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Linear feature detection and enhancement in noisy images via the Radon transform

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Abstract: A new approach to the problem of detecting and enhancing linear features in noisy digital images is described. The method, which is based on the Radon transform, can be efficiently implemented on a general-purpose digital computer.

Key words: Linear feature detection, linear feature enhancement, Radon transform.

1. Introduction

In recent years many algorithms for detecting linear features in digital images have been suggested. Unfortunately these techniques often produce poor results when applied to noisy images and this characteristic limits their usefulness in many application areas. In remote sensing, for example, images generated by Synthetic Aperture Radars (SARs) suffer from an inherent type of noise, known as 'speckle', which can only be reduced at the expense of image resolution. SAR image analysts have therefore been motivated to investigate image processing algorithms which are robust when applied to noisy images. In particular, research has been concentrated on the search for a noise-resistant line detection technique.

This paper describes an approach to the problem of linear feature detection which is based on the Radon transform. The Radon transform of a function $f(x,y)$ on two-dimensional Euclidean space is defined (Rosenfeld, 1982, p. 355) by

$$P_\theta(\varrho) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(\varrho - x\cos\theta - y\sin\theta)\,\mathrm{d}x\,\mathrm{d}y$$

where $\delta(r)$ is the Dirac delta function which is zero except when $r=0$. The presence of the term $\delta(\varrho - x\cos\theta - y\sin\theta)$ in the definition of the Radon transform forces the integration of $f(x,y)$ along the line

$$\varrho - x\cos\theta - y\sin\theta = 0.$$

Consequently if $f(x,y)$ is a two-dimensional image intensity function, computation of its Radon transform yields the projections across the image at varying orientations θ and offsets ϱ (relative to a parallel line passing through the image centre). Bright (dark) lines in an image are therefore mapped by the Radon transform to bright (dark) points in feature space (Figure 1), enabling the values of ϱ and θ which parameterise each line to be easily identified by a straightforward thresholding operation in feature space.

The Radon transform approach to linear feature detection is described in this paper and illustrated by application to a noisy SAR image. In applications where feature detection is the only requirement, mapping of an image from image space to feature space via the Radon transform, with subsequent processing in feature space, is sufficient. In some situations, however, it may be desirable to generate a modified image in which the linear features are accentuated. This can be achieved by applying an 'enhancement operator' in feature space and then mapping the result from feature

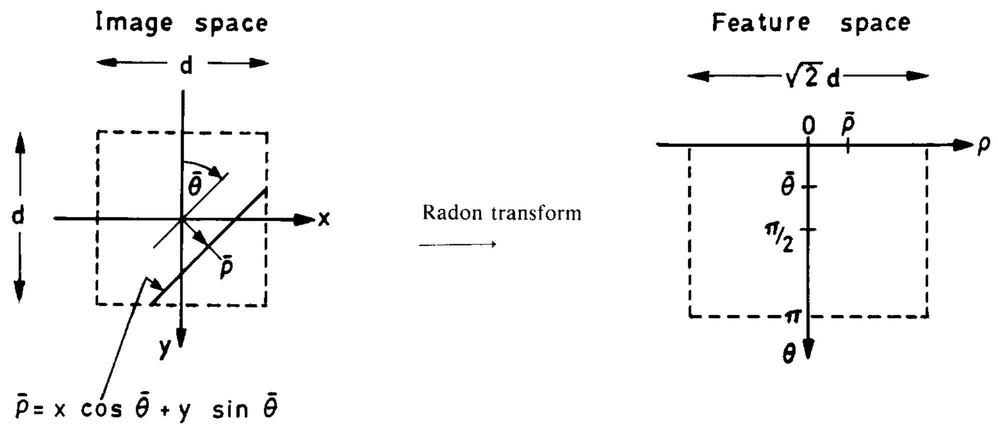

Figure 1. Image space (left) and feature space (right).

space back to the original image space. The invertibility property of the Radon transform (see Barrett (1984)) ensures that this mapping is well-defined; furthermore, the Filtered-Backprojection algorithm (Rosenfeld, 1982, p. 369) which has previously been used in applications such as computerised X-ray tomography and radio astronomy to reconstruct images from their projections, can be used to compute the Inverse Radon transform. The linear feature enhancement algorithm is described and the process illustrated in Section 3 of this paper.

2. Linear feature detection

The Radon transform defined on two-dimensional Euclidean space has been shown by Deans (1981) to have the same major properties as the Hough transform used for finding line segments in digital pictures. Both transforms essentially involve integration of an image intensity function along a set of lines and are therefore well suited for application to noisy images. This is because intensity fluctuations due to noise tend to be cancelled out by the process of integration. One consequence of this tendency to attenuate noise is that the signal-to-noise ratio of a Radon or Hough transform of a noisy image containing a linear feature may be expected to be greater than that of the image itself. Thus better results should be obtainable by carrying out the line detection task in feature space rather than in the original image space.

Duda and Hart (1972) have described how the Hough transform may be used to detect lines and curves in pictures. The technique generally used to compute the Hough transform used for line detection involves considering each point (x, y) in an image in turn, and incrementing a set of accumulators by an amount equal to the intensity at (x, y). The number of accumulators that have to be incremented equals the number of angles for which projection data is required. The computational efficiency of the Hough transform technique therefore falls sharply as the image dimensions and/or the number of angles increases.

A major advantage of the Radon transform technique over the conceptually similar Hough transform approach is the increase in computational efficiency which it affords. This efficiency is achieved by use of an algorithm which is based on the Fourier Slice Theorem. The Fourier Slice Theorem states that if $F(u, v)$ is the Fourier transform of a two-dimensional function $f(x, y)$, $P_\theta(\varrho)$ is a projection at an angle θ across $f(x, y)$, and $S_\theta(\omega)$ is the Fourier transform of $P_\theta(\varrho)$, then

$$S_\theta(\omega) = F(\omega\cos\theta, \omega\sin\theta).$$

In order to compute a Radon transform it is therefore necessary to:

(i) Compute the two-dimensional Fourier transform $F(u, v)$ of the image $f(x, y)$.

(ii) Interpolate the two-dimensional Fourier transform to obtain a set of functions $\{S_\theta(\omega)\}$, each defined in the frequency domain along a radial line orientated at an angle θ. Several orientations in the range 0 to 180 degrees should be considered.

(iii) Compute the Inverse Fourier transform of each function $S_\theta(\omega)$. The result is a set of projection functions $P_\theta(\varrho)$ which together constitute the Radon transform of the image.

The algorithm's efficiency is due to the fact that the transformation from image space to feature space is made via the frequency domain. As a result, most of the computation can be carried out by repeated application of an efficient Fast Fourier transform routine.

The Radon transform technique for linear feature detection has been applied to the Synthetic Aperture Radar (SAR) image shown in Figure 2. This image provides a good example of the 'speckle' noise which is inherent in a coherent imaging system such as a SAR. The dimensions of the image are 512×512 pixels (i.e. picture elements), corresponding to a 1.536 km $\times$ 1.536 km area of ocean. Thus each pixel, for which a grey level in the range 0 (black) to 255 (white) is stored, represents the signal received from a 3 m $\times$ 3 m square area.

A fairly broad, dark, horizontal line is visible in the lower part of the image shown in Figure 2. Close examination also reveals a slightly thinner dark line extending from the top-left to the bottom-right of the image. The latter feature is one arm of the characteristic V-shaped wake pattern which is generated by a ship, while the former is due to the turbulent water directly behind the ship.

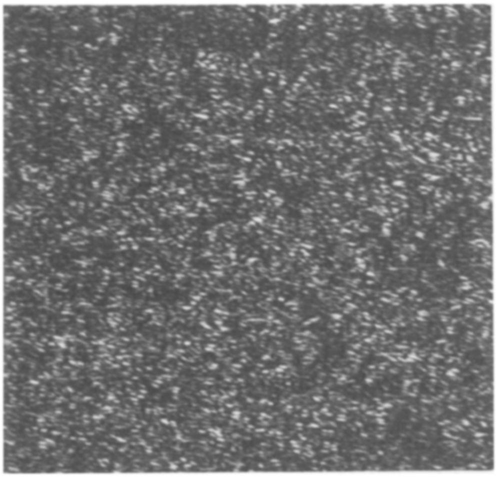

Figure 2. A synthetic aperture radar image of a ship's wake (512×512 pixels).

The Radon transform of a completely uniform, finite sized image is given simply by the length of the projections across the image, since the grey level of the area beyond the image is taken to be 0 (i.e. black). Consequently a characteristic background pattern is generated whenever the Radon transform of a finite image is computed, and this must be eliminated in order to discern the feature space peaks and troughs which are due to linear features. One way in which this can be achieved is to subtract the mean intensity of an image from each pixel value before computing the Radon transform. Figure 3 shows the Radon transform of the SAR image in Figure 2 after this has been done. Note that the feature space axes (ϱ and θ) for this figure are as shown in Figure 1. The angular (θ) and offset (ϱ) sampling intervals are respectively 0.25 degree and $\sqrt{2}\,\Delta x$, where Δx is the sampling interval for the SAR image.

The Radon transform contains two dark patches whose peaks are located in feature space at the points corresponding to parameter values

$$\varrho = 198.0 \times \Delta x = 594 \text{ m}, \quad \theta = 89 \text{ degrees},$$

and

$$\varrho = -4.2 \times \Delta x = -12.6 \text{ m}, \quad \theta = 143 \text{ degrees},$$

respectively. Thus the Radon transform technique succeeds in detecting both lines despite the fact that they appear relatively indistinct against the noisy background. This may be attributed to the

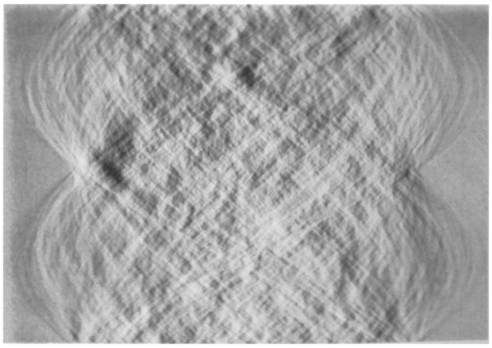

Figure 3. The Radon transform of the SAR image in Figure 2 (512 pixels from left to right, 720 pixels from top to bottom).

tendency of the Radon transform to cancel out intensity fluctuations due to noise by integrating along lines across the image, and thus to increase the signal-to-noise ratio.

The Radon transform technique may be expected to generate prominent and easily detectable peaks in feature space when it is applied to images which contain linear features extending right across the image. The information provided by the Radon transform includes line orientation (θ) and offset (ϱ). In addition, the width in the ϱ direction of a peak in a Radon transform provides an estimate of the mean width of the corresponding linear feature.

One deficiency of the Radon transform technique for line detection is that it cannot be used to detect linear features of dimensions much less than those of the image itself. This is due to the fact that short lines may not generate peaks or troughs in the Radon transform despite being very bright or dark, since less prominent but longer lines may produce Radon transform values of equal magnitude. Even in cases where a small local maximum (minimum) is generated, it is difficult to determine whether this is due to a very bright (dark) short line or a much longer but less prominent line. A second deficiency of the Radon transform technique for line detection is its inability to provide information on the positions of the end-points of detected lines, or on line length. Further processing is necessary after a peak has been identified in order to extract this information.

3. Linear feature enhancement

Use of the Radon transform for line detection, combined with the invertibility property of the Radon transform operator, leads naturally to a linear feature enhancement technique in which an enhancement operator is applied in feature space to the Radon transform of an image. The basis of the method is as follows. First, the Radon transform (r) of an image (i) is computed, i.e.

$$r = Ri$$

where R is the Radon transform operator. An enhancement operator E which accentuates the peaks and troughs in the Radon transform is then applied in feature space to give a modified Radon transform (r'), i.e.

$$r' = Er = ERi.$$

An image i' with Radon transform r' exists as a consequence of the invertibility property of the Radon transform operator. Application of the Inverse Radon transform operator, R^{-1}, will therefore yield the image i', i.e.

$$i' = R^{-1}r' = R^{-1}ERi.$$

Any bright (dark) linear features in the original image should appear in the filtered image i' with increased (decreased) average intensity, provided the enhancement operator is suitably chosen. Suitable enhancement operators are those which increase and decrease the magnitudes of the peaks and troughs in a Radon transform by significant amounts while having less effect on intermediate values. Operators which square and cube the difference between each Radon transform value and the mean may be appropriate. The main problem in using either of these operators is, as may be expected, their non-linearity, which hinders analytic investigation of their effects when used to enhance an image.

The linear feature enhancement technique can be implemented using the algorithm described in Section 2 to compute the Radon transform and the Filtered-Backprojection algorithm for parallel-projection data (Rosenfeld, 1982, p. 369) to compute the Inverse Radon transform. The latter algorithm has previously been used in a number of diverse applications, including computerised X-ray tomography and radio astronomy, to reconstruct images from their projections. The algorithm is based on the following equations which enable an image $f(x, y)$ to be expressed in terms of its Radon transform $P_\theta(\varrho)$:

$$f(x, y) = \int_0^\pi Q_\theta(t)\,\mathrm{d}\theta,$$

where

$$Q_\theta(t) = \int_{-\infty}^{\infty} S_\theta(\omega)|\omega|\exp[j\omega t]\,\mathrm{d}\omega,$$

$$S_\theta(\omega) = \int_{-\infty}^{\infty} P_\theta(\varrho)\exp[-j\omega\varrho]\,\mathrm{d}\varrho,$$

and

$$t = x \cos \theta + y \sin \theta.$$

The principal stages of the Filtered-Backprojection algorithm may therefore be summarised as follows:

(i) Compute the Fourier transforms, $S_\theta(\omega)$, of each projection function $P_\theta(\varrho)$.

(ii) Filter the results by multiplying by $|\omega|$ in the frequency domain.

(iii) Compute the Inverse Fourier transforms, $Q_\theta(t)$, of each product $S_\theta(\omega)|\omega|$.

(iv) 'Backproject' the functions $Q_\theta(t)$ over the image plane to form the image $f(x, y)$. This process involves computing the contribution made by each $Q_\theta(t)$ to the reconstructed image. Since each function $Q_\theta(t)$ makes the same contribution to the image at all points (x, y) on the line $t = x \cos \theta + y \sin \theta$, the image is effectively reconstructed by superimposing a large number of lines.

The Filtered-Backprojection algorithm generally produces extremely good reconstructions. For example, very small-scale features in the background speckle pattern are reconstructed when the algorithm is applied to the (unenhanced) Radon transform of a SAR image.

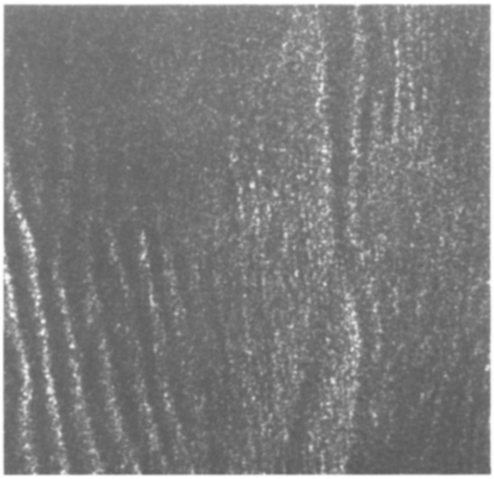

Figure 4. A synthetic aperture radar image of ocean waves (512×512 pixels).

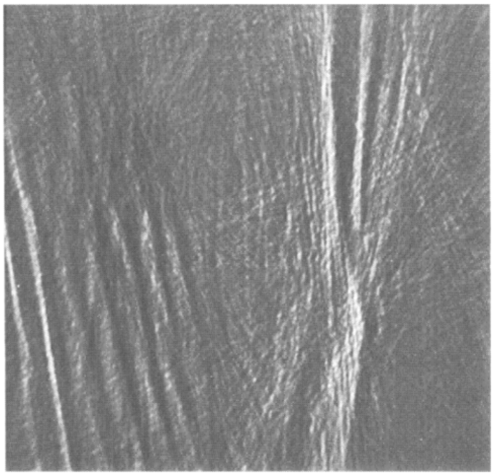

Figure 5. The result of applying the linear feature enhancement technique to the SAR image in Figure 4 (512×512 pixels).

The 512×512 pixel SAR image in Figure 4 shows a sea-surface pattern generated by internal ocean waves. The Radon transform approach to linear feature enhancement has been applied to this image using an enhancement operator E_2 which squares the difference between each Radon transform value and the mean. Figure 5 illustrates the result of the process. It can be seen that the linear features appear far more prominent against the background and that the speckle noise is significantly reduced.

The filtered image shown in Figure 5 provides an illustration of the 'artifact problem' associated with the Radon transform approach to linear feature enhancement. Although the linear features in the original image are enhanced, it can be seen that the filtered image contains some lines which are not present in the original. Studies have shown that these artifacts can be reduced to some extent by increasing the number of angular samples used in the computation of the Radon transform. A sampling interval of 0.25 degree was used in this example, so that the filtered image was reconstructed from a 720×512 Radon transform. Increasing the number of angular samples does not remove all the linear artifacts, however. This is because the Filtered-Backprojection algorithm constructs an image by superposition of a set of lines. Therefore, when the enhancement operator is applied to increase the magnitude of a peak or trough in the Radon transform, the algorithm generates a corresponding increase or decrease in the intensities of some of the lines which are superimposed during the reconstruction process. This results in the type of artifacts exhibited in Figure 5.

The artifact problem may be reduced or worsened by changing the enhancement operator. The operator E_3, for example, which cubes the difference between each Radon transform value and the mean, results in reconstructions in which linear features arc more highly accentuated but in which the artifacts are similarly intensified. In contrast, an operator which reduced the artifact problem may be expected to be less effective at enhancing those features of interest.

4. Concluding remarks

The Radon transform approach to line detection is an effective technique provided the linear features of interest are of dimensions approaching or exceeding those of the image. Qualitative assessments of the results of applying the technique to several SAR images have indicated that Radon transformation causes an increase in signal-to-noise ratio and thus eases the task of detecting lines against noisy backgrounds. It is hoped that further work will enable this improvement to be quantified. The method yields a pair of values (ϱ and θ) which parameterise each detected line, plus an estimate of mean line width. It fails, however, to provide an indication of line length or end-point positions, and cannot be relied upon to detect linear features of short extent.

The technique suggested for linear feature enhancement has produced some good results but requires further study. In particular, the performance of various enhancement operators needs to be investigated. The aim is to identify an 'optimum' operator which is capable of accentuating linear features without generating artifacts in the reconstructed image.

A significant advantage of both techniques is that they can be efficiently implemented on a general-purpose digital computer. In addition, the algorithms are well suited for implementation on a machine with parallel processing capability, and thus further improvements in computational efficiency should be achievable.

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