Orbital mechanics theory notes

cxcorp

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Contents

1 Keplerian orbital elements

The parameters required to uniquely identify a specific orbit. Assuming a two-body system.

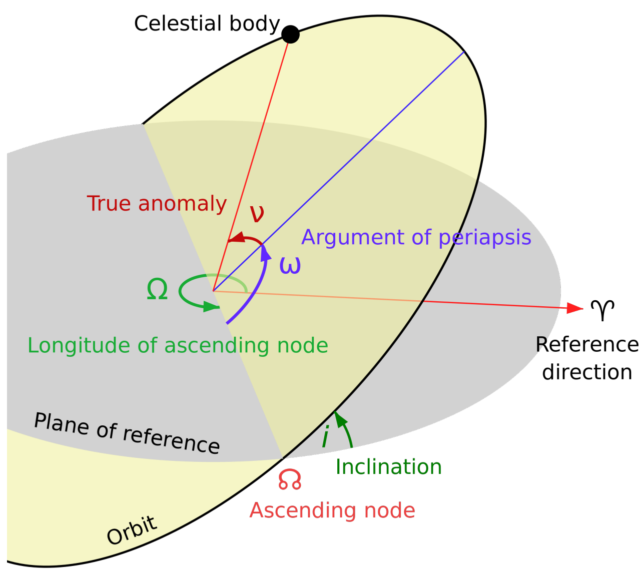

Figure 1: Diagram illustrating and explaining various terms in relation to Orbits of Celestial bodies.
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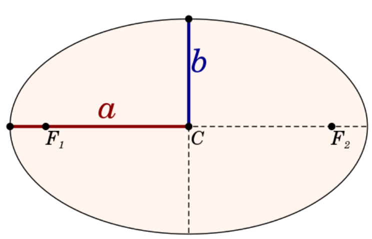
2 Shape and size of the ellipse

2.1 a – Length of semi-major axis

The **major axis** is the longest diameter of an ellipse. The semi-major axis is half of the diameter.



$$\begin{aligned} &\boxed{\boldsymbol{a} = \text{length of semi-major axis}} \\ b &= \text{length of semi-minor} \\ c &= \text{center point} \\ F_1, F_2 &= \text{focus points} \end{aligned}$$

2.2 e – Eccentricity

Amount by which orbit deviates from a perfect circle. More $e \Rightarrow$ more squished.

Eccentricity	Shape
$e = 0$	Circular orbit
$0 < e < 1$	Elliptic orbit
$e = 1$	Parabolic trajectory
$e > 1$	Hyperbolic trajectory

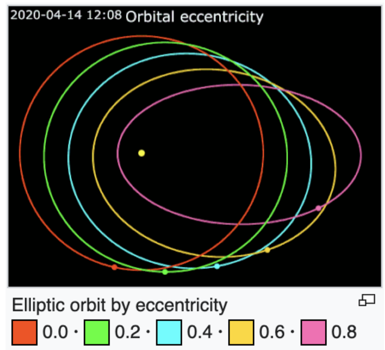


Figure 2: Elliptic orbit by eccentricity. By Phoenix7777 at the English Wikipedia. Licensed under CC BY-SA 4.0.

2.2.1 Relation to apoapsis and periapsis

$$\boxed{e = \frac{r_a - r_p}{r_a + r_p}} = 1 - \frac{2}{\frac{r_a}{r_p} + 1} \tag{1}$$

$$\Leftrightarrow \quad \boxed{r_p = \frac{(1-e)r_a}{1+e}} \tag{2}$$

$$\Leftrightarrow \quad \boxed{r_a = -\frac{(1+e)r_p}{e-1}} \tag{3}$$

$e =$ eccentricity
$a =$ semi-major axis length
$r_a =$ apoapsis
$r_p =$ periapsis

3 Orientation of the orbital plane

3.1 i – Inclination

"vertical tilt of the ellipse with respect to the reference plane, measured at the ascending node (where the orbit passes upward through the reference plane, the green angle i in the diagram [3]"

3.2 Ω – Longitude of the ascending node

Horizontal tilt. Angle from reference direction to the ascending node.

4 Other orbital parameters

4.1 T – Orbital period

How long it takes to complete one orbit.

For all orbits with a specific length of semi-major axis, the orbital period is the same. The shape of the ellipsis (eccentricity) does not matter.

$$\boxed{T = \sqrt{\frac{\pi^2 (r_p + r_a)^3}{2\mu}}} = 2\pi\sqrt{\frac{a^3}{\mu}} \quad (4)$$

$a =$ semi-major axis length
$\mu =$ *see below*

[2].

4.2 $r(\phi)$ – Radius of orbit at specific angle

This is the distance between the craft and the body [2] at a specific angle.

$$\boxed{r(\phi) = 2\frac{r_p r_a}{r_p + r_a + (r_a - r_p)\cos\phi}} \quad (5)$$

$r_a =$ apoapsis
$r_p =$ periapsis
$\phi =$ orbital angle

4.3 μ – Standard gravitational parameter

Specific to a celestial body. It is the gravitational constant G multiplied by the mass of the body M:

$$\mu = GM$$

4.3.1 Standard gravitational parameters of KSP celestial objects

Celestial object	$\mu \quad (\frac{m^3}{s^2})$
Mun	$6.5138398 \cdot 10^{10}$
$e > 1$	Hyperbolic trajectory

5 todo

- Geosynchronous orbits [1]

6 References

[1] Kerbal Space Program Wiki contributors. *Geosynchronous Orbit (Math) — Kerbal Space Program Wiki*. Accessed 6 August 2023. 2017. URL: https://wiki.kerbalspaceprogram.com/wiki/Geosynchronous_Orbit_(Math).

[2] Kerbal Space Program Wiki contributors. *Tutorial: Basic Orbiting (Math) — Kerbal Space Program Wiki*. Accessed 6 August 2023. 2019. URL: https://wiki.kerbalspaceprogram.com/wiki/Tutorial:_Basic_Orbiting_(Math).

[3] *Orbital Elements*. In: *Wikipedia*. May 16, 2023. URL: https://en.wikipedia.org/w/index.php?title=Orbital_elements&oldid=1155075030 (visited on 08/06/2023).