Designing Track Alignment Worksheet

April 2025

Aim

In this workshop session you are going to work through some of the equations and calculations you were introduced to in the preparatory videos. You will then go on to look at how to design a horizontal and a vertical curve.

Learning Outcomes

- To become familiar in some frequently used calculations

- To improve confidence in using these calculations

1 Calculating Cant

There are 3 measures related to cant, E_q equilibrium cant, E_a applied cant, and, D cant deficiency. All are measured in mm.

As shown in equation 1, equilibrium cant is always equal to the sum of applied cant and cant deficiency.

$$E_q = E_a + D \tag{1}$$

The equations in 2 gives an approximation for calculating applied cant and cant deficiency that shall be used in this worksheet. The applied cant will be two thirds of the calculated equilibrium cant and therefore one third of the calculated equilibrium cant will be cant deficiency.

$$E_a = \frac{2}{3}E_q, \quad D = \frac{E_a}{2} \tag{2}$$

Equation 3 gives the calculation for equilibrium cant.

$$E_q = \frac{11.82V^2}{R} \tag{3}$$

Where:

- E_q: equilibrium cant (mm)

- V: velocity (km/h)

- R: curve radius (m)

1. What is the equilibrium cant for a track for a curve with a rolling stock speed of 85 km/h and a radius of 500 m?

> **Solution:** $E_q = 170.8$ mm
>
> $$E_q = \frac{11.82 \cdot 85^2}{500} = 170.8 \tag{4}$$

2. What is the equilibrium cant for a track for a curve with a rolling stock speed of 56 km/h and a radius of 300 m?

> **Solution:** $E_q = 123.6$ mm
>
> $$E_q = \frac{11.82 \cdot 56^2}{300} = 123.6 \tag{5}$$

3. What is the equilibrium cant for a track for a curve with a rolling stock speed of 120 km/h and a radius of 800 m?

Solution: $E_q = 212.8\,\text{mm}$

$$E_q = \frac{11.82 \cdot 120^2}{800} = 212.8 \tag{6}$$

4. Using equation 2 and a maximum allowed cant $E_{a(max)}$ of $150\,\text{mm}$:

   - What would the applied cant, E_a be in each of the previous questions?
   - In which cases would a speed limit need to be applied?

Solution:

Question 1:

$V = 85\,\text{km/h}$ and $R = 500\,\text{m}$

$E_a = 170.8 \cdot \frac{2}{3} = 133.9\,\text{mm}$

No speed limit needed.

Question 2:

$V = 56\,\text{km/h}$ and $R = 300\,\text{m}$

$E_a = 123.6 \cdot \frac{2}{3} = 82.4\,\text{mm}$

No speed limit needed.

Question 3:

$V = 120\,\text{km/h}$ and $R = 800\,\text{m}$

$E_a = 212.8 \cdot \frac{2}{3} = 141.8\,\text{mm}$

No speed limit needed.

2 Designing an Alignment

Rearranging equation 3, it is possible to calculate the radius of a curve and the maximum velocity.

1. For all questions in this section assume that the maximum applied cant for a curve is $150\,\text{mm}$.

   Using equation 2, if a curve uses the maximum applied cant, what is the equilibrium cant E_q?

Solution: $E_q = 225\,\text{mm}$

$$E_q = \frac{3}{2} \cdot E_a = \frac{3}{2} \cdot 150 = 225 \tag{7}$$

Radius

2. You are designing a curve for a new alignment. The track speed will be $180\,\text{km/h}$ and a maximum applied cant will be used.

   What should the radius of this curve be?

Solution: $R = 1702\,\text{m}$

$$R = \frac{11.82 \cdot V^2}{E_q} = \frac{11.82 \cdot 180^2}{225} = 1702 \tag{8}$$

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3. The designer of a new track would like to cut costs by reducing the radius of a curve on a $270\,\text{km/h}$ railway alignment to $3000\,\text{m}$.

   What is your advice?

Solution: $R = 3830\,\text{m}$

$$R = \frac{11.82 \cdot V^2}{E_q} = \frac{11.82 \cdot 270^2}{225} = 3830 \tag{9}$$

As the radius of the curve should be 3830 m, reducing it may be unadvisable as it may increase the risk of derailment. You could argue about the use of tilting trains or a risk assessment needing to be written.

Velocity

4. What is the maximum speed on a corner of $750\,\text{m}$ with the maximum applied cant?

> **Solution:** $V = 119\,\text{km/h}$
>
> $$V = \sqrt{\frac{E_q \cdot R}{11.82}} = \sqrt{\frac{225 \cdot 750}{11.82}} = 119 \tag{10}$$

5. How would this speed change if the applied cant were reduced to $110\,\text{mm}$?

> **Solution:** $V = 102\,\text{km/h}$
>
> $$E_q = \frac{3}{2} \cdot E_a = \frac{3}{2} \cdot 110 = 165 \tag{11}$$
>
> $$V = \sqrt{\frac{E_q \cdot R}{11.82}} = \sqrt{\frac{165 \cdot 750}{11.82}} = 102 \tag{12}$$

3 Jerk

The physical property of excessive jerk is what we want to avoid, as it makes a journey uncomfortable for passengers. Some limits for jerk include:

Lateral jerk limit: $0.2\,\mathrm{m\,s^{-3}}$

Longitudinal jerk (comfortable limit): $0.3\,\mathrm{m\,s^{-3}}$

Longitudinal jerk (extreme limit): $0.5\,\mathrm{m\,s^{-3}}$

jerk is the third derivative of displacement, equations 13, 14, and, 15 are used to calculate velocity, acceleration and jerk. These assume that velocity, acceleration and jerk are constant, for all questions in this section assume that they are.

$$v = \frac{\Delta s}{t} = \frac{s_2 - s_1}{t} \tag{13}$$

$$a = \frac{\Delta v}{t} = \frac{v_2 - v_1}{t} \tag{14}$$

$$j = \frac{\Delta a}{t} = \frac{a_2 - a_1}{t} \tag{15}$$

Where:

- t time (s)

- s displacement (m)

- v velocity ($\mathrm{m\,s^{-1}}$)

- a acceleration ($\mathrm{m\,s^{-2}}$)

- j jerk ($\mathrm{m\,s^{-3}}$)

NOTE: In these, Δ is used to show change, so Δv is change in velocity, where v_1 is the initial velocity and v_2 is the final velocity

1. A track fault causes a lateral movement of a train of $2\,\mathrm{mm}$ to the left in $0.5\,\mathrm{s}$. Is this acceptable? Show your working.

Solution: $j = 0.016\,\mathrm{m\,s^{-3}}$

This is an acceptable value.

$$v = \frac{\Delta s}{t} = \frac{0.002}{0.5} = 0.004$$
$$a = \frac{\Delta v}{t} = \frac{0.004}{0.5} = 0.008 \tag{16}$$
$$j = \frac{\Delta a}{t} = \frac{0.008}{0.5} = 0.016$$

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2. A new design for a braking system accelerates from $30\,\mathrm{m\,s^{-1}}$ to $0\,\mathrm{m\,s^{-1}}$ in $10\,\mathrm{s}$. Is this acceptable? Show your working.

Solution: $j = -0.3\,\mathrm{m\,s^{-3}}$

This may be acceptable, it exceeds the comfort limit but is inside the extreme limit and therefore could be appropriate for an emergency brake but not a service brake.

$$
\begin{aligned}
a &= \frac{\Delta v}{t} = \frac{0 - 30}{10} = -3 \\
j &= \frac{\Delta a}{t} = \frac{-3}{10} = -0.3
\end{aligned}
\tag{17}
$$

NOTE: The minus simply shows the vehicle is slowing down, a negative acceleration

3. A new design for a metro train accelerates from $0\,\mathrm{km/h}$ to $60\,\mathrm{km/h}$ in $10\,\mathrm{s}$. Is this acceptable? Show your working.

Solution: $j = 0.167\,\mathrm{m\,s^{-3}}$

This is an acceptable value.

Firstly, unit conversion, $\frac{60 \cdot 60}{1000} = 3.6$, $60\,\mathrm{km/h}$ is equal to $16.667\,\mathrm{m\,s^{-1}}$

$$
\begin{aligned}
a &= \frac{\Delta v}{t} = \frac{16.667 - 0}{10} = 1.667 \\
j &= \frac{\Delta a}{t} = \frac{1.667}{10} = 0.167
\end{aligned}
\tag{18}
$$

4. The design is later changed so that it accelerates from $0\,\mathrm{km/h}$ to $60\,\mathrm{km/h}$ in $5.8\,\mathrm{s}$. Is this acceptable? Show your working.

Solution: $j = 0.495\,\mathrm{m\,s^{-3}}$

This is no longer within the comfortable limits, and is very close to the extreme limit. This will probably require a risk assessment.

$$
\begin{aligned}
a &= \frac{\Delta v}{t} = \frac{16.667 - 0}{5.88} = 2.874 \\
j &= \frac{\Delta a}{t} = \frac{2.874}{10} = 0.495
\end{aligned}
\tag{19}
$$

4 Determining Transition Length

To determine transition length equations 20 and 21 are used.

$$L_{tr} = \frac{E_a \cdot V}{\Delta E \cdot 3.6} \tag{20}$$

$$L_{tr} = \frac{D \cdot V}{\Delta E \cdot 3.6} \tag{21}$$

Where:

- L_{tr} transition length (m)

- ΔE rate of change in applied cant (mm/s) (not to be confused with E_a or E_q

The highest value for L_{tr} calculated from E_a (equation 20) and D (equation 21) is used, and is rounded to the nearest 5 m.
For this section, use equation 2 as an assumption.

1. What would be the transition length for a curve with a running speed of 80 km/h? Take the rate of change in cant to be 35 mm/s and a cant deficiency of 45 mm.

> **Solution:** $L_{tr} = 60$ m
>
> $$E_a = 2 \cdot D = 2 \cdot 45 = 90 \tag{22}$$
>
> $$L_{tr} = \frac{E_a \cdot V}{\Delta E \cdot 3.6} = \frac{90 \cdot 80}{35 \cdot 3.6} = 57.1 \tag{23}$$
>
> $$L_{tr} = \frac{D \cdot V}{\Delta E \cdot 3.6} = \frac{45 \cdot 80}{35 \cdot 3.6} = 28.6 \tag{24}$$

2. What would be the transition length for a curve with a running speed of 120 km/h? Take the rate of change in cant to be 22.6 mm/s and a cant deficiency of 38.9 mm.

> **Solution:** $L_{tr} = 115$ m
>
> $$E_a = 2 \cdot D = 2 \cdot 38.9 = 77.8 \tag{25}$$
>
> $$L_{tr} = \frac{E_a \cdot V}{\Delta E \cdot 3.6} = \frac{77.8 \cdot 120}{22 \cdot 3.6} = 114.7 \tag{26}$$
>
> $$L_{tr} = \frac{D \cdot V}{\Delta E \cdot 3.6} = \frac{38.9 \cdot 120}{22 \cdot 3.6} = 57.4 \tag{27}$$

3. A curve has a transition length of 80 m and a running speed of 130 km/h. If we assume the maximum allowed cant of 150 mm is used, what is the rate of change of cant? Comment on your answer.

Solution: $\Delta E = 68\,\text{mm/s}$

With Network Rail, the maximum allowed change in cant under normal operations is $55\,\text{mm/s}$. $70\,\text{mm/s}$ is the exceptional maximum and therefore if you intended to use this figure you would need to justify your reasoning.

$$\Delta E = \frac{E_a \cdot V}{L_{tr} \cdot 3.6} = \frac{150 \cdot 130}{80 \cdot 3.6} = 68 \tag{28}$$
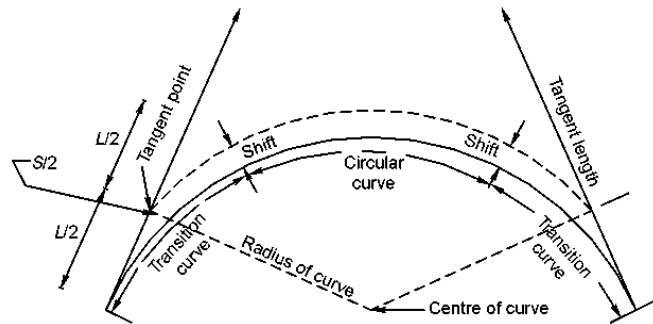
5   Designing Horizontal Curves

Figure 1: Annotated geometry of a horizontal curve

The aim of this section is to work through the method for designing a transition curve. Generally, this will be done using a software programme, but it is always useful to know how this would be done manually.

This will also demonstrate how some of the equations you have already met can be used in designing a track alignment

Worked example

A metro has the following design parameters:

- Velocity $V = 80\,\text{km/h}$

- Radius $R = 400\,\text{m}$

- Max applied cant $E_{a(max)} = 150\,\text{mm}$

- Max cant deficiency $D_{(max)} = 90\,\text{mm}$

- Max cant gradient $CG = 1\,\text{in}\,100$

- Max rate of change in cant $\Delta E = 55\,\text{mm/s}$

Calculate:

- equilibrium cant E_q (mm)

- applied cant E_a (mm)

- transition length L_{tr} (m)

- curve offset S_c (m)

- x & y co-ordinate of the transition at 4 locations along its length (m)

Equilibrium cant

$$E_q = \frac{11.82 \cdot V^2}{R} = \frac{11.82 \cdot 80^2}{400} = 189 \tag{29}$$

$E_q = 189\,\text{mm}$

Applied cant

Previously, where we used the assumption in equation 2 that applied cant was two thirds of the equilibrium cant, we are now going to use a ratio between the maximum specified applied cant and cant deficiency ($E_{a(max)}$ and $D_{(max)}$) in order to calculate E_a and D.

$$E_a = \frac{E_{a(max)}}{E_{a(max)} + D_{(max)}} E_q = \frac{150}{150 + 90} E_q = 118 \tag{30}$$

Rounded to nearest $5\,\text{mm}$, $E_a = 120\,\text{mm}$

Cant deficiency

$$D = E_q - E_a = 189 - 120 = 69 \tag{31}$$

$D = 69\,\text{mm}$

Transition curve length

This can be derived from either the max cant gradient (equation 32) or the max rate of change in cant (equation 33).

$$L_{tr} = E_a \cdot CG = 120 \cdot 500 = 60 \tag{32}$$

$$L_{tr} = \frac{E_a \cdot V}{\Delta E \cdot 3.6} = \frac{120 \cdot 80}{55 \cdot 3.6} = 48.5 \tag{33}$$

The actual transition length we want to design to is the higher result value of these equation, so: $L_{tr} = 60\,\text{m}$

Curve shift

This is how far the curve is moved inwards.

$$S_c = \frac{L_{tr}^2}{24 \cdot R} = \frac{60^2}{24 \cdot 400} = 0.375 \tag{34}$$

$S_c = 0.375\,\text{m}$

Figure 2 shows how the length of the radius is reduced by $0.375\,\text{m}$ at the start of the transition curve.
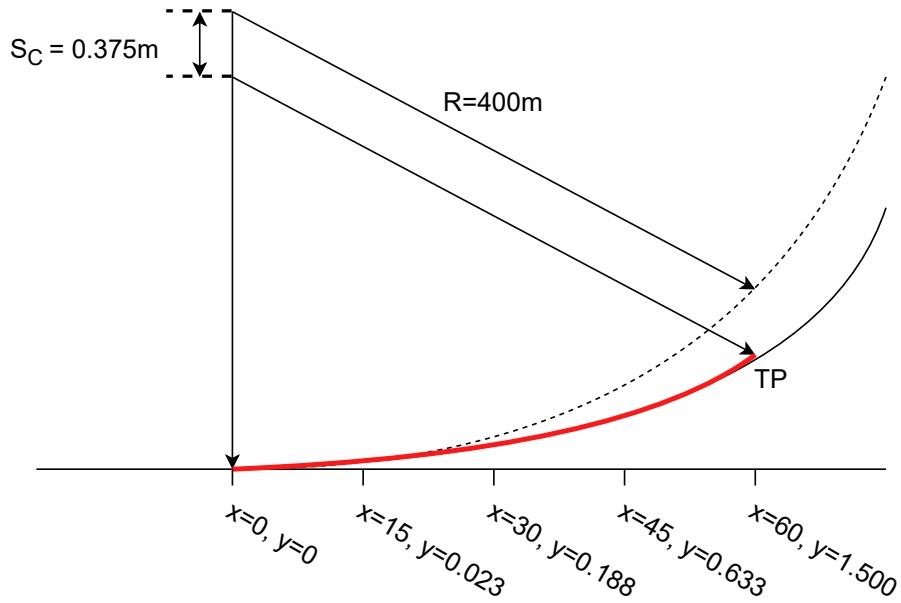
Figure 2: Diagram of horizontal curve worked example

Curve Co-ordinates

To calculate the co-ordinates along the transition to a circular curve, the following two equations are used.

$$x = s \tag{35}$$

$$y = \frac{s^3}{6 \cdot R \cdot L_{tr}} \tag{36}$$

The results at 4 co-ordinates along the transition curve are shown in the table below and in figure 2.

x	0	15	30	45	60
y	0.000	0.023	0.188	0.633	1.500

Questions

1. A railway has the following design parameters:

   - Velocity $V = 160\,\text{km/h}$
   - Radius $R = 1250\,\text{m}$
   - Max applied cant $E_{a(max)} = 150\,\text{mm}$
   - Max cant deficiency $D_{(max)} = 110\,\text{mm}$
   - Max cant gradient $CG = 1\,\text{in}\,400$

- Max rate of change in cant $\Delta E = 55$ mm/s

Calculate the equilibrium cant, applied cant, transition length, curve offset and the x & y co-ordinates of the transition at 4 locations along its length.

Solution:

$$E_q = \frac{11.82 \cdot V^2}{R} = \frac{11.82 \cdot 160^2}{1250} = 242 \tag{37}$$

$E_q = 242$ mm

$$E_a = \frac{E_{a(max)}}{E_{a(max)} + D_{(max)}} E_q = \frac{150}{150 + 110} E_q = 140 \tag{38}$$

Rounded to nearest 5 mm, $E_a = 140$ mm

$$D = E_q - E_a = 242 - 140 = 102 \tag{39}$$

$D = 102$ mm

$$L_{tr} = E_a \cdot CG = 140 \cdot 400 = 56 \tag{40}$$

$$L_{tr} = \frac{E_a \cdot V}{\Delta E \cdot 3.6} = \frac{140 \cdot 160}{55 \cdot 3.6} = 113 \tag{41}$$

$L_{tr} = 113$ m

$$S_c = \frac{L_{tr}^2}{24 \cdot R} = \frac{160^2}{24 \cdot 1250} = 0.427 \tag{42}$$

$S_c = 0.375$ m

x	0	15	30	45	60
y	0.000	0.004	0.032	0.107	0.255

2. A railway has the following design parameters:

- Velocity $V = 120$ km/h
- Radius $R = 1050$ m
- Max applied cant $E_{a(max)} = 110$ mm
- Max cant deficiency $D_{(max)} = 90$ mm
- Max cant gradient $CG = 1$ in 450
- Max rate of change in cant $\Delta E = 55$ mm/s

Calculate the equilibrium cant, applied cant, transition length, curve offset and the x & y co-ordinates of the transition at 4 locations along its length.

Solution:

$$E_q = \frac{11.82 \cdot V^2}{R} = \frac{11.82 \cdot 120^2}{1050} = 162 \tag{43}$$

$E_q = 162\,\text{mm}$

$$E_a = \frac{E_{a(max)}}{E_{a(max)} + D_{(max)}} E_q = \frac{110}{90} E_q = 89 \tag{44}$$

Rounded to nearest $5\,\text{mm}$, $E_a = 90\,\text{mm}$

$$D = E_q - E_a = 162 - 90 = 72 \tag{45}$$

$D = 72\,\text{mm}$

$$L_{tr} = E_a \cdot CG = 90 \cdot 450 = 40.5 \tag{46}$$

$$L_{tr} = \frac{E_a \cdot V}{\Delta E \cdot 3.6} = \frac{90 \cdot 120}{55 \cdot 3.6} = 54.5 \tag{47}$$

$L_{tr} = 54.5\,\text{m}$

$$S_c = \frac{L_{tr}^2}{24 \cdot R} = \frac{120^2}{24 \cdot 1050} = 0.118 \tag{48}$$

$S_c = 0.118\,\text{m}$

x	0	15	30	45	60
y	0.000	0.010	0.079	0.265	0.629

3. Design a curve for HS2. Highlight any assumptions you have made in order to be able to complete this task.

6   Designing Vertical Curves

A vertical curve is a parabolic curve between two grades on a railway. Its aim is to make the ride smooth.

Table 1: Vertical acceleration (a_v) limits for vertical curves

Criteria	Standard Limit ($\mathrm{m\,s^{-2}}$)	Maximum Limit ($\mathrm{m\,s^{-2}}$)	Exceptional Limit ($\mathrm{m\,s^{-2}}$)
Freight	0.10	0.22	0.31
Passenger	0.19	0.22	0.31

Worked Example

Calculate the vertical curve between an upward gradient of $1\,\%$ and a downward gradient of $2.5\,\%$ for a passenger service at the standard limit.

- Initial height $H_0 = -35\,\mathrm{m}$

- Velocity $V = 80\,\mathrm{km/h}$

- Minimum vertical curve radius $R_{(min)} = 2500\,\mathrm{m}$

Vertical acceleration

Using table 1:

- Vertical acceleration $a_v = 0.19\,\mathrm{m\,s^{-2}}$

Vertical Curve Radius

First, work out the length of the radius.

$$R_{vc} = \frac{V^2}{33.1776 \cdot a_v} = \frac{80^2}{33.1776 \cdot 0.19} = 1015 \tag{49}$$

However, this is below the minimum limit for vertical curve radius, therefore.
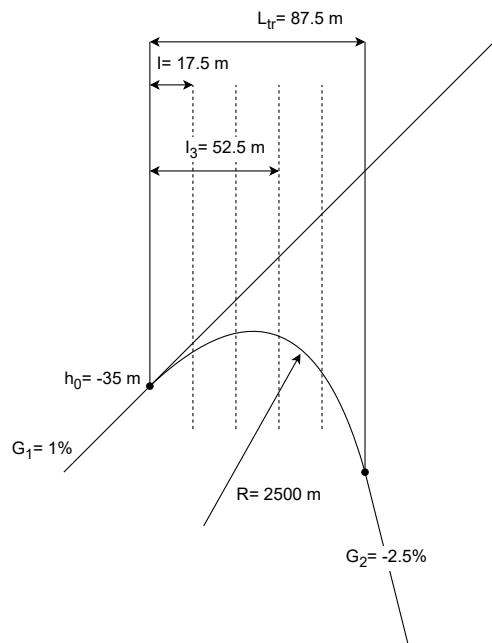$R_{vc} = 2500\,\mathrm{m}$

Curve Length

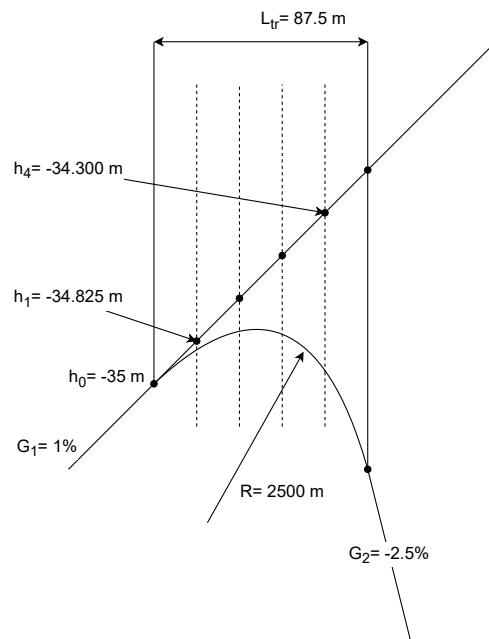Next, work out the longitudinal linear length of the vertical curve.

- Initial gradient $G_1 = 1\,\%$

- Final gradient $G_2 = -2.5\,\%$

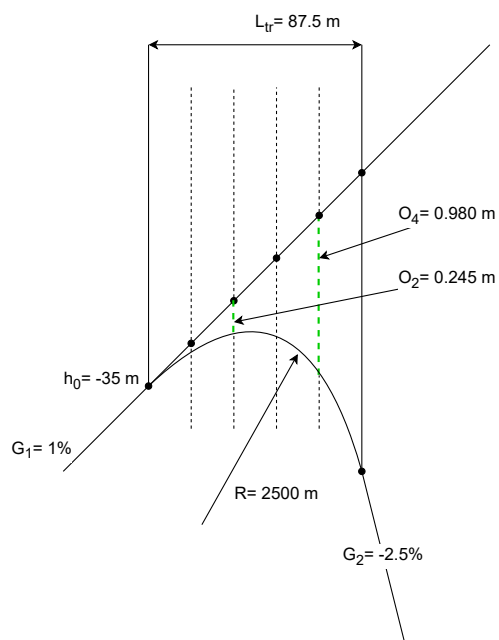$$L_{vc} = \frac{R_{vc}(G_1 - G_2)}{100} = \frac{2500(1 - (-2.5))}{100} = 87.5 \tag{50}$$
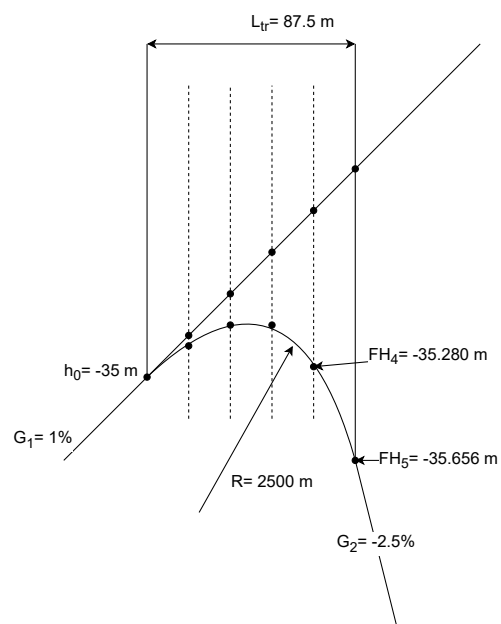
$L_{vc} = 87.5\,\mathrm{m}$

(a) interval

(b) grade levels

© extension

(d) final height

Figure 3: 4 Diagrams showing the states of calculating

Intervals

Next, assume there are 5 intervals and calculate the longitudinal length between each interval, this measure is shown in figure 3a.

- Number of intervals $N = 5$

$$I = \frac{L_{vc}}{N} = \frac{87.5}{5} = 17.5 \tag{51}$$

Next, calculate how far each interval is from the initial transition point at h_0.

$$I_n = I \cdot n \tag{52}$$

For the third interval $n = 3$

$$I_3 = 17.5 \cdot 3 = 52.5 \tag{53}$$

Grade level

Then, calculate the height of each interval point (on the linear project line of the initial gradient), this measure is shown in figure 3b.

$$h_n = h_0 + \frac{G_1 \cdot I_n}{100} \tag{54}$$

for the first interval $n = 1$

$$h_1 = h_0 + \frac{G_1 \cdot I_1}{100} = -35 + \frac{1 \cdot 17.5}{100} = -34.825 \tag{55}$$

Extension

Next, we now have the height of each individual point, so calculate the distance from the interval point to the radial curve, these are the sections shown in green in figure 3c.

$$O_n = \frac{(G_1 - G_2) \cdot I_n^2}{200 \cdot L_{vc}} \tag{56}$$

for the second interval n=2

$$O_2 = \frac{(G_1 - G_2) \cdot I_2^2}{200 \cdot L_{vc}} = \frac{(1 - (-2.5)) \cdot 35^2}{200 \cdot 87.5} = 0.245 \tag{57}$$

Final Height

The final step is therefore to subtract the extension from the grade level at each interval to find the final height, as is shown in figure 3d.

$$FH_n = h_n - O_n \tag{58}$$

for the fourth interval n=4

$$FH_n = h_n - O_n = -34.300 - 0.980 = -35.280 \tag{59}$$

Results Table

With interval

Table 2: Worked Example Results Table

Interval number (n)	Interval length I (m)	Grade level h (m)	Extension O (m)	Final Height FH (m)
0	0.000	−35.000	0.000	−35.000
1	17.500	−34.825	0.061	−34.886
2	35.000	−34.650	0.245	−34.895
3	52.500	−34.475	0.551	−35.026
4	70.000	−34.300	0.980	−35.280
5	87.500	−34.125	1.531	−35.656

Question

1. Calculate the x and y co-ordinates of a vertical curve at 5 intervals based on the following parameters for a passenger service with standard limits:

   - Initial gradient $G_1 = 2.5\,\%$
   - Final gradient $G_2 = 0\,\%$
   - Initial height $H_0 = -35\,\text{m}$
   - Velocity $V = 80\,\text{km/h}$
   - Minimum vertical curve radius $R_{(min)} = 1000\,\text{m}$

Solution: Using the standard limit for a passenger service, $a_v = 0.19\,\text{m s}^{-2}$

$$R_{vc} = \frac{V^2}{33.1776 \cdot a_v} = \frac{80^2}{33.1776 \cdot 0.19} = 1015 \tag{60}$$

This is greater than the minimum vertical curve radius so $R_{vc} = 1015$

$$L_{vc} = \frac{R_{vc}(G_1 - G_2)}{100} = \frac{1015(2.5 - 0)}{100} = 25 \tag{61}$$

$L_{vc} = 25\,\text{m}$

$$I = \frac{L_{vc}}{N} = \frac{25}{5} = 5 \tag{62}$$

$$I_n = n \cdot I = 5n \tag{63}$$

$$h_n = h_0 + \frac{G_1 \cdot I_n}{100} = -35 + \frac{2.5 \cdot 5n}{100} \tag{64}$$

$$O_n = \frac{(G_1 - G_2) \cdot I_n^2}{200 \cdot L_{vc}} = \frac{(2.5 - 0) \cdot (5n)^2}{200 \cdot 25} \tag{65}$$

$$FH_n = h_n - O_n = -35 + \frac{2.5 \cdot 5n}{100} - \frac{(2.5 - 0) \cdot (5n)^2}{200 \cdot 25} \tag{66}$$

Interval number (n)	Interval length I (m)	Grade level h (m)	Extension O (m)	Final Height FH (m)
0	0	-35.000	0.000	-35.000
1	5	-34.873	0.013	-34.886
2	10	-34.746	0.051	-34.797
3	15	-34.619	0.114	-34.733
4	20	-34.492	0.203	-34.695
5	25	-34.365	0.317	-35.683