Homework 4

Quantum Mechanics

Sept 22nd, 2022 Clayton Seitz

Problem 1. *Problem 2.14 from Sakurai*

Solution.

We are given that the state vector is

$$|\alpha\rangle = \exp\left(\frac{-ipa}{\hbar}\right)|0\rangle$$

The Heisenberg equation of motion reads

$$\frac{dx}{dt} = \frac{1}{i\hbar}[x, H] = 0$$

Therefore $x = x_0$ for all $t \geq t_0$

$$\begin{aligned}
\langle x\rangle &= \int x_0 \langle x|\alpha\rangle \langle\alpha|x\rangle\, dx \\
&= \int x \exp\left(\frac{-ipa}{\hbar}\right)\langle x|0\rangle \exp\left(\frac{ipa}{\hbar}\right)\langle 0|x\rangle\, dx \\
&= \int x_0 \,|\langle x|0\rangle|^2\, dx \\
&= \int x_0 \,|\langle x|0\rangle|^2\, dx
\end{aligned}$$

We could write out $\langle x|0\rangle$, its complex conjugate, and do the integral. Instead recall the general expression for the matrix element of x

$$\langle n'| x |n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\delta_{n',n-1} + \sqrt{n+1}\delta_{n',n+1}\right)$$

which is zero when $n = n'$ which means that $\langle x \rangle = 0$

$\blacksquare$

Problem 2. *Problem 2.15 from Sakurai*

Solution. $\blacksquare$

Problem 3. *Problem 2.16 from Sakurai*

Solution. $\blacksquare$

Problem 4. *Problem 2.28 from Sakurai*

Solution. $\blacksquare$

Problem 5. *Problem 2.29 from Sakurai*

Solution. $\blacksquare$

Problem 6. *Problem 2.32 from Sakurai*

Solution. $\blacksquare$