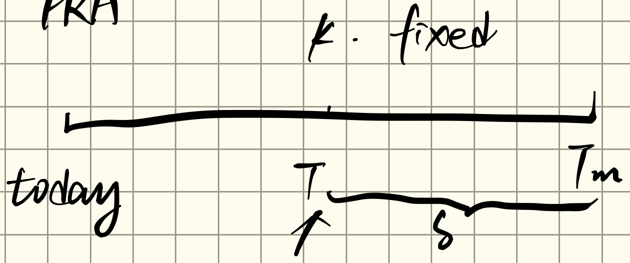
FRA



r is known at T.

so today we don't know r.

here. know the rate r.

if $r > K$. borrower will borrow at K, profit

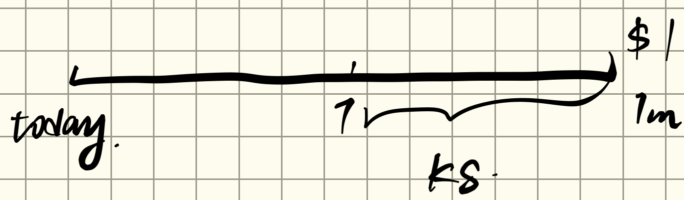
if $r < K$. lender will lend K. so profit & borrower loss.

$$\text{borrower } P/L = \frac{N \cdot (r-K)\delta}{1+r\delta}$$

→ Tm's value. 金个为 T.

long position of FRA. 以K买, ⇒ 以K利率借钱 borrower.

how to determine K?



$$P(T_m) = \frac{1}{(1+K\delta)} P(T)$$

$$K = \left(\frac{P(T)}{P(T_m)} - 1\right)\frac{1}{\delta}$$

Replicate:

borrower will borrow at K. from (T, T_m)

sell A Tm bond

buy T bond. nominal = 1.

st. [today, T] = 0.

Inial: sell $\frac{P(T)}{P(T_m)}$ Tm bond $+P(T_m) \cdot \frac{P(T)}{P(T_m)}$

buy T bond. $-P(T)$

T. Tm. 剩下的value为 $\frac{P(T)}{P(T_m)} \cdot \frac{1}{1+K\delta}$.

T. 收到 1 $

两者应该相当.

$$\Rightarrow \frac{P(T)}{P(T_m)} \cdot \frac{1}{1+K\delta} = 1$$

$$P(T_m) = \frac{P(T)}{1+K\delta}$$ ✓

if. 不当. T时. Tm —0.

T 收到 1 $. 将1用于投资.

T_m时. T_m 付 $\frac{P(T)}{P(T_m)}$.

T. 收到 $(1+r\delta)$.

$$\therefore P/L = 1+r\delta - \frac{P(T)}{P(T_m)}$$

$$= 1+r\delta-(1+K\delta) = (r-K)\delta.$$ (这是在T_m时)收益.

推到 T. $\frac{(r-K)\delta}{1+r\delta}$.

P51 - P77 zoom

loans: not tradeable.

banks → SPV → only P/L part is tranferred to SPV.

too much loans

future CF → Asset → CDO is 分级 debt

CDS is insurance

Investment funds: ask manager to manage money.

eg. SPV new investors.

increase shares numbers

unit trust

custodian: hold assets、保险箱

+ double checking NAV.

sticky work (不会随便更换).

fund {

public / mutual

hedge. / private

mutual: many regulations 透明.

hedge. less. regn. not open to everyone

Ito's formula

$$df(t, X_t) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dx + \frac{1}{2}\underbrace{\frac{\partial^2 f}{\partial x^2}dx\,dx}$$

没有 high order

For BM, high order $=0$

$f(t, X_t)$ martingale $\quad dX_t = a\,dt + b\,dW_t$

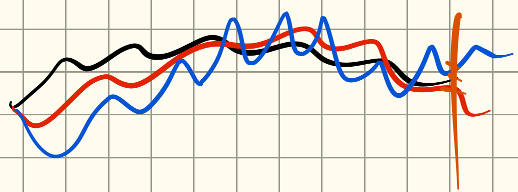
$$\Leftarrow \quad df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}(a\,dt + b\,dW_t) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} b^2 dt$$

$$= \underbrace{\left(\frac{\partial f}{\partial t} + a\frac{\partial f}{\partial x} + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} b^2\right)}_{=0} dt + \frac{\partial f}{\partial x} b\,dW_t$$

$\{\phi_t$ need to be known at the begin of period$\}$

$$\frac{dS_t}{dt} = \mu\,dt + \sigma\,dW_t. \qquad W_t = \sqrt{t}\,Z,\ Z \sim N(0,1)$$

S_0



all the points follow? distri

every point has its probility

⇒ 意味着 $P \neq Q$

ELN. $e^{-rT} E\left[N \cdot 1\{S_T > k \cdot S_0\} + \frac{N S_T}{k \cdot S_0} 1\{S_T \le k \cdot S_0\}\right]$

$= e^{-rT}\left(N P(S_T > k \cdot S_0) + \frac{N}{kS_0} \underbrace{E[S_T 1\{S_T \le k \cdot S_0\}]}\right.$ $\qquad W_T \le -$

$\cancel{\frac{N}{kS_0} E[S_T - S_T 1\{S_T > k S_0\}]}$ $\qquad \frac{N}{kS_0} S_0 E[Z_T 1]$

$= e^{-rT}\left(N P(Z ? d_1) + \cancel{\frac{N}{kS_0}(S_0 - S_0 P(Z < d_2))}\right)$ $\qquad = \frac{N}{kS_0} E[W_T^Q + \sigma T \le d_1]$

$(r - \frac{1}{2}\sigma^2)T + \sigma W_T$.

$e \qquad \le k S_0$

$= e^{-rT}\left(N P(Z < d_1) + \frac{N}{k} - \frac{N}{k} P(Z < d_2)\right)$ $\qquad E[S_T 1\{S_T > k \cdot S_0\}]$

$=$ $\qquad = E\left[S_T 1\left\{W_T \le \frac{\ln k - (r - \frac{1}{2}\sigma^2)T}{\sigma}\right\}\right]$

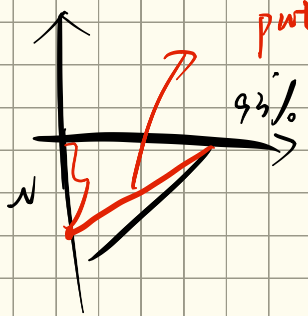
$W_T^Q = W_T + \sigma T.$ $\qquad = S_0 E[Z_T 1]$

$dW_T^Q = dW_T + \frac{\mu - r}{\sigma} dt.$ $\qquad = S_0 E[W_T^Q + \sigma t \le \quad]$

$$W_T^Q = W_T + \int_0^T \frac{\mu - r}{\sigma} dt$$

$$W_T^Q \le \frac{\ln K - (r - \frac{1}{2}\sigma^2)T - \sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln K - (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

put option.



ELN: $\boxed{\text{short put option.}}$

$$V_T = \underline{e^{rT} V_0} + \underbrace{\int_0^T e^{r(T-t)} \Delta_t (dS_t - rS_t dt)}_{=0}$$

$$EV_T = V_0 e^{rT}$$

option premium.

hedging P/L.

self-financing

but. if we take expectation. hedging P/L = 0

$\Rightarrow EV_T = V_0 e^{rT}$ $\therefore$ option premium is hedging cost.

expected return is under P.

not Q.

under Q. hedging cost.

$$d e^{-rt} S_t = S_t \cdot (-r) e^{-rt} dt + e^{-rt} dS_t.$$

$$= e^{-rt} (dS_t - rS_t dt)$$

$$= e^{-rt} \sigma S_t dW_t$$

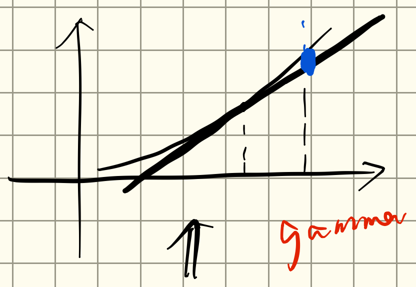
$\Rightarrow \tilde{S}_t$ martingale.
M_t

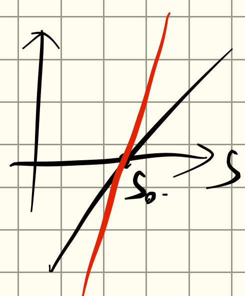
how come $e^{-rT} V_T = V_0 + \int_0^T \Delta_t \, d(e^{-rt} S_t)$.

is martingale.

我们假设 $e^{-rT} V_T$ is martingale.

why? BS equation.





这个曲线是 hold a stock

hold two stocks

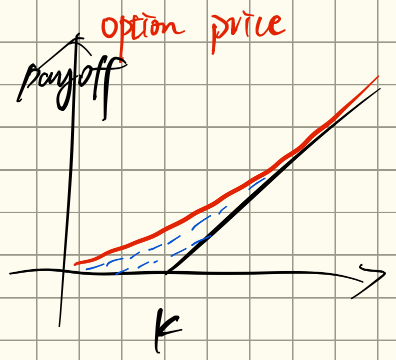
in a short period, don't have time to react.

so there is error

Can we have prefect hedge?

No. there is always one side error.

① how to explain?

② 

when time 越近于 maturity, option price 越趋于 S_t

⇒ how to hedge this? time value.

how to solve these two problem?

gamma & theta. offset each other.

$\frac{\partial^2 V}{\partial S^2}$ $\frac{\partial V}{\partial t} - rV$

where can see?

$$\underbrace{-rV_t + \frac{\partial V}{\partial t}}_{\text{theta}} + \underbrace{rS_t \frac{\partial V_t}{\partial S_t}}_{\text{delta}} + \underbrace{\frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S_t^2}}_{\text{gamma}} = 0$$

这是 dV.

如果我们将 V_t hedge. 构成一个 portfolio. $\frac{\partial V}{\partial S} = 0$

我们做了 delta hedging.

∴ theta + gamma = 0

In reality, can't offset

so we need human trader to judge.

As a trader, to hedge. 只都 hedge sensitivity delta: $\frac{\partial \Pi_t}{\partial S} = 0$ 我们想让它等于0.

$$\frac{d^2 \Pi_t}{\partial S_t^2} + q_t \frac{d^2 V_t}{\partial S_t^2} = 0.$$ self-financing

so hedge gamma.

对 delta. $\underline{V_t} + q_t S_t + C = 0$

令一阶导等于0.

delta. 是对S求导. 令它对价格变化不敏感.

$$\frac{\partial V}{\partial S} + q_t = 0 \qquad q_t = -\frac{\partial V}{\partial S}.$$

gamma. 是对 vol 不敏感. 对S求二阶导. 但 $\frac{d^2 S_t}{d S_t^2} = 0$

so we use option.

$$\Pi_t + q_t V_t + C = 0$$

$$\frac{\partial^2 \Pi_t}{\partial S^2} + q_t \frac{\partial^2 V_t}{\partial S^2} = 0.$$ for hedging gamma

vega $\frac{\partial V}{\partial \sigma}$

trader. if S_t don't move.

if S_t 变化. buy option and hedge option. ⇒

← hedge ⇒ make money

accumulator.

how to hedge

share price. ⇔ NAV. fundemental indicator.

exchange rate

对 foreign stock

$$\frac{dS_t}{S_t} = r_f dt + \sigma dW_t^f$$

how to write to domestic

$$\frac{dX_t}{X_t} = (r_d - r_f)dt + \sigma_X dW_X^d(t)$$

X_t. 汇率

1 foreign = _ domestic

相当于本币买了股票，有 continus. dividend.

对 $X_t S_t = Y_t$

$$\frac{dY_t}{Y_t} = r_d dt + \bigcirc dW_t^d$$

$$\frac{dY_t}{Y_t} = \frac{dX_tS_t}{X_tS_t} = \frac{X_t dS_t + S_t dX_t + dS_t dX_t}{X_tS_t}$$

$$= \cancel{X_tS_t}(r_f dt + \sigma dW_t^f) + \cancel{S_tX_t}((r_d - r_f)dt + \sigma_X dW_X^d(t))$$

$$+ \sigma_S\sigma_X \cdot dW_t^f \cdot dW_X^d(t)$$

$$= r_d dt + \sigma dW_t^f + \sigma_X dW_X^d + \sigma_S\sigma_X dW_t^f dW_X^d$$

$dW_t^f dW_X^d = \rho dt$

$$= r_d dt + \sigma(dW_S^d - \theta dt) + \sigma_X dW_X^d + \sigma_S\sigma_X \rho \cdot dt$$

$$= (r_d - \sigma_S\theta + \rho\sigma_S\sigma_X)dt + \sqrt{\sigma_S^2 + \sigma_X^2 + \rho\sigma_S\sigma_X}\, dW_t$$

$\Rightarrow$ 转换成 risk neutral

$$dW^d = \underline{dW^f} + \theta t.$$
$$dW_t^f$$

$\therefore \theta = \rho\sigma_X$

$$\therefore \frac{dS_t}{S_t} = r_f dt + \sigma_X(dW_X^d - \rho\sigma_S dt)$$

$$= (r_f - \rho\sigma_S\sigma_X)dt + \sigma_X dW_X^d$$

quanto call option. $(S_T - K)^+$. paying out in domestic currency

$$e^{-r_dT} E^d[(S_T - K)^+]$$

$$= e^{-r_dT} E^d[S_T 1_{\{S_T > K\}}]$$

$$+ e^{-r_dT} K E^d[1_{\{S_T > K\}}]$$

$$= A + B$$

composit $(X_T S_T - K)^+$

问题 $E^f[X_T(S_T - K)^+]$

& $E^d[(S_T - K)^+]$ 一样吗？

$$\frac{dS_t}{dS_t} = (r_f - \rho\sigma_S\sigma_X)dt + \sigma dW_S^d$$

$$S_t=S_0e^{(r_f-\rho\sigma_s\sigma_x-\frac{1}{2}\sigma^2)T+\sigma W_s^d}$$

$$B=e^{-r_dT}K\,N(d_2)$$

$$A=e^{-r_dT}E\left[S_0e^{(r_f-\rho\sigma_s\sigma_x-\frac{1}{2}\sigma^2)T+\sigma W_s^d}\,1_{\{W_s^d\le\sqrt{T}d_2\}}\right]$$

$$\ge K \quad \frac{\ln\frac{K}{S_0}-(r_f-\rho\sigma_s\sigma_x-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}=-d_2$$

$$=e^{-r_dT}S_0e^{r_f-\rho\sigma_s\sigma_x}\underbrace{E[Z_t1]}$$

$$Z\le \quad \frac{\ln\frac{S_0}{K}+(r_f-\rho\sigma_x\sigma_s-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}=d_2$$

$$=e^{(r_f-r_d-\rho\sigma_s\sigma_x)T}S_0\cdot P(W_s^Q+\sigma T\le\sqrt{T}d_2)$$

$$=e^{(r_f-r_d-\rho\sigma_s\sigma_x)T}S_0N(d_1)$$

$$W^Q=W^P+\sigma T$$

$$W_s^Q+\sigma T\ge -d_2\sqrt{T}$$

$$W_s^Q\ge -d_2\sqrt{T}-\sigma T$$

$$W_s^Q\le \underline{d_2\sqrt{T}+\sigma T}\ d_1$$

foreign exchange hedge

$$\Delta^S=\frac{1}{X}\frac{\partial V}{\partial S} \qquad \Delta^X=\frac{\partial V}{\partial X}-\Delta^S S$$

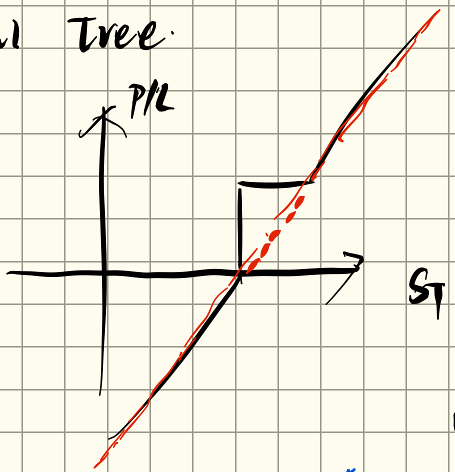
$$\Delta^X+\Delta^S S=\frac{\partial V}{\partial X}$$

$\Delta^S S$: exchange exposure through stocks

Δ^X: direct exchange exposure

Binomial Tree

BEN



dividend

用dividend buy a binary option

how to hedge: delta hedge

in business, call to ask if exit

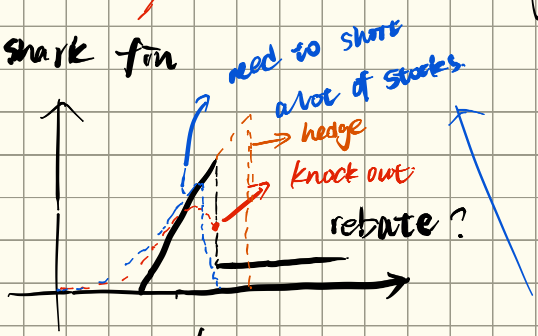
why we don't buy option from clients?

Clients will walk away

collateral

FCN price is N. collateral

shark fin



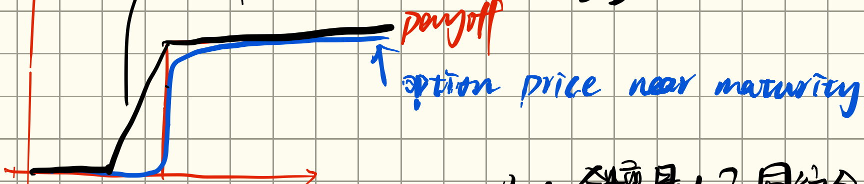
knock out call option

expected payoff is very small ⇒ cheap premium

欺骗航道 barrier option

up and out

斜率是什么时候最大的 $\frac{\partial V}{\partial S}$



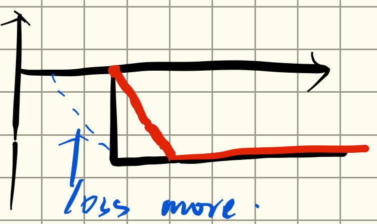
how to hedge?

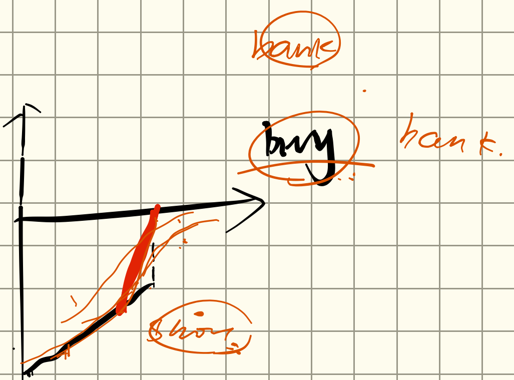
sell option

why 斜率最大？同行会竞争

if 斜率小，那确实也挣的多，但 premium大.

if buy





Cushion = ~~NAV - min~~ all cost (可以loss的钱)
max loss.

CPPI: \$10 to lose $\boxed{-20\%}$ gap risk. = $\frac{1}{5}$

Can invest 50. E_t = Multiplier × Cushion.

$\frac{1}{\text{Gap}}$ = Multiplier   = 5 × 10
= 50

the money we can invest.

Volatility: 从 market 推出来 ⇒ implied volatility.

exchange option: market will quote it

realized vol 不同

historical vol: not use in price & hedge.

only give infor to predict.

local volatility model: $\sigma \to \sigma(t, S_t)$

for almost all option.

BS. for call & put option.   except $\boxed{\text{forward skew}}$

stochastic model. time consuming

what is forward skew?

call spread

if close, $\frac{\partial V}{\partial \sigma}$ is not big.

$\boxed{\text{diff}(\sigma)}$ important.

K is determined by future price

$\boxed{\text{forward starting call spread}}$

can't local

where PDE? optimaze the parameters

$$\frac{dY_t}{Y_t} = r_d\,dt + \sigma_Y dW_t$$

$$\frac{dS_t}{S_t} = r_f\,dt + \sigma_S dW_t^f$$

$$\frac{dX_t}{X_t} = (r_d - r_f)dt + \sigma_X dW_X^d$$

EUR USD

stock

$Y_t = X_t S_t$

$$\frac{dX_tS_t}{X_tS_t} = X_t dS_t + S_t dX_t + dS_t dX_t$$

$$= \cancel{S_tX_t}(r_f\,dt + \sigma_S dW_t^f) + \cancel{X_tS_t}((r_d - r_f)dt + \sigma_X dW_X^d) + \cancel{S_tX_t}\,\sigma_S\sigma_X \rho\,dt$$

$$= (\cancel{r_f} + r_d - \cancel{r_f} + \rho\sigma_S\sigma_X)dt + \sigma_S \cancel{dW_t^f} + \sigma_X dW_X^d$$

$(dW_X^d + \theta dt)$

$$\therefore \theta = -\rho\sigma_X$$

$$\therefore dW_t^f = dW_X^d - \rho\sigma_X dt.$$

$$\frac{dS_t}{S_t} = r_f\,dt + \sigma_S dW_t^f$$

$$= (r_f - \rho\sigma_X\sigma_S)dt + \sigma_S dW_X^d$$

推导 $\Delta = \frac{\partial V}{\partial S}$

$$d\frac{S_t}{M_t} = \frac{1}{M_t}\sigma dW_t)$$

$$\frac{dV_t}{M_t} = \Delta_t\left(d\frac{S_t}{M_t}\right) = \Delta_t\left(\frac{1}{M_t}\cdot\sigma dW_t\right)$$

$a\,dt + \sigma dW_t$

$$\frac{dV_t}{M_t} = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S}dS + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\boxed{dS\cdot dS}$$

$$= \frac{1}{M_t}\left(\frac{\partial V}{\partial t}dt + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}b^2 dt\right) + \frac{\partial V}{\partial S}(a\,dt + \sigma dW_t)$$

$$= \frac{1}{M_t}\frac{\partial V}{\partial S}\sigma\,dW_t = \Delta_t \frac{1}{M_t}\sigma dW_t$$

$$\Delta_t = \frac{\partial V}{\partial S}$$

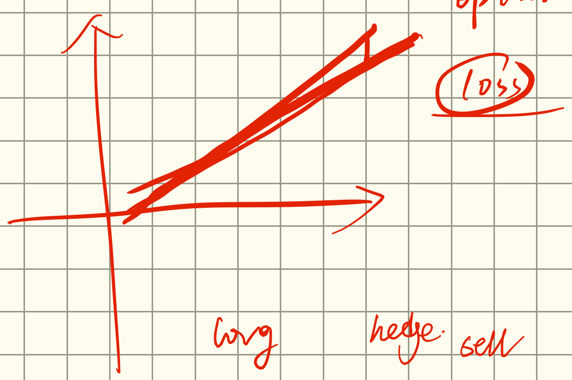
Question: 1. short an option. long an option

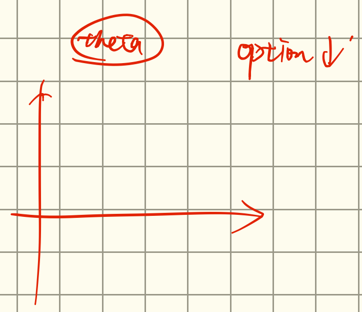
对 gamma. 对冲. lose. 对gamma. 对冲. lose.

theta. make theta loss

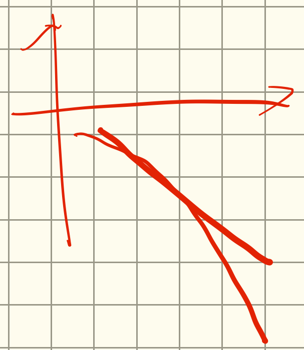
2. 不管怎样都 short (vol=0)

short call





long hedge sell



Repo:[illegible]回购,是 borrow 的一种,

if 市场liquid差. 要逆回购 lend

GMRA. 主协议.

SBL. 期间有interest. 个人只能 borrow stocks. can't lend

rear fixed rates: $1+\frac{N}{360}\cdot r = \prod\left(1+\frac{n_i r_i}{360}\right)$

$$r=\left(\prod\left(1+\frac{r_i n_i}{360}\right)-1\right)\frac{360}{N}$$

Duration: $\sum w_i t_i$. $w_i = \frac{CF_i}{(1+y)^i}/P$

$$MDur = \frac{\sum w_i t_i}{1+y} = -\frac{\partial P}{\partial y}/P$$

FRA

T_0 T T_m

A   B

$$P(T_m) = \frac{1}{1+ks}\cdot P(T)$$

$$k=\left(\frac{P(T)}{P(T_m)}-1\right)\frac{1}{s}$$

Longside. borrower 以K借钱.

replicate: Short $\frac{P(T)}{P(T_m)}$ B.

Long: 1 A

T时: A: 获得1

B: 价格为 $\frac{1}{1+rS} \cdot \frac{P(T)}{P(T_m)}$

$$\therefore P/L = 1 - \frac{1}{1+rS} \cdot \frac{P(T)}{P(T_m)}$$

$$= 1 - \frac{1+KS}{1+rS}$$

$$= \frac{(r-K)S}{1+rS}$$

Conversion Factor Calculation:

交割时没有⇒用另一种交割 cheapest to deliver. CTD.

IRS. floating 换 fixed.

开始时无交换，中间交换利息

$$\text{swap PV} = PV^{fixed} - PV^{floating}$$

$$= \sum P(t_j) S \tau_j - \sum P(t_i) F(t_{i-1}, t_i) \tau_i$$

对 floating part，相当于 FRA.

$$\therefore F(t_{i-1}, t_i) = \left(\frac{P(t_{i-1})}{P(t_i)} - 1\right) \frac{1}{\tau_i}$$

$$PV^{floating} = \sum P(t_i) \cdot \cancel{\frac{1}{\tau_i}} \left(\frac{P(t_{i-1})}{P(t_i)} - 1\right) \cancel{\tau_i}$$

$$= \sum P(t_{i-1}) - P(t_i)$$

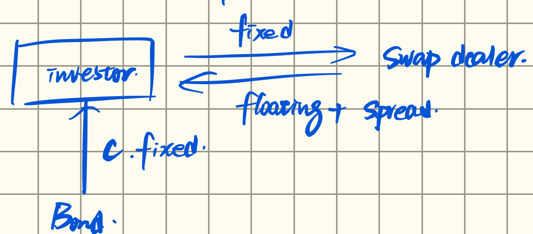
$$= 1 - P(t_n) = S \cdot \sum P(t_j) \tau_j$$

OIS: overnight Indexed swap

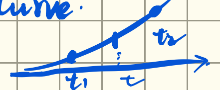
collateral. ⇒ OIS.

non-collateral. ⇒ repo.

Asset swap:



Yield Curve.



(t_2, e^{-rt_2})

(t_1, e^{-rt_1})

$P(t_2) = P(t_1)\cdot e^{-\frac{(t_2-t_1)}{365} r}$

$r = \ln\frac{P(t_1)}{P(t_2)} \times \frac{365}{t_2-t_1}$

∴对 t. $P(t) = P(t_1)\cdot e^{-\frac{(t-t_1)r}{365}}$

construct yield curve.

deposit. $P(t_2) = \frac{P(t_1)}{1+r\tau}$ $\tau = \frac{\text{end-start}}{360}$

1 month. $P(t_2) = \frac{P(t_1)}{1+r\tau}$ t_1: 01 Dec 14 t_2. 02 Jan 15 找 end date

Futures 可能插值.

Swap $P(t_0) - P(t_n) = s\sum P(t_j)\tau_j$

$P(t_n) = P(t_0) - s\sum P(t_j)\tau_j$

t_0 = 1-Dec-14 t_1 = 1-Dec-15 t_2 = 1-Dec-16

$P(2Y) = P(t_0) - sP(1Y)\cdot 1 - s\cdot P(2Y)$

Equity Market.

Equity index 不能直接交易 ⇒ ETF. mutual fund.

Equity forward $K = S_0 e^{(r-q)t} - Div$ (maturity)

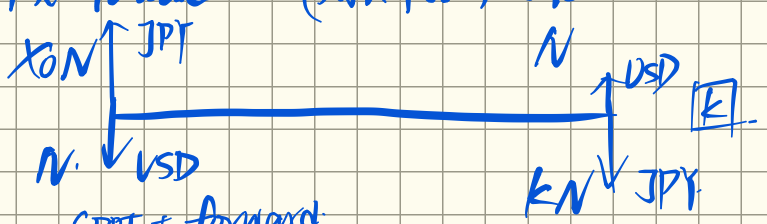
cross rate: not USD & EUR.

EURUSD domestic

foreign / base.

FX forward. (交换本金.) OTC USDJPY = X_0

per USD = X_0 JPY



1 USD = K JPY.

spot + forward

$(1+r^f T) = \frac{X_0(1+r^d T)}{K}$ $K = \frac{X_0(1+r^d T)}{1+r^f T} = \frac{X_0 P^f(T)}{P^d(T)}$

cross currency swap (也交换本金 + 利息).

本金给多少还是多少.

non deliverable swap.

用于无韩元、先转成美元，其实还是本金不变.（只是KRW对应的USD变了）

只是利息 KRW 是 fixed. 因为无 floating rate.

Commodities Market:

gold. $N(S_T-K)$ $F=S_0e^{(r-y+c)t}$.

Oil demand & offer.

rollover. 展期

CDS: 保险 $\overset{y}{\text{CDS}} + \boxed{\overset{x}{\text{cooperate bond}}} = \overset{z}{\text{treasury bond}}$.

$x-y=z$ 用 $y+z$ 来构造 x.

payout $= N(1-R)$ R是破产得到的部分.

CDO: tranches.

Investment fund. collective. eg. SPV.

custodian.

share price: demand & offer

NAV: book value / #share

ETF: PD

caplet payoff: call with interest rate

$$N\sum t_i(r_i-k)^+$$

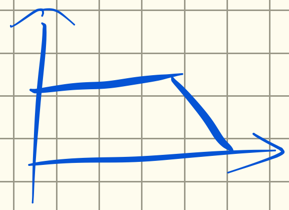
floorlet put.

$$N\sum t_i(k-r_i)^+$$

Swaption:

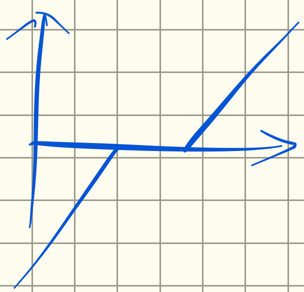
call spread

put spread



TRF target redemption structure.

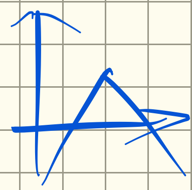
risk reversal



straddle

butterfly

pivot.



put & call parity: volatility same.

implied volatility.

theta = gamma.

$$S_0 \tilde{E}[e^{rT} S_T 1\{\tilde{W}_T > v\}]$$

$$Z_T = e^{-\frac{1}{2}\sigma^2 T + \sigma W_T}$$

$$S_0 \tilde{E}[Z_T 1\{\tilde{W}_T > v\}]$$

$$= S_0 \tilde{E}^{Q}[1\{\tilde{W}^{Q}_T + \sigma T > v\}]$$

foreign. option. $X_t = X_0 e^{(r_d - r_f)t}$.

EUR USD domestic.

$$\frac{dX_t}{X_t} = (r_d - r_f)dt + \sigma_x dW_x^d.$$

$$\frac{dS_t}{S_t} = r_f dt + \sigma_s dW_s^f$$

$$\frac{dS_t}{S_t} = (r_f - \rho_{sx}\sigma_s\sigma_x)dt + \sigma_s dW_s^d$$

$$\frac{dY_t}{Y_t} = r_d dt + \sqrt{\sigma_x^2 + \sigma_s^2 + 2\rho\sigma_s\sigma_x}\, d_Y^d$$

$dW_S^f = dW_S^d - \rho\sigma_X$

Composite option. $\mathbb{E}[(S_T X_T - K)^+]$ 用 Y_T

quanto option $\mathbb{E}^d[(S_T - K)^+]$ 用 S_T.

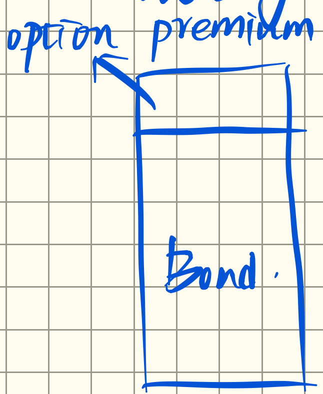
hedge: $\Delta^S = \frac{1}{X}\frac{\partial V}{\partial S}$ $\quad \Delta^X + \Delta^S S = \frac{\partial V}{\partial X}$

local volatility: 除了 forward starting call spread.

这个用 stochastic volatility model.

diff(σ) is important

not good for contracts depends on forward skew.



100 − 0.5% fees.

PR是买多少 option.

Payoff = $N + N \cdot PR \cdot \max\{\text{option}\}$

ELN.

$$\text{payoff} = \mathbb{E}\left[N \mathbb{1}_{\{S_T > K \cdot S_0\}} + \frac{N S_T}{K \cdot S_0} \mathbb{1}_{\{S_T < K \cdot S_0\}}\right]$$

$$= \mathbb{E}\left[N \cdot \mathbb{1}_{\{S_T > K \cdot S_0\}} + \left(N + \frac{N S_T}{K S_0} - N\right) \mathbb{1}_{\{S_T < K S_0\}}\right]$$

$$= \mathbb{E}\left[N - \frac{N}{K}\left(\frac{S_T}{S_0} - K\right) \mathbb{1}_{\{\frac{S_T}{S_0} < K\}}\right]$$

$$= N - \frac{N}{K} \max\left\{0, \frac{S_T}{S_0} - K\right)$$

$$= N - \frac{N}{K} \cdot \text{put}$$

short geared put

FCN ↑coupon

how? ① dividend → price ↓ → risk ↑.

② volatility ↑.

③ negative correlation

④ frequency ↓

Autocallable. knock out ↓

Snowball. knock out 才会给 coupon.

knock in 获得股票.

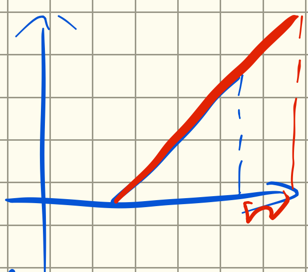
BEN

用dividend买 option.

accumulator.

→ knock out.

shark fin.



hedge. barrier shift

到什么时候 器 都小于 Q_{max} 即停.

binary option

Himalaya: can hedge easily.

Risk exposure = min{ Cap, $\frac{\sigma_{target}}{\sigma_{realized}}$ }

CPPI.

Multiplier = $\frac{1}{G}$ Gap risk.

Cushion : NAV − Floor − cost.

E_t = Multiplier × Cushion.

Monte Carlo , LSM [illegible]. issuer callable.

Binomial tree ✓

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exercise 1.1.

T_0 2024.3.11. — T 2024.6.11 — T_m 2024.9.11.

$P(T) = \frac{1}{1+rT}$ $P(T_m) = \frac{1}{1+rT_m}$.

$k = \left(\frac{P(T)}{P(T_m)} - 1\right)\frac{1}{\delta}$

$P/L = \frac{(r-k)\delta}{1+r\delta}$. long side / borrower.

2.1 $e^{-rT}\,\mathbb{E}[N\cdot(k-\frac{S_T}{S_0})\cdot \mathbb{1}\{k-\frac{S_T}{S_0}>0\}]$

$\frac{S_T}{S_0}<k.$

$= N\cdot e^{-rT}\cdot[\,k\cdot P(S_T<k\cdot S_0) + \frac{1}{S_0}\,\mathbb{E}[S_T\cdot \mathbb{1}\{\frac{S_T}{S_0}<k\}]\,]$

$= Ne^{-rT}\cdot k\cdot N(d_2) - \frac{S_0 N e^{-(d+q)T}}{S_0}\,\mathbb{E}[\,Z_T\cdot \mathbb{1}\{W_T<d_2\sqrt{T}\}]$

$S_T = S_0\, e^{(r-d-q-\frac{1}{2}\sigma^2)T+\sigma W_T}$ $<k\cdot S_0$

$z < \boxed{\frac{\ln k-(r-d-q-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}}\ -d_2.$

$= Ne^{-rT}\cdot k\cdot N C \;-\; N\cdot\mathbb{E}^{Q}[\,\mathbb{1}\{W_T^{Q}+\sigma T<d_2\sqrt{T}\}]\; e^{-(d+q)T}.$

$= Ne^{-rT}k\cdot N(-d_2) - N\cdot N(-d_1)\, e^{-(d+q)T}$