Question 1

a.) $5n^3 + 2n^2 + 3n = O(n^3) \rightarrow 5n^3 + 2n^2 + 3n \leq c \cdot n^3$

↳ $5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3 = 10n^3$

↳ $5n^3 + 2n^2 + 3n \leq 10n^3$ for $n \geq 1 \rightarrow c = 10,\ n_0 = 1$

b.) $\sqrt{7n^2 + 2n - 8} = \Theta(n) \rightarrow c_2 \cdot n \leq \sqrt{7n^2 + 2n - 8} \leq c_1 \cdot n$

↳ $O \rightarrow \sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2} = \sqrt{9n^2} = 3n$ (Upper)

↳ $\Omega \rightarrow \sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n^2 - 8n^2} \leq \sqrt{n^2} = n$ (Lower)

↳ $n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$ for $n \geq 1 \rightarrow c_2 = 1,\ c_1 = 3,\ n_0 = 1$

c.) $d(n) = O(f(n)),\ e(n) = O(g(n)) \rightarrow d(n) \cdot e(n) = O(f(n) \cdot g(n))$

↳ $d(n) \leq c_1 \cdot f(n)$ ↳ $e(n) \leq c_2 \cdot g(n)$

↳ $d(n) \cdot e(n) = c_1 \cdot f(n) \cdot c_2 \cdot g(n)$

$d(n) \cdot e(n) = O(f(n) \cdot g(n)) \rightarrow$ for $n \geq n_0$ and $c = c_1 \cdot c_2$

Question 2

```
def example1(lst):
    n = len(lst) ]-> θ(1)
    total = 0 ]-> θ(1)
θ(n) [for j in range(n):
        for k in range(1+j): ]-> θ(j)
            total += lst[k] ]-> θ(1)
    return total ]-> θ(1)
```

$$T(n) = 2 + (1+2+3+4+\cdots+n) + 1$$
$$= 3 + \left(\frac{n(n+1)}{2}\right)$$
$$= \frac{n^2}{2} + \frac{n}{2} + 3$$
$$= \cancel{\tfrac{1}{2}} n^2 + \cancel{\tfrac{1}{2}n} + \cancel{3}$$
$$= n^2$$
$$= \Theta(n^2)$$

```
def example2(lst):
    n = len(lst) ]-> θ(1)
    prefix = 0 ]-> θ(1)
    total = 0 ]-> θ(1)
    [for j in range(n):
θ(n)     prefix += lst[j] ]-> θ(1)
         total += prefix ]-> θ(1)
    return total ]-> θ(1)
```

$$T(n) = 3 + \underbrace{2+2+2+2+\cdots+2}_{n \text{ times}} + 1$$
$$= \cancel{2}n + \cancel{4}$$
$$= n$$
$$= \Theta(n)$$

```
def example3(n):
    i = 1 ]-> θ(1)
    sum = 0 ]-> θ(1)
    while (i < n*n):   ]
        i *= 2 ]       ]
        sum += i ]-> θ(1) ]
    return sum ]-> θ(1)
```

$$2^0, 2^1, 2^2, 2^3, 2^4, \ldots, 2^{k-1} \qquad k = \log_2(n)$$
$$\hookrightarrow T(n) = 2 + (1+2+3+\cdots+\log(n)) + 1$$
$$= 3 + \left(\frac{\log_2(n)\cdot(\log_2(n)+1)}{2}\right)$$
$$= \frac{\log_2^2(n)}{2} + \frac{\log_2(n)}{2} + 3$$
$$= \cancel{\tfrac{1}{2}} \log_2(n^{2}) + \cancel{\tfrac{1}{2}\log_2(n)} + \cancel{3}$$
$$= \log(n)$$
$$= \Theta(\log(n))$$