TTIC 31230, Fundamentals of Deep Learning

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Loopy Belief Propagation (Loopy BP)

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We design an algorithm that is correct for tree graphs and use it on non-tree (loopy) graphs.

Belief Propagation on Trees

Belief Propagation is a message passing procedure (actually dynamic programming).

For each edge $\{i, j\}$ and possible value $\tilde{y}$ for node i we define $Z_{j \to i}[c]$ to be the partition function for the subtree attached to i through j and with $\hat{y}[i]$ restricted to c.

The function $Z_{j \to i}$ on the possible values of node i is called the **message** from j to i.

The reverse direction message $Z_{i \to j}$ is defined similarly.

Dynamic Programming Computes the Messages

$$Z_{j \to i}[c] = \sum_{c'} e^{s_n[j,c'] + s_e[j,i,c',c]} \left(\prod_{k \in N(j),\ k \neq i} Z_{k \to j}[c'] \right)$$

Loopy BP

In a Loopy Graph we can initializing all message $Z_{i\to j}[c] = 1$ and then repeating (until convergence) the updates

$$\tilde{Z}_{j\to i}[c] = \frac{1}{Z_{j\to i}} Z_{j\to i}[c] \qquad Z_{j\to i} = \sum_c Z_{j\to i}[c]$$

$$Z_{j\to i}[c] = \sum_{c'} e^{s_n[j,c'] + s_e[j,i,c',c]} \left(\prod_{k\in N(j),\ k\neq i} \tilde{Z}_{k\to j}[c'] \right)$$

Computing Node Marginals from Messages

$$Z_i(c) \doteq \sum_{\hat{y}:\ \hat{y}[i]=c} e^{s(\hat{y})}$$

$$= e^{s_i[c]} \left(\prod_{j \in N(i)} Z_{j \to i}[c] \right)$$

$$P_i(c) = Z_i(c)/Z, \qquad Z = \sum_c Z_i(c)$$

Computing Edge Marginals from Messages

$$Z_{i,j}(c,c') \doteq \sum_{\hat{y}:\, \hat{y}[i]=c,\, \hat{y}[j]=c'} e^{s(\hat{y})}$$

$$= e^{s_n[i,c]+s_n[j,c']+s_e[i,j,c,c']} \prod_{k\in N(i),\, k\neq j} Z_{k\to i}[c] \prod_{k\in N(j),\, k\neq i} Z_{k\to j}[c']$$

$$P_{i,j}(c,c') = Z_{i,j}(c,c')/Z \quad Z = \sum_{c,c'} Z_{i,j}(c,c')$$

END