Homework 5

Quantum Mechanics
March 6, 2023 C Seitz

Problem 1. *Problem 4.4*

Solution.

$$
\begin{aligned}
H &= e^{i\alpha} R_z(\frac{\pi}{2}) R_x(\frac{\pi}{2}) \\
&= \frac{e^{i\alpha} e^{i\pi/4}}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}
\end{aligned}
$$

Therefore, $\alpha = -\pi/4$. ■

Problem 2. *Problem 4.5*

Solution.

$$
\begin{aligned}
(n \cdot \sigma)^2 &= (n_x \sigma_x + n_y \sigma_y + n_z \sigma_z)^2 \\
&= n_x^2 \sigma_x^2 + n_y^2 \sigma_y^2 + n_z^2 \sigma_z^2 \\
&= (n_x^2 + n_y^2 + n_z^2) I = I
\end{aligned}
$$

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Problem 3. *Problem 4.7*

Solution.

Simple matrix operations can confirm that $XYX = -Y$. It follows that

$$e^{\frac{i\theta}{2}(XYX)} = e^{-\frac{i\theta}{2}Y}$$

Now recall that $U^\dagger e^A U = e^{U^\dagger A U}$, which can be proven via a series expansion. Of course X is both unitary and hermitian, so we get that

$$X e^{\frac{i\theta}{2}Y} X = e^{-\frac{i\theta}{2}Y}$$

which is the desired result.

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Problem 4. *Problem 4.17*

Solution. For the first circuit, the matrix representation is

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h_{11} & h_{12} \\ 0 & 0 & h_{21} & h_{22} \end{pmatrix}$$

For the second circuit, the matrix representation is

$$A = \begin{pmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{21} & h_{22} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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