Chapter 1

- TS: a sequence of variable + all depend on time
- FTS: financial time series-->more uncertainty<--changing business and economic environment and the volatility is not directly observed.

1.1 Asset Returns

- one period simple return $R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$
  - gross return $1 + R_t = \frac{P_t}{P_{t-1}}$
- multi period simple return $R_t(k) = \frac{P_t - P_{t-k}}{P_{t-k}}$
  - gross return:

$$1+R_t(k) = \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{t-2}} \cdots \frac{P_{t-k+1}}{P_{t-k}}$$
$$= (1+R_t)(1+R_{t-1})\cdots(1+R_{t-k+1})$$
$$= \prod_{j=0}^{k-1} (1+R_{t-j})$$

- log return
  - $r_t = \ln(1+R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1}$
  - $r_t(k) = r_t + r_{t-1} + ... + r_{t-k+1}$
- Portfolio return
  - formula

$$w_i = \frac{P_{i,t}}{P_{p,t}} \quad P_{p,t} = \sum_{i=1}^{N} P_{i,t} \quad \text{t时的price}$$
$$R_{p,t} = \frac{P_{p,t} - P_{p,t-1}}{P_{p,t-1}} = \sum_{i=1}^{N} \frac{P_{i,t} - P_{i,t-1}}{P_{p,t-1}}$$
$$= \sum_{i=1}^{N} \frac{P_{i,t-1}}{P_{p,t-1}} \cdot \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$
$$= \sum_{i=1}^{N} w_i \cdot R_{i,t}$$

  - 因为x 约等于$\log(x+1)$

$$r_{p,t} \approx \sum_{i=1}^{N} w_i r_{it},$$

1.2 Distributional properties

moments of a random variable

- quantile: $x_p = \inf\{x | F_X(x) \geq p\} = \sup\{x | F_X(x) < p\}$
- why this is important? --> Actually the distribution is not normal, higher probability at the extreme data and less data in the middle.
- relate to long-term return and risk
- skewness: positive and negative --> about positions in risk management

$$S(x) = E\frac{(X-\mu_x)^3}{\sigma_x^3},$$

- kurtosis: fat tail,
  - kurtosis>3, 高峰肥尾，离群值多
  - kurtosis<3，离群值少

$$K(x) = E\frac{(X-\mu_x)^4}{\sigma_x^4}.$$

- t distribution

$$E(X) = 0, \quad v > 1$$
$$\mathrm{Var}(X) = \frac{v}{v-1}, \quad v > 2$$
$$\mathrm{S}(X) = 0, \quad v > 3$$
$$\mathrm{K}(X) = \frac{6}{v-4}, \quad v > 4.$$

- chi-square distribution

$$E(X) = k, \qquad \mathrm{Var}(X) = 2k$$

estimation

- symmetry

1. Test for H_0: symmetry v.s. H_1: asymmetry:

$$S^* = \frac{\hat{S}(x)}{\sqrt{6/T}} \sim N(0,1)$$

if normality holds.

- thick tail

2. Test for

$$H_0 : K(x) = 3(\text{thick tail}) \ v.s. \ H_1 : K(x) \neq 3.$$
$$K^* = \frac{\hat{K}(x) - 3}{\sqrt{24/T}} \sim N(0,1)$$

if normality holds.

- normal distribution

$$JB = (K^*)^2 + (S^*)^2 \sim \chi_2^2,$$

if normality holds, where χ_2^2 denotes a chi-squared distribution with 2 degrees of freedom.

$$H_0 : X \sim N(\mu, \sigma^2) \ v.s. H_1 : X \nsim N(\mu, \sigma^2).$$

Goodness-of-fit Tests

- empirical distribution, better than JB <-- use more information

$$F_n(x) = \frac{1}{n}\sum_{t=1}^{n} I\{r_t \leq x\} = \frac{1}{n}\sum_{i=1}^{n} I\{r_{(i)} \leq x\}$$
$$= \begin{cases} 0 & \text{if } x < r_{(1)} \\ \frac{i}{n} & \text{if } r_{(i)} \leq x < r_{(i+1)},\ i = 1, \cdots, n-1 \\ 1 & \text{if } x \geq r_{(n)}. \end{cases}$$

- methods
  - Kolmogorov-Smirnov (D)
  - Anderson-Darling $(A-sq)$
  - Cramr-von Mises $(W-sq)$