

Solving probability problems using simulation

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Question

What is the probability of getting three heads if a fair coin is tossed 10 times?

Analytical solution

This can be modelled by the binomial distribution $Bin(0.5, 10)$. The probability of getting three heads is

$$\begin{aligned} p(3 \text{ heads in } 10 \text{ tosses}) &= \binom{10}{3} * 0.5^3 * 0.5^7 \\ &= 120 * 0.125 * 0.0078125 \\ &= 0.1171875 \end{aligned}$$

Simulation

To solve this problem using simulation, we run the 10 times coin tossing process n times. If the analytical solution was correct, the simulation result should converge to it as $n \rightarrow \infty$. Below are the R codes for simulation.

```
nRuns = 100000 # the number of times to run a simulation
cnt = 0 # initialize a variable to count the number times we see three heads in ten tosses
for (i in 1:nRuns) {

  tenTosses = sample(c(0, 1), 10, replace = T) # 0 and 1 represent tail and head respectively
  headCount = sum(tenTosses) # count the number of heads in ten tosses
  if (headCount == 3) {

    cnt = cnt + 1 # only update cnt if you see exactly three heads in ten tosses

  }
}

(p = cnt / nRuns) # print the estimated probability

## [1] 0.11625
```