Solution to Selected Problems of Chapter 4

Xiaoshuang Chen

October 5, 2017

Problem 4.5-5. Consider the regularity condition $af(n/b) \leq cf(n)$ for some soncstant $c < 1$, which is part of case 3 of the master theorem. Give an example of constants $a \geq 1$ and $b > 1$ and a function $f(n)$ that satisfies all the conditions in case 3 of the master theorem except the regularity condition.

Solution:

Since $f(n) = \Omega(n^{\log_b a+\epsilon})$, we assume that $f(n) = n^{\log_b a+\epsilon}k(n)$, then

$$\frac{af(n/b)}{f(n)} = \frac{k(n/b)}{b^\epsilon k(n)}$$

Therefore, a solution exists if we can find some $k(n)$ so that

$$\forall N > 0, \exists n > N \text{ s.t. } \frac{k(n/b)}{b^\epsilon k(n)} \geq 1$$

Let $b = 2, \epsilon = 1$, and $k(n) = \sin(n)$, then

$$\frac{\sin(n/2)}{2sin(n)} = \frac{1}{4\cos(n/2)}$$

oscillates between $[1/4, \infty)$, thus is a solution to this problem.