Chapter 1

- TS: a sequence of variable + all depend on time
- FTS: financial time series-->more uncertainty<--changing business and economic environment and the volatility is not directly observed.
- 1.1 Asset Returns
  - one period simple return $R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$
    - gross return $1 + R_t = \frac{P_t}{P_{t-1}}$
  - multi period simple return $R_t(k) = \frac{P_t - P_{t-k}}{P_{t-k}}$
    - gross return：

      $$1+R_t(k) = \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}} \cdots \frac{P_{t-k+1}}{P_{t-k}}$$

      $$= (1+R_t)(1+R_{t-1}) \cdots (1+R_{t-k+1})$$

      $$= \prod_{j=0}^{k-1} (1+R_{t-j})$$
  - log return
    - $r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1}$
    - $r_t(k) = r_t + r_{t-1} + ... + r_{t-k+1}$
  - Portfolio return
    - formula

      $$w_i = \frac{P_{i,t}}{P_{p,t}} \quad P_{p,t} = \sum_{i=1}^{N} P_{i,t} \qquad t\text{时的 price}$$

      $$R_{p,t} = \frac{P_{p,t} - P_{p,t-1}}{P_{p,t-1}} = \sum_{i=1}^{N} \frac{P_{i,t} - P_{i,t-1}}{P_{p,t-1}}$$

      $$= \sum_{i=1}^{N} \frac{P_{i,t-1}}{P_{p,t-1}} \cdot \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

      $$= \sum_{i=1}^{N} w_i \cdot R_{i,t}$$
    - 因为x 约等于$\log(x+1)$

$$r_{p,t} \approx \sum_{i=1}^{N} w_i r_{it},$$

- 1.2 Distributional properties
    - quantile: $x_p = \inf\{x|F_X(x) \geq p\} = \sup\{x|F_X(x) < p\}$
    - moments of a random variable
        - why this is important? --> Actually the distribution is not normal, higher probability at the extreme data and less data in the middle.
        - relate to long-term return and risk
        - skewness: positive and negative --> about positions in risk management

          $$S(x) = E\frac{(X-\mu_x)^3}{\sigma_x^3},$$

        - kurtosis: fat tail,
            - kurtosis>3, 高峰肥尾， 离群值多
            - kurtosis<3， 离群值少

              $$K(x) = E\frac{(X-\mu_x)^4}{\sigma_x^4}.$$

        - t distribution

          $$\begin{aligned} &E(X) = 0, \quad v > 1 \\ &\mathrm{Var}(X) = \frac{v}{v-1}, \quad v > 2 \\ &\mathrm{S}(X) = 0, \quad v > 3 \\ &\mathrm{K}(X) = \frac{6}{v-4}, \quad v > 4. \end{aligned}$$

        - chi-square distribution

$$E(X) = k, \qquad \mathrm{Var}(X) = 2k$$

- estimation
  - symmetry

    1. Test for H_0: symmetry v.s. H_1: asymmetry:
    $$S^* = \frac{\widehat{S}(x)}{\sqrt{6/T}} \sim N(0,1)$$
    if normality holds.
  - thick tail

    2. Test for
    $$H_0 : K(x) = 3(\text{thick tail}) \ v.s. \ H_1 : K(x) \neq 3.$$
    $$K^* = \frac{\widehat{K}(x) - 3}{\sqrt{24/T}} \sim N(0,1)$$
    if normality holds.
  - normal distribution

    $$JB = (K^*)^2 + (S^*)^2 \sim \chi_2^2,$$
    if normality holds, where χ_2^2 denotes a chi-squared distribution with 2 degrees of freedom.
    $$H_0 : X \sim N(\mu, \sigma^2) \ v.s. H_1 : X \nsim N(\mu, \sigma^2).$$
  - Goodness-of-fit Tests
    - empirical distribution, better than JB <-- use more information

      $$\begin{aligned} F_n(x) &= \frac{1}{n}\sum_{t=1}^{n} I\{r_t \leq x\} = \frac{1}{n}\sum_{i=1}^{n} I\{r_{(i)} \leq x\} \\ &= \begin{cases} 0 & \text{if } x < r_{(1)} \\ \frac{i}{n} & \text{if } r_{(i)} \leq x < r_{(i+1)},\ i = 1, \cdots, n-1 \\ 1 & \text{if } x \geq r_{(n)}. \end{cases} \end{aligned}$$
    - methods

Kolmogorov-Smirnov (D)

Anderson-Darling ($A - sq$)

Cramr-von Mises ($W - sq$)