AARON KAW

# ACOUSTIC PROPAGATION MODELLING

# Part I

# Helmholtz Equation

Taken from Computational Ocean Acoustics[1].

[1] Jensen, F. B., Kuperman, W. A., Porter, M. B., and Schmidt, H. (2011). *Computational ocean acoustics*. Springer Science & Business Media

$$
\begin{aligned}
O(\omega^2): &\qquad |\nabla\tau|^2 = \frac{1}{c^2(\vec{x})} \\
O(\omega^1): &\qquad 2\nabla\tau\cdot\nabla A_0 + \left(\nabla^2\tau\right)A_0 = 0 \\
O(\omega^{1-j}): &\qquad 2\nabla\tau\cdot\nabla A_j + \left(\nabla^2\tau\right)A_j = -\nabla^2 A_{j-1}
\end{aligned}
$$

# *Eikonal Equation*

The eikonal equation

$$|\nabla\tau|^2 = \frac{1}{c^2(\vec{x})}$$

is a first-order nonlinear PDE for modelling the path taken by a ray.

## *First-Order System*

In cylindrical coordinates,

$$\frac{dr}{ds} = c\xi(s) \qquad \frac{d\xi}{ds} = \frac{-1}{c(r,z)^2}\frac{\partial c}{\partial r}$$

$$\frac{dz}{ds} = c\zeta(s) \qquad \frac{d\zeta}{ds} = \frac{-1}{c(r,z)^2}\frac{\partial c}{\partial z}$$

$$\frac{d\tau}{ds} = \frac{1}{c(r,z)}$$

with initial conditions

$$r = r_0 \qquad \xi = \frac{\cos(\theta_0)}{c(r_0,z_0)}$$

$$z = z_0 \qquad \zeta = \frac{\sin(\theta_0)}{c(r_0,z_0)}$$

$$\tau = 0$$

and boundary conditions defined as reflection off the bathymetry $z_{\text{bty}}(r)$ and altimetry $z_{\text{ati}}(r)$.

$$\begin{aligned}
\theta_i &= c(r,z)\cos^{-1}(\xi_i) \\
&= c(r,z)\sin^{-1}(\zeta_i) \\
\theta_r &= 2\theta_{\text{bnd}} - \theta_i \\
\xi_r &= \frac{\cos(\theta_r)}{c(r,z)} \\
\zeta_r &= \frac{\sin(\theta_r)}{c(r,z)}
\end{aligned}$$

# *Bibliography*

Jensen, F. B., Kuperman, W. A., Porter, M. B., and Schmidt, H. (2011). *Computational ocean acoustics*. Springer Science & Business Media.

# Part II

# Concocting Equations

# *Celerity*

$$
\begin{aligned}
c_{\max} &= 1600 \\
c_{\min} &= 1500 \\
c(r,z) &= c_0 + c_1 z + c_2 z^2 \\
c_0 + c_1 z_{\text{ati}} + c_2 z_{\text{ati}}^2 &= c(r, z_{\text{ati}}) = c_{\max} \\
c_0 + c_1 \frac{z_{\text{ati}} + z_{\text{bty}}}{2} + c_2 \left(\frac{z_{\text{ati}} + z_{\text{bty}}}{2}\right)^2 &= c(r, \frac{z_{\text{ati}} + z_{\text{bty}}}{2}) = c_{\min} \\
c_0 + c_1 z_{\text{bty}} + c_2 z_{\text{bty}}^2 &= c(r, z_{\text{bty}}) = c_{\max}
\end{aligned}
$$

$$
\begin{pmatrix} 1 & z_{\text{ati}} & z_{\text{ati}}^2 \\ 1 & \frac{z_{\text{ati}}+z_{\text{bty}}}{2} & \left(\frac{z_{\text{ati}}+z_{\text{bty}}}{2}\right)^2 \\ 1 & z_{\text{bty}} & z_{\text{bty}}^2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} c_{\max} \\ c_{\min} \\ c_{\max} \end{pmatrix}
$$