

# Ocean Acoustics Modelling

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# Chapter 1

## Ocean Parameters

### 1.1 Attenuation

#### 1.1.1 Units

TODO: Re-express all ocean parameters (except frequency) as complex. Julia implementation shows such.

We see from [Jensen et al., 2011] that the sound speed is often treated as a complex-valued parameter. However, Jensen is quite unclear as to how to define said parameter, regarding which other acoustic parameters are complex-valued and which are not.

From Jensen we take the normalized sound speed solution for plane wave attenuation

$$\exp\left(\frac{i\omega x}{c}\right)$$

with  $c \in \mathbb{C}$ , and from [Buckingham, 1997] we take the same normalized solution (with variables translated) to obtain

$$\exp\left(-\frac{i\omega x + \beta\omega x}{c(p)}\right)$$

and equate the (principle) arguments so

$$\begin{aligned}
& \frac{i\omega x}{c} = -\frac{i\omega x + \delta\omega x}{c^{(p)}} \\
\Rightarrow & \frac{c}{i} = -\frac{c^{(p)}}{i + \delta} \\
\Rightarrow & c = -\frac{ic_p}{i + \delta} \\
\Rightarrow & c = -\frac{-c^{(p)}}{-1 + i\delta} \\
\Rightarrow & c = \frac{c^{(p)}}{1 - i\delta} \\
\Rightarrow & c = c^{(p)} \frac{1 + i\delta}{1 + \delta^2}
\end{aligned}$$

It is of interest to note that the sound speed retains its magnitude,  $|c| = c^{(p)}$

From [Jensen et al., 2011] we have the relationship of the loss tangent as

$$\begin{aligned}
& ikx(1 + i\delta) = ikx - \alpha x \\
\Rightarrow & 1 + i\delta = 1 - \frac{\alpha}{ik} \\
\Rightarrow & i\delta = \frac{i\alpha}{k} \\
\Rightarrow & \delta = \frac{\alpha}{k}
\end{aligned}$$

where since the loss tangent  $\delta$  and attenuation  $\alpha$  are real,  $k$  must also be real here. Note that here, the units of  $\alpha$  are nepers/m.

For  $\alpha^{(\lambda)}$  which is in dB/ $\lambda$ ,

$$\alpha^{(\lambda)} = \alpha \lambda 20 \log_{10}(e)$$

so

$$\begin{aligned}
\delta &= \frac{\alpha}{k} \\
&= \frac{\alpha^{(\lambda)}}{k \lambda 20 \log_{10}(e)} \\
&= \frac{\alpha^{(\lambda)}}{40\pi \log_{10}(e)}
\end{aligned}$$

## 1.2 Bottom Loss

$$\mathcal{Z}_1 = \frac{\rho_1 \varsigma_1}{\sin(\theta_1)}$$

$$\mathcal{Z}_p = \frac{\rho_2 \varsigma_p}{\sin(\theta_p)}$$

$$\mathcal{Z}_s = \frac{\rho_2 \varsigma_s}{\sin(\theta_s)}$$

$$k_p \cos(\theta_p) = k_s \cos(\theta_s) = k_1 \cos(\theta_1)$$

$$\mathcal{Z}_{\text{tot}} = \mathcal{Z}_p \cos^2(2\theta_s) + \mathcal{Z}_s \sin^2(2\theta_s)$$

$$\mathcal{R} = \frac{\mathcal{Z}_{\text{tot}} - \mathcal{Z}_1}{\mathcal{Z}_{\text{tot}} + \mathcal{Z}_1}$$

$$\text{BL} = -10 \log_{10} |\mathcal{R}|^2$$

## Chapter 2

# Ray Tracing

$$\frac{dr}{dz} = \cot(\theta(z))$$

### 2.1 Eikonal Equation

# Bibliography

- [Buckingham, 1997] Buckingham, M. J. (1997). Theory of acoustic attenuation, dispersion, and pulse propagation in unconsolidated granular materials including marine sediments. *The Journal of the Acoustical Society of America*, 102(5):2579–2596.
- [Jensen et al., 2011] Jensen, F. B., Kuperman, W. A., Porter, M. B., and Schmidt, H. (2011). *Computational ocean acoustics*. Springer Science & Business Media.