Algorithms for Learning Probabilistic and Causal Models with Possible Imperfect Advice

Teamcore Postdoc Seminar

23rd August 2024

Davin Choo

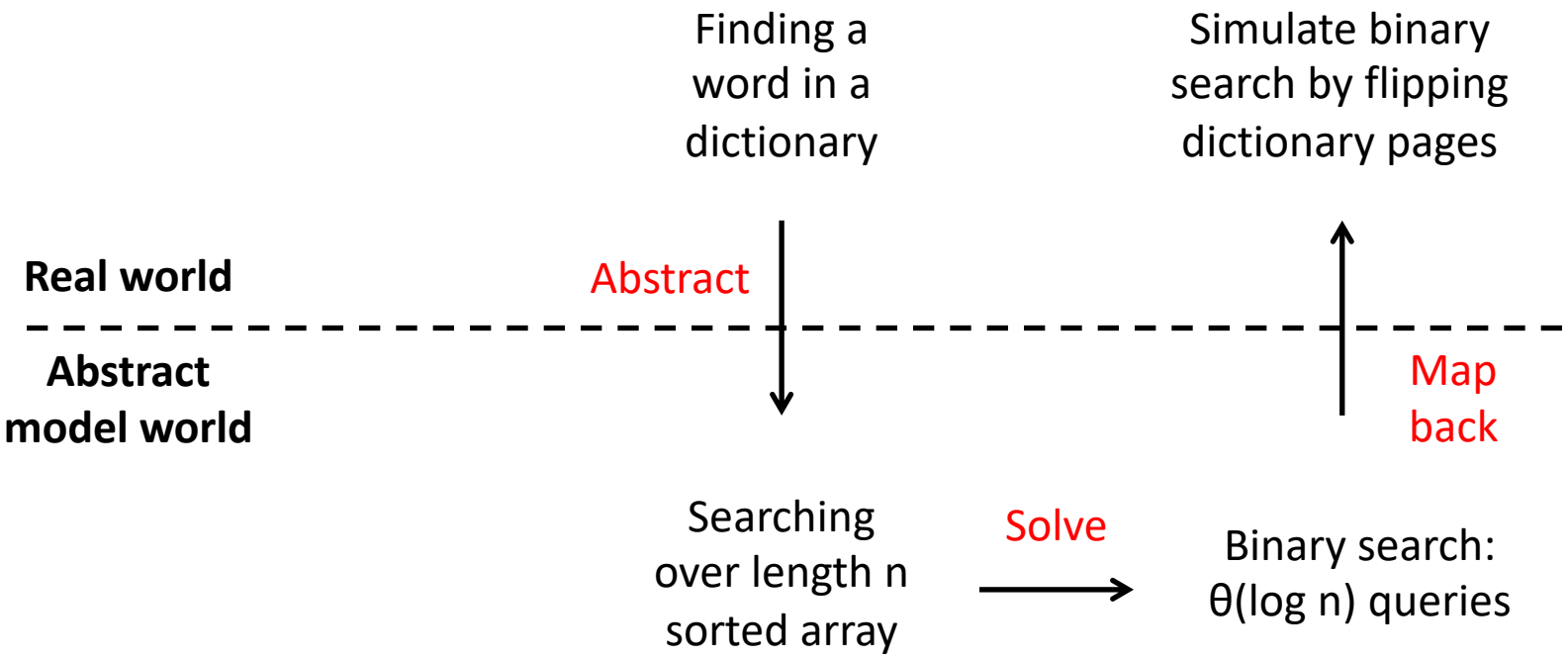
National University of Singapore

The world is complex

- **Learning a useful representation** of the world from data is a cornerstone of scientific discovery and the driving force behind successful modern machine learning methods
- This process often involves learning **probabilistic models for predictive tasks** and **causal models to understand interventional effects on systems**, which is crucial for informed downstream decision-making

https://huggingface.co/spaces/dalle-mini/dalle-mini on “A thoughtful person stands at the edge of a futuristic cityscape, filled with intricate technology, complex networks, and interconnected systems. They gaze intently at a holographic globe that displays dynamic data, charts, and code swirling around it, their expression a mix of curiosity and determination as they ponder how to interact and take action in this complex world”

A general problem-solving framework



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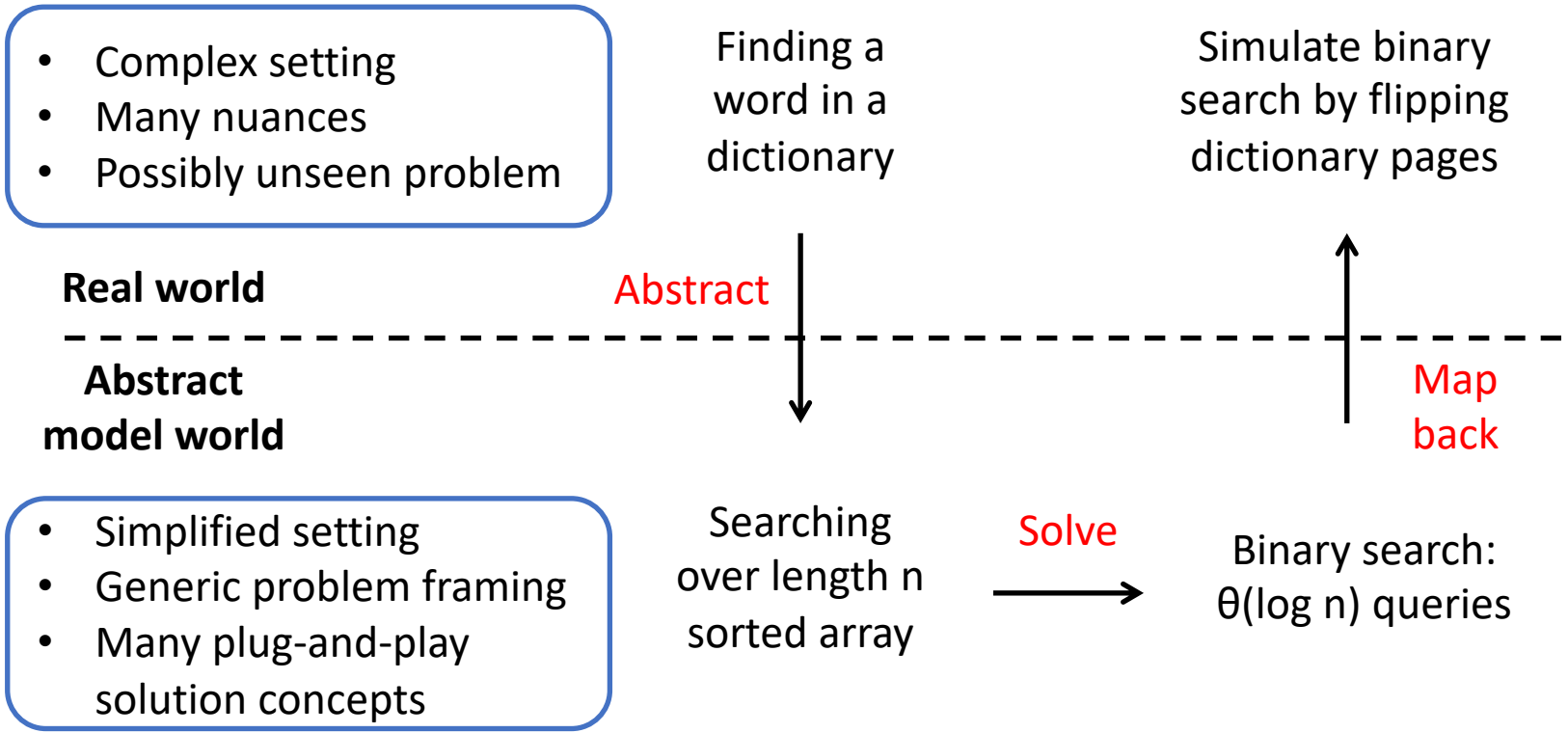
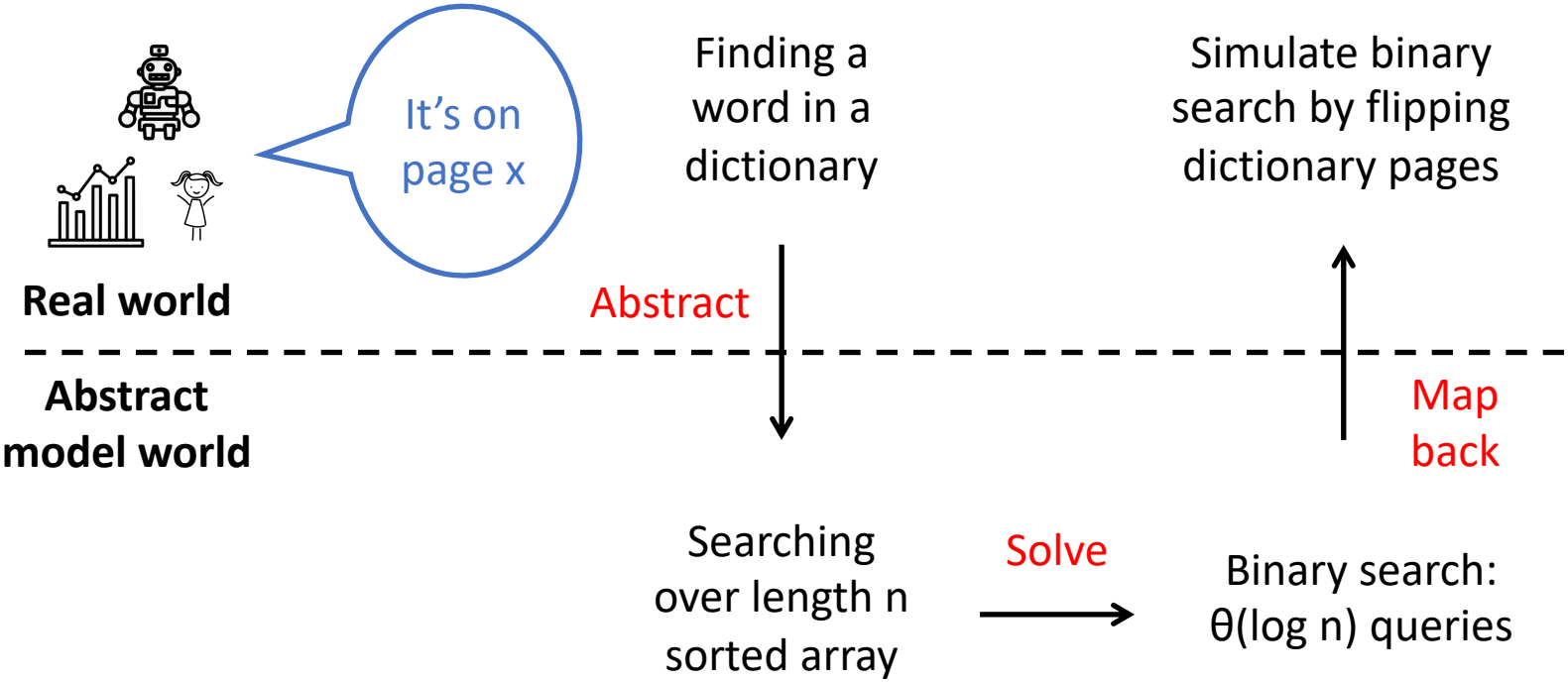


Side-information about problem instances



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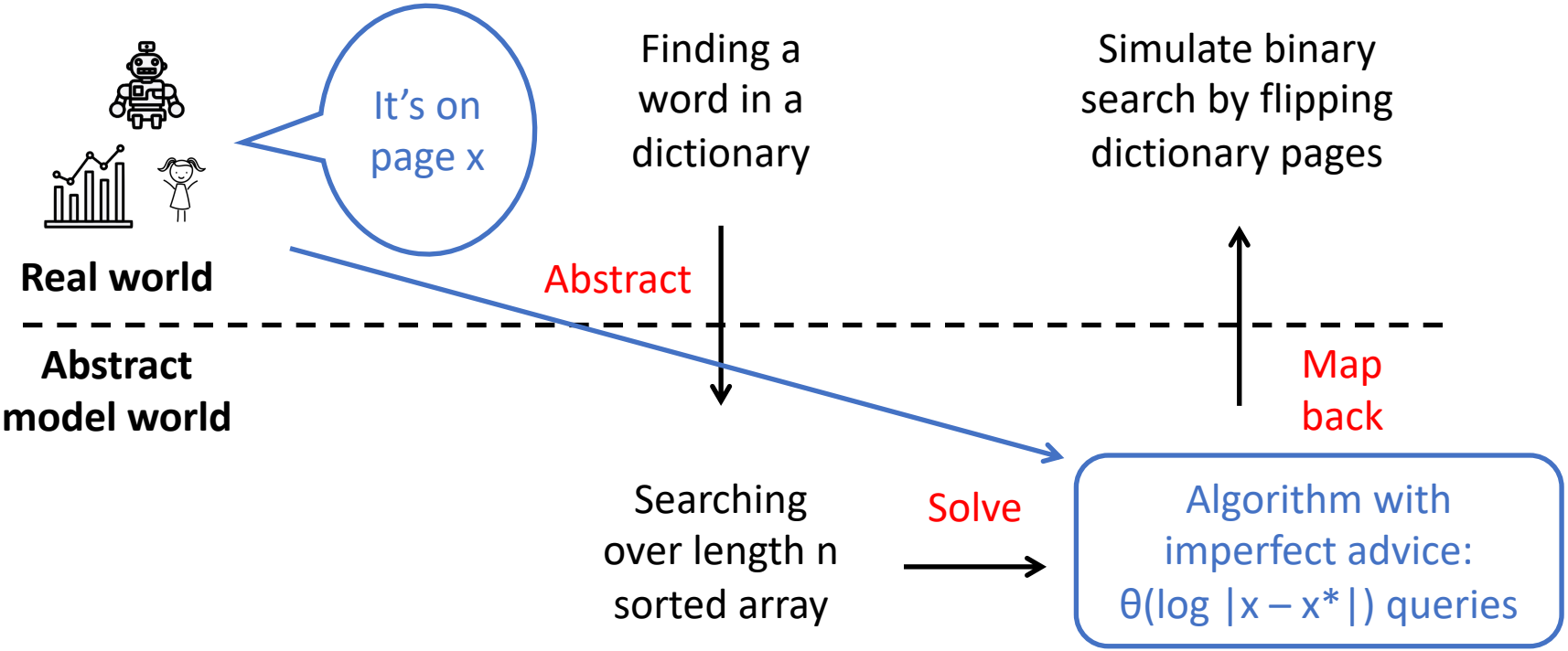
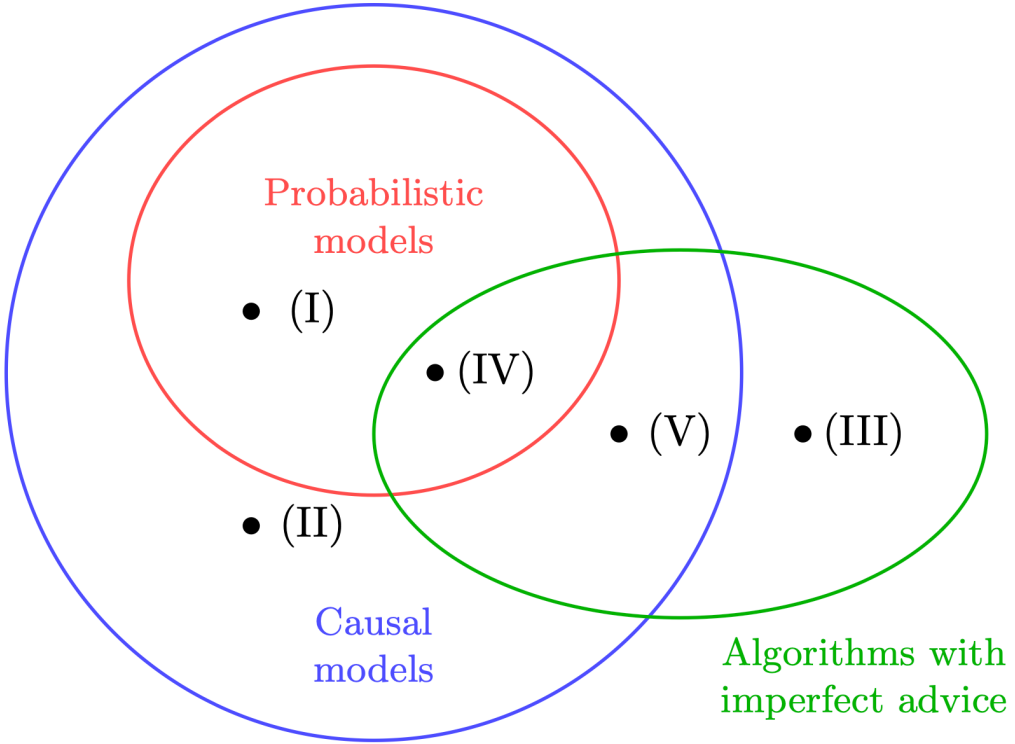
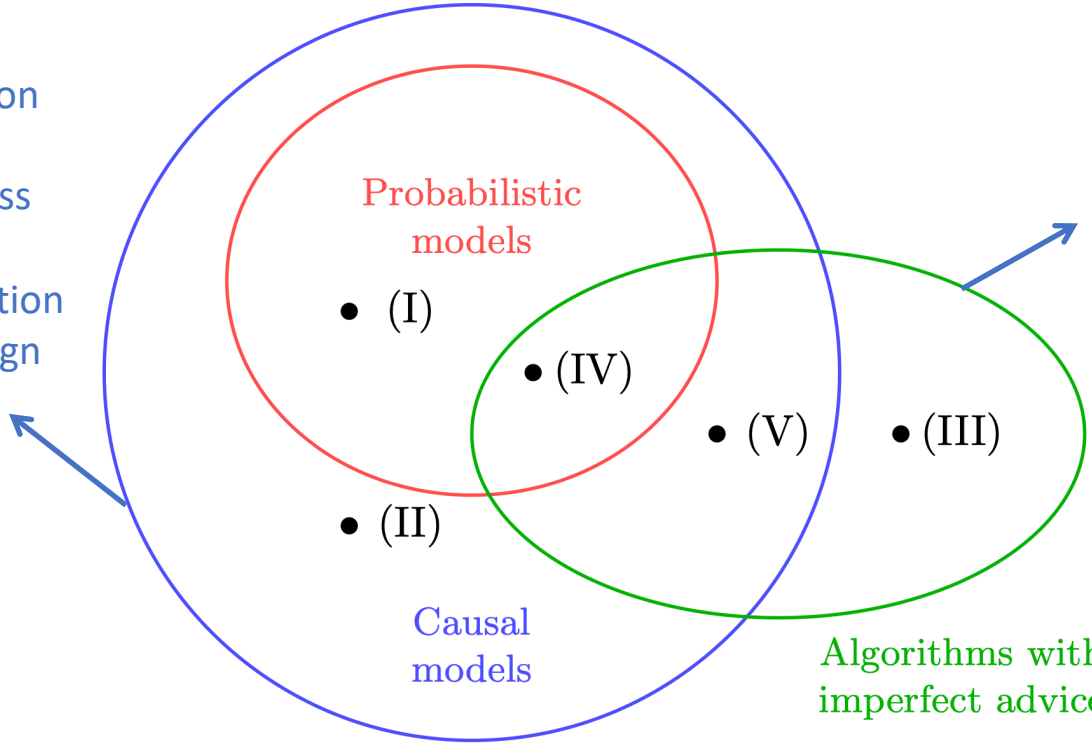


Main themes explored in my PhD



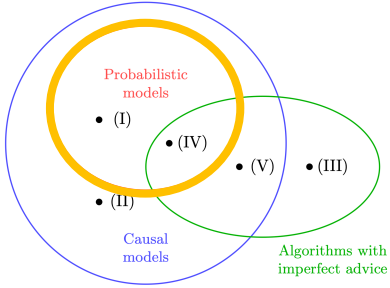
Relevance to “AI for social impact”

- Improve decision making
- Mitigate fairness and bias
- Aid in intervention and policy design



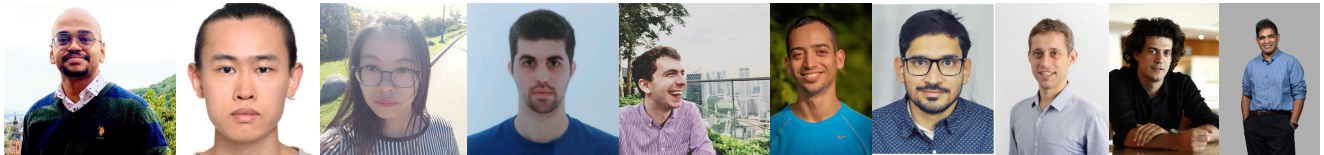
- Principally incorporate domain knowledge
- Enhance performance with provable guarantees

(I): Probabilistic models



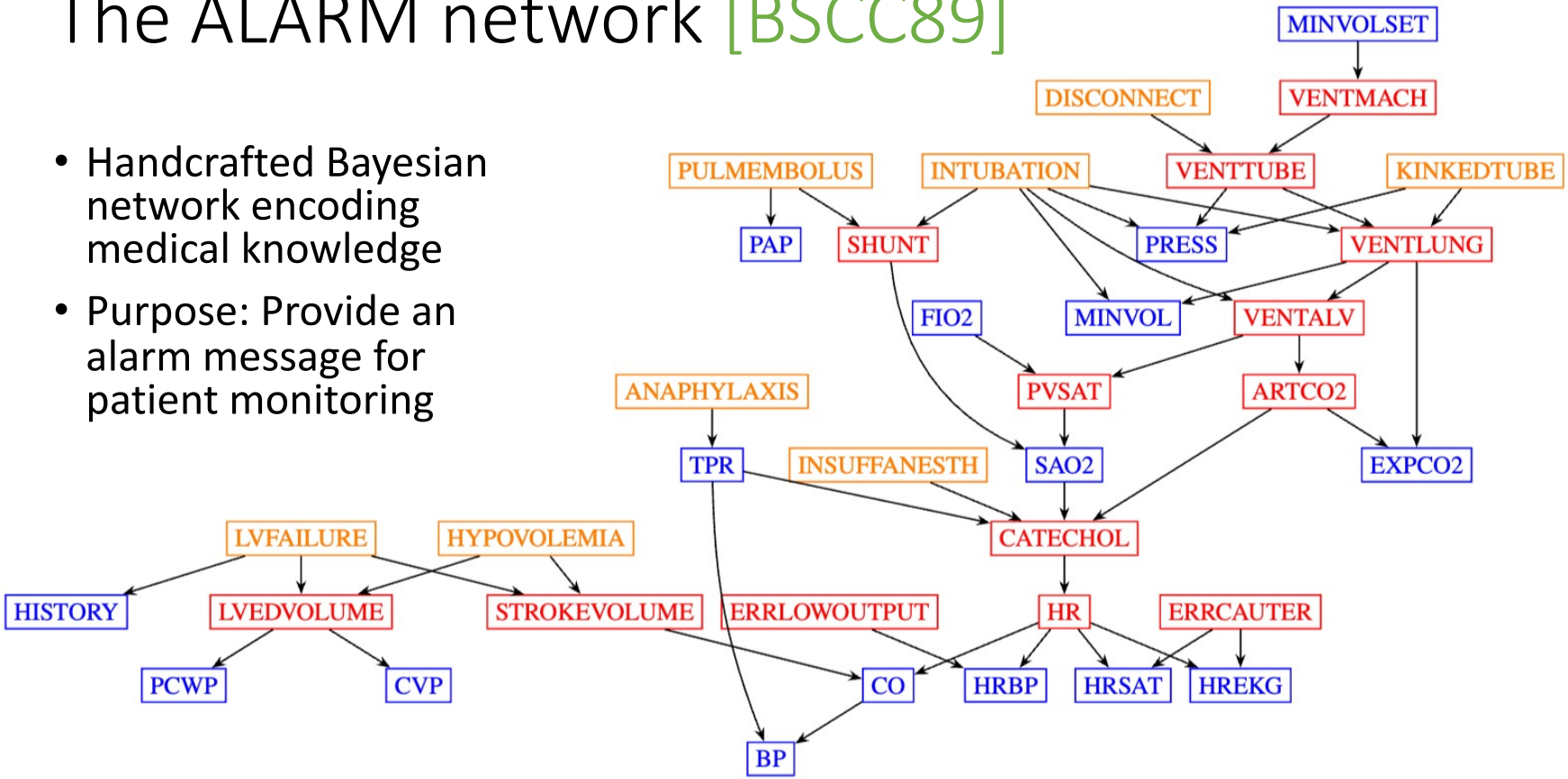
- Classic results in statistics show asymptotic convergence of estimators in the "infinite data" regime
- Probably Approximately Correct (PAC) learning model [Val84]
  - Given sample access to some underlying distribution $\mathcal{P}$, produce $\hat{\mathcal{P}}$ such that $\mathrm{TV}(\mathcal{P}, \hat{\mathcal{P}}) \leq \varepsilon$ with probability $\geq 1 - \delta$
- Bayesian networks [Pea88]
  - Probabilistic graphical model commonly used to model beliefs
  - $\approx 2^{n^2}$ candidate directed acyclic graphs (DAGs), one of which is $\mathcal{G}^*$

[Val84] Leslie G Valiant. *A theory of the learnable*. Communications of the ACM, 1984.
[Pea88] Judea Pearl. *Probabilistic reasoning in intelligent systems*. Morgan Kaufmann, 1988.



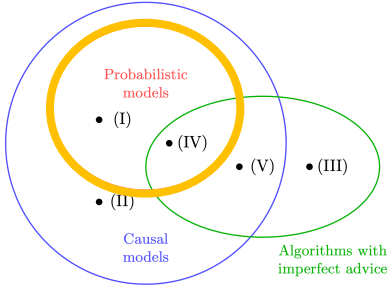
The ALARM network [BSCC89]

- Handcrafted Bayesian network encoding medical knowledge
- Purpose: Provide an alarm message for patient monitoring



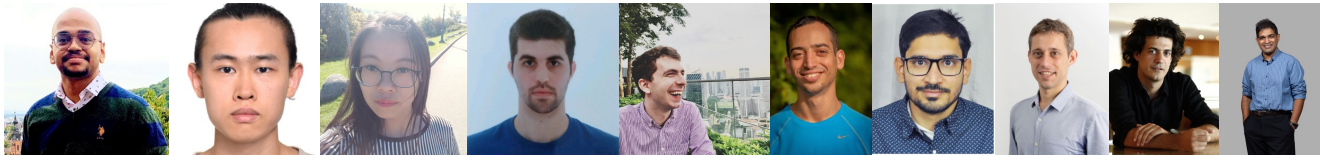
[BSCC89] Ingo A Beinlich, Henri Jacques Suermondt, R Martin Chavez, and Gregory F Cooper. *The alarm monitoring system: A case study with two probabilistic inference techniques for belief network*. Second European Conference on Artificial Intelligence in Medicine (AIME), 1989

(I): Probabilistic models

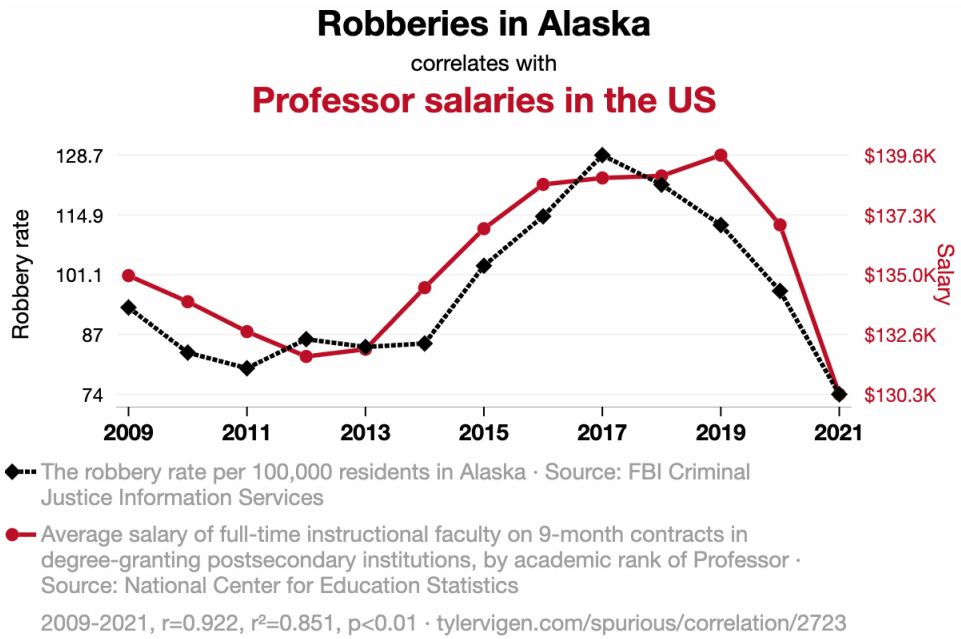


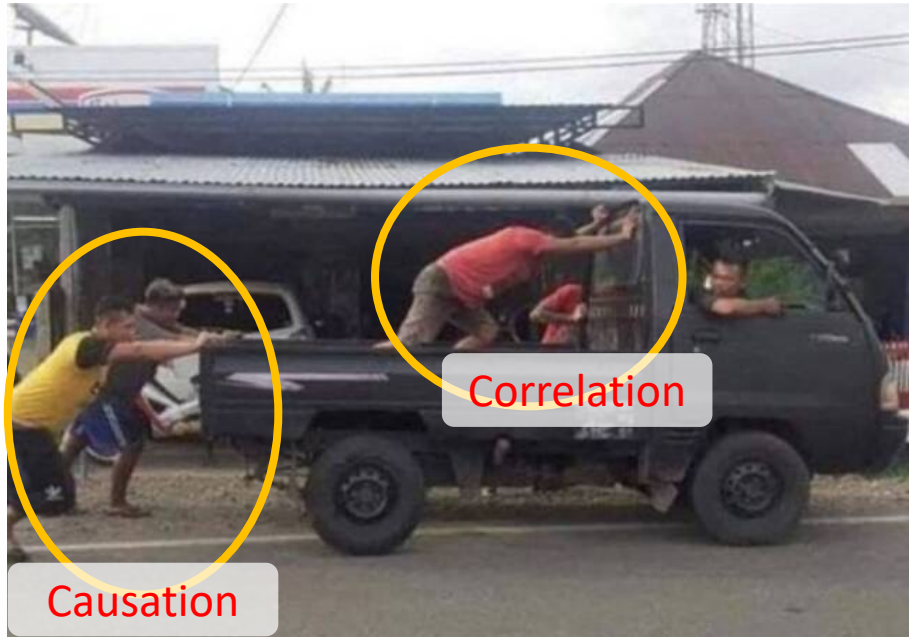
- Suppose data distribution $\mathcal{P}$ is described by Bayesian network
  - NP-hard to find "score maximizing" DAG from data [Chi96] and to decide whether $\mathcal{P}$ can be described by a DAG with p parameters [CHM04]
  - Even under the promise that $\mathcal{P}$ can be described by a DAG with p parameters, it is NP-hard to find such a parameter-bounded DAG [BCGM24]
  - We also have some PAC-style finite sample results in learning the structure and parameters of Bayesian network for $\mathcal{P}$ [BCG+22, DDKC23, CYBC24]
  - Insight: If network's in-degree is bounded, we can use less samples

[Chi96] David Maxwell Chickering. *Learning Bayesian networks is NP-complete*. Lecture Notes in Statistics, vol 112, 1996
[CHM04] Max Chickering, David Heckerman, and Chris Meek. *Large-sample learning of Bayesian networks is NP-hard*. Journal of Machine Learning Research (JMLR), 2004
[BCGM24] Arnab Bhattacharyya, Davin Choo, Sutanu Gayen, Dimitrios Myrisiotis. *Learnability of Parameter-Bounded Bayes Nets*. Structured Probabilistic Inference & Generative Modeling (ICML Workshop), 2024
[BCG+22] Arnab Bhattacharyya, Davin Choo, Rishikesh Gajjala, Sutanu Gayen, Yuhao Wang. *Learning Sparse Fixed-Structure Gaussian Bayesian Networks*. International Conference on Artificial Intelligence and Statistics (AISTATS), 2024
[DDKC23] Yuval Dagan, Constantinos Daskalakis, Anthimos-Vardis Kandiros, Davin Choo. *Learning and Testing Latent-Tree Ising Models Efficiently*. Conference on Learning Theory (COLT), 2023
[CYBC24] Davin Choo, Joy Qiping Yang, Arnab Bhattacharyya, Clément L. Canonne. *Learning bounded degree polytrees with samples*. International Conference on Algorithmic Learning Theory (ALT), 2024



Correlation does not imply causation





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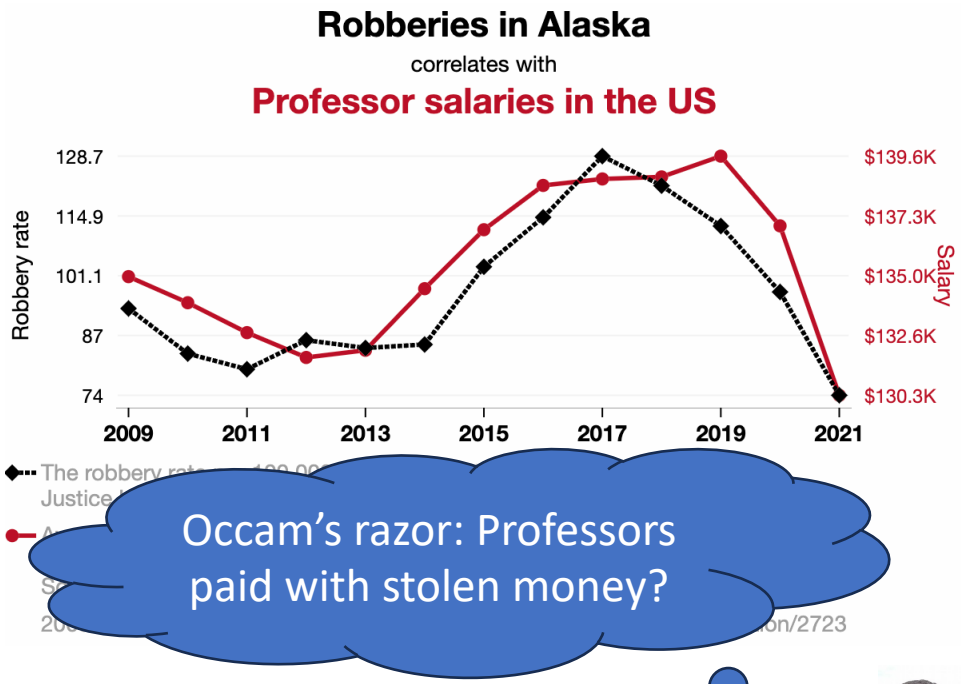
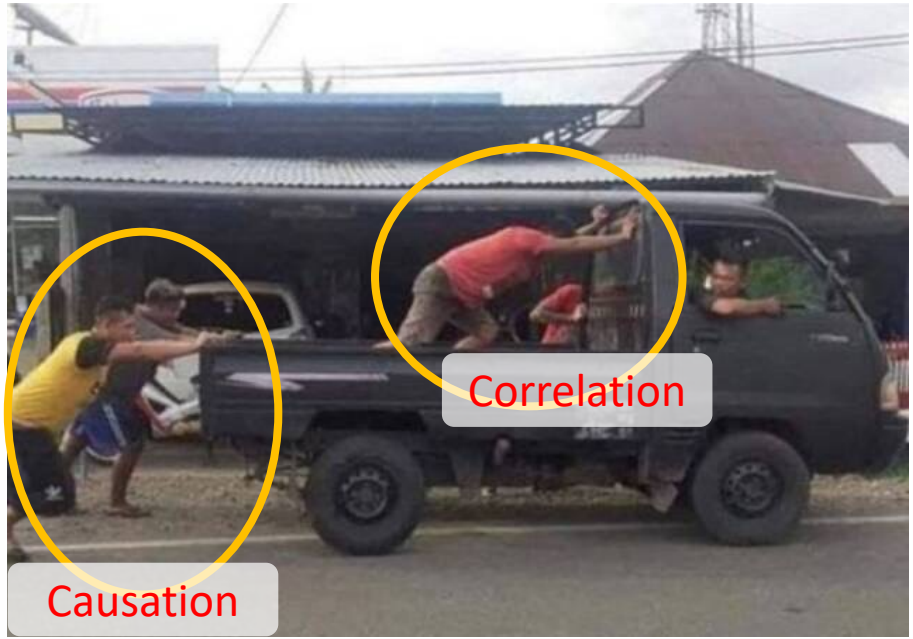
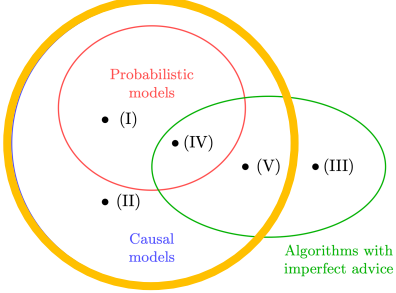


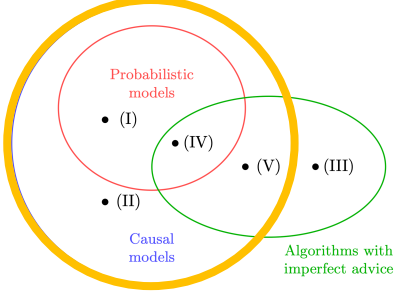


(II): Causal models



- Two fundamental problems in causal inference
  - Causal graph discovery: Recover true causal graph $\mathcal{G}^*$
  - Causal effect estimation: Estimate $\mathcal{P}(Y = y \mid do(X = x))$

(II): Causal models



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  - Causal graph discovery: Recover true causal graph $\mathcal{G}^*$
    - Even with infinite observational data, can only determine causal graph up to some equivalence class where all conditional independence relations agree
    - Use interventions and experiments!
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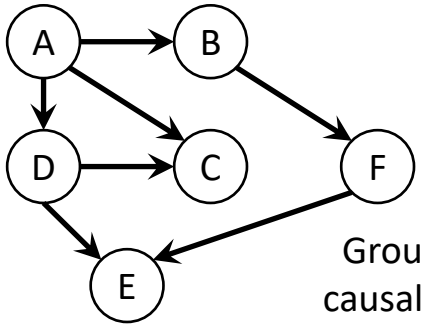
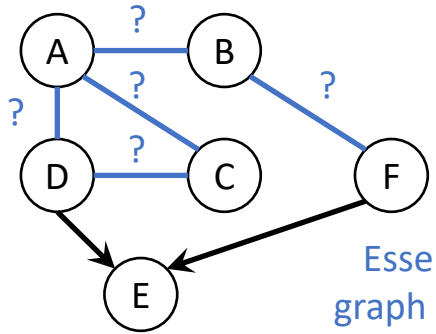
Causal discovery via interventions



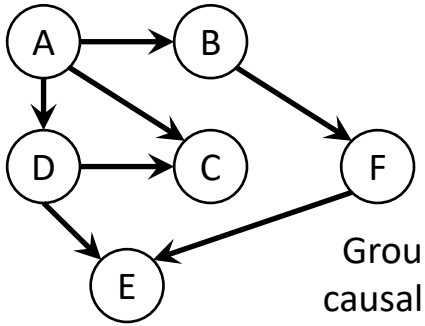
Ground truth causal graph $\mathcal{G}^*$



Essential graph $\mathcal{E}(\mathcal{G}^*)$

- Want: Recover $\mathcal{G}^*$ starting from partially oriented $\mathcal{E}(\mathcal{G}^*)$ from observational data

Causal discovery via interventions



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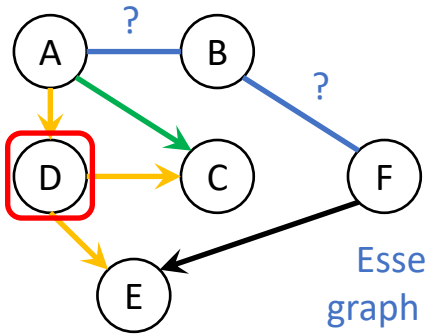

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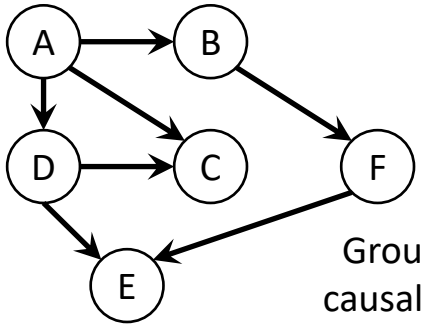
- Interventions reveal arc orientations (incident arcs + Meek rules)
- **Goal: Recover $\mathcal{G}^*$ using as few interventions as possible**

Causal discovery via interventions



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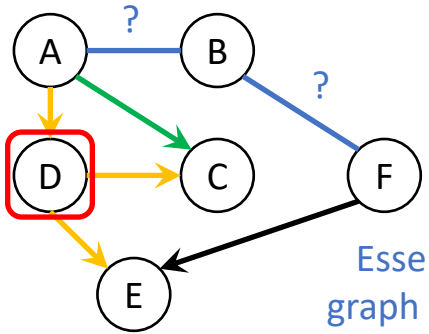

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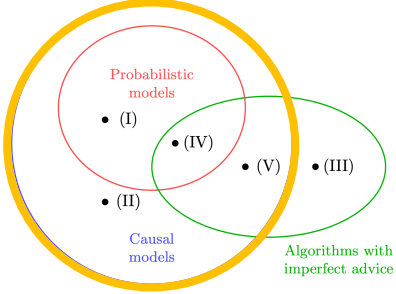
- Interventions reveal arc orientations (incident arcs + Meek rules)
- **Goal: Recover $\mathcal{G}^*$ using as few interventions as possible**
- We have some results regarding how to design algorithms to perform optimal adaptive interventions under various scenarios [CSB22, CS23a, CGB23, CS23b, CS23c, CSU24]
- Insight: Can abstract and treat this as a graph problem with specialized causal operations

[CSB22] Davin Choo, Kirankumar Shiragur, Arnab Bhattacharyya. *Verification and search algorithms for causal DAGs*. Conference on Neural Information Processing Systems (NeurIPS), 2022.
[CS23a] Davin Choo, Kirankumar Shiragur. *Subset verification and search algorithms for causal DAGs*. International Conference on Artificial Intelligence and Statistics (AISTATS), 2023.
[CGB23] Davin Choo, Themistoklis Gouleakis, Arnab Bhattacharyya. *Active causal structure learning with advice*. International Conference on Machine Learning (ICML), 2023.
[CS23b] Davin Choo, Kirankumar Shiragur. *New metrics and search algorithms for weighted causal DAGs*. International Conference on Machine Learning (ICML), 2023.
[CS23c] Davin Choo, Kirankumar Shiragur. *Adaptivity Complexity for Causal Graph Discovery*. Conference on Uncertainty in Artificial Intelligence (UAI), 2023.
[CSU24] Davin Choo, Kirankumar Shiragur, Caroline Uhler. *Causal discovery under off-target interventions*. International Conference on Artificial Intelligence and Statistics (AISTATS), 2024.

(II): Causal models



- Two fundamental problems in causal inference
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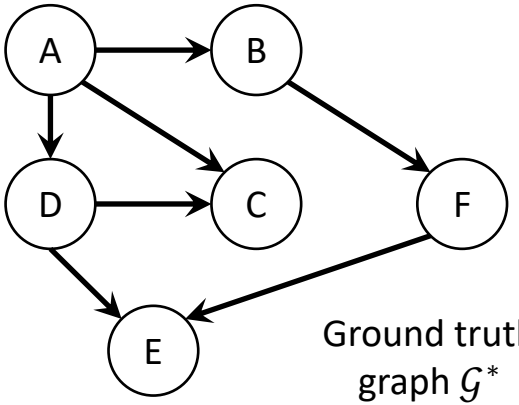
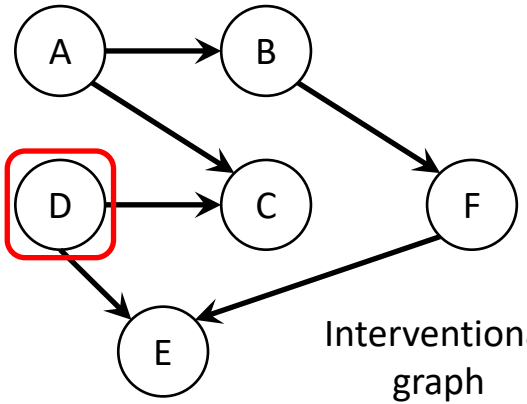
  - Causal effect estimation: Estimate $\mathcal{P}(Y = y \mid do(X = x))$
    - Typically, a 2-stage process: learn $\mathcal{G}^*$, then apply closed-form formulas

Causal identification (the 2nd step)



Ground truth graph $\mathcal{G}^*$



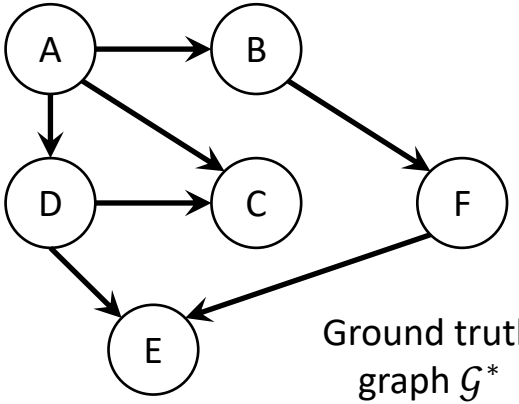
Interventional graph

$$\mathcal{P}(E = e \mid do(D = d^*))$$

Interventional query

What is probability of $E = e$ when we fix $D = d^*$?

Causal identification (the 2nd step)



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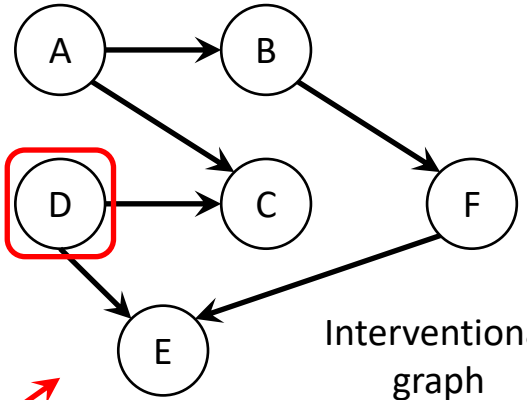


Interventional graph

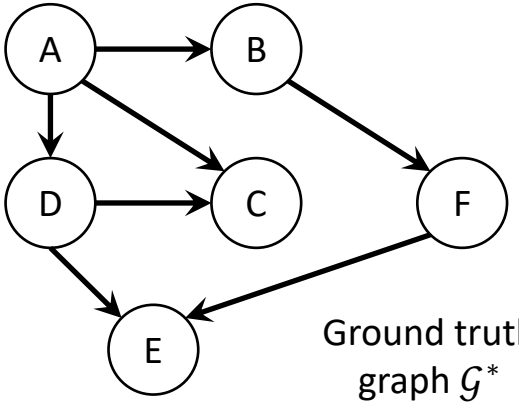
$$\mathcal{P}(E = e \mid do(D = d^*)) = \mathcal{P}(e \mid do(d^*)) \neq \mathcal{P}(e \mid d^*) \text{ in general}$$

Interventional query

What is probability of $E = e$ when we fix $D = d^*$?

Need to draw samples from interventional graph, i.e., perform experiment and measure

Causal identification (the 2nd step)



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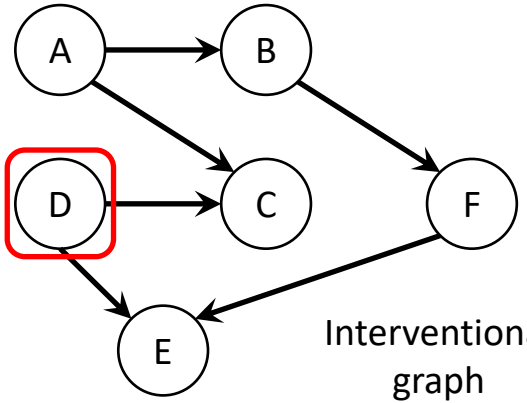


Interventional graph

Because of structure of $\mathcal{G}^*$

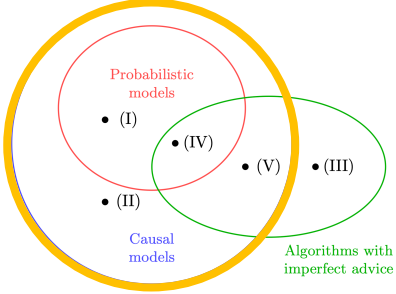
$$\mathcal{P}(E = e \mid do(D = d^*)) = \mathcal{P}(e \mid do(d^*)) = \int \mathcal{P}(e \mid d^*, a) \cdot \mathcal{P}(a)\, da$$

Interventional query

What is probability of $E = e$ when we fix $D = d^*$?

Just observational terms!

(II): Causal models



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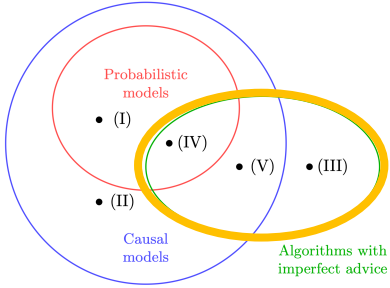
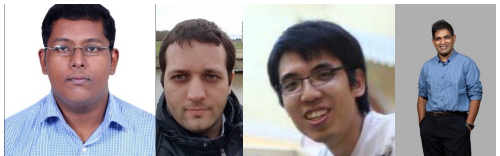
  - Causal effect estimation: Estimate $\mathcal{P}(Y = y \mid do(X = x))$
    - Typically, a 2-stage process: learn $\mathcal{G}^*$, then apply closed-form formulas
    - [CSBS24] This is suboptimal as it may require strong assumptions and a lot of samples
    - Insight: "weak edges" shouldn't affect much for PAC-style results

[CSBS24] Davin Choo, Chandler Squires, Arnab Bhattacharyya, and David Sontag. *Causal effect estimation via covariate adjustments without knowing an equivalent causal graph or relying on faithfulness.* In preparation, 2024.

(III/IV/V): Algorithms with advice

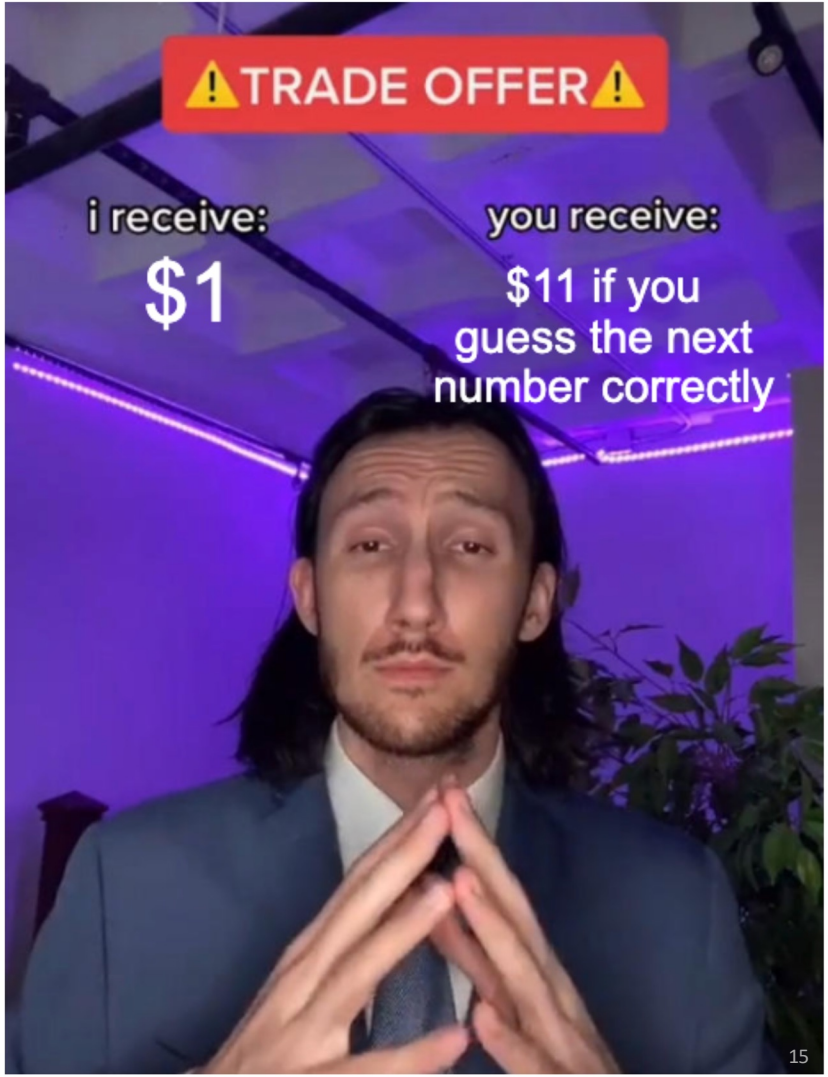


- Two key performance measures
  - Consistency: If advice is “perfect”, how good are things?
  - Robustness: If advice is “garbage”, how bad are things?
- Challenge: We don’t know how good the given advice is a priori!



Detour: Let's make a deal

- There are 10 numbers in the universe U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
- There is an underlying process $\mathcal{P}$ that generates IID samples from U
  - I.e., We can observe a sequence such as 1, 6, 3, 6, 2, 8, 0, 3, 9, 5, 4, …
- What property of $\mathcal{P}$ will make this deal profitable in expectation?



https://imgflip.com/memegenerator/309868304/Trade-Offer

Detour: Property testing land

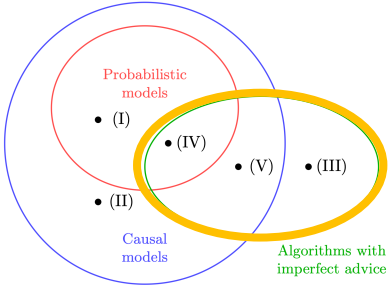
- How to test if $\mathcal{P}$ is the uniform distribution over U?
  - Say, we only care about constant success probability (can be amplified)
- Learning a ε-close $\hat{\mathcal{P}}$ then check: $\Theta\left(\frac{|U|}{\varepsilon^2}\right)$ IID samples from $\mathcal{P}$
- Uniformity testing requires $\Theta\left(\frac{\sqrt{|U|}}{\varepsilon^2}\right)$ IID samples from $\mathcal{P}$
  - If $\mathcal{P}$ is uniform, output YES w.p. $\geq \frac{2}{3}$
  - If $\mathcal{P}$ is ε-far from uniform, output NO w.p. $\geq \frac{2}{3}$
  - Many existing proofs for this bound. E.g., look at collisions in samples

→ Allowed to output arbitrarily if not uniform, yet not "far from uniform"

- See also [Can22] for an excellent property testing survey

[Can22] Clément L. Canonne. *Topics and Techniques in Distribution Testing: A Biased but Representative Sample*. Foundations and Trends® in Communications and Information Theory, 2022.

(III/IV/V): Algorithms with advice



- Two key performance measures
  - Consistency: If advice is "perfect", how good are things?
  - Robustness: If advice is "garbage", how bad are things?
- Challenge: We don't know how good the given advice is a priori!
- Insight: "Testing can be cheaper than learning" → `TestAndAct`
  - [CGLB24] TestAndMatch: Improve competitive ratio of online bipartite matching (III)
  - [BCGJ24] TestAndScheffe: Improve sample complexity of learning multivariate Gaussians (IV)
  - [CGB23] TestAndSubsetSearch: Reduce num of interventions required for causal graph discovery (V)

[CGLB24] Davin Choo, Themistoklis Gouleakis, Chun Kai Ling, and Arnab Bhattacharyya. *Online bipartite matching with imperfect advice*. International Conference on Machine Learning (ICML), 2024.
[BCGJ24] Arnab Bhattacharyya, Davin Choo, Themistoklis Gouleakis, and Philips George John. *Distribution learning with imperfect advice*. In preparation, 2024.
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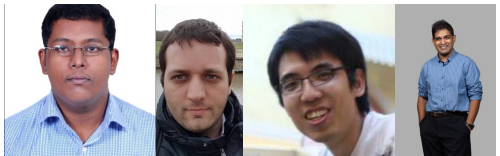
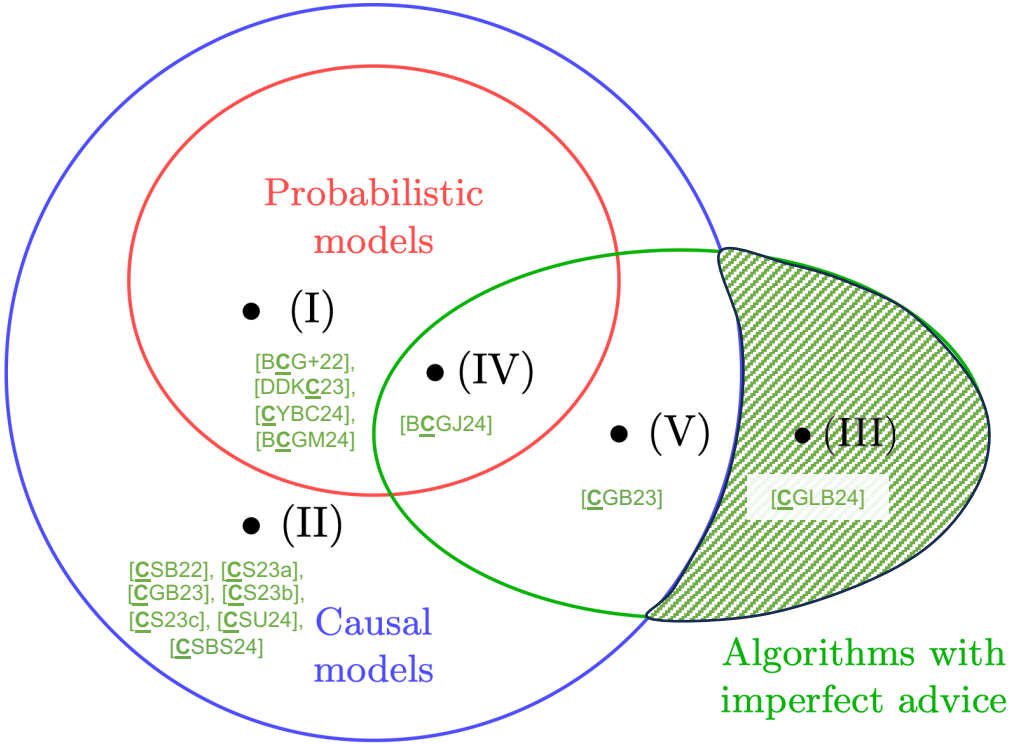


For the rest of this talk



Online bipartite matching

- Offline set $U = \{u_1, \ldots, u_n\}$ fixed and known
- Online set $V = \{v_1, \ldots, v_n\}$ arrive one by one

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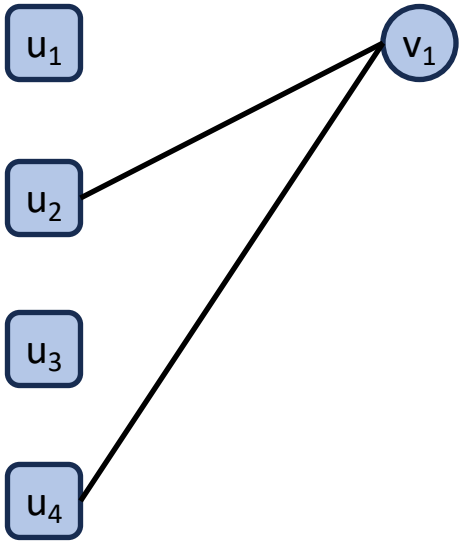

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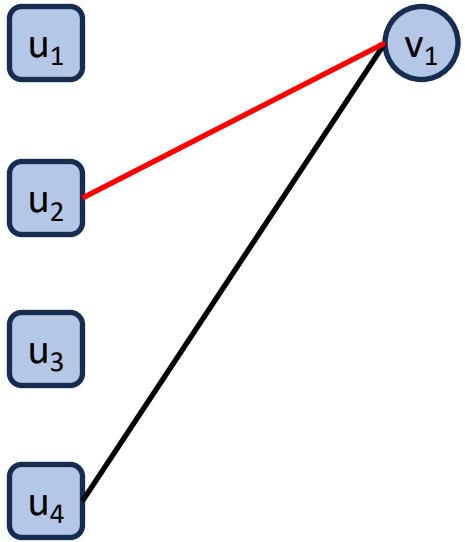

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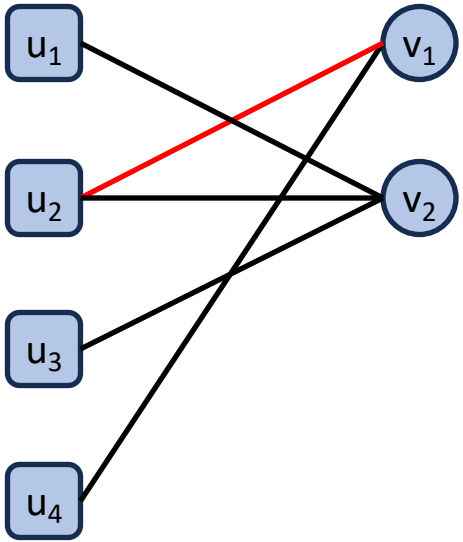

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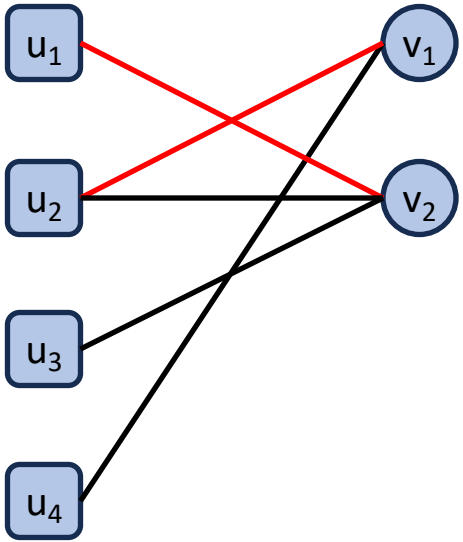

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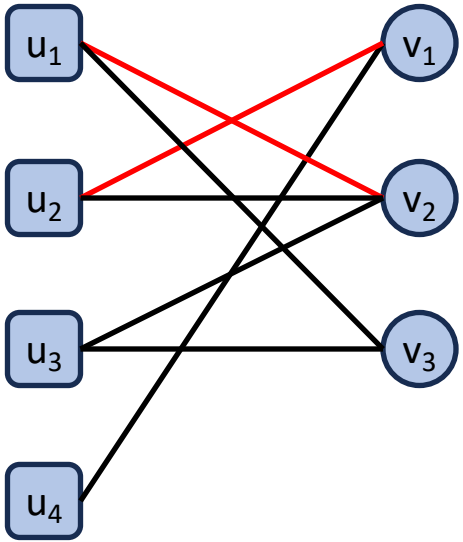

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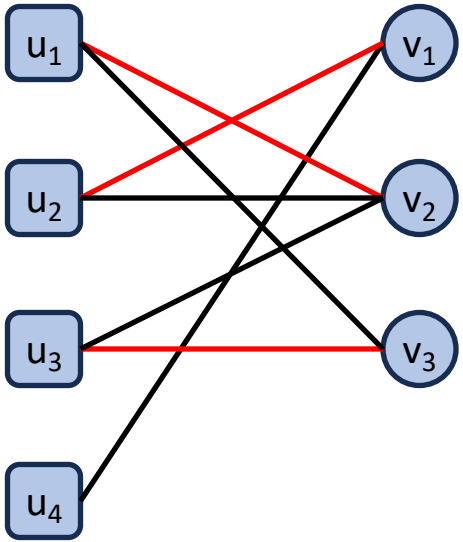

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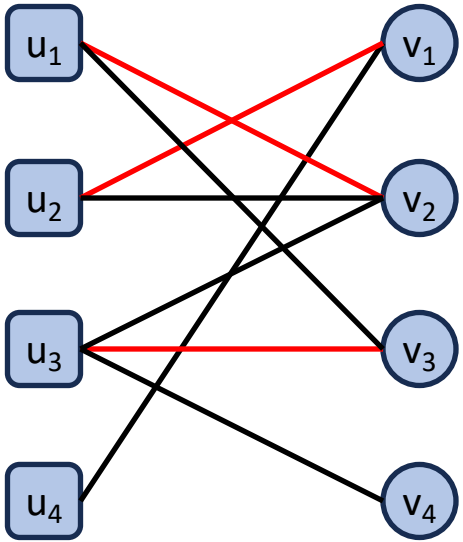

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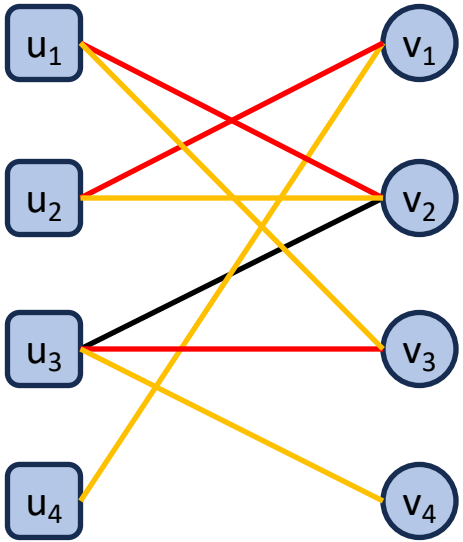
- Final offline graph $G^* = (U \cup V, E)$
  - $E = N(v_1) \cup \cdots \cup N(v_n)$
  - Maximum matching $M^* \subseteq E$ of size $|M^*| = n^* \leq n$



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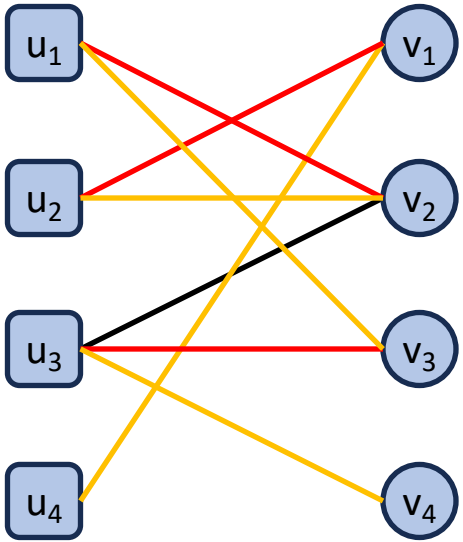

Here, the ratio is $3/4$

Goal of online bipartite matching problem

Produce a matching M such that the resulting competitive ratio $\frac{|M|}{|M^*|}$ is **maximized**

For this talk, let's treat $n^* = n$

What is known?

- Why is online bipartite matching hard?
  - Maximum bipartite matching is poly time computable…
  - But we don't know the future in the online setting!

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- Why is online bipartite matching hard?
  - Maximum bipartite matching is poly time computable…
  - But we don't know the future in the online setting!
- Any reasonable greedy algorithm has competitive ratio $\geq 1/2$
  - Size of maximal matching is at least half of size of maximum matching

What is known?

$$\min_{G} \min_{V\text{'s arrival sequence}} \frac{\text{(Expected) number of matches}}{n^*}$$

	(Expected) Competitive ratio
Deterministic algorithm	$\frac{1}{2}$ ← Greedy
Deterministic hardness	$\frac{1}{2}$
Randomized algorithm	$1-\frac{1}{e}$ [KVV90] ← Ranking
Randomized hardness	$1-\frac{1}{e}+o(1)$ [KVV90]

- The Ranking algorithm [KVV90]
  - Pick a random permutation π over the offline vertices U
  - When vertex v_i arrive with $N(v_i)$, match v_i to the smallest indexed (with respect to π) unmatched neighbor

[KVV90] Richard M. Karp, Umesh V. Vazirani, Vijay V. Vazirani. *An optimal algorithm for on-line bipartite matching*. Symposium on Theory of Computing (STOC), 1990

What if there is additional side information?

- Learning-augmented algorithms
  - Designing algorithms using advice, predictions, etc.
    - α-consistent: α-competitive with no advice error
    - β-robust: β-competitive with any advice error

A natural goal is to design an algorithm with $\alpha = 1$ while β being the best possible classically

Example settings with side information

Offline vertices	Online vertices	Presence of edge	Advice	Error
Advertisers	Ad slots	New ad slot fits the advertisers' requirements	Historical data	Data may have noise, bias, etc.
Job opening	Hiring company	Applicant's suitability for the job role	LinkedIn qualifications	May lie about credentials
Food bento boxes	Conference attendee	Attendee's dietary options match the food type	Food preferences	May change mind if see a tastier option

Online matching settings involving social good

This is a weighted and capacitated matching problem. Also, food banks may further have different nutritional targets to meet (e.g., 10kg of protein, 5kg of carbohydrates, etc.), making this a multidimensional instance

Offline vertices	Online vertices	Presence of edge	Advice	Error
Patients	Organ donors (e.g., upon death)	Compatibility	Historical data	Data may have noise, bias, etc.
Food banks	Food surplus from restaurants, etc.	Transportability	Restaurant information, seasonal trends	
Crisis response resource	Occurrence of a crisis request	Proximity	Nature of crisis, terrain, human experts, etc.	Murphy's law

The problem is more nuanced here: we might also be able to decide how to place resources in advice so that it can accommodate requests as they appear (think: k-server)

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Crisis resource	crisis request	Proximity	terrain, human experts, etc.	Murphy's law

Takeaway: Advice can come in many forms. Nuances in problem dictate which kind of advice are practical and useful

The problem is more nuanced here: we might also be able to decide how to place resources in advice so that it can accommodate requests as they appear (think: k-server)

Research question

- If we have “perfect information” about G^*, can we get n^* matches?
- Also, we know that Ranking achieves competitive ratio of $1 - \frac{1}{e}$

Can we get an algorithm that is both 1-consistent and $\left(1 - \frac{1}{e}\right)$-robust?

Prior related attempts

- [AGKK20] Prediction on edge weights adjacent to V under an optimal offline matching
  - Random vertex arrivals and weighted edges
  - Require hyper-parameter to quantify confidence in advice, so their consistency/robustness tradeoffs are not directly comparable
- [ACI22] Prediction of vertex degrees $\hat{d}(u_1), \ldots, \hat{d}(u_n)$ of the offline vertices in U
  - Adversarial arrival model
  - Optimal under the Chung-Lu-Vu random graph model [CLV03]
  - Unable to attain 1-consistency in general
- [JM22] Advice is a proposed matching for the first batch of arrived vertices
  - Two-staged arrival model [FNS21], where best possible robustness is ¾
  - For any $R \in [0, ¾]$, they can achieve consistency of $1 - \left(1 - \sqrt{1-R}\right)^2$
- [LYR23] Augment any "expert algorithm" with a pre-trained RL model
  - For any $\rho \in [0,1]$, their method is ρ-competitive to the given "expert algorithm"

[AGKK20] Antonios Antoniadis, Themis Gouleakis, Pieter Kleer, and Pavel Kolev. *Secretary and online matching problems with machine learned advice*. Neural Information Processing Systems (NeurIPS), 2020
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[CLV03] Fan Chung, Linyuan Lu, and Van Vu. *Spectra of random graphs with given expected degrees*. Proceedings of the National Academy of Sciences (PNAS), 2003
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Our first main result

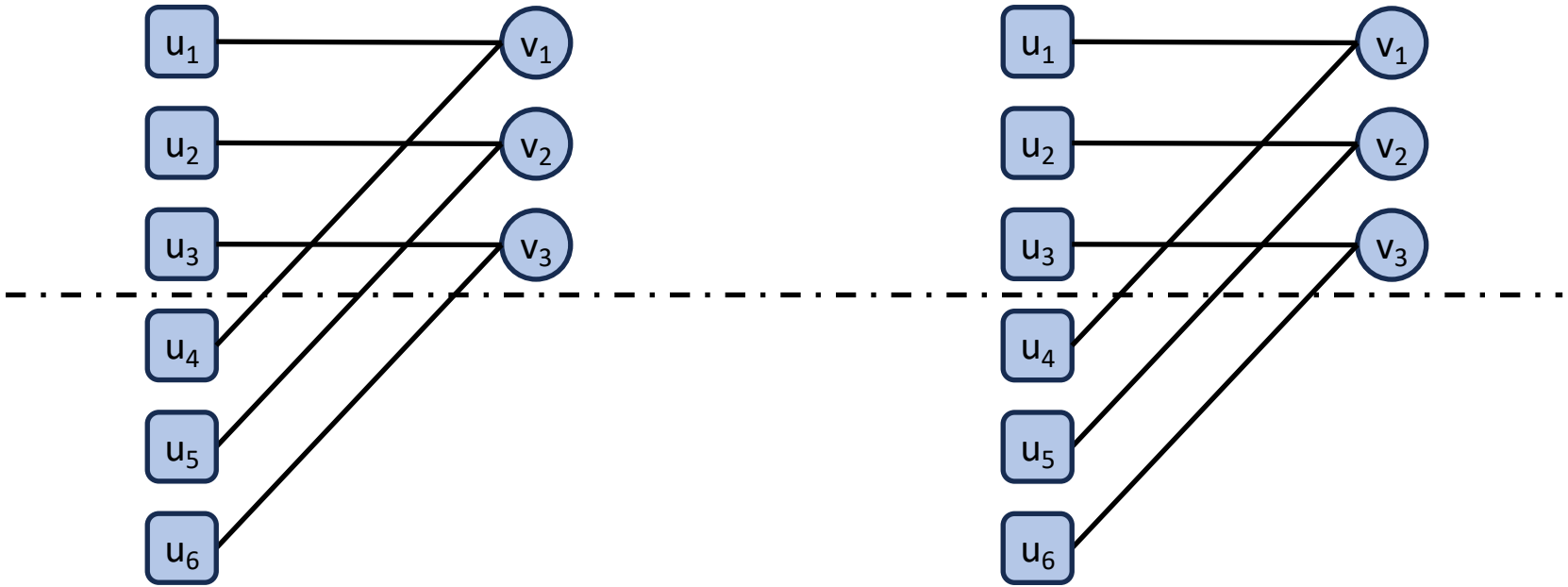
Impossibility result (Informal)

With adversarial vertex arrivals, no algorithm can be both 1-consistent and $> \frac{1}{2}$-robust, regardless of advice

- Extends to $(1 - a)$-consistent and $\left(\frac{1}{2} + a\right)$-robust, for any $a \in [0, ½]$
- Proof sketch (for $a = 0$ case):
  - Restrict G^* to be one of two possible graphs (next slide)
  - **Any** advice is equivalent to getting 1 bit of information
  - In first $\frac{n}{2}$ arrivals, no algorithm can distinguish between the two graphs
  - **Any** 1-consistent algorithm must behave as if the advice is perfect initially

Impossibility result (Informal)

With adversarial vertex arrivals, no algorithm can be both 1-consistent and $> \frac{1}{2}$-robust, regardless of advice



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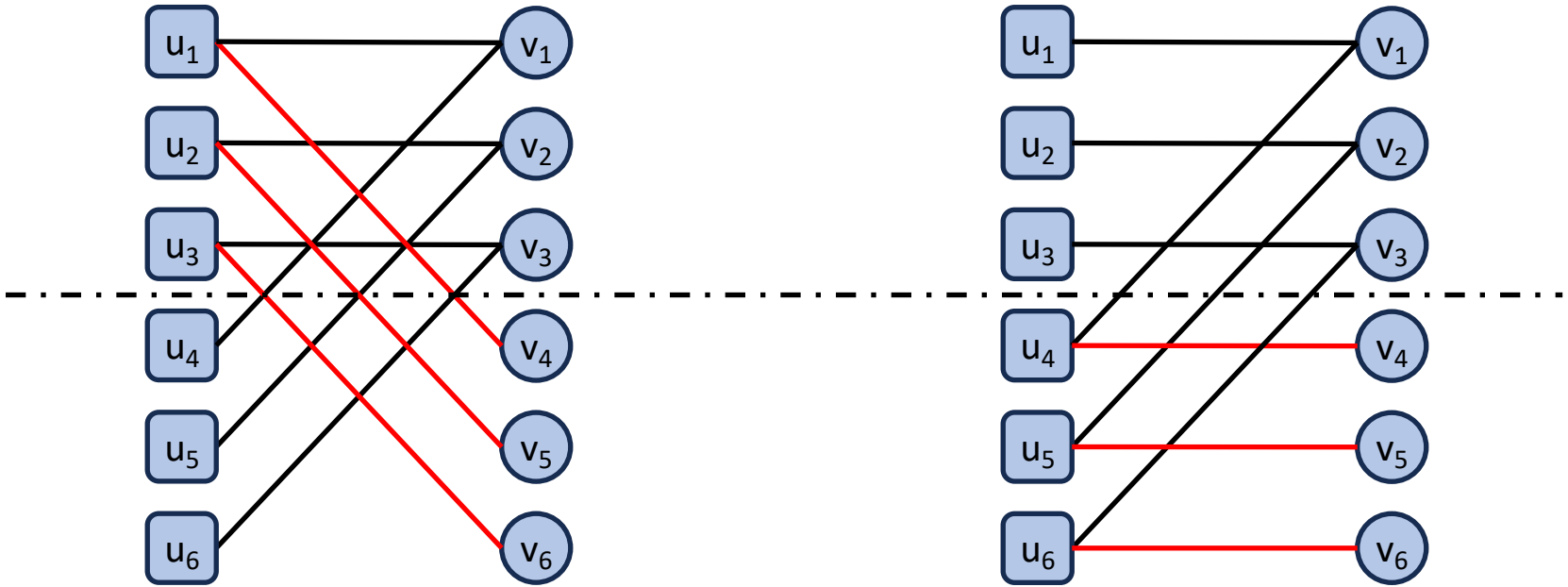


Hierarchy of arrival models [M13]

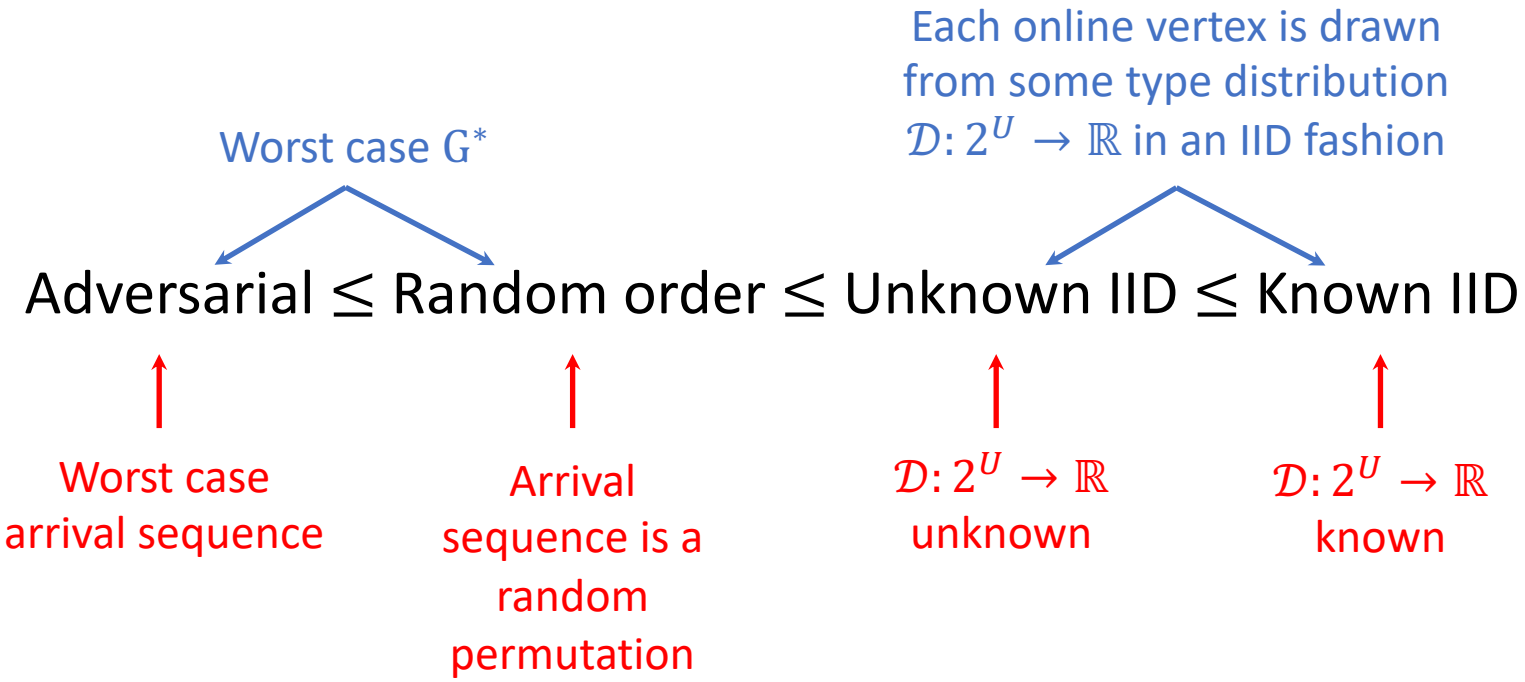
Harder Easier

Adversarial $\leq$ Random order $\leq$ Unknown IID $\leq$ Known IID

Easier models can achieve
higher competitive ratios

[M13] Aranyak Mehta. *Online matching and ad allocation*. Foundations and Trends in Theoretical Computer Science, 2013

Hierarchy of arrival models [M13]



What is known?

Adversarial $\leq$ Random order $\leq$ Unknown IID $\leq$ Known IID

	(Expected) Competitive ratio	
	Adversarial arrival	**Random order arrival**
Deterministic algorithm	$\frac{1}{2}$ Greedy	$1-\frac{1}{e}$ [GM08]
Deterministic hardness	$\frac{1}{2}$	$\frac{3}{4}$
Randomized algorithm	$1-\frac{1}{e}$ [KVV90] Ranking	0.696 [MY11]
Randomized hardness	$1-\frac{1}{e}+o(1)$ [KVV90]	0.823 [MGS12]

[GM08] Gagan Goel and Aranyak Mehta. *Online budgeted matching in random input models with applications to Adwords*. Symposium on Discrete Algorithms (SODA), 2008
[MY11] Mohammad Mahdian and Qiqi Yan. *Online Bipartite Matching with Random Arrivals: An Approach Based on Strongly Factor-Revealing LPs*. Symposium on Theory of Computing (STOC), 2011
[MGS12] Vahideh H Manshadi, Shayan Oveis Gharan, and Amin Saberi. *Online stochastic matching: Online actions based on offline statistics*. Mathematics of Operations Research, 2012

Research question

Can we get an algorithm that is both 1-consistent and $\left(1 - \frac{1}{e}\right)$-robust? β

- Let β denote the "best possible competitive ratio"
- Our first result says: This is not possible for adversarial arrivals!
- What about random order arrivals?

Adversarial $\leq$ Random order $\leq$ Unknown IID $\leq$ Known IID

Our second main result

Can we get an algorithm that is both 1-consistent and $\left(1 - \frac{1}{e}\right)$-robust?

β

Goal achievable in random order (Informal)

With random order, there is an algorithm achieves competitive ratio interpolating between 1 and $\beta \cdot \left(1 - o(1)\right)$, depending on advice quality

- Our method is a meta-algorithm that uses any `Baseline` that achieves β
- So, we are simultaneously 1-consistent and $\beta \cdot \left(1 - o(1)\right)$-robust
- For random arrival model, we know that $0.696 \leq \beta \leq 0.823$

e.g. use `Ranking`

Realized type counts as advice

- Classify online vertex in $G^* = (U \cup V, E)$ based on their types
  - Type of v_i is the set of offline vertices in $N(v_i)$ are adjacent to [BKP20]
- Define integer vector $c^* \in \mathbb{N}^{2^n}$ indexed by all possible types 2^U
  - $c^*(t)$ = Number of times the type $t \in 2^U$ occurs in G^*
- Define $T^* \subseteq 2^U$ as the subset of non-zero counts in c^*
  - Note: $|T^*| \leq n \ll 2^{|U|} = 2^n$

[BKP20] Allan Borodin, Christodoulos Karavasilis, and Denis Pankratov. *An experimental study of algorithms for online bipartite matching*. Journal of Experimental Algorithmics (JEA), 2020

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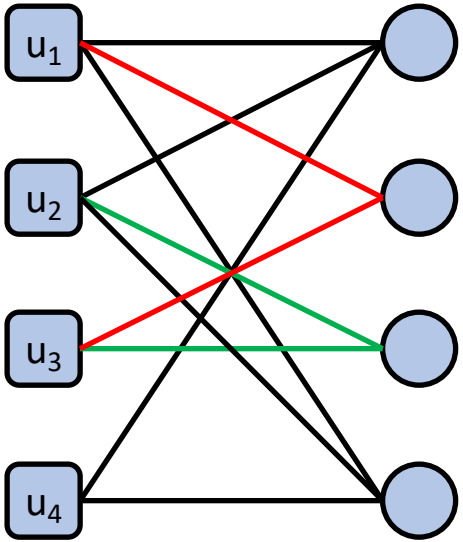
- Define $T^* \subseteq 2^U$ as the subset of non-zero counts in c^*
  - Note: $|T^*| \leq n \ll 2^{|U|} = 2^n$
- Advice is simply an estimate vector $\hat{c}$ which approximates c^*
  - Let $\hat{T}$ be non-zero counts in $\hat{c}$. Similarly, we have $\left|\hat{T}\right| \leq n$
  - Can represent $\hat{c}$ using $O(n)$ labels and numbers

Realized type counts as advice



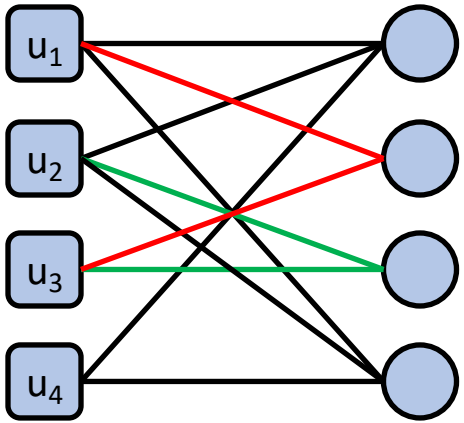
T^*

Type	c^*
$\{u_1, u_2, u_4\}$	2
$\{u_1, u_3\}$	1
$\{u_2, u_3\}$	1
$2^U \setminus T^*$	0

Here, $|T^*| = 3 \ll 2^4 = 16$

The Mimic algorithm

- Algorithm
  - Fix any arbitrary maximum matching $\widehat{M}$ on the graph defined by advice $\hat{c}$
  - Try to mimic edge matches in $\widehat{M}$ while tracking the types of each arrival
  - If unable to mimic, leave arrival unmatched



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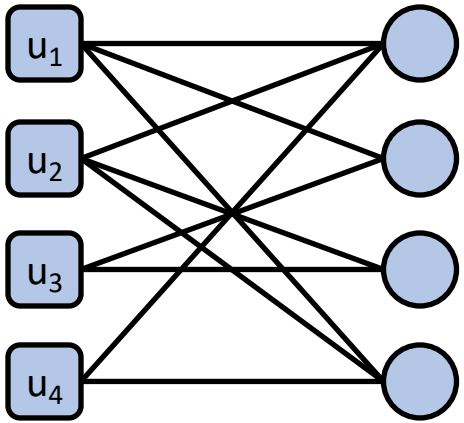

Type	c^*	$\hat{c}$
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The Mimic algorithm

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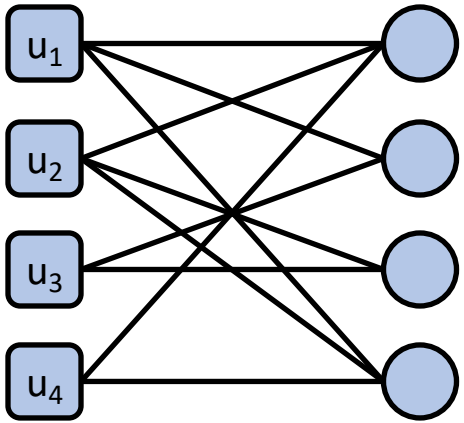

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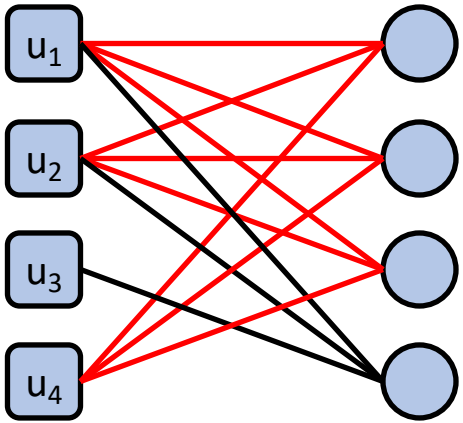

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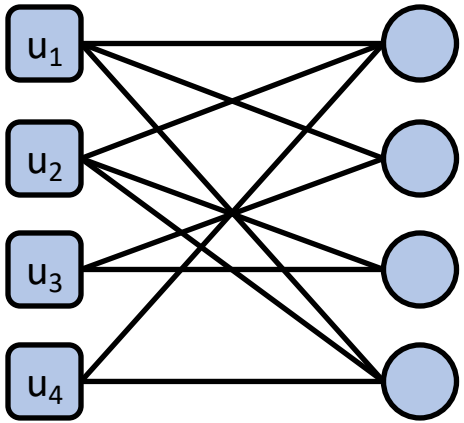

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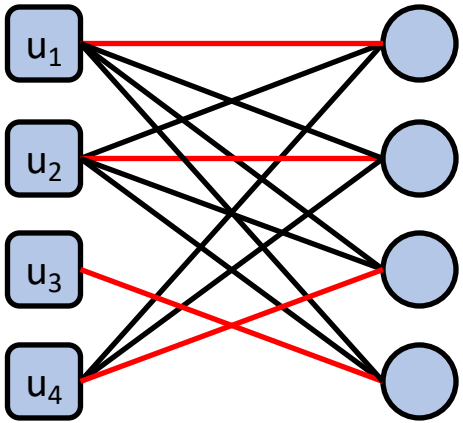

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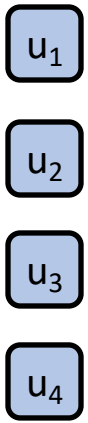

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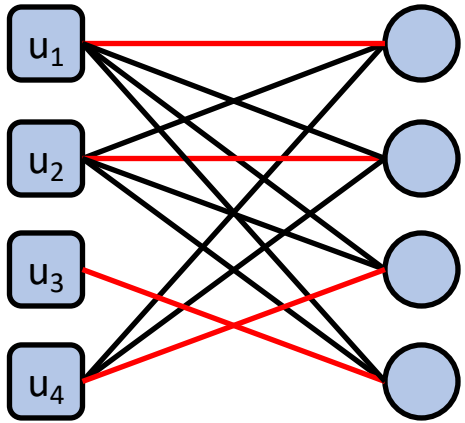

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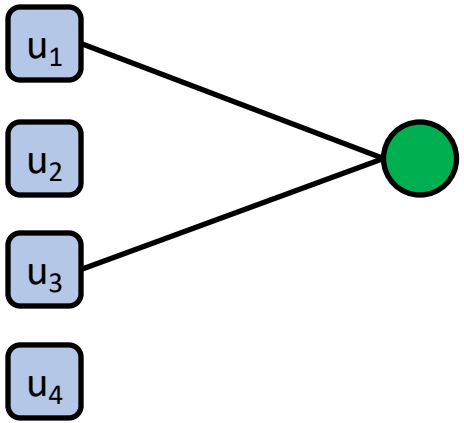

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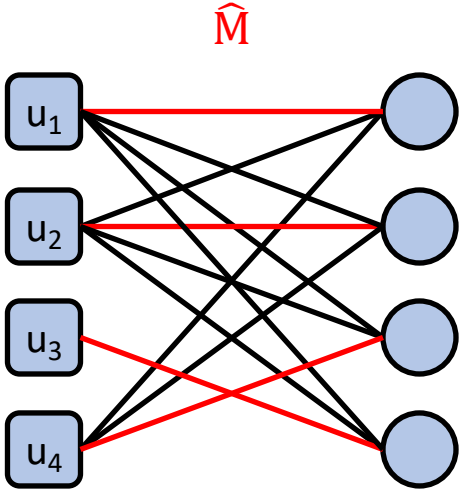

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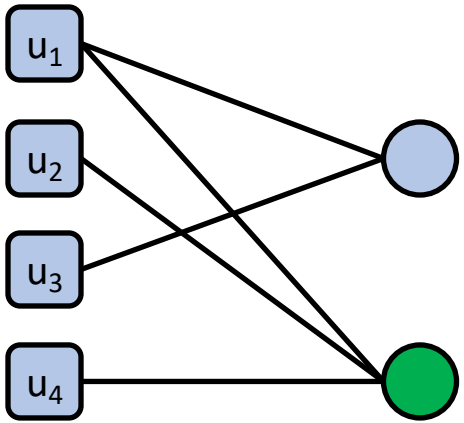

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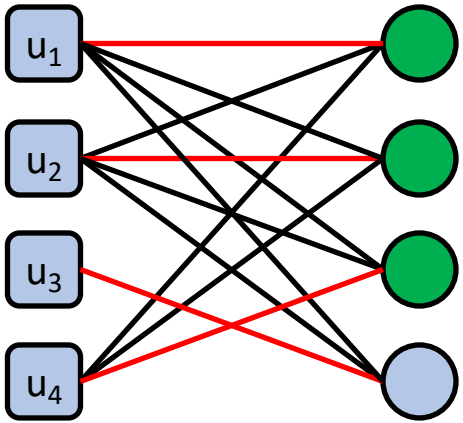

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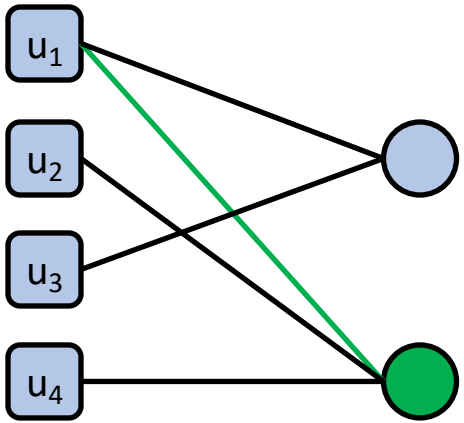

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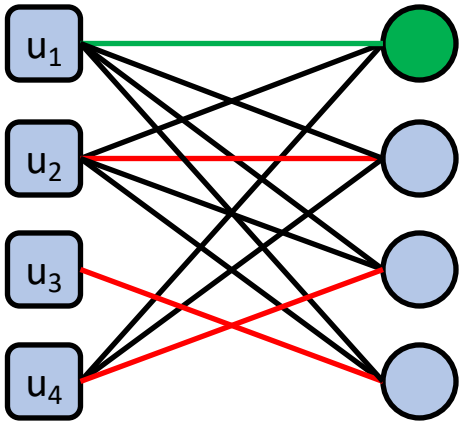

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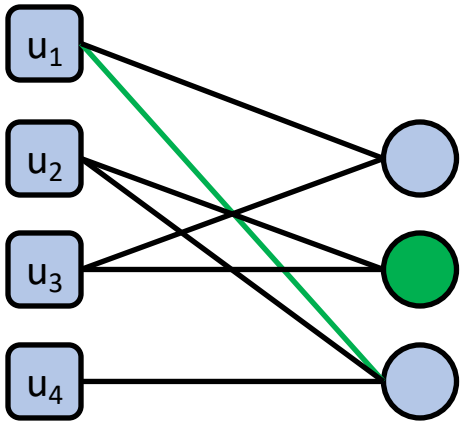

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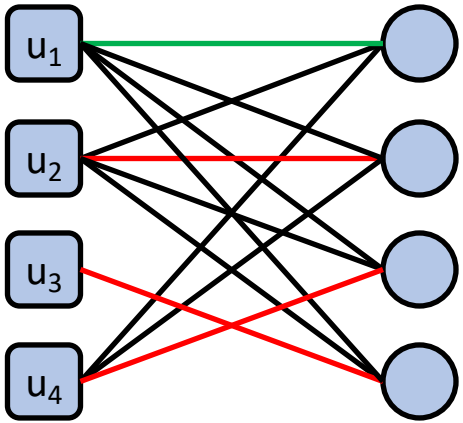

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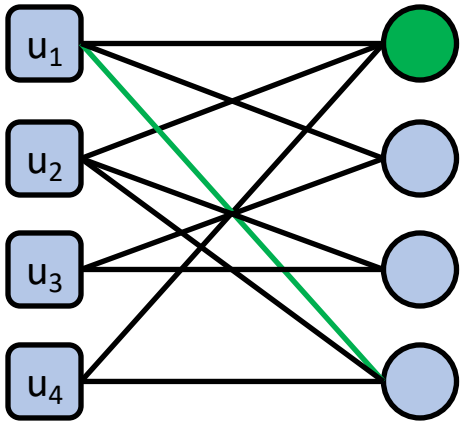

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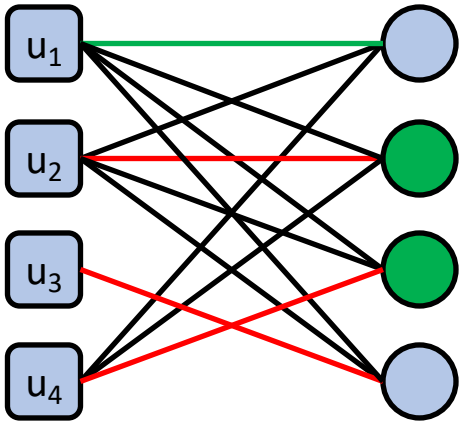

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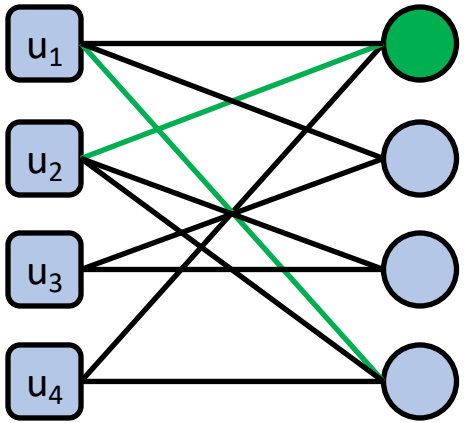

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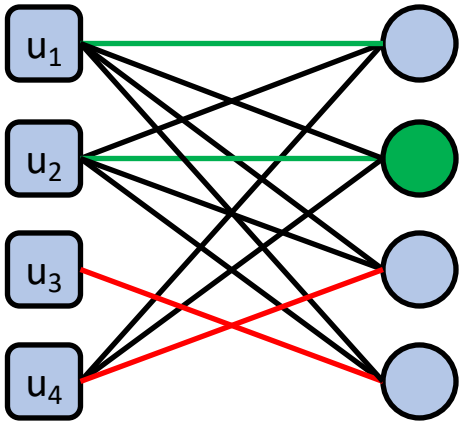

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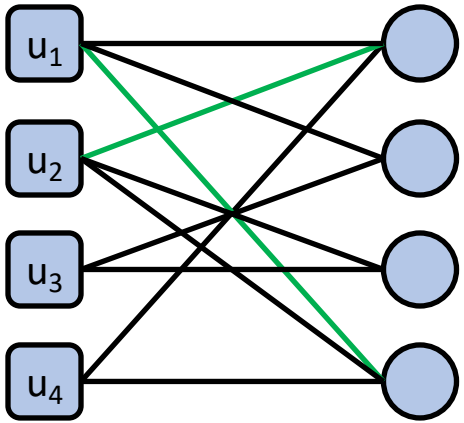

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Produced matching size = 2

The Mimic algorithm

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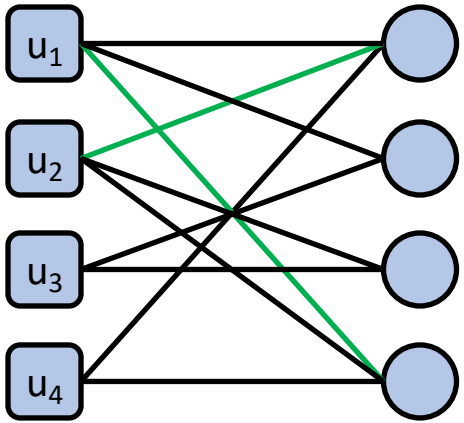

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Produced matching size

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$$L_1(c^*, \hat{c}) = |3-2| + |0-1| + |0-1| + |1-0| + 0 \ldots = 4$$

The Mimic algorithm

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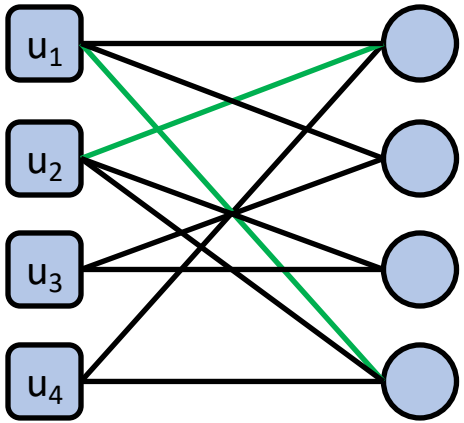

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Produced matching size

$= 2 = |\widehat{M}| - \frac{L_1(c^*, \hat{c})}{2}$

Error is "double counted" in L_1

$L_1(c^*, \hat{c})$
$= |3 - 2| + |0 - 1|$
$+ |0 - 1| + |1 - 0| + 0 \ldots$
$= 4$

The Mimic algorithm

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  - $0 \leq L_1(c^*, \hat{c}) \leq 2n$ measures how close $\hat{c}$ is to c^*
  - By blindly following advice, Mimic gets a matching of size $\left|\widehat{M}\right| - \frac{L_1(c^*, \hat{c})}{2}$
  - Mimic beats an advice-free Baseline whenever $\left|\widehat{M}\right| - \frac{L_1(c^*, \hat{c})}{2} > \beta \cdot n$

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  - Mimic beats an advice-free Baseline whenever $|\widehat{M}| - \frac{L_1(c^*,\hat{c})}{2} > \beta \cdot n$
  - Mimic beats an advice-free Baseline whenever $\frac{L_1(c^*,\hat{c})}{n} < 2(1-\beta)$

For this talk, let's treat $|\widehat{M}| = n$

How to test advice quality?

Insight: Use sublinear property testing to estimate $L_1(c^*, \hat{c})$!

- Define $p = \frac{c^*}{n}$ and $q = \frac{\hat{c}}{n}$ as distributions over the 2^U types

How to test advice quality?

Insight: Use sublinear property testing to estimate $L_1(c^*, \hat{c})$!

- Define $p = \frac{c^*}{n}$ and $q = \frac{\hat{c}}{n}$ as distributions over the 2^U types
- [VV11, JHW18]: Can estimate $L_1(p, q)$ "well" using $o(n)$ IID samples
  - Some adjustments needed to apply this property testing idea to our online bipartite matching setup, but it can be done (talk to me to find out more)

[VV11] Gregory Valiant and Paul Valiant. *The power of linear estimators*. Foundations of Computer Science (FOCS), 2011.
[JHW18] Jiantao Jiao, Yanjun Han, and Tsachy Weissman. *Minimax estimation of the L_1 distance*. IEEE Transactions on Information Theory, 2018.

The `TestAndMatch` algorithm

- Algorithm
  - Fix any arbitrary maximum matching $\widehat{M}$ on the graph defined by advice $\hat{c}$
  - Run `Mimic` while testing quality of $\hat{c}$ by estimating $L_1(c^*, \hat{c})$
  - If test declares $L_1(c^*, \hat{c})$ is "large", use `Baseline` for remaining arrivals
  - Otherwise, continue using `Mimic` for remaining arrivals

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  - Otherwise, continue using `Mimic` for remaining arrivals
- Analysis
  - If $\hat{L}_1 \lesssim 2(1-\beta)$, then `TestAndMatch` attains ratio of at least $1 - \frac{L_1(c^*,\hat{c})}{2n}$
  - Otherwise, `TestAndMatch` attains ratio of at least $\beta \cdot (1 - o(1))$

Our second main result

Can we get an algorithm that is both 1-consistent and $\left(1 - \frac{1}{e}\right)$-robust? β

Goal achievable in random order (Informal)

With random order, there is an algorithm achieves competitive ratio interpolating between 1 and $\beta \cdot \left(1 - o(1)\right)$, depending on advice quality

- Our method is a meta-algorithm that uses any `Baseline` that achieves β
- So, we are simultaneously 1-consistent and $\beta \cdot \left(1 - o(1)\right)$-robust
- For random arrival model, we know that $0.696 \leq \beta \leq 0.823$

Our second main result

Can we get an algorithm that is both 1-consistent and $\left(1 - \frac{1}{e}\right)$-robust? β

Goal achievable in random order (Informal)

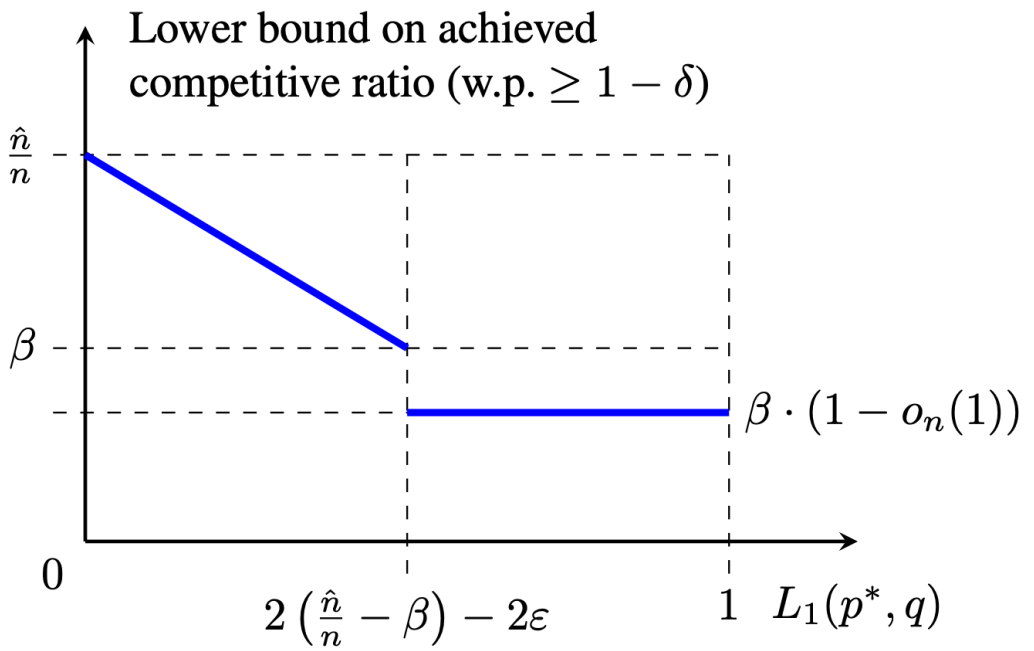
Let $\hat{L}_1$ be estimate of $L_1(c^*, \hat{c})$ from $o(n)$ vertex arrivals.
TestAndMatch achieves a competitive ratio of at least

- At least $1 - \frac{L_1(c^*, \hat{c})}{2n} \geq \beta$, when $\hat{L}_1$ "small"
- At least $\beta \cdot \left(1 - o(1)\right)$, when $\hat{L}_1$ "large"

i.e., TestAndMatch is 1-consistent and $\beta \cdot \left(1 - o(1)\right)$-robust

Our second main result

Can we ge… β …obust?

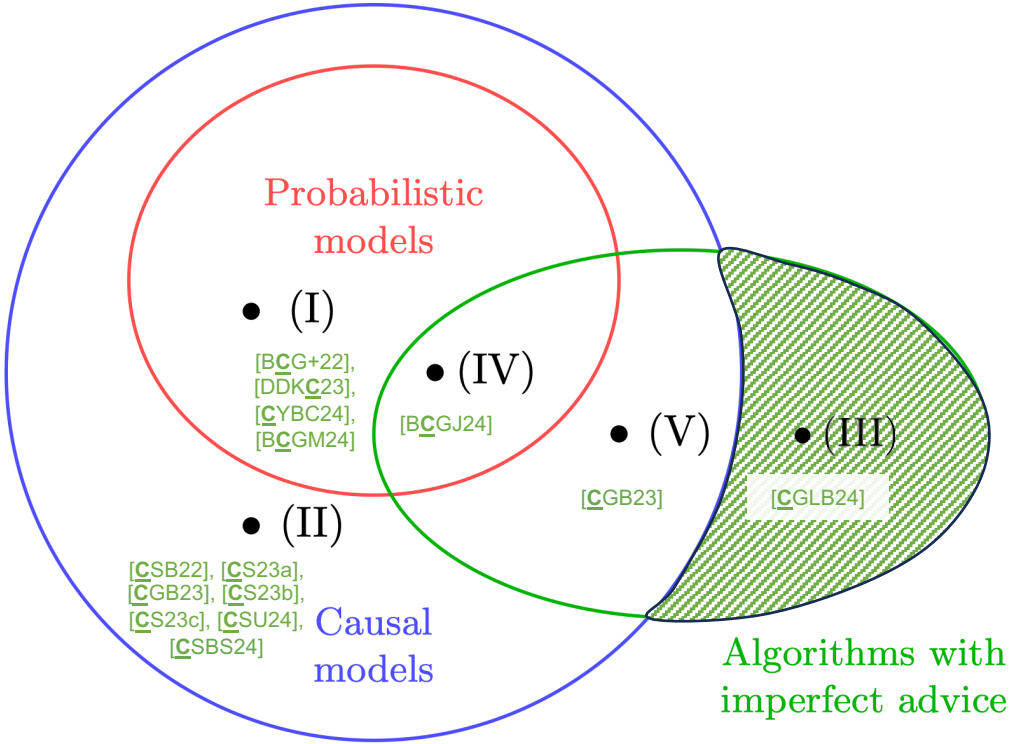


Let $\widehat{L}$ … ls.
Test … ast

- At lea…
- At lea…

i.e., `TestAndMatch` is 1-consistent and $\beta \cdot \big(1 - o(1)\big)$-robust

Recap: Main themes explored in my PhD



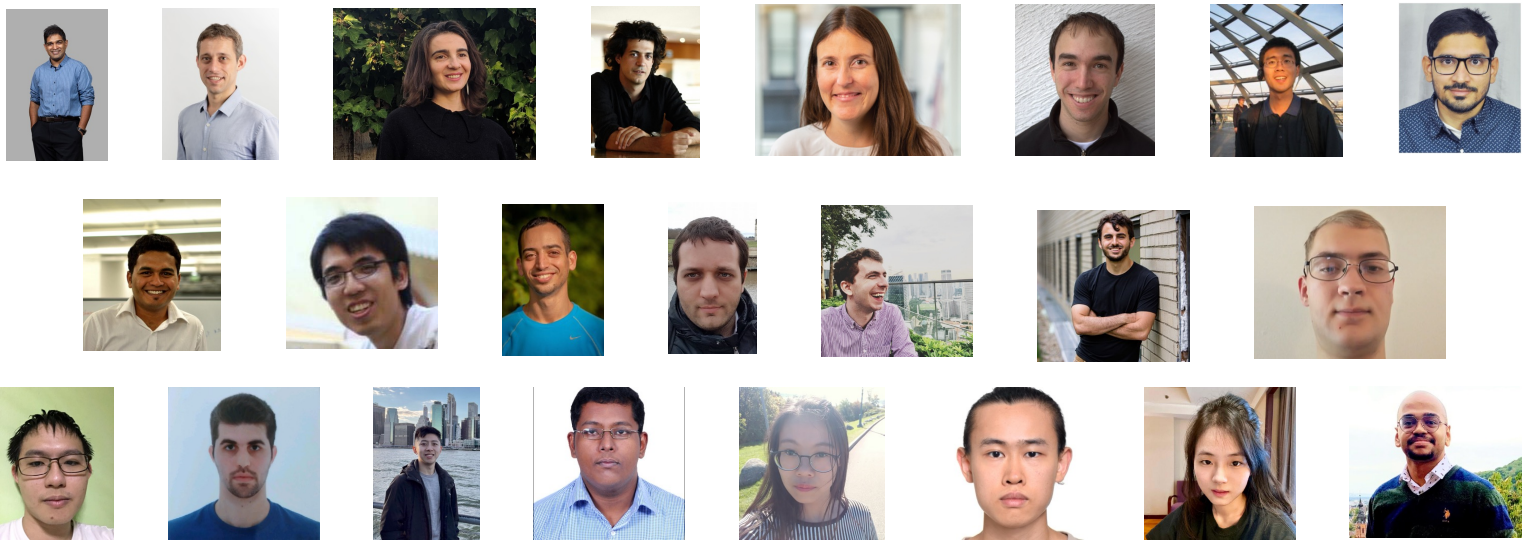
Future research directions

1. New directions and problems with real-world impact @ Teamcore
2. Developing causality-informed AI/ML methods
   - Most AI/ML methods are built on statistical/associational relations
   - Naively making real-world decisions can lead to unexpected outcomes

Future research directions

1. New directions and problems with real-world impact @ Teamcore
2. Developing causality-informed AI/ML methods
   - Most AI/ML methods are built on statistical/associational relations
   - Naively making real-world decisions can lead to unexpected outcomes
3. Incorporating human knowledge as imperfect advice
   - First came out of the study of online algorithms: main difficulty is not knowing the future and advice is partial future knowledge
   - I believe framework is much more broadly applicable
     - How can we principally elicit and incorporate human domain experts' knowledge?
     - Note: "Just Bayesian it" doesn't always work as an expert can be confidently wrong

Thank you to all my amazing collaborators during my PhD journey!



Thank you for your kind attention!

(Some of the works with them were not mentioned in this presentation)

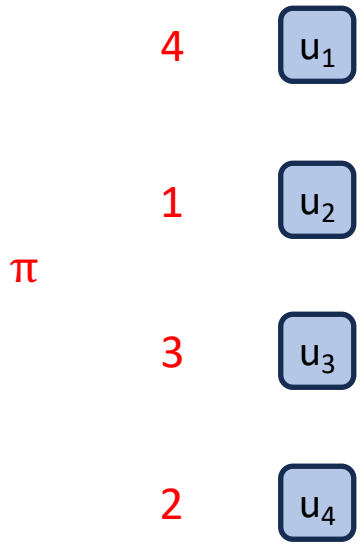
NUS National University of Singapore

AI SINGAPORE

BACK UP SLIDES

(from some of my older talks that may be relevant for discussion)

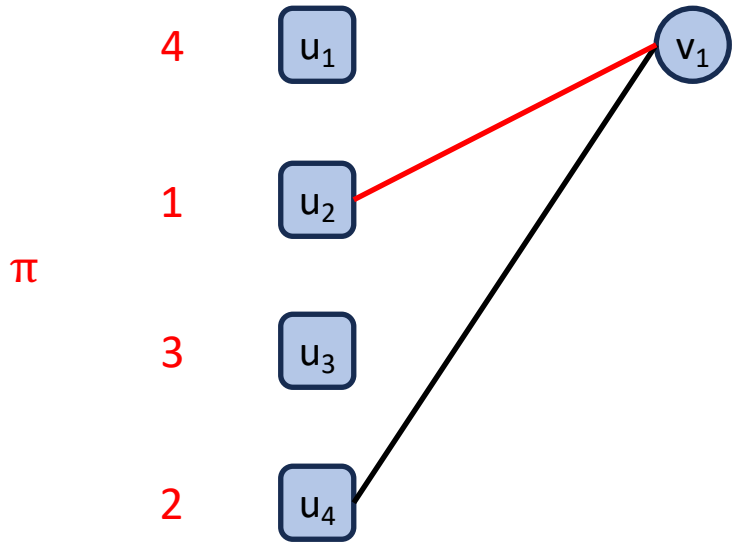
What is known?



- The Ranking algorithm [KVV90]
  - Pick a random permutation π over the offline vertices U
  - When vertex v_i arrive with $N(v_i)$, match v_i to the smallest indexed (with respect to π) unmatched neighbor

[KVV90] Richard M. Karp, Umesh V. Vazirani, Vijay V. Vazirani. *An optimal algorithm for on-line bipartite matching*. Symposium on Theory of Computing (STOC), 1990

What is known?



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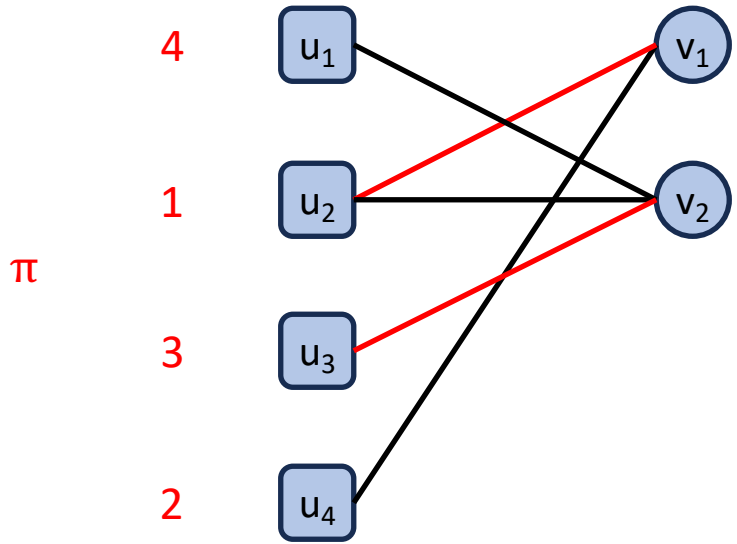

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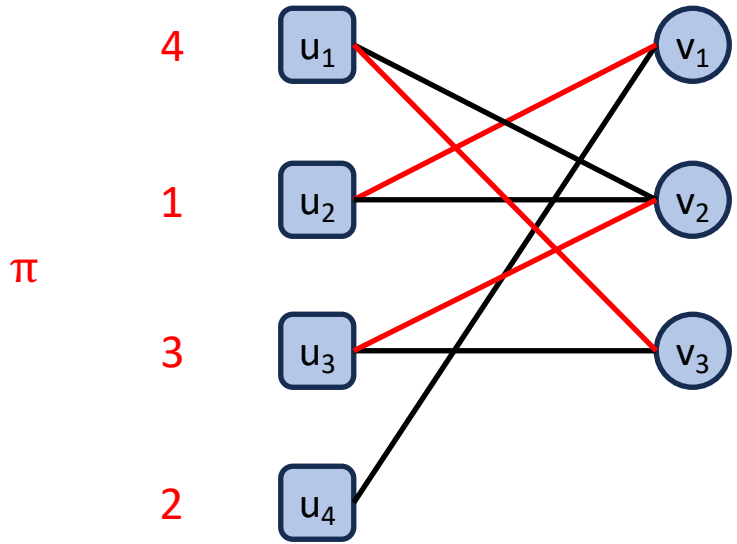

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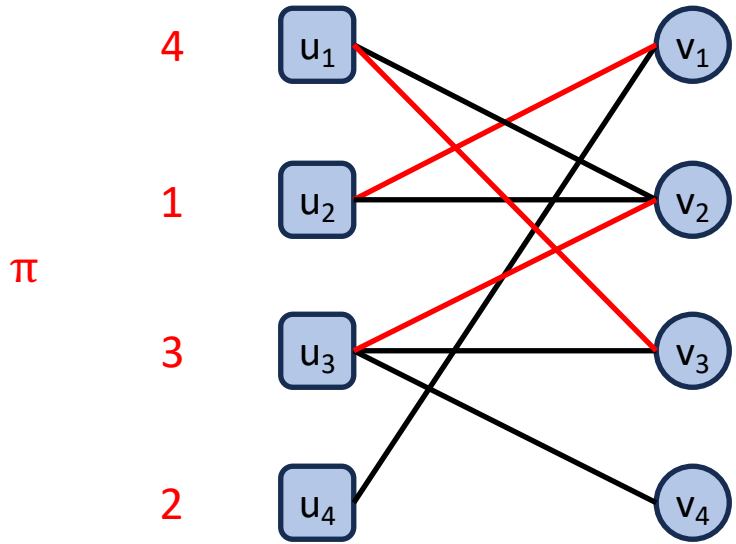

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What if there is additional side information?

- Learning-augmented algorithms
  - Designing algorithms using advice, predictions, etc.
    - α-consistent: α-competitive with no advice error
    - β-robust: β-competitive with any advice error
- Example: Binary search with advice
  - Want to find a word in an n page dictionary, say it is on page x^*
  - Classical binary search: $O(\log n)$ queries possible and worst case necessary
  - If someone provides an advice page $\hat{x}$, $O(\log |x^* - \hat{x}|)$ queries is possible

$\hat{x}$ obtained via letter frequency tables, someone who searched a "nearby" word, or asking ChatGPT, etc…

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  - Here, “best possible” is directly querying to page x^*
  - So, this algorithm is 1-consistent and $O(\log n)$-robust since $|x^* - \hat{x}| \leq n$

What is known?

- Hardness of $\frac{3}{4}$ for random order

	(Expected) Competitive ratio	
	Adversarial arrival	Random order arrival
Deterministic algorithm	$\frac{1}{2}$ Greedy	$1-\frac{1}{e}$ [GM08]
Deterministic hardness	$\frac{1}{2}$	$\frac{3}{4}$
Randomized algorithm	$1-\frac{1}{e}$ [KVV90] Ranking	0.696 [MY11]
Randomized hardness	$1-\frac{1}{e}+o(1)$ [KVV90]	0.823 [MGS12]

- [KMT11] showed that Ranking cannot beat 0.727 in general
- So, new ideas are needed if you believe the "right bound" is 0.823

[KMT11] Mohammad Mahdian and Qiqi Yan. *Online Bipartite Matching with Random Arrivals: An Approach Based on Strongly Factor-Revealing LPs.* Symposium on Theory of Computing (STOC), 2011

How to test advice quality?

Insight: Use sublinear property testing to estimate $L_1(c^*, \hat{c})$!

- Define $p = \frac{c^*}{n}$ and $q = \frac{\hat{c}}{n}$ as distributions over the 2^U types
- [VV11, JHW18]: Can estimate $L_1(p, q)$ "well" using $o(n)$ IID samples
  - To be precise, if p and q have domain size $r \leq n$, then $\Theta\left(\frac{r}{\varepsilon^2 \log r}\right)$ IID samples sufficient and necessary to estimate $\hat{L}_1$ such that $\left|\hat{L}_1 - L_1(p, q)\right| \leq \varepsilon$
  - c^* and $\hat{c}$ can be defined over $\left|\hat{T}\right| + 1$ elements with a "not in $\hat{T}$" bucket

[VV11] Gregory Valiant and Paul Valiant. *The power of linear estimators*. Foundations of Computer Science (FOCS), 2011.
[JHW18] Jiantao Jiao, Yanjun Han, and Tsachy Weissman. *Minimax estimation of the L_1 distance*. IEEE Transactions on Information Theory, 2018.

Some minor adjustments to our problem setting

- Adjustment 1
  - Random vertex arrivals are “sampling without replacement”
  - We can simulate IID samples by keeping track of what has arrived and then “reusing” arrivals with some probability proportional to number of arrivals
- Adjustment 2
  - L_1 estimator is in expectation, but can be made “with high probability”

Conclusions and future directions

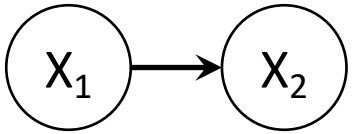
- Our paper also discussed some practical considerations while using the given advice $\hat{c}$
- Can our ideas such as using property testing extend to other versions of online bipartite matching and other online problems with random arrivals?
  - We suspect it extends with suitably chosen advice and quality metrics, e.g. Earthmover distance?
- Is there a smarter way using advice other than `Mimic`, leaving some arrivals unmatched?
  - [FMMM09] constructed two matchings to "load balance" in the known IID setting
  - In semi-online model, [KPSSV19] mimic matching on known arrivals and `Ranking` on adversarial arrivals
- Message to the learning-augmented community: Beyond consistency and robustness?
  - `TestAndMatch`'s guarantees is based on L_1 over the type histograms
  - This is sensitive to certain types of noise, e.g. $\hat{c}$ obtained after Erdős–Rényi edits to the offline graph G^*
  - We expect large L_1 in practice, but notions of advice practicality are not formally considered under the standard framework of consistency and robustness

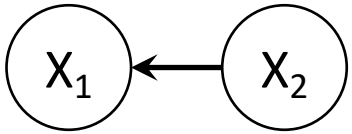
Thank you for your kind attention!

[FMMM09] Jon Feldman, Aranyak Mehta, Vahab Mirrokni, and Shan Muthukrishnan. *Online stochastic matching: Beating 1-1/e*. Foundations of Computer Science (FOCS), 2009.
[KPSSV19] Ravi Kumar, Manish Purohit, Aaron Schild, Zoya Svitkina, and Erik Vee. *Semi-Online Bipartite Matching*. Innovations in Theoretical Computer Science (ITCS), 2019.

Two equivalent causal models

X_1	-0.27	0.29	0.37	-0.09	0.34	0.33	0.30	-1.34	0.68
X_2	-0.10	1.65	0.47	1.92	2.04	1.67	0.11	-3.58	1.97





- $X_1 = \epsilon_1 \sim N(0,1)$
- $X_2 = X_1 + \epsilon_2 \sim N(0,2)$
- $\epsilon_1 \sim N(0,1)$
- $\epsilon_2 \sim N(0,1)$

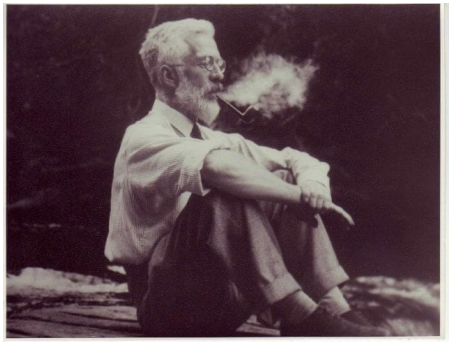
- $X_1 = \frac{1}{2} \cdot X_2 + \epsilon_3 \sim N(0,1)$
- $X_2 = \epsilon_4 \sim N(0,2)$
- $\epsilon_3 \sim N\left(0, \frac{1}{2}\right)$
- $\epsilon_4 \sim N(0,2)$

Example from: https://youtu.be/rE6IMfSkOU0?t=849. See https://github.com/csquires/6.S091-causality for full course

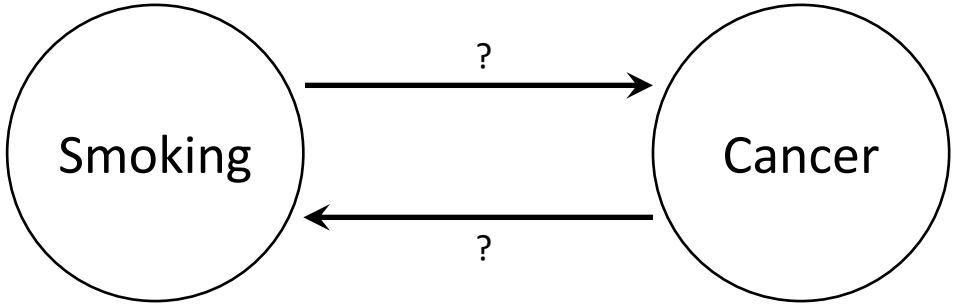
Smoking

Fisher's letter to Nature, 1958:

"The curious associations with lung cancer found in relation to smoking habits do not, in the minds of some of us, lend themselves easily to the simple conclusion that the products of combustion reaching the surface of the bronchus induce, though after a long interval, the development of a cancer... **Such results suggest that an error has been made, of an old kind, in arguing from correlation to causation**..."



Ronald Fisher

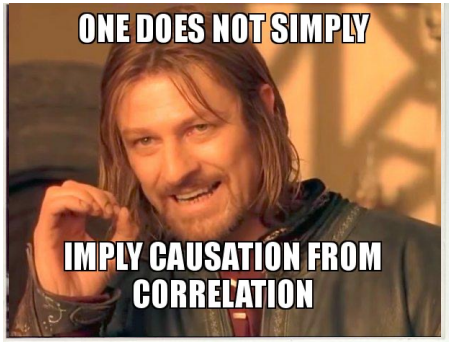


Fisher's letter to Nature, 1958: https://www.nature.com/articles/182596a0.pdf
Nice blog post: https://priceonomics.com/why-the-father-of-modern-statistics-didnt-believe/ (picture of Fisher from this link)

Smoking

Fisher's letter to Nature, 1958:

"… Such results suggest that an error has been made, of an old kind, in arguing from correlation to causation, and that the possibility should be explored that the different smoking classes, non-smokers, cigarette smokers, cigar smokers, pipe smokers, etc., have adopted their habits partly by reason of their personal temperaments and dispositions, and are not lightly to be assumed to be equivalent in their **genotypic composition**…"



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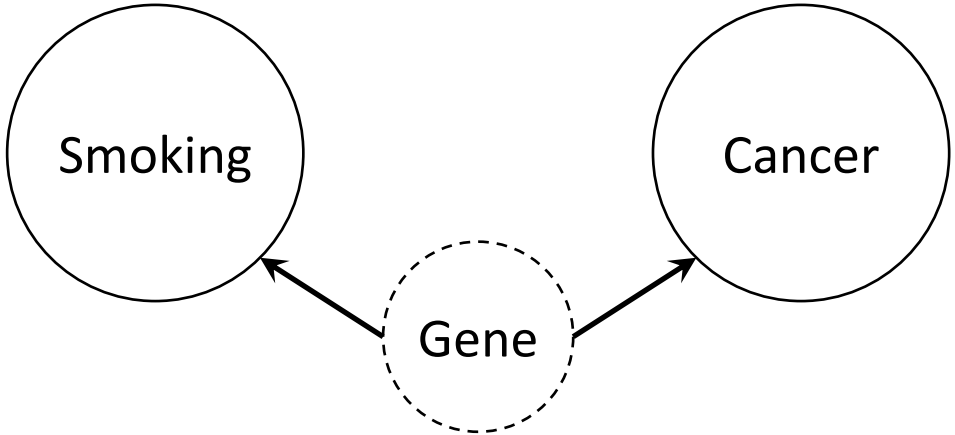
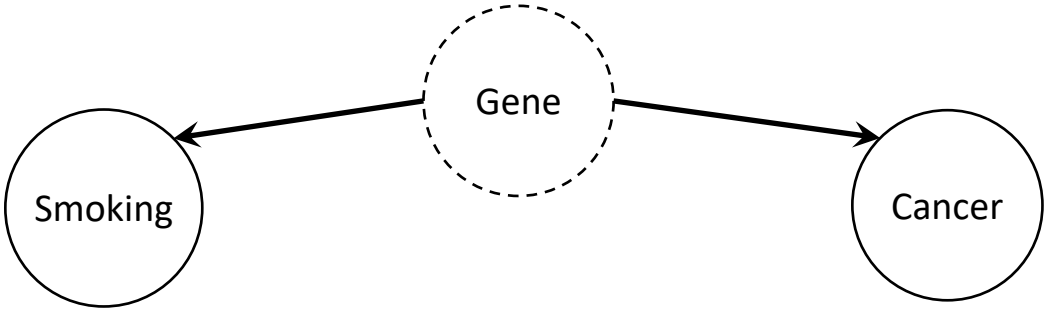
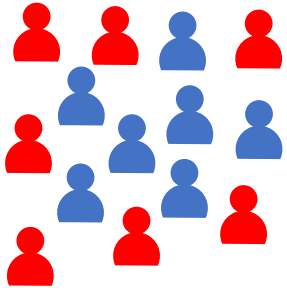


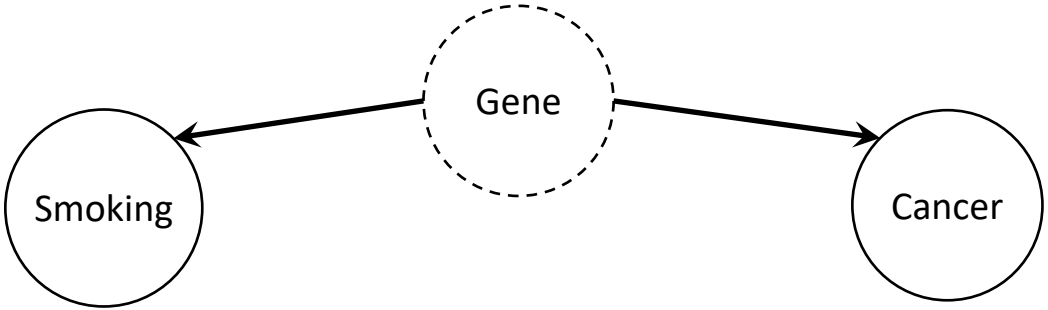
Maybe there's an unmeasured confounder?

From Google image search: https://makeameme.org/meme/one-does-not-5aed73



Hypothesis: There are two types of people in the world





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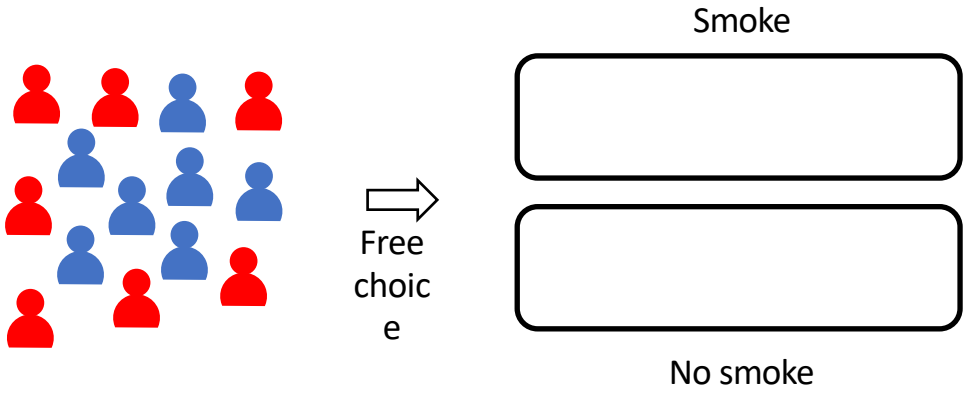

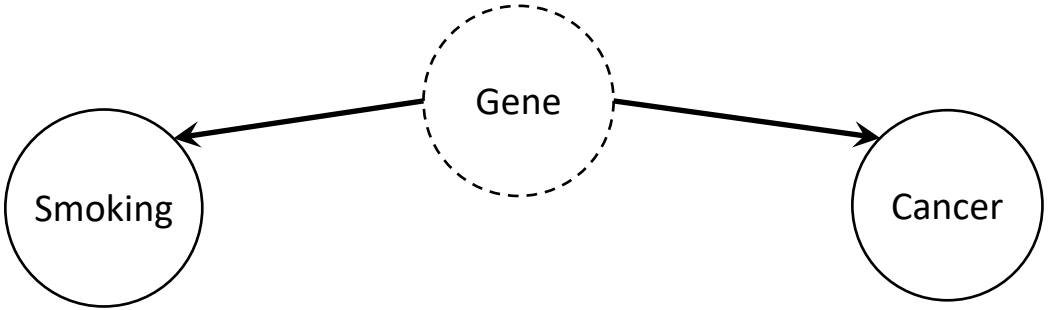

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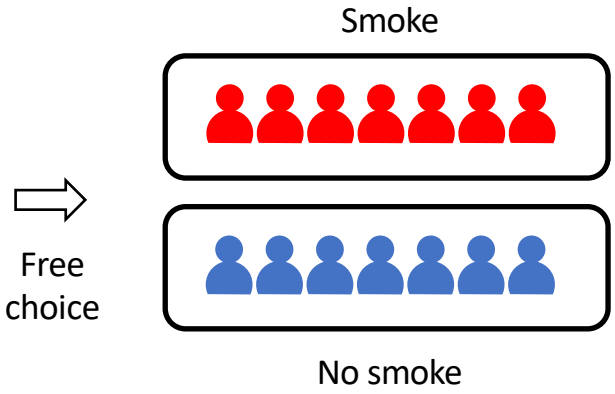

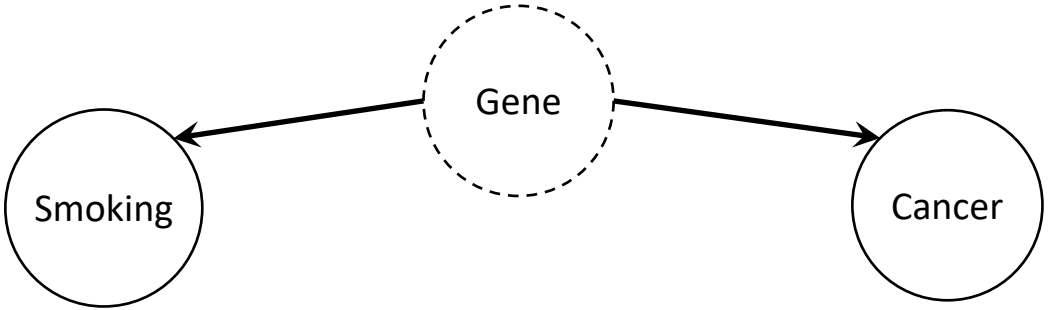

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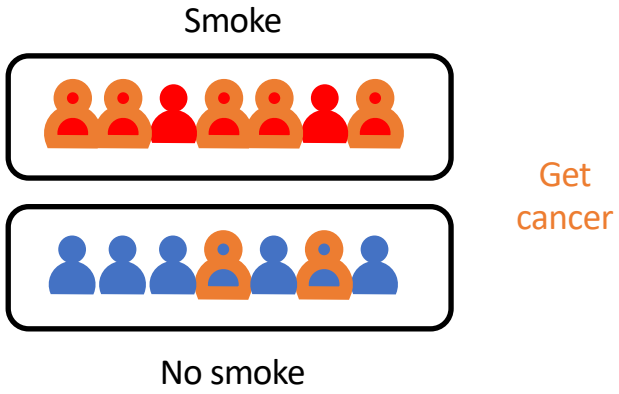
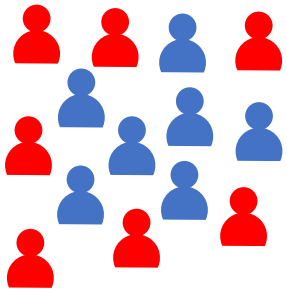
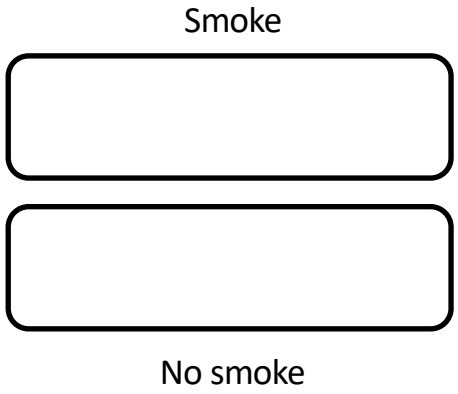


Randomized controlled trials

- Gold standard in scientific exploration
- RCTs ≡ Interventions in causality





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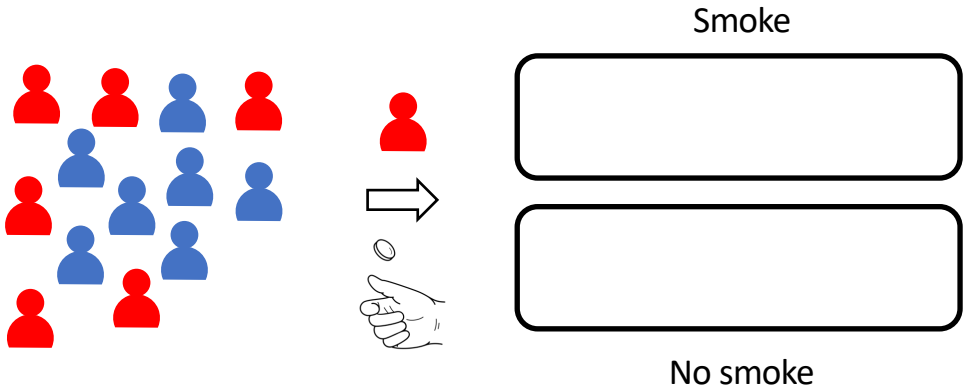

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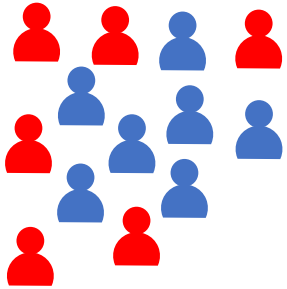

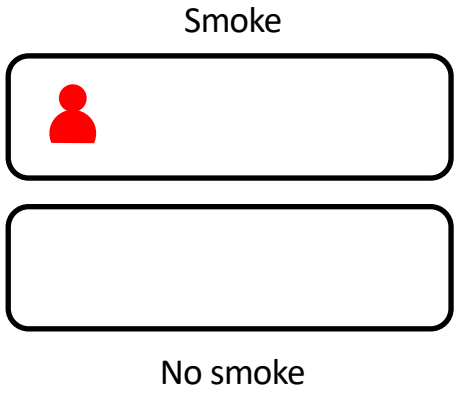

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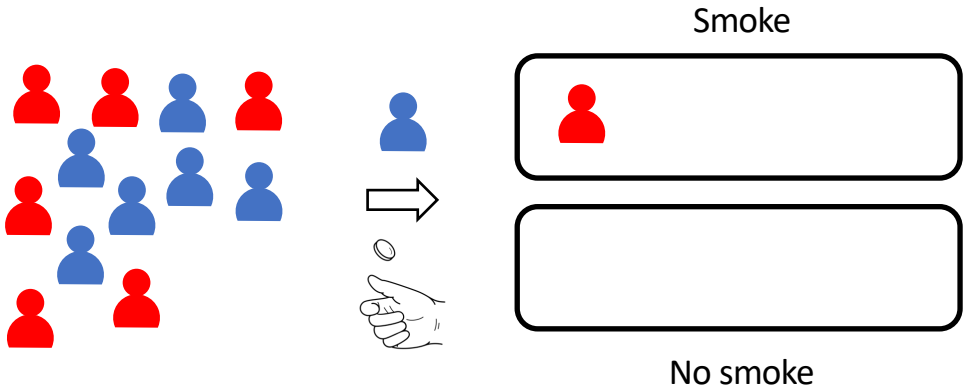

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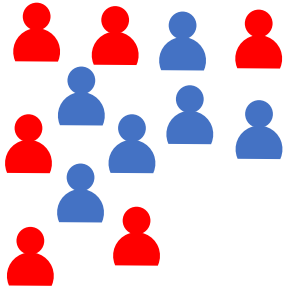

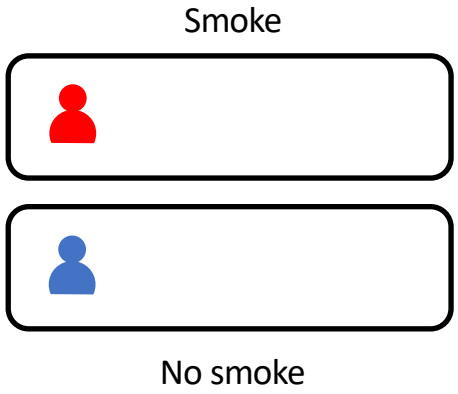

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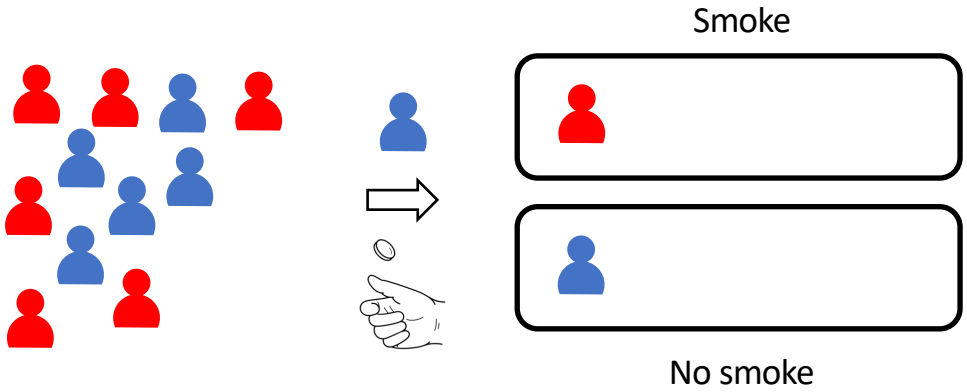

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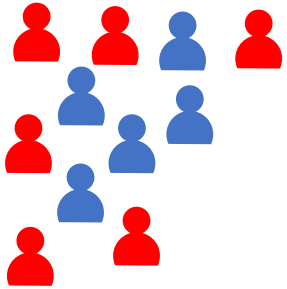

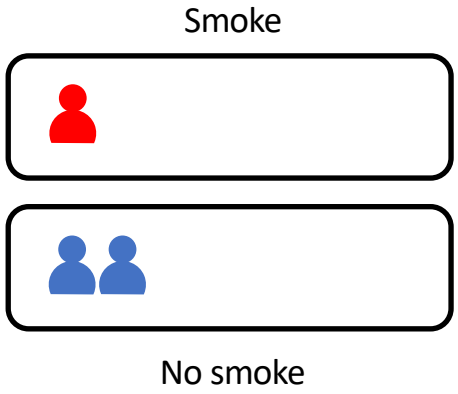

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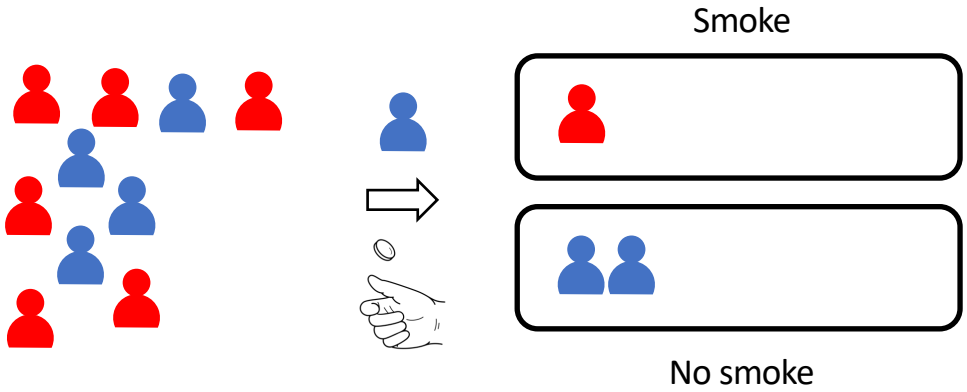

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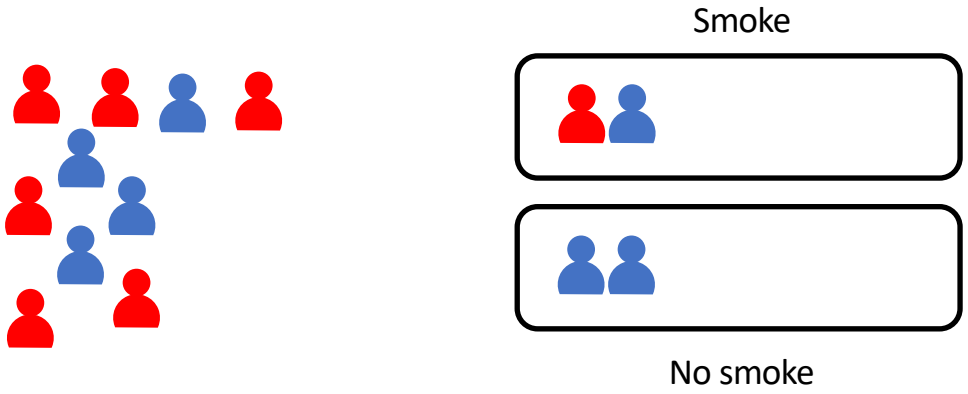

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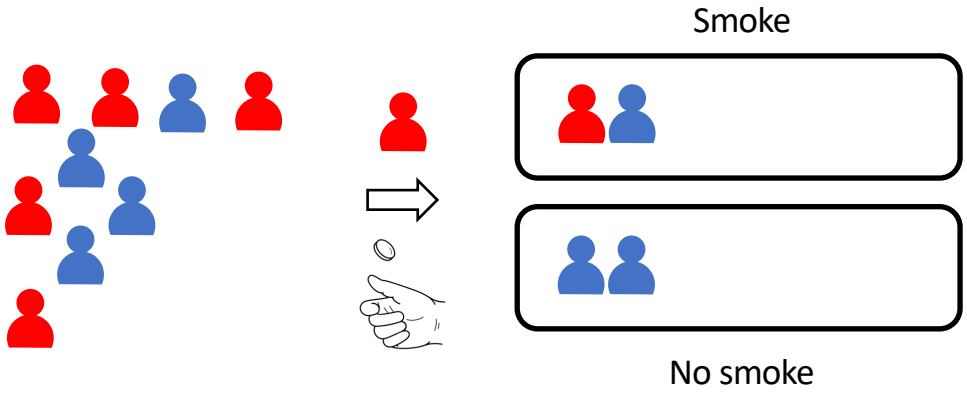

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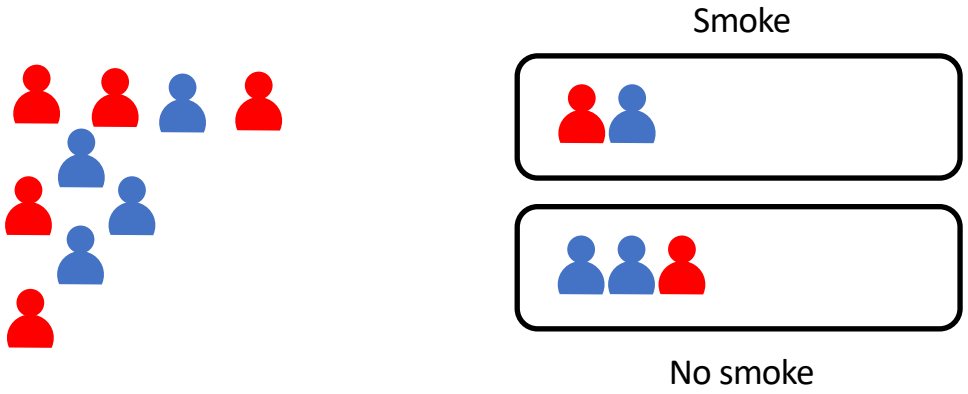

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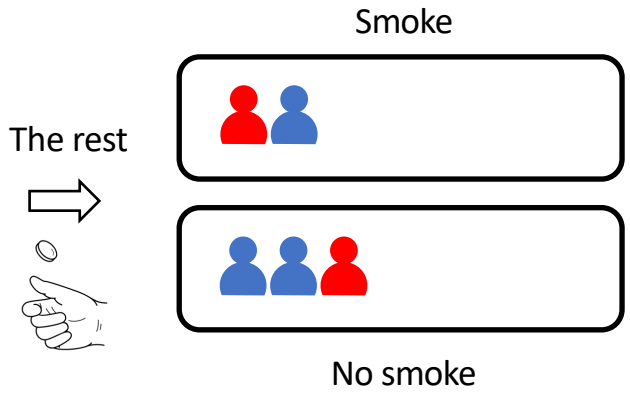

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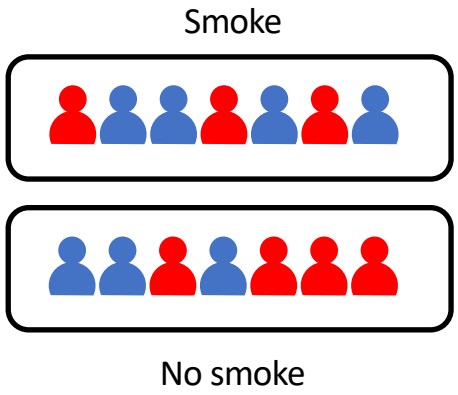

Randomized controlled trials

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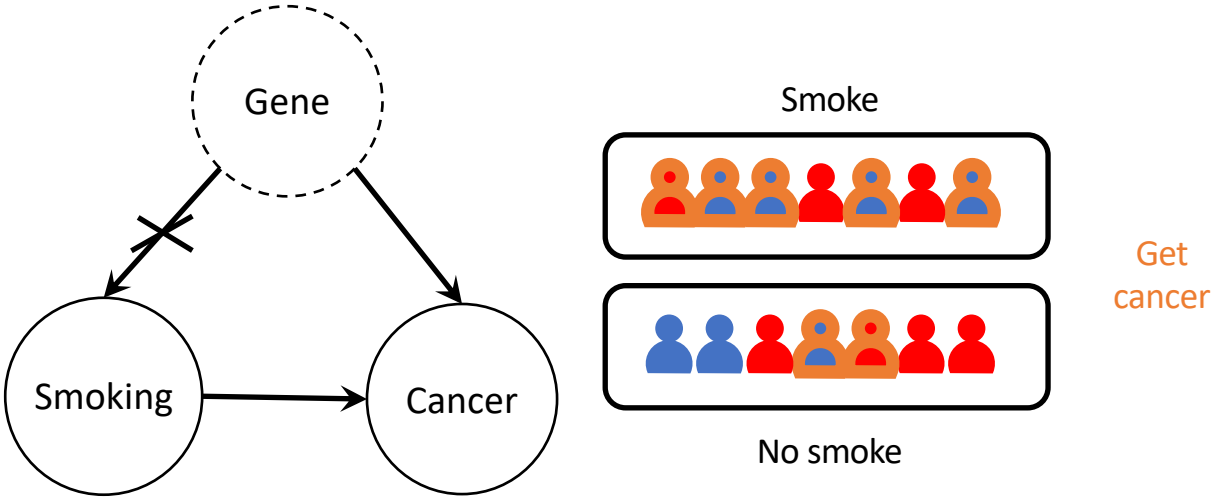

RCT removed causal link from “gene” to “smoking”
If smoking and cancer still highly correlated, then smoking causes cancer

Randomized controlled trials

- Gold standard in scientific exploration
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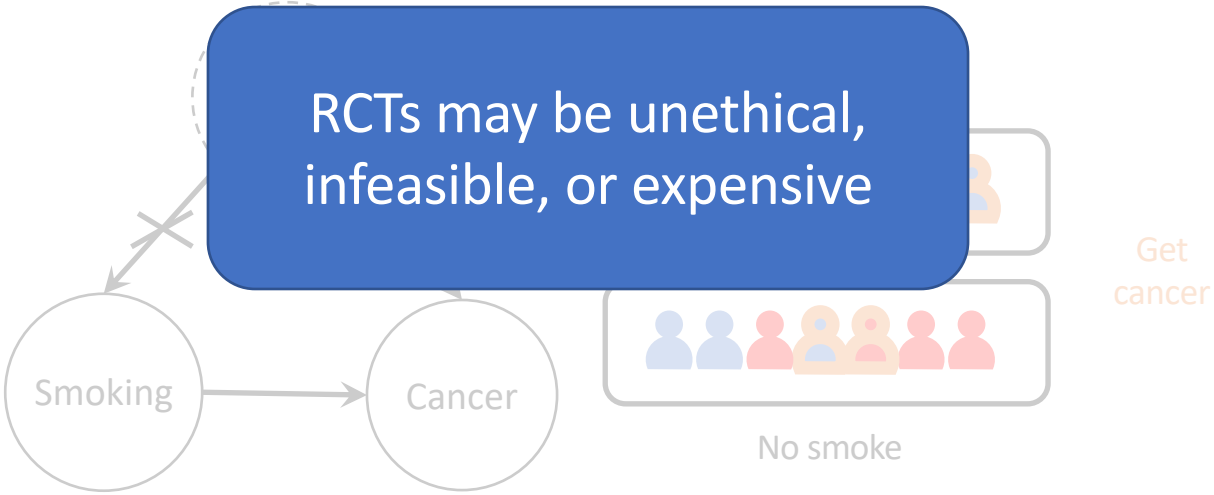

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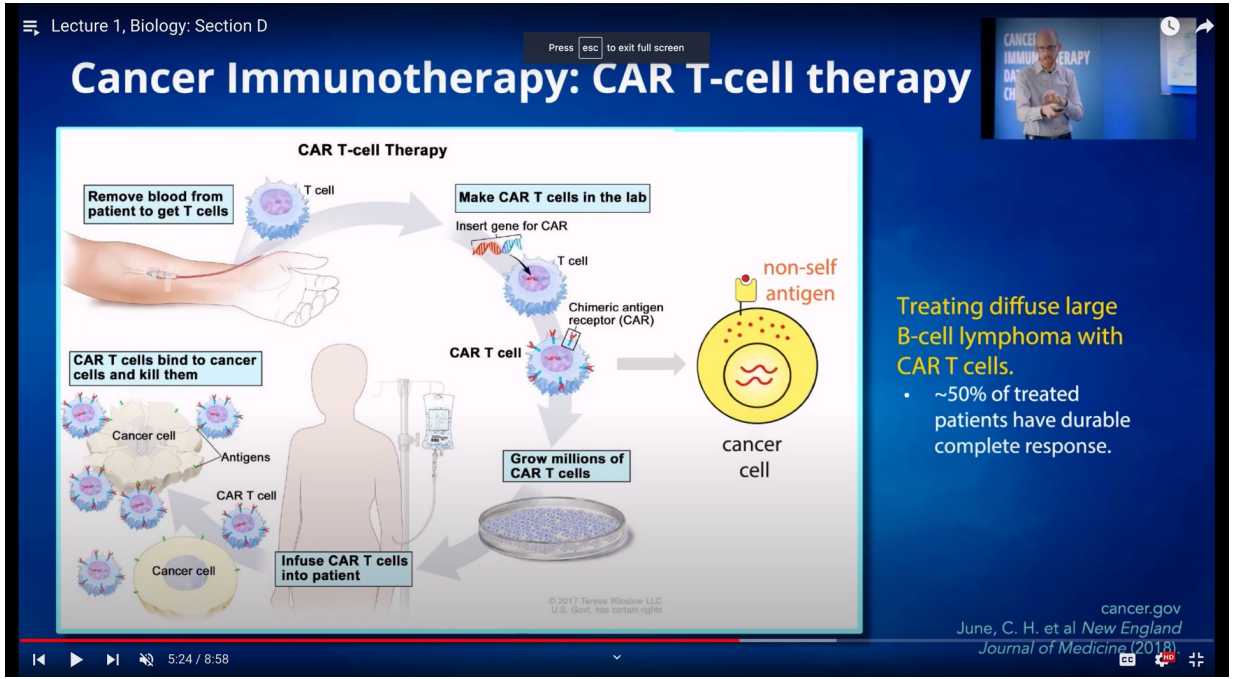
If smoking and cancer still highly correlated, then smoking causes cancer

ERIC AND WENDY SCHMIDT CENTER AT BROAD INSTITUTE

CANCER IMMUNOTHERAPY DATA SCIENCE GRAND CHALLENGE

2023



https://www.topcoder.com/challenges/25f60820-2e69-444b-bc03-490686af2c87
https://www.youtube.com/playlist?list=PL7Ioc09GvxoqmZOFJUc6ni5WS1FgpiIeN

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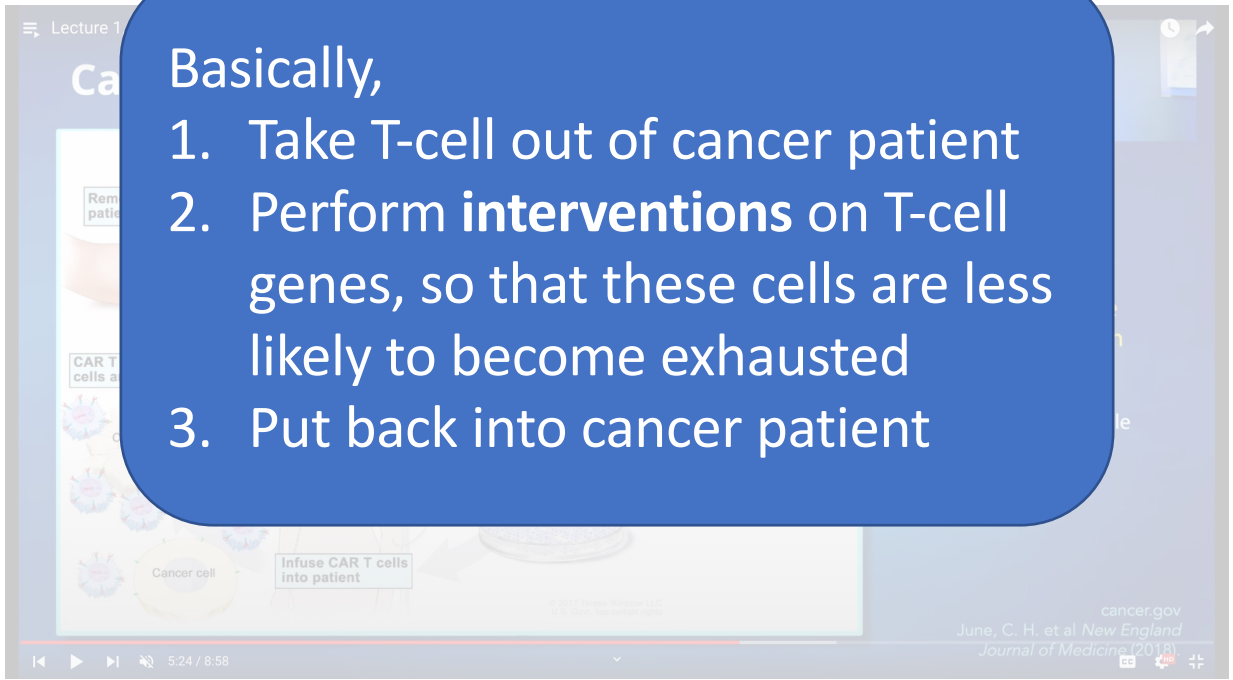


Basically,

1. Take T-cell out of cancer patient
2. Perform **interventions** on T-cell genes, so that these cells are less likely to become exhausted
3. Put back into cancer patient

https://www.topcoder.com/challenges/25f60820-2e69-444b-bc03-490686af2c87
https://www.youtube.com/playlist?list=PL7Ioc09GvxoqmZOFJUc6ni5WS1FgpiIeN

NEWS 07 October 2020

Pioneers of revolutionary CRISPR gene editing win chemistry Nobel

Emmanuelle Charpentier and Jennifer Doudna share the award for developing the precise genome-editing technology.

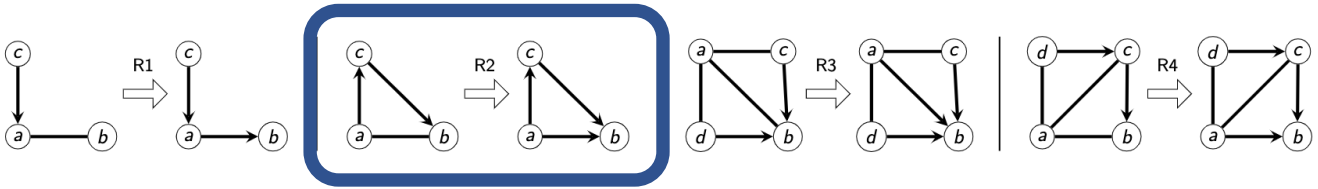
Heidi Ledford & Ewen Callaway

Jennifer Doudna and Emmanuelle Charpentier share the 2020 Nobel chemistry prize for their discovery of a game-changing gene-editing technique. Credit: Alexander Heinel/Picture Alliance/DPA

https://www.nature.com/articles/d41586-020-02765-9

Meek rules [Meek 1995]

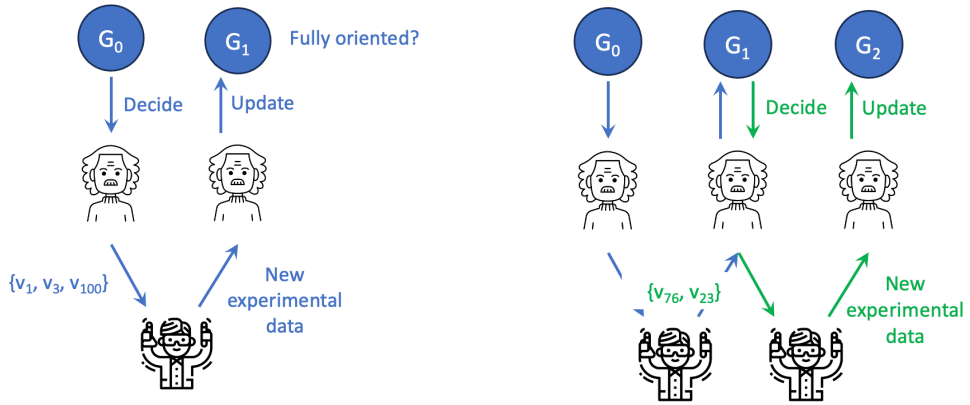
- **Sound** and **complete**
  (with respect to arc orientations with acyclic completion)



- Converge in polynomial time [Wienöbst, Bannach, Liśkiewicz 2021]

Adaptive interventions

- Intervene. Observe & decide next intervention. Repeat.



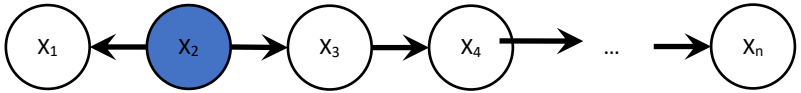
Remark: Non-adaptive may require **exponentially more interventions**

- For example, suppose the essential graph is a path
- Non-adaptive: $\Omega(n)$ interventions (must be a vertex cover)
- Fully adaptive: $\mathcal{O}(\log n)$ interventions (emulate binary search)

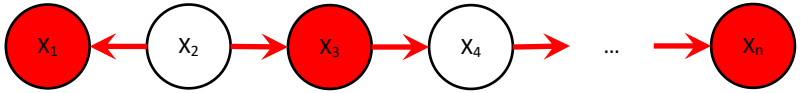
https://thenounproject.com/icon/science-4180127/
https://thenounproject.com/icon/albert-einstein-5297437/

The power of adaptivity

Hidden causal DAG



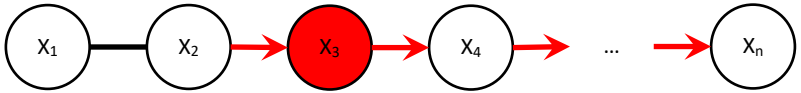
Naïve: Vertex cover

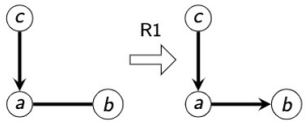


Need $\approx \frac{n}{2}$ interventions

We can do something like binary search and only use $\mathcal{O}(\log n)$ interventions

Suppose we intervene X_3





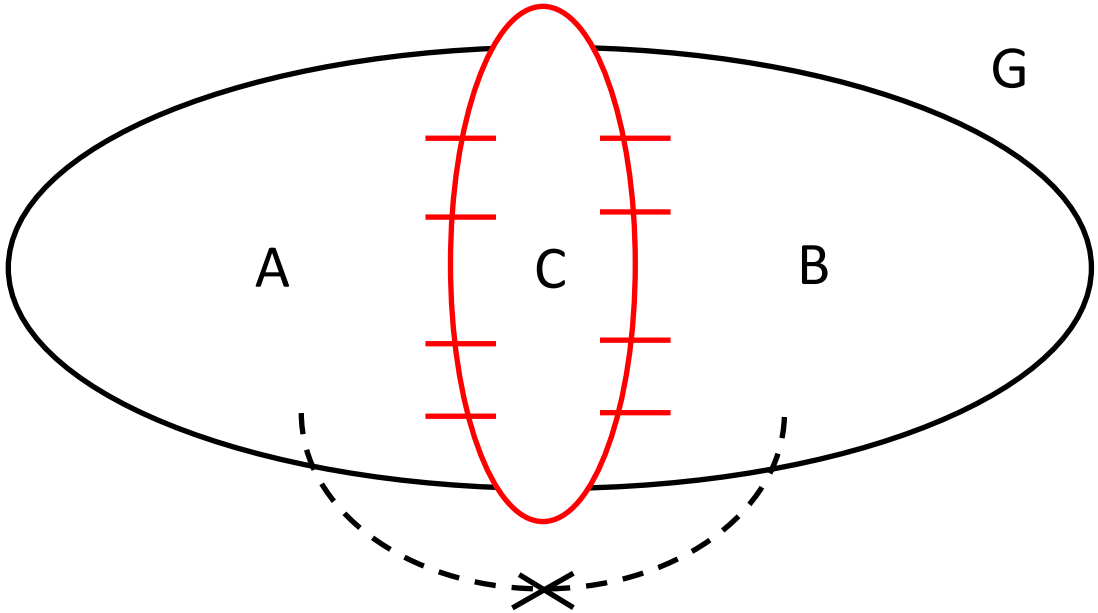
Meek R1

$\mathcal{O}(\log n)$ interventions suffice

The adaptive search problem

- What we know
  - We know at least $\nu(G^*)$ is necessary
  - Prior works only have guarantees for special classes of graphs: cliques, trees, intersection incomparable, etc.
- What we show [Choo, Shiragur, Bhattacharyya 2022]
  - Punchline: $\mathcal{O}\left(\log n \cdot \nu(G^*)\right)$ interventions suffice
  - "Search is almost as easy as verification"
  - Algorithm does not even know what $\nu(G^*)$ is!
  - $\Omega(\log n)$ is unavoidable when causal graph is a directed path on n nodes

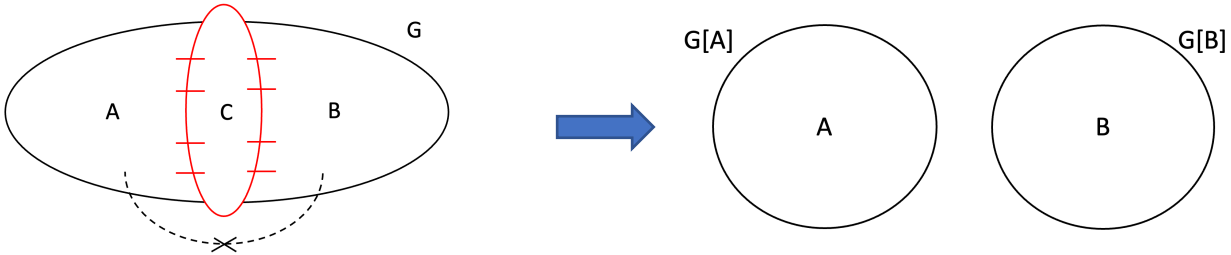
Key algorithmic idea: Graph separators



Partition vertex set V into A, B, C:

1. C separates A and B
2. $|A|, |B| \leq |V| / 2$

Key algorithmic idea: Graph separators



- Analysis:
  - $\mathcal{O}(\log n)$ rounds ← Chordal graph separator [Gilbert, Rose, Edenbrandt 1984]
  - $\mathcal{O}\left(\nu(G^*)\right)$ per round ← We prove new lower bound on $\nu(G^*)$

What is known?

- Verification number $v(G^*)$: A natural benchmark
  - How many interventions are required if you *know* G^* and want to verify it via experiments?

Recovery objective	Number of interventions†	Reference
Full graph G^*	$\mathcal{O}\left(\log n \cdot v(G^*)\right)$	[Choo, Shiragur, Bhattacharyya 2022]
Subgraph $\mathrm{H} \subseteq G^*$	$\mathcal{O}\left(\log \lvert H \rvert \cdot v(G^*)\right)$	[Choo, Shiragur 2023] §

- **Question: What if we have (imperfect) advice / side information from domain experts?**

† Atomic interventions. § To be precise, [Choo, Shiragur 2023] define "relevant vertices in H" instead of V(H).

Exploiting side information

- Learning-augmented algorithms
  - Designing algorithms using advice, predictions, etc.
    - α-consistent: α-competitive with no advice error
    - β-robust: β-competitive with any advice error
- Advice in causal structure learning
  - Prior works treat information from domain experts as ground truth about the underlying causal system*
    - i.e. No robustness guarantees
  - **For this talk, advice is a specific DAG $\widetilde{\boldsymbol{G}} \in [\boldsymbol{G}^*]$**
    - See our paper for discussion about partial advice

* Some prior works account for "expert confidence", but confidence and correctness are orthogonal issues: An expert can be confidently wrong.

Our main results

Recovery objective	Advice	Number of interventions†	Reference
G^*	-	$\mathcal{O}\left(\log n \cdot \nu(G^*)\right)$	[Choo, Shiragur, Bhattacharyya 2022]
$\mathrm{H} \subseteq G^*$	-	$\mathcal{O}\left(\log \lvert H \rvert \cdot \nu(G^*)\right)$	[Choo, Shiragur 2023] §
G^*	$\tilde{G} \in [G^*]$	$\mathcal{O}\left(\max\left\{1, \log \psi\left(\tilde{G}, G^*\right)\right\} \cdot \nu(G^*)\right)$	This work

- $0 \leq \psi\left(\tilde{G}, G^*\right) \leq n$ measures the quality of $\tilde{G}$ as advice to G^*. See paper for details.
- No advice error (when $\tilde{G} = G^*$): $\nu(G^*)$
- Arbitrary advice error: $\mathcal{O}\left(\log n \cdot \nu(G^*)\right)$

† Atomic interventions. § To be precise, [Choo, Shiragur 2023] define "relevant vertices in H" instead of V(H).

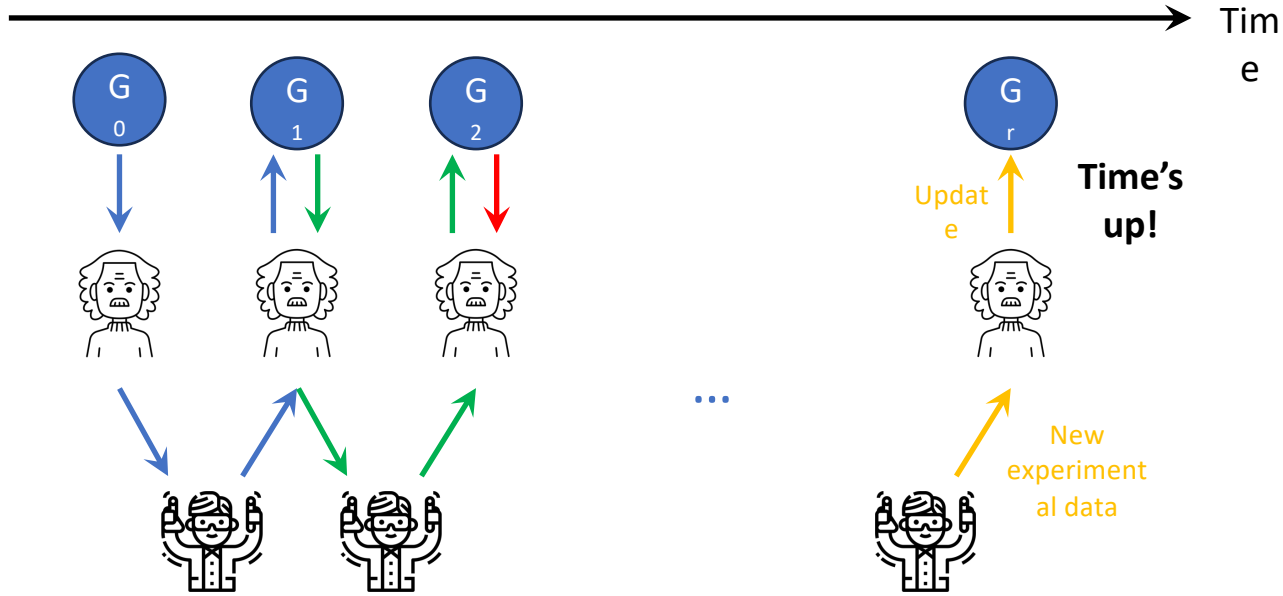
High-level technical details

Recovery objective	Advice	Number of interventions†	Reference
G^*	-	$\mathcal{O}\left(\log n \cdot \nu(G^*)\right)$	[Choo, Shiragur, Bhattacharyya 2022]
$\mathrm{H} \subseteq G^*$	-	$\mathcal{O}\left(\log \lvert H \rvert \cdot \nu(G^*)\right)$	[Choo, Shiragur 2023] §
G^*	$\tilde{G} \in [G^*]$	$\mathcal{O}\left(\max\left\{1, \log \psi\left(\tilde{G}, G^*\right)\right\} \cdot \nu(G^*)\right)$	This work

- For any two DAGs $G_1, G_2 \in [G^*]$, we have $\nu(G_1) \leq 2 \cdot \nu(G_2)$
- Insight from [Choo, Shiragur, Bhattacharyya 2022]
  - Minimum vertex cover (MVC) of covered edges orients G^*
- Our algorithm
  - Consider r -hop neighborhood of MVC of $\tilde{G}$'s covered edges
  - Invoke [Choo, Shiragur 2023] at suitable intervals as we increase r

† Atomic interventions. § To be precise, [Choo, Shiragur 2023] define "relevant vertices in H" instead of V(H).

What if we have limited rounds of adaptivity?



Given a budget of r adaptive rounds, how to minimize number of interventions?

(Note: A single round of n interventions is always trivially sufficient)

Adaptivity: Trading time for cost

- One-shot non-adaptive approach:
  - Interventions reveal incident arcs → $\mathcal{O}(n)$ interventions
  - Worst case necessary: "Need to recover all possible G*"
- Recent line of work: Adaptive interventions
  - Intervene. Observe & decide next intervention. Repeat.
  - $\mathcal{O}(\log n \cdot \nu(G^*))$ interventions† suffice
- Exists simple graphs where non-adaptive requires **exponentially** more interventions than adaptive
- Question: Why not just fully adaptive?
  - Adaptivity imposes sequentiality and hinders parallelism

† Atomic interventions

Our results

- Given 1 ≤ r ≤ n rounds of adaptivity

$$\Theta\left(\min\{r, \log n\} \cdot n^{\frac{1}{\min\{r, \log n\}}} \cdot \nu(G^*)\right)$$ atomic interventions

  - When $r = 1 \rightarrow \Theta(n)$ ← Match worst case bound of non-adaptive
  - When $r = \mathcal{O}(\log n) \rightarrow \Theta\left(\log n \cdot \nu(G^*)\right)$ ← Match worst case bound of fully adaptive
  - Key ideas:
    1. Chain components are chordal graphs → Chordal tree representation
    2. Balanced partitioning of trees
- Extension (see paper):
  - Bounded size interventions where each intervention involves up to 1 ≤ k ≤ n vertices

Thank you for your kind attention!