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ACOUSTIC PROPAGATION MODELLING

Part I

Helmholtz Equation

Taken from Computational Ocean Acoustics[1].

[1] Jensen, F. B., Kuperman, W. A., Porter, M. B., and Schmidt, H. (2011). *Computational ocean acoustics*. Springer Science & Business Media

$$
\begin{aligned}
O(\omega^2): \quad & |\nabla\tau|^2 = \frac{1}{c^2(\vec{x})} \\
O(\omega^1): \quad & 2\nabla\tau \cdot \nabla A_0 + \left(\nabla^2\tau\right) A_0 = 0 \\
O(\omega^{1-j}): \quad & 2\nabla\tau \cdot \nabla A_j + \left(\nabla^2\tau\right) A_j = -\nabla^2 A_{j-1}
\end{aligned}
$$

Eikonal Equation

The eikonal equation

$$|\nabla\tau|^2 = \frac{1}{c^2(\vec{x})}$$

is a first-order nonlinear PDE for modelling the path taken by a ray.

First-Order System

In cylindrical coordinates,

$$\begin{aligned}
\frac{dr}{ds} &= c\xi(s) & \frac{d\xi}{ds} &= \frac{-1}{c(r,z)^2}\frac{\partial c}{\partial r} \\
\frac{dz}{ds} &= c\zeta(s) & \frac{d\zeta}{ds} &= \frac{-1}{c(r,z)^2}\frac{\partial c}{\partial z} \\
\frac{d\tau}{ds} &= \frac{1}{c(r,z)}
\end{aligned}$$

with initial conditions

$$\begin{aligned}
r &= r_0 & \xi &= \frac{\cos(\theta_0)}{c(r_0,z_0)} \\
z &= z_0 & \zeta &= \frac{\sin(\theta_0)}{c(r_0,z_0)} \\
\tau &= 0
\end{aligned}$$

and boundary conditions defined as reflection off the bathymetry $z_{\mathrm{bty}}(r)$ and altimetry $z_{\mathrm{ati}}(r)$.

$$\begin{aligned}
\theta_{\mathrm{inc}} &= c(r,z)\cos^{-1}(\xi_{\mathrm{inc}}) \\
&= c(r,z)\sin^{-1}(\zeta_{\mathrm{inc}}) \\
\theta_{\mathrm{out}} &= 2\theta_{\mathrm{bnd}} - \theta_{\mathrm{inc}} \\
\xi_{\mathrm{out}} &= \frac{\cos(\theta_{\mathrm{out}})}{c(r,z)} \\
\zeta_{\mathrm{out}} &= \frac{\sin(\theta_{\mathrm{out}})}{c(r,z)}
\end{aligned}$$

Bibliography

Jensen, F. B., Kuperman, W. A., Porter, M. B., and Schmidt, H. (2011). *Computational ocean acoustics*. Springer Science & Business Media.