Quantum enhanced fluorescence microscopy with a SPAD array

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Introduction

Far-field optical microscopy is fundamentally limited by diffraction, with the maximum attainable resolution being limited to approximately half the wavelength of light. Several schemes to beat the diffraction limit have been developed in recent years. Many of these schemes utilize the concept of precise localization of isolated fluorescent emitters which blink over a time series of frames. One inherent problem of these methods is the requirement that fluorescent emitters be isolated, slowing down the acquisition of super-resolved images. To address this, gathering additional information on the number of active emitters by computing photon correlation statistics, enables localization in non-sparse scenes. However, photon correlation statistics indirectly provide information on the number of active fluorescent emitters in the sample. In this work, we introduce a model for precise counting of the number of active fluorescent emitters and demonstrate our method using a single photon avalanche diode (SPAD) array.

Molecular counting with photon statistics has a fairly simple motivation: coincidence of photons at multiple detector elements during high speed imaging provides evidence for the number of emitters present in the imaged region. Combining the ideas of conventional super-resolution approaches, with photon statistics may prove to be a powerful set of methods for bioimaging. Innovations in single photon detection technologies have begun to be integrated into fluorescence microscopes (Forbes 2019). Importantly, single photon detectors such as SPAD cameras have orders of magnitude higher temporal resolutions than standard CMOS cameras, single photon sensitivity, and theoretically zero readout noise. Such properties make these devices highly desirable for wide-field imaging applications; however, application of SPAD arrays in imaging have been limited to small bundles of detector elements combined with laser scanning (Israel 2017; Forbes 2019; Tenne 2019).

Results

We consider the case of pulsed excitation where the interval between pulses much longer than the fluorescence lifetime. Upon excitation, a single fluorophore emits at most one photon with a probability ζ. Therefore, the for N fluorophores within the region of interest, the number of photons emitted following a single excitation pulse follows a Binomial distribution: $x_{\text{signal}} \sim \text{Binom}(N, \zeta)$. Such behavior is profoundly different from the Poissonian statistics of classical light (Schwartz 2012). Background signal within the region of interest is modeled as shot noise: $x_{\text{background}} \sim \text{Poisson}(\lambda)$, where background photons arrive at single pixel with a rate λ. The total signal then will be a sum of Binomial and Poisson random variables i.e., $x = x_{\text{signal}} + x_{\text{background}}$, which is distributed by the likelihood

$$p(x_j = k \mid N, \zeta) = \sum_{i=0}^{\infty} \binom{N}{i} \zeta^i (1-\zeta)^{N-i} \frac{\lambda^{k-i}}{(k-i)!} e^{-\lambda} \quad (1)$$

Zero-lag second order coherence

Following (Israel 2017), we define

$$g^{(2)}(0) = \frac{G^{(2)}(0) - B}{\langle G^{(2)}(m) \rangle - B} \quad (2)$$

where $B = \langle x_{\text{background}} \rangle = N_{\text{frames}} \lambda \zeta$ is the expected number of background-signal coincidences in the region of interest. The quantities $G^{(2)}(0)$ represents the number of zero-lag signal-signal coincidences in the region of interest, over N_{frames}

$$G^{(2)}(0) \sim \text{Binomial}(N_{\text{frames}}, \mathbb{P}(\mathbf{x}_t \geq 2))$$
$$\mathbb{P}(\mathbf{x}_t \geq 2) = 1 - (1-\zeta)^n - n\zeta(1-\zeta)^{n-1}$$

The quantity $G^{(2)}(m)$ represents the number of signal-signal coincidences at lag m in the region of interest

$$G^{(2)}(m) \sim \text{Binomial}(N_{\text{frames}}, \mathbb{P}(\mathbf{x}_t \geq 1 \text{ and } \mathbf{x}_{t+m} \geq 1))$$
$$\mathbb{P}(\mathbf{x}_t \geq 1 \text{ and } \mathbf{x}_{t+m} \geq 1)) = (1 - (1-\zeta)^n)^2$$

Inference of the number of fluorescent emitters

We can write a posterior distribution on the Binomial parameters used in the likelihood (1) using Bayes rule

$$p(N, \zeta | x) \propto p(x|N, \zeta) p(N) p(\zeta)$$

$p(N)$ is taken to be uniform and $p(\zeta) = \mathcal{N}(\mu_\zeta, \sigma_\zeta)$

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APPENDIX

$$p(N=n|x) \propto \int_0^1 \prod_j p(x_j|n,\zeta)p(\zeta)d\zeta$$

which is estimated via MC integration. Minibatch the data and average the posterior $p(N|x)$ over minibatches of size M

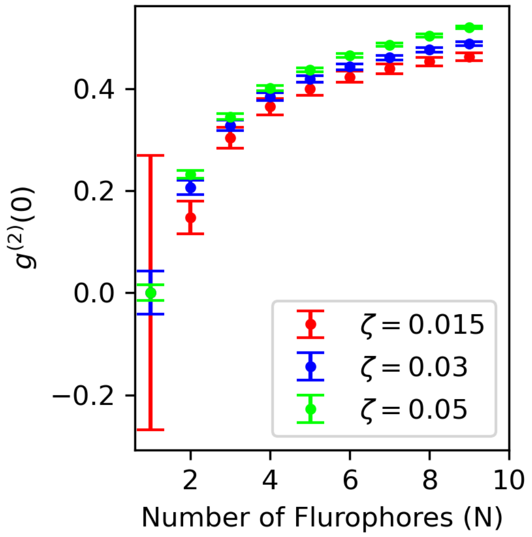


FIG. 1: Caption

When $N = 1$, we see some unique behavior of this function

$$g^{(2)}(0) = \frac{G^{(2)}(0) - B}{\langle G^{(2)}(m)\rangle - B} = -\frac{B}{N_{\text{frames}}\zeta(\zeta-\lambda)}$$

Basic Scheme

We consider a set of N point sources in the object plane labeled by continuous valued coordinates $r = (x, y)$ and labeled in the image plane $s = (u, v)$. respectively. The field is then treated by a simplified model consisting of a single mode (Vlasenko 2020). The positive frequency field operator in the object plane reads

$$E_0^+(r) \sim \sum_{i=1}^{N} \delta(r - r_i)a_j \tag{3}$$

The transfer function O of the field from object space to image space is assumed to have a Gaussian shape

$$O(s-r) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(s-r)^2}{2\sigma^2}\right) \tag{4}$$

The continuous field in image space is then

$$E^+(s) = \int d^2r E_0^+(r)O(s-r)$$

We take the point spread function O to be isotropic Gaussian. Since our detectors must be discrete, the total

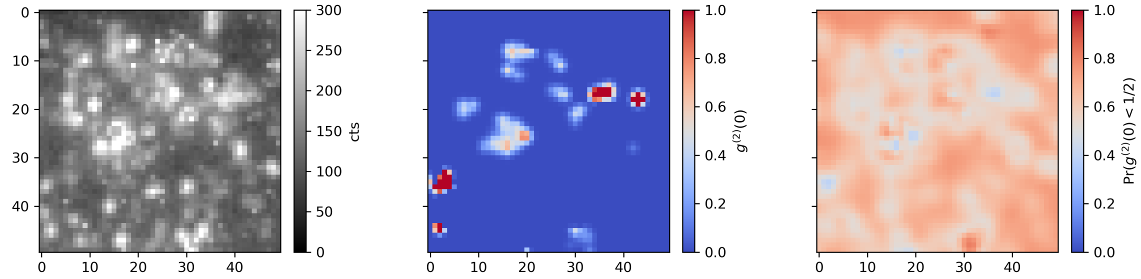


FIG. 2: Caption

field at a pixel $\tilde{E}^{+}(s_n)$ in image space is then given by integrating over pixels

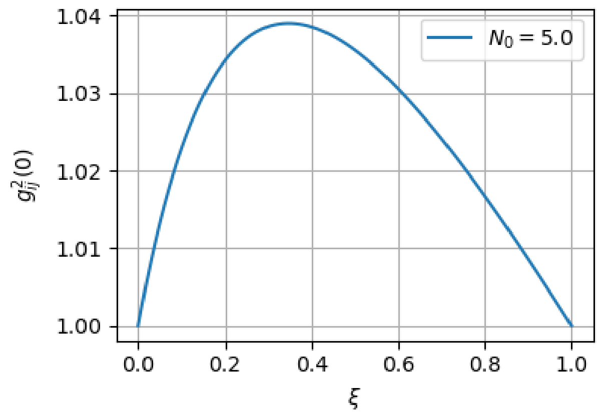
$$\begin{aligned}\tilde{E}^{+}(s_n) &= \int d^2 s E^{+}(s) \\ &= \int d^2 s \int d^2 r E_0^{+}(r) O(s_n - r) \\ &= \int d^2 r E_0^{+}(r) \int d^2 s \, O(s_n - r)\end{aligned}$$

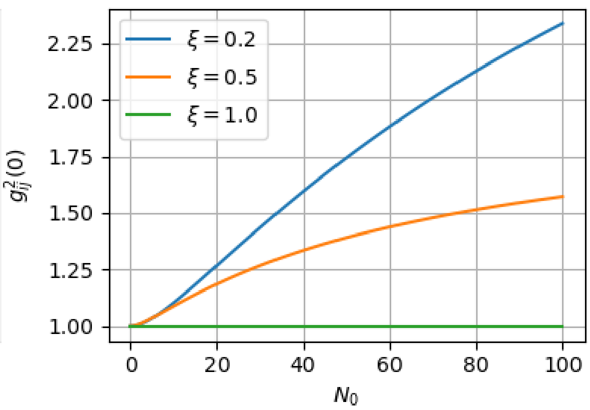
where $\int d^2 s \, O(s_n - r) = \frac{1}{2}\lambda_x \lambda_y$ and, for example,

$$\lambda_x = \mathrm{erf}\left(\frac{u_n + \frac{1}{2} - x_i}{\sqrt{2}\sigma}\right) - \mathrm{erf}\left(\frac{u_n - \frac{1}{2} - x_i}{\sqrt{2}\sigma}\right)$$

Combining this result with (1), we see the above integral is simply a sum of terms of the form

$$\tilde{E}^{+}(s_n) = \frac{a_i}{2}\lambda_x(u_n)\lambda_y(v_n)$$





Zero-lag order coherence for an isolated emitter

Photoswitching fluorescent emitters are described by the density matrices

$$\rho_S = \sum_k \xi_k \left|\alpha_k\right\rangle \left\langle\alpha_k\right| \quad \rho_B = \left|\beta\right\rangle \left\langle\beta\right|$$

We consider an isolated fluorescent emitter in the presence of a coherent background signal. The emitter can access a discrete set of states $|\alpha_k\rangle$, with occupancy probabilities ξ_k and amplitude α_k $\langle n\rangle = \langle\alpha_k|\, n \,|\alpha_k\rangle = |\alpha_k^2|$. We consider only zero-lag coherence, therefore neglecting the temporal structure of the jump process. For an isolated emitter, we have

$$\tilde{E}^{+}(s_n) = \Lambda_n \hat{a} + \hat{b} \tag{5}$$

where $\Lambda_n = \frac{1}{2}\lambda_x(u_n)\lambda_y(v_n)$. The intensity is

$$\begin{aligned}\langle I_n\rangle &= \langle \tilde{E}^{-}(s_n)\tilde{E}^{+}(s_n)\rangle \qquad (6)\\ &= \Lambda_n^2 \sum_k \xi_k \mu_k + \mu_B \qquad (7)\end{aligned}$$

Due to integration over finite size detector elements $\int d^2 s \, O^2(s_n - r) \neq \left(\int d^2 s \, O(s_n - r)\right)^2$ and we must define

$$\Lambda_n^2 := \int d^2 s \, O^2(s_n - r) \tag{8}$$

The non-normalized second order coherence in the image plane reads

$$G^2_{nm}(0) = \langle \tilde{E}^-(s_n)\tilde{E}^-(s_m)\tilde{E}^+(s_n)\tilde{E}^+(s_m)\rangle$$
$$= \Lambda_n^2\Lambda_m^2 \sum_k \xi_k\mu_k^2 + \mu_B(\Lambda_n^2+\Lambda_m^2)\sum_k \xi_k\mu_k + \mu_B^2$$

which in the normalized form reads

$$g^2_{nm}(0) = G^2_{nm}(0)/\langle I_n\rangle\langle I_m\rangle$$

where the intensity expectation at a detector element is of course $\langle I_n\rangle = \Lambda_n^2\sum_k \xi_k\mu_k + \mu_B$. For example, if we have a two-level system consisting of a fluorescent state with amplitude α and the vacuum state, this becomes

$$g^{(2)}_{ij}(0) = \frac{\xi|\alpha|^4}{\xi^2|\alpha|^4} = \frac{1}{\xi}$$

As $\xi \to 1$ (always on) we recover a coherent state. As $\xi \to 0$ we observe $g^{(2)}_{ij}(0) > 1$ i.e., bunching.

Expectations in a coherent state

$$\begin{aligned}
\mathrm{Tr}(a^\dagger a^\dagger aa\,(\xi_k\,|\alpha_k\rangle\,\langle\alpha_k|) &= \mathrm{Tr}\left(\xi_k e^{-|\alpha|^2}\sum_{n,m}^{\infty}\frac{\alpha^n}{n!}\,|n\rangle\,\langle m|\right)\\
&= \mathrm{Tr}\left(\xi_k e^{-|\alpha|^2}\sum_{n}^{\infty}\frac{|\alpha|^{2n}}{n!}n(n-1)\right)\\
&= \mathrm{Tr}\left(\xi_k e^{-|\alpha|^2}\sum_{n=2}^{\infty}\frac{|\alpha|^{2n}}{(n-2)!}\right)\\
&= \xi_k|\alpha_k|^4
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathrm{Tr}(a^\dagger a\,(\xi\,|\alpha\rangle\,\langle\alpha|)) &= \mathrm{Tr}\left(\xi e^{-|\alpha|^2}\sum_{n,m}^{\infty}\frac{\alpha^n(\alpha^m)^*}{\sqrt{n!}\sqrt{m!}}a^\dagger a\,|n\rangle\,\langle m|\right)\\
&= \xi e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{(|\alpha|^2)^n}{n!}n\\
&= \xi e^{-|\alpha|^2}\sum_{n=1}^{\infty}\frac{(|\alpha|^2)^n}{(n-1)!}\\
&= \xi e^{-|\alpha|^2}\left(|\alpha|^2+\frac{|\alpha|^4}{1!}+\frac{|\alpha|^6}{2!}+...\right)\\
&= \xi e^{-|\alpha|^2}|\alpha|^2\left(1+\frac{|\alpha|^2}{1!}+\frac{|\alpha|^3}{2!}+...\right)\\
&= \xi e^{-|\alpha|^2}e^{|\alpha|^2}|\alpha|^2 = \xi|\alpha|^2
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}(aa^\dagger\,(\xi\,|\alpha\rangle\,\langle\alpha|)) &= \mathrm{Tr}\left(\xi e^{-|\alpha|^2}\sum_{n,m}^{\infty}\frac{\alpha^n(\alpha^m)^*}{\sqrt{n!}\sqrt{m!}}aa^\dagger\,|n\rangle\,\langle m|\right)\\
&= \xi e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{(|\alpha|^2)^n}{n!}(n+1)\\
&= \xi e^{-|\alpha|^2}\left(\sum_{n=1}^{\infty}\frac{(|\alpha|^2)^n}{(n-1)!}+e^{|\alpha|^2}\right)\\
&= \xi e^{-|\alpha|^2}\left(|\alpha|^2e^{|\alpha|^2}+e^{|\alpha|^2}\right) = \xi(|\alpha|^2+1)
\end{aligned}$$