

Homework 5

Quantum Mechanics

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Problem 1. *Problem 4.4*

Solution.

$$\begin{aligned} H &= e^{i\alpha} R_z\left(\frac{\pi}{2}\right) R_x\left(\frac{\pi}{2}\right) \\ &= \frac{e^{i\alpha} e^{i\pi/4}}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \end{aligned}$$

Therefore, $\alpha = -\pi/4$. ■

Problem 2. *Problem 4.5*

Solution.

$$\begin{aligned} (n \cdot \sigma)^2 &= (n_x \sigma_x + n_y \sigma_y + n_z \sigma_z)^2 \\ &= n_x^2 \sigma_x^2 + n_y^2 \sigma_y^2 + n_z^2 \sigma_z^2 \\ &= (n_x^2 + n_y^2 + n_z^2) I = I \end{aligned}$$
■

Problem 3. *Problem 4.7*

Solution.

Simple matrix operations can confirm that $XYX = -Y$. It follows that

$$e^{\frac{i\theta}{2}(XYX)} = e^{-\frac{i\theta}{2}Y}$$

Now recall that $U^\dagger e^A U = e^{U^\dagger A U}$, which can be proven via a series expansion. Of course X is both unitary and hermitian, so we get that

$$X e^{\frac{i\theta}{2}Y} X = e^{-\frac{i\theta}{2}Y}$$

which is the desired result. ■