

# Algorithms for Learning Probabilistic and Causal Models with Possible Imperfect Advice

Teamcore Postdoc Seminar

23<sup>rd</sup> August 2024

Davin Choo

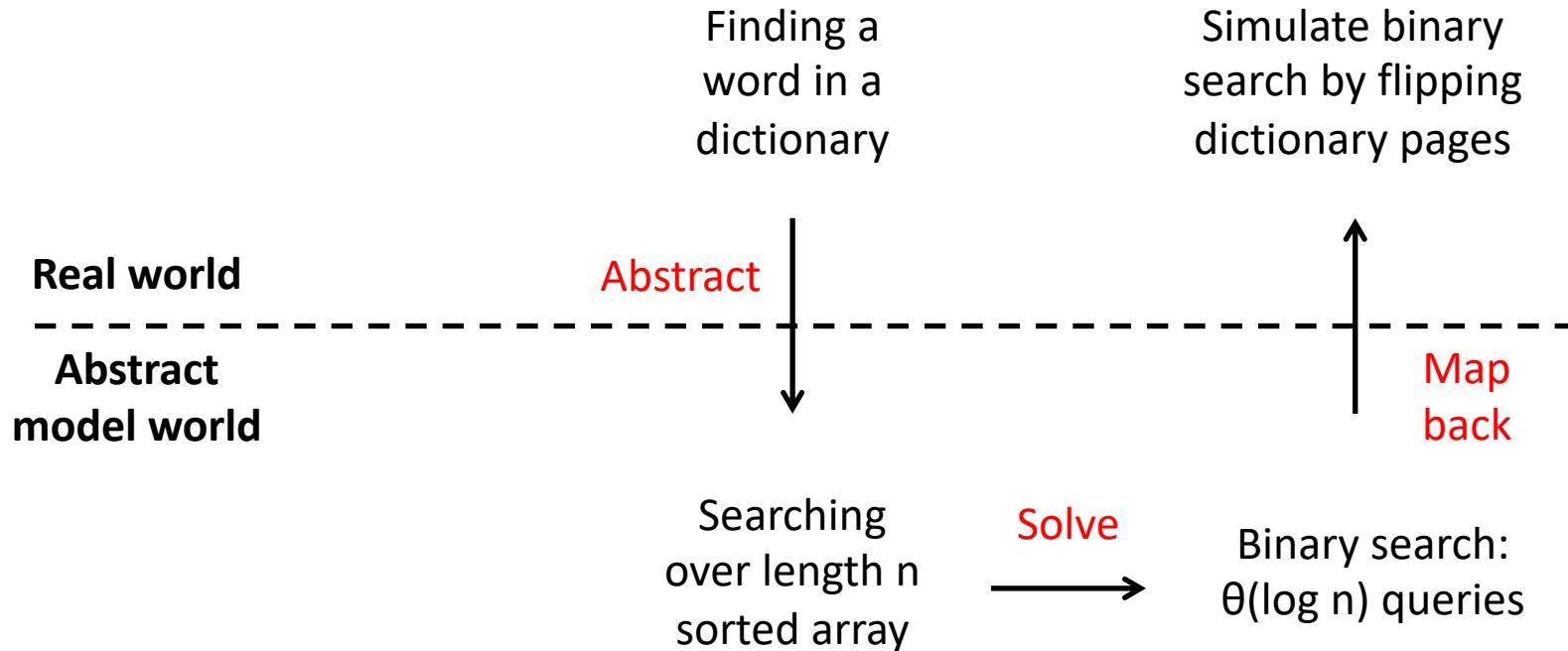
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# The world is complex

- **Learning a useful representation** of the world from data is a cornerstone of scientific discovery and the driving force behind successful modern machine learning methods
- This process often involves learning **probabilistic models for predictive tasks** and **causal models to understand interventional effects on systems**, which is crucial for informed downstream decision-making



# A general problem-solving framework



# A general problem-solving framework

- Complex setting
- Many nuances
- Possibly unseen problem

**Real world**

**Abstract  
model world**

- Simplified setting
- Generic problem framing
- Many plug-and-play solution concepts

Finding a word in a dictionary

Simulate binary search by flipping dictionary pages

**Abstract**



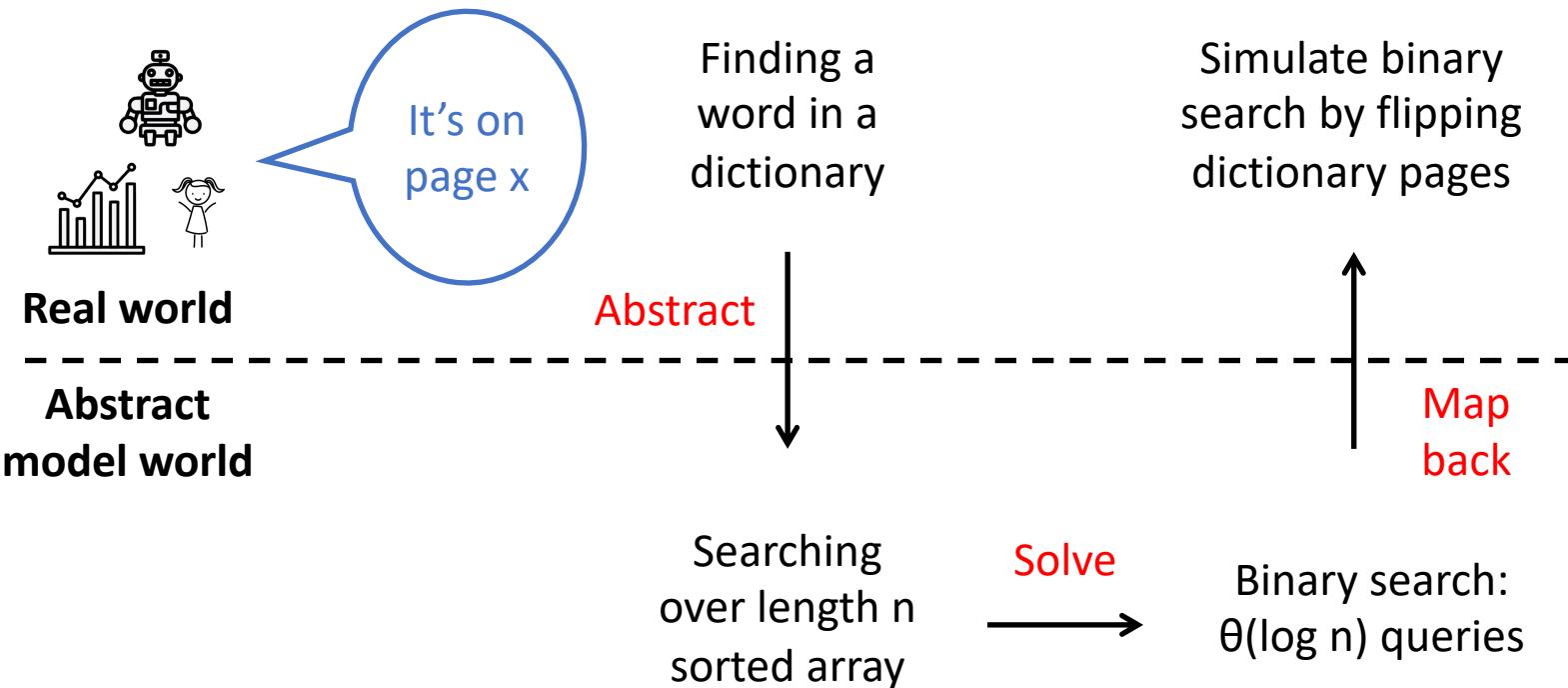
**Map back**

Searching over length n sorted array

**Solve**

Binary search:  
 $\Theta(\log n)$  queries

# Side-information about problem instances

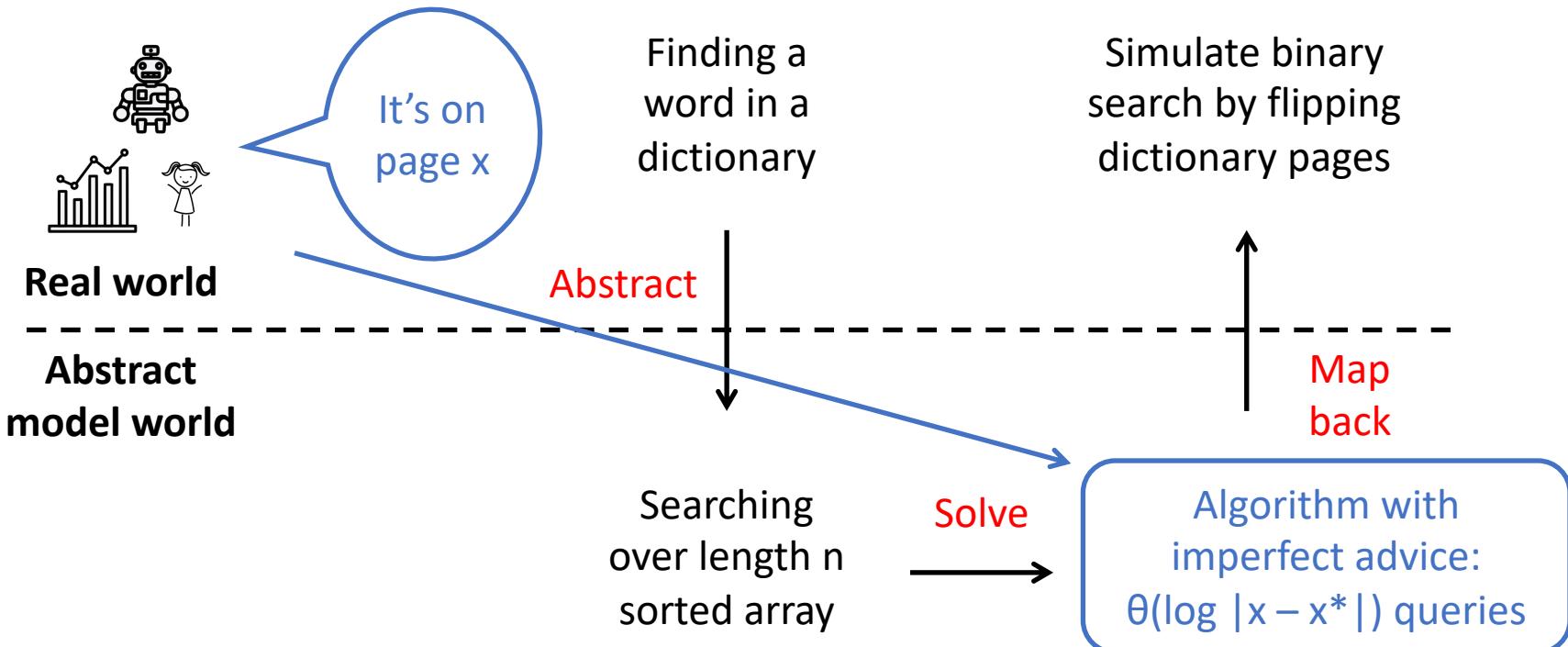


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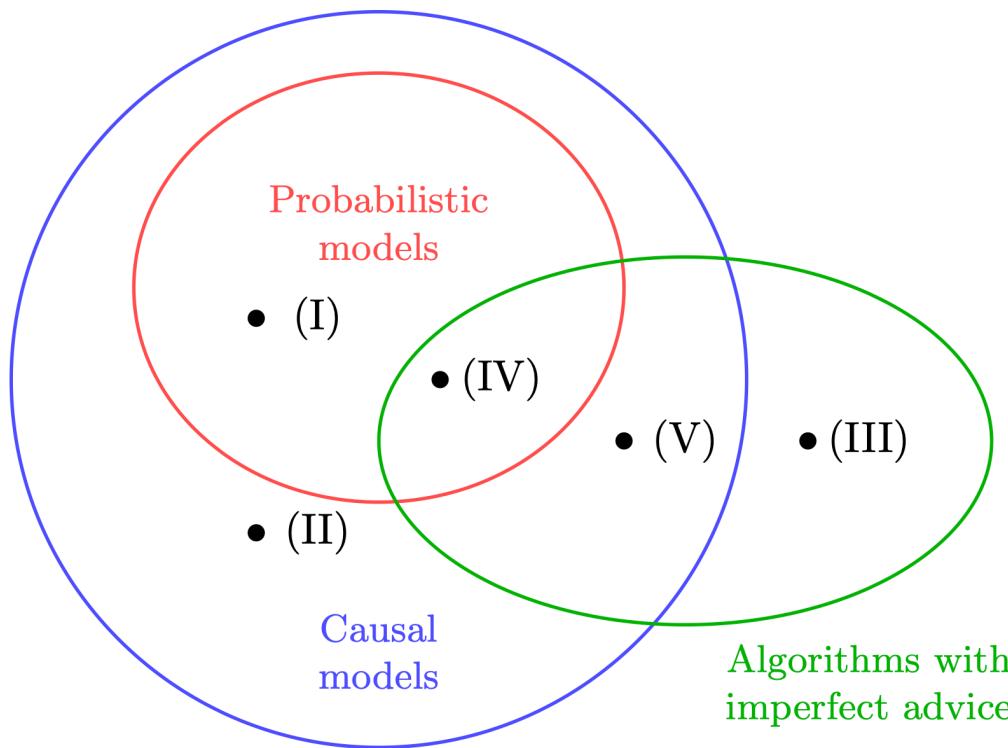
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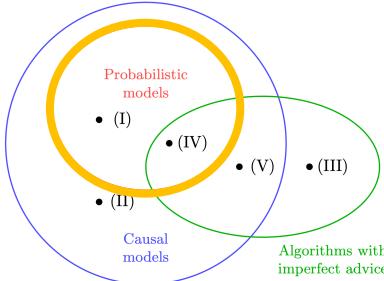
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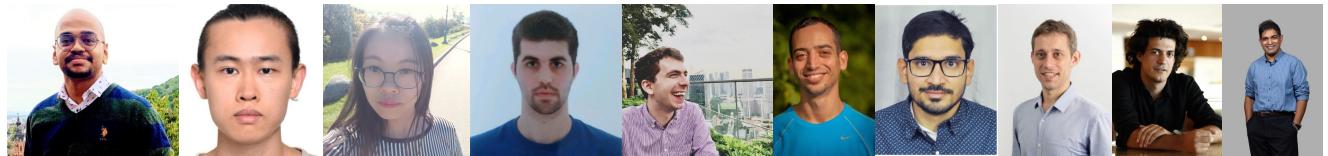
# Main themes explored in my PhD



# (I): Probabilistic models



- Classic results in statistics show asymptotic convergence of estimators in the “infinite data” regime
- Probably Approximately Correct (PAC) learning model [Val84]
  - Given sample access to some underlying distribution  $\mathcal{P}$ , produce  $\hat{\mathcal{P}}$  such that  $\text{TV}(\mathcal{P}, \hat{\mathcal{P}}) \leq \varepsilon$  with probability  $\geq 1 - \delta$
- Bayesian networks [Pea88]
  - Probabilistic graphical model commonly used to model beliefs
  - $\approx 2^{n^2}$  candidate directed acyclic graphs (DAGs), one of which is  $\mathcal{G}^*$



[Val84] Leslie G Valiant. *A theory of the learnable*.

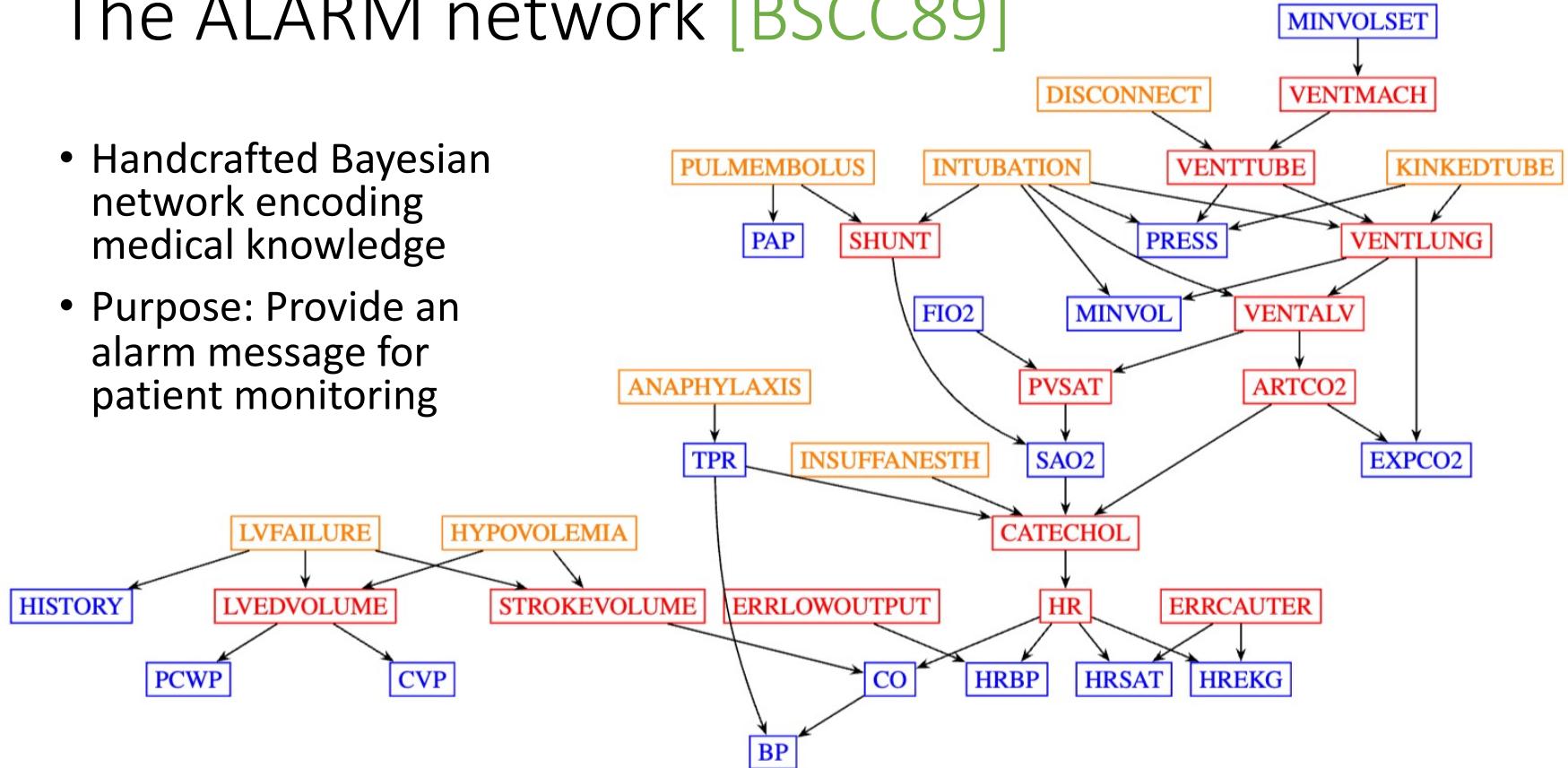
Communications of the ACM, 1984.

[Pea88] Judea Pearl. *Probabilistic reasoning in intelligent systems*.

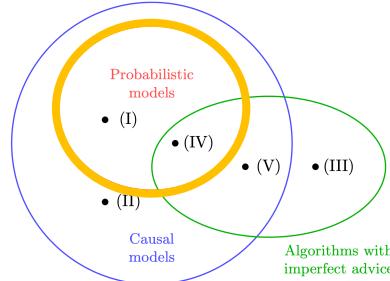
Morgan Kaufmann, 1988.

# The ALARM network [BSCC89]

- Handcrafted Bayesian network encoding medical knowledge
- Purpose: Provide an alarm message for patient monitoring



# (I): Probabilistic models



- Suppose data distribution  $\mathcal{P}$  is described by Bayesian network
  - NP-hard to find “score maximizing” DAG from data [Chi96] and to decide whether  $\mathcal{P}$  can be described by a DAG with  $p$  parameters [CHM04]
  - Even under the promise that  $\mathcal{P}$  can be described by a DAG with  $p$  parameters, it is NP-hard to find such a parameter-bounded DAG [BCGM24]
  - We also have some PAC-style finite sample results in learning the structure and parameters of Bayesian network for  $\mathcal{P}$  [BCG+22, DDKC23, CYBC24]
  - **Insight: If network's in-degree is bounded, we can use less samples**

[Chi96] David Maxwell Chickering. *Learning Bayesian networks is NP-complete*. Lecture Notes in Statistics, vol 112, 1996

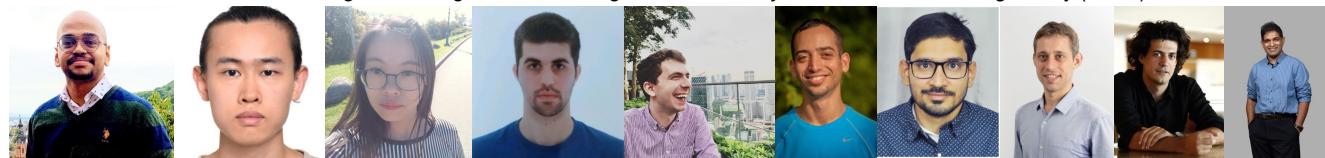
[CHM04] Max Chickering, David Heckerman, and Chris Meek. *Large-sample learning of Bayesian networks is NP-hard*. Journal of Machine Learning Research (JMLR), 2004

[BCGM24] Arnab Bhattacharya, Davin Choo, Sutanu Gayen, Dimitrios Myrisiotis. *Learnability of Parameter-Bounded Bayes Nets*. Structured Probabilistic Inference & Generative Modeling (ICML Workshop), 2024

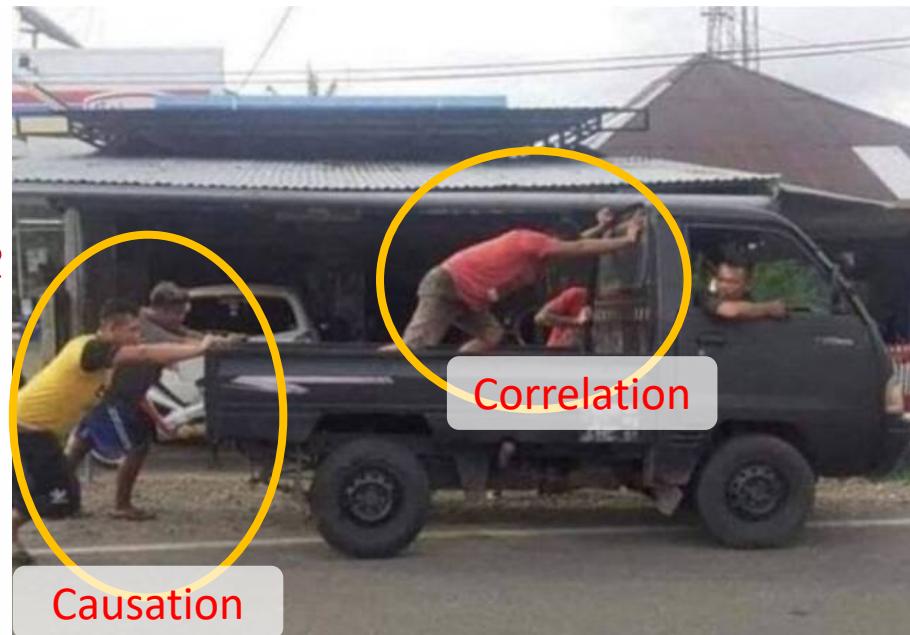
[BCG+22] Arnab Bhattacharya, Davin Choo, Rishikesh Gajjala, Sutanu Gayen, Yuhao Wang. *Learning Sparse Fixed-Structure Gaussian Bayesian Networks*. International Conference on Artificial Intelligence and Statistics (AISTATS), 2024

[DDKC23] Yuval Dagan, Constantinos Daskalakis, Anthimos-Vardis Kandiros, Davin Choo. *Learning and Testing Latent-Tree Ising Models Efficiently*. Conference on Learning Theory (COLT), 2023

[CYBC24] Davin Choo, Joy Qiping Yang, Arnab Bhattacharya, Clément L. Canonne. *Learning bounded degree polytrees with samples*. International Conference on Algorithmic Learning Theory (ALT), 2024



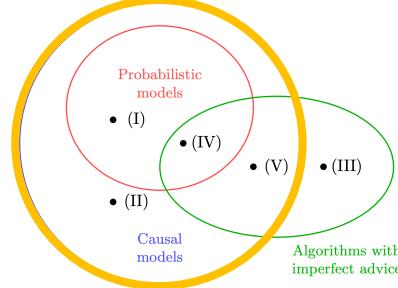
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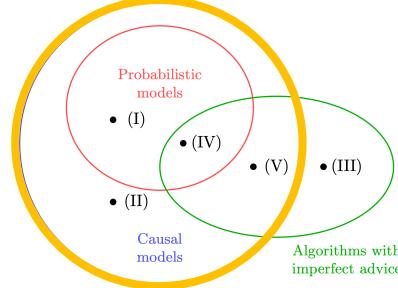
# (II): Causal models



- Two fundamental problems in causal inference
  - Causal graph discovery: Recover true causal graph  $\mathcal{G}^*$
  - Causal effect estimation: Estimate  $\mathcal{P}(Y = y \mid do(X = x))$



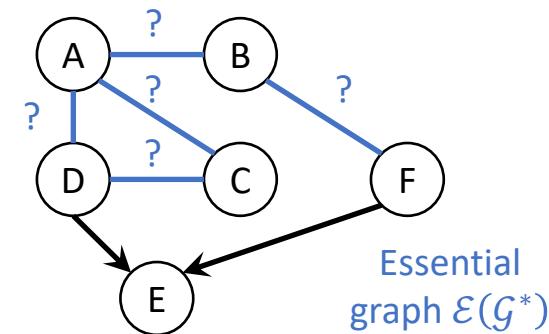
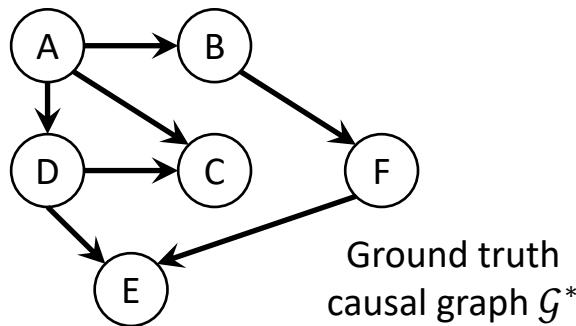
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    - Use interventions and experiments!
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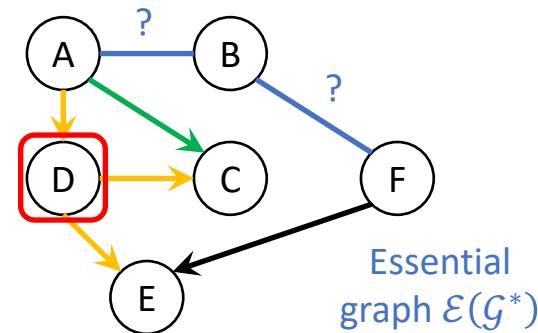
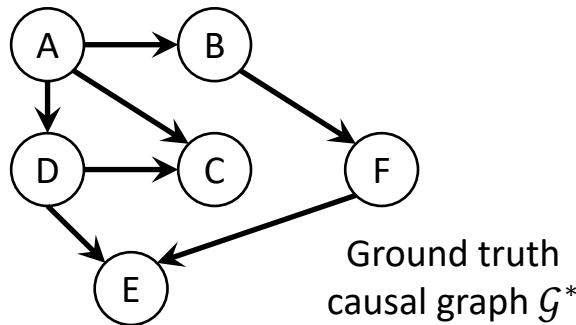


# Causal discovery via interventions



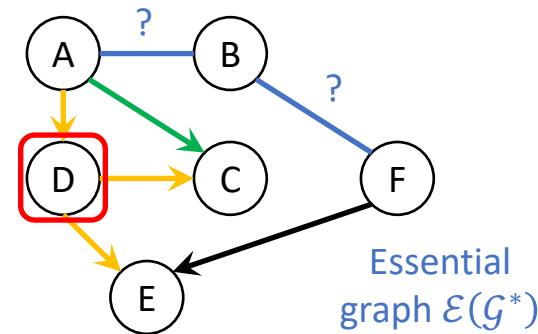
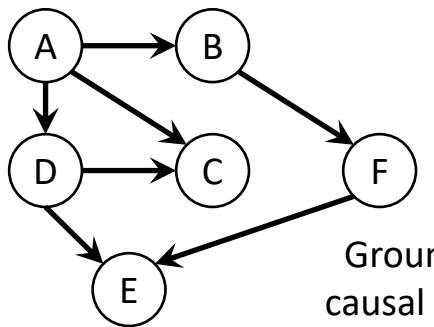
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# Causal discovery via interventions



- Want: Recover  $\mathcal{G}^*$  starting from **partially oriented**  $\mathcal{E}(\mathcal{G}^*)$  from observational data
- Interventions reveal arc orientations (**incident arcs** + **Meek rules**)
- Goal: Recover  $\mathcal{G}^*$  using as few interventions as possible**

# Causal discovery via interventions



- Want: Recover  $\mathcal{G}^*$  starting from **partially oriented  $\mathcal{E}(\mathcal{G}^*)$**  from observational data
- Interventions reveal arc orientations (**incident arcs** + **Meek rules**)
- Goal: Recover  $\mathcal{G}^*$  using as few interventions as possible**
- We have some results regarding how to design algorithms to perform optimal adaptive interventions under various scenarios [[CSB22](#), [CS23a](#), [CGB23](#), [CS23b](#), [CS23c](#), [CSU24](#)]
- Insight: Can abstract and treat this as a graph problem with specialized causal operations**

[[CSB22](#)] Davin Choo, Kirankumar Shiragur, Arnab Bhattacharyya. *Verification and search algorithms for causal DAGs*. Conference on Neural Information Processing Systems (NeurIPS), 2022.

[[CS23a](#)] Davin Choo, Kirankumar Shiragur. *Subset verification and search algorithms for causal DAGs*. International Conference on Artificial Intelligence and Statistics (AISTATS), 2023.

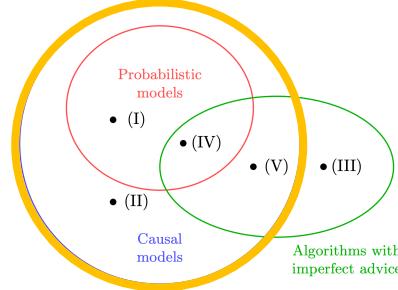
[[CGB23](#)] Davin Choo, Themistoklis Gouleakis, Arnab Bhattacharyya. *Active causal structure learning with advice*. International Conference on Machine Learning (ICML), 2023.

[[CS23b](#)] Davin Choo, Kirankumar Shiragur. *New metrics and search algorithms for weighted causal DAGs*. International Conference on Machine Learning (ICML), 2023.

[[CS23c](#)] Davin Choo, Kirankumar Shiragur. *Adaptivity Complexity for Causal Graph Discovery*. Conference on Uncertainty in Artificial Intelligence (UAI), 2023.

[[CSU24](#)] Davin Choo, Kirankumar Shiragur, Caroline Uhler. *Causal discovery under off-target interventions*. International Conference on Artificial Intelligence and Statistics (AISTATS), 2024.

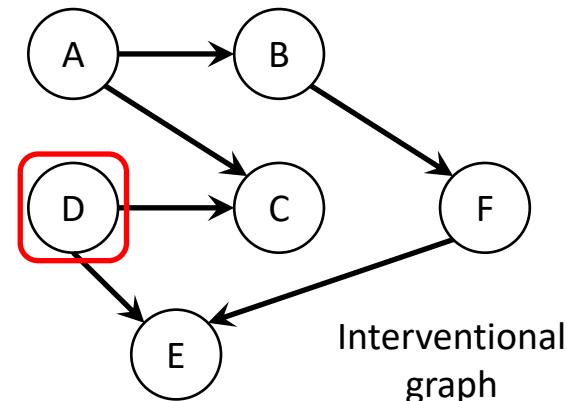
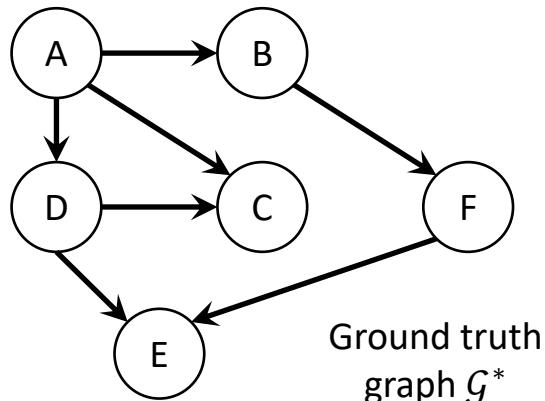
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- Two fundamental problems in causal inference
  - Causal graph discovery: Recover true causal graph  $\mathcal{G}^*$ 
    - Even with infinite observational data, can only determine causal graph up to some equivalence class where all conditional independence relations agree
    - Use interventions and experiments!
  - Causal effect estimation: Estimate  $\mathcal{P}(Y = y \mid do(X = x))$ 
    - Typically, a 2-stage process: learn  $\mathcal{G}^*$ , then apply closed-form formulas



# Causal identification (the 2<sup>nd</sup> step)

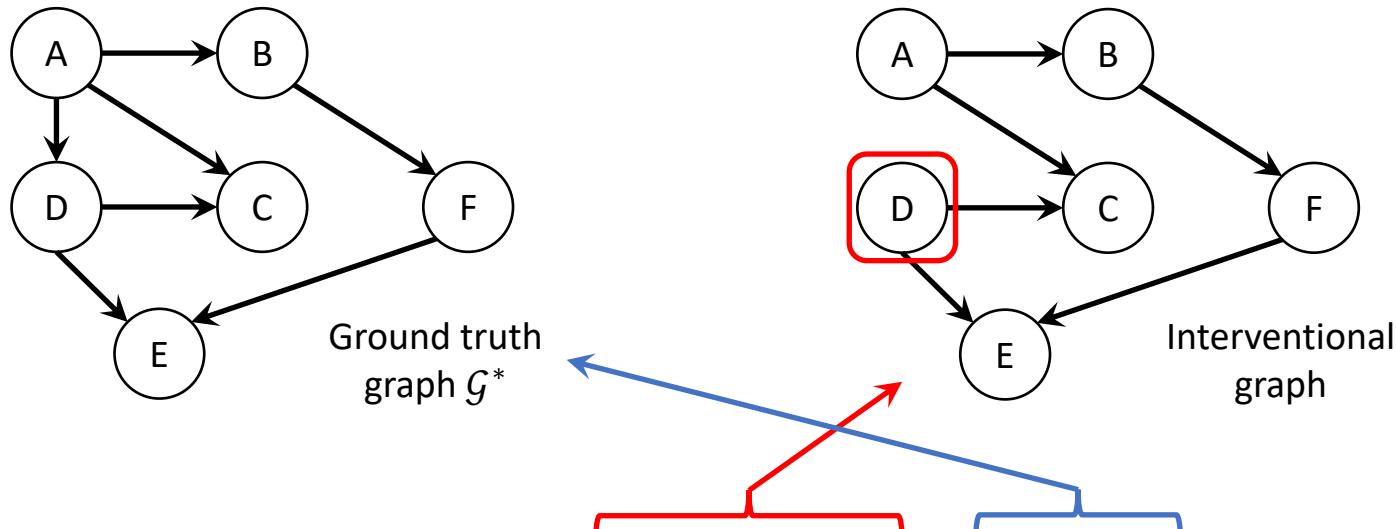


$$\mathcal{P}(E = e \mid do(D = d^*))$$

Interventional query

What is probability of  $E = e$  when we fix  $D = d^*$ ?

# Causal identification (the 2<sup>nd</sup> step)

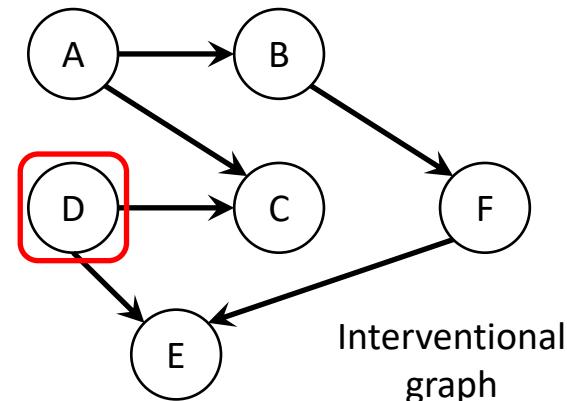
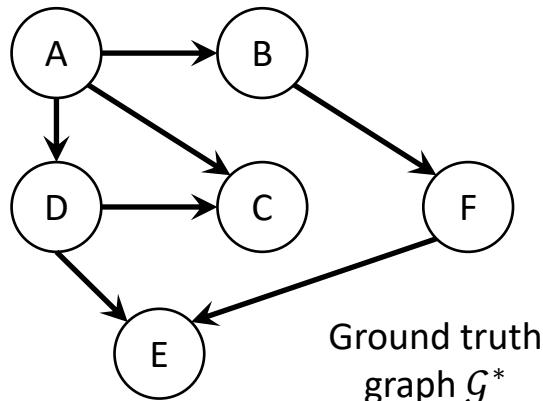


Interventional query

What is probability of  $E = e$  when we fix  $D = d^*$ ?

Need to draw samples from interventional graph, i.e., perform experiment and measure

# Causal identification (the 2<sup>nd</sup> step)



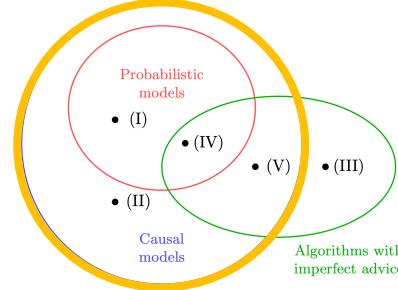
Because of structure of  $\mathcal{G}^*$

$$\mathcal{P}(E = e \mid do(D = d^*)) = \mathcal{P}(e \mid do(d^*)) = \int \mathcal{P}(e \mid d^*, a) \cdot \mathcal{P}(a) da$$

Interventional query

What is probability of  $E = e$  when we fix  $D = d^*$ ?

Just observational terms!

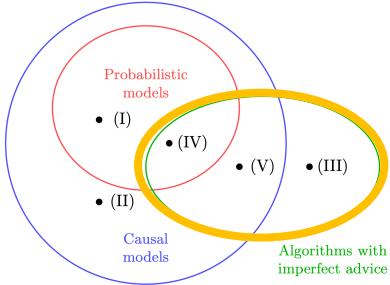


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    - Use interventions and experiments!
  - Causal effect estimation: Estimate  $\mathcal{P}(Y = y \mid do(X = x))$ 
    - Typically, a 2-stage process: learn  $\mathcal{G}^*$ , then apply closed-form formulas
    - [CSBS24] This is suboptimal as it may require strong assumptions and a lot of samples
    - Insight: “weak edges” shouldn’t affect much for PAC-style results



# (III/IV/V): Algorithms with advice

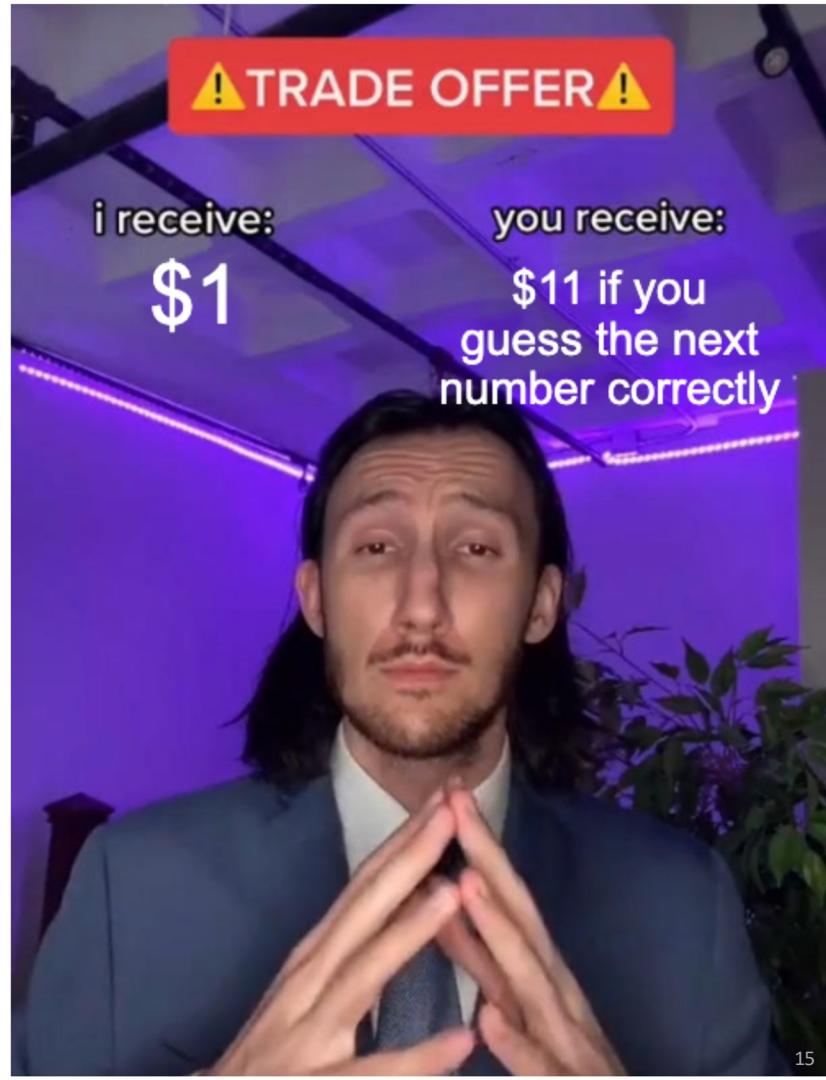


- Two key performance measures
  - Consistency: If advice is “perfect”, how good are things?
  - Robustness: If advice is “garbage”, how bad are things?
- Challenge: We don’t know how good the given advice is a priori!



# Detour: Let's make a deal

- There are 10 numbers in the universe  
 $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- There is an underlying process  $\mathcal{P}$  that generates IID samples from  $U$ 
  - I.e., We can observe a sequence such as  
1, 6, 3, 6, 2, 8, 0, 3, 9, 5, 4, ...
- What property of  $\mathcal{P}$  will make this deal profitable in expectation?

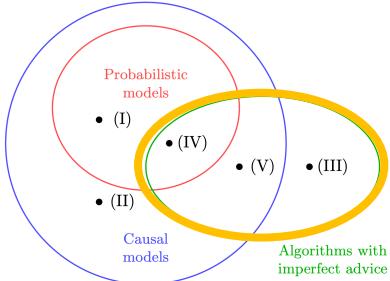


# Detour: Property testing land

- How to test if  $\mathcal{P}$  is the uniform distribution over  $U$ ?
  - Say, we only care about constant success probability (can be amplified)
- Learning a  $\varepsilon$ -close  $\hat{\mathcal{P}}$  then check:  $\Theta\left(\frac{|U|}{\varepsilon^2}\right)$  IID samples from  $\mathcal{P}$
- Uniformity testing requires  $\Theta\left(\frac{\sqrt{|U|}}{\varepsilon^2}\right)$  IID samples from  $\mathcal{P}$ 
  - If  $\mathcal{P}$  is uniform, output YES w.p.  $\geq \frac{2}{3}$
  - If  $\mathcal{P}$  is  $\varepsilon$ -far from uniform, output NO w.p.  $\geq \frac{2}{3}$
  - Many existing proofs for this bound. E.g., look at collisions in samples

→ Allowed to output arbitrarily if not uniform, yet not “far from uniform”
- See also [Can22] for an excellent property testing survey

# (III/IV/V): Algorithms with advice



- Two key performance measures
  - Consistency: If advice is “perfect”, how good are things?
  - Robustness: If advice is “garbage”, how bad are things?
- Challenge: We don’t know how good the given advice is a priori!
- Insight: “Testing can be cheaper than learning” → **TestAndAct**
  - [CGLB24] **TestAndMatch**: Improve competitive ratio of online bipartite matching (III)
  - [BCGJ24] **TestAndScheffe**: Improve sample complexity of learning multivariate Gaussians (IV)
  - [CGB23] **TestAndSubsetSearch**: Reduce no. of interventions required for causal graph discovery (V)

[CGLB24] Davin Choo, Themistoklis Gouleakis, Chun Kai Ling, and Arnab Bhattacharyya. *Online bipartite matching with imperfect advice*.

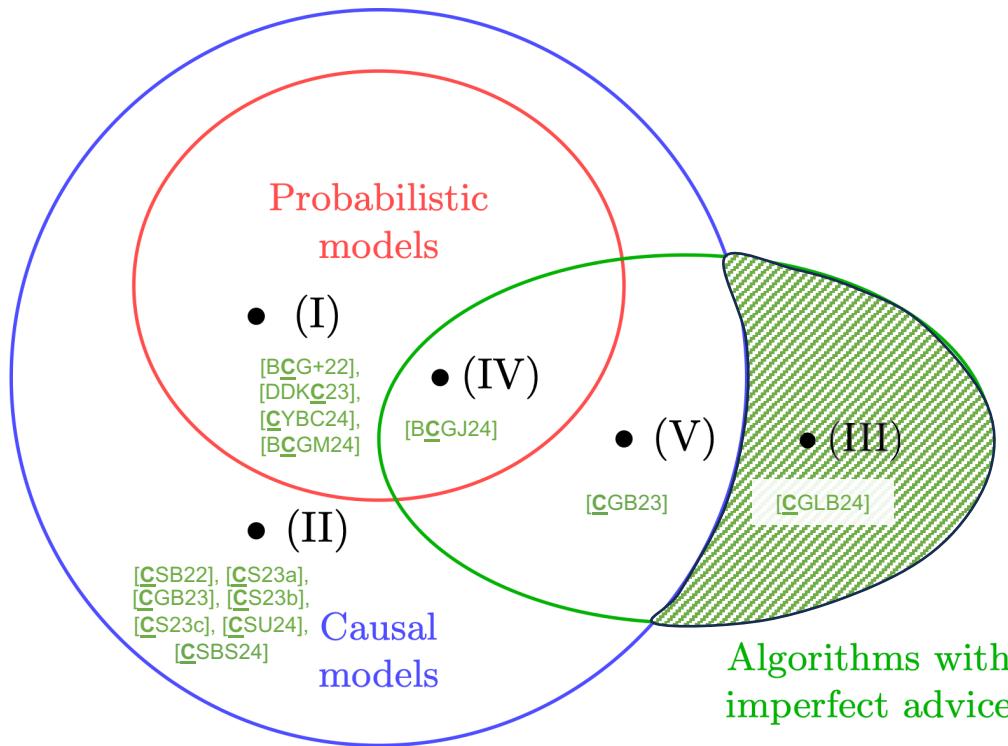
International Conference on Machine Learning (ICML), 2024.

[BCGJ24] Arnab Bhattacharyya, Davin Choo, Themistoklis Gouleakis, and Philips George John. *Distribution learning with imperfect advice*. In preparation, 2024.

[CGB23] Davin Choo, Themistoklis Gouleakis, Arnab Bhattacharyya. *Active causal structure learning with advice*. International Conference on Machine Learning (ICML), 2023.



# For the rest of this talk



# Online bipartite matching

- Offline set  $U = \{u_1, \dots, u_n\}$  fixed and known
- Online set  $V = \{v_1, \dots, v_n\}$  arrive one by one

$u_1$

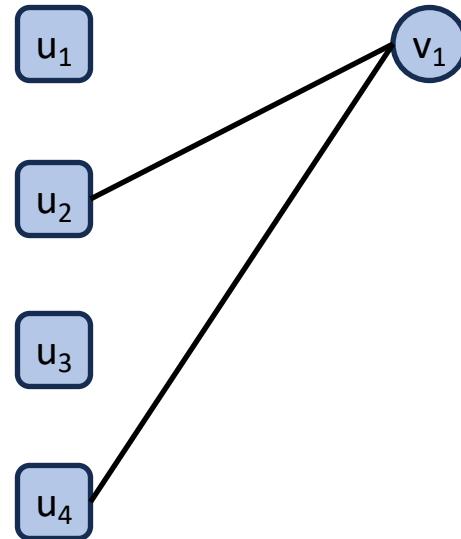
$u_2$

$u_3$

$u_4$

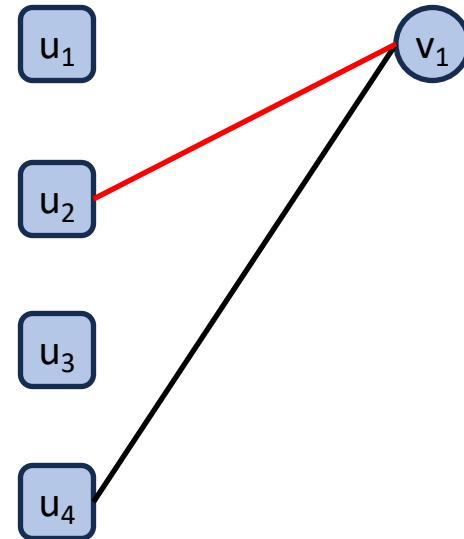
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- When an online vertex  $v_i$  arrives
  - Its neighbors  $N(v_i)$  are revealed
  - We must make an irrevocable decision whether, and how, to match  $v_i$  to something in  $N(v_i)$



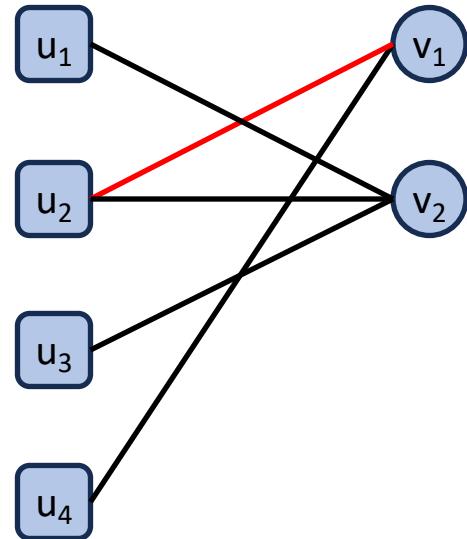
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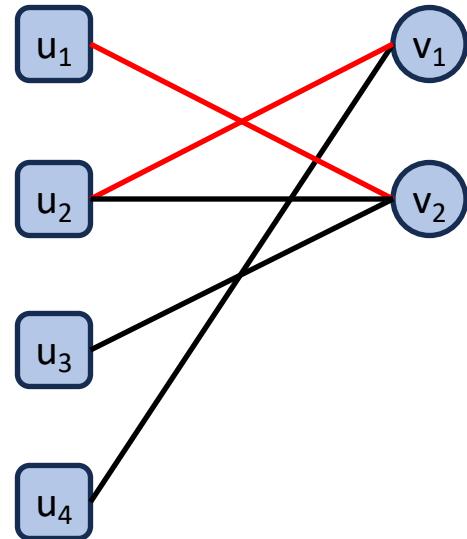
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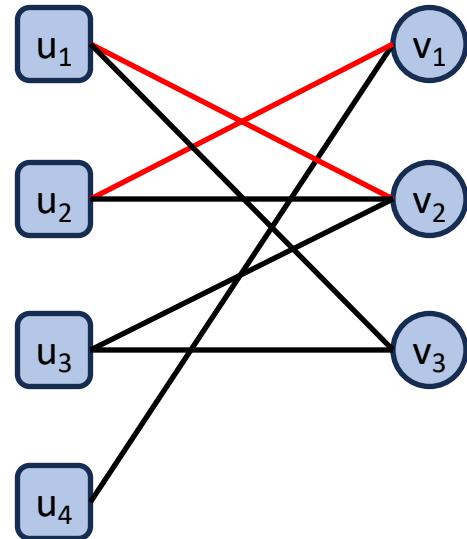
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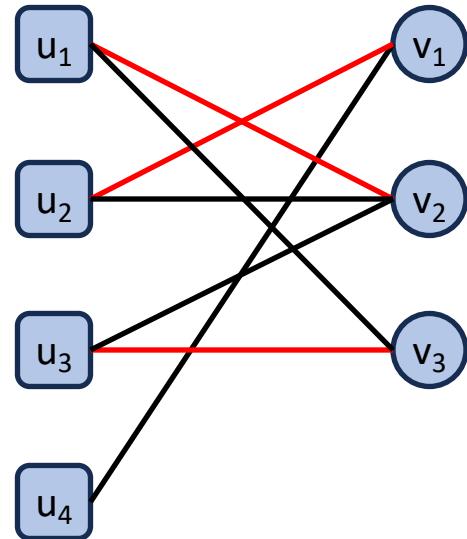
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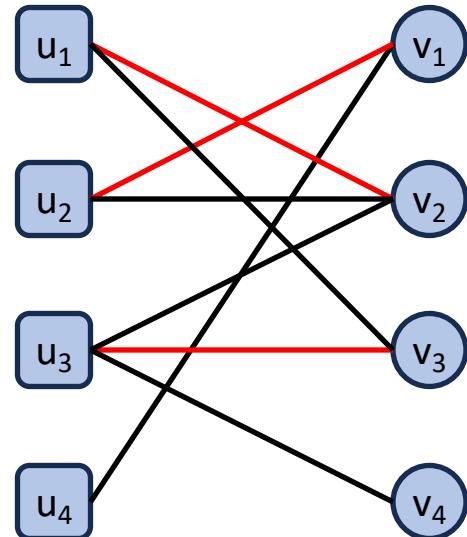
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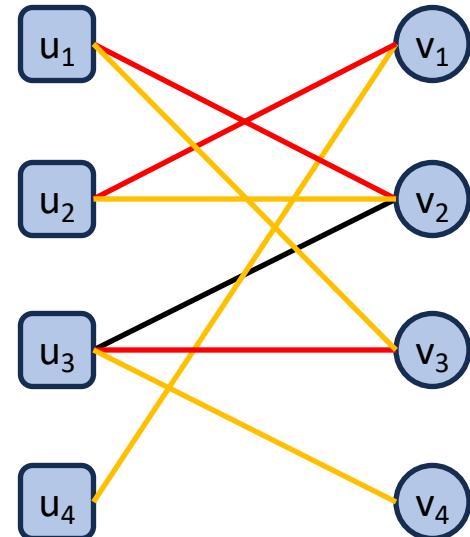
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- Final offline graph  $G^* = (U \cup V, E)$ 
  - $E = N(v_1) \cup \dots \cup N(v_n)$
  - Maximum matching  $M^* \subseteq E$  of size  $|M^*| = n^* \leq n$



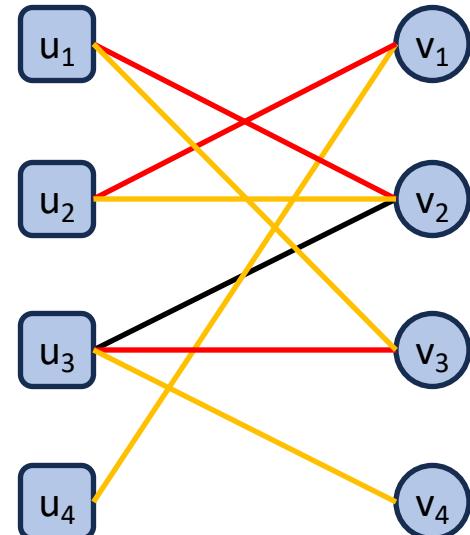
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Goal of online bipartite matching problem

Produce a matching  $M$  such that the resulting  
competitive ratio  $\frac{|M|}{|M^*|}$  is **maximized**

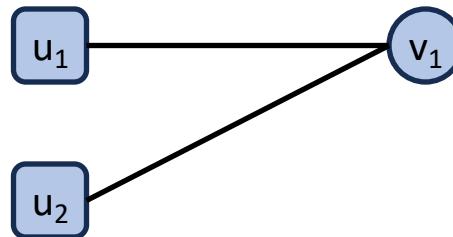
For this talk, let's treat  $n^* = n$



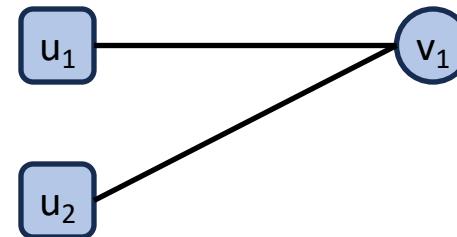
Here, the ratio is  $3/4$

# What is known?

- Why is online bipartite matching hard?
  - Maximum bipartite matching is poly time computable...
  - But we don't know the future in the online setting!

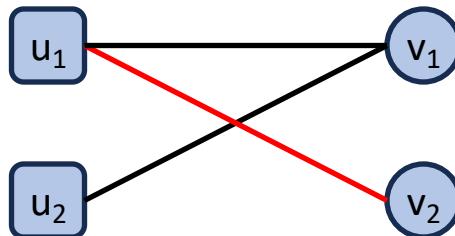


versus

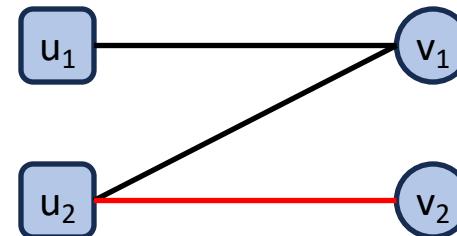


# What is known?

- Why is online bipartite matching hard?
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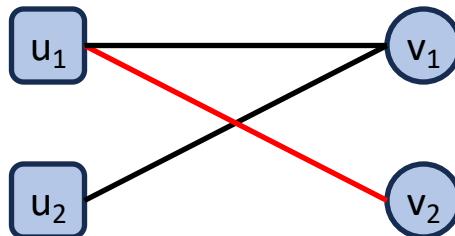


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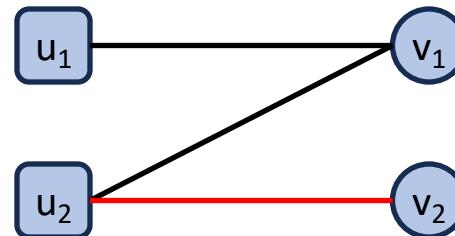


# What is known?

- Why is online bipartite matching hard?
  - Maximum bipartite matching is poly time computable...
  - But we don't know the future in the online setting!
- Any reasonable greedy algorithm has competitive ratio  $\geq 1/2$ 
  - Size of maximal matching is at least half of size of maximum matching



versus



# What is known?

$$\min_G \min_{V' \text{ s arrival sequence}} \frac{\text{(Expected) number of matches}}{n^*}$$

	(Expected) Competitive ratio
Deterministic algorithm	$\frac{1}{2}$
Deterministic hardness	$\frac{1}{2}$
Randomized algorithm	$1 - \frac{1}{e}$ [KVV90]
Randomized hardness	$1 - \frac{1}{e} + o(1)$ [KVV90]

- The **Ranking** algorithm [KVV90]

- Pick a random permutation  $\pi$  over the offline vertices  $U$
- When vertex  $v_i$  arrive with  $N(v_i)$ , match  $v_i$  to the smallest indexed (with respect to  $\pi$ ) unmatched neighbor

# What if there is additional side information?

- Learning-augmented algorithms
  - Designing algorithms using advice, predictions, etc.
    - $\alpha$ -consistent:  $\alpha$ -competitive with no advice error
    - $\beta$ -robust:  $\beta$ -competitive with any advice error

A natural goal is to design an algorithm with  $\alpha = 1$   
while  $\beta$  being the best possible classically

# Example settings with side information

Offline vertices	Online vertices	Presence of edge	Advice	Error
Advertisers	Ad slots	New ad slot fits the advertisers' requirements	Historical data	Data may have noise, bias, etc.
Job opening	Hiring company	Applicant's suitability for the job role	LinkedIn qualifications	May lie about credentials
Food bento boxes	Conference attendee	Attendee's dietary options match the food type	Food preferences	May change mind if see a tastier option

# Online matching settings involving social good

This is a weighted and capacitated matching problem. Also, food banks may further have different nutritional targets to meet (e.g., 10kg of protein, 5kg of carbohydrates, etc.), making this a multidimensional instance

Offline vertices	Online vertices	Presence of edge	Advice	Error
Patients	Organ donors (e.g., upon death)	Compatibility	Historical data	
Food banks	Food surplus from restaurants, etc.	Transportability	Restaurant information, seasonal trends	Data may have noise, bias, etc.
Crisis response resource	Occurrence of a crisis request	Proximity	Nature of crisis, terrain, human experts, etc.	Murphy's law



The problem is more nuanced here: we might also be able to decide how to place resources in advice so that it can accommodate requests as they appear (think: k-server)

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Food				
Crisis resource	crisis request	proximity	certainty, human experts, etc.	Murphy's law

**Takeaway:** Advice can come in many forms. Nuances in problem dictate which kind of advice are practical and useful

The problem is more nuanced here: we might also be able to decide how to place resources in advice so that it can accommodate requests as they appear (think: k-server)

# Research question

- If we have “perfect information” about  $G^*$ , can we get  $n^*$  matches?
- Also, we know that **Ranking** achieves competitive ratio of  $1 - \frac{1}{e}$

Can we get an algorithm that is both  
1-consistent and  $\left(1 - \frac{1}{e}\right)$ -robust?

# Prior related attempts

- [AGKK20] Prediction on edge weights adjacent to  $V$  under an optimal offline matching
  - Random vertex arrivals and weighted edges
  - Require hyper-parameter to quantify confidence in advice, so their consistency/robustness tradeoffs are not directly comparable
- [ACI22] Prediction of vertex degrees  $\hat{d}(u_1), \dots, \hat{d}(u_n)$  of the offline vertices in  $U$ 
  - Adversarial arrival model
  - Optimal under the Chung-Lu-Vu random graph model [CLV03]
  - Unable to attain 1-consistency in general
- [JM22] Advice is a proposed matching for the first batch of arrived vertices
  - Two-staged arrival model [FNS21], where best possible robustness is  $\frac{3}{4}$
  - For any  $R \in [0, \frac{3}{4}]$ , they can achieve consistency of  $1 - (1 - \sqrt{1 - R})^2$
- [LYR23] Augment any “expert algorithm” with a pre-trained RL model
  - For any  $\rho \in [0, 1]$ , their method is  $\rho$ -competitive to the given “expert algorithm”

[AGKK20] Antonios Antoniadis, Themis Gouleakis, Pieter Kleer, and Pavel Kolev. *Secretary and online matching problems with machine learned advice*. Neural Information Processing Systems (NeurIPS), 2020

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[CLV03] Fan Chung, Linyuan Lu, and Van Vu. *Spectra of random graphs with given expected degrees*. Proceedings of the National Academy of Sciences (PNAS), 2003

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- [ACI22]
  - Adversarial
  - Optimal
  - Unweighted
- [JM22]
  - Two-stage
  - Forwards
- [LYR23] Augment any “expert algorithm” with a pre-trained RL model
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Do not yield an algorithm that is both  
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# Our first main result

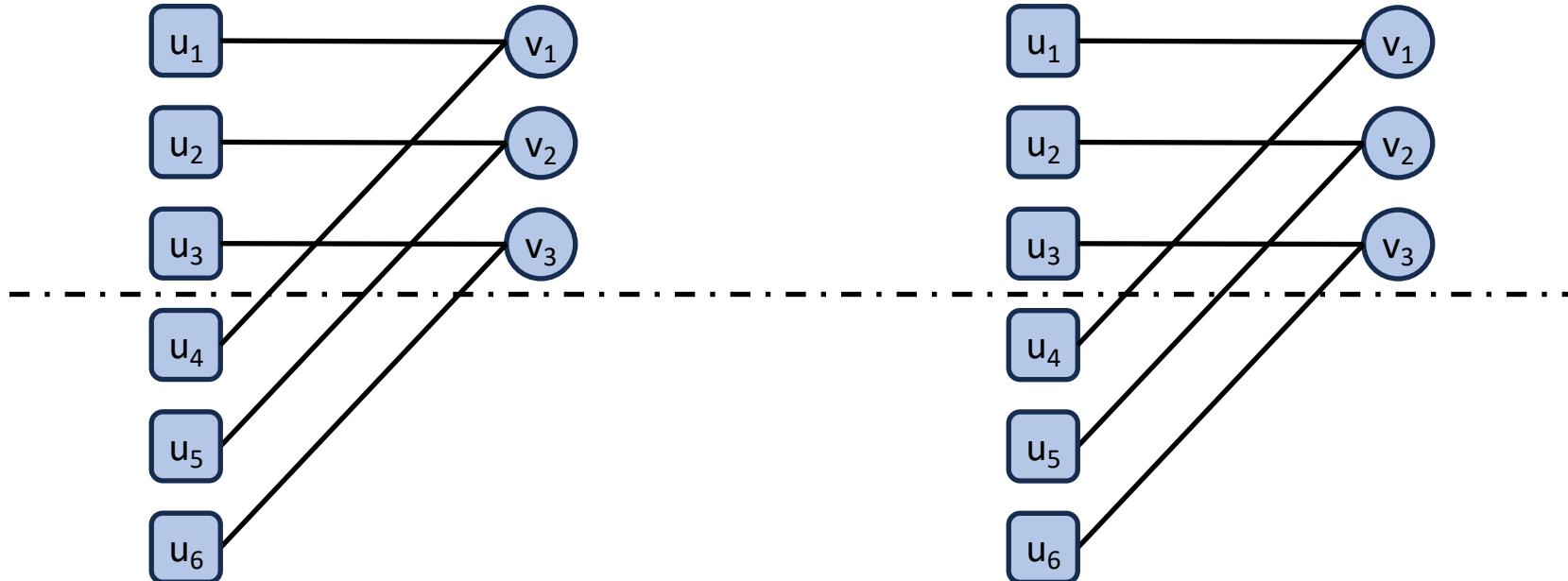
## Impossibility result (Informal)

With adversarial vertex arrivals, no algorithm can be both  
1-consistent and  $> \frac{1}{2}$ -robust, regardless of advice

- Extends to  $(1 - a)$ -consistent and  $\left(\frac{1}{2} + a\right)$ -robust, for any  $a \in [0, \frac{1}{2}]$
- Proof sketch (for  $a = 0$  case):
  - Restrict  $G^*$  to be one of two possible graphs (next slide)
  - **Any** advice is equivalent to getting 1 bit of information
  - In first  $\frac{n}{2}$  arrivals, no algorithm can distinguish between the two graphs
  - **Any** 1-consistent algorithm must behave as if the advice is perfect initially

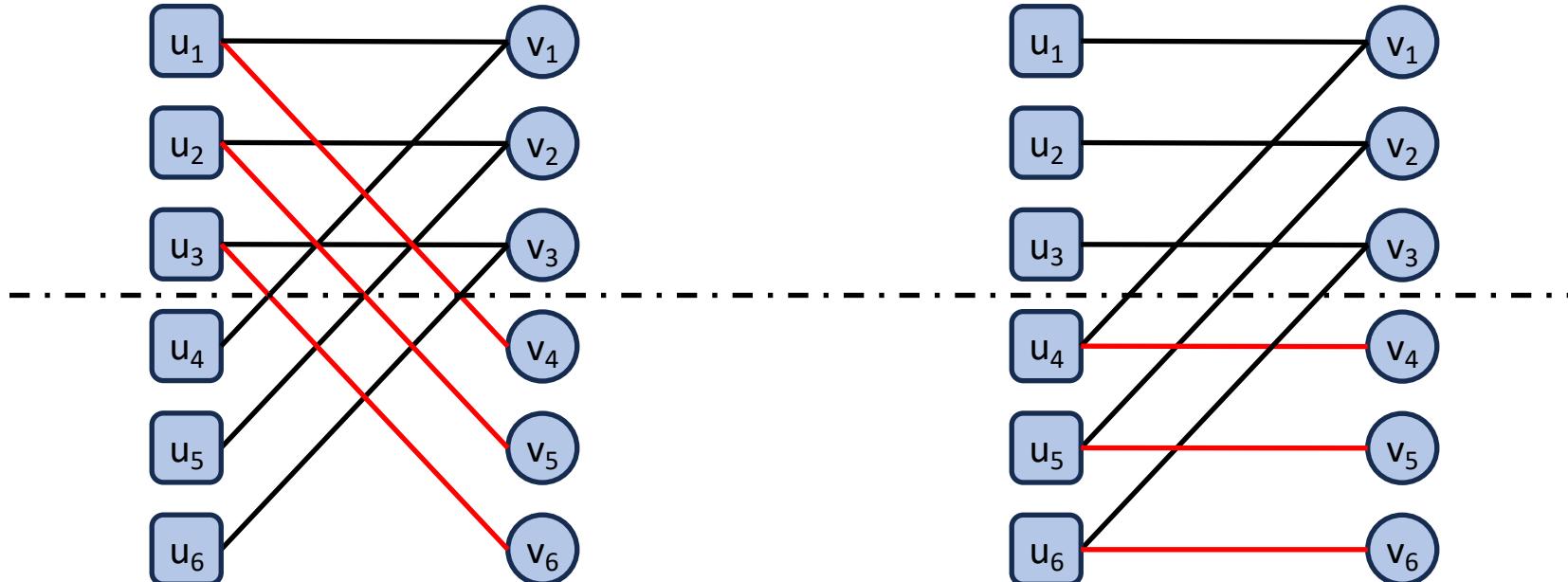
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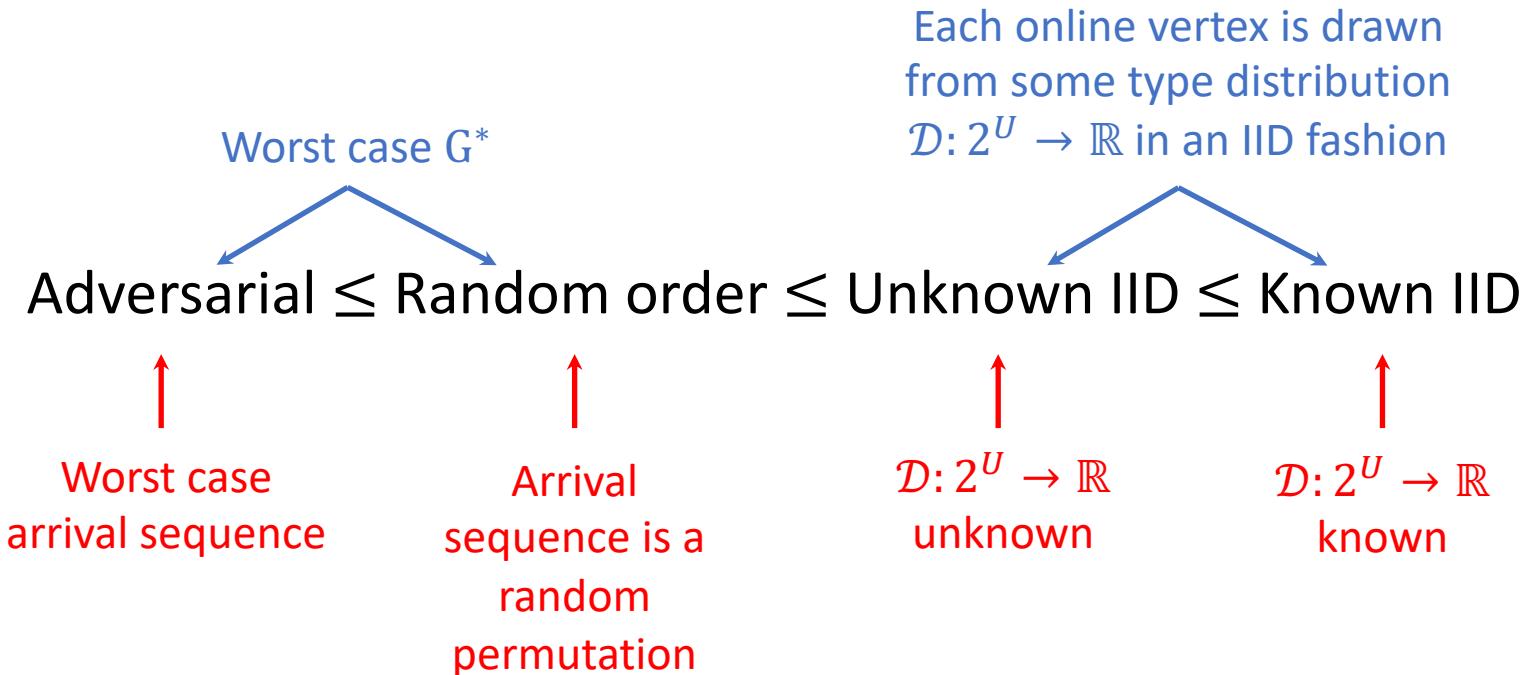
# Hierarchy of arrival models [M13]

Harder  Easier

Adversarial  $\leq$  Random order  $\leq$  Unknown IID  $\leq$  Known IID

Easier models can achieve  
higher competitive ratios

# Hierarchy of arrival models [M13]



# What is known?

Adversarial  $\leq$  Random order  $\leq$  Unknown IID  $\leq$  Known IID

	(Expected) Competitive ratio	
	Adversarial arrival	Random order arrival
Deterministic algorithm	$\frac{1}{2}$	Greedy
Deterministic hardness	$\frac{1}{2}$	$\frac{3}{4}$
Randomized algorithm	$1 - \frac{1}{e}$ [KVV90]	Ranking
Randomized hardness	$1 - \frac{1}{e} + o(1)$ [KVV90]	0.823 [MGS12]

[GM08] Gagan Goel and Aranyak Mehta. *Online budgeted matching in random input models with applications to Adwords*. Symposium on Discrete Algorithms (SODA), 2008

[MY11] Mohammad Mahdian and QiQi Yan. *Online Bipartite Matching with Random Arrivals: An Approach Based on Strongly Factor-Revealing LPs*. Symposium on Theory of Computing (STOC), 2011

[MGS12] Vahideh H Manshadi, Shayan Oveis Gharan, and Amin Saberi. *Online stochastic matching: Online actions based on offline statistics*. Mathematics of Operations Research, 2012

# Research question

Can we get an algorithm that is both 1-consistent and  $\left(1 - \frac{1}{e}\right)$ -robust?

$\beta$

- Let  $\beta$  denote the “best possible competitive ratio”
- Our first result says: This is not possible for adversarial arrivals!
- What about random order arrivals?

Adversarial  $\leq$  Random order  $\leq$  Unknown IID  $\leq$  Known IID

# Our second main result

Can we get an algorithm that is both 1-consistent and  $\left(1 - \frac{1}{e}\right)$ -robust?

$\beta$

Goal achievable in random order (Informal)

With random order, there is an algorithm achieves competitive ratio interpolating between 1 and  $\beta \cdot (1 - o(1))$ , depending on advice quality

- Our method is a meta-algorithm that uses any **Baseline** that achieves  $\beta$
- So, we are simultaneously 1-consistent and  $\beta \cdot (1 - o(1))$ -robust
- For random arrival model, we know that  $0.696 \leq \beta \leq 0.823$

e.g. use  
**Ranking**

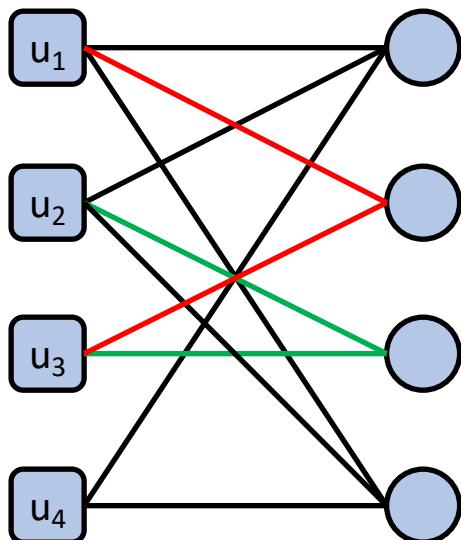
# Realized type counts as advice

- Classify online vertex in  $G^* = (U \cup V, E)$  based on their types
  - Type of  $v_i$  is the set of offline vertices in  $N(v_i)$  are adjacent to [BKP20]
- Define integer vector  $c^* \in \mathbb{N}^{2^n}$  indexed by all possible types  $2^U$ 
  - $c^*(t) =$  Number of times the type  $t \in 2^U$  occurs in  $G^*$
- Define  $T^* \subseteq 2^U$  as the subset of non-zero counts in  $c^*$ 
  - Note:  $|T^*| \leq n \ll 2^{|U|} = 2^n$

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- Define  $T^* \subseteq 2^U$  as the subset of non-zero counts in  $c^*$ 
  - Note:  $|T^*| \leq n \ll 2^{|U|} = 2^n$
- Advice is simply an estimate vector  $\hat{c}$  which approximates  $c^*$ 
  - Let  $\hat{T}$  be non-zero counts in  $\hat{c}$ . Similarly, we have  $|\hat{T}| \leq n$
  - Can represent  $\hat{c}$  using  $O(n)$  labels and numbers

# Realized type counts as advice

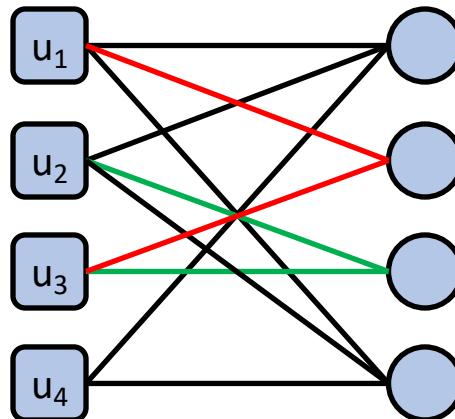


Type	$c^*$
$\{u_1, u_2, u_4\}$	2
$\{u_1, u_3\}$	1
$\{u_2, u_3\}$	1
$2^U \setminus T^*$	0

Here,  $|T^*| = 3 \ll 2^4 = 16$

# The Mimic algorithm

- Algorithm
  - Fix any arbitrary maximum matching  $\hat{M}$  on the graph defined by advice  $\hat{c}$
  - Try to mimic edge matches in  $\hat{M}$  while tracking the types of each arrival
  - If unable to mimic, leave arrival unmatched

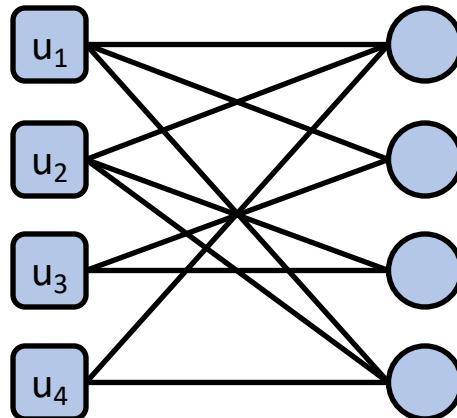


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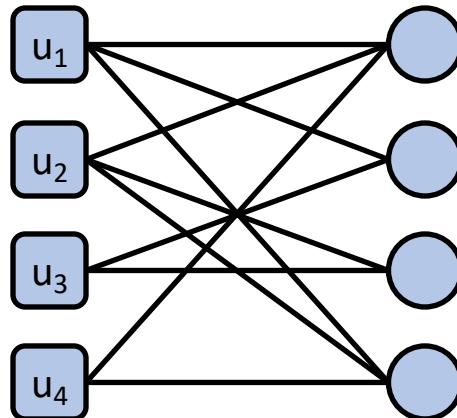
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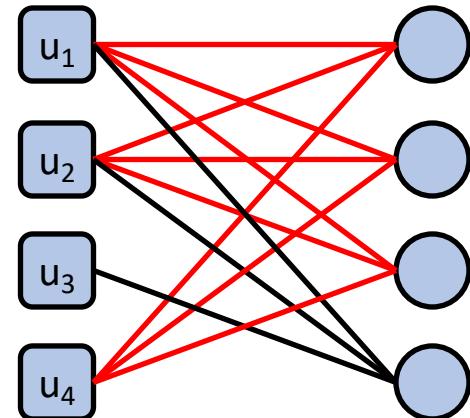
Type	$c^*$	$\hat{c}$
$\{u_1, u_2, u_4\}$	2	3
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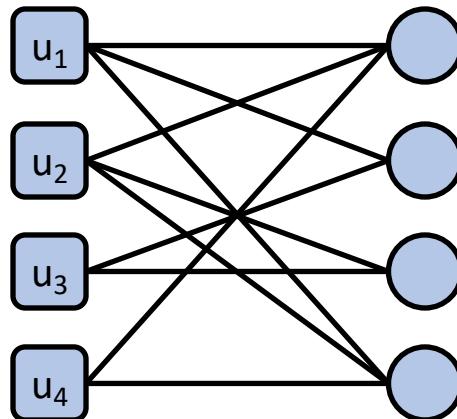


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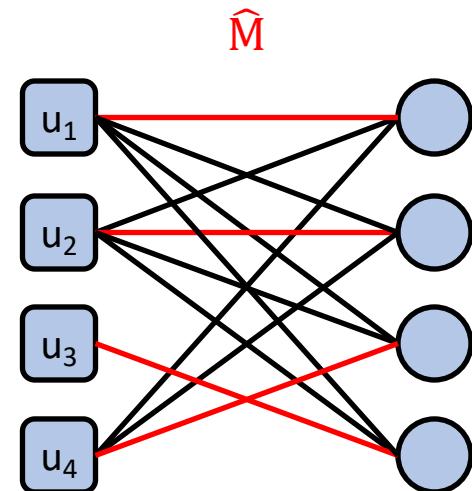


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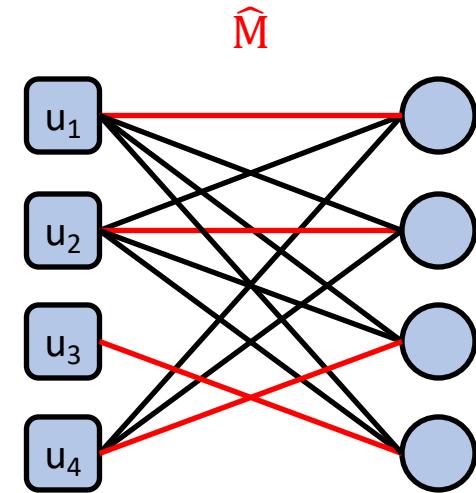
$u_1$

$u_2$

$u_3$

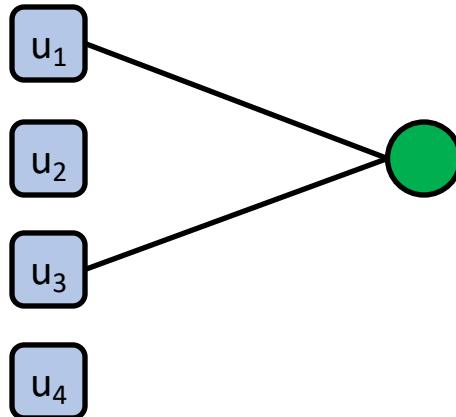
$u_4$

Type	$c^*$	$\hat{c}$
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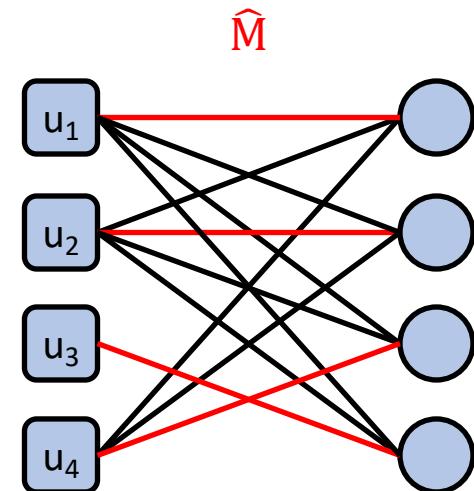


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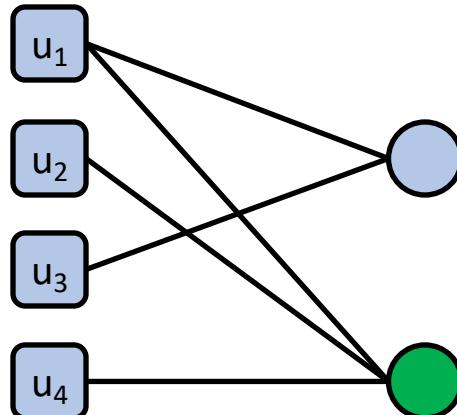


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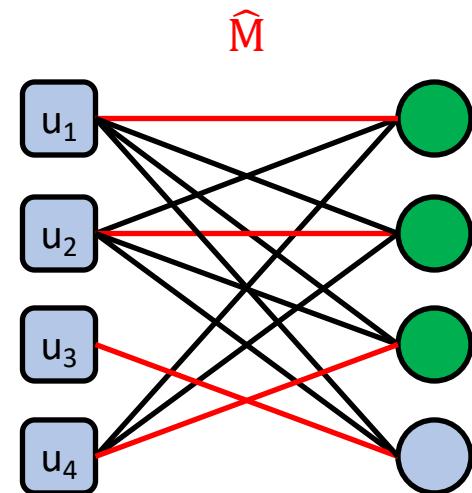


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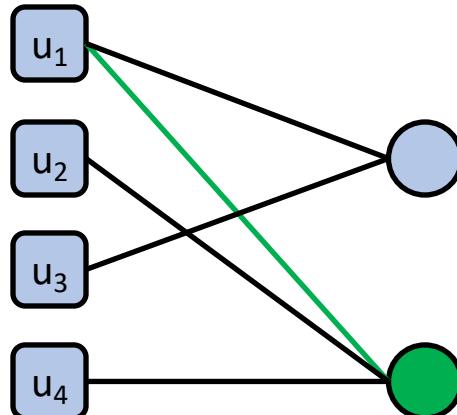


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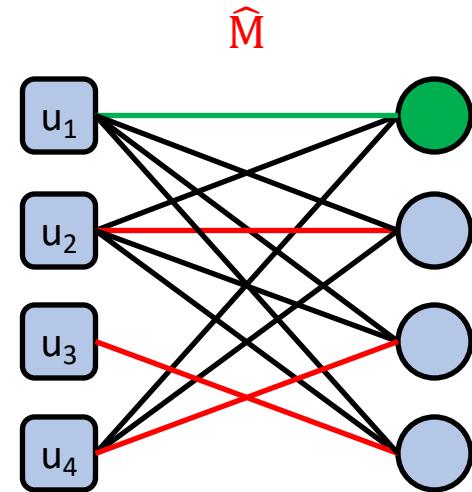


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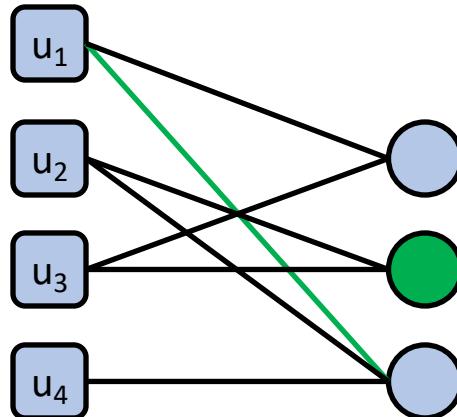


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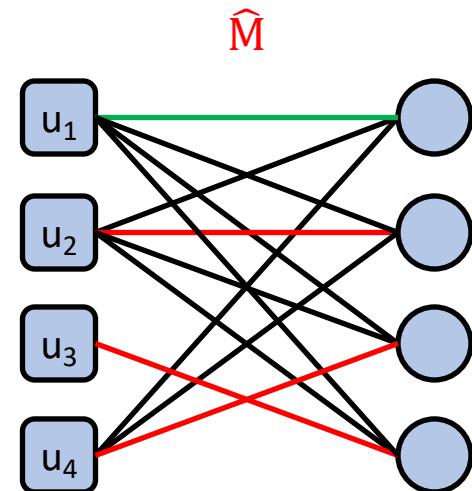


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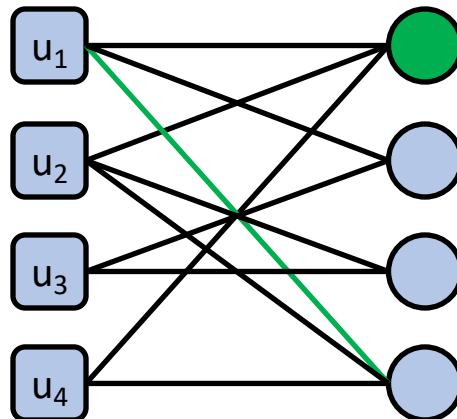


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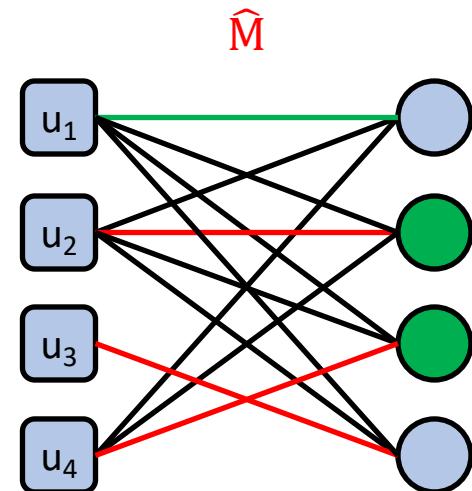


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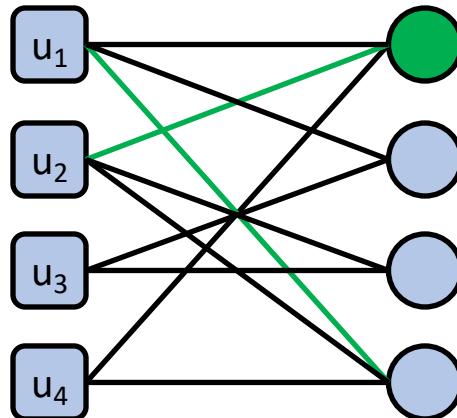


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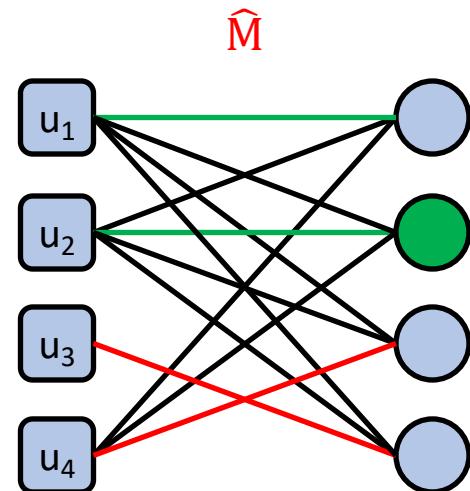


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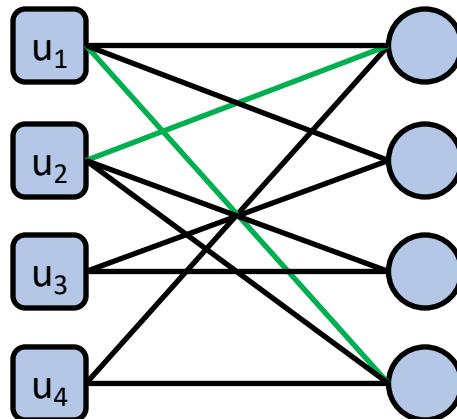


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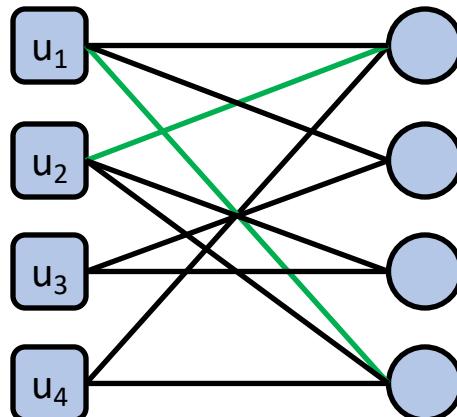


Type	$c^*$	$\hat{c}$
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Produced matching size  
= 2

# The Mimic algorithm

- Algorithm
  - Fix any arbitrary maximum matching  $\hat{M}$  on the graph defined by advice  $\hat{c}$
  - Try to mimic edge matches in  $\hat{M}$  while tracking the types of each arrival
  - If unable to mimic, leave arrival unmatched



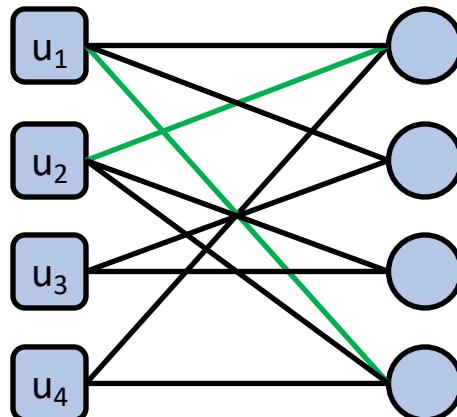
Type	$c^*$	$\hat{c}$
$\{u_1, u_2, u_4\}$	2	3
$\{u_1, u_3\}$	1	0
$\{u_2, u_3\}$	1	0
$\{u_1, u_2, u_3\}$	0	1

Produced matching size  
= 2

$$\begin{aligned} L_1(c^*, \hat{c}) &= |3 - 2| + |0 - 1| \\ &\quad + |0 - 1| + |1 - 0| + 0 \dots \\ &= 4 \end{aligned}$$

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$\{u_1, u_3\}$	1	0
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$\{u_1, u_2, u_3\}$	0	1

Produced matching size

$$= 2 = |\hat{M}| - \frac{L_1(c^*, \hat{c})}{2}$$

Error is "double counted" in  $L_1$

$$L_1(c^*, \hat{c})$$

$$\begin{aligned} &= |3 - 2| + |0 - 1| \\ &\quad + |0 - 1| + |1 - 0| + 0 \dots \\ &= 4 \end{aligned}$$

# The **Mimic** algorithm

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- Analysis
  - $0 \leq L_1(c^*, \hat{c}) \leq 2n$  measures how close  $\hat{c}$  is to  $c^*$
  - By blindly following advice, **Mimic** gets a matching of size  $|\hat{M}| - \frac{L_1(c^*, \hat{c})}{2}$
  - **Mimic** beats an advice-free **Baseline** whenever  $|\hat{M}| - \frac{L_1(c^*, \hat{c})}{2} > \beta \cdot n$

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  - **Mimic** beats an advice-free **Baseline** whenever  $\frac{L_1(c^*, \hat{c})}{n} < 2(1 - \beta)$

For this talk, let's treat  $|\hat{M}| = n$

# How to test advice quality?

**Insight:** Use sublinear property testing to estimate  $L_1(c^*, \hat{c})$ !

- Define  $p = \frac{c^*}{n}$  and  $q = \frac{\hat{c}}{n}$  as distributions over the  $2^U$  types

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- Define  $p = \frac{c^*}{n}$  and  $q = \frac{\hat{c}}{n}$  as distributions over the  $2^U$  types
- [VV11, JHW18]: Can estimate  $L_1(p, q)$  “well” using  $o(n)$  IID samples
  - Some adjustments needed to apply this property testing idea to our online bipartite matching setup, but it can be done (talk to me to find out more)

# The **TestAndMatch** algorithm

- Algorithm
  - Fix any arbitrary maximum matching  $\hat{M}$  on the graph defined by advice  $\hat{c}$
  - Run **Mimic** while testing quality of  $\hat{c}$  by estimating  $L_1(c^*, \hat{c})$
  - If test declares  $L_1(c^*, \hat{c})$  is “large”, use **Baseline** for remaining arrivals
  - Otherwise, continue using **Mimic** for remaining arrivals

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  - If test declares  $L_1(c^*, \hat{c})$  is “large”, use **Baseline** for remaining arrivals
  - Otherwise, continue using **Mimic** for remaining arrivals
- Analysis
  - If  $\hat{L}_1 \lesssim 2(1 - \beta)$ , then **TestAndMatch** attains ratio of at least  $1 - \frac{L_1(c^*, \hat{c})}{2n}$
  - Otherwise, **TestAndMatch** attains ratio of at least  $\beta \cdot (1 - o(1))$

# Our second main result

Can we get an algorithm that is both 1-consistent and  $\left(1 - \frac{1}{e}\right)$ -robust?

$\beta$

Goal achievable in random order (Informal)

With random order, there is an algorithm achieves competitive ratio interpolating between 1 and  $\beta \cdot (1 - o(1))$ , depending on advice quality

- Our method is a meta-algorithm that uses any **Baseline** that achieves  $\beta$
- So, we are simultaneously 1-consistent and  $\beta \cdot (1 - o(1))$ -robust
- For random arrival model, we know that  $0.696 \leq \beta \leq 0.823$

# Our second main result

Can we get an algorithm that is both 1-consistent and  $\left(1 - \frac{1}{e}\right)$ -robust?

$\beta$

Goal achievable in random order (Informal)

Let  $\hat{L}_1$  be estimate of  $L_1(c^*, \hat{c})$  from  $o(n)$  vertex arrivals.

**TestAndMatch** achieves a competitive ratio of at least

- At least  $1 - \frac{L_1(c^*, \hat{c})}{2n} \geq \beta$ , when  $\hat{L}_1$  “small”
  - At least  $\beta \cdot (1 - o(1))$ , when  $\hat{L}_1$  “large”
- i.e., **TestAndMatch** is 1-consistent and  $\beta \cdot (1 - o(1))$ -robust

# Our second main result

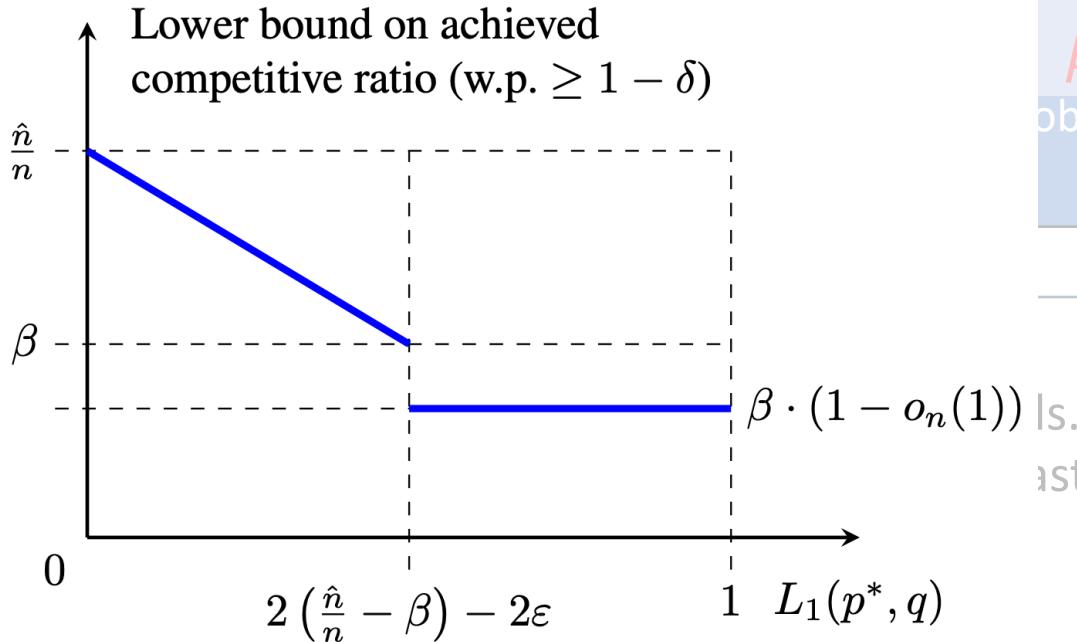
Can we get

Lower bound on achieved  
competitive ratio (w.p.  $\geq 1 - \delta$ )

$\beta$   
robust?

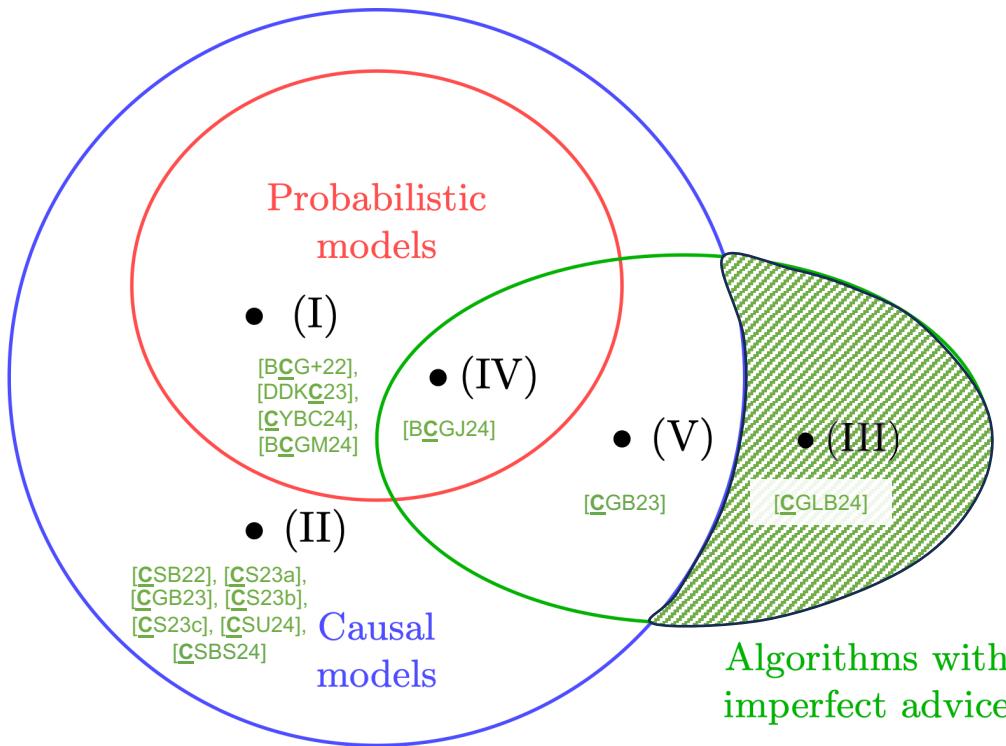
Let  $\hat{L}$   
**Test**

- At least
- At least



i.e., **TestAndMatch** is 1-consistent and  $\beta \cdot (1 - o(1))$ -robust

# Recap: Main themes explored in my PhD



# Future research directions

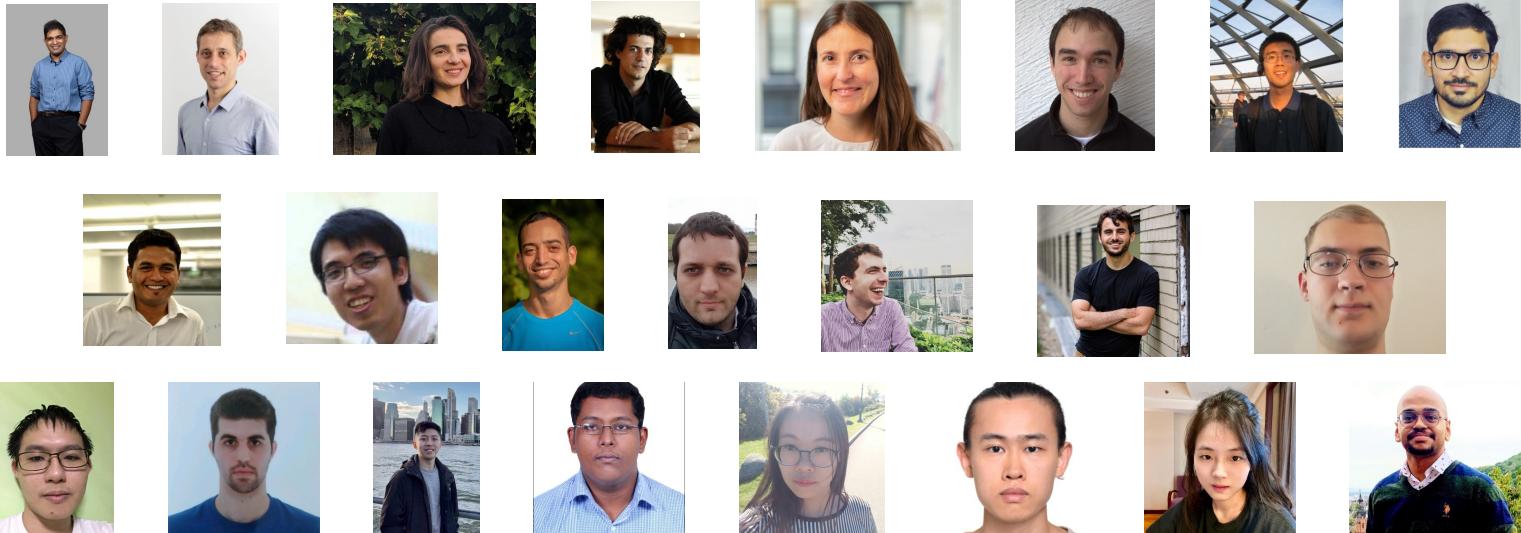
1. New directions and problems with real-world impact @ Teamcore
2. Developing causality-informed AI/ML methods
  - Most AI/ML methods are built on statistical/associational relations
  - Naively making real-world decisions can lead to unexpected outcomes



# Future research directions

1. New directions and problems with real-world impact @ Teamcore
2. Developing causality-informed AI/ML methods
  - Most AI/ML methods are built on statistical/associational relations
  - Naively making real-world decisions can lead to unexpected outcomes
3. Incorporating human knowledge as imperfect advice
  - First came out of the study of online algorithms: main difficulty is not knowing the future and advice is partial future knowledge
  - I believe framework is much more broadly applicable
    - How can we principally elicit and incorporate human domain experts' knowledge?
    - Note: "Just Bayesian it" doesn't always work as an expert can be confidently wrong

# Thank you to all my amazing collaborators during my PhD journey!



**Thank you for your kind attention!**

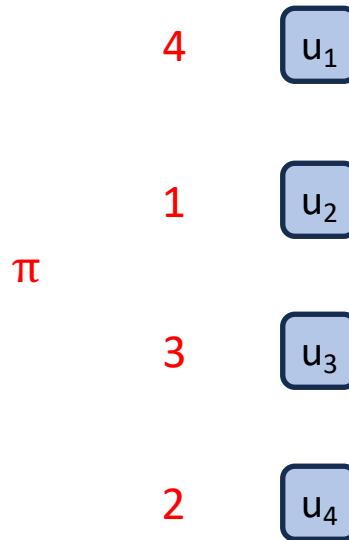
(Some of the works with them were not mentioned in this presentation)



# BACK UP SLIDES

(from some of my older talks that may be relevant for discussion)

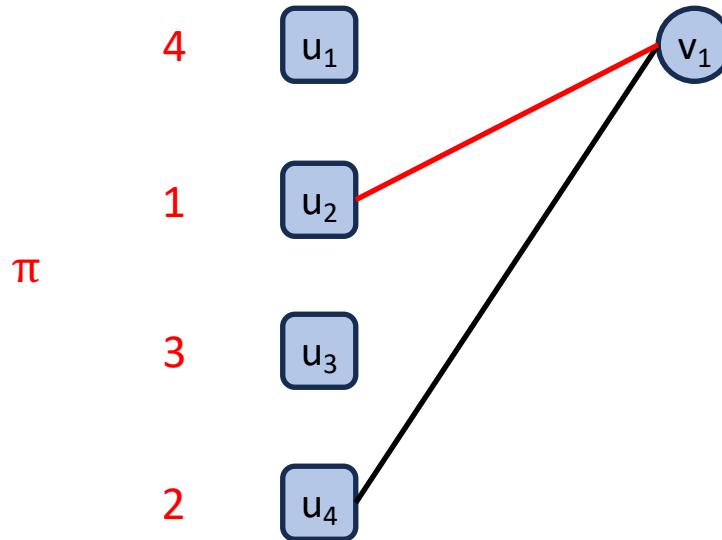
# What is known?



- The **Ranking** algorithm [KVV90]

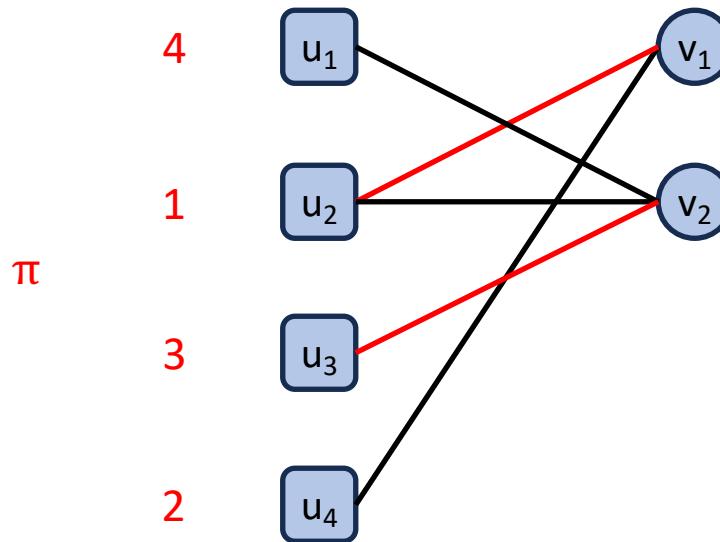
- Pick a random permutation  $\pi$  over the offline vertices  $U$
- When vertex  $v_i$  arrive with  $N(v_i)$ , match  $v_i$  to the smallest indexed (with respect to  $\pi$ ) unmatched neighbor

# What is known?



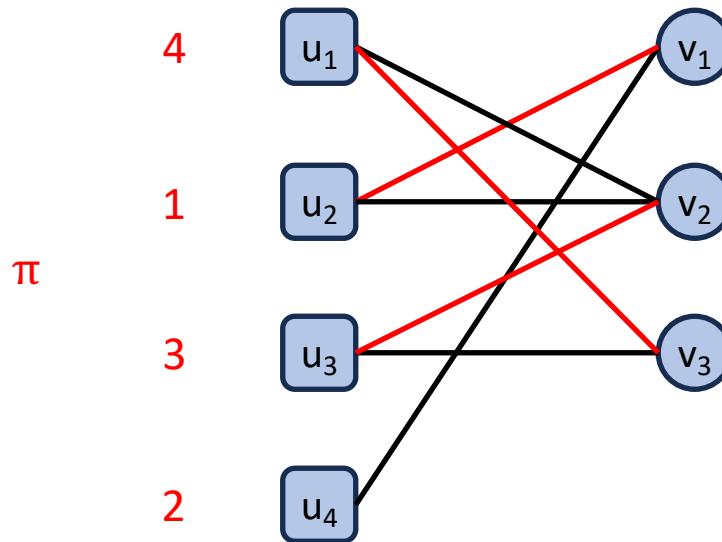
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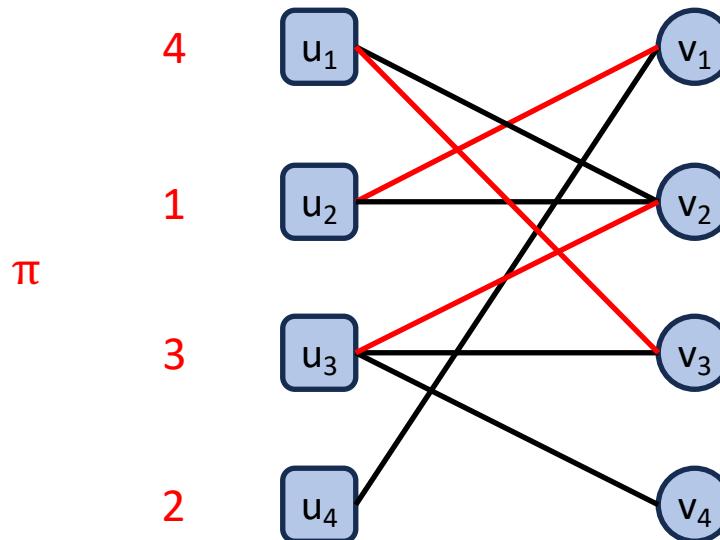
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# What if there is additional side information?

- Learning-augmented algorithms
  - Designing algorithms using advice, predictions, etc.
    - $\alpha$ -consistent:  $\alpha$ -competitive with no advice error
    - $\beta$ -robust:  $\beta$ -competitive with any advice error
- Example: Binary search with advice
  - Want to find a word in an  $n$  page dictionary, say it is on page  $x^*$
  - Classical binary search:  $O(\log n)$  queries possible and worst case necessary
  - If someone provides an advice page  $\hat{x}$ ,  $O(\log |x^* - \hat{x}|)$  queries is possible



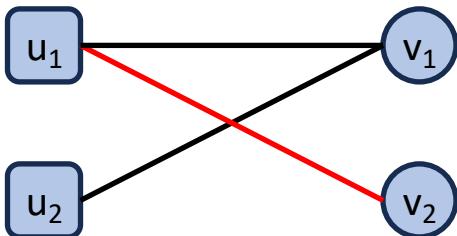
$\hat{x}$  obtained via letter frequency tables, someone who searched a “nearby” word, or asking ChatGPT, etc...

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  - If someone provides an advice page  $\hat{x}$ ,  $O(\log |x^* - \hat{x}|)$  queries is possible
  - Here, “best possible” is directly querying to page  $x^*$
  - So, this algorithm is 1-consistent and  $O(\log n)$ -robust since  $|x^* - \hat{x}| \leq n$

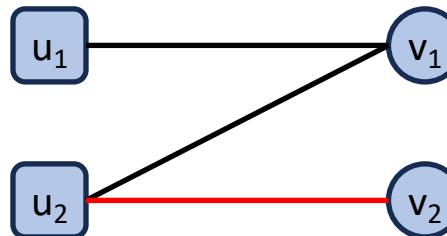
# What is known?

- Hardness of  $\frac{3}{4}$  for random order



	(Expected) Competitive ratio	
	Adversarial arrival	Random order arrival
Deterministic algorithm	$\frac{1}{2}$	Greedy $1 - \frac{1}{e}$ [GM08]
Deterministic hardness	$\frac{1}{2}$	$\frac{3}{4}$
Randomized algorithm	$1 - \frac{1}{e}$ [KVV90]	Ranking $0.696$ [MY11]
Randomized hardness	$1 - \frac{1}{e} + o(1)$ [KVV90]	$0.823$ [MGS12]

versus



- [KMT11] showed that Ranking cannot beat 0.727 in general
- So, new ideas are needed if you believe the “right bound” is 0.823

# How to test advice quality?

**Insight:** Use sublinear property testing to estimate  $L_1(c^*, \hat{c})$ !

- Define  $p = \frac{c^*}{n}$  and  $q = \frac{\hat{c}}{n}$  as distributions over the  $2^U$  types
- [VV11, JHW18]: Can estimate  $L_1(p, q)$  “well” using  $o(n)$  IID samples
  - To be precise, if  $p$  and  $q$  have domain size  $r \leq n$ , then  $\Theta\left(\frac{r}{\varepsilon^2 \log r}\right)$  IID samples sufficient and necessary to estimate  $\hat{L}_1$  such that  $|\hat{L}_1 - L_1(p, q)| \leq \varepsilon$
  - $c^*$  and  $\hat{c}$  can be defined over  $|\hat{T}| + 1$  elements with a “not in  $\hat{T}$ ” bucket

# Some minor adjustments to our problem setting

- Adjustment 1
  - Random vertex arrivals are “sampling without replacement”
  - We can simulate IID samples by keeping track of what has arrived and then “reusing” arrivals with some probability proportional to number of arrivals
- Adjustment 2
  - $L_1$  estimator is in expectation, but can be made “with high probability”

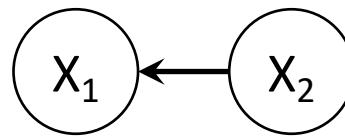
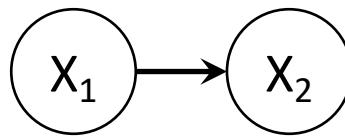
# Conclusions and future directions

- Our paper also discussed some practical considerations while using the given advice  $\hat{c}$
- Can our ideas such as using property testing extend to other versions of online bipartite matching and other online problems with random arrivals?
  - We suspect it extends with suitably chosen advice and quality metrics, e.g. Earthmover distance?
- Is there a smarter way using advice other than **Mimic**, leaving some arrivals unmatched?
  - [FMMM09] constructed two matchings to “load balance” in the known IID setting
  - In semi-online model, [KPSSV19] mimic matching on known arrivals and **Ranking** on adversarial arrivals
- Message to the learning-augmented community: Beyond consistency and robustness?
  - **TestAndMatch**'s guarantees is based on  $L_1$  over the type histograms
  - This is sensitive to certain types of noise, e.g.  $\hat{c}$  obtained after Erdős–Rényi edits to the offline graph  $G^*$
  - We expect large  $L_1$  in practice, but notions of advice practicality are not formally considered under the standard framework of consistency and robustness

**Thank you for your kind attention!**

# Two equivalent causal models

$X_1$	-0.27	0.29	0.37	-0.09	0.34	0.33	0.30	-1.34	0.68
$X_2$	-0.10	1.65	0.47	1.92	2.04	1.67	0.11	-3.58	1.97

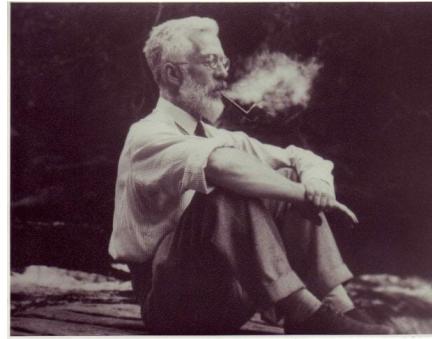


- $X_1 = \epsilon_1 \sim N(0, 1)$
- $X_2 = X_1 + \epsilon_2 \sim N(0, 2)$
- $\epsilon_1 \sim N(0, 1)$
- $\epsilon_2 \sim N(0, 1)$
- $X_1 = \frac{1}{2} \cdot X_2 + \epsilon_3 \sim N(0, 1)$
- $X_2 = \epsilon_4 \sim N(0, 2)$
- $\epsilon_3 \sim N\left(0, \frac{1}{2}\right)$
- $\epsilon_4 \sim N(0, 2)$

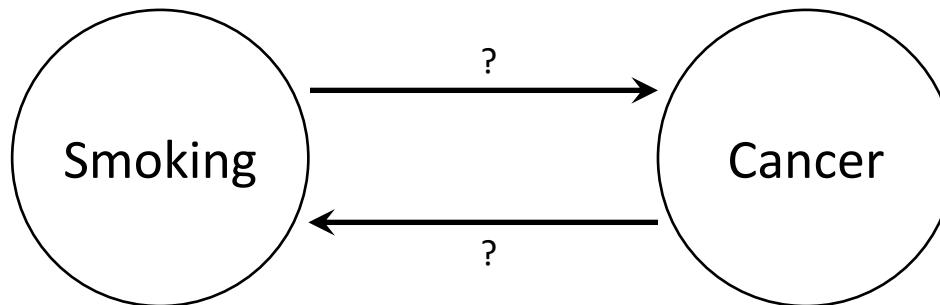
# Smoking

Fisher's letter to Nature, 1958:

"The curious associations with lung cancer found in relation to smoking habits do not, in the minds of some of us, lend themselves easily to the simple conclusion that the products of combustion reaching the surface of the bronchus induce, though after a long interval, the development of a cancer... **Such results suggest that an error has been made, of an old kind, in arguing from correlation to causation..."**



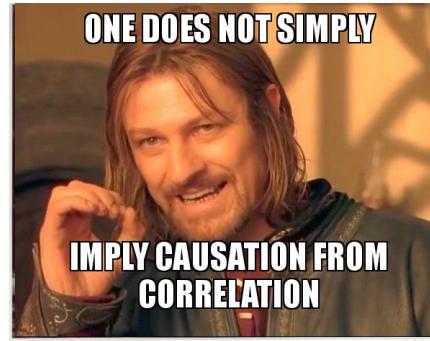
Ronald Fisher



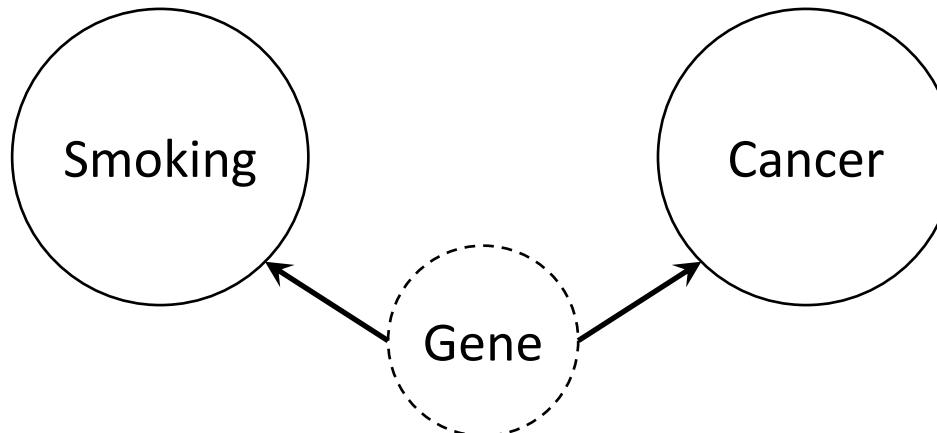
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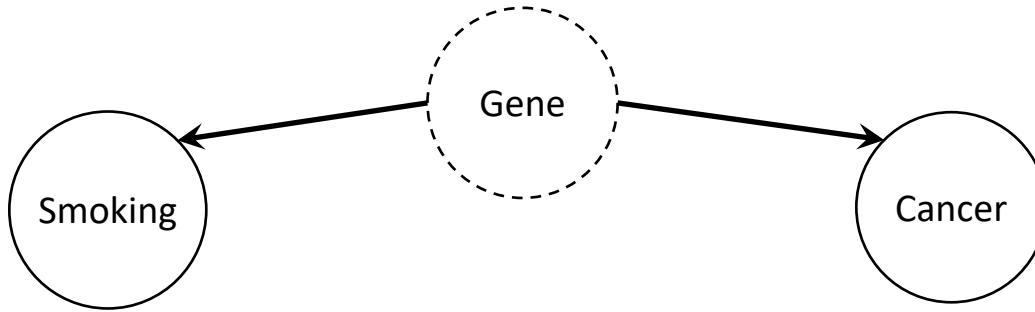
"... Such results suggest that an error has been made, of an old kind, in arguing from correlation to causation, and that the possibility should be explored that the different smoking classes, non-smokers, cigarette smokers, cigar smokers, pipe smokers, etc., have adopted their habits partly by reason of their personal temperaments and dispositions, and are not lightly to be assumed to be equivalent in their **genotypic composition**..."



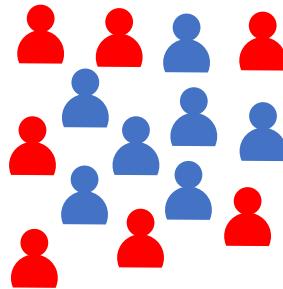
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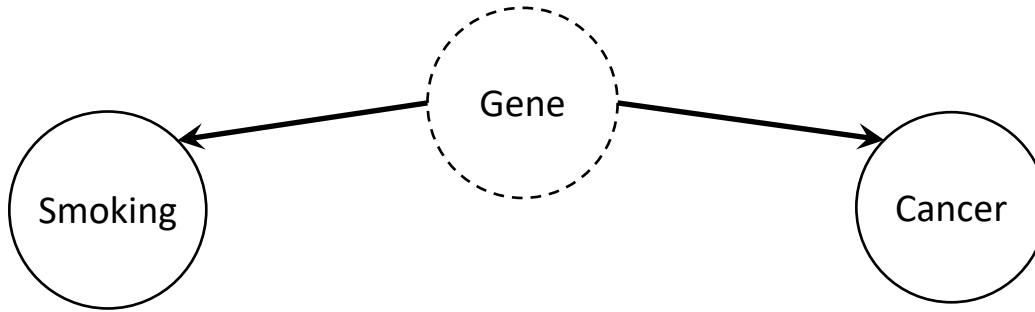


Maybe there's an unmeasured confounder?

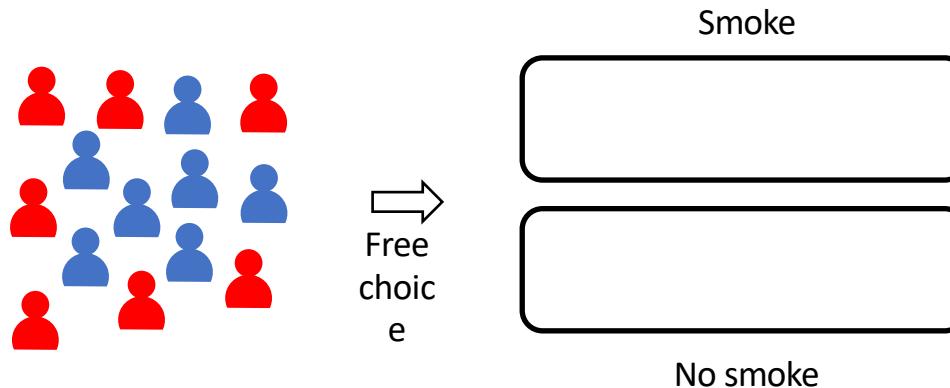


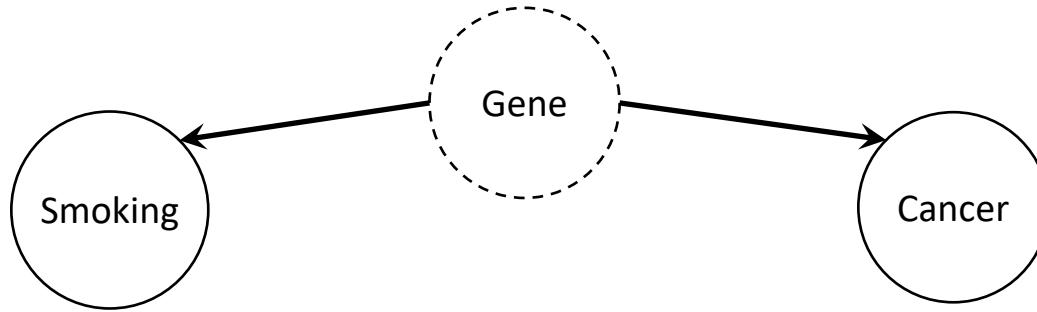
Hypothesis: There are two types of people in the world



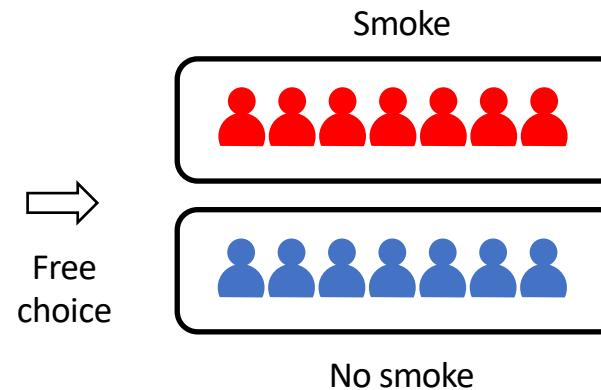


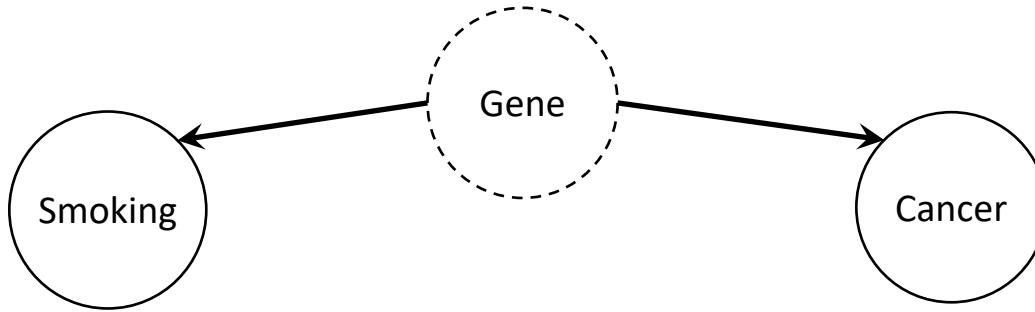
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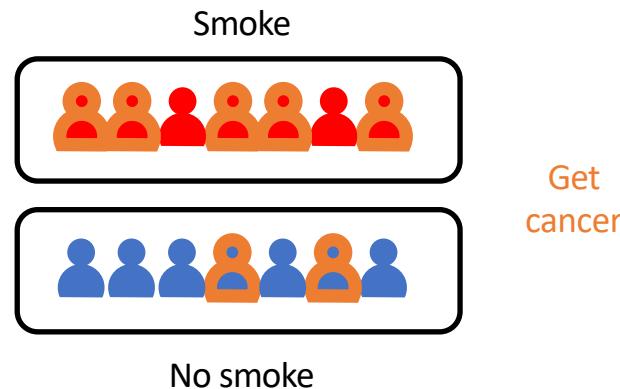


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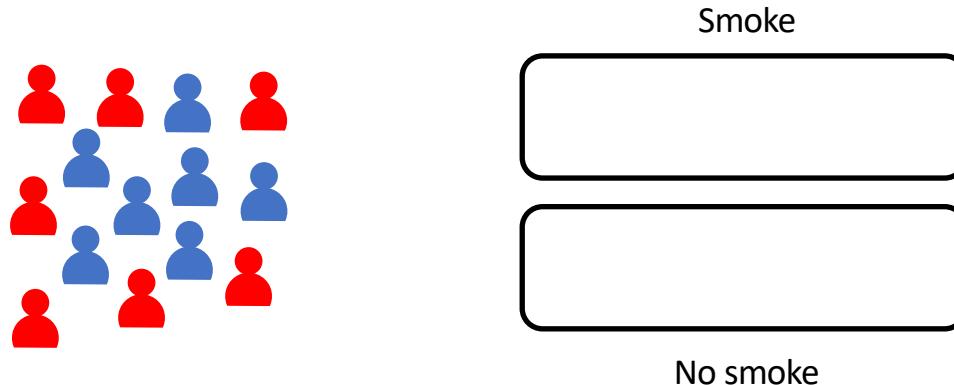


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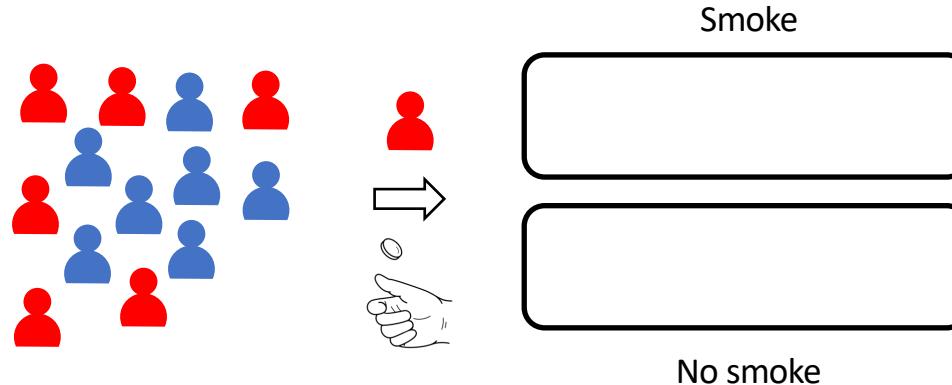
# Randomized controlled trials

- Gold standard in scientific exploration
- RCTs  $\equiv$  Interventions in causality



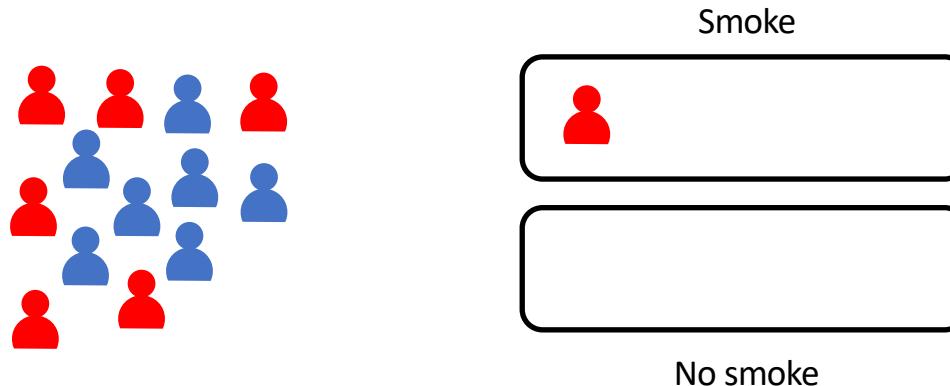
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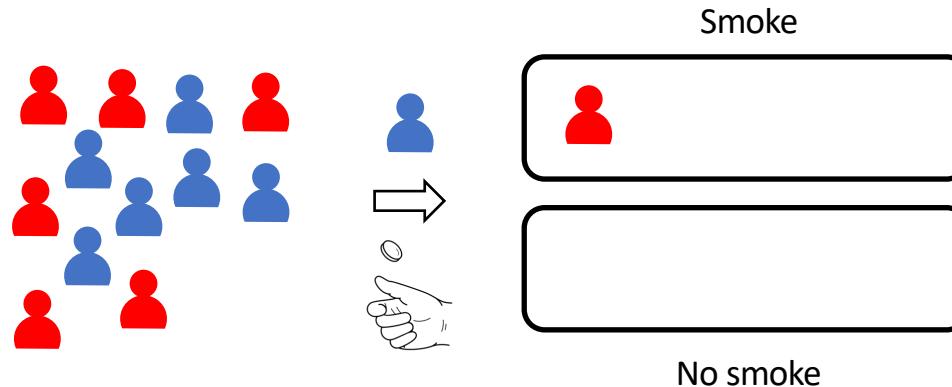
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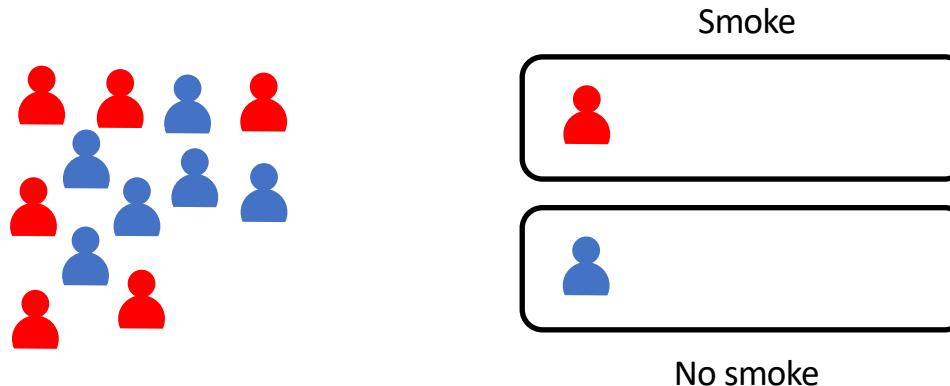
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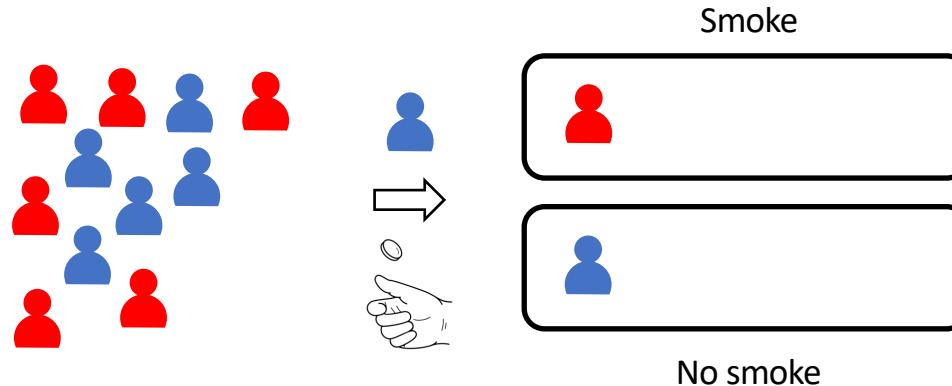
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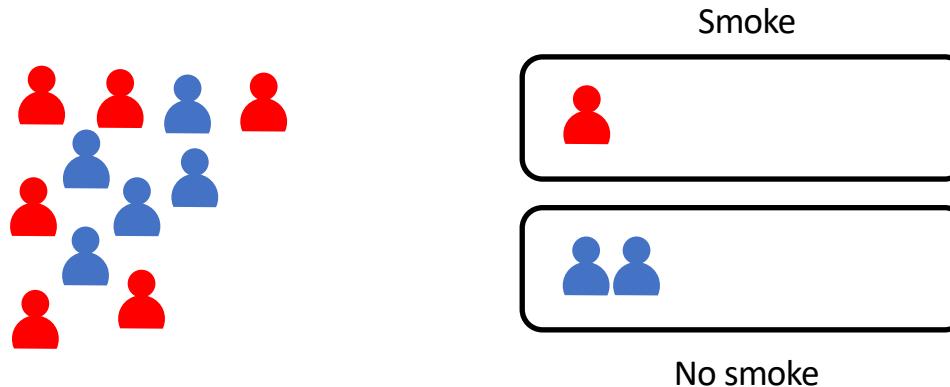
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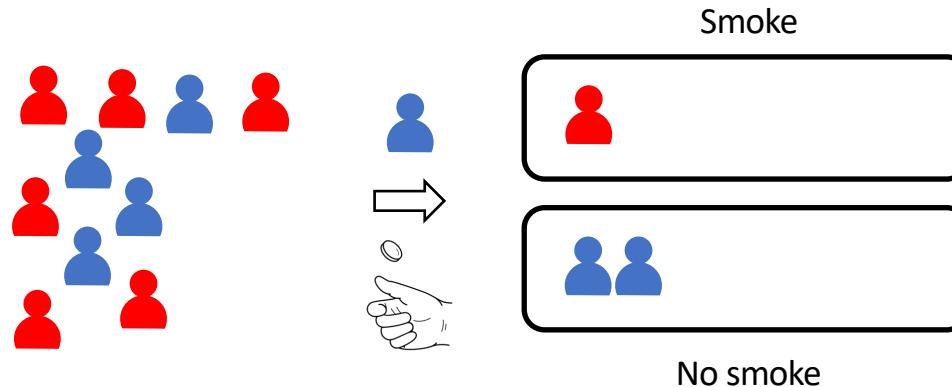
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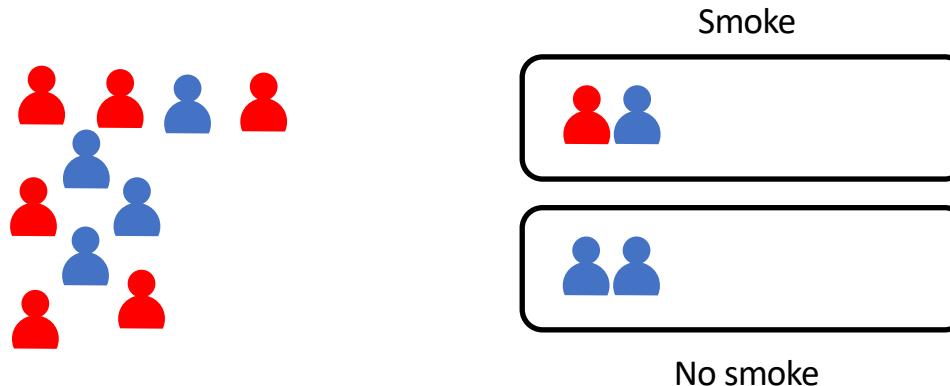
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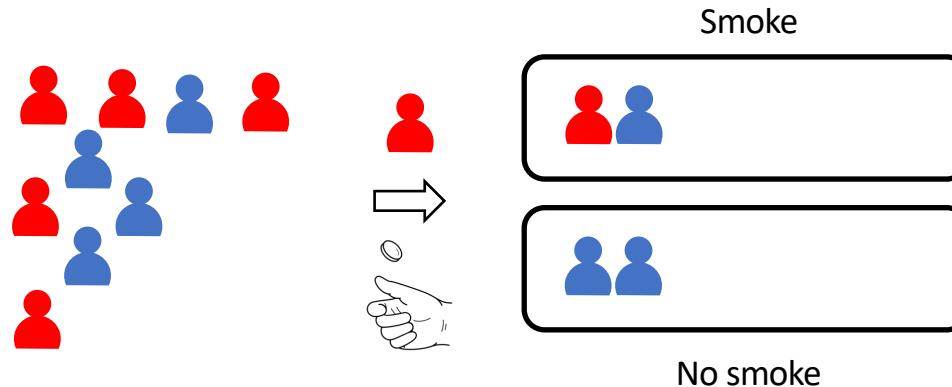
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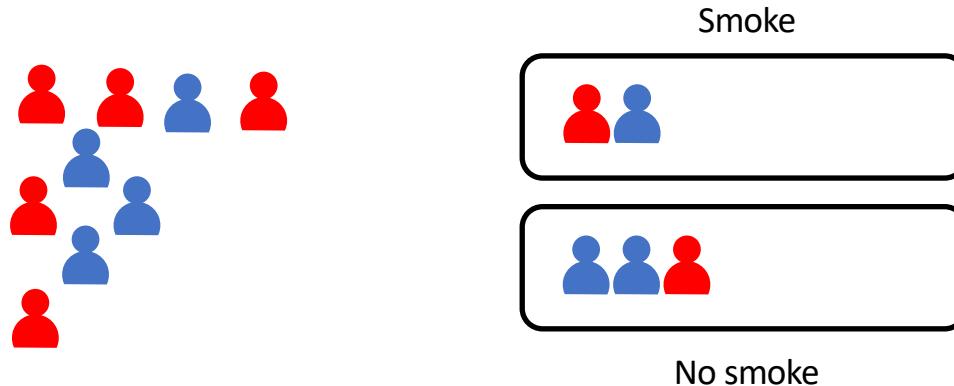
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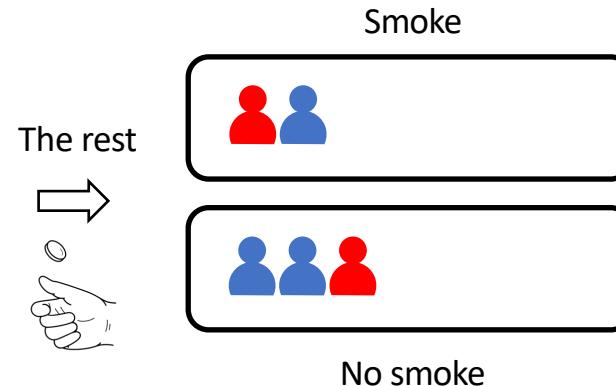
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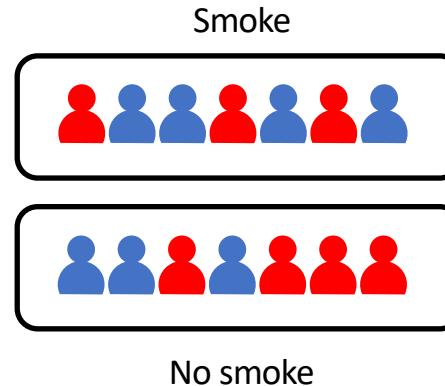
# Randomized controlled trials

- Gold standard in scientific exploration
- RCTs  $\equiv$  Interventions in causality



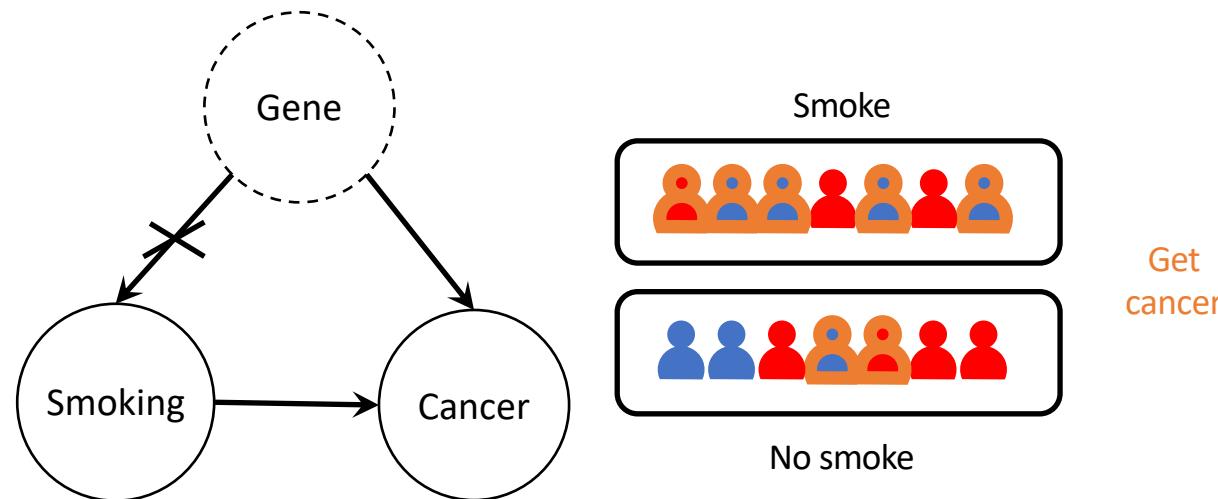
# Randomized controlled trials

- Gold standard in scientific exploration
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# Randomized controlled trials

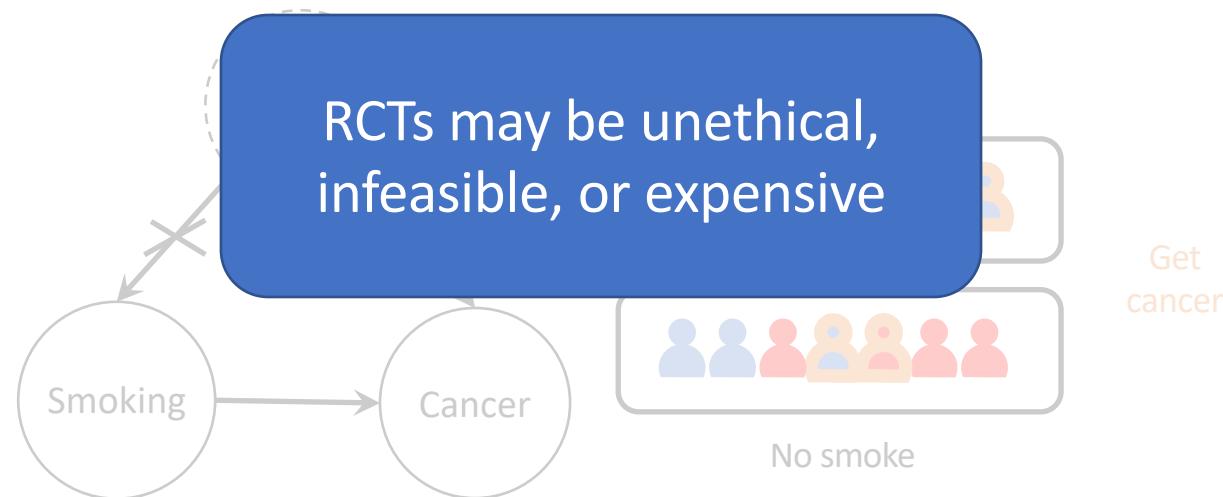
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RCT removed causal link from “gene” to “smoking”  
If smoking and cancer still highly correlated, then smoking causes cancer

# Randomized controlled trials

- Gold standard in scientific exploration
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If smoking and cancer still highly correlated, then smoking causes cancer

# CANCER IMMUNOTHERAPY DATA SCIENCE GRAND CHALLENGE

2023

Lecture 1, Biology: Section D

Press esc to exit full screen

## Cancer Immunotherapy: CAR T-cell therapy

The diagram illustrates the CAR T-cell therapy process:

- Remove blood from patient to get T cells:** Shows a hand with a needle drawing blood.
- Make CAR T cells in the lab:** Shows a T cell being modified. A gene for CAR (Chimeric antigen receptor) is inserted into the T cell. The resulting **CAR T cell** is shown with a yellow oval containing red dots labeled "non-self antigen".
- Grow millions of CAR T cells:** Shows a petri dish with many blue dots representing CAR T cells.
- Infuse CAR T cells into patient:** Shows a person receiving an infusion of CAR T cells.
- CAR T cells bind to cancer cells and kill them:** Shows CAR T cells binding to cancer cells (yellow ovals with red wavy lines) and killing them.

**Treating diffuse large B-cell lymphoma with CAR T cells.**

- ~50% of treated patients have durable complete response.

cancer.gov

June, C. H. et al *New England Journal of Medicine* (2018)

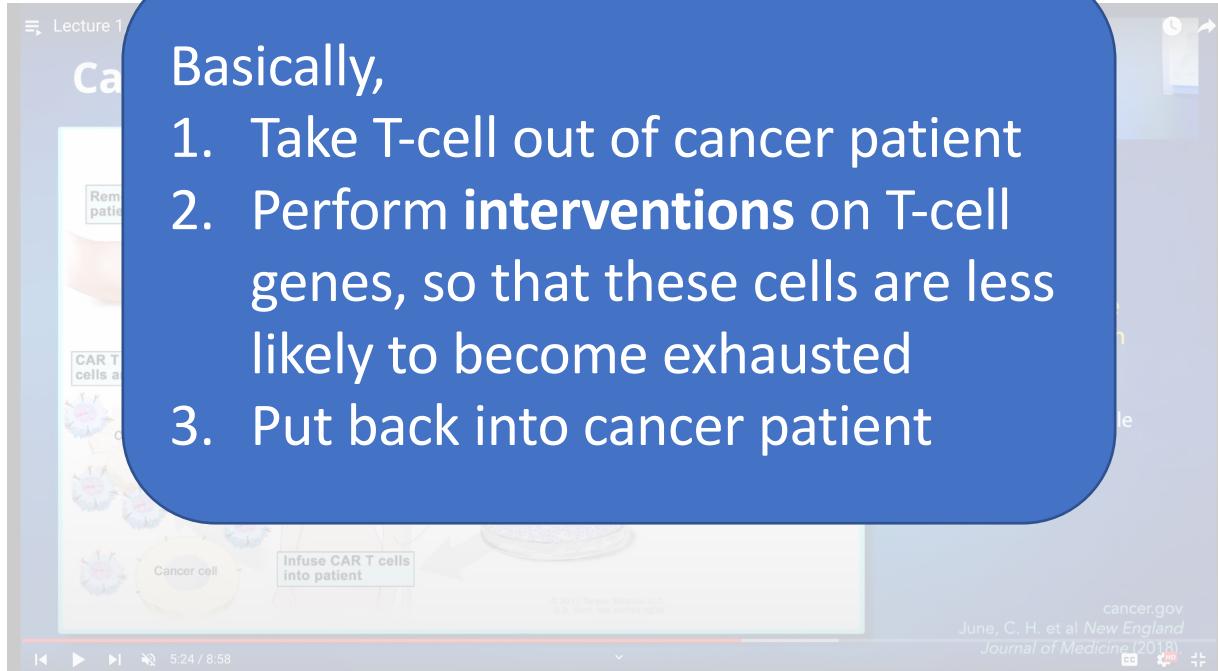
<https://www.topcoder.com/challenges/25f60820-2e69-444b-bc03-490686af2c87>

<https://www.youtube.com/playlist?list=PL7loc09GvxoqmZOFJUc6ni5WS1FgpileN>

2023

Basically,

1. Take T-cell out of cancer patient
2. Perform **interventions** on T-cell genes, so that these cells are less likely to become exhausted
3. Put back into cancer patient



<https://www.topcoder.com/challenges/25f60820-2e69-444b-bc03-490686af2c87>

<https://www.youtube.com/playlist?list=PL7loc09GvxoqmZOFJUc6ni5WS1FgpileN>

NEWS | 07 October 2020



# Pioneers of revolutionary CRISPR gene editing win chemistry Nobel

Emmanuelle Charpentier and Jennifer Doudna share the award for developing the precise genome-editing technology.

Lecture 1

Ca

- 1.
- 2.
- 3.

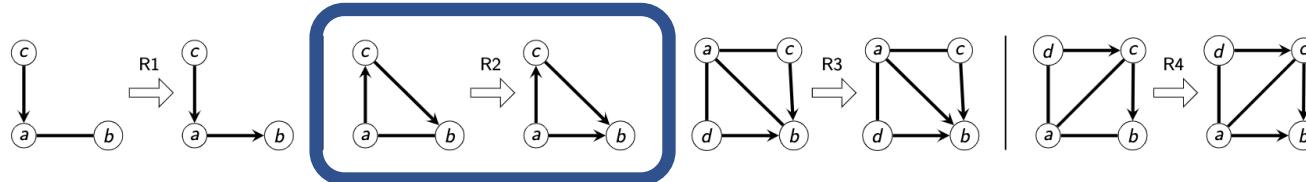


Jennifer Doudna and Emmanuelle Charpentier share the 2020 Nobel chemistry prize for their discovery of a game-changing gene-editing technique. Credit: Alexander Heine/Picture Alliance/DPA

# Meek rules

[Meek 1995]

- **Sound and complete**  
(with respect to arc orientations with acyclic completion)

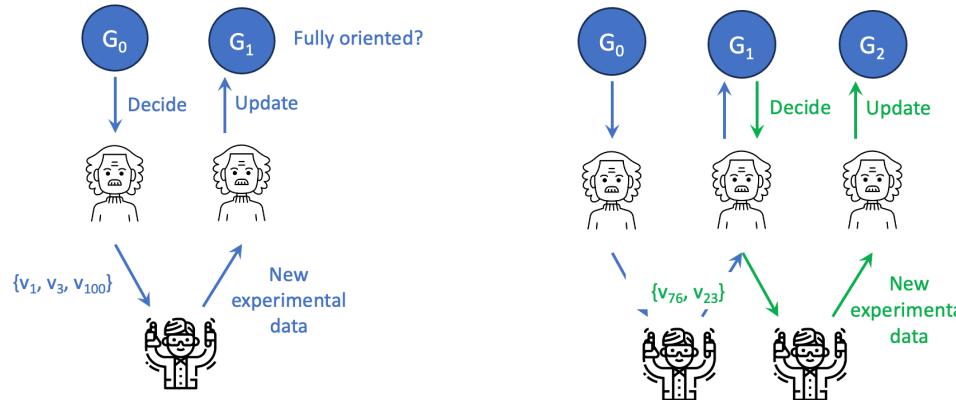


If  $b \leftarrow a$ , then cycle

- Converge in polynomial time [Wienöbst, Bannach, Liśkiewicz 2021]

# Adaptive interventions

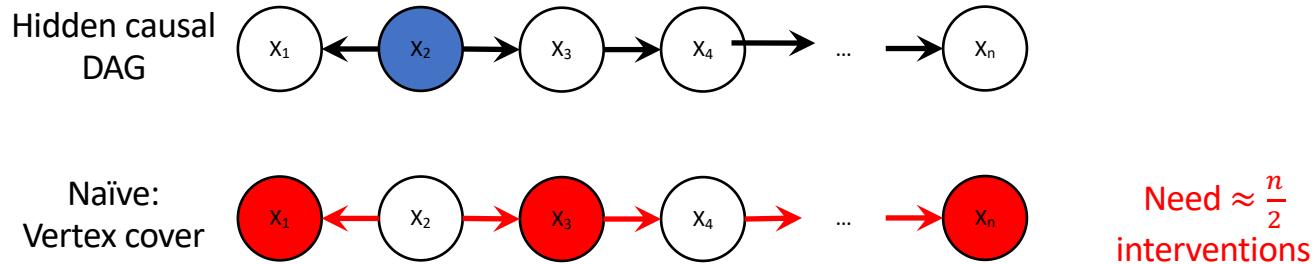
- Intervene. Observe & decide next intervention. Repeat.



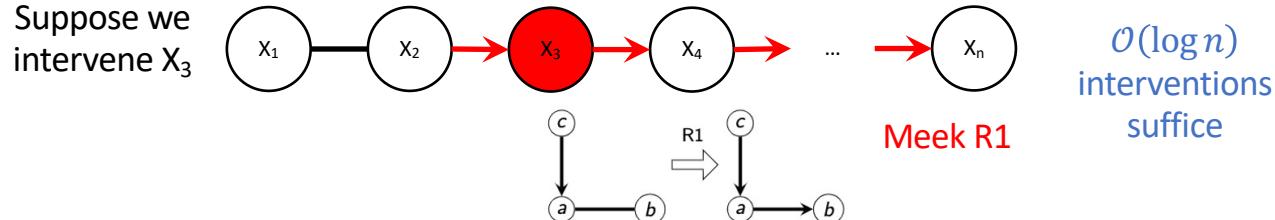
Remark: Non-adaptive may require **exponentially more interventions**

- For example, suppose the essential graph is a path
- Non-adaptive:  $\Omega(n)$  interventions (must be a vertex cover)
- Fully adaptive:  $\mathcal{O}(\log n)$  interventions (emulate binary search)

# The power of adaptivity



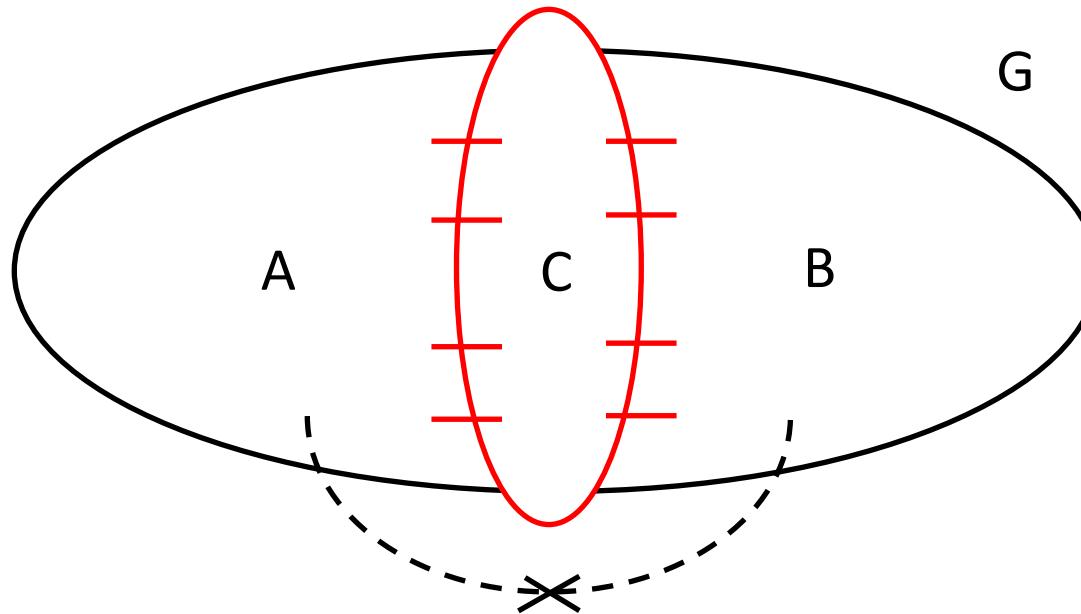
We can do something like binary search and only use  $\mathcal{O}(\log n)$  interventions



# The adaptive search problem

- What we know
  - We know at least  $\nu(G^*)$  is necessary
  - Prior works only have guarantees for special classes of graphs: cliques, trees, intersection incomparable, etc.
- What we show [Choo, Shiragur, Bhattacharyya 2022]
  - Punchline:  $\mathcal{O}(\log n \cdot \nu(G^*))$  interventions suffice
  - “Search is almost as easy as verification”
  - Algorithm does not even know what  $\nu(G^*)$  is!
  - $\Omega(\log n)$  is unavoidable when causal graph is a directed path on  $n$  nodes

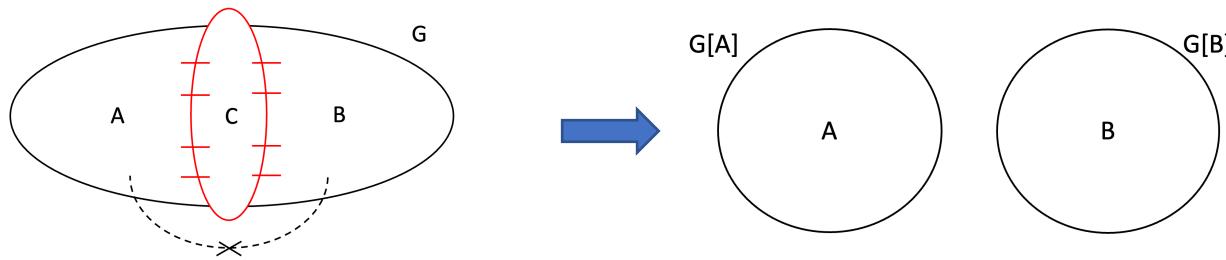
# Key algorithmic idea: Graph separators



Partition vertex set  $V$  into  $A$ ,  $B$ ,  $C$ :

1.  $C$  separates  $A$  and  $B$
2.  $|A|, |B| \leq |V| / 2$

# Key algorithmic idea: Graph separators



- Analysis:
  - $\mathcal{O}(\log n)$  rounds ← Chordal graph separator [Gilbert, Rose, Edenbrandt 1984]
  - $\mathcal{O}(v(G^*))$  per round ← We prove new lower bound on  $v(G^*)$

# What is known?

- Verification number  $\nu(G^*)$ : A natural benchmark
  - How many interventions are required if you *know*  $G^*$  and want to verify it via experiments?

Recovery objective	Number of interventions <sup>†</sup>	Reference
Full graph $G^*$	$\mathcal{O}(\log n \cdot \nu(G^*))$	[Choo, Shiragur, Bhattacharyya 2022]
Subgraph $H \subseteq G^*$	$\mathcal{O}(\log  H  \cdot \nu(G^*))$	[Choo, Shiragur 2023] <sup>§</sup>

- **Question: What if we have (imperfect) advice / side information from domain experts?**

<sup>†</sup> Atomic interventions. <sup>§</sup> To be precise, [Choo, Shiragur 2023] define "relevant vertices in  $H$ " instead of  $V(H)$ .

# Exploiting side information

- Learning-augmented algorithms
  - Designing algorithms using advice, predictions, etc.
    - $\alpha$ -consistent:  $\alpha$ -competitive with no advice error
    - $\beta$ -robust:  $\beta$ -competitive with any advice error
- Advice in causal structure learning
  - Prior works treat information from domain experts as ground truth about the underlying causal system<sup>\*</sup>
    - i.e. No robustness guarantees
  - **For this talk, advice is a specific DAG  $\tilde{G} \in [G^*]$** 
    - See our paper for discussion about partial advice

<sup>\*</sup> Some prior works account for “expert confidence”, but confidence and correctness are orthogonal issues: An expert can be confidently wrong.

# Our main results

Recovery objective	Advice	Number of interventions <sup>†</sup>	Reference
$G^*$	-	$\mathcal{O}(\log n \cdot v(G^*))$	[Choo, Shiragur, Bhattacharyya 2022]
$H \subseteq G^*$	-	$\mathcal{O}(\log  H  \cdot v(G^*))$	[Choo, Shiragur 2023] <sup>§</sup>
$G^*$	$\tilde{G} \in [G^*]$	$\mathcal{O}\left(\max\{1, \log \psi(\tilde{G}, G^*)\} \cdot v(G^*)\right)$	This work

- $0 \leq \psi(\tilde{G}, G^*) \leq n$  measures the quality of  $\tilde{G}$  as advice to  $G^*$ . See paper for details.
- No advice error (when  $\tilde{G} = G^*$ ):  $v(G^*)$
- Arbitrary advice error:  $\mathcal{O}(\log n \cdot v(G^*))$

<sup>†</sup> Atomic interventions. <sup>§</sup> To be precise, [Choo, Shiragur 2023] define "relevant vertices in  $H$ " instead of  $V(H)$ .

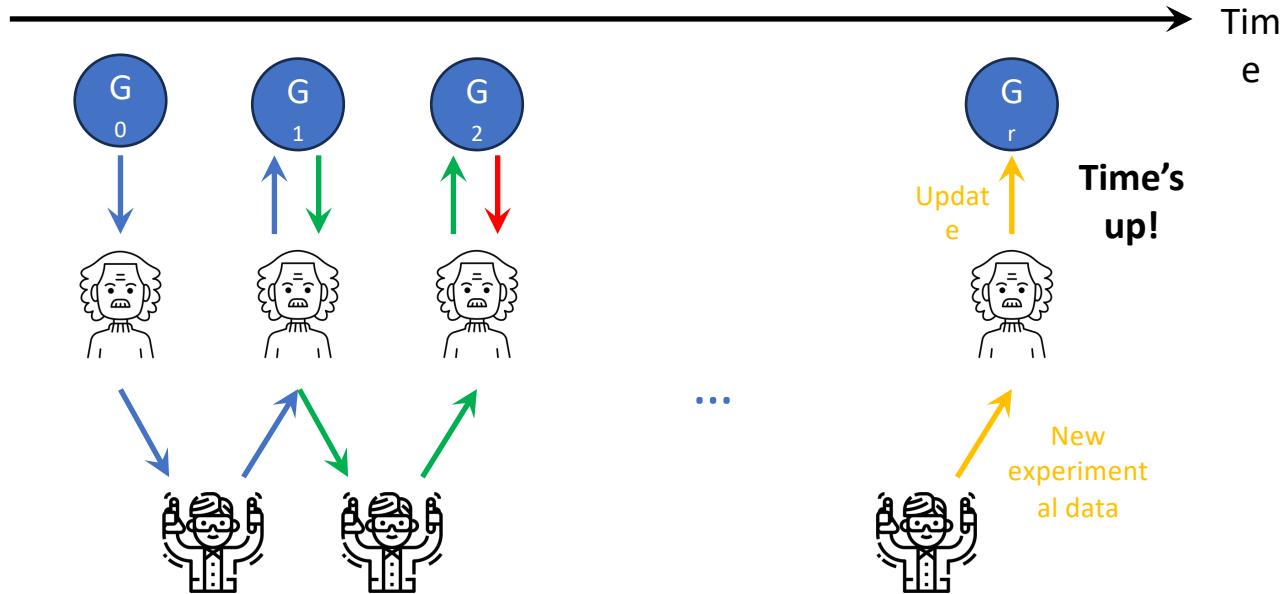
# High-level technical details

Recovery objective	Advice	Number of interventions <sup>†</sup>	Reference
$G^*$	-	$\mathcal{O}(\log n \cdot v(G^*))$	[Choo, Shiragur, Bhattacharyya 2022]
$H \subseteq G^*$	-	$\mathcal{O}(\log  H  \cdot v(G^*))$	[Choo, Shiragur 2023] <sup>§</sup>
$G^*$	$\tilde{G} \in [G^*]$	$\mathcal{O}\left(\max\{1, \log \psi(\tilde{G}, G^*)\} \cdot v(G^*)\right)$	This work

- For any two DAGs  $G_1, G_2 \in [G^*]$ , we have  $v(G_1) \leq 2 \cdot v(G_2)$
- Insight from [Choo, Shiragur, Bhattacharyya 2022]
  - Minimum vertex cover (MVC) of covered edges orients  $G^*$
- Our algorithm
  - Consider  $r$ -hop neighborhood of MVC of  $\tilde{G}$ 's covered edges
  - Invoke [Choo, Shiragur 2023] at suitable intervals as we increase  $r$

<sup>†</sup> Atomic interventions. <sup>§</sup> To be precise, [Choo, Shiragur 2023] define "relevant vertices in  $H$ " instead of  $V(H)$ .

# What if we have limited rounds of adaptivity?



Given a budget of  $r$  adaptive rounds, how to minimize number of interventions?  
(Note: A single round of  $n$  interventions is always trivially sufficient)

# Adaptivity: Trading time for cost

- One-shot non-adaptive approach:
  - Interventions reveal incident arcs  $\rightarrow \mathcal{O}(n)$  interventions
  - Worst case necessary: “Need to recover all possible  $G^*$ ”
- Recent line of work: Adaptive interventions
  - Intervene. Observe & decide next intervention. Repeat.
  - $\mathcal{O}(\log n \cdot \nu(G^*))$  interventions<sup>†</sup> suffice
- Exists simple graphs where non-adaptive requires **exponentially** more interventions than adaptive
- Question: Why not just fully adaptive?
  - Adaptivity imposes sequentiality and hinders parallelism

<sup>†</sup> Atomic interventions

# Our results

- Given  $1 \leq r \leq n$  rounds of adaptivity

$$\Theta\left(\min\{r, \log n\} \cdot n^{\frac{1}{\min\{r, \log n\}}} \cdot v(G^*)\right) \text{ atomic interventions}$$

- When  $r = 1 \rightarrow \Theta(n)$  ← Match worst case bound of non-adaptive
- When  $r = \mathcal{O}(\log n) \rightarrow \Theta(\log n \cdot v(G^*))$  ← Match worst case bound of fully adaptive
- Key ideas:
  - Chain components are chordal graphs → Chordal tree representation
  - Balanced partitioning of trees

- Extension (see paper):

- Bounded size interventions where each intervention involves up to  $1 \leq k \leq n$  vertices

Thank you for your kind attention!