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# Notes

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## 1 Tpyst symbols

Tpyst symbols.

## 2 Random effects model for election polls

Andrew Gelman's recent blog post, [Polls & Betting Odds & Nonsampling Errors & Win Probabilities & Vote Margins](#), discusses how to estimate winning probabilities based on polling data. A ChatGPT summary is available here: [chatgpt-history](#).

He presents three examples: the New Jersey governor race, the Virginia governor race, and the New York City mayoral race. Individual polls are often unreliable, so systematic differences between polls must be considered. A simple random-effects model can capture this phenomenon:

$$Y_{i,t} = \theta + \eta_{i,t} + \varepsilon_i$$

Here,  $Y_{i,t}$  is the  $t$ -th poll for the  $i$ -th media,  $\theta$  is the true underlying support level,  $\varepsilon_i$  represents the systematic error for poll  $i$ , and  $\eta_{i,t}$  represents the sampling error of the poll.

When focusing on a single media's polls (fixing  $i = i_0$ ), Gelman uses historical information to estimate  $\varepsilon_t$ , assuming  $\varepsilon_t \sim \mathcal{N}(0, 2\%)$ . He then uses polling data of media  $i_0$  to estimate the sample variance  $\sigma_{i_0}^2$  of  $\eta_{i,i_0}$ .

Let  $\bar{Y}_{i_0} = \frac{1}{n} \sum_{i=1}^n Y_{i,i_0}$ . Under his Bayesian viewpoint, the posterior distribution of  $\theta$  is approximated by:

$$\theta | Y \sim \mathcal{N}\left(\bar{Y}_{i_0}, \sqrt{\sigma_{i_0}^2 + 2\%}\right)$$

The winning probability can then be computed as:

$$\Pr(\theta > 0.5 | Y).$$

## 3 Writing

### 3.1 Good title: The unreasonable effectiveness of XX in/of YY

Good title from [The unreasonable effectiveness of mathematics in the natural sciences](#) by Eugene Wigner in 1960 [1].

Richard Hamming in computer science, "The Unreasonable Effectiveness of Mathematics".

Arthur Lesk in molecular biology, "The Unreasonable Effectiveness of Mathematics in Molecular Biology"

Peter Norvig in artificial intelligence, "The Unreasonable Effectiveness of Data"

Vela Velupillai in economics, "The Unreasonable Ineffectiveness of Mathematics in Economics"

Terrence Joseph Sejnowski in Artificial Intelligence: The Unreasonable Effectiveness of Deep Learning in Artificial Intelligence".

### 3.2 Good title: Rebel With a Cause

From the famous movie [Rebel Without a Cause](#).

The special issue Volume 8, Issue 2, 2022 Issue of *Observational Studies* titled [Rebel With a Cause](#)

### 3.3 Fun example

#### 3.3.1 on overparameterized models

Form comment in [Impossible statistical problems](#) of Andrew Gelman by Phil, November 14, 2024.

"I'm imagining a political science student coming in for statistical advice:  
 Student: I'm trying to predict the Democratic percentage of the two-party vote in U.S. Presidential elections, six months before Election Day. I want to use just the past ten elections because I think the political landscape was too different before that.  
 Statistician: Sounds interesting. What predictive variables do you have?  
 Student: I've got the Democratic share in the last election, and the change in unemployment rate over the past year and the past three years, and the inflation rate over the past year and the past three years, and the change in median income over the past year and past three years.  
 Statistician: That's a lot of predictors for not many elections, we are going to have some issues, but maybe we can use lasso or a regularization scheme or something. Let's get started.  
 Student: I also own an almanac.  
 Statistician: Oh. Sorry, I can't help you, your problem is impossible."

10 data points and 7 predictors, there are something to do, with a almanac, 1000+ predictors, the problem is impossible since the model is overparameterized and can not give any prediction power for future.

Thus, in tiny sample point, give too much useless predictors may indeed pollute the data and make the problem impossible.

**Question 3.1.** In dense high dimensional model, and number of samples  $n$  is not so big,

$$\mathbb{E}[Y | X] = \rho(X^\top \beta), \beta_j \sim O\left(\frac{1}{p}\right), j = 1, \dots, p, \frac{p}{n} \rightarrow (0, \infty)$$

If the model is misspecified, it just like the above example, may too many useless predictors and give a useless prediction. "Is there an example of such dense high dimensional model?"

## 4 On the undistinguishable or identification of statistical models

Based on talk with Ruiqi Zhang, Lin Liu and Chatgpt.

The undistinguishable means you can not distinguish two models from data. Which will corresponds to 3 cases.

One and two are both in the modeling stage. When you consider one model  $P_\theta$ , this corresponds to non-identifiable model. When you consider two models, this corresponds to two models sharing some common distributions.

The third case is in the hypothesis testing stage, two models can not be distinguished by data since they are too close.

In a word, two models  $\mathcal{P}_1$  and  $\mathcal{P}_2$  as two distribution classes are undistinguishable if  $\mathcal{P}_1 \cap \mathcal{P}_2 \neq \emptyset$  or they are too close under some metric.

### 4.1 On latent structure models

The potential outcome model is an example of latent structure model. The observed random variable is determined by some unobservable/latent variable in this class.

**Definition 4.1.** *The observed random variable  $X$  is determined by a high dimensional latent variable  $Z$  by a map  $X = f(Z)$ .*

**Example 4.2.** The observed random variable is  $(A, Y)$  determined by three latent variable  $(A, Y(0), Y(1))$ ,  $A \in \{0, 1\}$ ,  $Y = AY(1) + (1 - A)Y(0)$ , consider two submodels:

- $\mathcal{P}_1$ :  $Y(1) - Y(0) = 0$ ,  $Y(1) \perp A$ ,  $Y(0) \perp A$
- $\mathcal{P}_2$ :  $Y(1) - Y(0) = Z \neq 0$ , but  $Y(1) \stackrel{d}{=} Y(0)$ ,  $Y(1) \perp A$ ,  $Y(0) \perp A$ .

The model 2 is not empty, take

$$Y(0), \varepsilon \sim \mathcal{N}(0, 1), Y(0) \perp \varepsilon, Z = -\frac{1}{2}Y(0) + \frac{\sqrt{3}}{2}\varepsilon$$

then  $Y(1) \sim \mathcal{N}(0, 1)$  and  $Y(1) \stackrel{d}{=} Y(0)$ .

On the observed data level, we can not distinguish these two models since they both have the same conditional distribution of  $Y|A$ , therefore they are undistinguishable in modeling stage.

This example is the reason why the sharp null hypothesis can not be tested in randomized experiment, and also the joint distribution of  $(Y(0), Y(1))$  is not identifiable.

[2] talk about the identification of joint distribution of potential outcomes under some assumptions.

## 5 On the cluster analysis

## 6 Causal Inference

### 6.1 Causal Example

#### 6.1.1 Yule–Simpson’s Paradox

##### 6.1.1.1 Healty worker effect

##### 6.1.2 Smoke cause lung cancer

##### 6.1.2.1 Mediation analysis

##### 6.1.2.2 Sensitivity analysis

### 6.2 Identification

#### 6.2.1 G-formula

#### 6.2.2 Joint distribution of potential outcomes

[2] consider a case with multiple randomized controlled trials(RCTs), where data are  $(G, A, Y)$ ,  $G$  is the indicator of RCTs,  $A$  is the treatment,  $Y$  is the outcome.

Under consistency, positivity, and exchangeability, Adding one assumption called “transportability”:

$$Y(1) \perp G \mid Y(0)$$

We can then identify the conditional distribution  $Y(1) \mid Y(0)$ .

$$\begin{aligned} \Pr(Y(1) = b \mid G = g) &= \sum_a \Pr(Y(1) = b, Y(0) = a \mid G = g) \Pr(Y(0) = a \mid G = g) \\ &= \sum_a \Pr(Y(1) = b \mid Y(0) = a) \Pr(Y(0) = a \mid G = g) \end{aligned}$$

Here  $\Pr(Y(1) = b \mid G = g)$  and  $\Pr(Y(0) = a \mid G = g)$  can be identified from data by the consistency, positivity and unconfounder assumption, using them to solve the above equation system, we can identify  $\Pr(Y(1) = b \mid Y(0) = a)$ .

### **6.2.3 Instrumental variable**

[3]

- data are  $(X, A, Z, Y)$  only assume consistency, positivity, unconfounder, exclusion, no monotonicity, provide a bound estimation on ATE.
- The identification assumption of IV:

“Critically, under the four assumptions introduced in the previous section, the ATE is not point identified. Analysts typically take one of two approaches for point identification. The first approach invokes some type of homogeneity assumptions and places various restrictions on how the effects of A and Z vary from unit to unit in the study population. See Hernan and Robins (2019) and Wang and Tchetgen Tchetgen (2018) for prominent examples. However, homogeneity assumptions are often implausible or difficult to verify in specific applications. The second approach invokes an assumption known as monotonicity, which has the following form:  $A(z = 1) \geq A(z = 0)$ , i.e., if  $A(z = 0) = 1$  then  $A(z = 1) = 1$  (Imbens and Angrist, 1994). Under monotonicity, the target estimand is no longer the ATE, but instead is the local average treatment effect (LATE):”

- The lower bound and upper bound is not a differentiable functional, thus an assumption is invoked to make the bound functional differentiable and thus have inference function to faster convergence rate.

## **6.3 The equivalence between DAG and potential outcome framework**

### **6.3.1 The equivalence between nonparametric structural equation model(NPSEM) and potential outcome framework**

### **6.3.2 The equivalence between SWIG and FFRCISTG**

## **6.4 ADMG : the inequality constraints**

### **6.4.1 Bell inequality**

## **7 Semiparametric theory**

### **7.1 Parametric theory**

#### **7.1.1 Efficiency**

##### **7.1.1.1 Convolution theorem**

### **7.2 nonparametric theory**

### **7.3 Regularity**

### **7.4 Influence function**

#### **7.4.1 Numerical calculation of influence function**

[4], [5]

#### **7.4.2 Von mises representation**

#### **7.4.3 Tangent space**

S8 in [6]

Formulation in [7]

#### 7.4.4 Higher order influence function

## 8 U statistics

### 8.1 Computation

#### 8.1.1 Motif counts

- For GPU:
  - Gunrock [8]
  - Cugraph [9]
- For CPU:
  - Peregrine [10]
  - Automine [11]

## 9 Applications

### 9.1 Umbrella review

See [Belief in the law of small numbers as a way to understand the replication crisis and silly researchers who continue to cite discredited behavioral research](#) by Andrew Gelman, but I just see the paper involved a title “umbrella review” [12], so I search more about umbrella review.

Umbrella review consider the evidence form multiple meta-analysis and system review studies on the same topic.

using [13](10K+ citation!) to assess the quality of included meta-analysis/system review studies, but they did not provide a way to aggregate the quality scores and just give a subjective criteria, the overall confidence is rating as “high”, “moderate”, “low”, “critically low”. Produce an conclusion such that “In the 60 meta-analyses/systematic reviews included on the treatment for type 2 diabetes, treatments A have 5 strong evidence to support its effectiveness, while treatment B has only 1 moderate evidence to support its effectiveness...”

## 10 multiple testing

### 10.1 Varibale selection

A talk given by Zhong Wei: [A unified stability approach to false discovery rate control](#)

Knockoff is a randomized method for variable selection controlling false discovery rate(FDR) [14]. The key idea is to construct knockoff variables  $\tilde{X} \in R^p$  that mimic the correlation structure of the original variables  $X \in R^p$ , such that for any subset  $S \subset \{1, \dots, p\}$ , swapping the variables in  $S$  with their knockoffs does not change the joint distribution of  $(X, \tilde{X})$ , while also ensuring that the knockoff variables are conditionally independent of the response variable  $Y$  given the original variables  $X$ .

randomized method have a problem in reproduciblity, e-value developed by [15] is a way to solve this problem, the e-value is a function of data that has an expected value at most 1 under the null hypothesis, thus can be used to control type I error in multiple testing, it can combine evidence from multiple independent tests easily, by running the knockoff procedure multiple times and averaging the resulting e-values for each variable, we can obtain a more stable measure of evidence against the null hypothesis for each variable, that's the work in [16].

Zhong Wei proposed a general framework of stability for FDR control, which includes derandomized knockoff as a special case, and provide theoretical guarantee for FDR control under this framework.

Their another work is consider the inference after variable selection, using the same sample for variable selection and inference will lead to selection bias [17].

**Question 10.1.** A question is deose there have some minimax optimal method or other criteria for variable selection controlling FDR? e-value BH method or p-value BH method? or here derandomized knockoff BH method? Which is best?

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“For fundamental contributions to statistical sparsity and selective inference in high-dimensional problems, for the creative and novel knockoff filter to cope with correlated coefficients, for contributions to compressed sensing, the jackknife, and conformal predictive inference; for the encouragement and training of graduate and undergraduate students.”

e-valuede 的王若度 (U of Waterloo, Chair profess) 曾经是星际争霸职业选手。

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