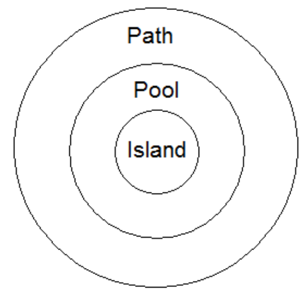
Question #1

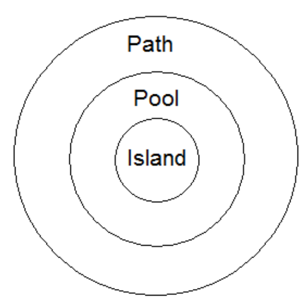
I need your help with my new plans for my backyard. I want a pool with a central island and a surrounding path, as shown in the diagram, which is not drawn to scale. Of course, the pool, island, and path are all concentric circles. The island is 4 ft wide with a uniform depth of 2 ft. The pool is going to be 4 ft wide with a uniform depth of 8 ft, and the surrounding pathway is going to be 3 ft wide with a uniform depth of 1ft. Now the cost: The sand for the island costs \$.40 per π ft^3; the water for the pool costs $\$\frac{1}{4\pi}$ per ft^3; the bricks for the pathway cost \$.50 per π ft^3. Note that there will be no water under the island column in the pool or under the pathway. So with your help, tell me: How much will this backyard structure cost me?



February Regional Geometry Team

Question #1

I need your help with my new plans for my backyard. I want a pool with a central island and a surrounding path, as shown in the diagram, which is not drawn to scale. Of course, the cross-sections of the pool, island, and path are all concentric circles. The island is 4 ft wide with a uniform depth of 2 ft. The pool is going to be 4 ft wide with a uniform depth of 8 ft, and the surrounding pathway is going to be 3 ft wide with a uniform depth of 1ft. Now the cost: The sand for the island costs \$.40 per π ft^3; the water for the pool costs $\$\frac{1}{4\pi}$ per ft^3; the bricks for the pathway cost \$.50 per π ft^3. Note that there will be no water under the island column in the pool or under the pathway. So with your help, tell me: How much will this backyard structure cost me?



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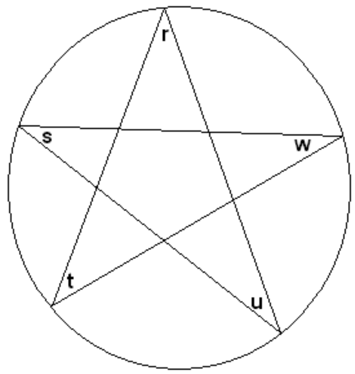
Question #2

In the circle, a five-point star has inscribed angles with vertices on the circumference.



Find: r + s + t + u + w

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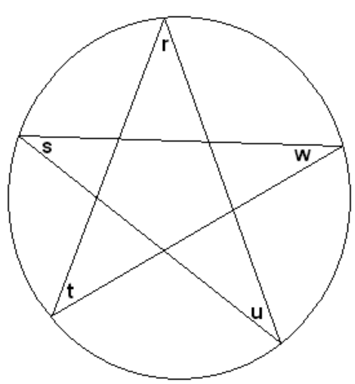
Question #2

In the circle, a five-point star has inscribed angles with vertices on the circumference.



Find: r + s + t + u + w

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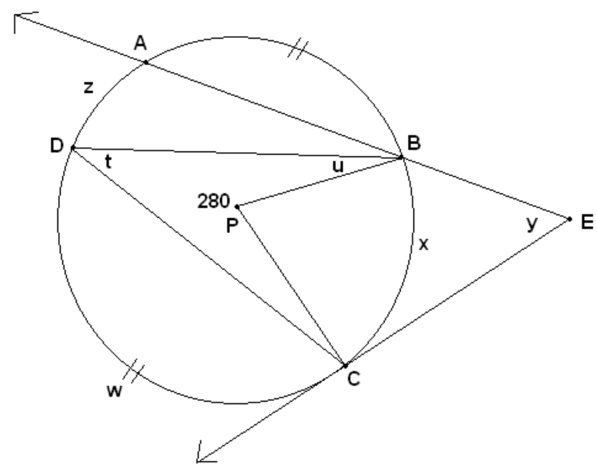
Question #3

Observe the following figure. Angles listed are in degrees (NOTE: Figure may not be drawn to scale)

Given: Arc BAD $\cong$ Arc CD
Arc AD is one-fifth of Arc BAD
P is the center of the circle.
$\overrightarrow{EC}$ is tangent to the circle at point C



t= angle measure of BDC
u= angle measure of DBP
w= arc measure of CD
x= arc measure of BC
y= angle measure of BEC
z= arc measure of AD

Find: $\frac{t}{u}+\left(\frac{w}{x}\right)\left(\frac{y}{z}\right)$

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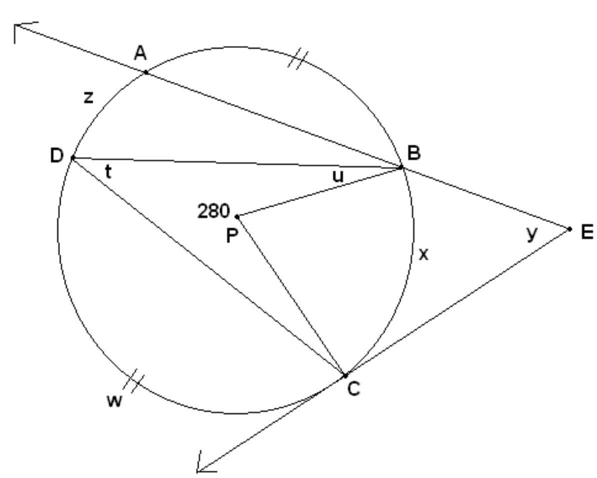

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Question #4

Part 1) A hexagon has interior angle measures that are consecutive odd integers. What is the ratio of the smallest exterior angle to the largest interior angle?

Part 2) A pentagon has exterior angle measure that are consecutive integers. What is the ratio of the smallest exterior angle to the largest interior angle?

The answer to this problem is the product of the solutions from Part A and Part B.

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February Regional Geometry Team

Question #5

On the Cartesian Coordinate Plane, the points A(-1,2), B(3,4) and C(5,2) lie on circle P. Coincidentally, those same points form a triangle that is inscribed in this circle. Find the area that is inside the circle, yet outside of the triangle.

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Question #5

On the Cartesian Coordinate Plane, the points A(-1,2), B(3,4) and C(5,2) lie on circle P. Coincidentally, those same points form a triangle that is inscribed in this circle. Find the area that is inside the circle, yet outside of the triangle.

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Question #6

Given the line $2x + y + 4 = 0$, find the lines perpendicular at (1,-6) and at (-3,2) and the line with an undefined slope through the point (5,4). Find the area of the shape enclosed by these four lines.

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Question #6

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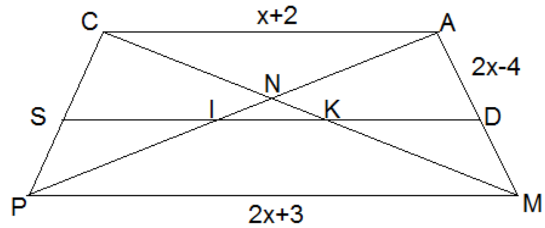
Question #7



Isosceles trapezoid CAMP, with $\overline{CA} \parallel \overline{MP}$, CP=AM, and CS=SP, has base lengths of CA=x+2 and MP=2x+3. SD=x^2 and AD=2x-4. Diagonals CM and AP intersect at N, and the diagonals intersect SD at K and I, respectively. SD is the median of trapezoid CAMP.

A=the area of the trapezoid. B=the length of IK

Evaluate $\frac{A}{B}$ and you get the solution in the form $\frac{F\sqrt{G}}{H}$, with F, G, and H relatively prime to one another. The answer to this problem is F+G+H

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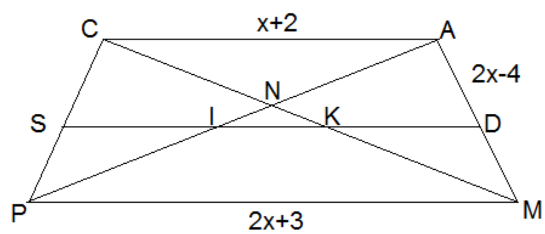
Question #7



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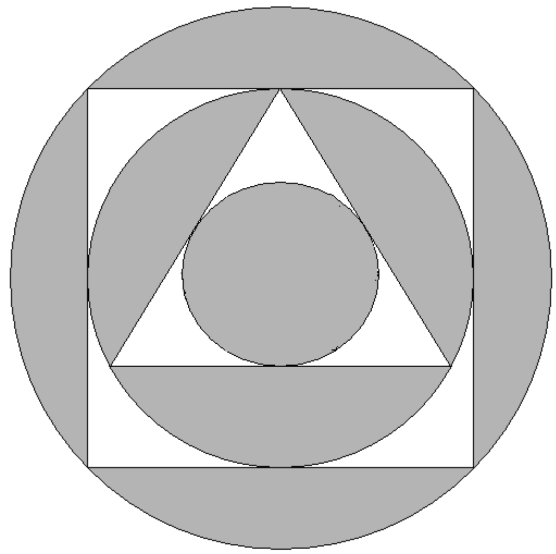
February Regional Geometry Team

Question #8

In the following diagram, all circles and polygons are concentric. The largest circle is circumscribed about a square, which is circumscribed about another circle, which is circumscribed about an equilateral triangle, which is finally circumscribed about a circle. Given that the radius of the largest circle is 6, find the area of the shaded region.



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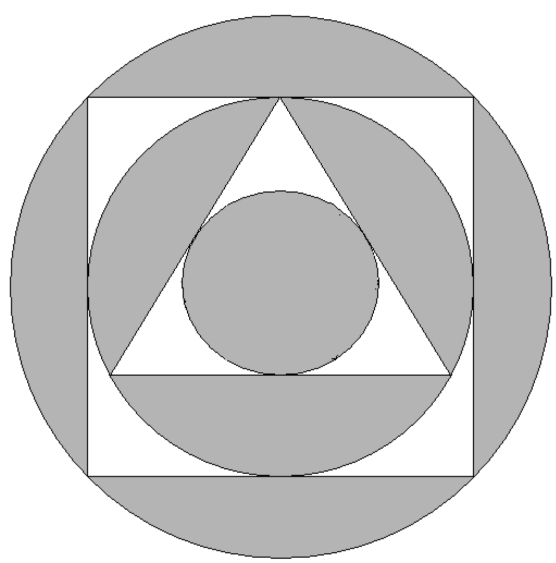

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<u>Question #9</u>

This problem is going to have some fun with the very important Golden Ratio, which is denoted by the greek letter phi (φ).

First, you need the exact value of the Golden Ratio. If you don't have it memorized, then solve for φ in this equation to get the value: $\varphi^2 - \varphi - 1 = 0$. The Golden Ratio is positive.

Next, take an isosceles triangle with side lengths 1, 1 and φ. Evaluate the cosine of the angle opposite the side of length φ.

Finally, take the opposite of the value from the above step, and have that value equal the length of the sides of an equilateral triangle. The answer to this problem is the area of this equilateral triangle.

February Regional Geometry Team

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Next, take an isosceles triangle with side lengths 1, 1 and φ. Evaluate the cosine of the angle opposite the side of length φ. Use the law of Cosines.

Finally, take the opposite of the value from the above step, and have that value equal the length of the sides of an equilateral triangle. The answer to this problem is the area of this equilateral triangle.

February Regional Geometry Team

Question #10

Determine which of the following statements are true or false:

A) Three unique, coplanar, and non-collinear points are required to determine a unique circle.
B) It is possible to construct a 112.5° angle, using only a compass and a straightedge.
C) If two lines in space intersect, the lines are said to be skew.
D) Two circles that are externally tangent can have a maximum of two common tangent lines.
E) On the Cartesian Coordinate plane, the product of the slopes of two perpendicular lines with defined slopes is -1.
F) Given a triangle with sides a, b, and c, $a<c$, $b<c$, and $a+b>c$, if $a^2+b^2<c^2$, then the triangle is obtuse.
G) If two similar figures have a side ratio of 3:4, then the area ratio will also be 3:4.

List the letter(s) of the true statement(s) for your answer to this problem.

February Regional Geometry Team

Question #10

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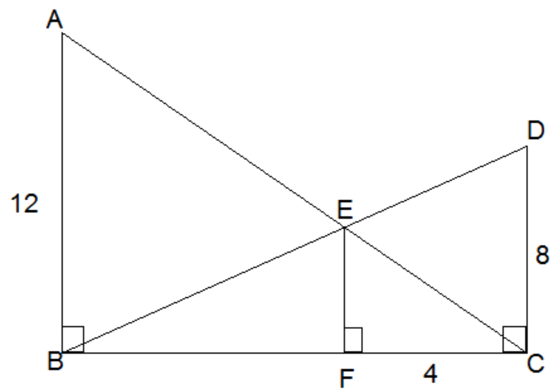
Question #11



Evaluate the area of trapezoid ABFE.

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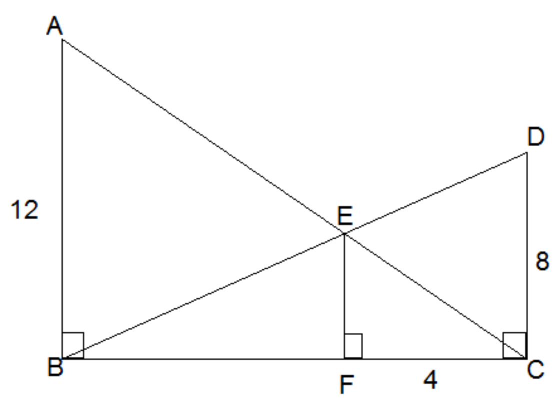
Question #11



Evaluate the area of trapezoid ABFE.

February Regional Geometry Team

Question #12

A= The diameter of a sphere with equal volume and surface area

B= The volume of a cylinder that has its lateral surface area equal to the volume of a cone. The cone and cylinder both have a height of 1.

C= The factor that the volume of a rectangular prism changes when the length is increased by 150% of the original length, the width is decreased by 75% of the original width, and the height is tripled.

Evaluate A+B-8C

February Regional Geometry Team

Question #12

A= The diameter of a sphere with equal volume and surface area

B= The total surface area of a cylinder that has its lateral surface area equal to the volume of a cone. The cone and cylinder both have a height of 1.

C= The factor that the volume of a rectangular prism changes when the length is increased by 150% of the original length, the width is decreased by 75% of the original width, and the height is tripled.

Evaluate A+B-8C

February Regional Geometry Team

Question #13

A triangle has sides of consecutive odd integer lengths. For this problem, use the triangle with the smallest area that satisfies the above condition.

A= the smallest possible area of this triangle

B= the length of the altitude to the shortest base of the triangle

C=the cosine of the largest angle (Hint: Use Law of Cosines)

Evaluate: $\frac{AB}{C}$

February Regional Geometry Team

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C=the cosine of the largest angle (Hint: Use Law of Cosines)

Evaluate: $\frac{AB}{C}$

February Regional Geometry Team

Question #14

A= the number of sides in a nonagon

B= the measure of one interior angle of a regular nonagon

C= the measure of one exterior angle of a regular nonagon

D= the number of diagonals in a nonagon

Evaluate A+B-C+D

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Question #14

A= the number of sides in a nonagon

B= the measure of one interior angle of a regular nonagon

C= the measure of one exterior angle of a regular nonagon

D= the number of diagonals in a nonagon

Evaluate A+B-C+D

February Regional Geometry Team

Question #15

A regular octahedron has an edge length of 6 cm. Let:

A= the surface area of the octahedron

B= the volume of the octahedron

C= the Euler Characteristic of this regular octahedron

D= the maximum area of any cross-section taken from this octahedron

The quantity $A+B+CD$ can be factored into $w\left(x+\sqrt{y}+\sqrt{z}\right)$, where $y<z$ and $w,x,y,$ and z are relatively prime.

Evaluate $w+x+y+z$

February Regional Geometry Team

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