A Bit of Progress in Language Modeling
Extended Version

Joshua T. Goodman
Machine Learning and Applied Statistics Group
Microsoft Research
One Microsoft Way
Redmond, WA 98052
joshuago@microsoft.com

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Microsoft Research
Microsoft Corporation
One Microsoft Way
Redmond, WA 98052
http://www.research.microsoft.com

1 Introduction

1.1 Overview

Language modeling is the art of determining the probability of a sequence of words. This is useful in a large variety of areas including speech recognition, optical character recognition, handwriting recognition, machine translation, and spelling correction (Church, 1988; Brown et al., 1990; Hull, 1992; Kernighan et al., 1990; Srihari and Baltus, 1992). The most commonly used language models are very simple (e.g. a Katz-smoothed trigram model). There are many improvements over this simple model however, including caching, clustering, higher-order n-grams, skipping models, and sentence-mixture models, all of which we will describe below. Unfortunately, these more complicated techniques have rarely been examined in combination. It is entirely possible that two techniques that work well separately will not work well together, and, as we will show, even possible that some techniques will work better together than either one does by itself. In this paper, we will first examine each of the aforementioned techniques separately, looking at variations on the technique, or its limits. Then we will examine the techniques in various combinations, and compare to a Katz smoothed trigram with no count cutoffs. On a small training data set, 100,000 words, we can get up to a 50% perplexity reduction, which is one bit of entropy. On larger data sets, the improvement declines, going down to 41% on our largest data set, 284,000,000 words. On a similar large set without punctuation, the reduction is 38%. On that data set, we achieve an 8.9% word error rate reduction. These are perhaps the largest reported perplexity reductions for a language model, versus a fair baseline.

The paper is organized as follows. First, in this section, we will describe our terminology, briefly introduce the various techniques we examined, and describe our evaluation methodology. In the following sections, we describe each technique in more detail, and give experimental results with variations on the technique, determining for each the best variation, or its limits. In particular, for caching, we show that trigram caches have nearly twice the potential of unigram caches. For clustering, we find variations that work slightly better than traditional clustering, and examine the limits. For n-gram models, we examine up to 20-grams, but show that even for the largest models, performance has plateaued by 5 to 7 grams. For skipping models, we give the first detailed comparison of different skipping techniques, and the first that we know of at the 5-gram level. For sentence mixture models, we show that mixtures of up to 64 sentence types can lead to improvements. We then give experiments comparing all techniques, and combining all techniques in various ways. All of our experiments are done on three or four data sizes, showing which techniques improve with more data, and which get worse. In the concluding section, we discuss our results. Finally, in the appendices, we give a proof helping to justify Kneser-Ney smoothing and we describe implementation tricks and details for handling large data sizes, for optimizing parameters, for clustering, and for smoothing.

1.2 Technique introductions

The goal of a language model is to determine the probability of a word sequence $w_1...w_n$, $P(w_1...w_n)$. This probability is typically broken down into its component probabilities:

$$P(w_1...w_i) = P(w_1) \times P(w_2|w_1) \times ... \times P(w_i|w_1...w_{i-1})$$

Since it may be difficult to compute a probability of the form $P(w_i|w_1...w_{i-1})$ for large i, we typically assume that the probability of a word depends on only the two previous words, the *trigram* assumption:

$$P(w_i|w_1...w_{i-1}) \approx P(w_i|w_{i-2}w_{i-1})$$

which has been shown to work well in practice. The trigram probabilities can then be estimated from their counts in a training corpus. We let $C(w_{i-2}w_{i-1}w_i)$ represent the number of occurrences of $w_{i-2}w_{i-1}w_i$ in our training corpus, and similarly for $C(w_{i-2}w_{i-1})$. Then, we can approximate:

$$P(w_i|w_{i-2}w_{i-1}) \approx \frac{C(w_{i-2}w_{i-1}w_i)}{C(w_{i-2}w_{i-1})}$$

Unfortunately, in general this approximation will be very noisy, because there are many three word sequences that never occur. Consider, for instance, the sequence *"party on Tuesday"*. What is $P(\textit{Tuesday}|\textit{party on})$? Our training corpus might not contain any instances of the phrase, so $C(\textit{party on Tuesday})$ would be 0, while there might still be 20 instances of the phrase *"party on"*. Thus, we would predict $P(\textit{Tuesday}|\textit{party on}) = 0$, clearly an underestimate. This kind of 0 probability can be very problematic in many applications of language models. For instance, in a speech recognizer, words assigned 0 probability cannot be recognized no matter how unambiguous the acoustics.

Smoothing techniques take some probability away from some occurrences. Imagine that we have in our training data a single example of the phrase *"party on Stan Chen's birthday"*.[1] Typically, when something occurs only one time, it is greatly overestimated. In particular,

$$P(\textit{Stan}|\textit{party on}) \ll \frac{1}{20} = \frac{C(\textit{party on Stan})}{C(\textit{party on})}$$

By taking some probability away from some words, such as *"Stan"* and redistributing it to other words, such as *"Tuesday"*, zero probabilities can be avoided. In a smoothed trigram model, the extra probability is typically distributed according to a smoothed bigram model, etc. While the most commonly used smoothing techniques, Katz smoothing (Katz, 1987) and Jelinek-Mercer smoothing (Jelinek and Mercer, 1980) (sometimes called deleted interpolation) work fine, even better smoothing techniques exist. In particular, we have previously shown (Chen and Goodman, 1999) that versions of Kneser-Ney smoothing

[1] Feb. 25. Stan will provide cake. You are all invited.

(Ney et al., 1994) outperform all other smoothing techniques. In the appendix, we give a proof partially explaining this optimality. In Kneser-Ney smoothing, the backoff distribution is modified: rather than a normal bigram distribution, a special distribution is used. Using Kneser-Ney smoothing instead of more traditional techniques is the first improvement we used.

The most obvious extension to trigram models is to simply move to *higher-order n-grams*, such as four-grams and five-grams. We will show that in fact, significant improvements can be gotten from moving to five-grams. Furthermore, in the past, we have shown that there is a significant interaction between smoothing and n-gram order (Chen and Goodman, 1999): higher-order n-grams work better with Kneser-Ney smoothing than with some other methods, especially Katz smoothing. We will also look at how much improvement can be gotten from higher order n-grams, examining up to 20-grams.

Another simple extension to n-gram models is *skipping* models (Rosenfeld, 1994; Huang et al., 1993; Ney et al., 1994), in which we condition on a different context than the previous two words. For instance, instead of computing $P(w_i|w_{i-2}w_{i-1})$, we could instead compute $P(w_i|w_{i-3}w_{i-2})$. This latter model is probably not as good, but can be combined with the standard model to yield some improvements.

Clustering (also called *classing*) models attempt to make use of the similarities between words. For instance, if we have seen occurrences of phrases like "*party on Monday*" and "*party on Wednesday*", then we might imagine that the word "*Tuesday*", being similar to both "*Monday*" and "*Wednesday*", is also likely to follow the phrase "*party on*." The majority of the previous research on word clustering has focused on how to get the best clusters. We have concentrated our research on the best way to *use* the clusters, and will report results showing some novel techniques that work a bit better than previous methods. We also show significant interactions between clustering and smoothing.

Caching models (Kuhn, 1988; Kuhn and De Mori, 1990; Kuhn and De Mori, 1992) make use of the observation that if you use a word, you are likely to use it again. They tend to be easy to implement and to lead to relatively large perplexity improvements, but relatively small word-error rate improvements. We show that by using a trigram cache, we can get almost twice the improvement as from a unigram cache.

Sentence Mixture models (Iyer and Ostendorf, 1999; Iyer et al., 1994) make use of the observation that there are many different sentence types, and that making models for each type of sentence may be better than using one global model. Traditionally, only 4 to 8 types of sentences are used, but we show that improvements can be gotten by going to 64 mixtures, or perhaps more.

1.3 Evaluation

In this section, we first describe and justify our use of perplexity or entropy as an evaluation technique. We then describe the data and experimental techniques used in the experiments in the following sections.

The most commonly used method for measuring language model performance is *perplexity*. A language model that assigned equal probability to 100 words would have perplexity 100. In general, the perplexity of a language model is equal to the geometric average of the inverse probability of the words measured on test data:

$$\sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1...w_{i-1})}}$$

Perplexity has many properties that make it attractive as a measure of language model performance; among others, the "true" model for any data source will have the lowest possible perplexity for that source. Thus, the lower the perplexity of our model, the closer it is, in some sense, to the true model. While several alternatives to perplexity have been shown to correlate better with speech recognition performance, they typically have free variables that need to be optimized for a particular speech recognizer; others are significantly more difficult to compute in our framework.

An alternative, but equivalent measure to perplexity is entropy, which is simply $\log_2$ of perplexity. Entropy has the nice property that it is the average number of bits per word that would be necessary to encode the test data using an optimal coder. For those familiar with information theory, what we actually measure is the cross entropy of the test data given the model. Since this is by far the most common type of entropy measured, we abuse the term by simply saying entropy, when what we really mean is a particular cross entropy.

We will use both entropy and perplexity measurements in this paper: entropy reductions have several nice properties, including being additive and graphing well, but perplexity reductions are more common in the literature.[2] The following table may be helpful. Notice that the relationship between entropy and perplexity reductions is roughly linear up through about .2 bits.

reduction									
entropy	.01	.1	.16	.2	.3	.4	.5	.75	1
perplexity	0.69%	6.7%	10%	13%	19%	24%	29%	41%	50%

All of our experiments were performed on the NAB (North American Business news) corpus (Stern, 1996). We performed most experiments at 4 different training data sizes: 100,000 words, 1,000,000 words, 10,000,000 words, and the whole corpus – except 1994 WSJ data – approximately 284,000,000 words. In all cases, we performed parameter optimization on a separate set of heldout data, and then performed testing on a set of test data. None of the three data sets overlapped. The heldout and test sets were always every fiftieth sentence from two non-overlapping sets of 1,000,000 words, taken from the 1994 section. In the appendix, we describe implementation tricks we used; these tricks made it possible to train very complex models on very large amounts of training data,

[2]Probably because it sounds much better to get a 20% perplexity reduction than to get a 0.32 bit entropy reduction.

but made it hard to test on large test sets. For this reason, we used only 20,000 words total for testing or heldout data. On the other hand, we did not simply want to use, say, a 20,000 word contiguous test or heldout set, since this would only constitute a few articles, and thus risk problems from too much homogeneity; thus we chose to use every 50'th sentence from non-overlapping 1,000,000 word sets. All of our experiments were done using the same 58,546 word vocabulary. End-of-sentence, end-of-paragraph, and end-of-article symbols were included in perplexity computations, but out-of-vocabulary words were not.

It would have been interesting to try our experiments on other corpora, as well as other data sizes. In our previous work (Chen and Goodman, 1999), we compared both across corpora and across data sizes. We found that different corpora were qualitatively similar, and that the most important differences were across training data sizes. We therefore decided to concentrate our experiments on different training data sizes, rather than on different corpora.

Our toolkit is unusual in that it allows all parameters to be jointly optimized. In particular, when combining many techniques, there are many interpolation and smoothing parameters that need to be optimized. We used Powell's algorithm (Press et al., 1988) over the heldout data to jointly optimize all of these parameters.

2 Smoothing

This section is a recap of Chen and Goodman (1999). If you have read that, skip this.

There are many different smoothing techniques that can be used, and the subject is a surprisingly subtle and complicated one. Those interested in smoothing should consult our previous work (Chen and Goodman, 1999), where detailed descriptions and detailed comparisons of almost all commonly used smoothing algorithms are done. We will limit our discussion here to four main techniques: simple interpolation, Katz smoothing, Backoff Kneser-Ney smoothing, and Interpolated Kneser-Ney smoothing. In this section, we describe those four techniques, and recap previous results, including the important result that Interpolated Kneser-Ney smoothing, or minor variations on it, outperforms all other smoothing techniques.

The simplest way to combine techniques in language modeling is to simply interpolate them together. For instance, if one has a trigram model, a bigram model, and a unigram model, one can use

$$\begin{aligned}&P_{\texttt{interpolate}}(w|w_{i-2}w_{i-1}) = \\ &\qquad \lambda P_{\texttt{trigram}}(w|w_{i-2}w_{i-1}) + (1-\lambda)[\mu P_{\texttt{bigram}}(w|w_{i-1}) + (1-\mu)P_{\texttt{unigram}}(w)]\end{aligned}$$

where λ and μ are constants such that $0 \leq \lambda, \mu \leq 1$. In practice, we also interpolate with the uniform distribution $P_{\texttt{uniform}}(w) = \frac{1}{\text{size of vocabulary}}$; this ensures that no word is assigned probability 0. Also, we need to deal with the case when, for instance, the trigram context $w_{i-2}w_{i-1}$ has never been seen, $C(w_{i-2}w_{i-1}) = 0$. In this case, we use an interpolated bigram model, etc. Given its simplicity, simple interpolation works surprisingly well, but other techniques, such as Katz smoothing, work even better.

Katz smoothing (Katz, 1987) is based on the Good-Turing formula (Good, 1953). Notice that if a particular word sequence (i.e. "*party on Stan*") occurs only once (out of perhaps a billion words) it is probably significantly overestimated – it probably just showed up by chance, and its true probability is much less than one one billionth. It turns out that the same thing is true to a lesser degree for sequences that occurred twice, and so on. Let n_r represent the number of n-grams that occur r times, i.e.

$$n_r = |\{w_{i-n+1}...w_i | C(w_{i-n+1}...w_i) = r\}|$$

Good proved that under some very weak assumptions that for any n-gram that occurs r times, we should *discount* it, pretending that it occurs $disc(r)$ times where

$$disc(r) = (r+1)\frac{n_{r+1}}{n_r}$$

($disc(r)$ is more typically written as r^*). In language modeling, the estimate $disc(r)$ will almost always be less than r. This will leave a certain amount of probability "left-over." In fact, letting N represent the total size of the training set, this left-over probability will be equal to $\frac{n_1}{N}$; this represents the amount of probability to be allocated for events that were never seen. This is really quite an amazing and generally useful fact, that we can predict how often we expect something to happen that has never happened before, by looking at the proportion of things that have occurred once.

For a given context, Katz smoothing uses one of two formulae. If the word sequence $w_{i-n+1}...w_i$ has been seen before, then Katz smoothing uses the discounted count of the sequence, divided by the the counts of the context $w_{i-n+1}...w_{i-1}$. On the other hand, if the sequence has never been seen before, then we back off to the next lower distribution, $w_{i-n+2}...w_{i-1}$. Basically, we use the following formula:

$$P_{\texttt{Katz}}(w_i | w_{i-n+1}...w_{i-1})$$
$$= \begin{cases} \frac{disc(C(w_{i-n+1}...w_i))}{C(w_{i-n+1}...w_{i-1})} & \text{if } C(w_{i-n+1}...w_i) > 0 \\ \\ \alpha(w_{i-n+1}...w_{i-1}) \times P_{\texttt{Katz}}(w_i | w_{i-n+1}...w_{i-1}) & \text{otherwise} \end{cases}$$

where $\alpha(w_{i-n+1}...w_{i-1})$ is a normalization constant chosen so that the probabilities sum to 1.[3]

Katz smoothing is one of the most commonly used smoothing techniques, but it turns out that other techniques work even better. Chen and Goodman (1999) performed a detailed comparison of many smoothing techniques and found that a modified interpolated form of Kneser-Ney smoothing (Ney et al., 1994) consistently outperformed all other smoothing techniques. The basic insight behind

[3] Chen and Goodman (1999) as well as the appendix give the details of our implementation of Katz smoothing. Briefly, we also smooth the unigram distribution using additive smoothing; we discount counts only up to k, where we determine k to be as large as possible, while still giving reasonable discounts according to the Good-Turing formula; we add pseudo-counts β for any context with no discounted counts. Tricks are used to estimate n_r.

Kneser-Ney smoothing is the following. Consider a conventional bigram model of a phrase such as $P_{\texttt{Katz}}(\mathit{Francisco}|\mathit{on})$. Since the phrase *San Francisco* is fairly common, the conventional unigram probability (as used by Katz smoothing or techniques like deleted interpolation) $\frac{C(\mathit{Francisco})}{\sum_w C(w)}$ will also be fairly high. This means that using, for instance, a model such as Katz smoothing, the probability

$$\begin{aligned} P_{\texttt{Katz}}(\mathit{on\ Francisco}) &= \begin{cases} \frac{\mathit{disc}(C(\mathit{on\ Francisco}))}{C(\mathit{on})} & \text{if } C(\mathit{on\ Francisco}) > 0 \\ \alpha(\mathit{on}) \times P_{\texttt{Katz}}(\mathit{Francisco}) & \text{otherwise} \end{cases} \\ &= \alpha(\mathit{on}) \times P_{\texttt{Katz}}(\mathit{Francisco}) \end{aligned}$$

will also be fairly high. But, the word *Francisco* occurs in exceedingly few contexts, and its probability of occuring in a new one is very low. Kneser-Ney smoothing uses a modified backoff distribution based on the number of contexts each word occurs in, rather than the number of occurrences of the word. Thus, a probability such as $P_{\texttt{KN}}(\mathit{Francisco}|\mathit{on})$ would be fairly low, while for a word like *Tuesday* that occurs in many contexts, $P_{\texttt{KN}}(\mathit{Tuesday}|\mathit{on})$ would be relatively high, even if the phrase *on Tuesday* did not occur in the training data. Kneser-Ney smoothing also uses a simpler discounting scheme than Katz smoothing: rather than computing the discounts using Good-Turing, a single discount, D, (optimized on held-out data) is used. In particular, Backoff Kneser-Ney smoothing uses the following formula (given here for a bigram) where $|\{v|C(vw_i) > 0\}|$ is the number of words v that w_i can occur in the context of.

$$P_{\texttt{BKN}}(w_i|w_{i-1}) = \begin{cases} \frac{C(w_{i-1}w_i) - D}{C(w_{i-1})} & \text{if } C(w_{i-1}w_i) > 0 \\ \alpha(w_{i-1}) \frac{|\{v|C(vw_i)>0\}|}{\sum_w |\{v|C(vw)>0\}|} & \text{otherwise} \end{cases}$$

Again, α is a normalization constant such that the probabilities sum to 1. The formula can be easily extended to higher order n-grams in general. For instance, for trigrams, both the unigram and bigram distributions are modified.

Chen and Goodman (1999) showed that methods like Katz smoothing and Backoff Kneser-Ney smoothing that backoff to lower order distributions only when the higher order count is missing do not do well on low counts, such as one counts and two counts. This is because the estimates of these low counts are fairly poor, and the estimates ignore useful information in the lower order distribution. *Interpolated* models always combine both the higher-order and the lower order distribution, and typically work better. In particular, the basic formula for Interpolated Kneser-Ney smoothing is

$$P_{\texttt{IKN}}(w_i|w_{i-1}) = \frac{C(w_{i-1}w_i) - D}{C(w_{i-1})} + \lambda(w_{i-1}) \frac{|\{v|C(vw_i)>0\}|}{\sum_w |\{v|C(vw)>0\}|}$$

where $\lambda(w_{i-1})$ is a normalization constant such that the probabilities sum to 1. Chen and Goodman (1999) proposed one additional modification to Kneser-Ney smoothing, the use of multiple discounts, one for one counts, another for two counts, and another for three or more counts. This formulation, Modified Kneser-Ney smoothing, typically works slightly better than Interpolated

Kneser-Ney. However, in our experiments on combining techniques, it would have nearly tripled the number of parameters our system needed to search, and in a pilot study, when many techniques were combined, it did not work better than Interpolated Kneser-Ney. Thus, in the rest of this paper, we use Interpolated Kneser-Ney instead of Modified Kneser-Ney. In the appendix, we give a few more details about our implementation of our smoothing techniques, including standard refinements used for Katz smoothing. We also give arguments justifying Kneser-Ney smoothing, and example code, showing that interpolated Kneser-Ney smoothing is easy to implement.

For completeness, we show the exact formula used for an interpolated Kneser-Ney smoothed trigram. In practice, to avoid zero probabilities, we always smooth the unigram distribution with the uniform distribution, but have ommitted unigram smoothing from other formulas for simplicity; we include it here for completeness. Let $|V|$ represent the size of the vocabulary.

$$P_{\texttt{IKN}}(w_i|w_{i-2}w_{i-1}) = \frac{C(w_{i-2}w_{i-1}w_i)-D_3}{C(w_{i-2}w_{i-1})} + \lambda(w_{i-2}w_{i-1})P_{\texttt{ikn-mod-bigram}}(w_i|w_{i-1})$$

$$P_{\texttt{ikn-mod-bigram}}(w_i|w_{i-1}) = \frac{|\{v|C(vw_{i-1}w_i)>0\}|-D_2}{\sum_w |\{v|C(vw_{i-1}w)>0\}|} + \lambda(w_{i-1})P_{\texttt{ikn-mod-unigram}}(w_i)$$

$$P_{\texttt{ikn-mod-unigram}}(w_i|w_{i-1}) = \frac{|\{v|vw_i)>0\}|-D_1}{\sum_w |\{v|C(vw)>0\}|} + \lambda\frac{1}{|V|}$$

In Figure 1, we repeat results from Chen and Goodman (1999). These are the only results in this paper not run on exactly the same sections of the corpus for heldout, training, and test as the rest of the paper, but we expect them to be very comparable. The baseline used for these experiments was a simple version of Jelinek-Mercer smoothing, using a single bucket; that version is identical to the first smoothing technique we described, simple interpolation. Kneser-Ney smoothing is the interpolated version of Kneser-Ney smoothing used throughout this paper, and Kneser-Ney mod is the version with three discounts instead of a single discount. Katz smoothing is essentially the same as the version in this paper. j-m is short for Jelinek Mercer smoothing, sometimes called deleted interpolation elsewhere; abs-disc-interp is the interpolated version of absolute discounting. Training set size was measured in sentences, rather than in words, with about 20 words per sentence. Notice that Jelinek-Mercer smoothing and Katz smoothing cross, one being better at lower data sizes, the other at higher sizes. This was part of our motivation for running all experiments in this paper on multiple data sizes. On the other hand, in those experiments, which were done on multiple corpora, we did not find any techniques where one technique worked better on one corpus, and another worked better on another one. Thus, we feel reasonably confident in our decision not to run on multiple corpora. Chen and Goodman (1999) give a much more complete comparison of these techniques, as well as much more in depth analysis. Chen and Goodman (1998) gives a superset that also serves as a tutorial introduction.

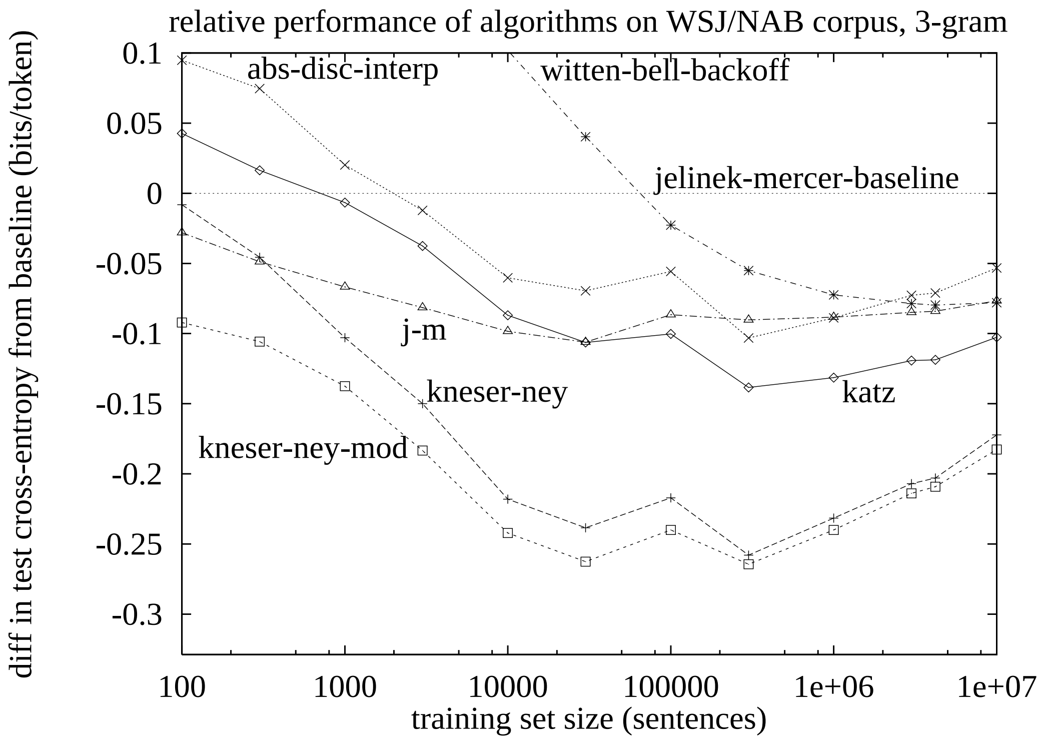

Figure 1: Smoothing results across data sizes

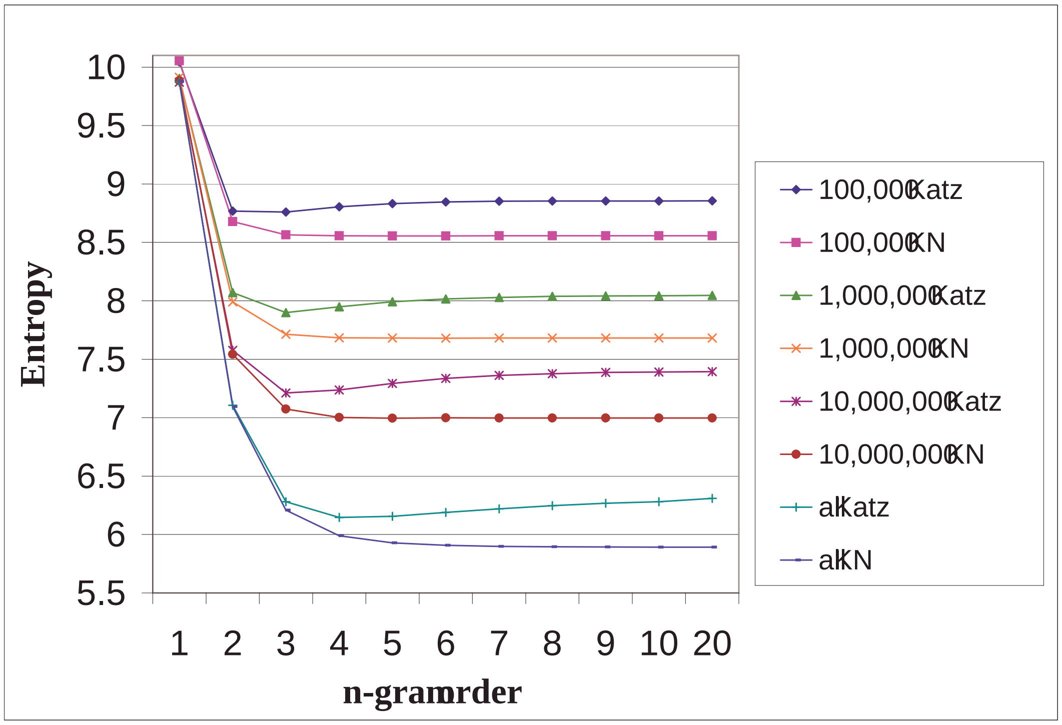

Figure 2: N-gram Order vs. Entropy

3 Higher-order n-grams

While the trigram assumption has proven, in practice, to be reasonable, there are many cases in which even longer contexts can be helpful. It is thus natural to relax the trigram assumption, and, rather than computing $P(w_i|w_{i-2}w_{i-1})$, use a longer context, such as $P(w_i|w_{i-4}w_{i-3}w_{i-2}w_{i-1})$, a five-gram model. In many cases, no sequence of the form $w_{i-4}w_{i-3}w_{i-2}w_{i-1}$ will have been seen in the training data, and the system will need to backoff to or interpolate with four-grams, trigrams, bigrams, or even unigrams, but in those cases where such a long sequence has been seen, it may be a good predictor of w_i.

Some earlier experiments with longer contexts showed little benefit from them. This turns out to be partially due to smoothing. As shown by Chen and Goodman (1999), some smoothing methods work significantly better with higher-order n-grams than others do. In particular, the advantage of Interpolated Kneser-Ney smoothing is much larger with higher-order n-grams than with lower-order ones.

We performed a variety of experiments on the relationship between n-gram order and perplexity. In particular, we tried both Katz smoothing and Interpolated Kneser-Ney smoothing on n-gram orders from 1 to 10, as well as 20, and over our standard data sizes. The results are shown in Figure 2.

As can be seen, and has been previously observed (Chen and Goodman, 1999), the behavior for Katz smoothing is very different than the behavior for Kneser-Ney smoothing. Chen and Goodman determined that the main cause of this difference was that backoff smoothing techniques, such as Katz smoothing, or even the backoff version of Kneser-Ney smoothing (we use only interpolated Kneser-Ney smoothing in this work), work poorly on low counts, especially one counts, and that as the n-gram order increases, the number of one counts increases. In particular, Katz smoothing has its best performance around the trigram level, and actually gets worse as this level is exceeded. Kneser-Ney smoothing on the other hand is essentially monotonic even through 20-grams.

The plateau point for Kneser-Ney smoothing depends on the amount of training data available. For small amounts, 100,000 words, the plateau point is at the trigram level, whereas when using the full training data, 280 million words, small improvements occur even into the 6-gram (.02 bits better than 5-gram) and 7-gram (.01 bits better than 6-gram.) Differences of this size are interesting, but not of practical importance. The difference between 4-grams and 5-grams, .06 bits, is perhaps important, and so, for the rest of our experiments, we often use models built on 5-gram data, which appears to give a good tradeoff between computational resources and performance.

Note that in practice, going beyond trigrams is often impractical. The tradeoff between memory and performance typically requires heavy pruning of 4-grams and 5-grams, reducing the potential improvement from them. Throughout this paper, we ignore memory-performance tradeoffs, since this would overly complicate already difficult comparisons. We seek instead to build the single best system possible, ignoring memory issues, and leaving the more practical, more interesting, and very much more complicated issue of finding the best system at a given memory size, for future research (and a bit of past research, too (Goodman and Gao, 2000)). Note that many of the experiments done in this section could not be done at all without the special tool described briefly at the end of this paper, and in more detail in the appendix.

4 Skipping

As one moves to larger and larger n-grams, there is less and less chance of having seen the exact context before; but the chance of having seen a similar context, one with most of the words in it, increases. Skipping models (Rosenfeld, 1994; Huang et al., 1993; Ney et al., 1994; Martin et al., 1999; Siu and Ostendorf, 2000) make use of this observation. There are also variations on this technique, such as techniques using lattices (Saul and Pereira, 1997; Dupont and Rosenfeld, 1997), or models combining classes and words (Blasig, 1999).

When considering a five-gram context, there are many subsets of the five-gram we could consider, such as $P(w_i|w_{i-4}w_{i-3}w_{i-1})$ or $P(w_i|w_{i-4}w_{i-2}w_{i-1})$. Perhaps we have never seen the phrase "*Show John a good time*" but we have seen the phrase "*Show Stan a good time.*" A normal 5-gram predicting $P(\textit{time}|\textit{show John a good})$ would back off to $P(\textit{time}|\textit{John a good})$ and from there to $P(\textit{time}|\textit{a good})$, which would have a relatively low probability. On the other hand, a skipping model of the form $P(w_i|w_{i-4}w_{i-2}w_{i-1})$ would assign high probability to $P(\textit{time}|\textit{show} \_\_\_ \textit{a good})$.

These skipping 5-grams are then interpolated with a normal 5-gram, forming models such as

$$\lambda P(w_i|w_{i-4}w_{i-3}w_{i-2}w_{i-1}) + \mu P(w_i|w_{i-4}w_{i-3}w_{i-1}) + (1-\lambda-\mu)P(w_i|w_{i-4}w_{i-2}w_{i-1})$$

where, as usual, $0 \leq \lambda \leq 1$ and $0 \leq \mu \leq 1$ and $0 \leq (1-\lambda-\mu) \leq 1$.

Another (and more traditional) use for skipping is as a sort of poor man's higher order n-gram. One can, for instance, create a model of the form

$$\lambda P(w_i|w_{i-2}w_{i-1}) + \mu P(w_i|w_{i-3}w_{i-1}) + (1-\lambda-\mu)P(w_i|w_{i-3}w_{i-2})$$

In a model of this form, no component probability depends on more than two previous words, like a trigram, but the overall probability is 4-gram-like, since it depends on w_{i-3}, w_{i-2}, and w_{i-1}. We can extend this idea even further, combining in all pairs of contexts in a 5-gram-like, 6-gram-like, or even 7-gram-like way, with each component probability never depending on more than the previous two words.

We performed two sets of experiments, one on 5-grams and one on trigrams. For the 5-gram skipping experiments, all contexts depended on at most the previous four words, w_{i-4}, w_{i-3}, w_{i-2}, w_{i-1}, but used the four words in a variety of ways. We tried six models, all of which were interpolated with a baseline 5-gram model. For readability and conciseness, we define a new notation, letting $v = w_{i-4}$, $w = w_{i-3}$, $x = w_{i-2}$ and $y = w_{i-1}$, allowing us to avoid numerous subscripts in what follows. The results are shown in Figure 3.

The first model interpolated dependencies on vw_y and v_xy. This simple model does not work well on the smallest training data sizes, but is competitive for larger ones. Next, we tried a simple variation on this model, which also interpolated in $vwx\_$. Making that simple addition leads to a good-sized improvement at all levels, roughly .02 to .04 bits over the simpler skipping model. Our next variation was analogous, but adding back in the dependencies on the missing words. In particular, we interpolated together $xvwy$, $wvxy$, and $yvwx$; that is, all models depended on the same variables, but with the interpolation order modified. For instance, by $xvwy$, we refer to a model of the form $P(z|vwxy)$ interpolated with $P(z|vw\_y)$ interpolated with $P(z|w\_y)$ interpolated with $P(z|y)$ interpolated with $P(z|y)$ interpolated with $P(z)$. All of these experiments were done with Interpolated Kneser-Ney smoothing, so all but the first probability uses the modified backoff distribution. This model is just like the previous one, but for each component starts the interpolation with the full 5-gram. We had hoped that in the case where the full 5-gram had occurred in the training data, this would make the skipping model more accurate, but it did not help at all.[4]

[4]In fact, it hurt a tiny bit, 0.005 bits at the 10,000,000 word training level. This turned

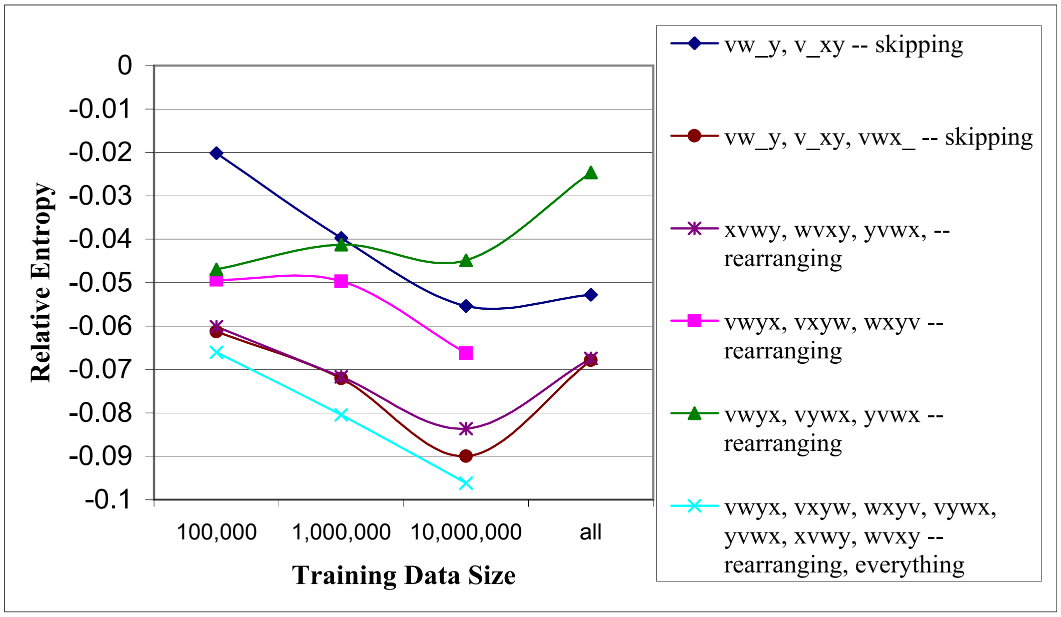

Figure 3: 5-gram Skipping Techniques versus 5-gram Baseline

We also wanted to try more radical approaches. For instance, we tried interpolating together $vwyx$ with $vxyw$ and $wxyv$ (along with the baseline $vwxy$). This model puts each of the four preceding words in the last (most important) position for one component. This model does not work as well as the previous two, leading us to conclude that the y word is by far the most important. We also tried a model with $vwyx$, $vywx$, $yvwx$, which puts the y word in each possible position in the backoff model. This was overall the worst model, reconfirming the intuition that the y word is critical. However, as we saw by adding $vwx\_$ to vw_y and v_xy, having a component with the x position final is also important. This will also be the case for trigrams.

Finally, we wanted to get a sort of upper bound on how well 5-gram models could work. For this, we interpolated together $vwyx$, $vxyw$, $wxyv$, $vywx$, $yvwx$, $xvwy$ and $wvxy$. This model was chosen as one that would include as many pairs and triples of combinations of words as possible. The result is a marginal gain – less than 0.01 bits – over the best previous model.

We do not find these results particularly encouraging. In particular, when compared to the sentence mixture results that will be presented later, there seems to be less potential to be gained from skipping models. Also, while sentence mixture models appear to lead to larger gains the more data that is used, skipping models appear to get their maximal gain around 10,000,000 words. Presumably, at the largest data sizes, the 5-gram model is becoming well trained, and there are fewer instances where a skipping model is useful but the 5-gram is not.

We also examined trigram-like models. These results are shown in Figure 4. The baseline for comparison was a trigram model. For comparison, we also show the relative improvement of a 5-gram model over the trigram, and the relative improvement of the skipping 5-gram with vw_y, v_xy and $vwx\_$. For the trigram skipping models, each component never depended on more than two of the previous words. We tried 5 experiments of this form. First, based on the intuition that pairs using the 1-back word (y) are most useful, we interpolated xy, wy, vy, uy and ty models. This did not work particularly well, except at the largest sizes. Presumably at those sizes, a few appropriate instances of the 1-back word had always been seen. Next, we tried using all pairs of words through the 4-gram level: xy, wy and wx. Considering its simplicity, this worked very well. We tried similar models using all 5-gram pairs, all 6-gram pairs and all 7-gram pairs; this last model contained 15 different pairs. However, the improvement over 4-gram pairs was still marginal, especially considering the large number of increased parameters.

The trigram skipping results are, relative to their baseline, much better than the 5-gram skipping results. They do not appear to have plateaued when more data is used and they are much more comparable to sentence mixture models in terms of the improvement they get. Furthermore, they lead to more

out to be due to technical smoothing issues. In particular, after some experimentation, this turned out to be due to our use of Interpolated Kneser-Ney smoothing with a single discount, even though we know that using multiple discounts is better. When using multiple discounts, the problem goes away.

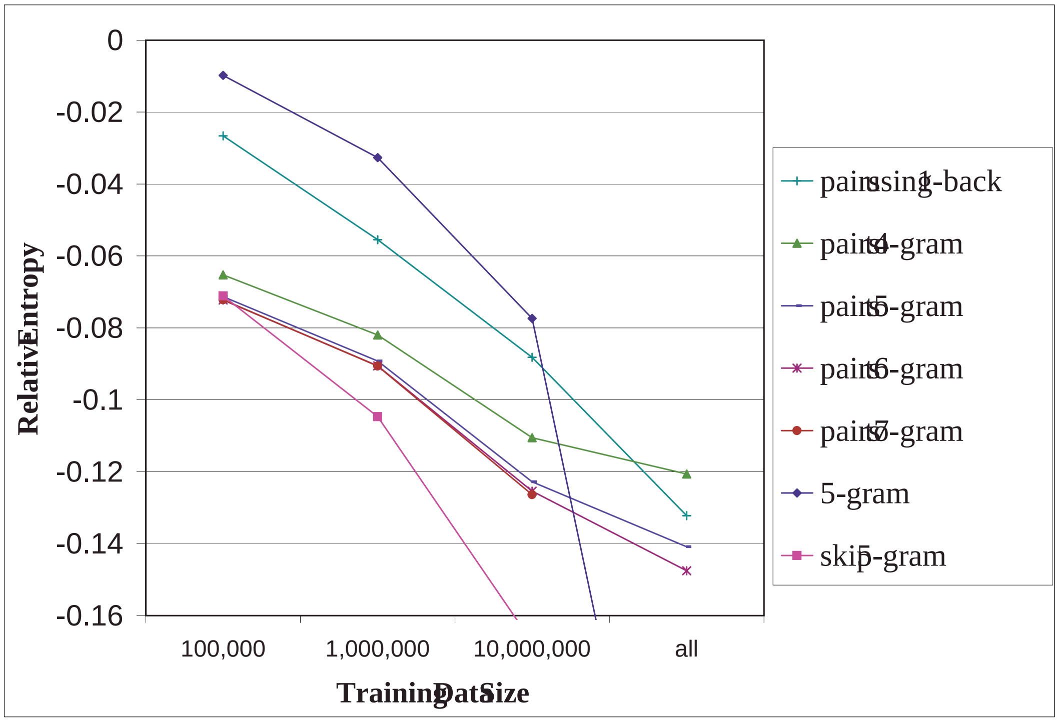

Figure 4: Trigram Skipping Techniques versus Trigram Baseline

improvement than a 5-gram alone does when used on small amounts of data (although, of course, the best 5-gram skipping model is always better than the best trigram skipping model.) This makes them a reasonable technique to use with small and intermediate amounts of training data, especially if 5-grams cannot be used.

5 Clustering

5.1 Using Clusters

Next, we describe our clustering techniques, which are a bit different (and, as we will show, slightly more effective) than traditional clustering (Brown et al., 1992; Ney et al., 1994). Consider a probability such as $P(\textit{Tuesday}|\textit{party on})$. Perhaps the training data contains no instances of the phrase *"party on Tuesday"*, although other phrases such as *"party on Wednesday"* and *"party on Friday"* do appear. We can put words into classes, such as the word *"Tuesday"* into the class *WEEKDAY*. Now, we can consider the probability of the word *"Tuesday"* given the phrase *"party on"*, and also given that the next word is a *WEEKDAY*. We will denote this probability by $P(\textit{Tuesday}|\textit{party on WEEKDAY})$. We can then decompose the probability

$$\begin{aligned}&P(\textit{Tuesday}|\textit{party on})\\ &\quad= P(\textit{WEEKDAY}|\textit{party on}) \times P(\textit{Tuesday}|\textit{party on WEEKDAY})\end{aligned}$$

When each word belongs to only one class, which is called hard clustering, this decompostion is a strict equality a fact that can be trivially proven. Let W_i represent the cluster of word w_i. Then,

$$\begin{aligned}P(W_i|w_{i-2}w_{i-1}) \times P(w_i|w_{i-2}w_{i-1}W_i) &= \frac{P(w_{i-2}w_{i-1}W_i)}{P(w_{i-2}w_{i-1})} \times \frac{P(w_{i-2}w_{i-1}W_iw_i)}{P(w_{i-2}w_{i-1}W_i)}\\ &= \frac{P(w_{i-2}w_{i-1}W_iw_i)}{P(w_{i-2}w_{i-1})} \qquad (1)\end{aligned}$$

Now, since each word belongs to a single cluster, $P(W_i|w_i) = 1$, and thus

$$\begin{aligned}P(w_{i-2}w_{i-1}W_iw_i) &= P(w_{i-2}w_{i-1}w_i) \times P(W_i|w_{i-2}w_{i-1}w_i)\\ &= P(w_{i-2}w_{i-1}w_i) \times P(W_i|w_i)\\ &= P(w_{i-2}w_{i-1}w_i) \qquad (2)\end{aligned}$$

Substituting Equation 2 into Equation 1, we get

$$P(W_i|w_{i-2}w_{i-1}) \times P(w_i|w_{i-2}w_{i-1}W_i) = \frac{P(w_{i-2}w_{i-1}w_i)}{P(w_{i-2}w_{i-1})} = P(w_i|w_{i-2}w_{i-1}) \qquad (3)$$

Now, although Equation 3 is a strict equality, when smoothing is taken into consideration, using the clustered probability will be more accurate than the non-clustered probability. For instance, even if we have never seen an example of *"party on Tuesday"*, perhaps we have seen examples of other phrases, such as *"party on Wednesday"* and thus, the probability $P(\mathit{WEEKDAY}|\mathit{party\ on})$ will be relatively high. And although we may never have seen an example of *"party on WEEKDAY Tuesday"*, after we backoff or interpolate with a lower order model, we may be able to accurately estimate $P(\mathit{Tuesday}|\mathit{on\ WEEKDAY})$. Thus, our smoothed clustered estimate may be a good one. We call this particular kind of clustering *predictive clustering.* (On the other hand, we will show that if the clusters are poor, predictive clustering can also lead to degradation.)

Note that predictive clustering has other uses as well as for improving perplexity. Predictive clustering can be used to significantly speed up maximum entropy training (Goodman, 2001), by up to a factor of 35, as well as to compress language models (Goodman and Gao, 2000).

Another type of clustering we can do is to cluster the words in the contexts. For instance, if *"party"* is in the class *EVENT* and *"on"* is in the class *PREPOSITION*, then we could write

$$P(\mathit{Tuesday}|\mathit{party\ on}) \approx P(\mathit{Tuesday}|\mathit{EVENT\ PREPOSITION})$$

or more generally

$$P(w|w_{i-2}w_{i-1}) \approx P(w|W_{i-2}W_{i-1}) \tag{4}$$

Combining Equation 4 with Equation 3 we get

$$P(w|w_{i-2}w_{i-1}) \approx P(W|W_{i-2}W_{i-1}) \times P(w|W_{i-2}W_{i-1}W) \tag{5}$$

Since Equation 5 does not take into account the exact values of the previous words, we always (in this work) interpolate it with a normal trigram model. We call the interpolation of Equation 5 with a trigram *fullibm* clustering. We call it fullibm because it is a generalization of a technique invented at IBM (Brown et al., 1992), which uses the approximation $P(w|W_{i-2}W_{i-1}W) \approx P(w|W)$ to get

$$P(w|w_{i-2}w_{i-1}) \approx P(W|W_{i-2}W_{i-1}) \times P(w|W) \tag{6}$$

which, when interpolated with a normal trigram, we refer to as *ibm* clustering. Given that fullibm clustering uses more information than regular ibm clustering, we assumed that it would lead to improvements.[5] As will be shown, it works about the same, at least when interpolated with a normal trigram model.

Alternatively, rather than always discarding information, we could simply change the backoff order, called *index* clustering:

$$P_{\mathtt{index}}(\mathit{Tuesday}|\mathit{party\ on}) = P(\mathit{Tuesday}|\mathit{party\ EVENT\ on\ PREPOSITION}) \tag{7}$$

Here, we abuse notation slightly to use the order of the words on the right side of the | to indicate the backoff/interpolation order. Thus, Equation 7

[5] In fact, we originally used the name goodibm instead of fullibm on the assumption that it *must* be better.

implies that we would go from $P(\textit{Tuesday}|\textit{party EVENT on PREPOSITION})$ to $P(\textit{Tuesday}|\textit{EVENT on PREPOSITION})$ to $P(\textit{Tuesday}|\textit{on PREPOSITION})$ to $P(\textit{Tuesday}|\textit{PREPOSITION})$ to $P(\textit{Tuesday})$. Notice that since each word belongs to a single cluster, some of these variables are redundant. For instance, in our notation

$$C(\textit{party EVENT on PREPOSITION}) = C(\textit{party on})$$

and

$$C(\textit{EVENT on PREPOSITION}) = C(\textit{EVENT on})$$

We generally write an index clustered model as $P(w_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1})$.

There is one especially noteworthy technique, *fullibmpredict*. This is the best performing technique we have found (other than combination techniques.) This technique makes use of the intuition behind predictive clustering, factoring the problem into prediction of the cluster, followed by prediction of the word given the cluster. In addition, at each level, it smooths this prediction by combining a word-based and a cluster-based estimate. It is not interpolated with a normal trigram model. It is of the form

$$\begin{aligned} P_{\texttt{fullibmpredict}}(w|w_{i-2}w_{i-1}) = \; & (\lambda P(W|w_{i-2}w_{i-1}) + (1-\lambda)P(W|W_{i-2}W_{i-1}))\times \\ & (\mu P(w|w_{i-2}w_{i-1}W) + (1-\mu)P(w|W_{i-2}W_{i-1}W) \end{aligned}$$

There are many variations on these themes. As it happens, none of the others works much better than ibm clustering, so we describe them only very briefly. One is *indexpredict*, combining index and predictive clustering:

$$\begin{aligned} & P_{\texttt{indexpredict}}(w_i|w_{i-2}w_{i-1}) = \\ & \quad P(W_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1}) \times P(w_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1}W_i) \end{aligned}$$

Another is *combinepredict*, interpolating a normal trigram with a predictive clustered trigram:

$$\begin{aligned} & P_{\texttt{combinepredict}}(w_i|w_{i-2}w_{i-1}) = \\ & \quad \lambda P(w_i|w_{i-1}w_{i-2}) + (1-\lambda)P(W_i|w_{i-2}w_{i-1}) \times P(w_i|w_{i-2}w_{i-1}W_i) \end{aligned}$$

Finally, we wanted to get some sort of upper bound on how much could be gained by clustering, so we tried combining all these clustering techniques together, to get what we call *allcombinenotop*, which is an interpolation of a normal trigram, a fullibm-like model, an index model, a predictive model, a true fullibm model, and an indexpredict model.

Begin boring details

$$\begin{aligned} & P_{\texttt{allcombinenotop}}(w_i|w_{i-2}w_{i-1}) = \\ & \quad \lambda P(w_i|w_{i-2}w_{i-1}) \\ & \quad +\mu P(w_i|W_{i-2}W_{i-1}) \\ & \quad +\nu P(w_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1}) \\ & \quad +\alpha P(W_i|w_{i-2}w_{i-1}) \times P(w_i|w_{i-2}w_{i-1}W_i) \\ & \quad +\beta P(W_i|W_{i-2}W_{i-1}) \times P(w_i|W_{i-1}W_{i-2}W_i) \\ & \quad +(1-\lambda-\nu-\alpha-\beta)P(W_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1}) \times P(w_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1}W_i) \end{aligned}$$

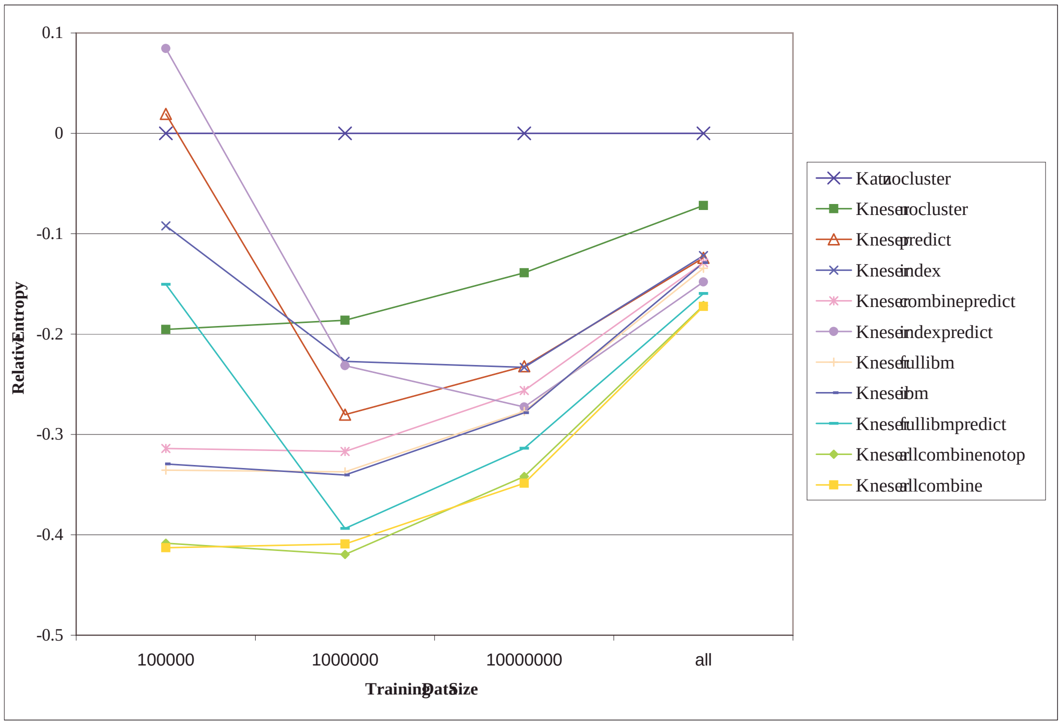

Figure 5: Comparison of nine different clustering techniques, Kneser-Ney smoothing

or, for a different way of combining, *allcombine*, which interpolates the predict-type models first at the cluster level, before inteprolating with the word level models.

$$
\begin{aligned}
&P_{\texttt{allcombine}}(w_i|w_{i-2}w_{i-1}) = \\
&\quad \lambda P(w_i|w_{i-2}w_{i-1}) \\
&\quad +\mu P(w_i|W_{i-2}W_{i-1}) \\
&\quad +\nu P(w_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1}) \\
&\quad + \ (1-\lambda-\mu-\nu) \\
&\qquad [\times \alpha P(W_i|w_{i-2}w_{i-1}) + \beta P(W_i|W_{i-2}W_{i-1}) + (1-\alpha-\beta)P(W_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1})] \\
&\qquad \times [\gamma P(w_i|w_{i-2}w_{i-1}W_i) + \rho P(w_i|W_{i-2}W_{i-1}W_i) + (1-\gamma-\rho)P(w_i|w_{i-2}W_{i-2}w_{i-1}W_{i-1}W_i)]
\end{aligned}
$$

End boring details

In Figure 5, we show a comparison of nine different clustering techniques, all using Kneser-Ney smoothing. The clusters were built separately for each training size. To keep these comparable to our next chart, we use Katz smoothing as the baseline for the relative entropy comparisons. Notice that the value

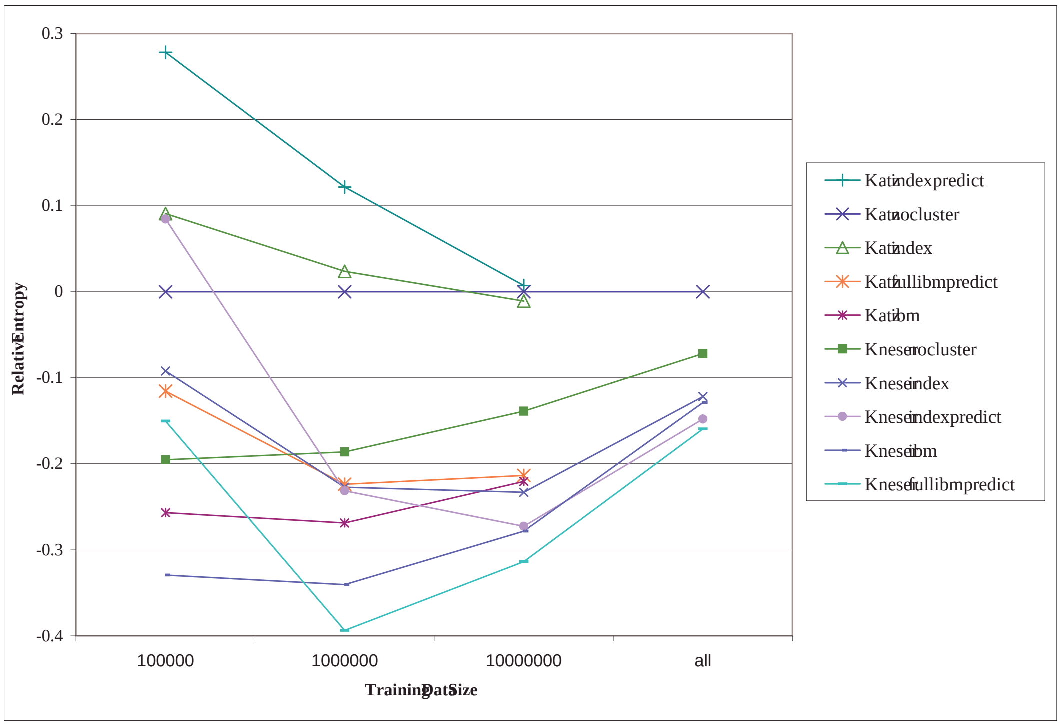

Figure 6: Comparison of Kneser-Ney smoothed Clustering to Katz smoothed

of clustering decreases with training data; at small data sizes, it is about 0.2 bits for the best clustered model; at the largest sizes, it is only about 0.1 bits. Since clustering is a technique for dealing with data sparseness, this is unsurprising. Next, notice that ibm clustering consistently works very well. Of all the other techniques we tried, only 4 others worked as well or better: fullibm clustering, which is a simple variation; allcombine and allcombinenotop, which interpolate in a fullibm; and fullibmpredict. Fullibmpredict works very well – as much as 0.05 bits better than ibm clustering. However, it has a problem at the smallest training size, in which case it is worse. We believe that the clusters at the smallest training size are very poor, and that predictive style clustering gets into trouble when this happens, since it smooths across words that may be unrelated, while ibm clustering interpolates in a normal trigram model, making it more robust to poor clusters. All of the models that use predict clustering and do not interpolate an unclustered trigram are actually worse than the baseline at the smallest training size.

Note that our particular experiments, which use a fixed vocabulary, are a severe test of clustering at the smallest levels. Many of the words in the 58,000 word vocabulary do not occur at all in 100,000 words of training data. We attempted to partially deal with this by adding a "pseudo-count" to every word, a co-occurrence with a fake word. This would have the property of making all unseen words, and words with very low counts, similar, hopefully putting them in the same cluster. The data with 100,000 words of training should be interpreted more as how well a system will work with bad clusters, and less about how a realistic system, in which the vocabulary would match the training data, would work.

In Figure 6 we show a comparison of several techniques using Katz smoothing and the same techniques with Kneser-Ney smoothing. The results are similar, with some interesting exceptions: in particular, indexpredict works well for the Kneser-Ney smoothed model, but very poorly for the Katz smoothed model. This shows that smoothing can have a significant effect on other techniques, such as clustering. The other result is that across all nine clustering techniques, at every size, the Kneser-Ney version always outperforms the Katz smoothed version. In fact, the Kneser-Ney smoothed version also outperformed both interpolated and backoff absolute discounting versions of each technique at every size.

There are two other ways to perform clustering, which we will not explore here. First, one can cluster groups of words – complete contexts – instead of individual words. That is, to change notation for a moment, instead of computing

$$P(w|\textit{word-cluster}(w_{i-2})\textit{word-cluster}(w_{i-1}))$$

one could compute

$$P(w|\textit{context-cluster}(w_{i-2}w_{i-1}))$$

For instance, in a trigram model, one could cluster contexts like *"New York"* and *"Los Angeles"* as *"CITY"*, and *"on Wednesday"* and *"late tomorrow"* as *"TIME"*. There are many difficult issues to solve for this kind of clustering.

Another kind of conditional clustering one could do is to empirically determine, for a given context, the best combination of clusters and words to use, the *varigram* approach (Blasig, 1999).

5.2 Finding Clusters

A large amount of previous research has focused on how best to find the clusters (Brown et al., 1992; Kneser and Ney, 1993; Yamamoto and Sagisaka, 1999; Ueberla, 1995; Pereira et al., 1993; Bellegarda et al., 1996). Most previous research has found only small differences between different techniques for finding clusters. One result however is that automatically derived clusters outperform part-of-speech tags (Niesler et al., 1998), at least when there is enough training data (Ney et al., 1994). We did not explore different techniques for finding clusters, but simply picked one we thought would be good, based on previous research.

There is no need for the clusters used for different positions to be the same. In particular, for a model like ibm clustering, with $P(w_i|W_i) \times P(W_i|W_{i-2}W_{i-1})$, we will call the W_i cluster a *predictive* cluster, and the clusters for W_{i-1} and W_{i-2} *conditional* clusters. The predictive and conditional clusters can be different (Yamamoto and Sagisaka, 1999). For instance, consider a pair of words like *a* and *an*. In general, *a* and *an* can follow the same words, and so, for predictive clustering, belong in the same cluster. But, there are very few words that can follow both *a* and *an* – so for conditional clustering, they belong in different clusters. We have also found in pilot experiments that the optimal number of clusters used for predictive and conditional clustering are different; in this paper, we always optimize both the number of conditional and predictive clusters separately, and reoptimize for each technique at each training data size. This is a particularly time consuming experiment, since each time the number of clusters is changed, the models must be rebuilt from scratch. We always try numbers of clusters that are powers of 2, e.g. 1, 2, 4, etc, since this allows us to try a wide range of numbers of clusters, while never being more than a factor of 2 away from the optimal number. Examining charts of performance on heldout data, this seems to produce numbers of clusters that are close enough to optimal.

The clusters are found automatically using a tool that attempts to minimize perplexity. In particular, for the conditional clusters we try to minimize the perplexity of training data for a bigram of the form $P(w_i|W_{i-1})$, which is equivalent to maximizing

$$\prod_{i=1}^{N} P(w_i|W_{i-1})$$

For the predictive clusters, we try to minimize the perplexity of training data of $P(W_i|w_{i-1}) \times P(w_i|W_i)$. (We do not minimize $P(W_i|w_{i-1}) \times P(w_i|w_{i-1}W_i)$, because we are doing our minimization on unsmoothed training data, and the latter formula would thus be equal to $P(w_i|w_{i-1})$ for any clustering. If we were to use the method of leaving-one-out (Kneser and Ney, 1993), then we could use

Begin boring details

the latter formula, but that approach is more difficult.) Now,

$$\begin{aligned}
\prod_{i=1}^{N} P(W_i|w_{i-1}) \times P(w_i|W_i) &= \prod_{i=1}^{N} \frac{P(w_{i-1}W_i)}{P(w_{i-1})} \times \frac{P(W_i w_i)}{P(W_i)} \\
&= \prod_{i=1}^{N} \frac{P(W_i w_i)}{P(w_{i-1})} \times \frac{P(w_{i-1}W_i)}{P(W_i)} \\
&= \prod_{i=1}^{N} \frac{P(w_i)}{P(w_{i-1})} \times P(w_{i-1}|W_i)
\end{aligned}$$

Now, $\frac{P(w_i)}{P(w_{i-1})}$ will be independent of the clustering used; therefore, it is sufficient to try to maximize $\prod_{i=1}^{N} P(w_{i-1}|W_i)$.[6] This is very convenient, since it is exactly the opposite of what was done for conditional clustering. It means that we can use the same clustering tool for both, and simply switch the order used by the program used to get the raw counts for clustering. We give more details about the clustering algorithm used in section 9.

End boring details

6 Caching

If a speaker uses a word, it is likely that he will use the same word again in the near future. This observation is the basis of caching (Kuhn, 1988; Kuhn and De Mori, 1990; Kuhn and De Mori, 1992; Kupiec, 1989; Jelinek et al., 1991). In particular, in a unigram cache, we form a unigram model from the most recently spoken words (all those in the same article if article markers are available, or a fixed number of previous words if not.) This unigram cache can then be linearly interpolated with a conventional n-gram.

Another type of cache model depends on the context. For instance, we could form a smoothed bigram or trigram from the previous words, and interpolate this with the standard trigram. In particular, we use

$$\begin{aligned}
&P_{\texttt{trigram-cache}}(w|w_1...w_{i-2}w_{i-1}) = \\
&\quad \lambda P_{\texttt{Smooth}}(w|w_{i-2}w_{i-1}) + (1-\lambda)P_{\texttt{tricache}}(w|w_1...w_{i-1})
\end{aligned}$$

where $P_{\texttt{tricache}}(w|w_1...w_{i-1})$ is a simple interpolated trigram model, using counts from the preceding words in the same document.

Yet another technique is to use conditional caching. In this technique, we weight the trigram cache differently depending on whether or not we have previously seen the context or not.

Begin boring details

We digress here for a moment to mention a trick that we use. When interpolating three probabilities $P_1(w)$, $P_2(w)$, and $P_3(w)$, rather than use

$$\lambda P_1(w) + \mu P_2(w) + (1-\lambda-\mu)P_3(w)$$

[6]Thanks to Lillian Lee

we actually use

$$\frac{\lambda}{\lambda+\mu+\nu}P_1(w) + \frac{\mu}{\lambda+\mu+\nu}P_2(w) + \frac{\nu}{\lambda+\mu+\nu}P_3(w)$$

This allows us to simplify the constraints of the search, and we also believe aids our parameter search routine, by adding a useful dimension to search through. It is particularly useful when sometimes we do not use one of the three components. In particular, for conditional caching, we use the following formula:

$$P_{\texttt{conditionaltrigram}}(w|w_1...w_{i-2}w_{i-1}) = \\ \begin{cases} \begin{array}{l} \frac{\lambda}{\lambda+\mu+\nu}P_{\texttt{Smooth}}(w|w_{i-2}w_{i-1}) \\ +\frac{\mu}{\lambda+\mu+\nu}P_{\texttt{unicache}}(w|w_1...w_{i-1}) \\ +\frac{\nu}{\lambda+\mu+\nu}P_{\texttt{tricache}}(w|w_1...w_{i-1}) \end{array} & \text{if } w_{i-1} \text{ in cache} \\ \\ \begin{array}{l} \frac{\lambda}{\lambda+\mu}P_{\texttt{Smooth}}(w|w_{i-2}w_{i-1}) \\ +\frac{\mu}{\lambda+\mu}P_{\texttt{unicache}}(w|w_1...w_{i-1}) \end{array} & \text{otherwise} \end{cases}$$

We tried one additional improvement. We assume that the more data we have, the more useful each cache is. Thus, we make λ, μ and ν be linear functions of the amount of data in the cache (number of words so far in the current document.)

$$\lambda(\mathit{wordsincache}) = \lambda_{\mathit{startweight}} + \lambda_{\mathit{multiplier}} \times \frac{\min(\mathit{wordsincache}, \lambda_{\mathit{maxwordsweight}})}{\lambda_{\mathit{maxwordsweight}}}$$

where, as usual, $\lambda_{\mathit{startweight}}$, $\lambda_{\mathit{multiplier}}$ and $\lambda_{\mathit{maxwordsweight}}$ are parameters estimated on heldout data. However, our parameter search engine nearly always set $\lambda_{\mathit{maxwordsweight}}$ to at or near the maximum value we allowed it to have, 1,000,000, while assigning $\lambda_{\mathit{multiplier}}$ to a small value (typically 100 or less) meaning that the variable weighting was essentially ignored.

Finally, we can try conditionally combinging unigram, bigram, and trigram caches.

$$P_{\texttt{conditionaltrigram}}(w|w_1...w_{i-2}w_{i-1}) = \\ \begin{cases} \begin{array}{l} \frac{\lambda}{\lambda+\mu+\nu+\kappa}P_{\texttt{Smooth}}(w|w_{i-2}w_{i-1}) \\ +\frac{\mu}{\lambda+\mu+\nu+\kappa}P_{\texttt{unicache}}(w|w_1...w_{i-1}) \\ +\frac{\nu}{\lambda+\mu+\nu+\kappa}P_{\texttt{bicache}}(w|w_1...w_{i-1}) \\ +\frac{\kappa}{\lambda+\mu+\nu+\kappa}P_{\texttt{tricache}}(w|w_1...w_{i-1}) \end{array} & \text{if } w_{i-2}w_{i-1} \text{ in cache} \\ \\ \begin{array}{l} \frac{\lambda}{\lambda+\mu+\nu}P_{\texttt{Smooth}}(w|w_{i-2}w_{i-1}) \\ +\frac{\mu}{\lambda+\mu+\nu}P_{\texttt{unicache}}(w|w_1...w_{i-1}) \\ +\frac{\nu}{\lambda+\mu+\nu}P_{\texttt{bicache}}(w|w_1...w_{i-1}) \end{array} & \text{if } w_{i-1} \text{ in cache} \\ \\ \begin{array}{l} \frac{\lambda}{\lambda+\mu}P_{\texttt{Smooth}}(w|w_{i-2}w_{i-1}) \\ +\frac{\mu}{\lambda+\mu}P_{\texttt{unicache}}(w|w_1...w_{i-1}) \end{array} & \text{otherwise} \end{cases}$$

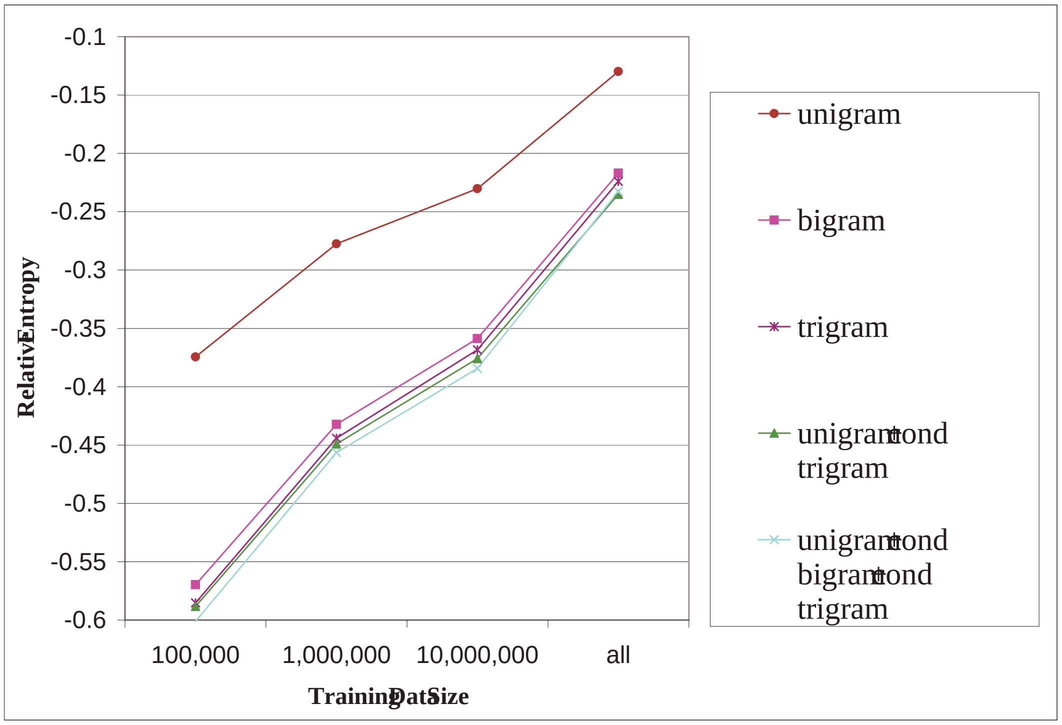

Figure 7: Five different cache models interpolated with trigram compared to trigram baseline

End boring details

Figure 7 gives results of running each of these five cache models. All were interpolated with a Kneser-Ney smoothed trigram. Each of the n-gram cache models was smoothed using simple interpolation, for technical reasons. As can be seen, caching is potentially one of the most powerful techniques we can apply, leading to performance improvements of up to 0.6 bits on small data. Even on large data, the improvement is still substantial, up to 0.23 bits. On all data sizes, the n-gram caches perform substantially better than the unigram cache, but which version of the n-gram cache is used appears to make only a small difference.

It should be noted that all of these results assume that the previous words are known exactly. In a speech recognition system however, many product scenarios do not include user correction. It is then possible for a cache to "lock-in" errors. For instance, if the user says "recognize speech" and the system hears "wreck a nice beach" then, later, when the user says "speech recognition" the system may hear "beach wreck ignition", since the probability of "beach" will be significantly raised. Thus, getting improvements from caching in a real product is potentially a much harder problem.

7 Sentence Mixture Models

Iyer and Ostendorf (1999; Iyer et al. (1994) observed that within a corpus, there may be several different sentence types; these sentence types could be grouped by topic, or style, or some other criterion. No matter how they are grouped, by modeling each sentence type separately, improved performance can be achieved. For instance, in Wall Street Journal data, we might assume that there are three different sentence types: financial market sentences (with a great deal of numbers and stock names), business sentences (promotions, demotions, mergers), and general news stories. We can compute the probability of a sentence once for each sentence type, then take a weighted sum of the probabilities across sentence types. Because long-distance correlations within a sentence (lots of numbers, or lots of promotions) are captured by such a model, the overall model is better. Of course, in general, we do not know the sentence type until we have heard the sentence. Therefore, instead, we treat the sentence type as a hidden variable.

Let s_j denote the condition that the sentence under consideration is a sentence of type j. Then the probability of the sentence, given that it is of type j can be written as

$$\prod_{i=1}^{N} P(w_i|w_{i-2}w_{i-1}s_j)$$

Sometimes, the global model (across all sentence types) will be better than any individual sentence type. Let s_0 be a special context that is always true:

$$P(w_i|w_{i-2}w_{i-1}s_0) = P(w_i|w_{i-2}w_{i-1})$$

Let there be S different sentence types ($4 \leq S \leq 8$ is typical); let $\sigma_0 ... \sigma_S$ be sentence interpolation parameters optimized on held-out data subject to the constraint $\sum_{j=0}^{S} \sigma_j = 1$. The overall probability of a sentence $w_1 ... w_n$ is equal to

$$\sum_{j=0}^{S} \sigma_j \prod_{i=1}^{N} P(w_i|w_{i-2}w_{i-1}s_j) \tag{8}$$

Equation 8 can be read as saying that there is a hidden variable, the sentence type; the prior probability for each sentence type is σ_j. We compute the probability of a test sentence once for each sentence type, and then sum these probabilities according to the prior probability of that sentence type.

The probabilities $P(w_i|w_{i-2}w_{i-1}s_j)$ may suffer from data sparsity, so they are linearly interpolated with the global model $P(w_i|w_{i-2}w_{i-1})$, using interpolation weights optimized on held-out data.

$$\sum_{j=0}^{S} \sigma_j \prod_{i=1}^{N} \lambda_j P(w_i|w_{i-2}w_{i-1}s_j) + (1 - \lambda_j) P(w_i|w_{i-2}w_{i-1})$$

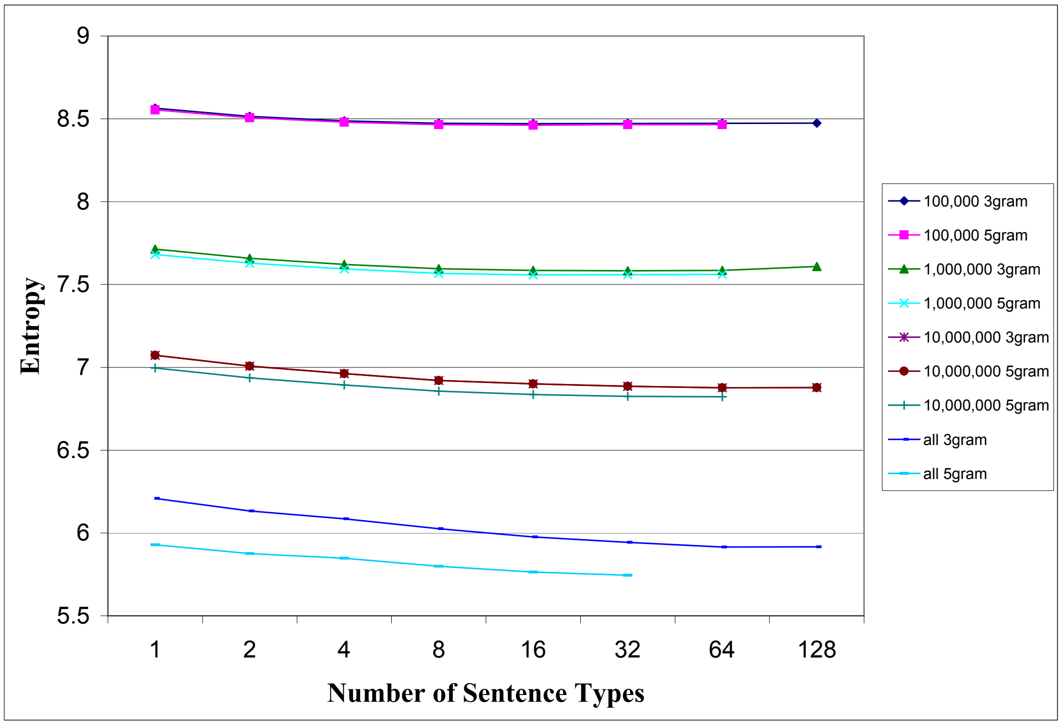

Figure 8: Number of sentence types versus entropy

Sentence types for the training data were found by using the same clustering program used for clustering words; in this case, we tried to minimize the sentence-cluster unigram perplexities. That is, let $s(i)$ represent the sentence type assigned to the sentence that word i is part of. (All words in a given sentence are assigned to the same sentence type.) We tried to put sentences into clusters in such a way that $\prod_{i=1}^{N} P(w_i|s(i))$ was maximized. This is a much simpler technique than that used by Iyer and Ostendorf (1999). They use a two stage process, the first stage of which is a unigram-similarity agglomerative clustering method; the second stage is an EM-based n-gram based reestimation. Also, their technique used soft clusters, in which each sentence could belong to multiple clusters. We assume that their technique results in better models than ours.

We performed a fairly large number of experiments on sentence mixture models. We sought to study the relationship between training data size, n-gram order, and number of sentence types. We therefore ran a number of experiments using both trigrams and 5-grams, at our standard data sizes, varying the number of sentence types from 1 (a normal model without sentence mixtures) to 128.

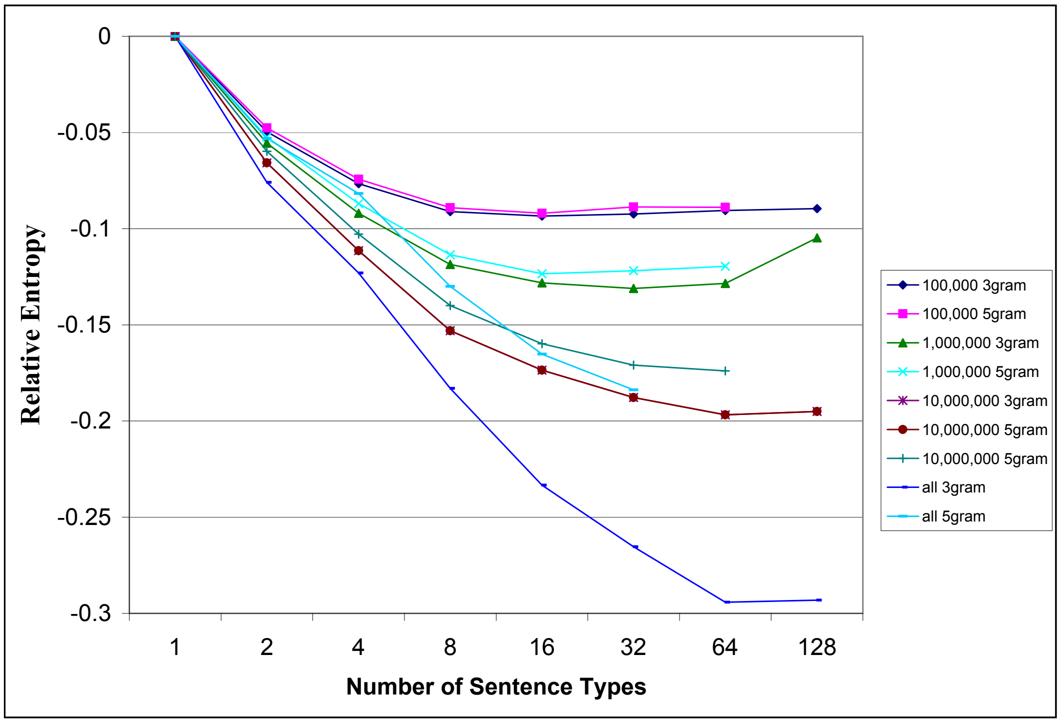

Figure 9: Number of sentence types versus entropy, relative to baseline

All experiments were done with Kneser-Ney smoothing. The results are shown in Figure 8. We give the same results again, but graphed relative to their respective n-gram model baselines, in Figure 9. Note that we do not trust results for 128 mixtures; for these experiments, we used the same 20,000 words of heldout data used in our other experiments. With 128 sentence types, there are 773 parameters, and the system may not have had enough heldout data to accurately estimate the parameters. In particular, the plateau shown at 128 for 10,000,000 and all training data does not show up in the heldout data. Ideally, we would run this experiment with a larger heldout set, but it already required 5.5 days with 20,000 words, so this is impractical.

The results are very interesting for a number of reasons. First, we suspected that sentence mixture models would be more useful on larger training data sizes, and indeed they are; with 100,000 words, the most improvement from a sentence mixture model is only about .1 bits, while with 284,000,000 words, it is nearly .3 bits. This bodes well for the future of sentence mixture models: as computers get faster and larger, training data sizes should also increase. Second, we had suspected that because both 5-grams and sentence mixture models attempt to model long distance dependencies, the improvement from their combination would be less than the sum of the individual improvements. As can be seen in Figure 8, for 100,000 and 1,000,000 words of training data, the difference between trigrams and 5-grams is very small anyway, so the question is not very important. For 10,000,000 words and all training data, there is some negative interaction. For instance, with 4 sentence types on all training data, the improvement is 0.12 bits for the trigram, and 0.08 bits for the 5-gram. Similarly, with 32 mixtures, the improvement is .27 on the trigram and .18 on the 5-gram. So, approximately one third of the improvement seems to be correlated.

Iyer and Ostendorf (1999) reported experiments on both 5-mixture components and 8 components and found no significant difference, using 38 million words of training data. However, our more thorough investigation shows that indeed there is substantial room for improvement by using larger numbers of mixtures, especially when using more training data, and that this potential extends at least to 64 sentence types on our largest size. This is an important result, leading to almost twice the potential improvement of using only a small number of components.

We think this new result is one of the most interesting in our research. In particular, the techniques we used here were relatively simple, and many extensions to these techniques might lead to even larger improvements. For instance, rather than simply smoothing a sentence type with the global model, one could create sentence types and supertypes, and then smooth together the sentence type with its supertype and with the global model, all combined. This would alleviate the data sparsity effects seen with the largest numbers of mixtures.

Begin boring details

Our sentence mixture model results are encouraging, but disappointing when compared to previous results. While Iyer and Ostendorf (1999) achieve about 19% perplexity reduction and about 3% word error rate reduction with 5 mixtures, on similar data we achieve only about 9% and (as we will show later) 1.3% reductions with 4 mixtures. The largest difference we are aware of between their system and ours is the difference in clustering technique: we used a fairly simple technique, and they used a fairly complex one. Other possibilities include the different smoothing that they used (Witten-Bell smoothing) versus our Katz or Interpolated Kneser-Ney smoothing; and the fact that they used five clusters while we used four. However, Katz and Interpolated Kneser-Ney are very different techniques, but, as we will report later, sentence mixture models produce about the same improvement with both, so we do not think the difference in smoothing explains the different results. Also, Iyer and Ostendorf (1999) found no significant difference when using 8 clusters instead of 5, and we found only a 4% difference when using 8 instead of 4 on similar data. It is noteworthy that Iyer and Ostendorf have a baseline perplexity of 211, versus our baseline perplexity of 95, with the same training set, but different vocabularies and test sets. Perhaps whatever unknown factor accounts for this difference in baseline perplexities gives more room for improvement from sentence mixture models. It is worth noting that our improvement from caching is also much less than Iyer and Ostendorf's: about 7.8% versus their 16.7%. Our cache implementations were very similar, the main difference being their exclusion of stop-words. This adds support to the "different baseline/test condition" explanation. If the difference in perplexity reduction is due to some difference in the mixture model implementation, rather than in the test conditions, then an additional 10% perplexity reduction could be achieved, an amount that merits additional exploration.

End boring details

Sentence mixture models can also be useful when combining very different language model types. For instance, Jurafsky et al. (1995) uses a sentence mixture model to combine a stochastic context-free grammar (SCFG) model with a bigram model, resulting in marginally better results than either model used separately. The model of Jurafsky et al. is actually of the form

$$\begin{aligned}
&P(w_i|w_1...w_{i-1}) = \\
&\quad P(\text{SCFG}|w_1...w_{i-1}) \times P(w_i|w_1...w_{i-1}, \text{SCFG}) \\
&\quad +P(\text{bigram}|w_1...w_{i-1}) \times P(w_i|w_1...w_{i-1}, \text{bigram})
\end{aligned}$$

which turns out to be equivalent to a model in the form of Equation 8. This version of the equations has the advantage that when used in a stack-decoder, it allows sentence mixture models to be used with relatively little overhead, compared to Equation 8 Charniak (2001), as discussed in Section 10.5, uses a sentence level mixture model to combine a linguistic model with a trigram model, achieving significant perplexity reduction.

8 Combining techniques

In this section, we present additional results on combining techniques. While each of the techniques we have presented works well separately, we will show that some of them work together synergistically, and that some of them are partially redundant. For instance, we have shown that the improvement from Kneser-Ney modeling and 5-gram models together is larger than the improvement from either one by itself. Similarly, as we have already shown, the improvement from sentence mixture models when combined with 5-grams is only about $\frac{2}{3}$ of the improvment of sentence mixture models by themselves, because both techniques increase data sparsity. In this section, we systematically study three issues: what effect does smoothing have on each technique; how much does each technique help when combined with all of the others; and how does each technique affect word error rate, separately and together.

There are many different ways to combine techniques. The most obvious way to combine techniques is to simply linearly interpolate them, but this is not likely to lead to the largest possible improvement. Instead, we try to combine concepts. To give a simple example, recall that a fullibmpredict clustered trigram is of the form:

$$\begin{array}{l}(\lambda P(W|w_{i-2}w_{i-1}) + (1-\lambda)P(W|W_{i-2}W_{i-1}))\times \\ (\mu P(w|w_{i-2}w_{i-1}W) + (1-\mu)P(w|W_{i-2}W_{i-1}W)\end{array}$$

One could simply interpolate this clustered trigram with a normal 5-gram, but of course it makes much more sense to combine the concept of a 5-gram with the concept of fullibmpredict, using a clustered 5-gram:

$$\begin{array}{l}(\lambda P(W|w_{i-4}w_{i-3}w_{i-2}w_{i-1}) + (1-\lambda)P(W|W_{i-4}W_{i-3}W_{i-2}W_{i-1}))\times \\ (\mu P(w|w_{i-4}w_{i-3}w_{i-2}w_{i-1}W) + (1-\mu)P(w|W_{i-4}W_{i-3}W_{i-2}W_{i-1}W)\end{array}$$

We will follow this idea of combining concepts, rather than simply interpolating throughout this section. This tends to result in good performance, but complex models.

Begin boring details

When we combine sentence mixture models with caches, we need to answer additional questions. Iyer and Ostendorf (1999) used separate caches for each sentence type, putting each word into the cache for the most likely sentence type. This would have required a great deal of additional work in our system; also, we did pilot experiments combining sentence mixture models and caches with only a single global cache, and found that the improvement from the combination was nearly equal to the sum of the individual improvements. Since Iyer and Ostendorf also get an improvement nearly equal to the sum, we concluded that not using separate caches was a reasonable combination technique.

End boring details

Our overall combination technique is somewhat complicated. At the highest level, we use a sentence mixture model, in which we sum over sentence-specific models for each sentence type. Within a particular sentence mixture model, we combine different techniques with predictive clustering. That is, we combine sentence-specific, global, cache, and global skipping models first to predict the cluster of the next word, and then again to predict the word itself given the cluster.

For each sentence type, we wish to linearly interpolate the sentence-specific 5-gram model with the global 5-gram model, the three skipping models, and the two cache models. Since we are using fullibmpredict clustering, we wish to do this based on both words and clusters. Let $\lambda_{1,j}...\lambda_{12,j}$, $\mu_{1,j}...\mu_{12,j}$ be interpolation parameters. Then, we define the following two very similar functions.

First,[7]

$$\begin{aligned}
&\mathit{sencluster}_j(W, w_{i-4}...w_{i-1}) = \\
&\quad \lambda_{1,j} P(W|w_{i-4}w_{i-3}w_{i-2}w_{i-1}s_j) + \lambda_{2,j} P(W|W_{i-4}W_{i-3}W_{i-2}W_{i-1}s_j) + \\
&\quad \lambda_{3,j} P(W|w_{i-4}w_{i-3}w_{i-2}w_{i-1}) + \lambda_{4,j} P(W|W_{i-4}W_{i-3}W_{i-2}W_{i-1}) + \\
&\quad \lambda_{5,j} P(W|w_{i-4}w_{i-3}w_{i-1}) + \lambda_{6,j} P(W|W_{i-4}W_{i-3}W_{i-1}) + \\
&\quad \lambda_{7,j} P(W|w_{i-4}w_{i-2}w_{i-1}) + \lambda_{8,j} P(W|W_{i-4}W_{i-2}W_{i-1}) + \\
&\quad \lambda_{9,j} P(W|w_{i-4}w_{i-3}w_{i-2}) + \lambda_{10,j} P(W|W_{i-4}W_{i-3}W_{i-2}) + \\
&\quad \lambda_{11,j} P_{\texttt{unicache}}(W) + \lambda_{12,j} P_{\texttt{tricache}}(W|w_{i-2}w_{i-1})
\end{aligned}$$

Next, we define the analogous function for predicting words given clusters:

$$\begin{aligned}
&\mathit{senword}_j(w, w_{i-4}...w_{i-1}, W) = \\
&\quad \mu_{1,j} P(w|w_{i-4}w_{i-3}w_{i-2}w_{i-1}Ws_j) + \mu_{2,j} P(w|W_{i-4}W_{i-3}W_{i-2}W_{i-1}Ws_j) + \\
&\quad \mu_{3,j} P(w|w_{i-4}w_{i-3}w_{i-2}w_{i-1}W) + \mu_{4,j} P(w|W_{i-4}W_{i-3}W_{i-2}W_{i-1}W) + \\
&\quad \mu_{5,j} P(w|w_{i-4}w_{i-3}w_{i-1}W) + \mu_{6,j} P(w|W_{i-4}W_{i-3}W_{i-1}W) + \\
&\quad \mu_{7,j} P(w|w_{i-4}w_{i-2}w_{i-1}W) + \mu_{8,j} P(w|W_{i-4}W_{i-2}W_{i-1}W) + \\
&\quad \mu_{9,j} P(w|w_{i-4}w_{i-3}w_{i-2}W) + \mu_{10,j} P(w|W_{i-4}W_{i-3}W_{i-2}W) + \\
&\quad \mu_{11,j} P_{\texttt{unicache}}(w|W) + \mu_{12,j} P_{\texttt{tricache}}(w|w_{i-2}w_{i-1}W)
\end{aligned}$$

Now, we can write out our probability model:

$$\begin{aligned}
&P_{\mathit{everything}}(w_1...w_N) = \\
&\quad \sum_{j=0}^{S} \sigma_j \prod_{i=1}^{N} \mathit{sencluster}_j(W_i, w_{i-4}...w_{i-1}) \times \mathit{senword}_j(w_i, w_{i-4}...w_{i-1}, W_i) \qquad (9)
\end{aligned}$$

Clearly, combining all of these techniques together is not easy, but as we will show, the effects of combination are very roughly additive, and the effort is worthwhile.

We performed several sets of experiments. In these experiments, when we perform caching, it is with a unigram cache and conditional trigram cache; when we use sentence mixture models, we use 4 mixtures; when we use trigram skipping, it is *w_y* and *wx_*; and when we use 5-gram skipping it is *vw_y* interpolated with *v_xy* and *vwx_*. Our word error rate experiments were done without punctuation, so, to aid comparisons, we perform additional entropy experiments in this section on "all-no-punc", which is the same as the "all" set, but without punctuation.

[7] This formula is actually an oversimplification because the values $\lambda_{11,j}$ and $\lambda_{12,j}$ depend on the amount of training data in a linear fashion, and if the context w_{i-1} does not occur in the cache, then the trigram cache is not used. In either case, the values of the λ's have to be renormalized for each context so that they sum to 1.

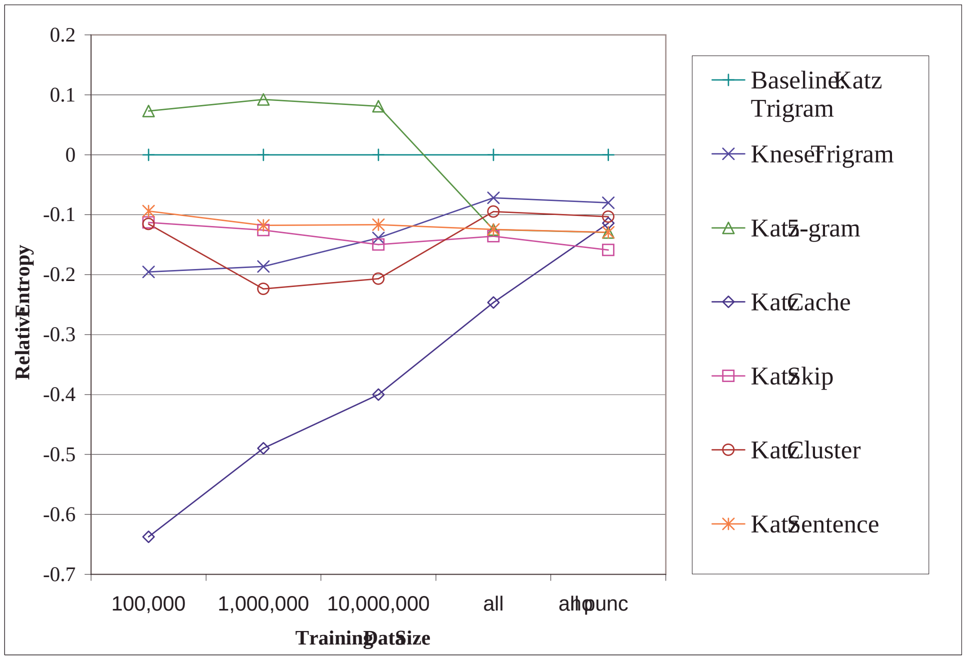

Figure 10: Relative Entropy of Each Technique versus Katz Trigram Baseline

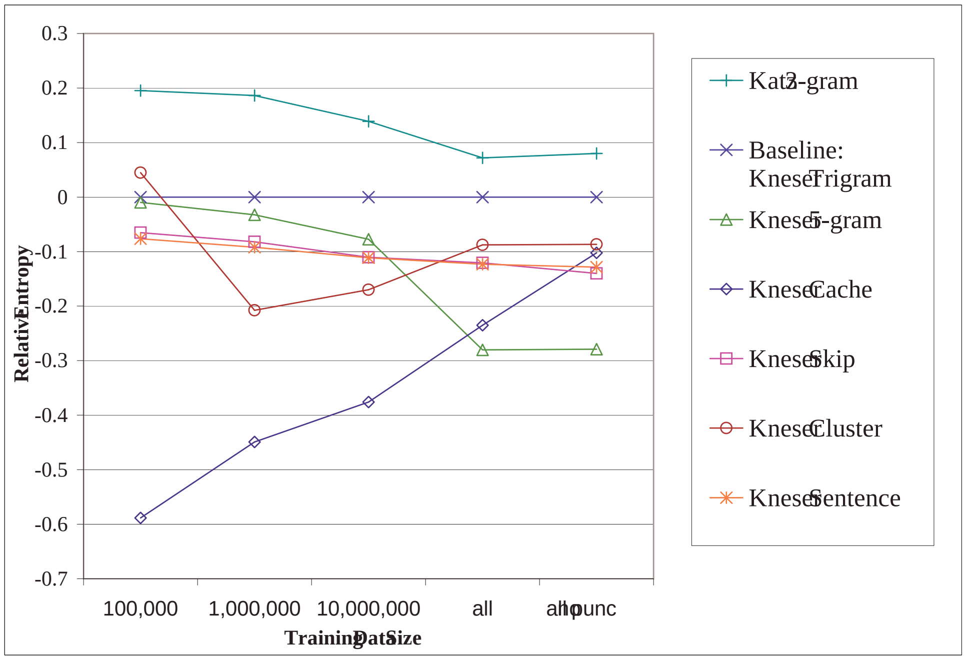

Figure 11: Relative Entropy of Each Technique versus Kneser-Ney Trigram Baseline

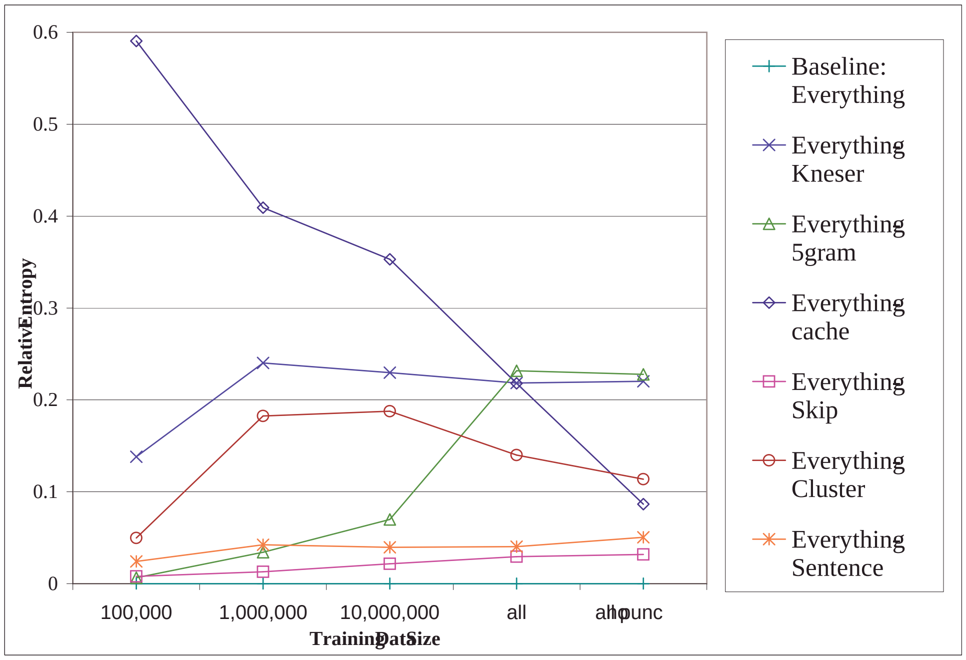

Figure 12: Relative Entropy of *Removing* Each Technique versus All Techniques Combined Baseline

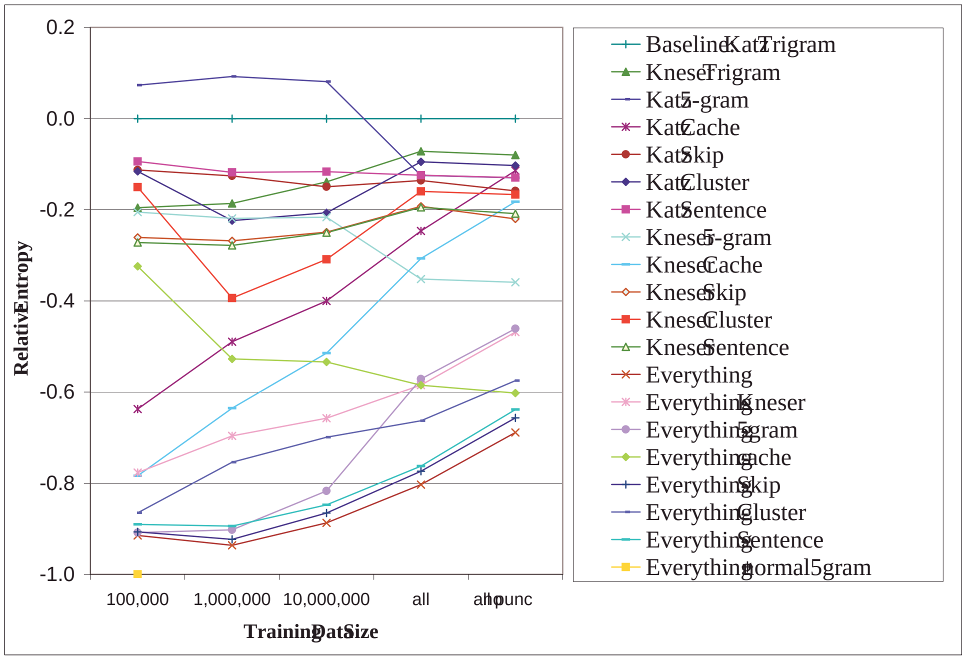

Figure 13: All Techniques Together versus Katz Trigram Baseline

In the first set of experiments, we used each technique separately, and Katz smoothing. The results are shown in Figure 10. Next, we performed experiments with the same techniques, but with Kneser-Ney smoothing; the results are shown in Figure 11. The results are similar for all techniques independent of smoothing, except 5-grams, where Kneser-Ney smoothing is clearly a large gain; in fact, without Kneser-Ney smoothing, 5-grams actually hurt at small and medium data sizes. This is a wonderful example of synergy, where the two techniques together help more than either one separately. Caching is the largest gain at small and medium data sizes, while, when combined with Kneser-Ney smoothing, 5-grams are the largest gain at large data sizes. Caching is still key at most data sizes, but the advantages of Kneser-Ney smoothing and clustering are clearer when they are combined with the other techniques.

In the next set of experiments, shown in Figure 12, we tried removing each technique from the combination of all techniques (Equation 9). The baseline is all techniques combined – "everything", and then we show performance of, for instance, everything except Kneser-Ney, everything except 5-gram models, etc. In Figure 12 we show all techniques together versus a Katz smoothed trigram. We add one additional point to this graph. With 100,000 words, our Everything model was at .91 bits below a normal Katz model, an excellent result, but we knew that the 100,000 word model was being hurt by the poor performance of fullibmpredict clustering at the smallest data size. We therefore interpolated in a normal 5-gram at the word level, a technique indicated as "Everything + normal5gram." This led to an entropy reduction of 1.0061 – 1 bit. This gain is clearly of no real-world value – most of the entropy gains at the small and medium sizes come from caching, and caching does not lead to substantial word error rate reductions. However, it does allow a nice title for the paper. Interpolating the normal 5-gram at larger sizes led to essentially no improvement.

We also performed word-error rate experiments rescoring 100-best lists of WSJ94 dev and eval, about 600 utterances. The 1-best error rate for the 100-best lists was 10.1% (our recognizer's models were slightly worse than even the baseline used in rescoring) and the 100-best error rate (minimum possible from rescoring) was 5.2%. We were not able to get word-error rate improvements by using caching (when the cache consisted of the output of the recognizer), and were actually hurt by the use of caching when the interpolation parameters were estimated on correct histories, rather than on recognized histories. Figure 14 shows word-error rate improvement of each technique, either with Katz smoothing, Kneser-Ney smoothing, or removed from Everything, except caching. The most important single factor for word-error rate was the use of Kneser-Ney smoothing, which leads to a small gain by itself, but also makes skipping, and 5-grams much more effective. Clustering also leads to significant gains. In every case except clustering, the Kneser-Ney smoothed model has lower word-error rate than the corresponding Katz smoothed model. The strange clustering result (the Katz entropy is higher) might be due to noise, or might be due to the fact that we optimized the number of clusters separately for the two systems, optimizing perplexity, perhaps leading to a number of clusters

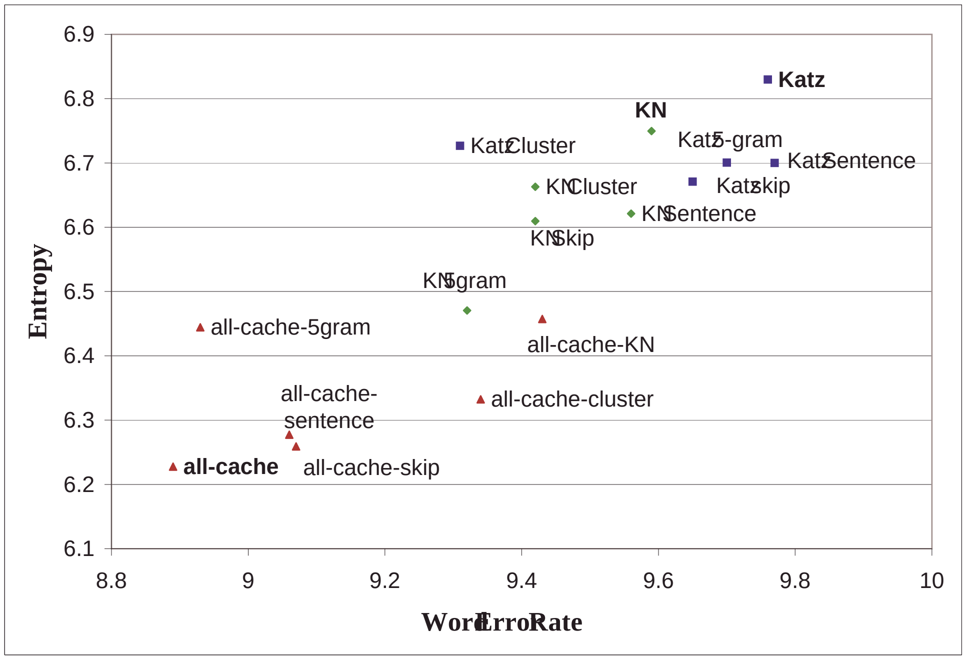

Figure 14: Word Error Rate versus Entropy

that was not optimal for word error rate reduction. Overall, we get an 8.9% word error rate reduction over a Katz smoothed baseline model. This is very good, although not as good as one might expect from our perplexity reductions. This is probably due to our rescoring of n-best lists rather than integrating our language model directly into the search, or rescoring large lattices.

9 Implementation Notes

Actually implementing the model described here is not straightforward. We give here a few notes on the most significant implementation tricks, some of which are reasonably novel, and in the appendix give more details. First, we describe our parameter search technique. Next, we discuss techniques we used to deal with the very large size of the models constructed. Then, we consider architectural strategies that made the research possible. Finally, we give a few hints on implementing our clustering methods.

The size of the models required for this research is very large. In particular, many of the techniques have a roughly multiplicative effect on data sizes: moving to five-grams from trigrams results in at least a factor of two increase; fullibmpredict clustering results in nearly a factor of 4 increase; and the combination of sentence-mixture models and skipping leads to about another factor of four. The overall model size then, is, very roughly, 32 times the size of a standard trigram model. Building and using such a complex model would be impractical.

Instead, we use a simple trick. We first make a pass through the test data (either text, or n-best lists), and the heldout data (used for parameter optimization), and determine the complete set of values we will need for our experiments. Then, we go through the training data, and record only these values. This drastically reduces the amount of memory required to run our experiments, reducing it to a managable 1.5 gigabytes roughly. Another trick we use is to divide the test data into pieces – the less test data there is, the fewer values we need to store. The appendix describes some ways that we verified that this “cheating” resulted in the same results that a non-cheating model would have gotten.

Careful design of the system was also necessary. In particular, we used a concept of a “model”, an abstract object, with a set of parameters, that could return the probability of a word or class given a history. We created models that could compose other models, by interpolating them at the word level, the class level, or the sentence level, or even by multiplying them together as done in predictive clustering. This allowed us to compose primitive models that implemented caching, various smoothing techniques, etc., in a large variety of ways.

Both our smoothing techniques and interpolation require the optimization of free parameters. In some cases, these free parameters can be estimated from the training data by leaving-one-out techniques, but better results are obtained by using a Powell search of the parameters, optimizing the perplexity of held-out data (Chen and Goodman, 1999), and that is the technique used here. This allowed us to optimize all parameters jointly, rather than say optimizing one model, then another, then their interpolation parameters, as is typically done. It also made it relatively easy to, in essentially every experiment in this paper, find the optimal parameter settings for that model, rather than use suboptimal guesses or results from related models.[8]

Although all smoothing algorithms were reimplemented for this research (reusing only a small amount of code), the details closely follow Chen and Goodman (1999). This includes our use of additive smoothing of the unigram distribution for both Katz smoothing and Kneser-Ney smoothing. That is, we found a constant b which was added to all unigram counts; this leads to better performance in small training-data situations, and allowed us to compare perplexities across different training sizes, since no unigram received 0 counts, meaning 0 probabilities were never returned.

There is no shortage of techniques for generating clusters, and there appears to be little evidence that different techniques that optimize the same criterion result in a significantly different quality of clusters. We note, however, that different algorithms may require significantly different amounts of run time. In particular, agglomerative clustering algorithms may require significantly more time than top-down, splitting algorithms. Within top-down, splitting algorithms, additional tricks can be used, including the techniques of Buckshot (Cutting et al., 1992). We also use computational tricks adapted from Brown et al. (1992). Many more details about the clustering techniques used are given in Appendix B.4.

10 Other techniques

In this section, we briefly discuss several other techniques that have received recent interest for language modeling; we have done a few experiments with some of these techniques. These techniques include maximum entropy models, whole sentence maximum entropy models, latent semantic analysis, parsing based models, and neural network based models. Rosenfeld (2000) gives a much broader, different perspective on the field, as well as additional references for the techniques discussed in this paper.

10.1 Maximum Entropy Models

Maximum entropy models (Darroch and Ratcliff, 1972) have received a fair amount of attention since their introduction for language modeling by Rosenfeld (1994). Maximum entropy models are models of the form

$$P_{\texttt{maxent}}(w|w_1...w_{i-1}) = \frac{\exp(\sum_k \lambda_k f_k(w, w_1...w_{i-1}))}{z(w_1...w_{i-1})}$$

[8]The only exception was that for Katz smoothed "everything" models we estimated the number of clusters from simple Katz clustered models; large Katz smoothed models are extremely time consuming because of the need to find the α's after each potential parameter change.

where z is a normalization function, simply set equal to

$$\sum_w \exp(\sum_k \lambda_k f_k(w, w_1...w_{i-1}))$$

The λ_k are real numbers, learned by a learning algorithm such as Generalized Iterative Scaling (GIS) (Darroch and Ratcliff, 1972). The f_k are arbitrary functions of their input, typically returning 0 or 1. They can be used to capture n-grams, caches, and clusters, and skipping models. They can also be used for more sophisticated techniques, such as triggers, described below. The generality and power of the f_k are one of the major attractions of maximum entropy models. There are other attractions to maximum entropy models as well. Given training data $w_1...w_T$, it is possible to find λs such that for each every k,

$$\sum_w \sum_{1 \le i \le T} P_{\texttt{maxent}}(w|w_1...w_{i-1}) f_k(w, w_1...w_{i-1}) = \sum_{1 \le i \le T} f_k(w, w_1...w_{i-1})$$

In other words, the number of times we expect f_k to occur given the model is the number of times we observe it in training data. That is, we can define arbitrary predicates over words w and histories $w_1...w_{i-1}$, and build a model such that all of these predicates are true as often as we observed. In addition, this model maximizes entropy, in the sense that it is also as smooth as possible (as close to the uniform distribution) while meeting these constraints. This is a quite seductive capability.

Furthermore, there have been two pieces of recent research that have made us especially optimistic about the use of maximum entropy models. The first is the the smoothing technique of Chen and Rosenfeld (1999b), which is the first maximum entropy smoothing technique that works as well (or better than) Kneser-Ney smoothing. The second is our own recent speedup techniques (Goodman, 2001), which lead to up to a factor of 35, or more, speedup over traditional maximum entropy training techniques, which can be very slow.

There are reasons both for optimism and pessimism with respect to maximum entropy models. On the one hand, maximum entropy models have lead to some of the largest reported perplexity reductions. Rosenfeld (1994) reports up to a 39% perplexity reduction when using maximum entropy models, combined with caching and a conventional trigram. On the other hand, our own pilot experiments with maximum entropy were negative, when we compared to comparable interpolated n-gram models: we are not aware of any research in which maximum entropy models yield a significant improvement over comparable n-gram models. Furthermore, maximum entropy models are extremely time consuming to train, even with our speedup techniques, and also slow to use during test situations. Overall, maximum entropy techniques may be too impractical for real-world use.

One special aspect of maximum entropy models worth mentioning is word triggers (Rosenfeld, 1994). Word triggers are the source of the most substantial gain in maximum entropy models. In trigger models, a word such as "school" increases the its own probability, as well as the probability of similar words, such as "teacher." Rosenfeld gets approximately a 25% perplexity reduction by using word triggers, although the gain is reduced to perhaps 7%-15% when combining with a model that already contains a cache. Tillmann and Ney (1996) achieves about a 7% perplexity reduction when combined with a model that already has a cache, and Zhang et al. (2000) reports an 11% reduction.

10.2 Whole Sentence Maximum Entropy Models

A recent variation of the maximum entropy approach is the whole sentence maximum entropy approach (Rosenfeld et al., 2001). In this variation, the probability of whole sentences is predicted, instead of the probabilities of individual words. This allows features of the entire sentence to be used, e.g. coherence (Cai et al., 2000) or parsability, rather than word level features. If s represents an entire sentence, then a whole sentence maximum entropy model is of the form

$$P(s) = \frac{1}{Z} P_0(s) \exp \sum_k \lambda_k f_k(s)$$

where $P_0(s)$ is some starting model, e.g. an n-gram model. Notice that the normalization factor Z is in this case a constant, eliminating the need to compute the normalization factor separately for each context. In fact, it is not necessary to compute it at all, for most uses. This is one of the main benefits of these models, according to their proponents, although other techniques (Goodman, 2001) also reduce the burden of normalization.

There are several problems with the whole sentence approach. First, training whole sentence maximum entropy models is particularly complicated (Chen and Rosenfeld, 1999a), requiring in practice sampling methods such as Monte Carlo Markov chain techniques. Second, the benefits of a whole sentence model may be small when divided over all the words. Consider a feature such as parsability: is the sentence parsable according to some grammar. Imagine that this feature contributes an entire bit of information per sentence. (Note that in practice, a grammar broad coverage enough to parse almost all grammatical sentences is broad coverage enough to parse many ungrammatical sentences as well, reducing the information it provides.) Now, in an average 20 word sentence, 1 bit of information reduces entropy by only .05 bits per word, a relatively small gain for a very complex feature. Another problem we see with these models is that most of their features can be captured in other ways. For instance, a commonly advocated feature for whole sentence maximum entropy models is "coherence", the notion that the words of a sentence should all be on similar topics. But other techniques, such as caching, triggering, and sentence mixture models are all ways to improve the coherence of a sentence that do not require the expense of a whole sentence approach. Thus, we are pessimistic about the long term potential of this approach.[9]

[9] Of course, we are pessimistic about almost everything.

10.3 Latent Semantic Analysis

Bellegarda (2000) shows that techniques based on Latent Semantic Analysis (LSA) are very promising. LSA is similar to Principle Components Analysis (PCA) and other dimensionality reduction techniques, and seems to be a good way to reduce the data sparsity that plagues language modeling. The technique leads to significant perplexity reductions (about 20%) and word error rate reductions (about 9% relative) when compared to a Katz trigram on 42 million words. It would be interesting to formally compare these results to conventional caching results, which exploit similar long term information. Bellegarda gets additional improvement over these results by using clustering techniques based on LSA; the perplexity reductions appear similar to the perplexity reductions from conventional IBM-style clustering techniques.

10.4 Neural Networks

There has been some interesting recent work on using Neural Networks for language modeling, by Bengio et al. (2000). In order to deal with data sparsity, they first map each word to a vector of 30 continuous features, and then the probability of various outputs is learned as a function of these continuous features. The mapping is learned by backpropagation in the same way as the other weights in the network. The best result is about a 25% perplexity reduction over a baseline deleted-interpolation style trigram.

Begin boring details

We wanted to see how the neural network model compared to standard clustering models. We performed some experiments on the same corpus, vocabulary, and splits into training, test, and heldout data as used by Bengio et al. and we are gratful to them for supplying us with the relevant data. First, we verified that their baseline was correct; we got a perplexity of 350.6 using simple interpolation, versus 348 for a deleted-interpolation style baseline, which seems very reasonable. On the other hand, a Kneser-Ney smoothed trigram had a perplexity of 316. The remaining experiments we did were with Kneser-Ney smoothing. We next tried an ibm-clustered 4-gram model, which is in some ways similar to the neural network model with three words of context. The clustered model has perplexity 271, compared to the neural network perplexity of 291. However, the 291 number does not interpolate with a trigram, while our clustered model does. We also ran a 6-gram ibm-clustered model. This model had a perplexity of 271, which is about 5% worse than the best neural network model score of 258; the 258 model was interpolated with a trigram, so this is a fair comparison. Finally, we tried to build a "pull-out-all-the-stops" model. This was not the very best model we could build – it did not include caching, for instance – but it was the best model we could build using the same inputs as the best neural network, the previous 5 words. Actually, we used only the previous 4 words, since we were not getting much benefit from the 5'th word back. This model was of the form

$$P_{\texttt{all-the-stops}}(w|w_{n-4}w_{n-3}w_{n-2}w_{n-1}) =$$

$$\begin{aligned}
&\lambda P(w|w_{n-4}w_{n-3}w_{n-2}w_{n-1}) + \mu P(W|W_{n-4}W_{n-3}W_{n-2}W_{n-1}) \times P(w|W) + \\
&\alpha P(w|w_{n-4}w_{n-3}w_{n-2}W_{n-1}) + \beta P(w|w_{n-4}w_{n-3}W_{n-2}W_{n-1}) + \\
&\gamma P(w|w_{n-4}W_{n-4}w_{n-3}W_{n-3}w_{n-2}W_{n-2}w_{n-1}W_{n-1}) + \\
&\epsilon P(W|w_{n-4}w_{n-3}w_{n-2}w_{n-1}) \times P(w|w_{n-4}w_{n-3}w_{n-2}w_{n-1}W) + \\
&(1-\lambda-\mu-\alpha-\beta-\gamma-\epsilon)\ P(W|w_{n-4}W_{n-4}w_{n-3}W_{n-3}w_{n-2}W_{n-2}w_{n-1}W_{n-1}) \times \\
&\qquad\qquad\qquad\qquad P(w|w_{n-4}W_{n-4}w_{n-3}W_{n-3}w_{n-2}W_{n-2}w_{n-1}W_{n-1}W)
\end{aligned}$$

and had a perplexity of 254.6, a meaningless 2% better than the best neural network. The best neural network model relied on interpolation with a trigram model, and used the trigram model exclusively for low frequency events. Since their trigram model had a relatively high perplexity of 348, compared to a Kneser-Ney smoothed trigram's 316, we assume that a significant perplexity reduction could be gotten simply from using a Kneser-Ney smoothed trigram. This means that the neural network model results really are very good.

It would be very interesting to see how much the neural network model has in common with traditional clustered models. One simple experiment would interpolate a neural network model with a clustered model, to see how much of the gains were additive. Given the relative simplicity of the neural network used, and the very good results, this seems like a very promising area of research.

End boring details

10.5 Structured Language Models

One of the most interesting and exciting new areas of language modeling research has been structured language models (SLMs). The first successful structured language model was the work of Chelba and Jelinek (1998), and other more recent models have been even more successful (Charniak, 2001). The basic idea behind structured language models is that a properly phrased statistical parser can be thought of as a generative model of language. Furthermore, statistical parsers can often take into account longer distance dependencies, such as between a subject and its direct or indirect objects. These dependencies are likely to be more useful than the previous two words, as captured by a trigram model. Chelba is able to achieve an 11% perplexity reduction over a baseline trigram model, while Charniak achieves an impressive 24% reduction.

Begin boring details

We hypothesized that much of the benefit of a structured language model might be redundant with other techniques, such as skipping, clustering, or 5-grams. The question of how much information one model has that another model already captures turns out to be a difficult one. While formulas for measuring the conditional entropy of a word given two different models are well known, they rely on computing joint probabilities. If the two models already are sparsely estimated, such as a conventional trigram and a structured language model, then computing these joint probabilities is hopeless. So, we decided to use more approximate measures. One simple, practical technique is to simply try interpolating the two models. It seems, at first glance, that if the interpolation leads to no gain, then the models must be capturing the same information, and if the gains are additive over a common baseline, then the information is independent. Unfortunately, at least the first assumption is incorrect. In particular, when comparing the structured language model to a Kneser-Ney smoothed trigram interpolated with a trigram cache, with which we assume the overlap in information is minimal, we end up getting almost no gain from the interpolation versus the cache model/Kneser-Ney trigram alone. This is simply because interpolation is such a crude way to combine information (although we don't know any that are much better). The cache model is so much better than the structured language model that the interpolation weight all goes to the cache model, and thus the structured language model cannot help; we learn nothing except that the cache model had a lower perplexity, which we already knew.

Our strategy then was a bit more complex, and a bit harder to interpret. First, we used versions of both systems with simple interpolation for the smoothing. This was the only smoothing technique that was already implemented in both the SLM program and our toolkit. This removed smoothing as a factor. Next, we tried comparing the SLM to a trigram with various cache sizes (although, of course, we never cached beyond document boundaries), and interpolating that with the SLM. We assumed that the SLM and the cache, of whatever size, were capturing roughly orthogonal information. This let us figure out the amount of perplexity reduction we would expect if the two models were uncorrelated. For instance, the baseline SLM, not interpolated with a trigram, had a perplexity of 167.5. A fullibm clustered model had a perplexity of 144.7; similarly, a trigram with trigram cache with 160 words of context had a perplexity of 143.4 – about the same as the clustered model. The combination of the SLM with the 160 word context cache had a perplexity of 130.7, a reduction of 8.8% over the cache alone. When combined with the fullibm clustered model, the perplexity was 137.0, a 5.3% reduction over the fullibm clustered model. So, if the improvements were uncorrelated, we would have assumed an 8.8% reduction, and instead we only saw a 5.3% reduction. This is 60% of the reduction we would have expected. Thus, we say that the overlap with a clustered model is very roughly 40%.

The following table shows the results of our various experiments. The "model" column describes the model we interpolated with the SLM. The first "perplex" column shows the perplexity of the model, and the first "reduction" column shows the reduction from interpolating the model with the SLM. The second "perplex" column shows the perplexity of the most similar cache model and the second "reduction" column shows the perplexity reduction from interpolating this cache model with the SLM. The final column, "overlap", shows the ration between the first reduction and the second: the presumed overlap between the model and the SLM.

	model		closest cache		
model	perplex	reduction	perplex	reduction	overlap
fullibm	144.7	5.3%	160	8.8%	40%
Kneser-Ney	144.9	6.7%	143.4	8.8%	23%
5-gram	160.0	9.9%	162.6	10.5%	6%
trigram skipping	157.0	8.5%	155.0	9.8%	13%
special cluster 1	141.4	4.5%	141.8	8.5%	45%
special cluster 2	143.6	5.0%	143.4	8.8%	43%

The trigram skipping model was a model with all pairs, through the 5-gram level. The special cluster 1 and special cluster 2 models were clustered skipping models designed to capture contexts similar to what we assumed the structured language model was doing.

We would have liked to have looked at combinations, such as Kneser-Ney smoothing with fullibm clustering, but the best cache model we tested had a perplexity of 141.8, while the Kneser-Ney clustered model had a much lower perplexity.

It is difficult to draw any too strong conclusions from these results. One odd result is the large overlap with Kneser-Ney smoothing – 23%. We suspect that somehow the breakdown of the language model into individual components in the structured LM has a smoothing effect. Or, perhaps our entire evaluation is flawed.

We also looked at the correlation of individual word probabilities. We examined for each model for each word the difference from the probability of the baseline trigram model. We then measured the correlation of the differences. These results were similar to the other results.

It would be interesting to perform these same kind of experiments with other structured language models. Unfortunately, the current best, Charniak's, does not predict words left to right. Therefore, Charniak can only interpolate at the sentence level. Sentence level interpolation would make these experiments even harder to interpret.

Overall, we are reasonably confident that some large portion of the increase from the SLM – on the order of 40% or more – comes from information similar to clustering. This is not surprising, given that the SLM has explicit models for part of speech tags and non-terminals, objects that behave similarly to word clusters. We performed these experiments with Ciprian Chelba, and reached opposite conclusions: we concluded that the glass was half empty, while Chelba concluded that the glass was half full.

End boring details

11 Conclusion

11.1 Previous combinations

There has been relatively little previous research that attempted to combine more than two techniques, and even most of the previous research combining two techniques was not particularly systematic. Furthermore, one of the two techniques typically combined was a cache-based language model. Since the cache model is simply linearly interpolated with another model, there is not much room for interaction.

A few previous papers do merit mentioning. The most recent is that of Martin et al. (1999). They combined interpolated Kneser-Ney smoothing, classes, word-phrases, and skipping. Unfortunately, they do not compare to the same baseline we use, but instead compare to what they call interpolated linear discounting, a poor baseline. However, their improvement over Interpolated Kneser-Ney is also given; they achieve about 14% perplexity reduction over this baseline, versus our 34% over the same baseline. Their improvement from clustering is comparable to ours, as is their improvement from skipping models; their improvement from word-phrases, which we do not use, is small (about 3%); thus, the difference in results is due mainly to our implementation of additional techniques: caching, 5-grams, and sentence-mixture models. Their word error rate reduction over Interpolated Kneser-Ney is 6%, while ours is 7.3%. We assume that the reason our word error rate reduction is not proportional to our perplexity reduction is two-fold. First, 4% of our perplexity reduction came from caching, which we did not use in our word error-rate results. Second, they were able to integrate their simpler model directly into a recognizer, while we needed to rescore n-best lists, reducing the number of errors we could correct.

Another piece of work well worth mentioning is that of Rosenfeld (1994). In that work, a large number of techniques are combined, using the maximum entropy framework and interpolation. Many of the techniques are tested at multiple training data sizes. The best system interpolates a Katz-smoothed trigram with a cache and a maximum entropy system. The maximum entropy system incorporates simple skipping techniques and triggering. The best system has perplexity reductions of 32% to 39% on data similar to ours. Rosenfeld gets approximately 25% reduction from word triggers alone (p. 45), a technique we do not use. Overall, Rosenfeld's results are excellent, and would quite possibly exceed ours if more modern techniques had been used, such as Kneser-Ney smoothing the trigram model (which is interpolated in), using smaller cutoffs made possible by faster machines and newer training techniques, or smoothing the maximum entropy model with newer techniques. Rosenfeld experiments with some simple class-based techniques without success; we assume that more modern classing technology could also be used. Rosenfeld achieves about a 10% word error rate reductionwhen using unsupervised adaptation (the same way we adapted. He achieves 13% assuming supervsion – users correct mistakes immediately.).

There is surprisingly little other work combining more than two techniques. The only other noteworthy research we are aware of is that of Weng et al. (1997), who performed experiments combining multiple corpora, fourgrams, and a class-based approach similar to sentence-mixture models. Combining all of these techniques leads to an 18% perplexity reduction from a Hub4-only language model. This model was trained and tested on a different text genre than our models, and so no comparison to our work can be made.

11.2 All hope abandon, ye who enter here

Begin long rambling cynical diatribe – no results or particularly novel ideas. Grad students thinking about research in language modeling *should* read this section

In this section,[10] we argue that meaningful, practical reductions in word error rate are hopeless. We point out that trigrams remain the de facto standard not because we don't know how to beat them, but because no improvements justify the cost. We claim with little evidence that entropy is a more meaningful measure of progress than perplexity, and that entropy improvements are small. We conclude that most language modeling research, including ours, by comparing to a straw man baseline and ignoring the cost of implementations, presents the illusion of progress without the substance. We go on to describe what, if any, language modeling research is worth the effort.

11.2.1 Practical word error rate reductions are hopeless

Most language modeling improvements require significantly more space than the trigram baseline compared to, and also typically require significantly more time. Most language modeling papers contain a paragraph such as "Trigram models are state-of-the-art. Trigram models are obviously braindamaged. We did something slightly less braindamaged. Our model has much lower perplexity and somewhat lower word error rates than a trigram." However, what the papers don't say, but what almost universally applies, is that the resulting model requires much more space and/or time than a simple trigram model. Trigram models are state-of-the-art in the sense that they are used in most commercial speech recognizers and as at least the first pass of most research recognizers. This is for the following two reasons: they are fairly space efficient, and all sorts of tricks can be used to integrate the trigram storage with the search. For instance, in our recognizers at Microsoft, we use a storage method in which the trigrams are stored as phonetic trees (Alleva et al., 1996). Many more complex language modeling techniques, especially clustering techniques, are incompatible with this type of storage, and thus much less efficient in a practical, optimized recognizer.

Consider in practice each of the techniques we implemented. Sentence mixture models require substantially more storage than corresponding global models, since different models have to be stored for each mixture, and a global model still needs to be stored, to interpolate with the mixture-specific models. Furthermore, if integrated into the first pass of a recognizer, a different path has to be explored for each mixture type, significantly slowing the recognizer. Caching has problems we have already discussed: while it does not significantly increase the storage for the recognizer, it requires users to correct all their mistakes after each sentence; otherwise, mistakes are "locked in." Clustered models of the sort we have described here substantially increase storage, and interact very poorly with the kinds of prefix trees used for search. 5-grams require substantially more

[10]Given that this extended version is a technical report, rather than a full paper, we can rant and rave without fear of editorial review, willy nilly titling sections, and scaring innocent graduate students. We can express our own bitterness and frustration and vent it on unsuspecting readers. Those with a natural predisposition to pessimism should skip this section, or risk the consequences.

space than trigrams and also slow the search. Kneser-Ney smoothing leads to improvements in theory, but in practice, most language models are built with high count cutoffs, to conserve space, and speed the search; with high count cutoffs, smoothing doesn't matter. Skipping models require both a more complex search and more space and lead to marginal improvements. In short, none of the improvements we studied is likely to make a difference in a real system in the forseeable future, with the exception of caching, which is typically available, but often turned off by default, because users don't like to or can't be trained to consistently correct recognizer errors.

Our results have surveyed and reimplemented the majority of promising language modeling techniques introduced since Katz (1987). The improvement in perplexity is one of the largest ever reported, but the improvement in word error rate is relatively small. This partially reflects our own emphasis on perplexity rather than word error rate, and flaws in the experiments. Perhaps rescoring lattices rather than n-best lists would have lead to larger improvements. Perhaps integrating the language modeling into the search would have helped. Perhaps trying to optimize parameters and techniques for word error rate reduction, rather than perplexity reduction would have worked better. But, despite all these flaws, we think our results are reasonably representative. Others who have tried their best to get language modeling improvements have had mixed results. For instance, Martin et al. (1999) gets a 12% relative word error rate reduction from a fair baseline (larger from an easier baseline), slightly better than ours. But the cost of most of their improvements is high – a quadrupling in size of the language model and 32 times the CPU time. Larger improvements could probably be gotten more easily and cheaply simply by increasing the number of mixture components or loosening search thresholds in the acoustic model. In fact, simply by using more language model training data, comparable improvements could probably be had.[11]

11.2.2 Perplexity is the wrong measure

In this subsubsection, we claim with little evidence that word error rate correlates better with entropy than with perplexity. Since relative entropy reductions are much smaller than relative perplexity reductions, it is difficult to get useful word error rate reductions.

We considered doing the following experiment: incrementally add portions of the test data to our training set, and use this to compute perplexity/entropy and word error rates. However, we noticed that actually conducting the experiment is unneccessary, since (in a slightly idealized world) we can predict the results. If we add in x% of the test data, then we will have perfect information for that x%. With a sufficiently powerful n-gram model (say a 1000-gram), and no search errors, our speech recognizer will get that portion exactly correct, leading to an

[11]These criticisms apply not just to the work of Martin et al. but to our own as well. We use Martin's system as an example because quirks of our tool (see Appendix B) make it hard for us to measure the size of the models we used and our acoustic modeling was done as offline n-best rescoring, rather than integrated with the search.

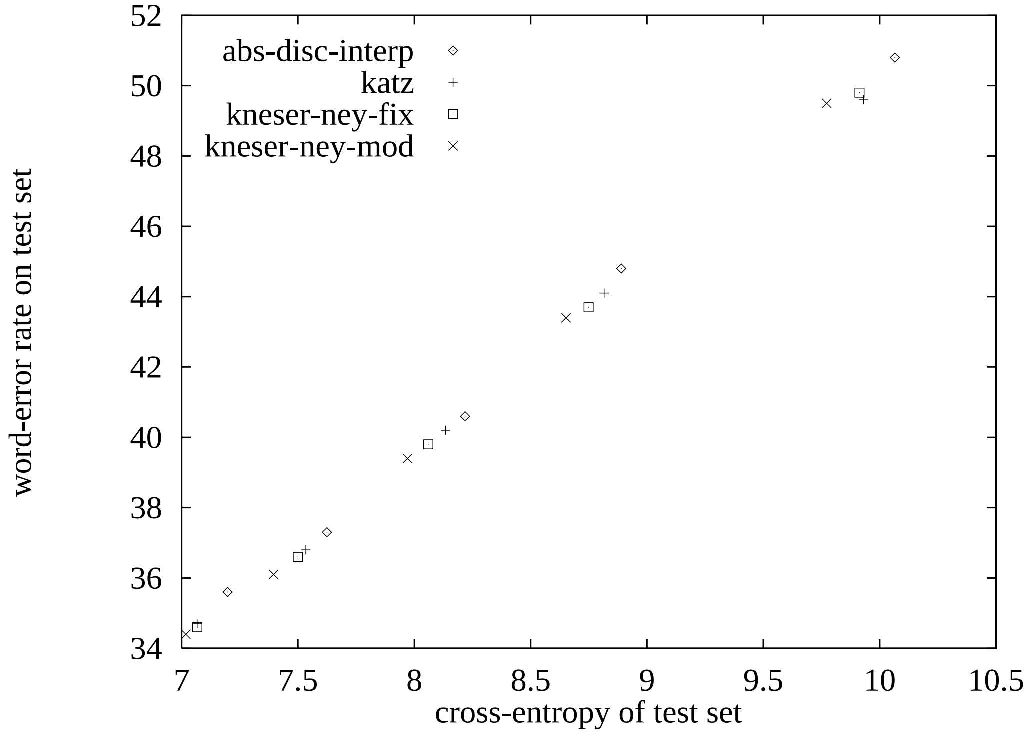

Figure 15: Word error rate versus entropy

x% relative reduction in word error rate. Similarly, it will require essentially 0 bits to predict the portion of the training data we have already seen, leading to an x% reduction in entropy. This theoretical analysis leads us to hypothesize a linear relationship between entropy and word error rate.

Of course, one could perform more realistic experiments to see whether perplexity or entropy corellates better with word error rate. Our own recent experiments do not shed light on this question – the correlation between word error rate and entropy, and the correlation between word error rate and perplexity are the same, over the relatively narrow range of perplexities we tested. However, consider previous work by Chen and Goodman (1998). In Figure 15, we show experiments done on the correlation between word error rate and entropy. Obviously, the relationship is very strong and appears linear over a large range, including a roughly 0 intercept, just as our analysis predicted.[12]

Consider also other domains, such as text compression. Here, trivially, the relationship between entropy and the objective function, number of bits required

[12]We prefer to make all of our predictions after seeing the data; this vastly increases their accuracy.

to represent the data, is again linear.

These are not strong arguments, especially in light of the many researchers who have gotten results where entropy and word error rate do not correlate. Still, they are worrisome. If true, they imply that we must get very large reductions in perplexity to get meaningful word error rate reductions. For instance, a 10% perplexity reduction, from say 100 to 90, corresponds to a reduction from 6.64 bits to 6.49 bits, which is only a 2% entropy reduction. What appears as a meaningful perplexity reduction is a trivial entropy reduction. We fear that this means that commercially useful reductions in perplexity are unlikely.

11.2.3 Progress is Illusory

A common and related pitfall in language modeling research is the straw man baseline. Most language modeling papers point out a problem with trigram models, typically that they cannot capture long distance dependencies, and then proceed to give some way to model longer contexts. These papers almost never even compare to a well-smoothed 5-gram, which would be a simple, obvious way to make a trigram capture longer contexts, never mind comparing to a trigram with a cache, a technique known for over 10 years, or to more recent models, such as sentence mixture models. The result is yet another way to beat trigrams, assuming we don't care about speed or space. Whether these new techniques are better than the previous ones, or can be combined with them for larger improvements, or offer a better speed or space tradeoff than other techniques, is only very rarely explored. When a cache comparison is used, it is typically in the context of combining the cache with their own model, rather than with the baseline trigram. Perplexity results in the introduction, conclusion, and abstract will usually compare to their technique with a cache, versus a trigram without one. Sometimes the cache does not look like a cache. It might, for instance, be a trigger model if maximum entropy is being done. But since the most useful triggers are just self triggers, which are basically cache features, it is just a cache after all.

Poor experimental technique and misleading papers are by no means unique to language modeling. But the prevalence of the trigram baseline in practice, allowing researchers to call it "state-of-the-art" with a straight face, makes it particularly easy to give the illusion of progress.[13]

11.2.4 So, what now?

Given the previous sections, should anyone do any language modeling research at all? Maybe. There are four areas where I see some hope: language model compression, language model adapatation, new applications of language modeling, and basic research.

[13]Wait a minute, doesn't this paper do all those same bad things? No. Our paper doesn't claim that trigrams are state of the art (although it does, misleadingly, call them a fair baseline), and it fairly compares all techniques. [14]

[14]Sounds like weasling to me. Hypocrite!

Language model compression or pruning is an area that has received relatively little research, although good work has been done by Kneser (1996), Seymore and Rosenfeld (1996), Stolcke (1998), Siu and Ostendorf (2000), and by us (Goodman and Gao, 2000). Care is needed here. For instance, the techniques of Seymore et al. and Stolcke can be used with almost any recognizer, but our own techniques use clustering, and interact poorly with the search in some speech recognizers. Still, this seems like an area where progress can be made. Similarly, there has been no comprehensive work showing the space/perplexity tradeoffs of all of the techniques we examined in this paper, nor has there been any work on pruning interpolated models, such as skipping models or sentence mixture models.

Another area for language modeling research is language model adaptation. A very common product scenario involves only a very small amount of in-domain training data, and lots of out-of-domain data. There has been a moderate amount of research in this area. Much of the research does not compare to simply interpolating together the in-domain and out-of-domain language models, which in our experiments works quite well. The best research in this area is probably that of Iyer and Ostendorf (1997). We suspect that better techniques could be found, but our own attempts have failed.

Language models work very well and are well understood, and can be applied to many domains (Church, 1988; Brown et al., 1990; Hull, 1992; Kernighan et al., 1990; Srihari and Baltus, 1992). Rather than trying to improve them, trying to use them in new ways can be fruitful. Almost any machine learning problem could potentially use language modeling techniques as a solution, and identifying new areas where language models work well is likely to be as useful as trying to do basic research.

There are many unfinished areas of language modeling basic research. None of these will have a huge practical impact, but they will help advance the field. One area is a continuous version of Kneser-Ney smoothing. Interpolated Kneser-Ney smoothing is wonderful. No matter what kind of model we have used, it has worked better with Kneser-Ney smoothing than with any other smoothing technique we have tried. But Kneser-Ney smoothing is limited to discrete distributions; it cannot handle fractional counts. Fractional counts are very common with, for instance, Expectation Maximization (EM) algorithms. This means that we do not currently know how to do smoothing for distributions learned through EM, such as most instances of Hidden Markov Models or of Probabilistic Context-Free Grammars. A continuous version of Kneser-Ney could be used in all of these areas. A related area that needs more research is soft-clustering versus hard clustering. Hard clustering assigns each word to a single cluster, while soft clustering allows the same word to be placed in different clusters. There has been essentially no work comparing hard clustering to soft clustering, but several soft-style techniques, including the work of Bengio et al. (2000) and of Bellegarda (2000) have had good success, hinting that these techniques may be more effective. One reason we have not tried soft clustering is because we do not know how to properly smooth it: soft clustered models have fractional counts and would work best with a continuous version of Kneser-Ney smoothing.

11.2.5 Some hope is left

To summarize, language modeling is a very difficult area, but not one that is completely hopeless. Basic research is still possible, and there continue to be new, if not practical, then certainly interesting language modeling results. There also appear to be a few areas in which useful language modeling research is promising. But language modeling, like most research, but perhaps more so, is not an area for the faint of heart or easily depressed.

End long rambling cynical diatribe

11.3 Discussion

We believe our results – a 50% perplexity reduction on a very small data set, and a 41% reduction on a large one (38% for data without punctuation) – are the best ever reported for language modeling, as measured by improvement from a fair baseline, a Katz smoothed trigram model with no count cutoffs. We also systematically explored smoothing, higher order n-grams, skipping, sentence mixture models, caching, and clustering.

Our most important result is perhaps the superiority of Interpolated Kneser-Ney smoothing in every situation we have examined. We previously showed (Chen and Goodman, 1998) that Kneser-Ney smoothing is always the best technique across training data sizes, corpora types, and n-gram order. We have now shown that it is also the best across clustering techniques, and that it is one of the most important factors in building a high performance language model, especially one using 5-grams.

We have carefully examined higher-order n-grams, showing that performance improvements plateau at about the 5-gram level, and we have given the first results at the 20-gram level, showing that there is no improvement to be gained past 7 grams.

We have systematically examined skipping techniques. We examined trigram-like models, and found that using pairs through to the 5-gram level captures almost all of the benefit. We also performed experiments on 5-gram skipping models, finding a combination of 3 contexts that captures most of the benefit.

We carefully explored sentence mixture models, showing that much more improvement can be had than was previously expected by increasing the number of mixtures. In our experiments, increasing the number of sentence types to 64 allows nearly twice the improvement over a small number of types.

Our caching results show that caching is by far the most useful technique for perplexity reduction at small and medium training data sizes. They also show that a trigram cache can lead to almost twice the entropy reduction of a unigram cache.

Next, we systematically explored clustering, trying 9 different techniques, finding a new clustering technique, fullibmpredict, that is a bit better than standard ibm clustering, and examining the limits of improvements from clustering. We also showed that clustering performance may depend on smoothing in some cases.

Our word-error rate reduction of 8.9% from combining all techniques except caching is also very good.

Finally, we put all the techniques together, leading to a 38%-50% reduction in perplexity, depending on training data size. The results compare favorably to other recently reported combination results (Martin et al., 1999), where, essentially using a subset of these techniques, from a comparable baseline (absolute discounting), the perplexity reduction is half as much. Our results show that smoothing can be the most important factor in language modeling, and its interaction with other techniques cannot be ignored.

In some ways, our results are a bit discouraging. The overall model we built is so complex, slow and large that it would be completely impractical for a product system. Despite this size and complexity, our word error rate improvements are modest. To us, this implies that the potential for practical benefit to speech recognizers from language model research is limited. On the other hand, language modeling is useful for many fields beyond speech recognition, and is an interesting test bed for machine learning techniques in general.

Furthermore, parts of our results are very encouraging. First, they show that progress in language modeling continues to be made. For instance, one important technique in our system, sentence mixture models, is only a few years old, and, as we showed, its potential has only been partially tapped. Similarly, the combination of so many techniques is also novel. Furthermore, our results show that the improvements from these different techniques are roughly additive: one might expect an improvement of .9 bits for the largest training size based on simply adding up the results of Figure 11, and instead the total is about .8 bits – very similar. This means that further incremental improvements may also lead to improvements in the best models, rather than simply overlapping or being redundant.

As we noted in Section 10, there are many other promising language modeling techniques currently being pursued, such as maximum entropy models, neural networks, latent semantic analysis, and structured language models. Figuring out how to combine these techniques with the ones we have already implemented should lead to even larger gains, but also yet more complex models.

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References

Fil Alleva, Xuedong Huang, and Mei-Yuh Hwang. 1996. Improvements on the pronunciation prefix tree search organization. In *ICASSP-96*, volume 1, pages 133–136.

J. R. Bellegarda, J. W. Butzberger, Yen-Lu Chow, N. B. Coccaro, and D. Naik. 1996. A novel word clustering algorithm based on latent semantic analysis. In *ICASSP-96*, volume 1, pages 172 – 175.

J.R. Bellegarda. 2000. Exploiting latent semantic information in statistical language modeling. *Proceedings of the IEEE*, 88:1279–1296, August.

Yoshua Bengio, Rejean Ducharme, and Pascal Vincent. 2000. A neural probabilistic language model. Technical Report 1178, Département d'informatique et recherche opérationnelle, Université de Montréal.

Reinhard Blasig. 1999. Combination of words and word categories in varigram histories. In *ICASSP-99*, volume 1, pages 529–532.

Peter F. Brown, John Cocke, Stephen A. DellaPietra, Vincent J. DellaPietra, Frederick Jelinek, John D. Lafferty, Robert L. Mercer, and Paul S. Roossin. 1990. A statistical approach to machine translation. *Computational Linguistics*, 16(2):79–85, June.

Peter F. Brown, Vincent J. DellaPietra, Peter V. deSouza, Jennifer C. Lai, and Robert L. Mercer. 1992. Class-based n-gram models of natural language. *Computational Linguistics*, 18(4):467–479, December.

Can Cai, Ronald Rosenfeld, and Larry Wasserman. 2000. Exponential language models, logistic regression, and semantic coherence. In *Proc. NIST/DARPA Speech Transcription Workshop*, May.

Eugene Charniak. 2001. Immediate-head parsing for language models. In *ACL-01*, pages 116–123, July.

Ciprian Chelba and Frederick Jelinek. 1998. Exploiting syntactic structure for language modeling. In *Proceedings of the 36th Annual Meeting of the ACL*, August.

Stanley F. Chen and Joshua T. Goodman. 1998. An empirical study of smoothing techniques for language modeling. Technical Report TR-10-98, Harvard University. Available from `http://www.research.microsoft.com/~joshuago`.

Stanley F. Chen and Joshua Goodman. 1999. An empirical study of smoothing techniques for language modeling. *Computer Speech and Language*, 13:359–394, October.

Stanley Chen and Ronald Rosenfeld. 1999a. Efficient sampling and feature selection in whole sentence maximum entropy language models. In *Proc. ICASSP '99*, March.

Stanley F. Chen and Ronald Rosenfeld. 1999b. A gaussian prior for smoothing maximum entropy models. Technical Report CMU-CS-99-108, Computer Science Department, Carnegie Mellon University.

Kenneth Church. 1988. A stochastic parts program and noun phrase parser for unrestricted text. In *Proceedings of the Second Conference on Applied Natural Language Processing*, pages 136–143.

Douglas R. Cutting, David R. Karger, Jan R. Pedersen, and John W. Tukey. 1992. Scatter/gather: A cluster-based approach to browsing large document collections. In *SIGIR 92*.

J.N. Darroch and D. Ratcliff. 1972. Generalized iterative scaling for log-linear models. *The Annals of Mathematical Statistics*, 43:1470–1480.

Pierre Dupont and Ronald Rosenfeld. 1997. Lattice based language models. Technical Report CMU-CS-97-173, School of Computer Science, Carnegie Mellon University, Pittsburgh, PA, September.

I.J. Good. 1953. The population frequencies of species and the estimation of population parameters. *Biometrika*, 40(3 and 4):237–264.

Joshua Goodman and Jianfeng Gao. 2000. Language model size reduction by pruning and clustering. In *ICSLP 2000*.

Joshua Goodman. 2001. Classes for fast maximum entropy training. In *ICASSP 2001*.

X. Huang, F. Alleva, H.-W. Hon, M.-Y. Hwang, K.-F. Lee, and R. Rosenfeld. 1993. The SPHINX-II speech recognition system: An overview. *Computer, Speech, and Language*, 2:137–148.

Jonathon Hull. 1992. Combining syntactic knowledge and visual text recognition: A hidden Markov model for part of speech tagging in a word recognition algorithm. In *AAAI Symposium: Probabilistic Approaches to Natural Language*, pages 77–83.

Rukmini Iyer and Mari Ostendorf. 1997. Transforming out-of-domain estimates to improve in-domain language models. In *Eurospeech97*, volume 4, pages 1975–1978.

Rukmini Iyer and Mari Ostendorf. 1999. Modeling long distance dependence in language: Topic mixtures versus dynamic cache models. *IEEE Transactions on Acoustics, Speech and Audio Processing*, 7:30–39, January.

Rukmini Iyer, Mari Ostendorf, and J. Robin Rohlicek. 1994. Language modeling with sentence-level mixtures. In *DARPA-HLT*, pages 82–86.

Frederick Jelinek and Robert L. Mercer. 1980. Interpolated estimation of Markov source parameters from sparse data. In *Proceedings of the Workshop on Pattern Recognition in Practice*, Amsterdam, The Netherlands: North-Holland, May.

F. Jelinek, B. Merialdo, S. Roukos, and M. Strauss. 1991. A dynamic lm for speech recognition. In *Proc. ARPA Workshop on Speech and Natural Language*, pages 293–295.

Daniel Jurafsky, Chuck Wooters, Jonathan Segal, Eric Fosler, Gary Tajchman, and Nelson Morgan. 1995. Using a stochastic context-free grammar as a language model for speech recognition. In *ICASSP-95*, pages 189–192.

Slava M. Katz. 1987. Estimation of probabilities from sparse data for the langauge model component of a speech recognizer. *IEEE Transactions on Acoustics, Speech and Signal Processing*, ASSP-35(3):400–401, March.

M.D. Kernighan, K.W. Church, and W.A. Gale. 1990. A spelling correction program based on a noisy channel model. In *Proceedings of the Thirteenth International Conference on Computational Linguistics*, pages 205–210.

Reinhard Kneser and Hermann Ney. 1993. Improved clustering techniques for class-based statistical language modeling. In *Eurospeech 93*, volume 2, pages 973–976.

Reinhard Kneser and Hermann Ney. 1995. Improved backing-off for m-gram language modeling. In *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing*, volume 1, pages 181–184.

Reinhard Kneser. 1996. Statistical language modeling using a variable context length. In *ICSLP-96*, volume 1, pages 494–497.

R. Kuhn and R. De Mori. 1990. A cache-based natural language model for speech reproduction. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(6):570–583.

R. Kuhn and R. De Mori. 1992. *Correction to* a cache-based natural language model for speech reproduction. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 14(6):691–692.

R. Kuhn. 1988. Speech recognition and the frequency of recently used words: A modified markov model for natural language. In *12th International Conference on Computational Linguistics*, pages 348–350, Budapest, August.

J. Kupiec. 1989. Probabilistic models of short and long distance word dependencies. In *Proc. ARPA Workshop on Speech and Natural Language*, pages 290–295.

David J. C. MacKay and Linda C. Peto. 1995. A hierarchical Dirichlet language model. *Natural Language Engineering*, 1(3):1–19.

Sven Martin, Christoph Hamacher, Jörg Liermann, Frank Wessel, and Hermann Ney. 1999. Assessment of smoothing methods and complex stochastic language modeling. In *6th European Conference on Speech Communication and Technology*, volume 5, pages 1939–1942, Budapest, Hungary, September.

Hermann Ney, Ute Essen, and Reinhard Kneser. 1994. On structuring probabilistic dependences in stochastic language modeling. *Computer, Speech, and Language*, 8:1–38.

T. R. Niesler, E. W. D. Whittaker, and P. C. Woodland. 1998. Comparison of part-of-speech and automatically derived category-based language models for speech recognition. In *ICASSP-98*, volume 1, pages 177–180.

Fernando Pereira, Naftali Tishby, and Lillian Lee. 1993. Distributional clustering of english words. In *Proceedings of the 31st Annual Meeting of the ACL*. To appear.

W.H. Press, B.P. Flannery, S.A. Teukolsky, and W.T. Vetterling. 1988. *Numerical Recipes in C.* Cambridge University Press, Cambridge.

Ronald Rosenfeld, Stanley F. Chen, and Xiaojin Zhu. 2001. Whole-sentence exponential language models: a vehicle for linguistic-statistical integration. *Computer Speech and Language.* To appear.

Ronald Rosenfeld. 1994. *Adaptive Statistical Language Modeling: A Maximum Entropy Approach.* Ph.D. thesis, Carnegie Mellon University, April.

Ronald Rosenfeld. 2000. Two decades of statistical language modeling: Where do we go from here? *Proc. of the IEEE*, 88:1270–1278, August.

Lawrence Saul and Fernando Pereira. 1997. Aggregate and mixed-order Markov models for statistical language processing. In *Proceedings of the Second Conference on Empirical Methods in Natural Language Processing*, pages 81–89.

Kristie Seymore and Ronald Rosenfeld. 1996. Scalable backoff language models. In *ICSLP'96*.

Manhung Siu and Mari Ostendorf. 2000. Variable n-grams and extensions for conversational speech language modeling. *IEEE Transactions on Speech and Audio Processing*, 8:63–75.

Rohini Srihari and Charlotte Baltus. 1992. Combining statistical and syntactic methods in recognizing handwritten sentences. In *AAAI Symposium: Probabilistic Approaches to Natural Language*, pages 121–127.

Richard M. Stern. 1996. Specification of the 1995 ARPA hub 3 evaluation: Unlimited vocabulary NAB news baseline. In *Proceedings of the DARPA Speech Recognition Workshop*, pages 5–7.

Andreas Stolcke. 1998. Entropy-based pruning of backoff language models. In *Proc. DARPA Broadcast News Transcription and Understanding Workshop*, pages 270–274. See online version for corrections.

Christoph Tillmann and Hermann Ney. 1996. Statistical language modeling and word triggers. In *Proceedings of the Int. Workshop "Speech and Computer" (SPECOM 96)*, pages 22–27, October.

Joeg P. Ueberla. 1995. More efficient clustering of n-grams for statistical language modeling. In *Eurospeech-95*, pages 1257–1260.

F. Weng, A. Stolcke, and A. Sankar. 1997. Hub4 language modeling using domain interpolation and data clustering. In *1997 DARPA Speech Recognition Workshop*, pages 147–151, February.

Hirofumi Yamamoto and Yoshinori Sagisaka. 1999. Multi-class composite n-gram based on connection direction. In *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing*, Phoenix, Arizona, May.

Ruiqiang Zhang, Ezra Black, Andrew Finch, and Yoshinori Sagisaka. 2000. Integrating detailed information into a language model. In *ICASSP-2000*, pages 1595–1598.

A Kneser-Ney Smoothing

In this section, we first briefly justify interpolated Kneser-Ney smoothing, giving a proof that helps explain why preserving marginals is useful. Then, we give some implementation tips for Kneser-Ney smoothing.

A.1 Proof

First, we give a short theoretical argument for why Kneser-Ney smoothing. Most of our argument is actually given by Chen and Goodman (1999), derived from work done by Kneser and Ney (1995). We prove one additional fact that was previously an assumption, somewhat strengthening our argument. In particular, we will show that any optimal smoothing algorithm must preserve known marginal distributions.

First, we mention that it is impossible to prove that any smoothing technique is optimal, without making at least a few assumptions. For instance, work by MacKay and Peto (1995) proves the optimality of a particular, simple interpolated model, subject to the assumption of a Dirichlet prior. Here, we prove the optimality of Kneser-Ney smoothing, based on other assumptions. We note however, that the assumption of a Dirichlet prior is probably, empirically, a bad one. The Dirichlet prior is a bit odd, and, at least in our work on language modeling, does not seem to correspond well to reality. The assumptions we make are empirically based, rather than based on mathematical convenience. We note also that we are not aware of any case in which another smoothing method outperforms Kneser-Ney smoothing, which is another excellent piece of evidence.

The argument for Kneser-Ney smoothing is as follows. First, absolute discounting empirically works reasonably well, and approximates the true discounts (which can be measured on real data.) Second, interpolated techniques fairly consistently outperform backoff techniques, especially with absolute-discounting style smoothing. These are empirical facts based on many experiments (Chen and Goodman, 1999). Next, Chen and Goodman (1999) and Kneser and Ney (1995) make an assumption that preserving marginal distributions is good. It is this assumption that we will prove below. Finally, Chen and Goodman (1999) prove (following an argument of Kneser and Ney (1995)) that the smoothing technique that uses absolute discounting, interpolation, and preserves marginals is Interpolated Kneser-Ney smoothing. Thus, the contribution of this section is to prove what was previously an assumption, strengthening the argument for Interpolated Kneser-Ney smoothing to one that relies on empirical facts and proofs derived from those facts, with only a very small amount of hand waving.

We previously assumed that smoothing techniques that preserve marginal distributions (e.g. smoothing the bigram such that the unigram distribution is preserved) is a good thing. In this section, we prove that any modeling technique that does not preserve known marginal distributions can be improved by preserving those distributions. Admittedly, when we smooth a Kneser-Ney bigram while preserving the observed unigram distribution, the observed unigram distribution is not the true unigram distribution; it is simply a good approximation to the true unigram.[15][16]

The proof is as follows. Let p represent an estimated distribution, and let P represent the true probability. Consider an estimated bigram distribution of the form $p(z|y)$ such that for some z', $\sum_y p(z'|y)P(y) \neq P(z')$. Then, we show that the entropy with respect to P is reduced by using a new distribution, with a multiplier λ. We simply take the old distribution, multiply the probability of $p(z'|y)$ by λ, and renormalize:

$$p'(z|y) = \begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases}$$

It will turn out that the optimal value for λ is the one such that $\sum_y p'(z'|y)P(y) = P(z')$.

The proof is as follows: consider the entropy of p' with respect to P. It will be convenient to measure entropy in nats instead of bits – that is, using e for the logarithm, instead of 2. (1 nat $= \frac{1}{\ln 2}$ bits). The entropy in nats is

$$\sum_{y,z} -P(y,z) \ln p'(z|y)$$

Consider the derivative of the entropy with respect to λ.

$$\begin{aligned}
&\frac{d}{d\lambda} \sum_{y,z} -P(y,z) \ln p'(z|y) \\
&= \frac{d}{d\lambda} \sum_{y,z} -P(y,z) \ln \begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases} \\
&= \sum_{y,z} P(y,z) - \frac{d}{d\lambda} \ln \begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases} \\
&= \sum_{y,z} \frac{-P(y,z)}{\begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases}} \frac{d}{d\lambda} \begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases} \\
&= \sum_{y,z} \frac{-P(y,z)}{\begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases}} \begin{cases} -\frac{p(z|y)\frac{d}{d\lambda}(1+(\lambda-1)p(z'|y))}{(1+(\lambda-1)p(z'|y))^2} & \text{if } z \neq z' \\ \frac{(1+(\lambda-1)p(z'|y))\frac{d}{d\lambda}\lambda p(z'|y) - \lambda p(z'|y)\frac{d}{d\lambda}(1+(\lambda-1)p(z'|y))}{(1+(\lambda-1)p(z'|y))^2} & \text{if } z = z' \end{cases}
\end{aligned}$$

[15]In fact, in practice, we smooth the unigram distribution with the uniform distribution because of its divergence from the true distribution.

[16]This is the very small amount of hand waving. Whether or not one agrees with this "proof", it is certainly an improvement over the previous argument which was simply "Preserving the marginal distribution is good."

$$
\begin{aligned}
&= \sum_{y,z} \frac{-P(y,z)}{\begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases}} \begin{cases} -\frac{p(z|y)p(z'|y)}{(1+(\lambda-1)p(z'|y))^2} & \text{if } z \neq z' \\ \frac{(1+(\lambda-1)p(z'|y))p(z'|y)-\lambda p(z'|y)p(z'|y)}{(1+(\lambda-1)p(z'|y))^2} & \text{if } z = z' \end{cases} \\
&= \sum_{y,z} \frac{-P(y,z)}{\begin{cases} \frac{p(z|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{\lambda p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z = z' \end{cases}} \begin{cases} -\frac{p(z|y)p(z'|y)}{(1+(\lambda-1)p(z'|y))^2} & \text{if } z \neq z' \\ \frac{(1-p(z'|y))p(z'|y)}{(1+(\lambda-1)p(z'|y))^2} & \text{if } z = z' \end{cases} \\
&= \sum_{y,z} -P(y,z) \begin{cases} -\frac{p(z'|y)}{1+(\lambda-1)p(z'|y)} & \text{if } z \neq z' \\ \frac{1-p(z'|y)}{\lambda(1+(\lambda-1)p(z'|y))} & \text{if } z = z' \end{cases}
\end{aligned}
$$

Now, if $\lambda = 1$, this reduces to

$$
\begin{aligned}
\sum_{y,z} -P(y,z) \begin{cases} -p(z'|y) & \text{if } z \neq z' \\ 1 - p(z'|y) & \text{if } z = z' \end{cases} &= -P(z') + \sum_y P(y) \sum_z P(z|y)p(z'|y) \\
&= -P(z') + \sum_y P(y)p(z'|y) \sum_z P(z|y) \\
&= -P(z') + \sum_y P(y)p(z'|y) \\
&= -P(z') + \sum_y P(y)p(z'|y)
\end{aligned}
$$

which will be zero only when the true marginal, $P(z')$ is equal to the marginal of p. If $\sum_y P(y)p(z'|y) > P(z')$, then the derivative will be > 0, meaning that we can decrease the entropy by decreasing λ, and if $\sum_y P(y)p(z'|y) < P(z')$, then the derivative will be < 0, meaning that we can decrease the entropy by increasing λ. Thus, whenever $\sum_y P(y)p(z'|y) \neq P(z')$, there will be some value of λ other than 1 that leads to lower entropy.

This means that if we have some smoothing algorithm that does not preserve the known marginal distributions, we can modify the resulting distribution by multiplying it in such a way that the marginal is preserved. Thus, any smoothing algorithm that does not preserve the known marginals is suboptimal.

(Notice that our argument was not really about smoothing, but just about preserving known marginal distributions in general. This argument is the same one used to justify maximum entropy models: the best model will be the one that preserves all the known marginals. Thus, this proof is not really new; the only novel part is noticing that this fact applies to smoothing algorithms, as well as to other distributions.)

A.2 Implementing Kneser-Ney Smoothing

The formula for Kneser-Ney smoothing is a bit complex looking. However, the algorithm for actually finding the counts for Kneser-Ney smoothing is actually quite simple, requiring only trivial changes from interpolated absolute discounting. In Figure 16 we give the code for training an interpolated absolute discounting model, and in Figure 17 we give the corresponding code for an interpolated Kneser-Ney model. The lines that change are marked with an asterisk (*) – notice that the only change is in the placement of two right curly braces (}). For completeness, Figure 18 gives the code for testing either interpolated absolute discounting or interpolated Kneser-Ney smoothing – the code is identical.

B Implementation Notes

While this paper makes a number of contributions, they are much less in the originality of the ideas, and much more in the completeness of the research: its thoroughness, size, combination, and optimality. This research was in many areas more thorough, than any other previous research: trying more variations on ideas; able to exceed previous size boundaries, such as examining 20-grams; able to combine more ideas; and more believable, because nearly all parameters were optimized jointly on heldout data. These contributions were made possible by a well designed tool, with many interesting implementation details. It is perhaps these implementation details that were the truly novel part of the work.

In this section, we sketch some of the most useful implementation tricks needed to perform this work.

B.1 Store only what you need

The single most important trick used here, part of which was previously used by Chen and Goodman (1999), was to store only those parameters that are actually needed for the models. In particular, the very first action the program takes is to read through the heldout and test data, and determine which counts will be needed to calculate the heldout and test probabilities. Then, during the training phase, only the needed counts are recorded. Note that the number of necessary counts may be much smaller than the total number of counts in the test data. For instance, for a test instance $P(z|xy)$, we typically need to store all training counts $C(xy*)$. However, even this depends on the particular model type. For instance, for our baseline smoothing, simple interpolation, it is only necessary to know $C(xy)$ and $C(xyz)$ (as well as the lower order counts). This fact was useful in our implementation of caching, which used the baseline smoothing for efficiency.

For Kneser-Ney smoothing, things become more complex: for the trigram, we need to temporarily store $C(xy*)$ so that we can compute $|\{v|C(xyv) > 0\}|$. Actually, we only need to store for each xyv whether we have seen it or not. Since we do not need to store the actual count, we can save a fair amount of space. Furthermore, once we know $|\{v|C(xyv) > 0\}|$, we can discard the table of which ones occurred. Partially for this reason, when we calculate the parameters of our models, we do it in steps. For instance, for a skipping model that interpolates two different kinds of trigrams, we first find the needed parameters for the first

```
# code for implementing interpolated absolute discounting
# usage: discount training test
# training data and test data are one word per line

$discount = shift; # must be between 0 and 1
$train = shift;
$test = shift;

$w2 = $w1 = "";

open TRAIN, $train;
while (<TRAIN>) {         # For each line in the training data
    $w0 = $_;
    $TD{$w2.$w1}++;       # increment Trigram Denominator
    if ($TN{$w2.$w1.$w0}++ == 0) { # increment Trigram Numerator
        $TZ{$w2.$w1}++;   # if needed, increment Trigram non-Zero counts
*   }                     # This curly brace will move for Kneser-Ney

    $BD{$w1}++;           # increment Bigram Denominator
    if ($BN{$w1.$w0}++==0){ # increment Bigram Numerator
        $BZ{$w1}++;       # if needed, increment Bigram non-Zero counts
*   }                     # This curly brace will move for Kneser-Ney

    $UD++;                # increment Unigram Denominator
    $UN{$w0}++;           # increment Unigram Numerator

    $w2 = $w1;
    $w1 = $w0;
}
close TRAIN;
```

Figure 16: Interpolated Absolute Discounting Perl code

```
# code for implementing interpolated Kneser-Ney
# usage: discount training test
# training data and test data are one word per line

$discount = shift; # must be between 0 and 1
$train = shift;
$test = shift;

$w2 = $w1 = "";

open TRAIN, $train;
while (<TRAIN>) {           # For each line in the training data
    $w0 = $_;
    $TD{$w2.$w1}++;         # increment Trigram Denominator
    if ($TN{$w2.$w1.$w0}++ == 0) { # increment Trigram Numerator
        $TZ{$w2.$w1}++;     # if needed, increment Trigram non-Zero counts

        $BD{$w1}++;         # increment Bigram Denominator
        if ($BN{$w1.$w0}++==0){ # increment Bigram Numerator
            $BZ{$w1}++;     # if needed, increment Bigram non-Zero counts

            $UD++;          # increment Unigram Denominator
            $UN{$w0}++;     # increment Unigram Numerator
*       }                   # This curly brace moved for Kneser-Ney
*   }                       # This curly brace moved for Kneser-Ney

    $w2 = $w1;
    $w1 = $w0;
}
close TRAIN;
```

Figure 17: Interpolated Kneser-Ney Perl Code

```
$w2 = $w1 = "";

open TEST, $test;
while (<TEST>) {              # For each line in the test data
    $w0 = $_;
    $unigram = $UN{$w0} / $UD;        # compute unigram probability
    if (defined($BD{$w1})) {  # non-zero bigram denominator
        if (defined($BN{$w1.$w0})) { # non-zero bigram numerator
            $bigram = ($BN{$w1.$w0} - $discount) /
                      $BD{$w1};
        } else {
            $bigram = 0;
        }
        $bigram = $bigram +
                 $BZ{$w1}* $discount / $BD{$w1} * $unigram;

        # Now, add in the trigram probability if it exists
        if (defined($TD{$w2.$w1})) { # non-zero trigram denominator
            if (defined($TN{$w2.$w1.$w0})) { # non-zero trigram numerator
                $trigram = ($TN{$w2.$w1.$w0} - $discount) /
                           $TD{$w2.$w1};
            } else {
                $trigram = 0;
            }
            $trigram = $trigram +
                      $TZ{$w2.$w1} * $discount / $TD{$w2.$w1} * $bigram;
            $probability = $trigram;
        } else {
            $probability = $bigram;
        }
    } else {
        $probability = $unigram;
    }

    print "Probability:\n  $w0    $w2    $w1       = $probability\n";

    $w2 = $w1;
    $w1 = $_;
}
close TEST;
```

Figure 18: Testing Code for Interpolated Absolute Discounting or Kneser-Ney

model, then discard any parts of the model we no longer need, then find the parameters for the second model. This results in a very significant savings in storage.

One problem with this "store only what you need" technique is that it interacts poorly with Katz smoothing. In particular, Katz smoothing needs to know the discount parameters. Recall that n_r represents the number of n-grams that occur r times, i.e.

$$n_r = |\{w_{i-n+1}...w_i | C(w_{i-n+1}...w_i) = r\}|$$

and that the Good-Turing formula states that for any n-gram that occurs r times, we should pretend that it occurs $disc(r)$ times where

$$disc(r) = (r+1)\frac{n_{r+1}}{n_r}$$

On its face, computing the n_r requires knowing all of the counts. However, typically, discounting is only done for counts that occur 5 or fewer times. In that case, we only need to know n_1, n_2, ..., n_6. In fact, however, we do not need to know n_r and n_{r+1} but only their ratio. This means that we can sample the counts and estimate their relative frequencies. By sampling the counts, we do *not* mean sampling the data: we mean sampling the counts. In particular, we define a random function of a sequence of words (a simple hash function); we store the count for a sequence $w_1w_2...w_n$ if and only if $hash(w_1w_2...w_n) \bmod s = 0$, where s is picked in such a way that we get 25,000 to 50,000 counts. This does not save any time, since we still need to consider ever count $C(w_1w_2...w_n)$ but it saves a huge amount of space. In pilot experiments, the effect of this sampling on accuracy was negligible.

One problem with the "store only what you need" technique is that it requires reading the test data. If there were a bug, then excellent (even perfect) performance could be achieved. It was therefore important to check that no such bugs were introduced. This can be done in various ways. For instance, we added an option to the program that would compute the probability of all words, not just the words in the test data. We then verified that the sum of the probabilities of all words was within roundoff error (10^{-9}) of 1. We also verified that the probability of the correct word in the special score-everything mode was the same as the probability in the normal mode. Also, none of the individual results we report are extraordinarily good; all are in the same range as typically reported for these techniques. The only extraordinarily good result comes from combining all technique, and even here, the total improvement is less than the total of the individual improvements, implying it is well within the reasonable range. Furthermore, when our code is used to rescore n-best lists, it reads through the lists and figures out which counts are needed to score them. However, the n-best lists have no indication of which is the correct output. (A separate, standard program scores the output of our program against the correct transcription.) Since we get good improvement on n-best rescoring, and the improvement is consistent with the perplexity reductions, there is further

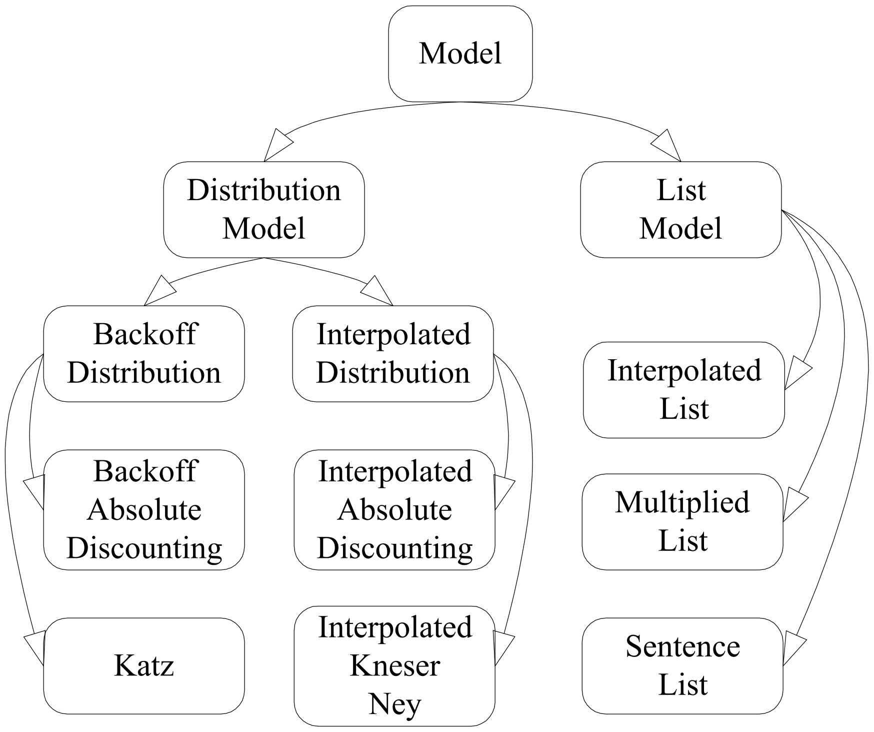

Figure 19: Class Structure

evidence that our implementation of "store only what you need" did not lead to any cheating.

B.2 System architecture

It turns out that language modeling is an ideal place to use object oriented methods, and inheritance. In particular, our language modeling tool was based on a virtual base class called Model. The single most important member function of a model provides the probability of a word or a cluster given a context. Models have all sorts of other useful functions; for instance, they can return the set of parameters that can be optimized. They can go through the test data and determine which counts they will need. They can go through the training data and accumulate those counts.

From this base class, we can construct all sorts of other subclasses. In particular, we can construct models that store probabilities – DistributionModels – and models that contain other models – ListModels.

The DistributionModels are, for instance, Kneser-Ney smoothed models, Katz smoothed models, simple interpolated models, and absolute discounting models. Even within these models, there is some substructure; for instance, absolute discounting backoff models, Kneser-Ney backoff models, and Katz models can share most of their code, while interpolated Kneser-Ney models and interpolated absolute discounting models can also share code.

The other type of model is a model containing other models. For instance, an InterpolateModel contains a list of other models, whose probability it interpolates. For clustered models of the form $P(Z|XY) \times P(z|Z)$, there is a model that contains a list of other models, and multiplies their probabilities. There is also a SentenceInterpolateModel that contains a list of other models, and interpolates them at the sentence level, instead of the word level. All of these container type models have similar behavior in many ways, and much of their code can be shared as well.

It was in part this architecture, providing a set of basic building blocks and tools for combining them, that allowed so many different experiments to be performed.

B.3 Parameter search

Another key tool was our parameter search engine. In many cases, it is possible to find EM algorithms to estimate the various parameters of models. In other cases it is possible to find good theoretical approximate closed form solutions, using leaving-one-out, such as for discounting parameters. But for the large number of different models used here, this approach would have lead to endless coding, and, for some of the more complex model types, perhaps to suboptimal parameter settings. Instead, we used Powell's method (Press et al., 1988) to search all parameters of our models. This had many advantages. First, there are some parameters, such as β in Katz smoothing, that are hard to find except through a general search procedure. Second, it meant that we could optimize all parameters jointly, rather than separately. For instance, consider a skipping model that interpolates two models together. We could, traditionally, optimize the smoothing parameters for one model, optimize the smoothing parameters for the other model, and then find the optimal interpolation interpolation of the two models, without reestimating the smoothing parameters. Instead, we find jointly optimal settings. Of course, like all gradient descent techniques, the parameters we find are probably only locally optimal, but that is still an improvement over traditional techniques.

We used certain tricks to speed up and improve the parameter search. While in theory Powell search should always find local minima, it can run into certain problems. Consider interpolating two distributions, $\lambda P_1(w) + (1 - \lambda)P_2(w)$. Imagine that the parameters for P_2 have not yet been optimized. Now, the Powell search routine optimizes λ. Since P_2 is awful, λ is set at 1 or nearly 1. Eventually, the routine searches the parameters of P_2, but, finding essentially no effect from changing these parameters, quickly gives up searching them. Instead, we search interpolation parameters last, and start with all distributions interpolated evenly. This ensures that each distribution has its parameters optimized.

Furthermore, searching the parameters of P_2 interpolated with P_1 is slow, since both distributions must be evaluated. We typically search the parameters of each one separately first, potentially halving the computation. Only once each one is optimized do we optimize them jointly, but since they are often already nearly optimal this search goes faster. In the case of two distributions, the savings may be small, but in the case of 10, it may be almost an order of magnitude. This technique must however be used with care, since sometimes the optimal settings of one distribution differ wildly from their settings when combined. In particular, with sentence mixture models, we found that an individual mixture component might significantly oversmooth, unless it was trained jointly.

B.4 Clustering

There is no shortage of techniques for generating clusters, and there appears to be little evidence that different techniques that optimize the same criterion result in a significantly different quality of clusters. We note, however, that different algorithms may require significantly different amounts of run time. We used several techniques to speed up our clustering significantly.

The basic criterion we followed was to minimize entropy. In particular, assume that the model we are using is of the form $P(z|Y)$; we want to find the placement of words y into clusters Y that minimizes the entropy of this model. This is typically done by swapping words between clusters whenever such a swap reduces the entropy.

The first important approach we took for speeding up clustering was to use a top-down approach. We note that agglomerative clustering algorithms – those which merge words bottom up – may require significantly more time than top-down, splitting algorithms. Thus, our basic algorithm is top-down. However, at the end, we perform four iterations of swapping all words between all clusters. This final swapping is typically the most time consuming part of the algorithm.

Another technique we use is Buckshot (Cutting et al., 1992). The basic idea is that even with a small number of words, we are likely to have a good estimate of the parameters of a cluster. So, we proceed top down, splitting clusters. When we are ready to split a cluster, we randomly pick a few words, and put them into two random clusters, and then swap them in such a way that entropy is decreased, until convergence (no more decrease can be found). Then we add a few more words, typically $\sqrt(2)$ more, and put each into the best bucket, then swap again until convergence. This is repeated until all words in the current cluster have been added and split. We haven't tested this particularly thoroughly, but our intuition is that it should lead to large speedups.

We use one more important technique that speeds computations, adapted from earlier work of Brown et al. (1992). We attempt to minimize the entropy of our clusters. Let v represent words in the vocabulary, and W represent a potential cluster. We minimize

$$\sum_v C(Wv) \log P(v|W)$$

The inner loop of this minimization considers adding (or removing) a word x to cluster W. What will the new entropy be? On it's face, this would appear to require computation proportional to the vocabulary size to recompute the sum. However, letting the new cluster, $W + x$ be called X,

$$\sum_v C(Xv) \log P(v|X) = \sum_{v|C(xv)\neq 0} C(Xv) \log P(v|X) + \sum_{v|C(xv)=0} C(Xv) \log P(v|X) \quad (10)$$

The first summation in Equation 10 can be computed relatively efficiently, in time proportional to the number of different words that follow x; it is the second summation that needs to be transformed:

$$\begin{aligned} &\sum_{v|C(xv)=0} C(Xv) \log P(v|X) \\ &= \sum_{v|C(xv)=0} C(Wv) \log \left(P(v|W) \frac{C(W)}{C(X)} \right) \\ &= \sum_{v|C(xv)=0} C(Wv) \log P(v|W) + \left(\log \frac{C(W)}{C(X)} \right) \sum_{v|C(xv)=0} C(Wv) \quad (11) \end{aligned}$$

Now, notice that

$$\sum_{v|C(xv)=0} C(Wv) \log P(v|W) = \sum_v C(Wv) \log P(v|W) - \sum_{v|C(xv)\neq 0} C(Wv) \log P(v|W) \quad (12)$$

and that

$$\sum_{v|C(xv)=0} C(Wv) = \left(C(W) - \sum_{v|C(xv)\neq 0} C(Wv) \right) \quad (13)$$

Substituting Equations 12 and 13 into Equation 11, we get

$$\begin{aligned} &\sum_{v|C(xv)=0} C(Xv) \log P(v|X) \\ &= \sum_v C(Wv) \log P(v|W) - \sum_{v|C(xv)\neq 0} C(Wv) \log P(v|W) \\ &\quad + \left(\log \frac{C(W)}{C(X)} \right) \left(C(W) - \sum_{v|C(xv)\neq 0} C(Wv) \right) \end{aligned}$$

Now, notice that $\sum_v C(Wv) \log P(v|W)$ is just the old entropy, before adding x. Assuming that we have precomputed/recorded this value, all the other summations only sum over words v for which $C(xv) > 0$, which, in many cases, is much smaller than the vocabulary size.

Many other clustering techniques (Brown et al., 1992) attempt to maximize $\sum_{Y}, ZP(YZ)\log\frac{P(Y|Z)}{P}(Z)$, where the same clusters are used for both. The original speedup formula uses this version, and is much more complex to minimize. Using different clusters for different positions not only leads to marginally lower entropy, but also leads to simpler clustering.

B.5 Smoothing

Although all smoothing algorithms were reimplemented for this research, the details closely follow Chen and Goodman (1999). This includes our use of additive smoothing of the unigram distribution for both Katz smoothing and Kneser-Ney smoothing. That is, we found a constant b which was added to all unigram counts; this leads to better performance in small training-data situations, and allowed us to compare perplexities across different training sizes, since no unigram received 0 counts, meaning 0 probabilities were never returned. For Katz smoothing, we found a maximum count to discount, based on when data sparsity prevented good estimates of the discount. As is typically done, we corrected the lower discounts so that the total probability mass for 0 counts was unchanged by the cutoff, and we also included a parameter β, which was added to a distribution whenever no counts were discounted.

We used a novel technique for getting the counts for Katz smoothing. As described in Appendix B.1, we do not record all counts, but only those needed for our experiments. This is problematic for Katz smoothing, where we need counts of counts (n_1, n_2, etc.) in order to compute the discounts. Actually, all we really need is the ratio between these counts. We use a statistical sampling technique, in which we randomly sample 25,000 to 50,000 counts (not word histories, but actual counts); this allows us to accurately estimate the needed ratios, with much less storage.