ISE224 Python for Engineers

Dr. Cheng-Bang Chen

Lab 9 - Forecasting

Forecasting

- Forecasting is a technique that uses historical data as inputs to make informed estimates that are predictive in determining the direction of future trends.

Week (t)	Y(t)	Y.hat(t)	e(t)
1	275	273	2
2	291	296	-5
3	307	314	-7
4	281	288	-7
5	295	301	-6
6	268	268	0
7	252	260	-8
8	279	272	7
9	264	274	-10
10	288	280	8

- Notations:
  - t: the time index
  - Y_t: the true value at time t
  - $\hat{Y}_t$: the predicted(estimated) value at time t
  - e_t: the forecast error of time t. ($e_t = Y_t - \hat{Y}_t$)

- Prediction Performance:
  - Mean Squared Error (MSE)

$$MSE = \frac{\sum_{t=1}^{n} \left(Y_t - \hat{Y}_t\right)^2}{n}$$

Naive Approach

- The naive method for prediction is a simple forecasting technique that assumes that the future value of a time series will be equal to the present value.

$$\hat{Y}_{t+1} = Y_t$$

- Using naïve method to predict the values of week 6 - 10. Calculate the MSE of this prediction.

Week (t)	Y(t)	Y.hat(t)	e(t)	e^2(t)
1	275			
2	291			
3	307			
4	281			
5	295			
6	268			
7	252			
8	279			
9	264			
10	288			

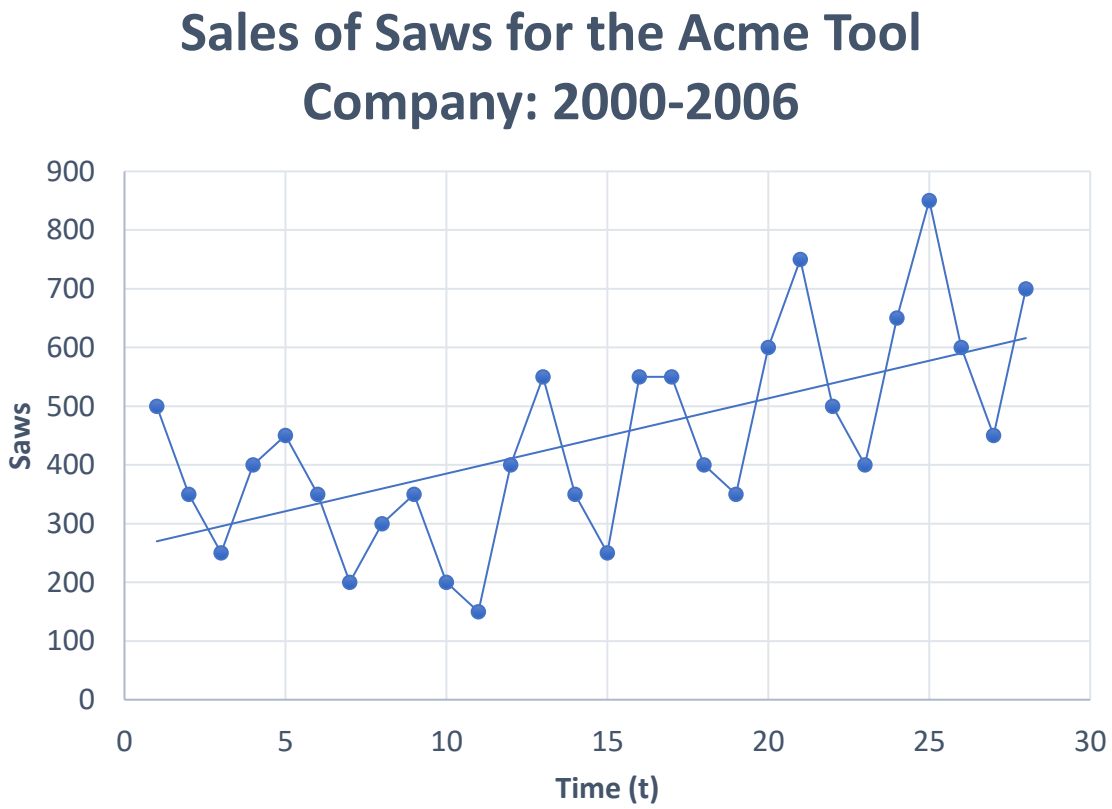
Naïve Trend Model

- When data values increase over time, they are said to be **nonstationary** in level or to have a **trend**.

- For the data with trend, if we apply the naïve method, the forecasting will be consistently low/high.

- **Naïve Trend Model:**
  - Adjust the naïve method to take trend into consideration by adding the difference between two periods.

$$\widehat{Y}_{t+1} = Y_t + (Y_t - Y_{t-1})$$



Naïve Trend Model

$$\widehat{Y}_{t+1} = Y_t + (Y_t - Y_{t-1}) = 2Y_t - Y_{t-1}$$

- Using naïve trend method to predict the values of week 6 - 10. Calculate the MSE of this prediction.

Week (t)	Y(t)	Y.hat(t)	e(t)	e^2(t)
1	275			
2	291			
3	307			
4	281			
5	295			
6	268			
7	252			
8	279			
9	264			
10	288			

Simple Moving Average

- Simple Moving Average is a commonly used technique for time series analysis and forecasting. It is a statistical method that calculates the average of a specified number of data points over a specific period of time.
  - Assumes an average is a good estimator of future behavior
  - Used if *little or no trend*
  - Used for smoothing

- N-period moving average:

$$\hat{Y}_{t+1} = \frac{(Y_t + Y_{t-1} + Y_{t-2} + \ldots + Y_{t-n+1})}{n}$$

Simple Moving Average

You're manager in Amazon's electronics department. You want to forecast ipad sales for months 4-6 using a 3-period moving average.

Month	Sales (1000)	
1	4	
2	6	
3	5	
4	?	
5	?	
6	?	

Simple Moving Average

- You're manager in Amazon's electronics department. You want to forecast ipad sales for months 4-6 using a 3-period moving average.

Month	Sales (1000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	?	(4+6+5)/3=5
5	?	
6	?	

Simple Moving Average

- What if ipad sales were actually 3 in month 4

Month	Sales (1000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	3	5
5	?	
6	?	

Simple Moving Average

- Forecast for Month 5?

Month	Sales (1000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	3	5
5	?	(6+5+3)/3=4.667
6	?	

Simple Moving Average

- Actual Demand for Month 5 = 7

Month	Sales (1000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	3	5
5	? 7	4.667
6	?	

Simple Moving Average

- Forecast for Month 6?

Month	Sales (1000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	3	5
5	7	4.667
6	?	(5+3+7)/3=5

Exercise: Using MA to forecast week 6-10

- Using 5-period Moving Average method to predict the values of week 6 - 10. Calculate the MSE of this prediction.

Week (t)	Y(t)	Y.hat(t)	e(t)	e^2(t)
1	275			
2	291			
3	307			
4	281			
5	295			
6	268			
7	252			
8	279			
9	264			
10	288			

Simple Exponential Smoothing Method

❖ Formally, the exponential smoothing equation is

$$\hat{Y}_{t+1} = \alpha Y_t + (1 - \alpha)\hat{Y}_t$$

- $\hat{Y}_t$ = forecast value for the period t.
- α = smoothing constant.
- Y_t = observed value of series in period t.

❑ The forecast $\hat{Y}_{t+1}$ is based on weighting the most recent observation Y_t with a weight α and weighting the most recent forecast $\hat{Y}_t$ with a weight of $1 - \alpha$

Simple Exponential Smoothing Method

- The implication of exponential smoothing can be better seen if the previous equation is expanded by replacing $\hat{Y}_t$ with its components as follows:

$$\begin{aligned}\hat{Y}_{t+1} &= \alpha\ Y_t + (1-\alpha)\hat{Y}_t \\ &= \alpha Y_t + (1-\alpha)[\alpha Y_{t-1} + (1-\alpha)\hat{Y}_{t-1}] \\ &= \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + (1-\alpha)^2\hat{Y}_{t-1}\end{aligned}$$

Simple Exponential Smoothing Method

- The value of smoothing constant α must be between 0 and 1
- α can not be equal to 0 or 1.
- To estimate the best α, Forecasts are computed for α equal to .1, .2, .3, …, .9 and the sum of squared forecast error is computed for each.
- The value of α with **the smallest MSE** is chosen for the best α using in producing the future forecasts.

Exercise: Using ES to forecast week 6-10

- Using Exponential Smoothing method with (α=0.2) to predict the values of week 6 - 10. Use $\hat{Y}_t = Y_t$ for the initial forecasting. Calculate the MSE of this prediction.

Week (t)	Y(t)	Y.hat(t)	e(t)	e^2(t)
1	275			
2	291			
3	307			
4	281			
5	295			
6	268			
7	252			
8	279			
9	264			
10	288			

Exercise: Using ES to forecast week 6-10

- Using Exponential Smoothing method with (α=0.3) to predict the values of week 6 - 10. Use $\hat{Y}_{t+1} = \frac{Y_t+Y_{t-1}+Y_{t-2}+Y_{t-3}+Y_{t-4}}{5}$ for the initial forecasting. Calculate the MSE of this prediction.

Week (t)	Y(t)	Y.hat(t)	e(t)	e^2(t)
1	275			
2	291			
3	307			
4	281			
5	295			
6	268			
7	252			
8	279			
9	264			
10	288			