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## 0 Basic

### 0A .vimrc

```
sy on
set si nu rnu cin cul et so=10 ts=4 sw=4 mouse=a
ino {<cr>} {<cr>} <esc>ko
ino jk <esc>
map <F7> :w<CR>:!g++ "%" -std
=c++17 -DLOCAL -Wall -Wextra -Wshadow -Wconversion
-fsanitize=address,undefined -g && ./a.out<CR>
ca Hash w !cpp -dD -P -fpreprocessed
  \| tr -d "[[:space:]]" \| md5sum \| cut -c-6
```

### 0B PBDS

```
// Tree and fast PQ
#include <bits/extc++.h>
using namespace __gnu_pbds;

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
tree<int, null_type, less<int>, rb_tree_tag
, tree_order_statistics_node_update> bst;
// order_of_key(n): # of elements <= n
// find_by_order(n): 0-indexed
```

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
__gnu_pbds::priority_queue<
<int, greater<int>, thin_heap_tag> pq;

0C pragma
#pragma GCC optimize("Ofast,unroll-loops")
#pragma GCC target("avx,avx2,sse,sse2
, sse3,ssse3,sse4,popcnt,abm,mmx,fma,tune=native")
// chrono
::steady_clock::now().time_since_epoch().count()

0D Default Code
using namespace std;

#define F first
#define S second
#define pii pair<int, int>
#define pll pair<ll, ll>
#define pdd pair<double, double>
#define ll long long
#define ld long double
#define i128 __int128

#define all(x) x.begin(), x.end()
#define pb emplace_back

#ifndef LOCAL
#define px(
    args...) LKJ("\033[1;32m" #args "\033[0m", args)
template<class I> void LKJ(I&&x){ cerr << x << '\n'; }
template<class I, class...T> void
LKJ(I&&x, T&...t){ cerr << x << ' ', LKJ(t...); }
template<class I> void OI(I a, I b){ while
(a < b) cerr << *a << " \n"[next(a) == b], ++a; }
#define pv(v) cerr
    << "\033[1;31m[" << #v << "]: \033[0m"; OI(all(v))
#else
#define px(...)
#define OI(...)
#define pv(v)
#endif

template<class A, class
B> ostream& operator<<(ostream &os, pair<A, B> p)
{ return os << '(' << p.F << ", " << p.S << ')'; }

void solve() {}

int main() {
cin.tie(0)->sync_with_stdio(0);
int T = 1;

// cin >> T;
while (T--) solve();
}
```

### 0E LambdaCompare

```
auto cmp = [] (int x, int y) { return x < y; };
std::set<int, decltype(cmp)> st(cmp);

1 Graph
1A 2SAT/SCC
struct SAT { // 0-base
int low[N], dfn[N], bln[N], n, Time, nScc;
bool instack[N], istrue[N];
stack<int> st;
vector<int> G[N], SCC[N];
void init(int _n) {
n = _n; // assert(n * 2 <= N);
for (int i = 0; i < n + n; ++i) G[i].clear();
}
void add_edge(int a, int b) { G[a].emplace_back(b); }
int rv(int a) {
if (a >= n) return a - n;
```

```

    return a + n;
}
void add_clause(int a, int b) {
    add_edge(rv(a), b), add_edge(rv(b), a);
}
void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for (int i : G[u])
        if (!dfn[i])
            dfs(i), low[u] = min(low[i], low[u]);
        else if (instack[i] && dfn[i] < dfn[u])
            low[u] = min(low[u], dfn[i]);
    if (low[u] == dfn[u]) {
        int tmp;
        do {
            tmp = st.top(), st.pop();
            instack[tmp] = 0, bln[tmp] = nScc;
        } while (tmp != u);
        ++nScc;
    }
}
bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
        SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
        if (!dfn[i]) dfs(i);
    for (int i =
        0; i < n + n; ++i) SCC[bln[i]].emplace_back(i);
    bool flag = true;
    for (int i = 0; i < n; ++i) {
        if (bln[i] == bln[i + n]) flag = false;
        istrue[i] = bln[i] < bln[i + n];
        istrue[i + n] = !istrue[i];
    }
    return flag;
    // return whether there is a set of solutions
    // istrue[] are one set of solutions if any
}
};


```

## 1B Vertex BCC

```

struct vertex_cc {
    struct edge {
        int to, nt;
    } e[M << 1];
    int hd[N], tot = 1;

    void add(int u, int v) { e[++tot] = edge{v, hd[u]}, hd[u] = tot; }
    void uadd(int u, int v) { add(u, v), add(v, u); }

    int ans, top, cnt, ord, root;
    int dfn[N], low[N], sta[N];
    bool cut[N];
    vector<int> dcc[N];

    void tarjan(int u) {
        dfn[u] = low[u] = ++ord, sta[++top] = u;
        if (u == root && hd[u] == 0) {
            dcc[++cnt].push_back(u);
            return;
        }
        int f = 0;
        for (int i = hd[u]; i; i = e[i].nt) {
            int v = e[i].to;
            if (!dfn[v]) {
                tarjan(v);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    if (++f > 1 || u != root) cut[u] = true;
                    cnt++;
                    do dcc[cnt].push_back(sta[top--]);
                    while (sta[top + 1] != v);
                    dcc[cnt].push_back(u);
                }
            }
        }
    }
};


```

```

    }
} else low[u] = min(low[u], dfn[v]);
}
int solve
    (int L, int R) { // vertex index range [L, R)
    for (int i = L; i < R; ++i) dfn[i] = 0;
    for (int i = L; i < R; ++i)
        if (!dfn[i]) { root = i; tarjan(i); }
    return cnt;
} // answer in dcc, BCCs are 1-based
};


```

## 1C Edge BCC

```

namespace bridge_cc { // vertex 0-based
vector<int> tim, low;
stack<int, vector<int>> st;
int t, bcc_id;
void dfs(int u, int p, const
         vector<vector<pii>> &edge, vector<int> &pa) {
    tim[u] = low[u] = t++;
    st.push(u);
    for (const auto &[v, id] : edge[u]) {
        if (id == p)
            continue;
        if (tim[v])
            low[u] = min(low[u], tim[v]);
        else {
            dfs(v, id, edge, pa);
            if (low[v] > tim[u]) {
                int x;
                do {
                    pa[x = st.top()] = bcc_id;
                    st.pop();
                } while (x != v);
                bcc_id++;
            }
        }
        low[u] = min(low[u], low[v]);
    }
}
vector<int> solve
    (const vector<vector<pii>> &edge) { // (to, id)
    int n = edge.size();
    tim.resize(n);
    low.resize(n);
    t = bcc_id = 1;
    vector<int> pa(n);

    for (int i = 0; i < n; i++) {
        if (!tim[i]) {
            dfs(i, -1, edge, pa);
            while (!st.empty()) {
                pa[st.top()] = bcc_id;
                st.pop();
            }
            bcc_id++;
        }
    }
    return pa;
} // return bcc id(start from 1)
};


```

## 1D Virtual Tree

```

// requires DFS io, lca, is_child
vector<int> tre[N];
bool cmp(int a, int b){ return in[a] < in[b]; }
void add_edge(int a, int b){
    tre[a].emplace_back(b);
    tre[b].emplace_back(a);
}
void virtual_tree(vector<int> arr, int k){
    vector<int> sta;
    sort(arr.begin(), arr.end(), cmp);
    for (int i = 1; i < k; i++)
        tre[in[sta[i]]].push_back(sta[i]);
}


```

```

    arr.emplace_back(lca(arr[i], arr[i - 1]));
sort(arr.begin(), arr.end(), cmp);
arr.resize
    (unique(arr.begin(), arr.end()) - arr.begin());
for (auto i : arr){
    while (!sta.empty)
        () && !is_child(sta.back(), i)) sta.pop_back();
    if (!sta.empty()) add_edge(sta.back(), i);
    sta.push_back(i);
}
}

```

## 1E Maximum Clique

```

struct MaxClique
    { // max complete subgraph, fast when N <= 100
bitset<N> G[N], cs[N];
int ans, sol[N], q, cur[N], d[N], n;
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) G[i].reset();
}
void add_edge(int u, int v) {
    G[u][v] = G[v][u] = 1;
}
void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
    if (l < 4) {
        for (int i : r) d[i] = (G[i] & mask).count();
        sort(all(r));
        , [&](int x, int y) { return d[x] > d[y]; });
    }
    vector<int> c(r.size());
    int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
    cs[1].reset(), cs[2].reset();
    for (int p : r) {
        int k = 1;
        while ((cs[k] & G[p]).any()) ++k;
        if (k > rgt) cs[++rgt + 1].reset();
        cs[k][p] = 1;
        if (k < lft) r[tp++] = p;
    }
    for (int k = lft; k <= rgt; ++k)
        for (int p = cs[k]._Find_first
            (); p < N; p = cs[k]._Find_next(p))
            r[tp] = p, c[tp] = k, ++tp;
    dfs(r, c, l + 1, mask);
}
void dfs(vector<
    int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pop_back(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr;
        for (int i : r) if (G[p][i]) nr.emplace_back(i);
        if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
        else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back(), --q;
    }
}
int solve() {
    vector<int> r(n);
    ans = q = 0, iota(all(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
} // first ans elements in sol form the vertex set
};

```

## 1F MinimumSteinerTree

```

struct SteinerTree { // 0-base
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcst[N]; // the cost of vertexs
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {

```

```

            fill_n(dst[i], n, INF);
            dst[i][i] = vcst[i] = 0;
        }
    }
    void chmin(int &x, int val) {
        x = min(x, val);
    }
    void add_edge(int ui, int vi, int wi) {
        chmin(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    chmin(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();
        int t = ter.size(), full = (1 << t) - 1;
        for (int i = 0; i <= full; ++i)
            fill_n(dp[i], n, INF);
        copy_n(vcst, n, dp[0]);
        for (int msk = 1; msk <= full; ++msk) {
            if (!(msk & (msk - 1))) {
                int who = __lg(msk);
                for (int i = 0; i < n; ++i)
                    dp[msk]
                        [i] = vcst[ter[who]] + dst[ter[who]][i];
            }
            for (int i = 0; i < n; ++i)
                for (int sub =
                    msk - 1) & msk; sub; sub = (sub - 1) & msk
                    chmin(dp[msk][i],
                        dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
            for (int i = 0; i < n; ++i)
                tdst[i] = INF;
            for (int j = 0; j < n; ++j)
                chmin(tdst[i], dp[msk][j] + dst[j][i]);
        }
        copy_n(tdst, n, dp[full]);
    }
    return *min_element(dp[full], dp[full] + n);
}
}; // O(V 3^T + V^2 2^T)

```

## 1G Dominator Tree

```

struct DominatorTree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].emplace_back(v), rG[v].emplace_back(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i)
            dfn[i] = idom[i] = 0;

```

```

        tree[i].clear();
        best[i] = semi[i] = i;
    }
    dfs(root);
    for (int i = Time; i > 1; --i) {
        int u = id[i];
        for (auto v : rG[u])
            if (v = dfn[v]) {
                find(v, i);
                semi[i] = min(semi[i], semi[best[v]]);
            }
        tree[semi[i]].emplace_back(i);
        for (auto v : tree[pa[i]]) {
            find(v, pa[i]);
            idom[v] =
                semi[best[v]] == pa[i] ? pa[i] : best[v];
        }
        tree[pa[i]].clear();
    }
    for (int i = 2; i <= Time; ++i) {
        if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
        tree[idom[i]].emplace_back(id[i]);
    }
}
}

```

## 1H Directed MST (slow)

```

struct DMST { // O(VE)
    struct edge {
        int u, v;
        ll w;
    };
    vector<edge> E; // O-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() { E.clear(); }
    void add_edge(int u, int v, ll w) {
        if (u != v) E.emplace_back(edge{u, v, w});
    }
    ll build(int root, int n) {
        ll ans = 0;
        for (;;) {
            fill_n(in, n, INF);
            for (int i = 0; i < (int)E.size(); ++i)
                if (E[i].u != E[i].v && E[i].w < in[E[i].v])
                    pe[E[i].v] = i, in[E[i].v] = E[i].w;
            for (int u = 0; u < n; ++u) // no solution
                if (u != root && in[u] == INF) return -INF;
            int cntnode = 0;
            fill_n(id, n, -1), fill_n(vis, n, -1);
            for (int u = 0; u < n; ++u) {
                if (u != root) ans += in[u];
                int v = u;
                while (vis[v] != u && !~id[v] && v != root)
                    vis[v] = u, v = E[pe[v]].u;
                if (v != root && !~id[v]) {
                    for (int x = E[pe[v]].u; x != v;
                        x = E[pe[x]].u)
                        id[x] = cntnode;
                    id[v] = cntnode++;
                }
            }
            if (!cntnode) break; // no cycle
            for (int u = 0; u < n; ++u)
                if (!~id[u]) id[u] = cntnode++;
            for (int i = 0; i < (int)E.size(); ++i) {
                int v = E[i].v;
                E[i].u = id[E[i].u], E[i].v = id[E[i].v];
                if (E[i].u != E[i].v) E[i].w -= in[v];
            }
            n = cntnode, root = id[root];
        }
        return ans;
    }
}

```

## 1I Directed MST (fast)

```

// O(E + V log V)
#define rep(i, a, b) for (int i = a; i < (b); ++i)
#define sz(x) (int)(x).size()
typedef vector<int> vi;
struct RollbackUF {
    vi e;
    vector<pii> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return sz(st); }
    void rollback(int t) {
        for (int i = time(); i-- > t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    }
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]});
        st.push_back({b, e[b]});
        e[a] += e[b];
        e[b] = a;
        return true;
    }
};
struct Edge {
    int a, b;
    ll w;
};
struct Node { // lazy skew heap node
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    Edge top() {
        prop();
        return key;
    }
};
Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}
void pop(Node *&a) {
    a->prop();
    a = merge(a->l, a->r);
}
pair<ll, vi> dmst(int n, int r, vector<Edge> &g) {
    RollbackUF uf(n);
    vector<Node *> heap(n);
    for (Edge e : g)
        heap[e.b] = merge(heap[e.b], new Node{e});
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1, -1}), comp;
    deque<tuple<int, int, vector<Edge>>> cycs;
    rep(s, 0, n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}};
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
        }
    }
}

```

```

res += e.w, u = uf.find(e.a);
if (seen[u] == s) { /// found cycle, contract
    Node *cyc = 0;
    int end = qi, time = uf.time();
    do cyc = merge(cyc, heap[w = path[--qi]]);
    while (uf.join(u, w));
    u = uf.find(u), heap[u] = cyc, seen[u] = -1;
    cycs.push_front({u, time, {&Q[qi], &Q[end]}});
}
rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
}

for (auto &[u, t, cmp] : cycs) {
    // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto &e : cmp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
}
rep(i, 0, n) par[i] = in[i].a;
return {res, par};
}

```

## 1J Minimum Clique Cover

```

struct CliqueCover {
    // 0-base, O(n2^n), 用最少數量的團覆蓋無向圖
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n, 1); // needs FWHT
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0; // probabilistic
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] *= co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

## 1K Count Maximal Clique

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];

```

```

        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsn = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsn++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsn, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

## 1L Theorems

$|\max \text{ independent edge set}| = |V| - |\min \text{ edge cover}|$   
 $|\max \text{ independent set}| = |V| - |\min \text{ vertex cover}|$   
 $\max \text{ independent set} = \max \text{ clique in the complement graph}$

## 2 Flow-Matching

### 2A Hopcroft-Karp

```

struct HopcroftKarp { // 0-based, return btoa to
    get matching, O(E sqrt(V)) bipartite graph matching
    bool dfs(int a, int L, vector<vector<int>> &g,
              vector<int> &btoa, vector<int> &A,
              vector<int> &B) {
        if (A[a] != L) return 0;
        A[a] = -1;
        for (int b : g[a])
            if (B[b] == L + 1) {
                B[b] = 0;
                if (btoa[b] == -1 ||
                    dfs(btoa[b], L + 1, g, btoa, A, B))
                    return btoa[b] = a, 1;
            }
        return 0;
    }
    int solve(vector<vector<int>> &g, int m) {
        int res = 0;
        vector<int> btoa(m, -1), A(g.size()),
                    B(btoa.size()), cur, next;
        for (;;) {
            fill(all(A), 0), fill(all(B), 0);
            cur.clear();
            for (int a : btoa)
                if (a != -1) A[a] = -1;
            for (int a = 0; a < (int)g.size(); a++)
                if (A[a] == 0) cur.push_back(a);
            for (int lay = 1;; lay++) {
                bool islast = 0;
                next.clear();
                for (int a : cur)
                    for (int b : g[a])
                        if (btoa[b] == -1) {
                            B[b] = lay;
                            islast = 1;
                        } else if (btoa[b] != a && !B[b]) {
                            B[b] = lay;
                            next.push_back(btoa[b]);
                        }
                if (islast) break;
                if (next.empty()) return res;
                for (int a : next) A[a] = lay;
                cur.swap(next);
            }
        }
    }

```

```

        for (int a = 0; a < (int)g.size(); a++)
            res += dfs(a, 0, g, btoa, A, B);
    }
    return res;
// or btoa
}
};

```

## 2B KM (Hungarian Alg.)

```

struct KM { // 0-base, maximum matching
    ll w[N][N], hl[N], hr[N], slk[N];
    int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
    bool vl[N], vr[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            fill_n(w[i], n, -INF);
    }
    void add_edge(int a, int b, ll wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] = 1, ~fl[x])
            return vr[qu[qr++]] = fl[x] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill_n(slk
            , n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        for (ll d;;) {
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] && slk
                        [x] >= (d = hl[x] + hr[y] - w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!Check(x)) return;
                    }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x)
                if (!vl[x] && !slk[x] && !Check(x)) return;
        }
    }
    ll solve() {
        fill_n(fl
            , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
        for (int i = 0; i < n; ++i)
            hl[i] = *max_element(w[i], w[i] + n);
        for (int i = 0; i < n; ++i) bfs(i);
        ll res = 0;
        for (int i = 0; i < n; ++i) res += w[i][fl[i]];
        return res;
    }
};

```

## 2C Max Flow (Dinic)

```

struct BoundedFlow { // 0-base
    struct edge { // note int!
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
};

```

```

void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].emplace_back(
        edge{v, rcap, lcap, (int)G[v].size()});
    G[v].emplace_back(
        edge{u, 0, 0, (int)G[u].size() - 1});
}
void add_edge(int u, int v, int cap) {
    G[u].emplace_back(
        edge{v, cap, 0, (int)G[v].size()});
    G[v].emplace_back(
        edge{u, 0, 0, (int)G[u].size() - 1});
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < (int)G[u].size(); ++i) {
        edge &e = G[u][i];
        if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df, G[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (edge &e : G[u])
            if (!~dis[e.to] && e.flow != e.cap)
                q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n + 3, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            add_edge(n + 1, i, cnt[i]), sum += cnt[i];
        else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            G[n + 1].pop_back(), G[i].pop_back();
        else if (cnt[i] < 0)
            G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}

```

## 2D MCMF

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    };
}
```

```

} *past[N];
vector<Edge> G[N];
int inq[N], n, s, t;
ll dis[N], up[N], pot[N];
bool BellmanFord() {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
        if (cap > 0 && dis[u] > d) {
            dis[u] = d, up[u] = cap, past[u] = e;
            if (!inq[u]) inq[u] = 1, q.push(u);
        }
    };
    relax(s, 0, INF, 0);
    while (!q.empty()) {
        int u = q.front();
        q.pop(), inq[u] = 0;
        for (auto &e : G[u]) {
            ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
            relax(
                e.to, d2, min(up[u], e.cap - e.flow), &e);
        }
    }
    return dis[t] != INF;
}
bool Dijkstra() {
    fill_n(dis, n, INF);
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
        if (cap > 0 && dis[u] > d) {
            dis[u] = d, up[u] = cap, past[u] = e;
            pq.push(pll(d, u));
        }
    };
    relax(s, 0, INF, 0);
    while (!pq.empty()) {
        auto [d, u] = pq.top();
        pq.pop();
        if (dis[u] != d) continue;
        for (auto &e : G[u]) {
            ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
            relax(
                e.to, d2, min(up[u], e.cap - e.flow), &e);
        }
    }
    return dis[t] != INF;
}
void solve(int _s, int _t, ll &flow, ll &cost,
bool neg = true) {
    s = _s, t = _t, flow = 0, cost = 0;
    if (neg) BellmanFord(), copy_n(dis, n, pot);
    // do BellmanFord() if time isn't tight
    for (; Dijkstra(); copy_n(dis, n, pot)) {
        for (int i = 0; i < n; ++i)
            dis[i] += pot[i] - pot[s];
        flow += up[t], cost += up[t] * dis[t];
        for (int i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], G[e.to][e.rev].flow -= up[t];
        }
    }
    void init(int _n) {
        n = _n, fill_n(pot, n, 0);
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].emplace_back(
            Edge{a, b, cap, 0, cost, (int)G[b].size()});
        G[b].emplace_back(
            Edge{b, a, 0, 0, -cost, (int)G[a].size() - 1});
    }
};

```

## 2E General Graph Matching

```

struct Matching
    { // 0-base, O(VE^2), but somehow very fast
queue<int> q; int n;
vector<int> fa, s, vis, pre, match;
vector<vector<int>> G;
int Find(int u)
{ return u == fa[u] ? u : fa[u] = Find(fa[u]); }
int LCA(int x, int y) {
    static int tk = 0; tk++; x = Find(x); y = Find(y);
    for (;;) swap(x, y) if (x != n) {
        if (vis[x] == tk) return x;
        vis[x] = tk;
        x = Find(pre[match[x]]);
    }
}
void Blossom(int x, int y, int l) {
    for (; Find(x) != l; x = pre[y]) {
        pre[x] = y, y = match[x];
        if (s[y] == 1) q.push(y), s[y] = 0;
        for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
    }
}
bool Bfs(int r) {
    iota(fa.begin
        (), fa.end(), 0); fill(s.begin(), s.end(), -1);
    q = queue<int>(); q.push(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
        for (int x = q.front(); int u : G[x])
            if (s[u] == -1) {
                if (pre[u] = x, s[u] = 1, match[u] == n) {
                    for (int a = u, b = x, last;
                        b != n; a = last, b = pre[a])
                        last =
                            match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.push(match[u]); s[match[u]] = 0;
            } else if (!s[u] && Find(u) != Find(x)) {
                int l = LCA(u, x);
                Blossom(x, u, l); Blossom(u, x, l);
            }
    }
    return false;
}
Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
    (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
void add_edge(int u, int v)
{ G[u].emplace_back(v), G[v].emplace_back(u); }
int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
        if (match[x] == n) ans += Bfs(x);
    return ans;
} // match[x] == n means not matched
};

2F Max Weight Matching
#define rep(i, l, r) for (int i = (l); i <= (r); ++i)
struct WeightGraph { // 1-based, note int!
    struct edge {
        int u, v, w;
    };
    int n, nx;
    vector<int> lab;
    vector<vector<edge>> g;
    vector<int> slack, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from;
    queue<int> q;
    WeightGraph(int n_)
        : n(n_), nx(n * 2), lab(nx + 1),
        g(nx + 1, vector<edge>(nx + 1)), slack(nx + 1),
        flo(nx + 1), flo_from(nx + 1, vector<int>(n + 1, 0)) {
            match = st = pa = S = vis = slack;
            rep(u, 1, n) rep(v, 1, n) g[u][v] = {u, v, 0};
        }
    int ED(edge e) {
        return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
    }
}
```

```

}

void update_slack(int u, int x, int &s) {
    if (!s || ED(g[u][x]) < ED(g[s][x])) s = u;
}

void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
            update_slack(u, x, slack[x]);
}

void q_push(int x) {
    if (x <= n) q.push(x);
    else
        for (int y : flo[x]) q_push(y);
}

void set_st(int x, int b) {
    st[x] = b;
    if (x > n)
        for (int y : flo[x]) set_st(y, b);
}

vector<int> split_flo(auto &f, int xr) {
    auto it = find(all(f), xr);
    if (auto pr = it - f.begin(); pr % 2 == 1)
        reverse(1 + all(f)), it = f.end() - pr;
    auto res = vector(f.begin(), it);
    return f.erase(f.begin(), it), res;
}

void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
    auto &f = flo[u], z = split_flo(f, xr);
    rep(i, 0, (int)z.size() - 1)
        set_match(z[i], z[i ^ 1]);
    set_match(xr, v);
    f.insert(f.end(), all(z));
}

void augment(int u, int v) {
    for (;;) {
        int xnv = st[match[u]];
        set_match(u, v);
        if (!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    }
}

int lca(int u, int v) {
    static int t = 0;
    ++t;
    for (++t; u || v; swap(u, v))
        if (u) {
            if (vis[u] == t) return u;
            vis[u] = t;
            u = st[match[u]];
            if (u) u = st[pa[u]];
        }
    return 0;
}

void add_blossom(int u, int o, int v) {
    int b = find(n + 1 + all(st), 0) - begin(st);
    lab[b] = 0, S[b] = 0;
    match[b] = match[o];
    vector<int> f = {o};
    for (int x = u, y; x != o; x = st[pa[y]])
        f.emplace_back(x),
        f.emplace_back(y = st[match[x]]), q.push(y);
    reverse(1 + all(f));
    for (int x = v, y; x != o; x = st[pa[y]])
        f.emplace_back(x),
        f.emplace_back(y = st[match[x]]), q.push(y);
    flo[b] = f;
    set_st(b, b);
    for (int x = 1; x <= nx; ++x)
        g[b][x].w = g[x][b].w = 0;
    fill(all(flo_from[b]), 0);
}

```

```

for (int xs : flo[b]) {
    for (int x = 1; x <= nx; ++x)
        if (g[b][x].w == 0 || ED(g[xs][x]) < ED(g[b][x]))
            g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
}
set_slack(b);

void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
        if (xs == -1) {
            xs = x;
            continue;
        }
        pa[xs] = g[x][xs].u;
        S[xs] = 1, S[x] = 0;
        slack[xs] = 0;
        set_slack(x);
        q.push(x);
        xs = -1;
    }
    for (int x : flo[b])
        if (x == xr) S[x] = 1, pa[x] = pa[b];
        else S[x] = -1, set_slack(x);
    st[b] = 0;
}

bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
        int nu = st[match[v]];
        pa[v] = e.u;
        S[v] = 1;
        slack[v] = slack[nu] = 0;
        S[nu] = 0;
        q.push(nu);
    } else if (S[v] == 0) {
        if (int o = lca(u, v)) add_blossom(u, o, v);
        else return augment(u, v), augment(v, u), true;
    }
    return false;
}

bool matching() {
    fill(all(S), -1), fill(all(slack), 0);
    q = queue<int>();
    for (int x = 1; x <= nx; ++x)
        if (st[x] == x && !match[x])
            pa[x] = 0, S[x] = 0, q.push(x);
    if (q.empty()) return false;
    for (;;) {
        while (q.size()) {
            int u = q.front();
            q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (ED(g[u][v]) != 0)
                        update_slack(u, st[v], slack[st[v]]);
                    else if (on_found_edge(g[u][v]))
                        return true;
                }
        }
        int d = INF;
        for (int b = n + 1; b <= nx; ++b)
            if (st[b] == b && S[b] == 1)
                d = min(d, lab[b] / 2);
        for (int x = 1; x <= nx; ++x)
            if (int s = slack[x];
                st[x] == x && s && S[x] <= 0)
                d = min(d, ED(g[s][x]) / (S[x] + 2));
        for (int u = 1; u <= n; ++u)
            if (S[st[u]] == 1) lab[u] += d;
            else if (S[st[u]] == 0) {

```

```

        if (lab[u] <= d) return false;
        lab[u] -= d;
    }
    rep(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
        lab[b] += d * (2 - 4 * S[b]);
    for (int x = 1; x <= nx; ++x)
        if (int s = slack[x]; st[x] == x && s &&
            st[s] != x && ED(g[s][x]) == 0)
            if (on_found_edge(g[s][x])) return true;
    for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
            expand_blossom(b);
    }
    return false;
}
pair<ll, int> solve() {
    fill(all(match), 0);
    rep(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    rep(u, 1, n) rep(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    fill(all(lab), w_max);
    int n_matches = 0;
    ll tot_weight = 0;
    while (matching()) ++n_matches;
    rep(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int u, int v, int w) {
    g[u][v].w = g[v][u].w = w;
}
};

```

## 2G Global Min Cut

```

struct StoerWagner { // O(V^3), is it O(VE + V log V)?
    int vst[N], edge[N][N], wei[N];
    void init(int n) {
        for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
    }
    void addEdge(int u, int v, int w) {
        edge[u][v] += w;
        edge[v][u] += w;
    }
    int search(int &s, int &t, int n) {
        fill_n(vst, n, 0), fill_n(wei, n, 0);
        s = t = -1;
        int mx, cur;
        for (int j = 0; j < n; ++j) {
            mx = -1, cur = 0;
            for (int i = 0; i < n; ++i)
                if (wei[i] > mx) cur = i, mx = wei[i];
            vst[cur] = 1, wei[cur] = -1;
            s = t;
            t = cur;
            for (int i = 0; i < n; ++i)
                if (!vst[i]) wei[i] += edge[cur][i];
        }
        return mx;
    }
    int solve(int n) {
        int res = INF;
        for (int x, y; n > 1; n--) {
            res = min(res, search(x, y, n));
            for (int i = 0; i < n; ++i)
                edge[i][x] = (edge[x][i] += edge[y][i]);
            for (int i = 0; i < n; ++i) {
                edge[y][i] = edge[n - 1][i];
                edge[i][y] = edge[i][n - 1];
            } // edge[y][y] = 0;
        }
        return res;
    }
} sw;

```

## 2H Gomory-Hu Tree

```

// 最小割樹，樹上兩點最小割 = 樹上兩點最小割
BoundedFlow Dinic;
int g[N];
void add_edge(int u, int v, int w); // TODO
void GomoryHu(int n) { // 0-base
    fill_n(g, n, 0);
    for (int i = 1; i < n; ++i) {
        Dinic.init(n);
        // build the graph
        add_edge(i, g[i], Dinic.maxflow(i, g[i]));
        for (int j = i + 1; j <= n; ++j)
            if (g[j] == g[i] && ~Dinic.dis[j])
                g[j] = i;
    }
}

2I Min Cost Circulation

struct MinCostCirculation { // 0-base
    struct Edge {
        ll from, to, cap, fcap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    ll dis[N], inq[N], n;
    void BellmanFord(int s) {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, Edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
    }
    void try_edge(Edge &cur) {
        if (cur.cap > cur.flow) return ++cur.cap, void();
        BellmanFord(cur.to);
        if (dis[cur.from] + cur.cost < 0) {
            cur.flow, --G[cur.to][cur.rev].flow;
            for (int i = cur.from; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                ++e.flow, --G[e.to][e.rev].flow;
            }
            ++cur.cap;
        }
        void solve(int mxlg) {
            for (int b = mxlg; b >= 0; --b) {
                for (int i = 0; i < n; ++i)
                    for (auto &e : G[i])
                        e.cap *= 2, e.flow *= 2;
                for (int i = 0; i < n; ++i)
                    for (auto &e : G[i])
                        if (e.fcap > b & 1)
                            try_edge(e);
            }
        }
        void init(int _n) { n = _n;
            for (int i = 0; i < n; ++i) G[i].clear();
        }
        void add_edge(ll a, ll b, ll cap, ll cost) {
            G[a].emplace_back(Edge{a, b,
                0, cap, 0, cost, (ll)G[b].size() + (a == b)});
            G[b].emplace_back(Edge
                {b, a, 0, 0, 0, -cost, (ll)G[a].size() - 1});
        }
    }
}
```

```
| }
} mcmf; // O(VE * ElogC)
```

## 2J Flow Models Building

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source  $S$  and sink  $T$ .
  2. For each edge  $(x,y,l,u)$ , connect  $x \rightarrow y$  with capacity  $u-l$ .
  3. For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  5. The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X,Y)$ 
  1. Redirect every edge:  $y \rightarrow x$  if  $(x,y) \in M$ ,  $x \rightarrow y$  otherwise.
  2. DFS from unmatched vertices in  $X$ .
  3.  $x \in X$  is chosen iff  $x$  is unvisited.
  4.  $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  1. Construct super source  $S$  and sink  $T$
  2. For each edge  $(x,y,c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  3. For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
  4. For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  5. For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  6. Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C+K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer  $T$
  2. Construct a max flow model, let  $K$  be the sum of all weights
  3. Connect source  $s \rightarrow v$ ,  $v \in G$  with capacity  $K$
  4. For each edge  $(u,v,w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
  5. For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K+2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6.  $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
  1. For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u,v)$ .
  2. Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  3. Find the minimum weight perfect matching on  $G'$ .
- Project selection problem
  1. If  $p_v > 0$ , create edge  $(s,v)$  with capacity  $p_v$ ; otherwise, create edge  $(v,t)$  with capacity  $-p_v$ .
  2. Create edge  $(u,v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  1. Capacity  $c_{uv}$ , Flow  $f_{uv}$ , Cost  $w_{uv}$ , Required Flow difference for vertex  $b_u$ .
  2. If all  $w_{uv}$  are integers, then optimal solution can happen when all  $p_u$  are integers.

$$\min \sum_{uv} w_{uv} f_{uv}$$

$$-f_{uv} \geq -c_{uv} \Leftrightarrow \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$\sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0$$

## 3 Data Struture

### 3A LichaoTree

```
#define INF 4e18
struct Line {
    ll a, b;
    Line(ll _a = 0, ll _b = INF) : a(_a), b(_b) {}
    ll f(ll x) const { return a * x + b; }
};

struct Node {
    Line line;
    Node *left = nullptr, *right = nullptr;
    Node(Line l) : line(l) {}
};

struct LiChaoTree {
    const ll MIN_X = -1e9;
    const ll MAX_X = 1e9;
    Node* root = nullptr;
    void insert(Line new_line, ll L, ll R) {
        insert(root, MIN_X, MAX_X, new_line, L, R);
    }
    ll query(ll x) const {
        return query(root, MIN_X, MAX_X, x);
    }
private:
    void insert(Node*& node, ll l, ll r, Line new_line, ll L, ll R) {
        if (r < L || R <= l) return;
        if (!node) node = new Node(Line());
        // don't use (l + r) / 2, it may overflow
        ll mid = (l + r) >> 1;
        if (L <= l && r < R) {
            bool left_better
                = new_line.f(l) < node->line.f(l);
            bool mid_better
                = new_line.f(mid) < node->line.f(mid);
            if (mid_better) swap(node->line, new_line);
            if (r == l) return;
            if (left_better != mid_better) insert
                (node->left, l, mid, new_line, L, R);
            else insert(node
                ->right, mid + 1, r, new_line, L, R);
            return;
        }
        insert(node->left, l, mid, new_line, L, R);
        insert
            (node->right, mid + 1, r, new_line, L, R);
    }
    ll query(Node* node, ll l, ll r, ll x) const {
        if (!node) return INF;
        ll res = node->line.f(x);
        if (l == r) return res;
        ll mid = (l + r) >> 1;
        if (x <= mid) return
            min(res, query(node->left, l, mid, x));
        else return
            min
                (res, query(node->right, mid + 1, r, x));
    }
} seg;
```

### 3B Treap

```
mt19937 rd(1);
#define sz(t) ((t) == 0 ? 0 : (t)->size)
struct Treap {
    int pri, size;
    Treap *l, *r;
    Treap(ll val = 0)
        : pri(rd()), size(1), l(0), r(0) {};
    void push();
    void pull() { size = 1 + sz(l) + sz(r); }
};
void spilt(int k, Treap *rt, Treap *&a, Treap *&b) {
    if (!rt) return a = b = 0, void();
    rt->push();
    int lsz = 1 + sz(rt->l);
    if (k >= lsz)
```

```

    a = rt, spilt(k - lsz, a->r, a->r, b), a->pull();
  else b = rt, spilt(k, b->l, a, b->l), b->pull();
}
Treap *merge(Treap *l, Treap *r) {
  if (!l) return r;
  if (!r) return l;
  if (l->pri < r->pri) {
    l->push(), l->r = merge(l->r, r), l->pull();
    return l;
  } else {
    r->push(), r->l = merge(l, r->l), r->pull();
    return r;
  }
}

```

### 3C LinkCutTree

```

#define ls(x) Tree[x].son[0]
#define rs(x) Tree[x].son[1]
#define fa(x) Tree[x].fa
struct node {
  int son[2], Min, id, fa, lazy;
} Tree[N];
int n, m, q, w[N], Min;
struct Node {
  int u, v, w;
} a[N];
inline bool IsRoot(int x) {
  return (ls(fa(x)) == x || rs(fa(x)) == x) ? false
                                                : true;
}
inline void PushUp(int x) {
  Tree[x].Min = w[x], Tree[x].id = x;
  if (ls(x) && Tree[ls(x)].Min < Tree[x].Min) {
    Tree[x].Min = Tree[ls(x)].Min;
    Tree[x].id = Tree[ls(x)].id;
  }
  if (rs(x) && Tree[rs(x)].Min < Tree[x].Min) {
    Tree[x].Min = Tree[rs(x)].Min;
    Tree[x].id = Tree[rs(x)].id;
  }
}
inline void Update(int x) {
  Tree[x].lazy ^= 1;
  swap(ls(x), rs(x));
}
inline void PushDown(int x) {
  if (!Tree[x].lazy) return;
  if (ls(x)) Update(ls(x));
  if (rs(x)) Update(rs(x));
  Tree[x].lazy = 0;
}
inline void Rotate(int x) {
  int y = fa(x), z = fa(y), k = rs(y) == x,
  w = Tree[x].son[!k];
  if (!IsRoot(y)) Tree[z].son[rs(z)] = y;
  fa(x) = z, fa(y) = x;
  if (w) fa(w) = y;
  Tree[x].son[!k] = y, Tree[y].son[k] = w;
  PushUp(y);
}
inline void Splay(int x) {
  stack<int> Stack;
  int y = x, z;
  Stack.push(y);
  while (!IsRoot(y)) Stack.push(y = fa(y));
  while (!Stack.empty())
    PushDown(Stack.top()), Stack.pop();
  while (!IsRoot(x)) {
    y = fa(x), z = fa(y);
    if (!IsRoot(y))
      Rotate((ls(y) == x) ^ (ls(z) == y) ? x : y);
    Rotate(x);
  }
  PushUp(x);
}

```

```

inline void Access(int root) {
  for (int x = 0; root; x = root, root = fa(root))
    Splay(root), rs(root) = x, PushUp(root);
}
inline void MakeRoot(int x) {
  Access(x), Splay(x), Update(x);
}
inline int FindRoot(int x) {
  Access(x), Splay(x);
  while (ls(x)) x = ls(x);
  return Splay(x), x;
}
inline void Link(int u, int v) {
  MakeRoot(u);
  if (FindRoot(v) != u) fa(u) = v;
}
inline void Cut(int u, int v) {
  MakeRoot(u);
  if (FindRoot(v) != u || fa(v) != u || ls(v)) return;
  fa(v) = rs(u) = 0;
}
inline void Split(int u, int v) {
  MakeRoot(u), Access(v), Splay(v);
}
inline bool Check(int u, int v) {
  return MakeRoot(u), FindRoot(v) == u;
}
inline int LCA(int root, int u, int v) {
  MakeRoot(root), Access(u), Access(v), Splay(u);
  if (!fa(u)) {
    Access(u), Splay(v);
    return fa(v);
  }
  return fa(u);
}
/* ETT
每次進入節點和走邊都放入一次共  $3n - 2$ 
node(u) 表示進入節點 u 放入 treap 的位置
edge(u, v) 表示  $u \rightarrow v$  的邊放入 treap 的位置 (push v)
MakeRoot u :
 $L1 = [\text{begin}, \text{node}(u) - 1], L2 = [\text{node}(u), \text{end}]$ 
 $\rightarrow L2 + L1$ 
*/

```

*Insert u, v :*  
 $T_u \rightarrow L1 = [\text{begin}, \text{node}(u) - 1], L2 = [\text{node}(u), \text{end}]$   
 $T_v \rightarrow L3 = [\text{begin}, \text{node}(v) - 1], L4 = [\text{node}(v), \text{end}]$   
 $\rightarrow L2 + L1 + \text{edge}(u, v) + L4 + L3 + \text{edge}(v, u)$

*Select u, v :*  
*maybe need swap u, v*  
 $T \rightarrow L1 + \text{edge}(u, v) + L2 + \text{edge}(v, u) + L3$   
 $\rightarrow L1 + L3, L2$

\*/

### 3D CentroidDecomposition

```

struct Cent_Dec { // 1-base
  vector<pll> G[N];
  pll info[N]; // store info. of itself
  pll upinfo[N]; // store info. of climbing up
  int n, pa[N], layer[N], sz[N], done[N];
  ll dis[_lg(N) + 1][N];
  void init(int _n) {
    n = _n, layer[0] = -1;
    fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
    for (int i = 1; i <= n; ++i) G[i].clear();
  }
  void add_edge(int a, int b, int w) {
    G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
  }
  void get_cent(
    int u, int f, int &mx, int &c, int num) {
    int mxsz = 0;
    sz[u] = 1;
    for (pll e : G[u])
      if (!done[e.X] && e.X != f) {
        get_cent(e.X, u, mx, c, num);
      }
  }
}

```

```

        sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
    }
    if (mx > max(mxsz, num - sz[u]))
        mx = max(mxsz, num - sz[u]), c = u;
}
void dfs(int u, int f, ll d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pll e : G[u])
        if (!done[e.X] && e.X != f)
            dfs(e.X, u, d + e.Y, org);
}
int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pll e : G[c])
        if (!done[e.X]) {
            if (sz[e.X] > sz[c])
                lc = cut(e.X, c, num - sz[c]);
            else lc = cut(e.X, c, sz[e.X]);
            upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
        }
    return done[c] = 0, c;
}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -=
                upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};
```

### 3E HeavyLightDecomposition

```

struct Heavy_Light_Decomposition { // 1-base
    int n, ulink[N], deep[N], mxson[N], w[N], pa[N];
    int t, pl[N], data[N], dt[N], bln[N], edge[N], et;
    vector<pii> G[N];
    void init(int _n) {
        n = _n, t = 0, et = 1;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pii(b, et));
        G[b].pb(pii(a, et));
        edge[et++] = w;
    }
    void dfs(int u, int f, int d) {
        w[u] = 1, pa[u] = f, deep[u] = d++;
        for (auto &i : G[u])
            if (i.X != f) {
                dfs(i.X, u, d), w[u] += w[i.X];
                if (w[mxson[u]] < w[i.X]) mxson[u] = i.X;
            } else bln[i.Y] = u, dt[u] = edge[i.Y];
    }
    void cut(int u, int link) {
        data[pl[u] = t++] = dt[u], ulink[u] = link;
        if (!mxson[u]) return;
        cut(mxson[u], link);
        for (auto i : G[u])
            // sa: order, rk[i]: pos of s[i...],
```

```

if (i.X != pa[u] && i.X != mxson[u])
    cut(i.X, i.X);
}
void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
int query(int a, int b) {
    int ta = ulink[a], tb = ulink[b], re = 0;
    while (ta != tb)
        if (deep[ta] < deep[tb])
            /*query*/, tb = ulink[b = pa[tb]];
        else /*query*/, ta = ulink[a = pa[ta]];
    if (a == b) return re;
    if (pl[a] > pl[b]) swap(a, b);
    /*query*/
    return re;
}
};
```

## 4 String

### 4A KMP

```

vector<int> pi(const string& s) {
    vector<int> p((int)s.size());
    for(int i=1; i<(int)s.size(); i++) {
        int g = p[i-1];
        while (g && s[i] != s[g]) g = p[g-1];
        p[i] = g + (s[i] == s[g]);
    }
    return p;
}
```

### 4B Z

```

vector<int> Z(const string& S) {
    vector<int> z((int)S.size());
    z[0] = (int)S.size(); // or 0
    int l = -1, r = -1;
    for(int i=1; i<(int)S.size(); i++) {
        z[i] = i >= r ? 0 : min(r - i, z[i - l]);
        while (i +
            z[i] < (int)S.size() && S[i + z[i]] == S[z[i]]);
            z[i]++;
        if (i + z[i] > r)
            l = i, r = i + z[i];
    }
    return z;
}
```

### 4C Manacher

```

vector<int> manacher(const string& s) {
    int n = s.size();
    array<vector<int>, 2> p = {vector<int>(n+1), vector<int>(n)};
    for(int z
        = 0; z<2; z++)
        for (int i=0, l=0, r=0; i < n; i++) {
            int t = r-i+z;
            if (i<r) p[z][i] = min(t, p[z][l+t]);
            int L = i-p[z][i], R = i+p[z][i]-z;
            while (L>=1 && R+1<n && s[L-1] == s[R+1])
                p[z][i]++, L--, R++;
            if (R>r) l=L, r=R;
        }
    vector<int> res(n*2+1);
    for(int i=0; i<n; i++){
        res[2*i]=p[0][i];
        res[2*i+1]=p[1][i];
    }
    res[2*n]=p[0][n];
    return res;
}
```

### 4D SuffixArray

```

struct SuffixArray {
#define add(x, k) (x + k + n) % n
    vector<int> sa, cnt, rk, tmp, lcp;
    // sa: order, rk[i]: pos of s[i...],
```

```

// lcp[i]: LCP of sa[i], sa[i-1]
void SA(string s) { // remember to append '\1'
    int n = (int)s.size();
    sa.resize(n), cnt.resize(n);
    rk.resize(n), tmp.resize(n);
    iota(all(sa), 0);
    sort(all(sa),
        [&](int i, int j) { return s[i] < s[j]; });
    rk[0] = 0;
    for (int i = 1; i < n; i++)
        rk[sa[i]] =
            rk[sa[i - 1]] + (s[sa[i - 1]] != s[sa[i]]);
    for (int k = 1; k <= n; k <= 1) {
        fill(all(cnt), 0);
        for (int i = 0; i < n; i++)
            cnt[rk[add(sa[i], -k)]]++;
        for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
        for (int i = n - 1; i >= 0; i--)
            tmp[--cnt[rk[add(sa[i], -k)]]] =
                add(sa[i], -k);
        sa.swap(tmp);
        tmp[sa[0]] = 0;
        for (int i = 1; i < n; i++)
            tmp[sa[i]] = tmp[sa[i - 1]] +
                (rk[sa[i - 1]] != rk[sa[i]] ||
                 rk[add(sa[i - 1], k)] !=
                    rk[add(sa[i], k)]);
        rk.swap(tmp);
    }
}
void LCP(string s) {
    int n = (int)s.size(), k = 0;
    lcp.resize(n);
    for (int i = 0; i < n; i++)
        if (rk[i] == 0) lcp[rk[i]] = 0;
        else {
            if (k) k--;
            int j = sa[rk[i] - 1];
            while (
                max(i, j) + k < n && s[i + k] == s[j + k])
                k++;
            lcp[rk[i]] = k;
        }
    }
};

```

## 4E SA-IS(Induced Sorting)

```

auto sais(const auto &s) {
    const int n = (int)s.size(), z = ranges::max(s) + 1;
    if (n == 1) return vector{0};
    vector<int> c(z);
    for (int x : s) ++c[x];
    partial_sum(all(c), begin(c));
    vector<int> sa(n);
    auto I = views::iota(0, n);
    vector<bool> t(n, true);
    for (int i = n - 2; i >= 0; --i)
        t[i] =
            (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    auto is_lms = views::filter(
        [&t](int x) { return x && t[x] && !t[x - 1]; });
    auto induce = [&] {
        for (auto x = c; int y : sa)
            if (y--)
                if (!t[y]) sa[x[s[y] - 1]++] = y;
        for (auto x = c; int y : sa | views::reverse)
            if (y--)
                if (t[y]) sa[--x[s[y]]] = y;
    };
    vector<int> lms, q(n);
    lms.reserve(n);
    for (auto x = c; int i : I | is_lms)
        q[i] = (int)lms.size(),
        lms.emplace_back(sa[--x[s[i]]] = i);
    induce();
}

```

```

vector<int> ns((int)lms.size());
for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
        int len = min({n - i, n - j, lms[q[i] + 1] - i});
        ns[q[i]] = nz += lexicographical_compare(
            begin(s) + j, begin(s) + j + len, begin(s) + i,
            begin(s) + i + len);
    }
    j = i;
}
fill(all(sa), 0);
auto nsa = sais(ns);
for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
return induce(), sa;
}

// sa[i]: sa[i]-th suffix is the i-th lexicographically
// smallest suffix. hi[i]: LCP of suffix sa[i] and
// suffix sa[i - 1].
struct Suffix {
    int n;
    vector<int> sa, hi, ra;
    Suffix(const auto &s, int _n)
        : n(_n), hi(n), ra(n) {
        vector<int> s(n + 1); // s[n] = 0;
        copy_n(_s, n, begin(s)); // _s shouldn't contain 0
        sa = sais(s);
        sa.erase(sa.begin());
        for (int i = 0; i < n; ++i) ra[sa[i]] = i;
        for (int i = 0, h = 0; i < n; ++i) {
            if (!ra[i]) {
                h = 0;
                continue;
            }
            for (int j = sa[ra[i] - 1];
                  max(i, j) + h < n && s[i + h] == s[j + h];)
                ++h;
            hi[ra[i]] = h ? h-- : 0;
        }
    }
};


```

## 4F ACAutomaton

```

#define sigma 26
#define base 'a'
struct AhoCorasick { // N: sum of length
    int ch[N][sigma] = {{}}, f[N] = {-1}, tag[N],
        mv[N][sigma], jump[N], cnt[N];
    int idx = 0, t = -1;
    vector<int> E[N], q;
    pii o[N];
    int insert(string &s) {
        int j = 0;
        for (int i = 0; i < (int)s.size(); i++) {
            if (!ch[j][s[i] - base])
                ch[j][s[i] - base] = ++idx;
            j = ch[j][s[i] - base];
        }
        tag[j] = 1;
        return j;
    }
    int next(int u, int c) {
        return u < 0 ? 0 : mv[u][c];
    }
    void dfs(int u) {
        o[u].F = ++t;
        for (auto v : E[u]) dfs(v);
        o[u].S = t;
    }
    void build() {
        int k = -1;
        q.emplace_back(0);
        while (++k < (int)q.size()) {
            int u = q[k];
            for (int v = 0; v < sigma; v++) {
                if (ch[u][v]) {

```

```

        f[ch[u][v]] = next(f[u], v);
        q.emplace_back(ch[u][v]);
    }
    mv[u][v] =
        (ch[u][v] ? ch[u][v] : next(f[u], v));
}
if (u) jump[u] = (tag[f[u]] ? f[u] : jump[f[u]]);
}
reverse(q.begin(), q.end());
for (int i = 1; i <= idx; i++)
    E[f[i]].emplace_back(i);
dfs(0);
}
void match(string &s) {
    fill(cnt, cnt + idx + 1, 0);
    for (int i = 0, j = 0; i < (int)s.size(); i++)
        cnt[j = next(j, s[i] - base)]++;
    for (int i : q)
        if (f[i] > 0) cnt[f[i]] += cnt[i];
}
} ac;

```

## 4G MinRotation

```

int mincyc(string s) {
    int n = (int)s.size();
    s = s + s;
    int i = 0, ans = 0;
    while (i < n) {
        ans = i;
        int j = i + 1, k = i;
        while (j < 2 * n && s[j] >= s[k]) {
            k = (s[j] > s[k] ? i : k + 1);
            ++j;
        }
        while (i <= k) i += j - k;
    }
    return ans;
}

```

## 4H ExtSAM

```

#define CNUM 26
struct exSAM {
    int len[N * 2], link[N * 2]; // maxlen, suflink
    int next[N * 2][CNUM], tot; // [0, tot), root = 0
    int lenSorted[N * 2]; // topo. order
    int cnt[N * 2]; // occurrence
    int newnode() {
        fill_n(next[tot], CNUM, 0);
        len[tot] = cnt[tot] = link[tot] = 0;
        return tot++;
    }
    void init() { tot = 0, newnode(), link[0] = -1; }
    int insertSAM(int last, int c) {
        int cur = next[last][c];
        len[cur] = len[last] + 1;
        int p = link[last];
        while (p != -1 && !next[p][c])
            next[p][c] = cur, p = link[p];
        if (p == -1) return link[cur] = 0, cur;
        int q = next[p][c];
        if (len
            [p] + 1 == len[q]) return link[cur] = q, cur;
        int clone = newnode();
        for (int i = 0; i < CNUM; ++i)
            next[
                clone][i] = len[next[q][i]] ? next[q][i] : 0;
        len[clone] = len[p] + 1;
        while (p != -1 && next[p][c] == q)
            next[p][c] = clone, p = link[p];
        link[link[cur] = clone] = link[q];
        link[q] = clone;
        return cur;
    }
    void insert(const string &s) {
        int cur = 0;

```

```

        for (auto ch : s) {
            int &nxt = next[cur][int(ch - 'a')];
            if (!nxt) nxt = newnode();
            cnt[cur = nxt] += 1;
        }
    }
    void build() {
        queue<int> q;
        q.push(0);
        q.push(0);
        while (!q.empty()) {
            int cur = q.front();
            q.pop();
            for (int i = 0; i < CNUM; ++i)
                if (next[cur][i])
                    q.push(insertSAM(cur, i));
        }
        vector<int> lc(tot);
        for (int i = 1; i < tot; ++i) ++lc[len[i]];
        partial_sum(all(lc), lc.begin());
        for (int i
            = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
    }
    void solve() {
        for (int i = tot - 2; i >= 0; --i)
            cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
    }
};

```

## 5 Number Theory

### 5A Primes

```

12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771 1001010013
1000512343 987654361 999991231 999888733 98789101
98777733 999991921 1010101333 1010102101 1000000000039
1000000000000037 2305843009213693951 4611686018427387847
9223372036854775783 18446744073709551557

```

### 5B ExtGCD

```

// beware of negative numbers!
void extgcd(ll a, ll b, ll c, ll &x, ll &y) {
    if (b == 0) x = c / a, y = 0;
    else {
        extgcd(b, a % b, c, y, x);
        y -= x * (a / b);
    }
} // |x| <= b/2, |y| <= a/2

```

### 5C FloorCeil

```

int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }

```

### 5D FloorSum

Computes

$$f(a,b,c,n) = \sum_{i=0}^n \left\lfloor \frac{a \cdot i + b}{m} \right\rfloor$$

Furthermore, Let  $m = \left\lfloor \frac{an+b}{c} \right\rfloor$ :

$$\begin{aligned}
 g(a,b,c,n) &= \sum_{i=0}^n i \left\lfloor \frac{ai+b}{c} \right\rfloor \\
 &= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}
 \end{aligned}$$

$$h(a,b,c,n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor^2$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor^2 \cdot (n+1) \\ + \left\lfloor \frac{a}{c} \right\rfloor \cdot \left\lfloor \frac{b}{c} \right\rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{a}{c} \right\rfloor \cdot g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ + 2 \left\lfloor \frac{b}{c} \right\rfloor \cdot f(a \bmod c, b \bmod c, c, n), & n < 0 \vee a = 0 \\ 0, & n = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

```
ll floorsum(ll A, ll B, ll C, ll N) {
    if (A == 0) return (N + 1) * (B / C);
    if (A > C || B > C)
        return (N + 1) * (B / C) +
            N * (N + 1) / 2 * (A / C) +
            floorsum(A % C, B % C, C, N);
    ll M = (A * N + B) / C;
    return N * M - floorsum(C, C - B - 1, A, M - 1);
}
```

## 5E MillerRabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : primes <= 13
// n < 2^64                7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
ll mul(ll a, ll b, ll mod) {
    return (ll)(__int128(a) * b % mod);
}
bool Miller_Rabin(ll a, ll n) { // O(log(n)^3)
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}
```

## 5F PollardRho

```
map<ll, int> cnt;
void PollardRho(ll n) { // O(n^(1/4))
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2 == 0)
        return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
#define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}
```

## 5G Fraction

```
struct fraction {
    ll n, d;
    fraction(const ll &n = 0, const ll &d = 1)
        : n(_n), d(_d) {
        ll t = __gcd(n, d);
        n /= t, d /= t;
        if (d < 0) n = -n, d = -d;
    }
```

```
}
fraction
operator-() const { return fraction(-n, d); }
fraction operator+(const fraction &b) const {
    return fraction(n * b.d + b.n * d, d * b.d); }
fraction operator-(const fraction &b) const {
    return fraction(n * b.d - b.n * d, d * b.d); }
fraction operator*(const fraction &b) const {
    return fraction(n * b.n, d * b.d); }
fraction operator/(const fraction &b) const {
    return fraction(n * b.d, d * b.n); }
bool is_zero() const { return n == 0; }
void print() {
    cout << n;
    if (d != 1) cout << "/" << d;
}
```

## 5H ChineseRemainder

```
ll solve(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    ll x, y;
    extgcd(m1, m2, __gcd(m1, m2), x, y);
    ll lcm = m1 * m2 * g;
    ll res = x * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return (res % lcm + lcm) % lcm;
}
```

## 5I FactorialModp<sup>k</sup>

```
// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k
```

## 5J QuadraticResidue

```
// Berlekamp-Rabin, log^2(p)
ll trial(ll y, ll z, ll m) {
    ll a0 = 1, a1 = 0, b0 = z, b1 = 1, p = (m - 1) / 2;
    while (p) {
        if (p & 1)
            tie(a0, a1) =
                make_pair((a1 * b1 % m * y + a0 * b0) % m,
                          (a0 * b1 + a1 * b0) % m);
            tie(b0, b1) =
                make_pair((b1 * b1 % m * y + b0 * b0) % m,
                          (2 * b0 * b1) % m);
            p >>= 1;
        if (a1) return inv(a1, m);
        return -1;
    }
    mt19937 rd(49);
    ll psqrt(ll y, ll p) { // sqrt(y) mod p
        if (y == 0) return 0;
        if (fpow(y, (p - 1) / 2, p) != 1) return -1;
        for (int i = 0; i < 30; i++) {
            ll z = rd() % p;
            if (z * z % p == y) return z;
            ll x = trial(y, z, p);
            if (x == -1) continue;
            return x;
        }
    }
```

```
    return -1;
}
```

## 5K MeisselLehmer

```
ll PrimeCount(ll n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<ll> larges(s);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i = 0; i < s; ++i) {
        roughs[i] = 2 * i + 1;
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    }
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            ++pc;
            if (1LL * q * q > n) break;
            skip[p] = 1;
            for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                ll d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges
                    [smalls[d] - pc] : smalls[n / d]) + pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c =
                    smalls[j] - pc, e = min(j * p + p, v + 1);
                for (int i = j * p; i < e; ++i) smalls[i] -= c;
            }
        }
    }
    for (int k = 1; k < s; ++k) {
        const ll m = n / roughs[k];
        ll t = larges[k] - (pc + k - 1);
        for (int l = 1; l < k; ++l) {
            int p = roughs[l];
            if (1LL * p * p > m) break;
            t -= smalls[m / p] - (pc + l - 1);
        }
        larges[0] -= t;
    }
    return larges[0];
}
```

## 5L DiscreteLog

```
int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}
int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
}
```

```
    if (fpow(x, p, m) != y) return -1;
    return p;
}
```

## 5M Möbius Function

```
vector<int> mobius_up_to(int N) {
    vector<int> mu(N + 1, 1), prm;
    vector<bool> is_prm(N + 1, 1);
    mu[0] = 0; // place holder
    for (int i = 2; i <= N; ++i) {
        if (is_prm[i]) {
            prm.push_back(i);
            mu[i] = -1;
        }
        for (auto &p : prm) {
            if (i * p > N) break;
            is_prm[i * p] = 0;
            if (i % p == 0) {
                mu[i * p] = 0;
                break;
            } else mu[i * p] = -mu[i];
        }
    }
    return mu;
}
```

## 5N Theorems

- Cramer's Rule

$$\begin{aligned} ax+by=e \\ cx+dy=f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed-bf}{ad-bc} \\ y &= \frac{af-ec}{ad-bc} \end{aligned}$$

- Vandermonde's Identity

$$C(n+m,k) = \sum_{i=0}^k C(n,i)C(m,k-i)$$

- Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii}=d(i)$ ,  $L_{ij}=-c$  where  $c$  is the number of edge  $(i,j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

- Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij}=x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i,j) \in E$ , otherwise  $d_{ij}=-d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

- Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each labeled vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of labeled forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k}=kn^{n-k-1}$ .

- Erdős-Gallai Theorem

A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + \dots + d_n$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$  holds for every  $1 \leq k \leq n$ .

- Gale-Ryser Theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic (degree sequence of bipartite graph) if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

- Fulkerson-Chen-Anstee Theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic (in, out degree of a directed graph) if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

- Möbius Inversion Formula

- $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right)$
- $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d)$

### Lagrange Multiplier

- Optimize  $f(x_1, \dots, x_n)$  when  $k$  constraints  $g_i(x_1, \dots, x_n) = 0$ .
- Lagrangian function  $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$ .
- The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

## 50 Estimation

- Number of divisors
 

$n \leq$	100	10 <sup>3</sup>	10 <sup>6</sup>	10 <sup>9</sup>	10 <sup>12</sup>	10 <sup>15</sup>	10 <sup>18</sup>
$\max d(n)$	12	32	240	1344	6720	26880	103680
- Unordered integer partition
 

$n$	2	3	4	5	6	7	8	9	20	30	40	50	100
$p(n)$	2	3	5	7	11	15	22	30	627	5604	$4 \cdot 10^4$	$2 \cdot 10^5$	$2 \cdot 10^8$
- Ways of partitions of  $n$  distinct elements
 

$n$	2	3	4	5	6	7	8	9	10	11	12	13
$B_n$	2	5	15	52	203	877	4140	21147	115975	$7 \cdot 10^5$	$4 \cdot 10^6$	$3 \cdot 10^7$

## 5P Numbers

- Bernoulli numbers
 
$$B_0 - 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$
- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.
 
$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$
- Pentagonal number theorem
 
$$\prod_{n=1}^{\infty} (1-x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$
- Catalan numbers
 
$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$
- Eulerian numbers
 

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 5Q GeneratingFunctions

- Ordinary Generating Function  $A(x) = \sum_{i \geq 0} a_i x^i$ 
  - $A(rx) \Rightarrow r^n a_n$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
  - $xA(x)' \Rightarrow na_n$
  - $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$
- Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$ 
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A^{(k)}(x) \Rightarrow a_{n+k}$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
  - $xA(x) \Rightarrow na_n$
- Special Generating Function
  - $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
  - $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{i}{n-1} x^i$
  - $S_k = \sum_{x=1}^n x^k: S = \sum_{p=0}^{\infty} x^p = \frac{e^x - e^{x(n+1)}}{1-e^x}$

## 6 Linear Algebra

### 6A GaussianElimination

```
struct Matrix {
    int m, n;
    vector<vector<fraction>> A;

    Matrix(int rows, int cols) : m(rows), n(cols), A(rows, vector<fraction>(cols)) {}

    void print() {
        for (int i = 0; i < m; ++i) {
            for (int j = 0; j < n; ++j) {
                cout << A[i][j];
            }
            cout << endl;
        }
    }
};
```

```
A[i][j].print();
cout << "\t";
}
cout << "\n";
}

// Reduced Row Echelon Form (over rationals)
void rref() {
    int row = 0;
    for (int col = 0; col < n && row < m; ++col) {
        int pivot = -1;
        for (int i = row; i < m; ++i)
            if (!A[i][col].is_zero()) {
                pivot = i;
                break;
            }
        if (pivot == -1) continue;

        swap(A[row], A[pivot]);

        // normalize pivot row
        fraction div = A[row][col];
        for (int j = 0; j < n; ++j)
            A[row][j] = A[row][j] / div;

        // eliminate other rows
        for (int i = 0; i < m; ++i) {
            if (i == row) continue;
            fraction factor = A[i][col];
            if (factor.is_zero()) continue;
            for (int j = 0; j < n; ++j)
                A[i][j] = A[i][j] - factor * A[row][j];
        }
        ++row;
    }
};
```

## 6B BerlekampMassey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(output.size() + 1), me, he;
    for (int f = 0, i = 1; i <= output.size(); ++i) {
        for (int j = 0; j < me.size(); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] -= output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f];
        o.emplace_back(-k);
        for (T x : he) o.emplace_back(x * k);
        o.resize(max(o.size(), me.size()));
        for (int j = 0; j < me.size(); ++j) o[j] += me[j];
        if (i - f + (int)he.size() >= (int)me.size()) he = me, f = i;
        me = o;
    }
    return me;
}
```

## 6C Simplex

Standard form: maximize  $\mathbf{c}^T \mathbf{x}$  subject to  $A\mathbf{x} \leq \mathbf{b}$  and  $\mathbf{x} \geq 0$ .  
 Dual LP: minimize  $\mathbf{b}^T \mathbf{y}$  subject to  $A^T \mathbf{y} \geq \mathbf{c}$  and  $\mathbf{y} \geq 0$ .  
 $\bar{x}$  and  $\bar{y}$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$  holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

- In case of minimization, let  $c'_i = -c_i$
- $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
- $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$

$$\sum_{1 \leq i \leq n} A_{j_i} x_i \geq b_j$$

4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

```

struct Simplex {
    using T = long double;
    static const int N = 50, M = 100;
    const T eps = 1e-7;
    int n, m;
    int Left[M], Down[N];
    T a[M][N], b[M], c[N], v, sol[N];
    bool eq(T a, T b) { return fabs(a - b) < eps; }
    bool ls (T a, T b) { return a < b && !eq(a, b); }
    void init(int _n, int _m) {
        n = _n, m = _m, v = 0;
        for(int
            i = 0; i < m; ++i) for (int j = 0; j < n; ++j) {
                a[i][j] = 0;
            }
        for(int i = 0; i < m; ++i) b[i] = 0;
        for(int i = 0; i < n; ++i) c[i] = sol[i] = 0;
    }
    void pivot(int x, int y) {
        swap(Left[x], Down[y]);
        T k = a[x][y]; a[x][y] = 1;
        vector<int> nz;
        for(int i = 0; i < n; ++i) {
            a[x][i] /= k;
            if(!eq(a[x][i], 0)) nz.push_back(i);
        }
        b[x] /= k;
        for(int i = 0 ; i < m; ++i) {
            if(i == x || eq(a[i][y], 0)) continue;
            k = a[i][y], a[i][y] = 0;
            b[i] -= k * b[x];
            for(int j : nz) a[i][j] -= k * a[x][j];
        }
        if(eq(c[y], 0)) return;
        k = c[y], c[y] = 0, v += k * b[x];
        for(int i : nz) c[i] -= k * a[x][i];
    }
    int solve() {
        for(int i = 0; i < n; ++i) Down[i] = i;
        for(int i = 0; i < m; ++i) Left[i] = n + i;
        while(1) {
            int x = -1, y = -1;
            for(int i = 0; i < m; ++i) if(ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
            if(x == -1) break;
            for(int i = 0; i < n; ++i) if(ls(a[x][i], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;
            if(y == -1) return 1;
            pivot(x, y);
        }
        while(1) {
            int x = -1, y = -1;
            for(int i = 0; i < n; ++i) if(ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
            if(y == -1) break;
            for(int
                i = 0; i < m; ++i) if(ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
            if (x == -1) return 2;
            pivot(x, y);
        }
        for(int i = 0;
            i < m; ++i) if(Left[i] < n) sol[Left[i]] = b[i];
    return 0;
}
};
```

## 7 Polynomials

### 7A NTT (FFT)

Mod	$g$	Form
65	537	$2^{16} + 1$
998	244 353	$119 \cdot 2^{23} + 1$
1 315	962 881	$1255 \cdot 2^{20} + 1$
1 711	276 033	$51 \cdot 2^{25} + 1$
9 223	372 036 737 335 297	$549755813881 \cdot 2^{24} + 1$

```

#define base ll // complex<double>
// const double PI = acosl(-1);
const ll mod = 998244353, g = 3;
base omega[4 * N], omega_[4 * N];
int rev[4 * N];

ll fpow(ll b, ll p);

ll inverse(ll a) { return fpow(a, mod - 2); }

void calcW(int n) {
    ll r = fpow(g, (mod - 1) / n), invr = inverse(r);
    omega[0] = omega_[0] = 1;
    for (int i = 1; i < n; i++) {
        omega[i] = omega[i - 1] * r % mod;
        omega_[i] = omega_[i - 1] * invr % mod;
    }
    // double arg = 2.0 * PI / n;
    // for (int i = 0; i < n; i++)
    // {
    //     omega[i] = base(cos(i * arg), sin(i * arg));
    //     omega_[i] = base(cos(-i * arg), sin(-i * arg));
    // }
}

void calcrev(int n) {
    int k = __lg(n);
    for (int i = 0; i < n; i++) rev[i] = 0;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < k; j++)
            if (i & (1 << j)) rev[i] ^= 1 << (k - j - 1);
}

vector<base> NTT(vector<base> poly, bool inv) {
    base *w = (inv ? omega_ : omega);
    int n = (int)poly.size();
    for (int i = 0; i < n; i++)
        if (rev[i] > i) swap(poly[i], poly[rev[i]]);

    for (int len = 1; len < n; len <= 1) {
        int arg = n / len / 2;
        for (int i = 0; i < n; i += 2 * len)
            for (int j = 0; j < len; j++) {
                base odd =
                    w[j * arg] * poly[i + j + len] % mod;
                poly[i + j + len] =
                    (poly[i + j] - odd + mod) % mod;
                poly[i + j] = (poly[i + j] + odd) % mod;
            }
        if (inv)
            for (auto &a : poly) a = a * inverse(n) % mod;
        return poly;
    }

    vector<base> mul(vector<base> f, vector<base> g) {
        int sz = 1 << (__lg(f.size()) + g.size() - 1) + 1;
        f.resize(sz), g.resize(sz);
        calcrev(sz);
        calcW(sz);
        f = NTT(f, 0), g = NTT(g, 0);
        for (int i = 0; i < sz; i++)
            f[i] = f[i] * g[i] % mod;
        return NTT(f, 1);
    }
}
```

### 7B FHWT

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
op: 1, invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { // or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int P = 21; // power of max N
int f[P][1 << P], g[P][1 << P], h[P][1 << P],
ct[1 << P];
void subset_convolution(
    int *a, int *b, int *c, int L) {
    // c[k] = \sum_{i | j = k, i & j = 0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i) fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i) c[i] = h[ct[i]][i];
}

```

## 7C PolynomialOperations

```

#define poly vector<ll>
poly inv(poly A) {
    A.resize(1 << (__lg(A.size() - 1) + 1));
    poly B = {inverse(A[0])};
    for (int n = 1; n < (int)A.size(); n <= 1) {
        poly pA(A.begin(), A.begin() + 2 * n);
        calcRev(4 * n), calcW(4 * n);
        pA.resize(4 * n), B.resize(4 * n);
        pA = NTT(pA, 0);
        B = NTT(B, 0);
        for (int i = 0; i < 4 * n; i++)
            B[i] =
                ((B[i] * 2 - pA[i] * B[i] % mod * B[i]) % mod +
                 mod) %
                mod;
        B = NTT(B, 1);
        B.resize(2 * n);
    }
    return B;
}
pair<poly, poly> div(poly A, poly B) {
    if (A.size() < B.size()) return make_pair(poly(), A);
    int n = A.size(), m = B.size();
    poly revA = A, invrevB = B;
    reverse(all(revA)), reverse(all(invrevB));
    revA.resize(n - m + 1);
    invrevB.resize(n - m + 1);
    invrevB = inv(invrevB);
    poly Q = mul(revA, invrevB);
    Q.resize(n - m + 1);
    reverse(all(Q));
    poly R = mul(Q, B);
    R.resize(m - 1);
    for (int i = 0; i < m - 1; i++)
        R[i] = (A[i] - R[i] + mod) % mod;
    return make_pair(Q, R);
}
poly modulo(poly A, poly B) { return div(A, B).S; }
ll fast_kitamasa(ll k, poly A, poly C) {
    int n = A.size();
    C.emplace_back(mod - 1);
    poly Q, R = {0, 1}, F = {1};
    R = modulo(R, C);

```

```

    for (; k; k >= 1) {
        if (k & 1) F = modulo(mul(F, R), C);
        R = modulo(mul(R, R), C);
        k >= 1;
    }
    ll ans = 0;
    for (int i = 0; i < F.size(); i++)
        ans = (ans + A[i] * F[i]) % mod;
    return ans;
}
vector<ll> fpow(vector<ll> f, ll p, ll m) {
    int b = 0;
    while (b < f.size() && f[b] == 0) b++;
    f = vector<ll>(f.begin() + b, f.end());
    int n = f.size();
    f.emplace_back(0);
    vector<ll> q(min(m, b * p), 0);
    q.emplace_back(fpow(f[0], p));
    for (int k = 0; q.size() < m; k++) {
        ll res = 0;
        for (int i = 0; i < min(n, k + 1); i++)
            res = (res +
                    p * (i + 1) % mod * f[i + 1] % mod *
                    q[k - i + b * p]) %
                    mod;
        for (int i = 1; i < min(n, k + 1); i++)
            res = (res -
                    f[i] * (k - i + 1) % mod *
                    q[k - i + 1 + b * p]) %
                    mod;
        res = (res < 0 ? res + mod : res) *
            inv(f[0] * (k + 1) % mod) % mod;
        q.emplace_back(res);
    }
    return q;
}

```

## 7D NewtonMethod+MiscGF

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomial such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

- $A^{-1}$ :  $B_{k+1} = B_k(2 - AB_k) \pmod{x^{2^{k+1}}}$
- $\ln A$ :  $(\ln A)' = \frac{A'}{A}$
- $\exp A$ :  $B_{k+1} = B_k(1 + A - \ln B_k) \pmod{x^{2^{k+1}}}$
- $\sqrt{A}$ :  $B_{k+1} = \frac{1}{2}(B_k + AB_k^{-1}) \pmod{x^{2^{k+1}}}$

## 8 Geometry

### 8A Basic

```

typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double pi = acos(-1);
const double eps = 1e-8;
pll operator+(pll a, pll b)
{ return pll(a.F + b.F, a.S + b.S); }
pll operator-(pll a, pll b)
{ return pll(a.F - b.F, a.S - b.S); }
pll operator-(pll a)
{ return pll(-a.F, -a.S); }
pll operator*(pll a, ll b)
{ return pll(a.F * b, a.S * b); }
pdd operator/(pll a, double b)
{ return pdd(a.F / b, a.S / b); }
ll dot(pll a, pll b)
{ return a.F * b.F + a.S * b.S; }
ll cross(pll a, pll b)
{ return a.F * b.S - a.S * b.F; }
ll abs2(pll a)

```

```

{ return dot(a, a); }
double abs(pll a)
{ return sqrt(dot(a, a)); }
int sign(ll a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pll a, pll b, pll c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pll p1, pll p2, pll p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pll a, pll b, pll c) {
    return collinearity
        (a, b, c) && sign(dot(a - c, b - c)) <= 0;
}
bool seg_strict_intersect
    (pdd p1, pdd p2, pdd p3, pdd p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    return a123 * a124 < 0 && a341 * a342 < 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4
        * a123 - p3 * a124) / (a123 - a124); // C^3 / C^2
}
pdd orth(pdd p1)
{ return pdd(-p1.S, p1.F); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 +
    (p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + orth(p2 - p1)
    * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0
    , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(
        cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r

```

## 8B ConvexHull

```

vector<pll> hull(vector<pll> dots) { // n=1 => ans = {}
    sort(dots.begin(), dots.end());
    vector<pll> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(all(dots)))
        for (int i = 1, t = (int)ans.size();
            i < (int)dots.size();
            ans.emplace_back(dots[i++]))
            while ((int)ans.size() > t &&
                ori(ans.end()[-2], ans.back(), dots[i]) <= 0)
                    ans.pop_back(); // "<" for keeping collinear
    ans.pop_back();
    return ans;
}

```

## 8C SortByAngle

```

bool down(pll k) {
    return sign(k.S) < 0 ||
        (sign(k.S) == 0 && sign(k.F) < 0);
}
int cmp(pll a, pll b, bool same = true) {
    int A = down(a), B = down(b);
}

```

```

if (A != B) return A < B;
if (sign(cross(a, b)) == 0)
    return same ? abs2(a) < abs2(b) : -1;
return sign(cross(a, b)) > 0;
}

```

## 8D Formulas

- Rotation

$$M(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

90 degree:  $(x,y) = (Y-y, x)$

- Pick's theorem  
For simple integer-coordinate polygon,

$$A = B + \frac{I}{2} - 1$$

Where  $A$  is the area;  $B, I$  is #lattice points in the interior, on the boundary.

- Spherical Cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume =  $\pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$ .
- Area =  $2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$ .

- Nearest points of two skew lines

- Line 1:  $v_1 = p_1 + t_1 d_1$
- Line 2:  $v_2 = p_2 + t_2 d_2$
- $n = d_1 \times d_2$
- $n_1 = d_1 \times n$
- $n_2 = d_2 \times n$
- $c_1 = p_1 + \frac{(p_2 - p_1) \cdot n_2}{d_1 \cdot n_2} d_1$
- $c_2 = p_2 + \frac{(p_1 - p_2) \cdot n_1}{d_2 \cdot n_1} d_2$

## 8E TriangleHearts

```

pdd excenter(
pdd p0, pdd p1, pdd p2) { // radius = abs(center)
p1 = p1 - p0, p2 = p2 - p0;
auto [x1, y1] = p1;
auto [x2, y2] = p2;
double m = 2. * cross(p1, p2);
pdd center = pdd((x1 * x1 * y2 - x2 * x2 * y1 +
    y1 * y2 * (y1 - y2)),
    (x1 * x2 * (x2 - x1) - y1 * y1 * x2 +
    x1 * y2 * y2)) / m;
return center + p0;
}

pdd incenter(
pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
double a = abs(p2 - p3), b = abs(p1 - p3),
c = abs(p1 - p2);
double s = a + b + c;
return (p1 * a + p2 * b + p3 * c) / s;
}

pdd masscenter(pdd p1, pdd p2, pdd p3) {
    return (p1 + p2 + p3) / 3;
}

pdd orthcenter(pdd p1, pdd p2, pdd p3) {
    return masscenter(p1, p2, p3) * 3 -
        excenter(p1, p2, p3) * 2;
}

```

## 8F PointSegmentDist

```

double PointSegDist(pdd q0, pdd q1, pdd p) {
    if (abs(q0 - q1) <= eps) return abs(q0 - p);
    if (dot(q1 - q0,
        p - q0) >= -eps && dot(q0 - q1, p - q1) >= -eps)
        return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

## 8G PointInCircle

```
// return q'
  s relation with circumcircle of tri(p[0],p[1],p[2])
bool in_cc(const array<pll, 3> p, pll q) {
    __int128 det = 0;
    for (int i = 0; i < 3; ++i)
        det += __int128(abs2(p[i]) - abs2(q)) *
            cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out: <0
}
```

## 8H PointInConvex

```
bool PointInConvex
    (const vector<pll> &C, pll p, bool strict = true) {
    int a = 1, b = (int)C.size() - 1, r = !strict;
    if ((int)C.size() == 0) return false;
    if ((int)C.size() < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}
```

## 8I PointToConvexHull

```
double PointToHull(const vector<pdd> &C, pdd p) {
    bool flg
        = (cross(C[0] - C.back(), p - C.back()) > 0);
    double ans = PointSegDist(C.back(), C[0], p);
    for (int i = 1; i < (int)C.size(); ++i) {
        ans =
            min(ans, PointSegDist(C[i - 1], C[i], p));
        if ((cross(C[i]
            ] - C[i - 1], p - C[i - 1]) <= 0)) flg = 0;
    }
    return (flg ? 0 : ans);
}
```

## 8J PointTangentConvex

```
/* The point should be strictly out of hull
   return arbitrary point on the tangent line */
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv: pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}
pii get_tangent(vector<pll> &C, pll p) {
    auto gao = [&](int s) {
        return cyc_tsearch((int)C.size(), [&](int x, int y) {
            { return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0
```

## 8K CircTangentCirc

```
vector<Line> go(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = abs2(c1.0 - c2.0);
    if (sign(d_sq) == 0) return ret;
```

```
    double d = sqrt(d_sq);
    pdd v = (c2.0 - c1.0) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        pdd n = pdd(v.F * c - sign2 * h * v.S,
                     v.S * c + sign2 * h * v.F);
        pdd p1 = c1.0 + n * c1.R;
        pdd p2 = c2.0 + n * (c2.R * sign1);
        if (sign(p1.F - p2.F) == 0 and
            sign(p1.S - p2.S) == 0)
            p2 = p1 + perp(c2.0 - c1.0);
        ret.emplace_back(Line(p1, p2));
    }
    return ret;
}
```

## 8L LineCircleIntersect

```
vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
    pdd p
        = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
    double s = cross
        (b - a, c - a), h2 = r * r - s * s / abs2(b - a);
    if (h2 < 0) return {};
    if (h2 == 0) return {p};
    pdd h = (b - a) / abs(b - a) * sqrt(h2);
    return {p - h, p + h};
}
```

## 8M LineConvexIntersect

```
int cyc_tsearch(int n, auto pred); // ref: TanPointHull
int TangentDir(vector<pll> &C, pll dir) {
    return cyc_tsearch((int)C.size(), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
#define cmpL(i) sign(cross(C[i] - a, b - a))
pii lineHull(pll a, pll b, vector<pll> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = (int)C.size();
    if (cmpL(A) < 0 || cmpL(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for (int t = l; (l + 1) % n != r;) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (cmpL(m) == cmpL(t) ? l : r) = m;
        }
        return (l + !cmpL(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if (res.F == res.S) // touching the corner i
        return pii(res.F, -1);
    if (!cmpL(res.F) &&
        !cmpL(res.S)) // along side i, i+1
        switch ((res.F - res.S + n + 1) % n) {
        case 0: return pii(res.F, res.F);
        case 2: return pii(res.S, res.S);
    }
    /* crossing sides (i, i+1) and (j, j+1)
       crossing corner i is treated as side (i, i+1)
       returned in the same order as the line hits the
       convex */
    return res;
} // convex cut: (r, l]
```

## 8N CircIntersectCirc

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.0, o2 = b.0;
    double r1 =
        a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
    if (d < max
        (r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
```

```

pdd u = (o1 + o2) * 0.5
    + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
double A = sqrt((r1 + r2 + d) *
    (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
pdd v
    = pdd(o1.S - o2.S, -o1.F + o2.F) * A / (2 * d2);
p1 = u + v, p2 = u - v;
return 1;
}

```

## 80 PolyIntersectCirc

```

// Divides into multiple triangle, and sum up
const double PI = acos(-1);
double _area(pdd pa, pdd pb, double r) {
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    double S, h, theta;
    double a = abs(pb), b = abs(pa), c = abs(pb - pa);
    double cosB = dot(pb, pb - pa) / a / c,
        B = acos(cosB);
    double cosC = dot(pa, pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < PI / 2)
            S -= (acos(h / r) * r * r -
                h * sqrt(r * r - h * h));
    } else if (b > r) {
        theta = PI - B - asin(sin(B) / r * a);
        S = .5 * a * r * sin(theta) +
            (C - theta) / 2 * r * r;
    } else S = .5 * sin(C) * a * b;
    return S;
}
double area_poly_circle(const vector<pdd> poly,
    const pdd &O, const double r) {
    double S = 0;
    for (int i = 0; i < (int)poly.size(); ++i)
        S += _area(poly[i] - O,
            poly[(i + 1) % (int)poly.size()] - O, r) *
            ori(
                0, poly[i], poly[(i + 1) % (int)poly.size()]);
    return fabs(S);
}

```

## 8P PolyUnion

```

double rat(pll a, pll b) {
    return sign
        (b.F) ? (double)a.F / b.F : (double)a.S / b.S;
} // all poly. should be ccw
double polyUnion(vector<vector<pll>> &poly) {
    double res = 0;
    for (auto &p : poly)
        for (int a = 0; a < (int)p.size(); ++a) {
            pll A = p[a], B = p[(a + 1) % (int)p.size()];
            vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
            for (auto &q : poly) {
                if (&p == &q) continue;
                for (int b = 0; b < (int)q.size(); ++b) {
                    pll C = q[b], D = q[(b + 1) % (int)q.size()];
                    int sc = ori(A, B, C), sd = ori(A, B, D);
                    if (sc != sd && min(sc, sd) < 0) {
                        double sa = cross(D
                            - C, A - C), sb = cross(D - C, B - C);
                        segs.emplace_back
                            (sa / (sa - sb), sign(sc - sd));
                    }
                    if (!sc && !sd &&
                        &q < &p && sign(dot(B - A, D - C)) > 0) {
                        segs.emplace_back(rat(C - A, B - A), 1);
                        segs.emplace_back(rat(D - A, B - A), -1);
                    }
                }
            }
        }
}

```

```

sort(all(segs));
for (auto &s : segs) s.F = clamp(s.F, 0.0, 1.0);
double sum = 0;
int cnt = segs[0].second;
for (int j = 1; j < (int)segs.size(); ++j) {
    if (!cnt) sum += segs[j].F - segs[j - 1].F;
    cnt += segs[j].S;
}
res += cross(A, B) * sum;
}
return res / 2;
}

```

## 8Q MinkowskiSum

```

void shift(vector<pdd>& h) {
    int p = 0; // must be convex hull
    for (int i = 1; i < h.size(); i++) {
        if (h[i].S < h[p].S) p = i;
        if (h[i].S == h[p].S && h[i].F < h[p].F) p = i;
    }
    rotate(h.begin(), h.begin() + p, h.end());
}
void Minkowski
    (vector<pdd>& a, vector<pdd>& b, vector<pdd>& c) {
    shift(a), shift(b); c = {};// |A|, |B|>=3
    int A = a.size(), B = b.size(), i = 0, j = 0;
    a.pb(a[0]), a.pb(a[1]), b.pb(b[0]), b.pb(b[1]);
    while (i < A || j < B) {
        c.pb(a[i] + b[j]);
        ll c = cross(a[i + 1] - a[i], b[j + 1] - b[j]);
        if (c >= 0 && i < A) i++;
        if (c <= 0 && j < B) j++;
    }
    for (int
        i = 0; i < 2; i++) a.pop_back(), b.pop_back();
}

```

## 8R DistanceBetweenHull

```

double ConvexHullDist
    (vector<pdd>& A, vector<pdd>& B, vector<pdd>& C) {
    Minkowski(A, B, C); // assert (int)C.size() > 0
    return PointToHull(C, pdd(0, 0));
}

```

## 8S MinMaxEnclosingRect

```

const double qi = acos(-1) / 2 * 3;
pdd solve(vector<pll> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[0])
    hull(dots);
    double Max = 0, Min = INF, deg;
    int n = (int)dots.size();
    dots.emplace_back(dots[0]);
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        pll nw = vec(i + 1);
        while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
            u = (u + 1) % n;
        while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
            r = (r + 1) % n;
        if (!i) l = (r + 1) % n;
        while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
            l = (l + 1) % n;
        Min = min(Min, (double)(dot(nw, vec(r)) - dot
            (nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
        deg = acos(dot(diff(r
            , l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
        deg = (qi - deg) / 2;
        Max = max(Max, abs(diff
            (r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
    }
    return pdd(Min, Max);
}

```

## 8T CircleCover

```

// N ~= 1000
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[ N ];
    void init(int _c){ C = _c;}
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a
            , double _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a) const
        {return ang < a.ang;}
        Teve[N * 2];
    };
    // strict: x = 0, otherwise x = -1
    bool disjuct(Cir &a, Cir &b, int x)
    {return sign(abs(a.0 - b.0) - a.R - b.R) > x;}
    bool contain(Cir &a, Cir &b, int x)
    {return sign(a.R - b.R - abs(a.0 - b.0)) > x;}
    bool contain(int i, int j) {
        /* clj is non-strictly in ci]. */
        return (sign
            (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].R) == 0 && i < j)) && contain(c[i], c[j], -1);
    }
    void solve(){
        fill_n(Area, C + 2, 0);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                overlap[i][j] = contain(i, j);
        for(int i = 0; i < C; ++i)
            for(int j = 0; j < C; ++j)
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
        for(int i = 0; i < C; ++i){
            int E = 0, cnt = 1;
            for(int j = 0; j < C; ++j)
                if(j != i && overlap[j][i])
                    ++cnt;
            for(int j = 0; j < C; ++j)
                if(i != j && g[i][j]) {
                    pdd aa, bb;
                    CCinter(c[i], c[j], aa, bb);
                    double A =
                        atan2(aa.S - c[i].0.S, aa.F - c[i].0.F);
                    double B =
                        atan2(bb.S - c[i].0.S, bb.F - c[i].0.F);
                    eve[E++] = Teve
                        (bb, B, 1), eve[E++] = Teve(aa, A, -1);
                    if(B > A) ++cnt;
                }
            if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
            else{
                sort(eve, eve + E);
                eve[E] = eve[0];
                for(int j = 0; j < E; ++j){
                    cnt += eve[j].add;
                    Area[cnt]
                        ] += cross(eve[j].p, eve[j + 1].p) * .5;
                    double theta = eve[j + 1].ang - eve[j].ang;
                    if (theta < 0) theta += 2. * pi;
                    Area[cnt] += (theta
                        - sin(theta)) * c[i].R * c[i].R * .5;
                }
            }
        }
    }
};


```

## 8U LineCmp

```

struct lineCmp { // coordinates should be even!
    bool operator()(Line l1, Line l2) const {

```

```

        int X =
            (max(l1.F.F, l2.F.F) + min(l1.S.F, l2.S.F)) / 2;
        ll p1 =
            (X - l1.F.F) * l1.S.S + (l1.S.F - X) * l1.F.S,
        p2 =
            (X - l2.F.F) * l2.S.S + (l2.S.F - X) * l2.F.S,
        q1 = (l1.S.F - l1.F.F), q2 = (l2.S.F - l2.F.F);
        if (q1 == 0) p1 = l1.F.S + l1.S.S, q1 = 2;
        if (q2 == 0) p2 = l2.F.S + l2.S.S, q2 = 2;
        // for query a point: ask make_pair(P, P)
        if (l1.F == l2.F || l2.F == l2.S) l1 = l2;
        return make_tuple((__int128)(p1 * q2), l1) <
            make_tuple((__int128)(p2 * q1), l2);
    }
};


```

## 8V Trapezoidalization

```

template<class T>
struct SweepLine {
    struct cmp {
        cmp(const SweepLine &swp): swp(_swp) {}
        bool operator()(int a, int b) const {
            if (abs(swp.get_y(a) - swp.get_y(b)) <= swp.eps)
                return swp.slope_cmp(a, b);
            return swp.get_y(a) + swp.eps < swp.get_y(b);
        }
        const SweepLine &swp;
    } _cmp;
    T curTime, eps, curQ;
    vector<Line> base;
    multiset<int, cmp> sweep;
    multiset<pair<T, int>> event;
    vector<typename multiset<int, cmp>::iterator> its;
    vector<typename multiset<pair<T, int>>::iterator> eits;
    bool slope_cmp(int a, int b) const {
        assert(a != -1);
        if (b == -1) return 0;
        return sign(cross(base
            [a].S - base[a].F, base[b].S - base[b].F)) < 0;
    }
    T get_y(int idx) const {
        if (idx == -1) return curQ;
        Line l = base[idx];
        if (l.F.F == l.S.F) return l.S.S;
        return ((curTime - l.F.F) * l.S.S
            + (l.S.F - curTime) * l.F.S) / (l.S.F - l.F.F);
    }
    void insert(int idx) {
        its[idx] = sweep.insert(idx);
        if (its[idx] != sweep.begin())
            update_event(*prev(its[idx]));
        update_event(idx);
        event.emplace
            (base[idx].S.F, idx + 2 * (int)base.size());
    }
    void erase(int idx) {
        assert(eits[idx] == event.end());
        auto p = sweep.erase(its[idx]);
        its[idx] = sweep.end();
        if (p != sweep.begin())
            update_event(*prev(p));
    }
    void update_event(int idx) {
        if (eits[idx] != event.end())
            event.erase(eits[idx]);
        eits[idx] = event.end();
        auto nxt = next(its[idx]);
        if (nxt ==
            sweep.end() || !slope_cmp(idx, *nxt)) return;
        auto t = intersect(base[idx].F, base[idx].S, base[*nxt].F, base[*nxt].S);
        if (t + eps < curTime || t
            >= min(base[idx].S.F, base[*nxt].S.F)) return;
        eits[idx]
            ] = event.emplace(t, idx + (int)base.size());
    }
};


```

```

}

void swp(int idx) {
    assert(eits[idx] != event.end());
    eits[idx] = event.end();
    int nxt = *next(eits[idx]);
    swap((int&)*eits[idx], (int&)*eits[nxt]);
    swap(eits[idx], eits[nxt]);
    if (eits[nxt] != sweep.begin())
        update_event(*prev(eits[nxt]));
    update_event(idx);
}

// only expected to call the functions below
SweepLine(T t, T e, vector<Line> vec): _cmp
    (*this), curTime(t), eps(e), curQ(), base(vec),
    sweep(_cmp), event(), its((int)vec.size(), sweep
    .end()), eits((int)vec.size(), event.end()) {
        for (int i = 0; i < (int)base.size(); ++i) {
            auto &[p, q] = base[i];
            if (p > q) swap(p, q);
            if (p.F <= curTime && curTime <= q.F)
                insert(i);
            else if (curTime < p.F)
                event.emplace(p.F, i);
        }
    }

void setTime(T t, bool ers = false) {
    assert(t >= curTime);
    while (!event.empty() && event.begin()->F <= t) {
        auto [et, idx] = *event.begin();
        int s = idx / (int)base.size();
        idx %= (int)base.size();
        if (abs(et - t) <= eps && s == 2 && !ers) break;
        curTime = et;
        event.erase(event.begin());
        if (s == 2) erase(idx);
        else if (s == 1) swp(idx);
        else insert(idx);
    }
    curTime = t;
}

T nextEvent() {
    if (event.empty()) return INF;
    return event.begin()->F;
}

int lower_bound(T y) {
    curQ = y;
    auto p = sweep.lower_bound(-1);
    if (p == sweep.end()) return -1;
    return *p;
}
};

```

## 8W HalfPlaneIntersect

```

pll area_pair(Line a, Line b)
{ return pll(cross(a.S
    - a.F, b.F - a.F), cross(a.S - a.F, b.S - a.F)); }

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return (_int128)
        a02Y * a12X - (_int128) a02X * a12Y > 0; // C^4
}

/* Having solution, check size > 2 */
/* --^- Line.X --^- Line.Y --^- */
vector<Line> halfPlaneInter(vector<Line> arr) {
    sort(all(arr), [&](Line a, Line b) -> int {
        if (cmp(a.S - a.F, b.S - b.F, 0) != -1)
            return cmp(a.S - a.F, b.S - b.F, 0);
        return ori(a.F, a.S, b.S) < 0;
    });
    deque<Line> dq(1, arr[0]);
    for (auto p : arr) {

```

```

        if (cmp(
            dq.back().S - dq.back().F, p.S - p.F, 0) == -1)
            continue;
        while ((int)dq.size() >= 2
            && !isin(p, dq[(int)dq.size() - 2], dq.back()))
            dq.pop_back();
        while ((int)dq.size() >= 2 && !isin(p, dq[0], dq[1]))
            dq.pop_front();
        dq.emplace_back(p);
    }
    while ((int)dq.size() >= 3 &&
        !isin(dq[0], dq[(int)dq.size() - 2], dq.back()))
        dq.pop_back();
    while ((int)
        dq.size() >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    return vector<Line>(all(dq));
}

```

## 8X RotatingSweepLine

```

void rotatingSweepLine(vector<pii> &ps) {
    int n = (int)ps.size(), m = 0;
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1));
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            if (i != j) line[m++] = pii(i, j);
    sort(all(line), [&](pii a, pii b) {
        return cmp(ps[a.S] - ps[a.F], ps[b.S] - ps[b.F]);
    }); // cmp(): polar angle compare
    iota(all(id), 0);
    sort(all(id), [&](int a, int b) {
        if (ps[a].S != ps[b].S) return ps[a].S < ps[b].S;
        return ps[a] < ps[b];
    }); // initial order, since (1, 0) is the smallest
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (int i = 0; i < m; ++i) {
        auto l = line[i];
        // do something
        tie(pos[l.F], pos[l.S], id[pos[l.F]], id[pos[l.S
            ]]) = make_tuple(pos[l.S], pos[l.F], l.S, l.F);
    }
}

```

## 8Y DelaunayTriangulation

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle. */

struct Edge {
    int id; // oidx[id]
    list<Edge>::iterator twin;
    Edge(int _id = 0) : id(_id) {}
};

struct Delaunay { // θ-base
    int n, oidx[N];
    list<Edge> head[N]; // result udir. graph
    pll p[N];
    void init(int _n, pll _p[]) {
        n = _n, iota(oidx, oidx + n, 0);
        for (int i = 0; i < n; ++i) head[i].clear();
        sort(oidx, oidx + n,
            [&](int a, int b) { return _p[a] < _p[b]; });
        for (int i = 0; i < n; ++i) p[i] = _p[oidx[i]];
        divide(0, n - 1);
    }
    void addEdge(int u, int v) {
        head[u].push_front(Edge(v));
        head[v].push_front(Edge(u));
        head[u].begin()->twin = head[v].begin();
        head[v].begin()->twin = head[u].begin();
    }
    void divide(int l, int r) {
        if (l == r) return;

```

```

if (l + 1 == r) return addEdge(l, l + 1);
int mid = (l + r) >> 1, nw[2] = {l, r};
divide(l, mid), divide(mid + 1, r);
auto gao = [&](int t) {
    pll pt[2] = {p[nw[0]], p[nw[1]]};
    for (auto it : head[nw[t]]) {
        int v = ori(pt[1], pt[0], p[it.id]);
        if (v > 0 ||
            (v == 0 &&
             abs2(pt[t ^ 1] - p[it.id]) <
             abs2(pt[1] - pt[0])))
            return nw[t] = it.id, true;
    }
    return false;
};
while (gao(0) || gao(1));
addEdge(nw[0], nw[1]); // add tangent
while (true) {
    pll pt[2] = {p[nw[0]], p[nw[1]]};
    int ch = -1, sd = 0;
    for (int t = 0; t < 2; ++t)
        for (auto it : head[nw[t]])
            if (ori(pt[0], pt[1], p[it.id]) > 0 &&
                (ch == -1 ||
                 in_cc({pt[0], pt[1], p[ch]}, p[it.id])))
                ch = it.id, sd = t;
    if (ch == -1) break; // upper common tangent
    for (auto it = head[nw[sd]].begin();
         it != head[nw[sd]].end();)
        if (seg_strict_intersect(
            pt[sd], p[it->id], pt[sd ^ 1], p[ch]))
            head[it->id].erase(it->twin),
            head[nw[sd]].erase(it++);
        else ++it;
    nw[sd] = ch, addEdge(nw[0], nw[1]);
}
} tool;

```

## 8Z VoronoiDiagram

```

// all coord. is even
// you may want to call halfPlaneInter after then
vector<vector<Line>> vec;
void build_voronoi_line(int n, pll *arr) {
    tool.init(n, arr); // Delaunay
    vec.clear(), vec.resize(n);
    for (int i = 0; i < n; ++i)
        for (auto e : tool.head[i]) {
            int u = tool.oidx[i], v = tool.oidx[e.id];
            pll m = (arr[v] + arr[u]) / 2LL, d = perp(arr[v] - arr[u]);
            vec[u].emplace_back(Line(m, m + d));
        }
}

```

## 9 Misc

### 9A ManhattanMST

```

#define p3i tuple<int, int, int>
struct DSU {
    vector<int> v;
    DSU(int n);
    int query(int u);
    void merge(int x, int y);
};
vector<p3i> manhattanMST(vector<pll> ps) {
    vector<int> id(ps.size());
    iota(id.begin(), id.end(), 0);
    vector<p3i> edges;
    for (int k = 0; k < 4; ++k) {
        sort(id.begin(), id.end(), [&](int i, int j) {
            return (ps[i] - ps[j]).F < (ps[j] - ps[i]).S;
        });
        map<int, int> sweep;
        for (int i : id) {

```

```

            for (auto it = sweep.lower_bound(-ps[i].S);
                 it != sweep.end(); sweep.erase(it++)) {
                int j = it->second;
                pll d = ps[i] - ps[j];
                if (d.S > d.F) break;
                edges.emplace_back(d.S + d.F, i, j);
            }
            sweep[-ps[i].S] = i;
        }
        for (auto &p : ps)
            if (k & 1) p.F = -p.F;
            else swap(p.F, p.S);
    }
    return edges;
}

vector<int> MST(int n, const vector<p3i> &e) {
    vector<int> idx(e.size());
    iota(idx.begin(), idx.end(), 0);
    sort(idx.begin(), idx.end(), [&](int i, int j) {
        return get<0>(e[i]) < get<0>(e[j]);
    });
    vector<int> r;
    DSU dsu(n);
    for (int o : idx) {
        const auto &[w, i, j] = e[o];
        if (dsu.query(i) == dsu.query(j)) continue;
        r.push_back(o);
        dsu.merge(i, j);
    }
    return r;
}

```

## 9B SternBrocotTree

- Construction: Root  $\frac{1}{1}$ , left/right neighbor  $\frac{0}{1}, \frac{1}{0}$ , each node is sum of last left/right neighbor:  $\frac{a}{b}, \frac{c}{d} \rightarrow \frac{a+c}{b+d}$
- Property: Adjacent (mid-order DFS)  $\frac{a}{b}, \frac{c}{d} \rightarrow bc-ad=1$ .
- Search known  $\frac{p}{q}$ : keep L-R alternative. Each step can calculated in  $O(1) \Rightarrow$  total  $O(\log C)$ .
- Search unknown  $\frac{p}{q}$ : keep L-R alternative. Each step can calculated in  $O(\log C)$  checks  $\Rightarrow$  total  $O(\log^2 C)$  checks.

## 9C CDQ

```

void solve(int l, int r, vector<Point> &a) { // 三維偏序
    if (l == r) return;
    int mid = (l+r)>>1;
    solve(l, mid, a), solve(mid+1, r, a);
    int lp = l, rp = mid+1;
    vector<Point> tmp;
    while(lp <= mid && rp <= r) {
        if (a[lp].y > a[rp].y) {
            bit.upd(a[lp].z, 1);
            tmp.pb(a[lp++]);
        } else {
            ans[a[rp].id]
                ] += (bit.sum(maxn)-bit.sum(a[rp].z));
            tmp.pb(a[rp++]);
        }
    }
    while(lp <= mid) {
        bit.upd(a[lp].z, 1);
        tmp.pb(a[lp++]);
    }
    while(rp <= r) {
        ans[a[rp]
            ].id ] += (bit.sum(maxn)-bit.sum(a[rp].z));
        tmp.pb(a[rp++]);
    }
    for (int i = l; i <= mid; ++i) bit.upd(a[i].z, -1);
    for (int i = l; i <= r; ++i) a[i] = tmp[i-l];
}

```

## 9D SimulatedAnnealing

```
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans: answer, nw: current value
    if (exp(-(nw -
        ans) / factor) >= (double)(rd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

## 9E Python

```
import math
math.sqrt(2) # integer sqrt
from decimal import *
Decimal(str(0.1)) # prevent precision issue
getcontext().prec = 100
```