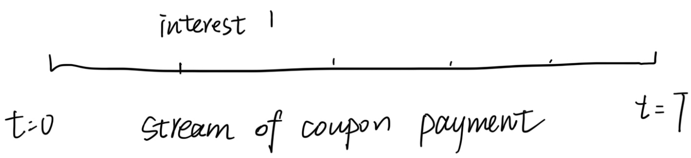
Topic 1 introduction

- 1.1 Basic derivative instruments
  - bond
    - procedure
      - 

        coupon rate $\Leftarrow r_f +$ risk premium
    - Features in bond indenture
      - risk higher, yield higher
    - Types of bond
      - Floating rate bond
      - callable bonds, have an initial call protection
      - put provision
      - convertible bond
        - issuers can raise capital at lower cost
        - bondholders can get potential profit
      - exchangeable bond
        - bond->common stocks of another company
    - Short rate
    - Money market account
      - just a bank account, but is beneficial to investment
      - 指数里的这一堆是增长因子，取倒数为折价因子

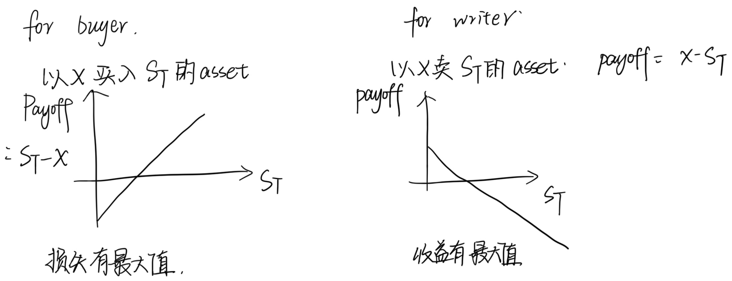
        $$M(T)=M(t)\cdot e^{\int_t^T r(u)\,du}$$

- Price of bond
- no dividend :记得取期望

$$M(T) = M(t)\cdot e^{\int_t^T r(u)du} = \frac{M(t)}{B(t,T)}$$

$$B(t,T) = e^{-\int_t^T r(u)du}$$

- Forward
  - 在t=0时签订的条约，约定在t=T时以约定价格K完成交易，而在t=T时资产的公允价值是ST
  - buyer can hedge the risk：避免太大的价格波动，将risk控制在一定范围内
  - hedge ratio=1, perfect hedging
  - Terminal payoff
    - zero-sum game, 一方亏钱，另一方一定赚钱（和swap进行比较）



  - 中间可能存在的cost
    - cost of carry: include cost of fund and storage cost
    - dividend
    - upfront cost(through borrowing): 属于cost of carry
  - Price
    - method 1->expectation pricing approach:
      - 如果已知未来ST价格的期望，就可以根据收益的期望=0来进行定价，即

$$E\left[(S_T-K)\cdot e^{-\int r(u)du}\right] = 0$$

    - method 2-> replication:
      - Portfolio A: long a forward and a discount bond with par value K

- Portfolio B: one unit of the underlying asset
- equal

> A: long a forward and a discount bond with par = K
>
> B: one unit of the underlying asset.
>
> t=t时 A的价值 $f + K \cdot B(t,T)$
>
> B的价值 S_0
>
> t=T时 A的价值 $S_T = K - K + S_T$ K是执行价格
>
> B的价值 S_T
>
> t=T时价值相等
>
> t=t时 价值相等. $f + K \cdot B(t,T) = S_0$ 由于 $f=0$ ⇐ no arbitrage opportunity
>
> $\therefore K = \frac{S_0}{B(t,T)}$

- if dividend

> A: long a forward and $\underbrace{\text{discount bond with par } K}$ (不收 interest)
>
> B: one unit of the underlying asset. 且借了 D dollar.
>
> ① B is a stock
>
> t=t时 value of A = $f + K \cdot B(t,T)$
>
> value of B = $S_0 - D$
>
> t→T时 A 无
>
> B: get dividends，并用dividend还 D dollar的贷款，在maturity时还清
>
> t=T时 value of A = $S_T - K + K = S_T$
>
> value of B = S_T
>
> ∴ t=t 时价值相等. $f + K \cdot B(t,T) = S_0 - \sum D B(t,t_i)$
>
> $f=0$ $\therefore K = \frac{S_0}{B(t,T)} - \sum \frac{D}{B(t_i,T)}$
>
> 如果 B is an 实物资产，则需要付 cost. 即
>
> change portfolio B → asset & lend C dollar.
>
> ∴ C dollar 收到的利息 恰好能支付 cost of carry.
>
> t=t时 value of A = $f + K \cdot B(t,T)$
>
> value of B = $S_0 + C$
>
> t→T时. A: 0
>
> B: 付 cost of carry. 收 interest. 恰好相抵
>
> t=T时 A: $K + S_T - K = S_T$
>
> B: S_T.
>
> $\therefore f + K \cdot B(t,T) = S_0 + C. \xrightarrow{f=0} K = \frac{S_0}{B(t,T)} + \sum \frac{C_i}{B(t_i,T)}$

- we can treat cost of carry as negative dividend

- method 3->procedure:
  - no dividend:

short a forward, long a stock.

t — T

在t=t时,从银行借S_0,买stock S_0 CF: $+S_0$ $-S_0$

在t-T时内 不产生分红.

在T时刻 需归还bank $\frac{S_0}{B(t,T)}$, 同时执行forward, 以X卖出stock.

$$\therefore \frac{S_0}{B(t,T)} = X = S_0 \cdot e^{rt}.$$

long a forward. short a stock

t — T

在t=t时 借stock 然后卖出, S_0. 将S_0存入bank.

t-T时. 不存在 cash flow

在t=T时. 执行forward, 必须以X买入stock, 且可以从银行获得 $S_0 e^{rt} = S_T$

$$\therefore S_0 e^{rt} = X = S_T$$

- have dividend:

long a forward, short a stock

$t=t$ — $t=T$

在t=t时 借stock卖出获得S_0, 将S_0存入bank.

在t-T内, 要归还stock的原主人 dividend. D_i

在T内. 可以从bank取出 $S_0 \cdot e^{rT}$. 花费X买入stock.

$$\therefore -D_i - X + S_0 \cdot e^{rT} = 0. \quad 即 \quad X = \frac{S_0}{B(t,T)} - \sum \frac{D_i}{B(t_i,T)}$$

若stock是asset,

则在t-T内不用归还divident, 且原主人少付了 cost. 相当于赚了cost

$$\therefore C - X + S_0 \cdot e^{rT} = 0. \quad 即 \quad X = \frac{S_0}{B(t,T)} + \sum \frac{C_i}{B(t_i,T)}$$

short a forward, long a stock

$t=t$ — $t=T$

在t=t时. 借钱S_0买stock $+S_0 - S_0$

在t-T时, 收到dividend, $+D$

在t=T时, 必须以X卖掉stock, 同时还给bank $S_0 \cdot e^{rt}$.

$$\therefore +D + X - S_0 \cdot e^{rt} = 0 \quad 即 \quad X = \frac{S_0}{B(t,T)} - \sum \frac{D_i}{B(t_i,T)}$$

若asset不是stock 而是实物资产, 即收不到dividend, 还要支付cost.

$$\therefore -C + X - S_0 \cdot e^{rt} = 0 \quad 即 \quad X = \frac{S_0}{B(t,T)} + \sum \frac{C_i}{B(t_i,T)}$$

- note: proportional cost of carry

      - u = cost per annum as a proportion of the spot price, 存储成本和当年的现货价格成比例，q是dividend支付的比例
      - 其实也可以说利息和principal成比例，所以只是compound的方式不同而已，也可以连续复利，在指数上面
      - 在这里，u和r可以在同等位置上
      - price:

        $$F = S \cdot e^{(r+u-q)\tau}$$

- Futures
  - 同样是在t=0时签订条约，约定在t=T时以执行价格K完成交易，而在t=T时资产的公允价值是ST，但是与forwards 不同的是，由于forward具有非常大的credit risk, futures将contracts标准化，同时加入了clearing house or exchange，使得credit risk大大降低，同时增加了远期的流动性
  - pay pr receive the change through the margin account in the process
  - on the maturity day, the payment is the spot price on the maturity day
  - Roles of the clearing house and margin account
    - minimize the counterparty risk through margin account
    - provide the platform to unwind (平仓) their position prior to the settlement date
  - Margin requirements
    - initial margin: paid at inception as a deposit for the contract
    - Maintenance margin: minimum level before the investor is required to deposit additional margin
    - Example: A 买了一个futures: 以2.1/kg 买5000kg， margin= 800, maintenance margin=600

      > t=0时，A在银行存了800
      > t=1时，价格2.07， 即A亏了0.03*5000=150， broker会拿走150给B，A的account剩650>600，不用call
      > t=2 时，价格2.05， A又亏了0.02*5000=100，broker拿走100，剩余550<600，需要给A打电话，交800-550=250块钱这个远期才能继续下去，不然这个保证金都不会还给A
  - price:
    - 虽然每天的利率都会改变，但是future和forward的价格一致
    - Fi和Gi表示第i天结束时的远期价格和期货价格，Sn表示到期日的资产价格，每天的常数利率是delta(!!!注意这里是常数利率)

- 证明future和forward定价一致

  第i天期货的收益是 $(G_i - G_{i-1})$，折价到第n天时 $(G_i - G_{i-1})e^{\delta(n-i)}$

  $\therefore$ 所有的收益折价到第n天为 $\sum_{i=1}^{n}(G_i - G_{i-1})\cdot e^{\delta(n-i)}$

  以上是一份期货，如果一共有α份期货，

  $\therefore$ 价值为 $\sum_{i=1}^{n}\alpha_i(G_i - G_{i-1})\cdot e^{\delta(n-i)}$

  portfolio A：一份forward + 第n天par value为 F_0 的一份多头

  portfolio B：到期时拥有 $e^{-\delta(n-i)}$ 份期货多头 + par value $= G_0$ 的债券多头

  $t=t$ 时　value of $A = F_0 \cdot B(t,T)$　value of $B = G_0 \cdot B(t,T)$

  $t=T$ 时　value of $A = F_0 + S_T - F_0 = S_T$

  value of $B = G_0 + \sum_{i=1}^{n}(G_i - G_{i-1})\cdot e^{\delta(n-i)}\cdot e^{-\delta(n-i)}$

  $= G_0 + \sum_{i=1}^{n}(G_i - G_{i-1}) = G_n = S_T$（期货到期日价格 = 当天的spot price）

  $\therefore$ $F_0 \cdot B(t,T) = G_0 \cdot B(t,T)$　$F_0 = G_0$. $\therefore$ 相等.

- 常数利率是futures 和forward价格相等的充分条件，但根本的充要条件是discount process 和terminal value of the underlying asset are uncorrelated.

  $$\text{forward price} = \mathbb{E}_P[S_T] = \frac{S_T}{B(t,T)}$$

  $$\text{future price} = \mathbb{E}_Q[S_T]$$

  $$B(t,T) = \mathbb{E}[e^{-\int r(u)du}]$$

  $$S_t = \mathbb{E}[e^{-\int r(u)du}\cdot S_T]$$

  $$\therefore G_t - F_t = \mathbb{E}_Q[S_T] - \frac{S_t}{B(t,T)}$$

  $$= \frac{\mathbb{E}_Q[S_T]\cdot\mathbb{E}[e^{-\int r(u)du}] - \mathbb{E}[e^{-\int r(u)du}\cdot S_T]}{B(t,T)}$$

  $$= \frac{-\text{cov}(S_T, e^{-\int r(u)du})}{B(t,T)}$$

  当cov=0时 $G_t = F_t$

- future's dynamic strategy-daily settlement
  - 注意，Gt,T 是futures的标的物的value，futures 本身是没有价值的

$t=t$时 付出 $G_{t,T}$ 的cash, 买一个risk-free的债券和 $\frac{1}{B(t,t+1)}$ 份 futures

根据动态调整 future的份数

$t=\tau$时 拥有 $\frac{1}{B(t,t+1)B(t+1,t+2)\dots B(\tau-1,\tau)}$ 份 futures

value of futures $= \frac{1}{B(t,t+1)B(t+1,t+2)\dots B(\tau-1,\tau)} \cdot (G_{\tau,T} - G_{\tau-1,T})$

bonds也要日结

value of bonds $= \frac{G_{\tau-1,T}}{B(t,t+1)B(t+1,t+2)\dots B(\tau-1,\tau)}$

value of cash $= \frac{G_{\tau,T}}{B(t,t+1)B(t+1,t+2)\dots B(\tau-1,\tau)}$ 再将cash投入bond.

$t=\tau+1$时 拥有 $\frac{1}{B(t,t+1)\dots B(\tau-1,\tau)B(\tau,\tau+1)}$ 份 futures.

value of bonds $= \frac{G_{\tau,T}}{B(t,t+1)B(t+1,t+2)\dots B(\tau-1,\tau)} \cdot \frac{1}{B(\tau,\tau+1)}$

日结 value of cash $= \frac{G_{\tau+1,T} - G_{\tau,T} + G_{\tau,T}}{B(t,t+1)\dots B(\tau,\tau+1)} = \frac{G_{\tau+1,T}}{B(t,t+1)\dots B(\tau,\tau+1)}$

$\therefore t=T$时 value of total $= \frac{G_{T,T}}{B(t,t+1)B(t+1,t+2)\dots B(T-1,T)} = \frac{S_T}{B(t,t+1)B(t+1,t+2)\dots B(T-1,T)}$

而 $\frac{1}{B(t,t+1)\tilde{B}(t+1,t+2)\dots\tilde{B}(T-1,T)}$ 份用asset 在 $t=T$ 时的价值 $= \frac{S_T}{B(t,t+1)\tilde{B}(t+1,t+2)\dots\tilde{B}(T-1,T)}$

$\therefore$ 这种 dynamic strategy 可以replicate $\frac{1}{B(t,t+1)\tilde{B}(t+1,t+2)\dots\tilde{B}(T-1,T)}$ 份asset

$\therefore$ asset 在 $t=t$ 时的现货价值 $S_t = G_{t,T}$ (通过 replicate)

$$\therefore G_{t,T} = \mathbb{E}\left[\frac{S_T}{B(t,t+1)\tilde{B}(t+1,t+2)\dots\tilde{B}(T-1,T)} \cdot B(t,t+1)\tilde{B}(t+1,t+2)\dots\tilde{B}(T-1,T)\right]$$
$$= \mathbb{E}[S_T] \quad \text{(discrete time)}$$

推广到 continuous time 也成立

$$V_t = \mathbb{E}\left[V_T\, e^{-\int_t^T r(u)du}\right]$$ V_T是 $\frac{S_T}{B(t,t+1)\tilde{B}(t+1,t+2)\dots\tilde{B}(T-1,T)}$ 的连续形

- Currency forward
  - The underlying asset is the exchange rate X, rd is the constant domestic interest rate, rf is the constant foreign interest rate
  - price
    - process:

      long a forward: short a stock. 1 foreign $= X$ domestic

      $t=t$ 时, 借 S_0 的外币, 卖掉 获得 S_0 的外币, 兑换成本币为 $X \cdot S_0$.

      将 XS_0 存入银行.

      在 $t=T$ 时, 可以从银行获得 $XS_0 e^{\int r_d(u)du}$

      还 $S_0 \cdot e^{\int r_f(u)du}$, 转化成本币为 $FS_0 e^{\int r_f(u)du}$

      $$\therefore XS_0 e^{\int r_d(u)du} = FS_0 e^{\int r_f(u)du}$$

      $$\therefore F = Xe^{(r_d - r_f)T}$$
    - replicate:
      - no dividend

portfolio A: currency forward with delivery price K $\boxed{1 \text{ unit}}$
and a domestic bond of par K.

portfolio B: foreign bond of unit par.

t=t时 value of A: $K \cdot e^{-r_d \tau} + f$ 1 foreign = X domestic

value of B = $(1 \cdot e^{-r_f \tau})X = X \cdot e^{-r_f \tau}$

t=T时 value of A = $K + S_T - K = S_T$ S_T是T时的汇率

value of B = $1 \cdot S_T = S_T$

$\therefore K \cdot e^{-r_d \tau} = X \cdot e^{-r_f \tau}$ $\therefore K = X \cdot e^{(r_d - r_f)\tau}$

- Interest Rate Parity Relation: 用相同货币衡量任意两种货币存款的预期收益率相等的条件，成为利率平价关系
  - 换种说法，两个汇率之间的差额可以由K和X决定
- flexible notional currency forward
  - bang-bang strategy: when the 交易成本和exercise the forward 独立时，最好的策略要不是全执行，要不是全不执行，因为不可能存在在一个时间点赎回60%在另一个时间点全赎回40%最划算的情况 <----不考虑time value

- bond forward
  - no dividend

no dividend:

t Forward maturity = T_F bond maturity = T_B.

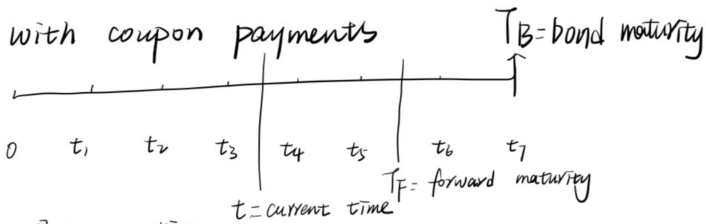
在T_F时执行 forward. 以 F 买一个bond. 而这个bond在T_B时到期.
par value = P

$\therefore NPV = -F \cdot B(t, T_F) + P \cdot B(t, T_B) = 0$

$$\therefore F = \frac{P \cdot B(t, T_B)}{B(t, T_F)} = \frac{P e^{-r(T_B - t)}}{e^{-r(T_F - t)}} = P \cdot e^{-rT_B + rT_F} = P \cdot e^{-r(T_B - T_F)}$$

$$= P \cdot B(T_F, T_B)$$

  - coupon payment



t是 current time.

在 T_F 时交割一个 forward. 以F买一个 bond.

在 T_B 时交割一个 bond. par value = P

$$\therefore MPV = \underset{=0}{-} FB(t,T_F) + C_6B(t,t_6) + C_7B(t,T_B) + PB(t,T_B)$$

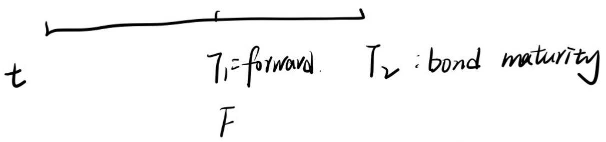
$$\therefore F = \frac{C_6B(t,t_6)+C_7B(t,T_B)+PB(t,T_B)}{B(t,T_F)}$$

而从t时看 forward 价格 $F = \frac{S_0}{B(t,T_F)} - \frac{C_4}{B(t_4,T_F)} - \frac{C_5}{B(t_4,T_F)}$

$$= \frac{S_0 - C_4B(t,t_4) - C_5B(t,t_4)}{B(t,t_F)}$$

$$\therefore S_0 = C_6B(t,t_6) + C_7B(t,T_B) + PB(t,T_B) + C_4B(t,t_4) + C_5B(t,t_5)$$

- Implied forward interest rate
  - 仍然是上面的过程，即TF到TB时间段内的等价利率



$$F(1+R(T_2-T_1)) = P$$

$$F = \frac{RB(t,T_2)}{B(t,T_1)}(1+R(T_2-T_1)) = \cancel{P}$$

$$R = \frac{\frac{B(t,T_1)}{B(t,T_2)} - 1}{T_2-T_1}$$

$$= \frac{1}{T_2-T_1}\left(\frac{B(t,T_1)}{B(t,T_2)} - 1\right)$$

  - Forward rate agreement(FRA)
    - 交换LIBOR和Fixed rate中，fixed rate应该等于implied rate
- Swap
  - Interest rate swap
    - exchange period interest payments
    - can be interpreted as a package of forwards
      - fixed- rate payer 付fixed interest，short fixed rate bond，long floating bond， both the values of the bonds are par value
    - price
      - process:

k = fixed interest rate

L_{i-1} 是 (t_{i-1}, t_i) 内在 t_i 支付的floating rate.

在 t_{i-1} 内约定的

在 (t_{i-1}, t_i) 内.

fixed interest = $N \cdot K \cdot \delta_i$

floating interest = $N \cdot L_{i-1} \cdot \delta_i$

$$\sum \frac{NK\delta_i}{B(t_i, t_n)} = \sum \frac{NL_{i-1}\delta_i}{B(t_i, t_n)}$$

N是notional principle. 不参与swap.

- replication:

Replication of cash flows

receive fixed interest

$KN\delta_i$ 可以看成是packages of forward.

$KN\delta_1$ $KN\delta_2$

必须以$KN\delta_i$ 卖 fixed interest bond (forward)

获得 $\sum KN\delta_i / B(t_i, t_n) \Rightarrow$ portfolio A.

而对floating 的刻画可以用两个fixed 债券replicate

t T_0 T_n

在t时刻. long a T_0-maturity bond par=N

short a T_n-maturity bond par=N.

在t-T_0时刻. 付的fixed 和收到的 fixed 相等 $(NK\delta_i)$

在T_0时. T_0-bond 得到N. 存入deposit 获得 floating rate. $\Rightarrow$ portfolio B.

$\because$ T_n时还N. $N + \sum NL_{i-1}\delta_i / B(t_i, t_n) - N$

$= \sum NL_{i-1}\delta_i / B(t_i, t_n)$

A和B在 T_n时获得的价值相等.

$\therefore$ 在$t=t$时的价值也相等

value of $A = \cancel{N}k\sum \delta_i B(t, T_i)$

value of $B = \cancel{N} \cdot B(t, T_0) - \cancel{N}B(t, T_n)$

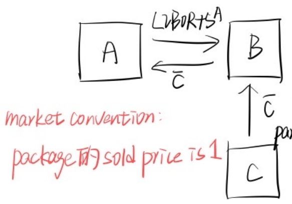
$$\therefore k = \frac{B(t, T_0) - B(t, T_n)}{\sum \delta_i B(t, T_i)}$$

若$n=1$. 而只交换1次interest rate

得到 $k = \dfrac{B(t, T_0) - B(t, T_1)}{(T_1 - T_0)B(t, T_1)}$ 和上式相等

- Asset swap
  - Combination of a defaultable bond and an interest rate swap
  - 可以看成t=0时，花费C(0)买了一个defaultable bond 然后将这个bond同时卖给了另一方，获得了C(0)，同时花费了1来获得floating rate，这个floating rate package 的sold value=1 , 事实上，这个package的value等于bond的面值，（通过调整SA实现）并且它的价值一直是bond的par value（所以t=0时刻的value等于C(0)-1），这个不像之前的swap的初始价值为0，因为之前的swap中，我们通过调整K(fixed rate)来使得initial value为0，但在这时，这里的fixed interest固定，SA取决于bond issuer 的credit risk，均是固定的，这是只能调整swap的初始价值使得fair deal

- price:



	defaultable bond	package	net
$t=0$时	$-\bar{C}(0)$	$\bar{C}(0)-1$	-1
$t=t_i$	$\bar{C}$	$-\bar{C}+L_{i-1}+S^A$	$L_{i-1}+S^A$
$t=t_n$	$1+\bar{C}$	$-C+L_{n-1}+S^A$	$L_{n-1}+S^A+1$

$\therefore$ 由package 得 $\bar{C}(0)-1-\bar{C}A(0)+S(0)A(0)+S^A(0)A(0)=0$

得 $\bar{C}(0)-\bar{C}A(0)+S(0)A(0)+S^A(0)A(0)=1$

调整 $S^A(0)=\dfrac{1+\bar{C}A(0)-\bar{C}(0)-S(0)A(0)}{A(0)}$

$=\dfrac{1}{A(0)}[1-\bar{C}(0)]+\bar{C}-S(0)$

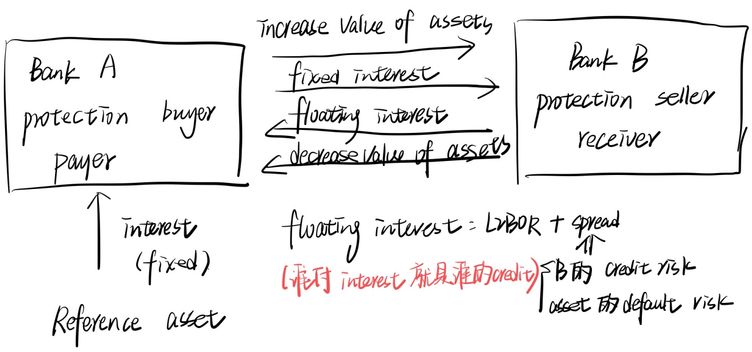
若这是一个 undefaultable bond

$\therefore C(0)-\bar{C}A(0)+S(0)A(0)=1$

再将上述式子 rearrange. $\underbrace{\bar{C}(0)+A(0)S^A(0)}=\underbrace{1+\bar{C}A(0)-S(0)A(0)}_{=C(0)}$

$S^A(0)=\dfrac{C(0)-\bar{C}(0)}{A(0)}$

  - alternative proof : defaultable bond 和 undefaultable bond 在 t=0 时的差价其实就是后面交的利息的差价，考虑到time value，也可以得到这个式子
- Total return swap
  - Exchange total economic performance



  - total return comprises the sum of interest, fees, and any change-in-value payments with respect to the reference asset
  - the receiver can invest the asset but not need to pay up front, improve the leverage, and get off B/S asset, reduce administrative fees
  - the payer can get profits when the asset depreciates
  - The payments received by the receiver
    - 每个阶段的fixed interest
    - increase value of the assets
    - recovery value of the bond (if there were default)
  - The payments made by the receiver
    - 每个阶段需要付floating interest

            - decrease value of the assets
            - The par value of the bond (if there were default)
    - IBM/World Bank
        - USD appreciate ; DM depreciate
        - IBM have DM debts, need to pay DM, because DM depreciates, IBM can get profits, if he can lock in the exchange rate
        - WB have US debts, and need to pay dollars, and he gets money from the DMers, want to lend out DM and borrow DM with low interest rate
        - So they swap, IBM receive DM and use DM pay the DM debts directly to the lenders using current interest -> lock the interest rate
        - WB pay DM to get dollars to pay the US debts, because current exchange rate is high and WB don't want to exchange
- Options
    - 价格的非负性
    - 美式看涨期权价格一定大于内在价值（否则借C+X买option，立即执行，就可以获得无风险利润）
        - 但是欧式看跌期权的价格可能低于内在价值,
    - 提前行使
        - 看涨期权：
            - 根据下面c的公式，当S-X>S-XB(T)-D时就成立，此时c可以小于内在价值且满足c的范围，即D>X-XB(T)
            - 当D不满足这个条件时，C>c>max(S-XB(T)-D,0)>S-X, C的价值一直大于立即执行的价值，所以不会提前行使权力
            - 当D=0时，天然不满足这个条件，不可能行权
        - 看跌期权
            - 同样，对于美式看跌期权，P>p>max(XB(T)+D-S,0)(利用平价关系推出)，当D<X-XB(T),XB(T)+D<X, max(XB(T)+D-S,0)<X-S, 此时，看跌期权的最小值小于它的内在价值，所以有可能可以提前执行
            - 当D足够大时，美式期权也不会提前执行，因为越不执行，asset分红，导致asset价格降低，而看跌期权拥有着会从中获利
            - 当r为0时，B=1，天然不满足D<的条件，所以也不可能行权
    - 美式期权的价格大于等于欧式期权价格
    - 美式看涨期权：
        - 执行价格越低，价格越高
        - 现货价格越低，价格越低
        - 到期时间越长，价格越高
    - 美式看跌期权：

    - 执行价格越低，价格越低
    - 现货价格越低，价格越高
    - 到期时间越长，价格越高
  - 欧式option price 是 strike price 的凸函数，且有线性齐次性
  - 欧式call option 还是asset prices 的凸函数
  - 欧式看涨-看跌平价关系：$p = c + D + XB(T) - S$， 但是欧式看涨期权没有这种关系（因为可以提前行权）
    - 但可以推导出美式看跌期权和看涨期权价格差的范围
    - $D=0$: $S-X < C-P < S-XB(T)$
    - $D>0$, $S-X-D < C-P < S-XB(T)$
  - 看涨期权的价值：
    - 没有分红
      - 美式看涨期权：$S >= C >= \max(S-X,0)$
      - 欧式看涨期权：$S >= c >= \max(S-XB(T),0)$
    - 有分红：
      - 美式看涨期权：$S >= C >= \max(S-X-D,0)$
      - 欧式看涨期权：$S >= c >= \max(S-XB(T)-D,0)$
  - 看跌期权的价值
    - $P>=p>=\max(XB(T)+D-S,0)$