PLASTIC NETWORKS OF BINARY SPIKING NEURONS

ABSTRACT

Fundamental questions in theoretical neuroscience pertain to how circuits in cortex store and process information. Recent decades have yielded a number of experimental and theoretical advances in our understanding of the mechanisms information storage, such as homosynaptic, heterosynaptic, and homeostatic plasticity. However, the precise mathematical relationship between synaptic weights and the stimuli or 'patterns' to which the network is exposed, remains elusive. At the same time, statistical physics and the theory of dynamical systems have provided substantial insight on network dynamics during processing. There remain questions on the optimum dynamical state or *phase* of the network during information processing and how that state is simultaneous with a particular class of synaptic connectivity. Here, we review the stream of research on the above topics.

1 INTRODUCTION

To present a comprehensive picture of recent research on information processing in spiking cortical circuits, we classify studies on their primary focus: information storage and network topology, network dynamics, or both. It is generally accepted that neural network models date back to McCulloch and Pitts [1] followed later by the Hopfield network [2] and many others. The purpose of the following sections is to summarize the important points in the development of neural networks with a unifying notation. All software for the following analysis can be found at [9].

2 THE HOPFIELD NETWORK

We begin by reviewing the Hopfield network as originally formulated by J.J. Hopfield in [2]. The Hopfield network appears primitive these days; however, it provides a useful starting point as it set the stage for its descendant models. We will conclude this section by pointing out some of the biological inaccuracies and reflect on the computational utility of those additional ingredients.

We describe the state of the neural network as a set of spins (neurons) $\{\sigma_i\}$ with $1 \leq i \leq n$. Each $\sigma_i \in \{0, 1\}$ which we will take to indicate whether or not a spike occured. The neurons σ_i and σ_j are coupled according to a connectivity matrix J_{ij}. If we let the network state $\{\sigma_i\}$ evolve in discrete time, it may relax to a particular state ξ_i with $1 \leq i \leq m$, in which case we say the pattern ξ_i is stored in the connectivity matrix J. We can be very general and write ξ as a matrix of m binary patterns i.e. $\xi \in \mathbb{F}_2^{n\times m}$ The mechanism to store all patterns ξ_i in the matrix simultaneously via an outer-product learning rule

$$J = \xi^T \xi - mI$$

where the second term serves to remove all n self connections $J_{ii} = 0$ for $1 \leq i \leq n$.

The Hopfield network is an attractive model because the theoretical treatment it requires is identical to that used for magnetic systems. However, when using such a system to directly describe a spiking neural network in cortex, there are notable gaps. Here are a few:

- Biological learning rules are much more complex
- Model neurons are not stateful objects
- Synaptic connectivity is sparse in cortex (low connection probability)

2.1 Storage Capacity

3 Phase transitions and critical phenomena

Statistical physics proves to be an invaluable tool when examining the relationship between synaptic connectivity and network dynamics. If the voltage of individual integrate and fire neurons is described by a diffusion process, we can make estimates of the time evolution of the instantaenous firing rate ν using Langevin dynamics. It turns out that the phase space of the network can be divided into four distinct phases [5]

- Asynchronous regular (AR), stationary global activity
- Asynchronous irregular (AI), stationary global activity but irregular firing at low rates
- Synchronous regular (SR), where neurons are almost fully synchronized in a few clusters
- Synchronous irregular (SI), oscillatory global activity but highly irregular firing at low rates

We can introduce a similiar parameterization to [5] to go beyond the Hopfield model and start to build more realistic networks.

It is thought that neural networks that lie at a critical point between order and chaos maximize basic processing properties such as sensitivity, dynamic range, correlation length, information transfer, and susceptibility [1]. Multiple signatures of criticality have been proposed in the literature, one being power-law distributions of neuronal avalanches following a branching process similar to that seen in physical theories of avalanches [2]. However, it has recently been demonstrated that criticality is not a universal determinant of computational power; rather, critical networks are only well-suited for more complex tasks such as those that require integration of stimulus information over long timescales [1].

Although various sources have analyzed the relationship between network topology and the dynamical state of the network, the relationship remains unclear. Here, we seek to accomplish two objectives (i) develop a mathematical framework to analyze the dynamics of coupled excitatory and inhibitory populations of leaky integrate and fire neurons (ii) use the framework in (i) to tune the spiking dynamics of the ensemble from asynchronous and chaotic to bursty and critical. Criticality of networks will be assessed according to established measures e.g., the branching parameter, autocorrelation time, and avalanche size distributions.

4 Diverse synaptic plasticity mechanisms

Previous models have been plagued with an inability to store memories efficiently. It is possible that a variety of plasticity mechanisms work in concert to form and consolidate memories over a different time scales [7].

References

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