Homework 11

Quantum Mechanics
January 20, 2023 C Seitz

Problem 1. *1.3.5 Calculations, No-cloning theorem*

Solution. Assume we have a unitary copying operator U and two quantum states $|\phi\rangle$ and $|\psi\rangle$. Suppose this unknown copying operator U could transform $|s\rangle$ to either $|\phi\rangle$ or $|\psi\rangle$.

$$|\psi\rangle \otimes |s\rangle \xrightarrow{U} |\psi\rangle \otimes |\psi\rangle$$
$$|\phi\rangle \otimes |s\rangle \xrightarrow{U} |\phi\rangle \otimes |\phi\rangle$$

If U is unitary, then it preserves inner products, so

$$(\langle\psi| \otimes \langle s|)(|\phi\rangle \otimes |s\rangle) = \langle\psi|\phi\rangle \otimes \langle s|s\rangle = \langle\psi|\phi\rangle$$

After the copying transformation, we have

$$\begin{aligned}(\langle\psi| \otimes \langle\psi|)(|\phi\rangle \otimes |\phi\rangle) &= \langle\psi|\phi\rangle \otimes \langle\psi|\phi\rangle \\ &= (\langle\psi|\phi\rangle)^2\end{aligned}$$

We demanded that the inner product be preserved, so these two results must be equivalent. However, there is only a solution when $|\psi\rangle = |\phi\rangle$ or $\langle\psi|\phi\rangle = 0$. Therefore, the copying circuit only works for orthogonal states, and not a general ket.

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Problem 2. *1.3.7 Calculations, Quantum Teleportation*

Solution.

The objective is for Alice to teleport to Bob a qubit in a state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, which can be done by using an entangled EPR pair. There three qubits in total: $|\psi\rangle$ and an entangled EPR pair $|\beta_{00}\rangle$. The first qubit in the EPR pair is kept by alice and the second is given to Bob. Since the EPR pair is entangled, the three qubits are in a state

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

Alice then sends this state through a CNOT gate, where the qubit $|\psi\rangle$ is the control bit and the first qubit of the EPR pair is the target bit. This of course flips the second bit for the second two terms:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$$

Then the first qubit is sent through a Hadamard gate. As a minor detour, the Hadamard gate, does

$$\begin{aligned} |0\rangle &\to (|0\rangle + |1\rangle)/\sqrt{2} \\ |1\rangle &\to (|0\rangle - |1\rangle)/\sqrt{2} \end{aligned}$$

Therefore, the effect on $|\psi_1\rangle$ is:

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{2}(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle \\ &+ \beta|010\rangle + \beta|001\rangle - \beta|110\rangle - \beta|101\rangle) \\ &= \frac{1}{2}(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\ &+ |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)) \end{aligned}$$

Therefore, if Alice measures her two qubits, say in state $|00\rangle$, she can communicate this to bob over a classical communication channel, and Bob then knows the superposition of his qubit. Bob can then apply the necessary quantum gate to transform his qubit to $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

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Problem 3. *1.4.3-1.4.4 Calculations, Deutsch Algorithms*

Solution. ■