Math 6767, Fall 2013 Homework 3

We consider the Binomial Tree Model for pricing American Options, particularly American Puts. The underlying is IWM which is an index tracking ETF. It attempts to track the Russell 2000 which is an index of small cap stocks. Here are some of the relevant market parameters:

initial spot: 106.14
dividend yield: 1.6% or 0.016
implied volatility: 22.72% or 0.2272
historical volatility: 12.66% or 0.1266
risk free rate: 1% or 0.01
time to expiration: 355 days

Historical volatility is essentially the annualized standard deviation of returns. We will talk about this soon. For now it is just a value we can give volatility.

Implied volatility is the value for volatility that makes the Black-Scholes function (or some closely related model) give the market price for the option. Since the prices are set by the market, the implied volatility accounts for the portion of the option price that does not depend on time to expiration or distance from strike. It is related to inverstors' sense of risk or fear. Again, for us for now it is just another parameter we can use for volatility.

Calibration with a European Pricer. In order to form a credible binomial tree for option pricing, one needs to specify the number of time steps, called N here, for the tree. A good way to do this is to choose N so that the tree produces values close to the values that the Black-Scholes function produces over a reasonable range of values of the underlying and of volatility. For the underlying here, we can take the underlying to vary over the range $[100, 110]$ and the volatility to vary over the range $[0.10, 0.30]$.

So after you write a binomial pricer that works for European options, the first thing to do is to vary the value of N so that your prices are within, say $0.05 of the values from the Black-Scholes function over the range given above.

Analysis. After you have selected a value of N that gives good results, use your pricer to value American options on the IWM with the following strike prices:

(1) 100
(2) 106
(3) 107
(4) 110

Do this for both values of volatility given above (implied and historical).

Make charts of the values of the option and estimates of the Delta of the option at the following instances of time:

(1) 100 days before expiration
(2) 200 days before expiration

You can do this by extracting the appropriate row from the stock and option trees.

Programming Considerations. Use vectors, not arrays. Try to make the pricing function be a template function so that general payoffs can be used. For example, something like this

```
template <typename T>
double pricer(stockTree st, optionTree ot, T f)
```

The stock tree would come in to this function completely filled and the option tree would be full of zeros waiting to have its last row filled with the values $f(s)$ where f is a payoff and s is a stock value. Note that the payoff takes only one variable. It should be a function object. Ideally you should be able tp price options with a variety of payoffs with only minimal modifications to your program. You might have to think about the details of this.

The accompanying program has some ideas that you might find useful.