Question 1a.

Discussion

- We will show that f is injective.
- We will show that f is surjective.

Proof

Proof. To show that f is bijective, we must show that it is both injective and surjective.

To prove that f is injective, let us first assume $f(s_1) = f(s_2)$. Then, the statement can be rewritten as follows: $ms_1 + b = ms_2 + b \Rightarrow ms_1 = ms_2 \Rightarrow s_1 = s_2$. Thus, f is injective.

Next, we will prove that f is surjective. Let t be an element in the co-domain of f. Assume that there exists a pre-image in the domain $s \in \mathbb{R}$. Then s must satisfy the equation $f(s) = ms + b = t$. Notice that if $s = \frac{-b+t}{m}$, the equation holds. Thus, every $t \in \mathbb{R}$ has the pre-image $\frac{-b+t}{m} \in \mathbb{R}$.

Since we have shown f to be both injective and surjective, we can conclude that f is bijective. $\square$

Question 1b.