ass3_wk4

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M1

(a)

```
print('Binomial distribution with n=500, theta=0.001')
```

```
## [1] "Binomial distribution with n=500, theta=0.001"
```

```
for (i in c(0:9)){
  cat('When k=',i,', the exact value p(X=k)=',dbinom(i,500,0.001),'\n')
}
```

```
## When k= 0 , the exact value p(X=k)= 0.6063789
## When k= 1 , the exact value p(X=k)= 0.303493
## When k= 2 , the exact value p(X=k)= 0.07579729
## When k= 3 , the exact value p(X=k)= 0.01259495
## When k= 4 , the exact value p(X=k)= 0.001566488
## When k= 5 , the exact value p(X=k)= 0.0001555512
## When k= 6 , the exact value p(X=k)= 1.284582e-05
## When k= 7 , the exact value p(X=k)= 9.074553e-07
## When k= 8 , the exact value p(X=k)= 5.597791e-08
## When k= 9 , the exact value p(X=k)= 3.063189e-09
```

```
cat('p(X<10)=',pbinom(9,500,0.001))
```

```
## p(X<10)= 1
```

(b) Using Gaussian as approximation

According to central limit theorem, binomial PMF converging to the normal density as n become larger. Hence, we can use Gaussian to yields the approximation:

$$E(X_i) = p, Var(X_i) = p(1-p)$$

$$X \sim B(n,p) \xrightarrow{approximate} X \sim N(np, p(1-p))$$

let:

$$Z = \frac{X - np}{\sqrt{np(1-p)}} \sim N(0,1)$$

Then:

$$Z = \frac{10 - 500 \times 0.001}{\sqrt{500 \times 0.001(1-0.001)}} = \frac{9.5}{\sqrt{0.4995}} = 8.793$$

$$P(X < 10) \approx \Phi(8.793) = 1$$

```
m=500*0.001
s=0.001*(1-0.001)
z=(10-m)/(sqrt(500*s))
cat("mean=n*p=",m,', variance=p(1-p)=',s,'\n')
```

```
## mean=n*p= 0.5 , variance=p(1-p)= 0.000999
```

```
cat("p(X<10)=",pnorm(10,m,s),"\n")
```

```
## p(X<10)= 1
```

```
cat("p(Z)=",pnorm(z))
```

```
## p(Z)= 1
```

(c) Use Poisson as approximation

Using Poisson approximation can yield s a good aprroximation when n is large and p is small:

$$P(X = i) = C_n^i p^i (1 - p)^{n-i} \approx e^{-\lambda} \frac{\lambda^i}{i!}$$

```
lambda=500*0.001
result=0
for (i in c(0:9)){
  cat('When i=',i,', p(X=k)=',dpois(i,lambda),'\n')
  result= result + dpois(i,lambda)
}
```

```
## When i= 0 , p(X=k)= 0.6065307
## When i= 1 , p(X=k)= 0.3032653
## When i= 2 , p(X=k)= 0.07581633
## When i= 3 , p(X=k)= 0.01263606
## When i= 4 , p(X=k)= 0.001579507
## When i= 5 , p(X=k)= 0.0001579507
## When i= 6 , p(X=k)= 1.316256e-05
## When i= 7 , p(X=k)= 9.401827e-07
## When i= 8 , p(X=k)= 5.876142e-08
## When i= 9 , p(X=k)= 3.264523e-09
```

```
cat('P(x<10)=',result)
```

```
## P(x<10)= 1
```

(d)

$$p = 0.001, 1 - p = 0.999$$

$$P(X = i) = C_n^i p^i (1-p)^{n-i}$$

$$P(X = 0) = C_{500}^0 p^0 (1-p)^{500} \approx 1 \times 0.606 = 0.606$$

$$P(X = 1) = C_{500}^1 p^1 (1-p)^{499} = 500 \times 6.069 \times 10^{-4} = 0.303$$

$$P(X = 2) = C_{500}^2 p^2 (1-p)^{498} \approx 1.248 \times 6.076 \times 10^{-2} = 0.076$$

$$P(X = 3) = C_{500}^3 p^3 (1-p)^{497} \approx 2.071 \times 6.082 \times 10^{-3} = 0.0126$$

$$P(X = 4) = C_{500}^4 p^4 (1-p)^{496} \approx 2.573 \times 6.088 \times 10^{-4} = 1.566 \times 10^{-3}$$

$$P(X = 5) = C_{500}^5 p^5 (1-p)^{495} \approx 2.552 \times 6.094 \times 10^{-5} = 1.555 \times 10^{-4}$$

$$P(X = 6) = C_{500}^6 p^6 (1-p)^{494} \approx 2.106 \times 6.100 \times 10^{-6} = 1.285 \times 10^{-5}$$

$$P(X = 7) = C_{500}^7 p^7 (1-p)^{493} \approx 1.486 \times 6.106 \times 10^{-7} = 9.074 \times 10^{-7}$$

$$P(X = 8) = C_{500}^8 p^8 (1-p)^{492} \approx 9.158 \times 6.113 \times 10^{-9} = 5.598 \times 10^{-8}$$

$$P(X = 9) = C_{500}^9 p^9 (1-p)^{491} \approx 5.006 \times \times 6.119 \times 10^{-10} = 3.063 \times 10^{-9}$$

$$P(X < 10) = \sum_{i=1}^{9} P(X = i) \approx 1$$

M2

$$IQ \sim N(100, 15^2)$$

Let's normalise it, say $Z_{IQ} \sim N(0, 1)$, then:

$$P(Z_{IQ} > x) = P(\frac{IQ - 100}{15} > \frac{x - 100}{15})$$

(1) Probability that 2 or more out of 40 would have IQs greater than 150

$$P(Z_{IQ} \leq 150) = P(\frac{IQ - 100}{15} \leq \frac{150 - 100}{15}) = P(Z_{IQ} \leq \frac{10}{3}) = \Phi(\frac{10}{3}) = 0.99957$$

$$P(Z_{IQ} > 150) = 1 - P(Z_{IQ} \leq 150) = 0.043\%$$

(2) Probability that 2 or more out of 40 would have IQs greater than 140

$$P(Z_{IQ} \leq 140) = P(\frac{IQ - 100}{15} \leq \frac{140 - 100}{15}) = P(Z_{IQ} \leq \frac{8}{3}) = \Phi(\frac{8}{3}) = 0.99617$$

$$P(Z_{IQ} > 140) = 1 - P(Z_{IQ} \leq 140) = 0.383\%$$