Dimensionality Reduction & Classification

Project 4—CISC-820 Quantitative Foundations

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I. Objective.

In this project, we worked with a database containing 400 facial images from 40 individuals, with 10 images per subject. Our task involved loading these images into MATLAB and applying PCA to extract eigenfaces. Subsequently, we utilized linear classification techniques for image reconstruction, face recognition, and face identification.

II. Dimensionality reduction.

1. Using PCA to Extract Eigenfaces on all 400 images

(a) How do the leading eigenfaces look like as an image?

In Figure 1, the leading eigenfaces demonstrate the general features of the human face, with the initial eigenfaces displaying a broad, template-like representation. These images exhibit varying contrasts that emphasize key facial features, such as hair, eyes, and mouth.

In particular, the first six eigenfaces appear to outline the structure of a human face. In the first eigenface, a circular shape is discernible, with contrasts that vaguely suggest facial features such as hair, eyes, mouth, and ears. However, this representation is largely indistinct, with defining characteristics remaining unrecognizable. By comparison, the sixth eigenface presents a more refined facial structure. Elements such as a chin, cheekbones, eyebrows, and a hair parting become evident. This progression indicates that as more eigenfaces are utilized, the detail and clarity of the reconstructed image improve, reflecting the linear combination of these eigenfaces.

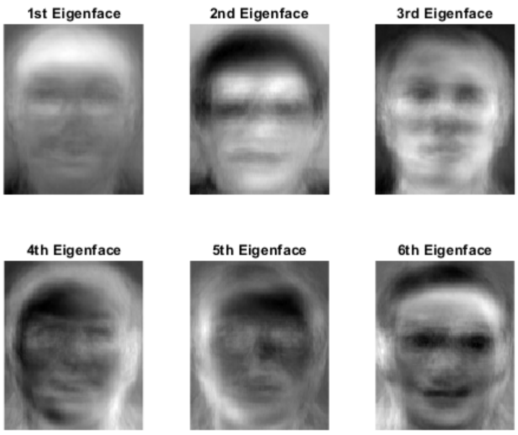

Figure 1: How do the leading eigenfaces look like as an image?

(b) How does the importance of the eigenfaces decrease?

The last few eigenfaces, when sorted by eigenvalues and visualized as images, convey minimal information, as shown in Figure 4. We can see that around 35-400 eigenfaces are able to completely explain the cumulative variance. In addition, visualizing the eigenfaces also confirms this. This decreasing significance is also evident in the reconstructed images, where the lower eigenfaces add only a small amount of detail to the reconstruction.

2. Face reconstruction with PCA: On a subset of images from different subjects, reconstruct the face image using different numbers of principal components.

(a) What is the difference between reconstructed and original images, as the number of eigenfaces used in reconstruction increases?

In Figure 2, as more eigenfaces are used, the difference between the original and reconstructed images gradually decreases. However, their contribution becomes increasingly negligible beyond a certain number of eigenfaces.

(b) How many eigenfaces are required to recover an original face with reasonable errors?

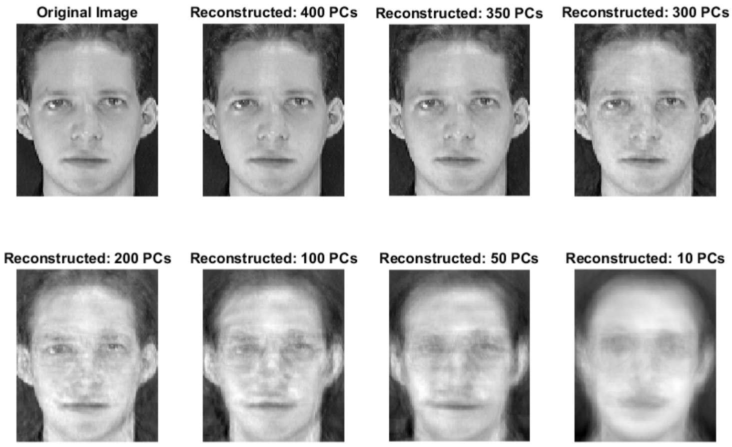

Figure 2: Effect of using different number of eigenfaces on the reconstructed face of participant 1

The decision regarding the optimal number of principal components can be guided through multiple approaches: analyzing reconstruction loss, cumulative variance explained, and visually inspecting the reconstructed images.

From Figure 3, it is evident that the loss significantly decreases as the number of principal components increases. At **350 eigenfaces**, the reconstruction loss (MSE) is approximately **5.14**, which is very reasonable. By increasing the components to **400 eigenfaces**, the reconstruction loss approaches near-zero levels, on the order of 10^{-27}.

From Figure 4, further supports this conclusion, as the cumulative variance plateaus around **350–400** principal components, nearing 100% explained variance. This indicates that the data is almost fully captured within this range.

In addition, visual inspection of the reconstructed images also helps verify the results: the quality and detail of the images improve significantly with more eigenfaces, particularly when approaching 350–400 components. Thus, 350 eigenfaces provide a strong trade-off between reconstruction quality and computational efficiency, with 400 components achieving near-perfect reconstruction.

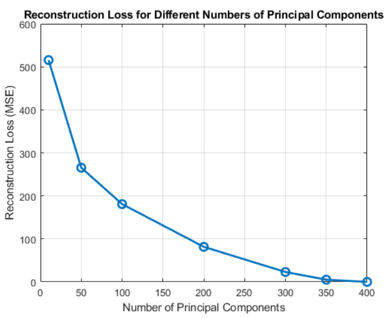

Figure 3: Reconstruction Loss

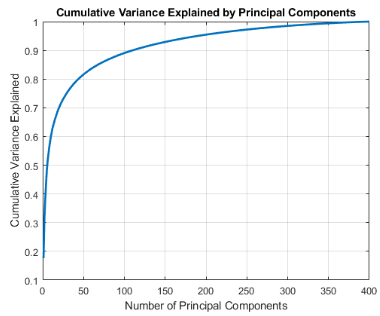

Figure 4: Cumulative Variance Explained

III. Classification

1. Face recognition: Face vs. "Non-Face" Classification

We calculated the PCA space using the training data and projected the test data onto this space. The test data was also normalized based on the parameters obtained from the training data. The test dataset comprised 120 face images and 30 "non-face" images, **while the training dataset contained only face images.**

The key challenge in this task was to classify face and non-face images without any exposure to non-face samples during training. To address this, we evaluated the confidence of the classifier at test time. If the classifier exhibited **low confidence** in an image being a face, we classified it as a **non-face** image. Unlike the method which does not provide normalized or comparable scores, we relied on methods that produce probabilities or comparable outputs.

We experimented with several methods, including Linear Regression(LR), KNN and K-means. However, KNN and K-means were the only methods that proved effective. The other methods misclassified non-face images as face images with similar confidence levels.

The **KNN** method achieved perfect classification after applying a **threshold** to the distance from the nearest neighbor. The clear separation between face and non-face images in the PCA space contributed to its success. The unsupervised **K-means** algorithm also perfectly separated the face and non-face images into **two clusters**.

In the case of KNN, if the distance to the closest neighbor exceeded the validation-derived threshold, the image was classified as non-face. Both methods successfully handled the classification task due to the distinct difference between face and non-face image features in the PCA-transformed space.

Method	**Task 1 Accuracy**	**Task 2 Accuracy**
LR Classifier	0.8	0.54167
KNN	0.99	0.56667
K-Means	1	-

Table 1: Different methods and Task Accuracies

2. Face identification

We framed this problem as a classification problem where we are identifying 35 known faces with a confidence score, and any image with low confidence is treated as an unknown face. We first tried to use sklearn methods to get the probabilities of our predictions. However, as in the first problem, the probabilities did not give us meaningful confidence levels for our predictions.

We countered this by considering the decision scores to estimate the confidence in predictions, where, applying softmax, we correctly identified five unknown face examples. We believe that with more validation and fine-tuning, this method will be more accurate.

In general, for Task 2 we find that if the problem isn't explicitly modeled as "35 known faces vs. unknown faces," most classifiers do pretty well. We don't show explicit quantitative results for K-means clustering, but from a qualitative perspective, the approach worked quite well. Quantitively: most image pairs ended up falling within the same cluster. In the last 50 images (10 images per subject), the majority of the images in a subject have a tendency to form one cluster. These results seem that K-means is able to create meaningful divisions, which show its efficiency for the task at hand.

V. Conclusion.

Because most of the information is packed into the top few components, PCA manages to reduce 10,304 dimensions down to 350–400 dimensions. At 350 dimensions, the MSE is very low, and at 400 dimensions, it is effectively zero, meaning very little is lost in terms of reconstruction quality.

Thus, by using PCA, we managed to reduce the dimensions of images from 10,304 to 350 with a very low reconstruction error. Also, when applied with 400 dimensions it has almost 0% reconstruction error. In conclusion, the size reduction rate is very good for many practical implementations. Another thing was being able to visualize the eigen-faces as images gave us an understanding about what it represents-for instance, the leading eigen-faces display fundamental facial features like chin, mouth, nose, ears, hair etc and all the images are the linear combination of these eigenfaces.