}{\hbar}\right)|-\rangle \\
&= \cos\frac{\beta}{2}\exp\left(\frac{-i\omega t}{2}\right)|+\rangle + \sin\frac{\beta}{2}\exp\left(\frac{i\omega t}{2}\right)|-\rangle
\end{aligned}
$$

where we have used E_+ and E_- to denote the energies in the eigenstates of S_z. In general, the probability of measuring $|+\rangle_x = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle$ is given by the inner product

$$
|\langle +; x|\psi(t)\rangle|^2 =
$$

The Hamiltonian is time-independent, therefore, in the Schrodinger picture, we can write the following

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Problem 3. *Problem 2.9 from Sakurai*

Solution.

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Problem 4. *Problem 2.10 from Sakurai*

Solution.

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Problem 5. *Problem 2.12 from Sakurai*

Solution.

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Problem 6. *Problem 2.13 from Sakurai*

Solution.

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