No.

Date . .

target : $L(x,c,q,\theta) = E_{z\sim q(z|x,c;\phi)} \log P(x|z,c;\theta) - KL(q(z|x,c;\phi) \| P(z|c))$

EM algo : $\log P(x;\theta) = \log P(x,z;\theta) - \log P(z|x;\theta)$

consider c $\Rightarrow \log P(x|c;\theta) = \log P(x,z|c;\theta) - \log P(z|x,c;\theta) = \log \frac{P(x,z|c;\theta)}{P(z|x,c;\theta)}$

introduce an arbitrary distribution $q(z|x,c;\phi)$ on both side and integrate over z

$\Rightarrow \int q(z|x,c;\phi) \log P(x|c;\theta) dz = \int q(z|x,c;\phi) \frac{P(x,z|c;\theta)}{P(z|x,c;\theta)} dz$

$= \int q(z|x,c;\phi) \log\left(\frac{P(x,z|c;\theta)}{q(z|x,c;\phi)} \times \frac{q(z|x,c;\phi)}{P(z|x,c;\theta)}\right) dz$

$= \int q(z|x,c;\phi) \log\left(\frac{P(x,z|c;\theta)}{q(z|x,c;\phi)}\right) dz + \int q(z|x,c;\phi) \log\left(\frac{q(z|x,c;\phi)}{P(z|x,c;\theta)}\right) dz$

$= L(x,c,q,\theta) + \underbrace{KL(q(z|x,c;\theta) \| P(z|x,c;\theta))}_{\geq 0} \geq L(x,c,q,\theta)$

$\Rightarrow$ lower bound

$L(x,c,q,\theta) = \int q(z|x,c;\phi) \log\left(\frac{P(x,z|c;\theta)}{q(z|x,c;\phi)}\right) dz$

$= \int q(z|x,c;\phi) \log\left(\frac{P(x|z,c;\theta) P(z|c;\theta)}{q(z|x,c;\phi)}\right) dz$

$= \int q(z|x,c;\phi) \log P(x|z,c;\theta) dz + \int q(z|x,c;\phi) \log\left(\frac{P(z|c;\theta)}{q(z|x,c;\phi)}\right) dz$

$= E_{z\sim q(z|x,c;\phi)} \log P(x|z,c;\theta) - KL(q(z|x,c;\phi) \| P(z|c))$ #