Multivariate Statistical Techniques
Matrix Operations in R

R is an open-source statistical programming package that is rich in vector and matrix operators. There are versions of R available for Windows, Mac OS and Unix that can be freely downloaded over the Internet.

The Matrix

```
# the matrix function
# R wants the data to be entered by columns starting with column one
# 1st arg: c(2,3,-2,1,2,2) the values of the elements filling the columns
# 2nd arg: 3 the number of rows
# 3rd arg: 2 the number of columns

> A <- matrix(c(2,3,-2,1,2,2),3,2)
> A

     [,1] [,2]
[1,]    2    1
[2,]    3    2
[3,]   -2    2
```

Is Something a Matrix

```
> is.matrix(A)

[1] TRUE

> is.vector(A)

[1] FALSE
```

Multiplication by a Scalar

```
> c <- 3
> c*A

     [,1] [,2]
[1,]    6    3
[2,]    9    6
[3,]   -6    6
```

Matrix Addition & Subtraction

```
> B <- matrix(c(1,4,-2,1,2,1),3,2)
> B

     [,1] [,2]
[1,]    1    1
[2,]    4    2
[3,]   -2    1

> C <- A + B
> C

     [,1] [,2]
[1,]    3    2
[2,]    7    4
[3,]   -4    3

> D <- A - B
> D

     [,1] [,2]
[1,]    1    0
[2,]   -1    0
[3,]    0    1
```

Matrix Multiplication

```
> D <- matrix(c(2,-2,1,2,3,1),2,3)
> D

     [,1] [,2] [,3]
[1,]    2    1    3
[2,]   -2    2    1

> C <- D %*% A
> C

     [,1] [,2]
```

```
[1,]    1   10
[2,]    0    4

> C <- A %*% D
> C

     [,1] [,2] [,3]
[1,]    2    4    7
[2,]    2    7   11
[3,]   -8    2   -4

> D <- matrix(c(2,1,3),1,3)
> D

     [,1] [,2] [,3]
[1,]    2    1    3

> C <- D %*% A
> C

     [,1] [,2]
[1,]    1   10

> C <- A %*% D

Error in A %*% D : non-conformable arguments
```

Transpose of a Matrix

```
> AT <- t(A)
> AT

     [,1] [,2] [,3]
[1,]    2    3   -2
[2,]    1    2    2

> ATT <- t(AT)
>ATT

     [,1] [,2]
[1,]    2    1
[2,]    3    2
[3,]   -2    2
```

Common Vectors

Unit Vector

```
> U <- matrix(1,3,1)
> U

     [,1]
[1,]    1
[2,]    1
[3,]    1
```

Zero Vector

```
> Z <- matrix(0,3,1)
> Z

     [,1]
[1,]    0
[2,]    0
[3,]    0
```

Common Matrices

Unit Matrix

```
> U <- matrix(1,3,2)
> U

     [,1] [,2]
[1,]    1    1
[2,]    1    1
[3,]    1    1
```

Zero Matrix

```
> Z <- matrix(0,3,2)
> Z

     [,1] [,2]
[1,]    0    0
[2,]    0    0
[3,]    0    0
```

Diagonal Matrix

```
> S <- matrix(c(2,3,-2,1,2,2,4,2,3),3,3)
> S

     [,1] [,2] [,3]
[1,]    2    1    4
[2,]    3    2    2
[3,]   -2    2    3

> D <- diag(S)
> D

[1] 2 2 3

> D <- diag(diag(S))
> D

     [,1] [,2] [,3]
[1,]    2    0    0
[2,]    0    2    0
[3,]    0    0    3
```

Identity Matrix

```
> I <- diag(c(1,1,1))
> I

     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

Symmetric Matrix

```
> C <- matrix(c(2,1,5,1,3,4,5,4,-2),3,3)
> C

     [,1] [,2] [,3]
[1,]    2    1    5
[2,]    1    3    4
[3,]    5    4   -2

> CT <- t(C)
> CT

     [,1] [,2] [,3]
[1,]    2    1    5
[2,]    1    3    4
[3,]    5    4   -2
```

Inverse of a Matrix

```
> A <- matrix(c(4,4,-2,2,6,2,2,8,4),3,3)
> A

     [,1] [,2] [,3]
[1,]    4    2    2
[2,]    4    6    8
[3,]   -2    2    4


> AI <- solve(A)
> AI

     [,1] [,2] [,3]
[1,]  1.0 -0.5  0.5
[2,] -4.0  2.5 -3.0
[3,]  2.5 -1.5  2.0

> A %*% AI

     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

```
> AI %*% A

     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

Inverse & Determinant of a Matrix

```
> C <- matrix(c(2,1,6,1,3,4,6,4,-2),3,3)
> C

     [,1] [,2] [,3]
[1,]    2    1    6
[2,]    1    3    4
[3,]    6    4   -2

> CI <- solve(C)
CI

           [,1]        [,2]        [,3]
[1,]  0.2156863 -0.25490196  0.13725490
[2,] -0.2549020  0.39215686  0.01960784
[3,]  0.1372549  0.01960784 -0.04901961

> d <- det(C)
> d

[1] -102
```

Rank of a MatrixM/h4>

```
> A <- matrix(c(2,3,-2,1,2,2,4,7,0),3,3)
> A

     [,1] [,2] [,3]
[1,]    2    1    4
[2,]    3    2    7
[3,]   -2    2    0

> matA <- qr(A)
> matA$rank

[1] 3

> A <- matrix(c(2,3,-2,1,2,2,4,6,-4),3,3)
> A

     [,1] [,2] [,3]
[1,]    2    1    4
[2,]    3    2    6
[3,]   -2    2   -4

> matA <- qr(A)
> matA$rank

[1] 2

# note column 3 is 2 times column 1
```

Number of Rows & Columns

```
> X <- matrix(c(3,2,4,3,2,-2,6,1),4,2)
> X

     [,1] [,2]
[1,]    3    2
[2,]    2   -2
[3,]    4    6
[4,]    3    1

> dim(X)

[1] 4 2

> r <- nrow(X)
> r

[1] 4

> c <- ncol(X)
> c

[1] 2
```

Computing Column & Row Sums

```
# note the uppercase S

> A <- matrix(c(2,3,-2,1,2,2),3,2)
> A

     [,1] [,2]
[1,]    2    1
[2,]    3    2
[3,]   -2    2

> c <- colSums(A)
> c

[1] 3 5

> r <- rowSums(A)
> r

[1] 3 5 0

> a <- sum(A)
> a

[1] 8
```

Computing Column & Row Means

```
# note the uppercase M

> cm <- colMeans(A)
> cm

[1] 1.000000 1.666667

> rm <- rowMeans(A)
> rm

[1] 1.5 2.5 0.0

> m <- mean(A)
> m

[1] 1.333333
```

Horizontal Concatenation

```
> A

     [,1] [,2]
[1,]    2    1
[2,]    3    2
[3,]   -2    2

> B <- matrix(c(1,3,2,1,4,2),3,2)
> B

     [,1] [,2]
[1,]    1    1
[2,]    3    4
[3,]    2    2

> C <- cbind(A,B)
> C

     [,1] [,2] [,3] [,4]
[1,]    2    1    1    1
[2,]    3    2    3    4
[3,]   -2    2    2    2
```

Vertical Concatenation (Appending)

```
> C <- rbind(A,B)
> C

     [,1] [,2]
[1,]    2    1
[2,]    3    2
[3,]   -2    2
[4,]    1    1
[5,]    3    4
[6,]    2    2
```

Multivariate Course Web Page

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