Pairs Trading Strategies

the Optimization in Decision-making Processes

Hung-Wei Chang
Jiaxin Li
Tianpei Zhu
Xiaohan Cheng
Yanlin Chen
Yuanhang Zhao

Duke University

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   - Pairs Selection
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A Combination of All Stages' Optimization.

Decision-Making Processes in Pairs Trading

1. Pairs Selection
2. Hedge Ratio & Spread Calculation
3. Optimal Training & Stop-Loss Boundaries
4. Performance Measure
5. ...

Different Stages, Several Methods to Optimize!

Stage 1

1. PCA
2. Machine Learning
3. Correlation
4. Empirical Criteria
5. ...

Stage 2

1. OLS
2. Kalman Filter
3. ...

Stage 3

1. Reinforced Learning
2. DQN
3. ...

Stage 4

1. Profit
2. Drawdown
3. Sharpe Ratio
4. ...

Stage Optimization

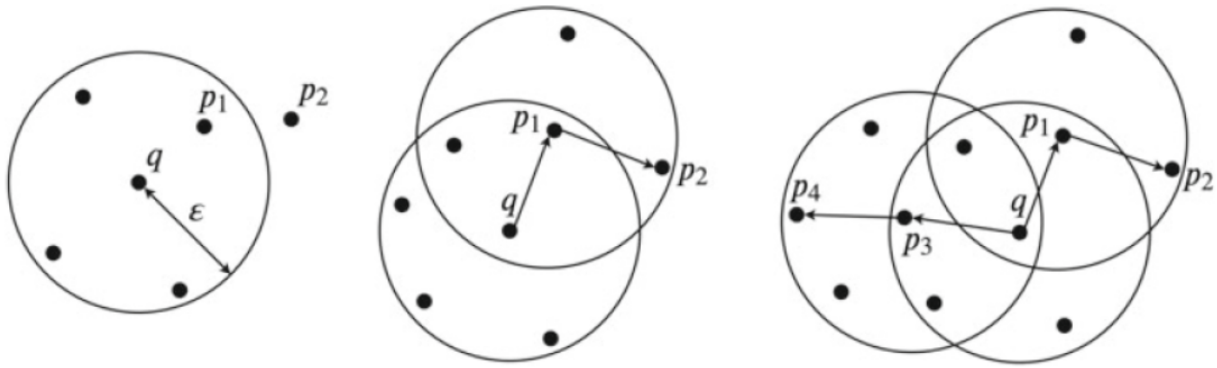
Pairs Selection - PCA

- PCA is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of linearly uncorrelated variables, the principal components.
- Each component can be seen as representing a risk factor.
- The number of features should not be large.
- Normalize the return series since PCA is sensitive to the relative scaling of the original variables.

$$Y_i = \frac{R_i - \bar{R}_i}{\sigma_i}$$

DBSCAN

1. Find the points in the ϵ-neighborhood of every point and identify the core points with more than minPts neighbours, where minPts is a parameter to be tuned.
2. Find the connected components of core points on the neighbour graph, ignoring all non-core points.
3. Assign each non-core point to a nearby cluster if the cluster is an ϵ-neighbor, otherwise assign it to noise.

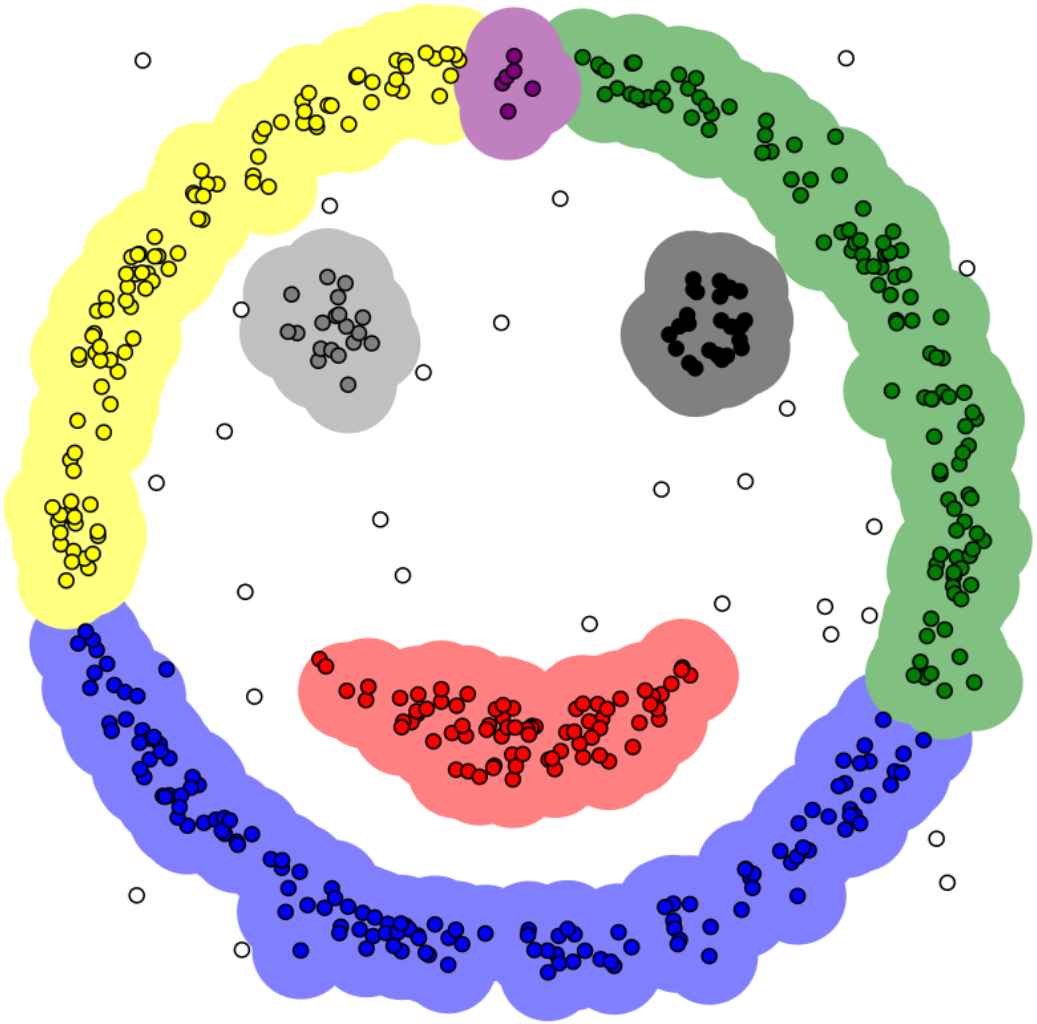


(a) Core points and outliers. **(b)** Density-reachable points. **(c)** Density-connected points.

DBSCAN illustration of basic concepts, with $minPts = 5$

Stage Optimization

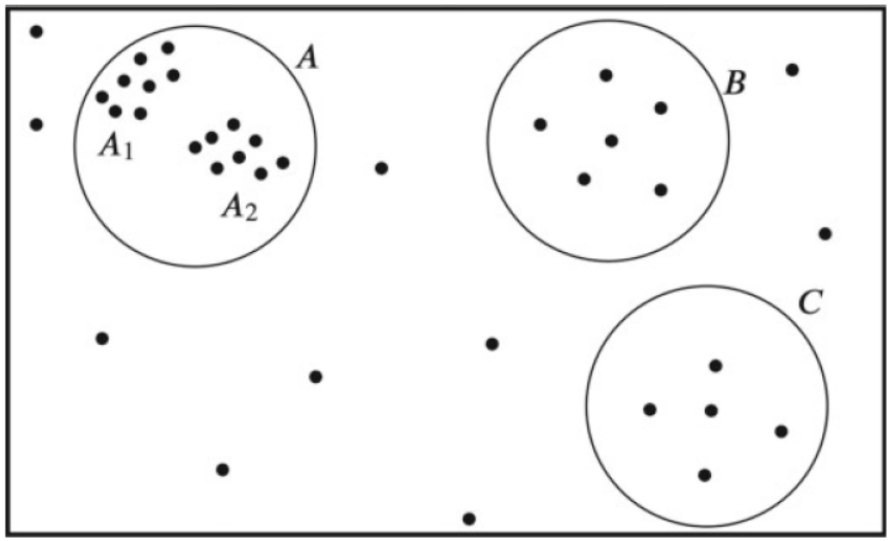
Pairs Selection - Clustering Methodologies



epsilon = 1.00
minPoints = 4

OPTICS

- OPTICS is based on DBSCAN.
- Appropriate under the assumption that clusters are not evenly dense.
- Only required to specify the parameter minPts, which is the minimum number of points to form a cluster
- OPTICS is capable of detecting the most appropriate parameter ϵ, which specifies how close points should be to each other to be considered neighbors.



Clusters with varying density

Stage Optimization

Pairs Selection - Other Criteria

- Correlation
- Cointegration
- Hurst exponent

$$\begin{aligned} H = 0.5 \quad & \textit{random walk} \\ H < 0.5 \quad & \textit{mean reversion} \\ H > 0.5 \quad & \textit{persistent trend} \end{aligned}$$

- Filter out pairs for which the half-life takes extreme values: less than one day or more than one year.

Stage Optimization

One method in Spread Calculation - Kalman Filter

A Three-step Process of Prediction, Observation, and Correction

corrected state = predicted state + k (observation - prediction)

- (observation - prediction) is called the observation innovation. A fraction of the observation innovation is added as a correction to the predicted state. The value of this fraction k is known as the Kalman gain.
- k is decided such that the corrected state has the least amount of error variance associated with it.
- k is indeed optimal in the case where the mathematical models of state and observation are both linear and the errors are drawn from independent Gaussian distributions.

Stage Optimization

One method in Spread Calculation - Kalman Filter

1. Evaluate $\hat{X}_{t|t-1}$ and $\hat{P}_{t|t-1}$ using the state equation.

$$\hat{X}_{t|t-1} = A\hat{X}_{t-1|t-1}$$

$$\hat{P}_{t|t-1} = A\hat{P}_{t-1|t-1}A^T$$

2. Find the observation Y_t and R by observing the system. Note we have the matrix H defined as follows:

$$Y_t = HX_t + v_t$$

3. Compute the Kalman gain K_t.

$$K_t = \hat{P}_t H^T (H\hat{P}_t H^T + R)^{-1}$$

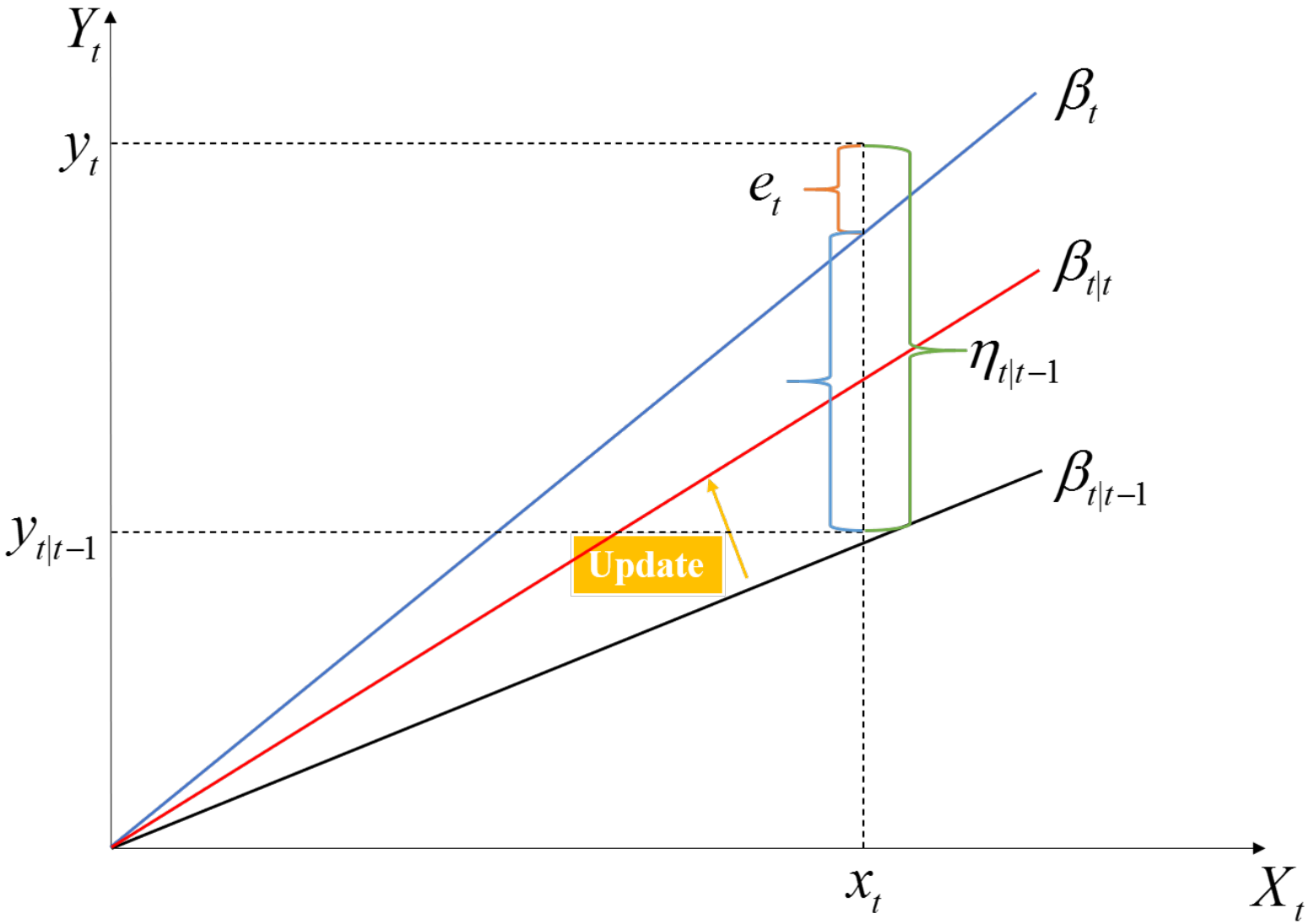
4. Evaluate $\hat{X}_{t|t}$ given by

$$\hat{X}_{t|t} = \hat{X}_{t|t-1} + K_t(Y_t - H\hat{X}_{t|t-1})$$

5. Evaluate $\hat{P}_{t|t}$

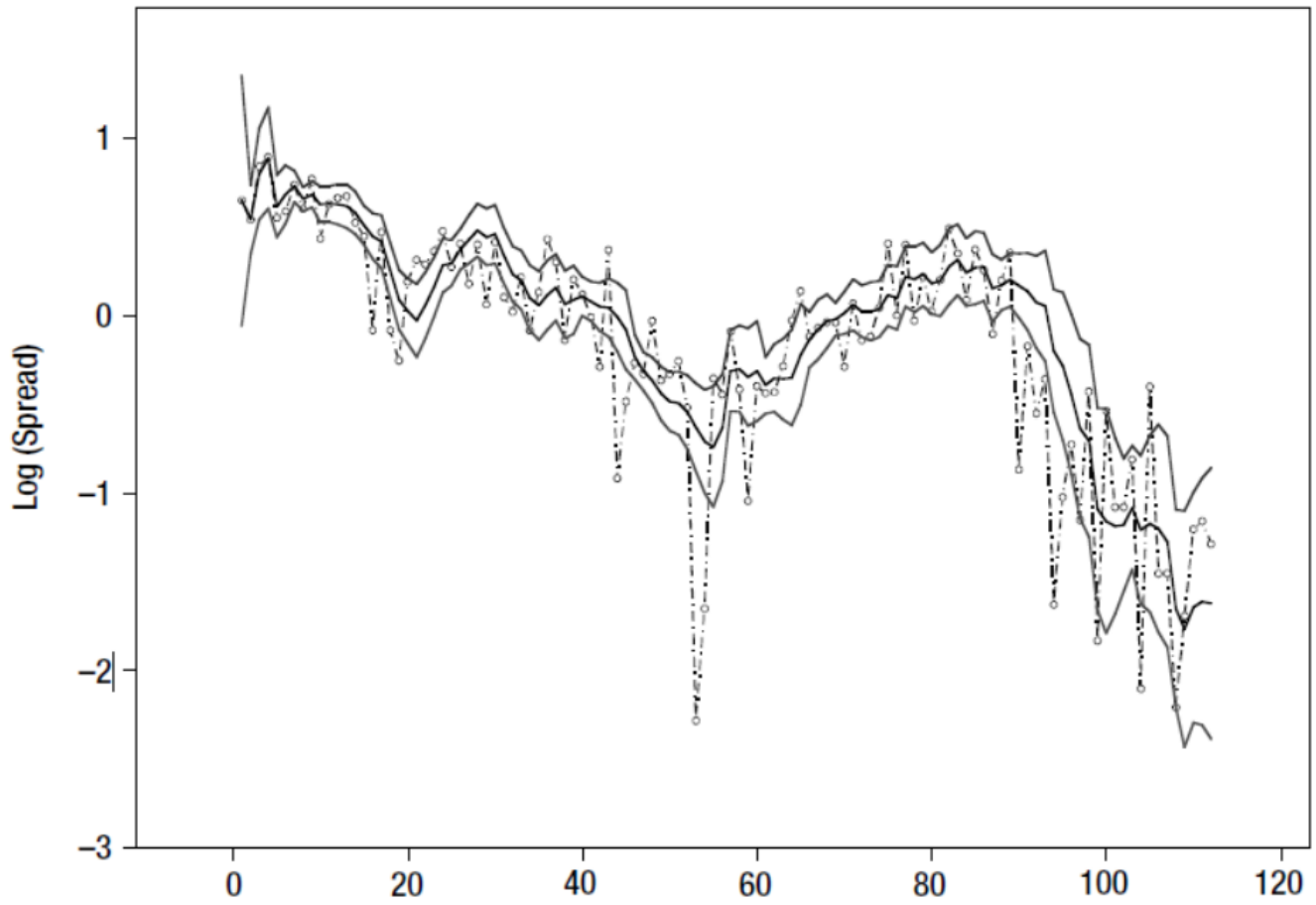
Stage Optimization

One method in Spread Calculation - Kalman Filter



Stage Optimization

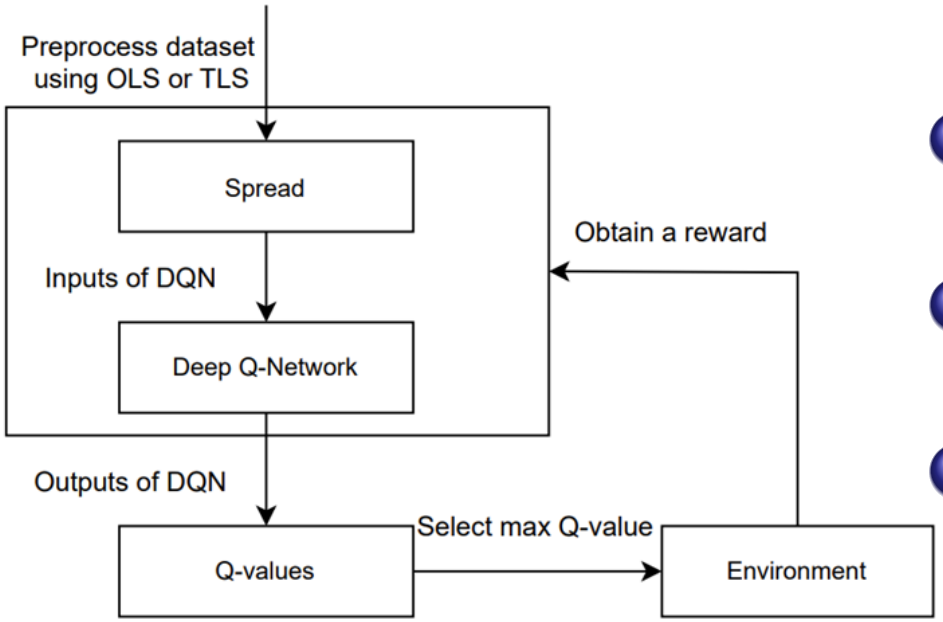
One method in Spread Calculation - Kalman Filter



Kalman Smoothing with Confidence Bands

Stage Optimization

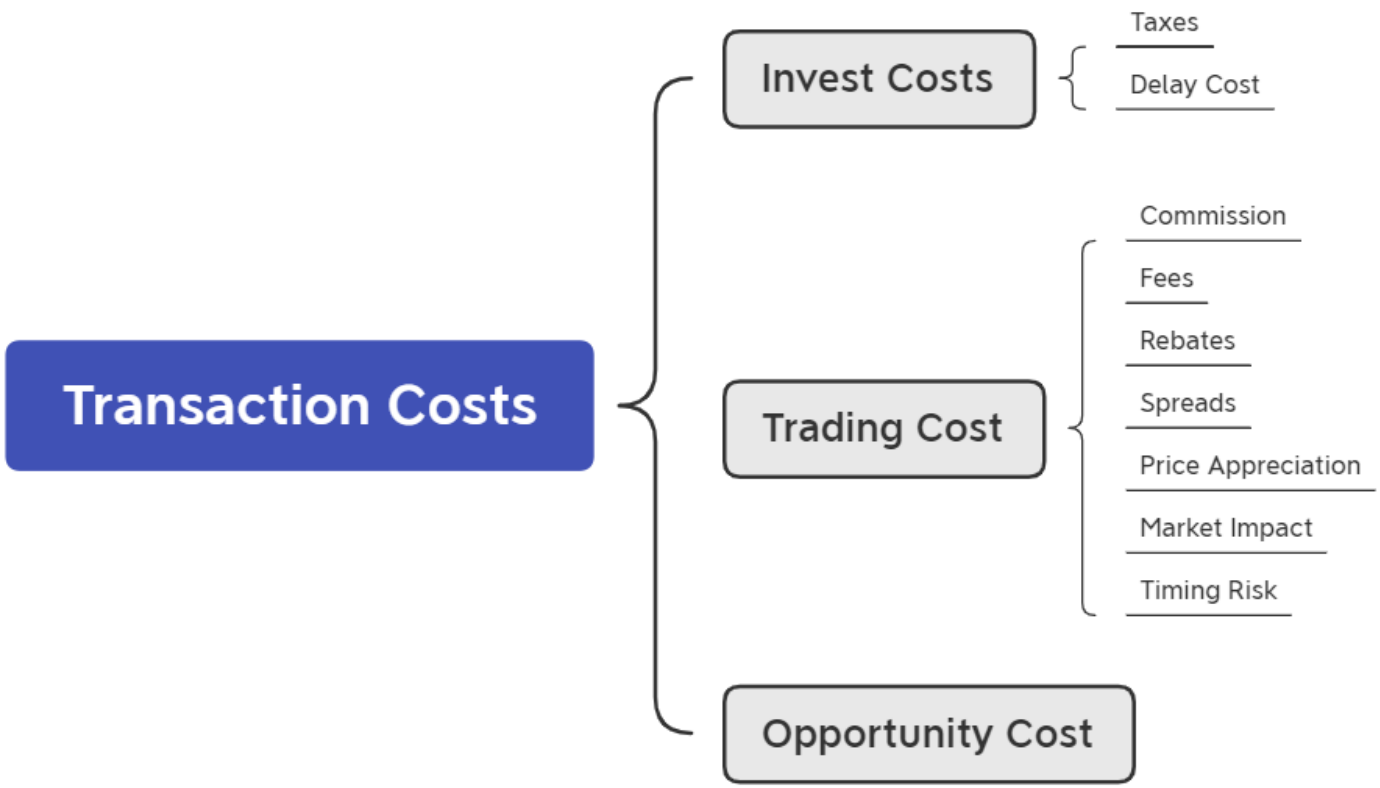
Optimal Training & Stop-Loss Boundaries - DQN Algorithm



1. Get the Spread from OLS or Kalman Filter.
2. Use DQN algorithm do the iteration for states.
3. Get the optimal trading actions by reinforced learning

Stage Optimization

Performance Measure - From Aspect of Cost management



Slippage Cost

- A subset of transaction cost
- The cost between listed price and actual pay

Eg. Spread Cost

- Main risk factor: Market Order
  advantage: liquidity
  drawback: bid ask spread may be different at different stock price level
- Solution: Limit Order
  advantage: full control of the bid-ask spread (we control the execution price)
  drawback: order may never be filled

Key point is the balance between execution speed and the control of bid-ask spread!

Performance Measure

1. Sharpe Ratio
2. Calmar Ratio
3. Jensen's Alpha
4. Treynor Ratio
5. Information Ratio
6. Sortino Ratio
7. Drawdown & Maximum Drawdown
8. Annualized Volatility
9. VaR & Expected Shortfall
10. Compare our NAV per unit with funds implementing similar strategies.

Future work

Plan

1. Compare the results from Stage 1 to Stage 3 using different optimal methods.
2. Find a **COMBINATION** of all the Optimal Strategies. Child strategies with different assets and weights result in diversification.
3. We might try different machine learning methods like XGBoost.

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