

## Background

• Intertemporal choices refer to decisions involving consequences and relative preferences - tradeoff between cost and benefits - that occur at different time points (Frederick et al., 2002). For example, a choice with two alternative options: \$10 today versus \$20 in a week, which is a standard *money-earlier-or-later* (MEL) task paradigm.

• Several models have been proposed to address individuals' decision between money and time. These models can be generally categorized into two groups: delay discounting models and heuristic models. This research compares three delay discounting models: exponential model, hyperbolic model, quasi-hyperbolic model, and three heuristic models: ITCH model (Ericson, et al., 2015), Trade-off model (Read et al., 2010), and DRIFT model (Scholten et al., 2013). This research also tries to develop new models nested on the best model.

## Data & Methods

• The data is from the experiment conducted by Ericson et al., (2015), which recruited 940 participants from the MTurk. Each participant was randomly assigned to one of five conditions and answer some MEL tasks:

Condition 1: Absolute Money Value, Delay Framing  
Condition 2: Relative Money Value, Delay Framing  
Condition 3: Standard MEL Format  
Condition 4: Absolute Money, Speedup Framing  
Condition 5: Relative Money, Speedup Framing

• The method is to perform cross-validation analyses to compare six models. The measure of model fit is the Mean Absolute Deviation between a model's prediction and the actual choice, both of which are probabilities of choosing the larger, later option. To estimate the parameters of the models, maximum likelihood estimation is performed. Based on these analyses, several new models are proposed to compete with the original six models.

## Classical Models

The intertemporal choice models have various versions. The version here are adapted from Ericson et al., (2015). In each of these models, the notation,  $L(z)$ , represents the inverse logistic function of  $z$ :  $L(z) = (1 + e^{-z})^{-1}$

$P(LL)$  represents the probability that individuals prefer the larger, later option over the smaller, sooner option.

### Delay Discounting Models

**Exponential Model:**  $P(LL) = L(a(x_2\delta^{t_2} - x_1\delta^{t_1}))$

**Hyperbolic Model:**

$P(LL) = L(a(x_2(1 + \alpha t_2)^{-1} - x_1(1 + \alpha t_1)^{-1}))$

**Quasi-hyperbolic Model:**

$P(LL) = L(a(x_2\beta^{I(t_2>0)}\delta^{t_2} - x_1\beta^{I(t_1>0)}\delta^{t_1}))$

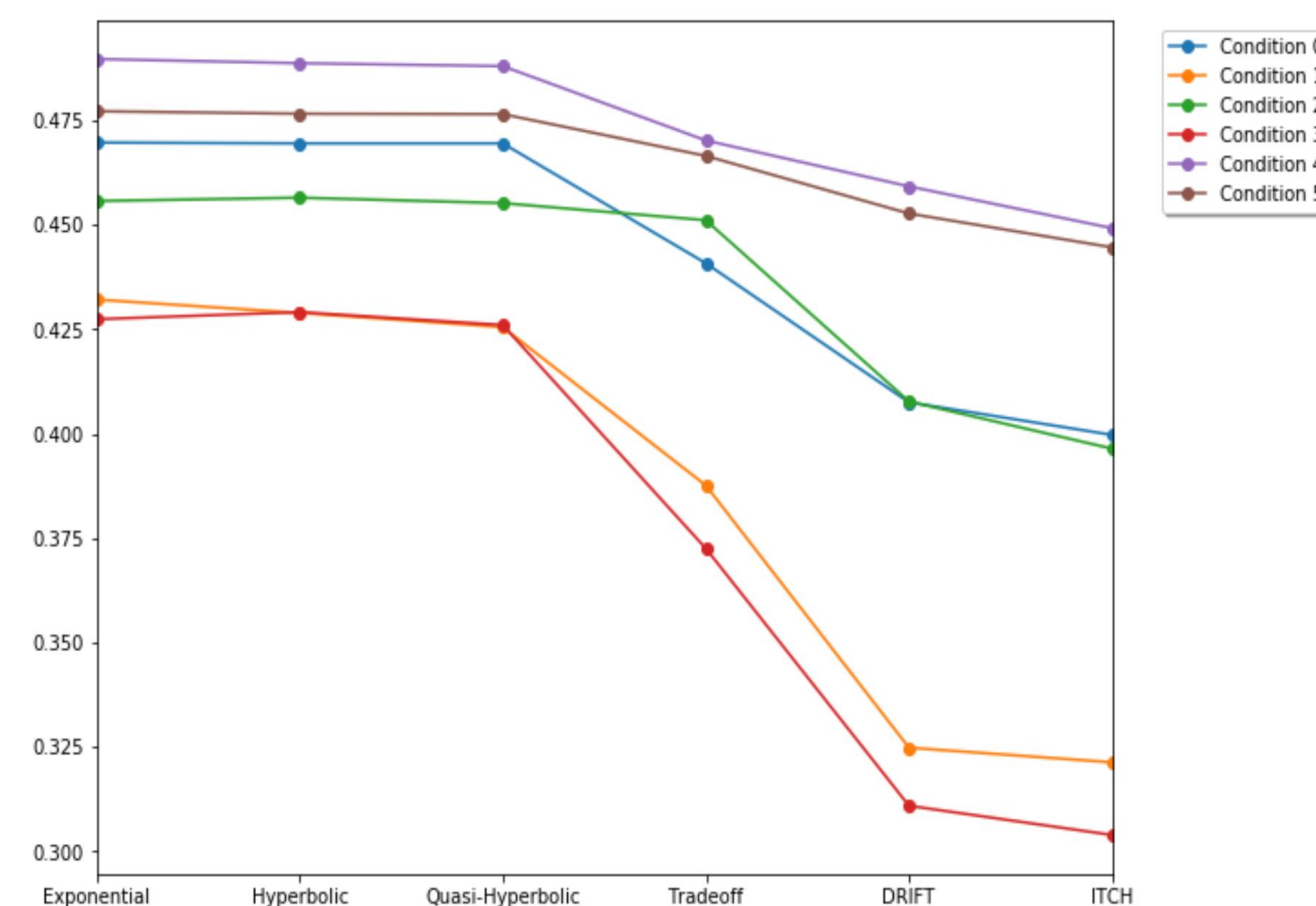
### Heuristic Models

**ITCH Model:**  $P(LL) = L(\beta_l + \beta_{xA}(x_2 - x_1) + \beta_{xR}\frac{x_2 - x_1}{x^*} + \beta_{tA}(t_2 - t_1) + \beta_{tR}\frac{t_2 - t_1}{t^*})$

**Trade-off Model:**  $P(LL) = L(a((\frac{\log(1+\gamma_x x_2)}{\gamma_x} - \frac{\log(1+\gamma_x x_1)}{\gamma_x} - k(\log(1 + \gamma_x t_2) - \frac{\log(1+\gamma_x t_1)}{\gamma_x})))$

**DRIFT Model:**  $P(LL) = L(\beta_0 + \beta_1(x_2 - x_1) + \beta_2\frac{x_2 - x_1}{x^*} + \beta_3\frac{x_2}{x_1}\frac{1}{t_2 - t_1} + \beta_4(t_2 - t_1))$

## Comparing Classical Models by Mean Absolute Deviation across Conditions



## Examining the Terms in the ITCH Model

Model Manipulation: Removing ...	Model Fit (Mean Absolute Deviations/MAD)					
	Pooled Data	Condition 1	Condition 2	Condition 3	Condition 4	Condition 5
Baseline / ITCH	0.3997	0.3213	0.3899	0.3158	0.4527	0.4410
Relative Time Term	0.4034	0.3225	0.3924	0.3172	0.4619	0.4460
Absolute Time Term	0.4034	0.3240	0.3948	0.3197	0.4562	0.4440
Relative Money Term	0.4524	0.3963	0.4578	0.3938	0.4761	0.4779
Absolute Money Term	0.4063	0.3320	0.3933	0.3277	0.4572	0.4464
Relative Terms	0.4554	0.3976	0.4592	0.3950	0.4839	0.4829
Absolute Terms	0.4099	0.3350	0.3974	0.3327	0.4603	0.4493
Constant Term	0.4318	0.4097	0.4199	0.4029	0.4585	0.4466

## New Models

**New Model 1:**  $P(LL) = L(\beta_l + \beta_2(x_2 - x_1) + \beta_3(t_2 - t_1) + \beta_4(d_2 - d_1))$

**New Model 2:**  $P(LL) = L(\beta_l + \beta_2(x_2 - x_1) + \beta_3\frac{x_2 - x_1}{x^*} + \beta_4(t_2 - t_1) + \beta_5(d_2 - d_1))$

**New Model 3:**

$P(LL) = L(\beta_l(x_2\delta^{t_2} - x_1\delta^{t_1}) + \beta_2(x_2 - x_1) + \beta_3\frac{x_2 - x_1}{x^*} + \beta_4(t_2 - t_1) + \beta_5(d_2 - d_1))$

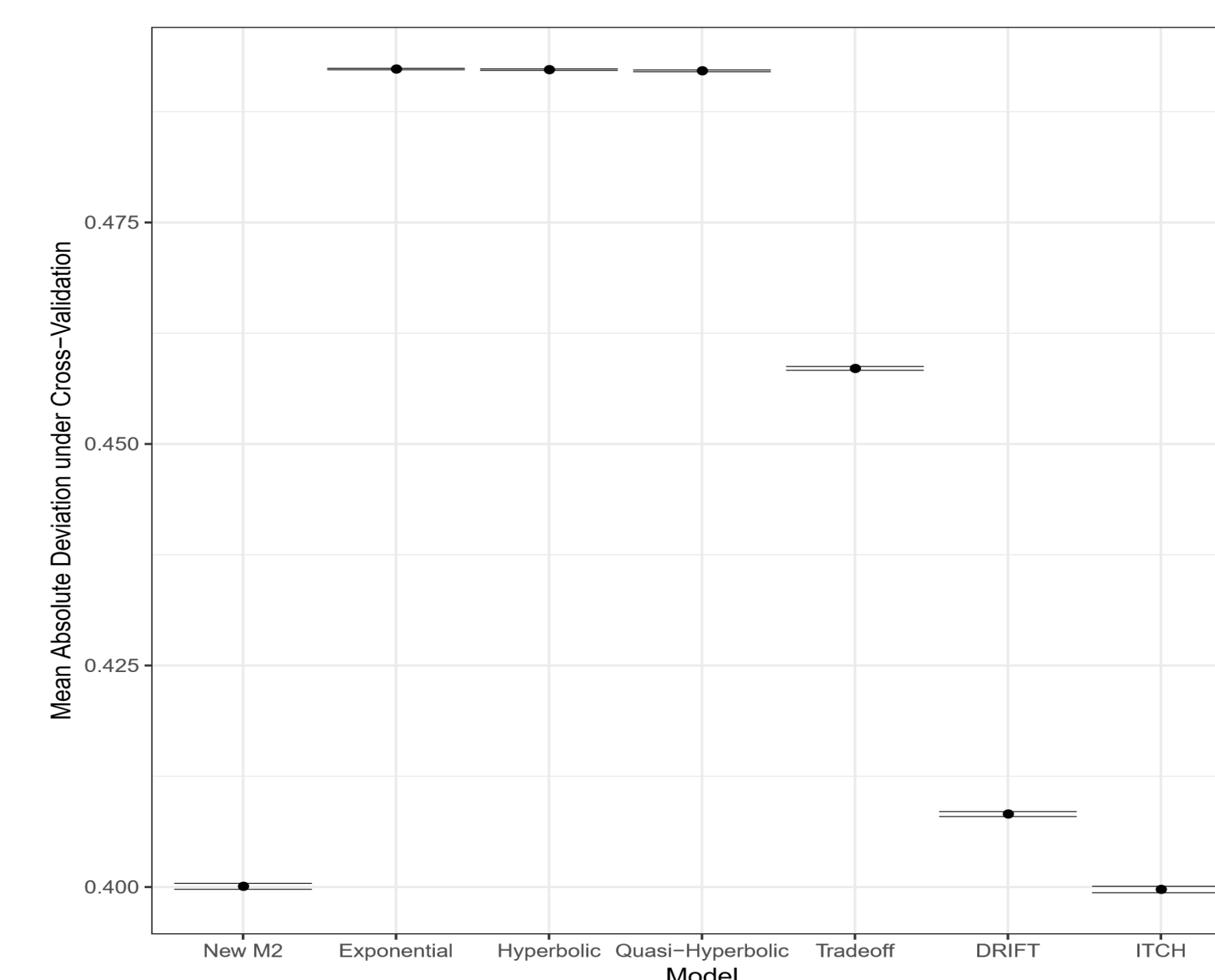
**New Model 4:**  $P(LL) = L(\beta_l(x_2^\alpha\delta^{t_2} - x_1^\alpha\delta^{t_1}) + \beta_2(d_2 - d_1))$

**New Model 5:**  $P(LL) = L(\beta_l[(\beta_2x_2 + \beta_3\frac{x_2}{x_1})\delta^{t_2 - t_1} - x_1] + \beta_4(d_2 - d_1))$

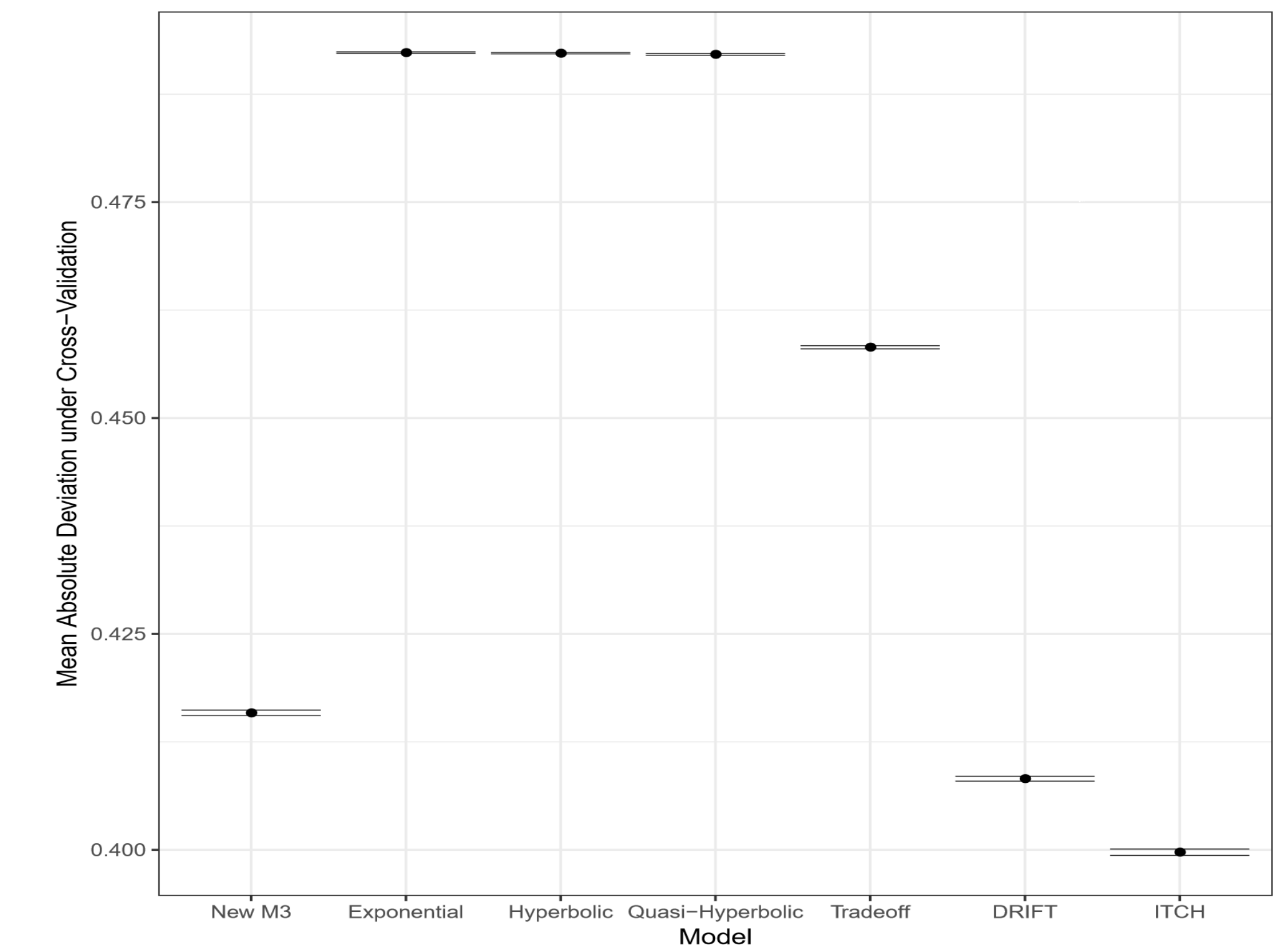
**New Model 6:**

$P(LL) = L(\beta_1(v_2 - v_1) + \beta_2\frac{v_2 - v_1}{v^*})$  where  $v_1 = x_1\delta^{t_1}$ ,  $v_2 = x_2\delta^{t_2}$ , and  $v^* = \frac{v_2 + v_1}{2}$ .

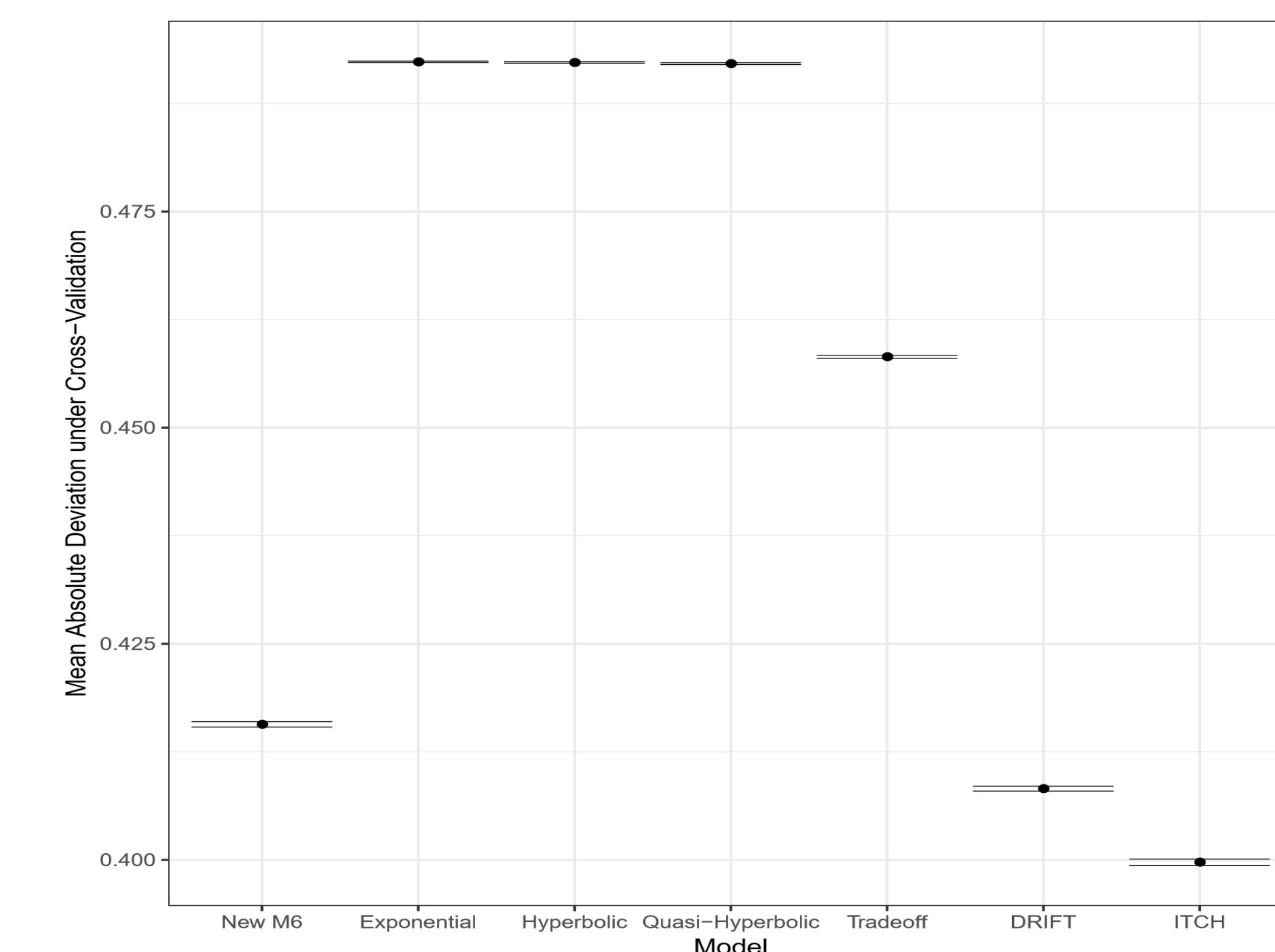
## Comparing Model Fit between the New Model 2 and Classical Models



## Comparing Model Fit between the New Model 3 and Classical Models



## Comparing Model Fit between the New Model 6 and Classical Models



## Conclusion

• ITCH model has the best performance among the classical models. The results from examining each term in this model show that both of the two relative terms:  $\frac{x_2 - x_1}{x^*}$ ,  $\frac{t_2 - t_1}{t^*}$  are of great importance; the relative money term:  $\frac{x_2 - x_1}{x^*}$  is especially important.

• Among the new models, Model 2 performs as well as the ITCH model; Model 3 and 6 have similar performance, and they both perform better than all the delay discounting models and one of the heuristic models. The future direction is to develop another model which may outperform all the other models.