Homework 11

Quantum Mechanics

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Problem 1. *No-cloning theorem*

Solution. Assume we have a unitary copying operator U and two quantum states $|\phi\rangle$ and $|\psi\rangle$. Suppose this unknown copying operator U could transform $|s\rangle$ to either $|\phi\rangle$ or $|\psi\rangle$.

$$|\psi\rangle \otimes |s\rangle \xrightarrow{U} |\psi\rangle \otimes |\psi\rangle$$
$$|\phi\rangle \otimes |s\rangle \xrightarrow{U} |\phi\rangle \otimes |\phi\rangle$$

If U is unitary, then it preserves inner products, so

$$(\langle\psi| \otimes \langle s|)(|\phi\rangle \otimes |s\rangle) = \langle\psi|\phi\rangle \otimes \langle s|s\rangle = \langle\psi|\phi\rangle$$

After the copying transformation, we have

$$\begin{aligned}(\langle\psi| \otimes \langle\psi|)(|\phi\rangle \otimes |\phi\rangle) &= \langle\psi|\phi\rangle \otimes \langle\psi|\phi\rangle \\ &= (\langle\psi|\phi\rangle)^2\end{aligned}$$

We demanded that the inner product be preserved, so these two results must be equivalent. However, there is only a solution when $|\psi\rangle = |\phi\rangle$ or $\langle\psi|\phi\rangle = 0$. Therefore, the copying circuit only works for orthogonal states, and not a general ket.

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Problem 2. *Calculations from section 1.3.7*

Solution. ■

Problem 3. *Carry out calculations from 1.4.3-1.4.4*

Solution. ■