# Single Queue/Server Simulation

Yichao Yao

Single Queue/Server problem is a popular problem in queuing networks. This model is also called M/M/1 model, which means the time between arrivals and the size of jobs follow Poisson process and there is only one server in the node[1]. For parts in the queue, they follow first-in-first-out(FIFO) rule, so that the functions developed for three stations simulation can be partly used in this simulation.

M/M/1 queue, introduced by U. Yechiali and P.Naor[2], have three parameters describing this system, mean service time $\frac{1}{\mu}$, and mean interarrival time $\frac{1}{\lambda}$. And a third parameter the gate rate or the probability that a station is busy, $\rho = \frac{\lambda}{\mu}$ is defined for further calculation. And in most researches, $\rho$ is defined as less than one, because only under this situation, the system will be stable[3]. In the simulation conducted in this report, only $\rho$ with value less than 1 is considered.

For this model, researchers are interested in number of parts in the system, busy time of server, response time and so on. Only response or the waiting time will be discussed in this report. Little's Law proposed by Little, J.D.[4] in 1961 is the key rule to solve this problem. It states that the service time and arrival rate determines the average number of parts in the system (the stable one). This statement can be applied to any queuing system, thus it can be used to analyze M/M/1 system. With this rule, it can be easily get the average running time is

$$\frac{1}{\mu - \lambda}$$

Therefore the average waiting time can be calculated as:

$$\frac{1}{\mu - \lambda} - \frac{1}{\mu} = \frac{\rho}{\mu - \lambda}$$

In recent researches, besides the mean value the distribution of waiting time is also described by some researchers such as PG Harrison[5], which is really useful for analyzing the queuing problem in real life.

In our simulation, a short event driven program is written based on FIFO queue implemented by linked list.

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[1] Kendall, D. G. (1953). Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded Markov chain. *The Annals of Mathematical Statistics*, 338-354.
[2] Yechiali, U., & Naor, P. (1971). Queuing problems with heterogeneous arrivals and service. *Operations Research*, *19*(3), 722-734.
[3] Abate, J., & Whitt, W. (1987). Transient behavior of the M/M/l queue: Starting at the origin. *Queueing Systems*, *2*(1), 41-65.
[4] Little, J. D. (1961). A proof for the queuing formula: L= λ W. *Operations research*, *9*(3), 383-387.
[5] Harrison, P. G. (1993). Response time distributions in queueing network models. In *Performance Evaluation of Computer and Communication Systems* (pp. 147-164). Springer Berlin Heidelberg.

Figure 1 shows the comparison between theoretical waiting time and experimental waiting time, twenty trials are plotted and in this case interarrival time is 20 and service time is 15.

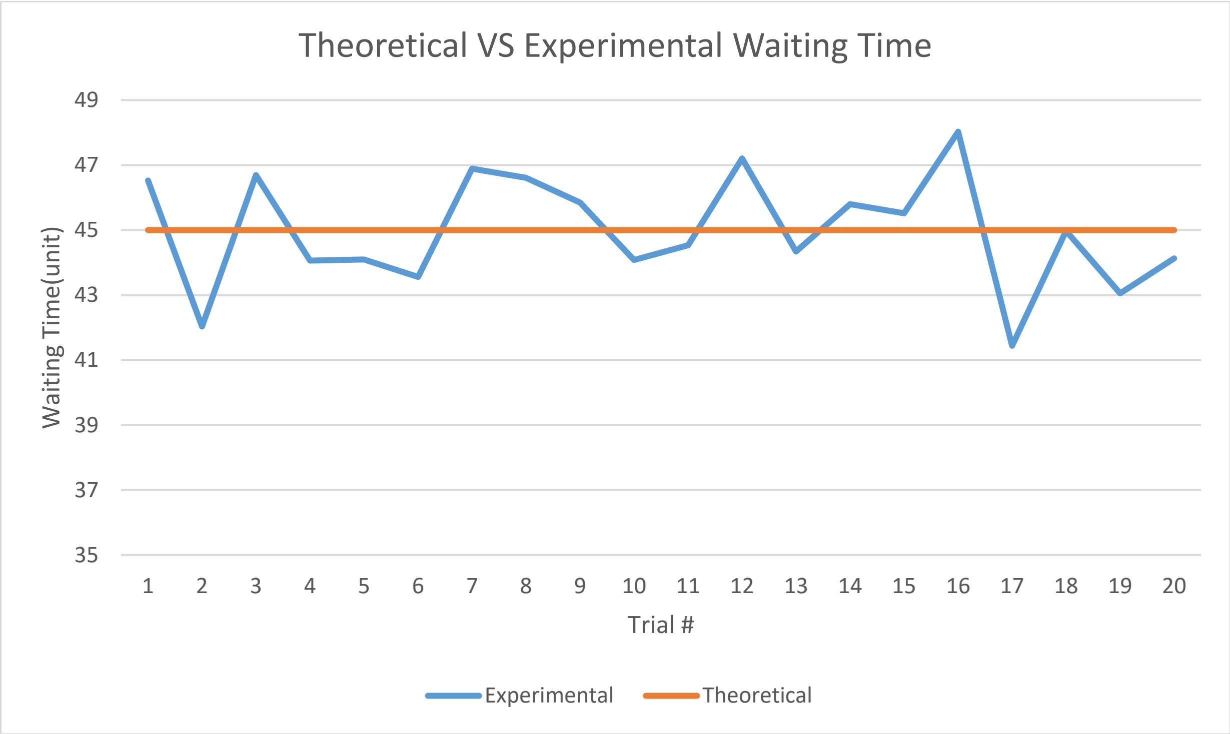


*Figure 1: Comparison between theoretical and experimental value*

In order to get the accurate mean value the program is running by two thousand iterations, and the average waiting is the average value of these two thousand iteration, table 1 shows the result

*Table 1: Comparison between theoretical and experimental value(average )*

| | A(Interarrival time) | S(Service time) | Theoretical waiting time | Average experimental waiting time |
|---|---|---|---|---|
| 1 | 5 | 1 | 0.25 | 0.25037 |
| 2 | 5 | 2 | 1.33 | 1.31386 |
| 3 | 5 | 3 | 4.50 | 4.47959 |
| 4 | 10 | 4 | 2.67 | 2.63384 |
| 5 | 10 | 5 | 5.00 | 4.89736 |
| 6 | 10 | 6 | 9.00 | 9.07347 |

From figure 1, the single value of experimental waiting time is vibrating around the theoretical value of the waiting time. And table 1 shows the mean value after huge amount of trials are very close theoretical model.

In conclusion, the simulation program in this assignment of single queue/server gets the really close result to the theoretical value got from the formula in literature.