Contents

0 Basic

0A .vimrc

```
sy on
set si nu rnu cin cul et so=10 ts=4 sw=4 mouse=a
ino {<cr> {<cr>}<esc>ko
ino jk <esc>
map <F7> :w<CR>:!g++ "%" -std
    =c++17 -DLOCAL -Wall -Wextra -Wshadow -Wconversion
     -fsanitize=address,undefined -g && ./a.out<CR>
ca Hash w !cpp -dD -P -fpreprocessed
     \| tr -d "[:space:]" \| md5sum \| cut -c-6
```

0B PBDS

```
// Tree and fast PQ
#include <bits/extc++.h>
using namespace __gnu_pbds;

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
tree<int, null_type, less<int>, rb_tree_tag
    , tree_order_statistics_node_update> bst;
// order_of_key(n): # of elements <= n
// find_by_order(n): 0-indexed

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
__gnu_pbds::priority_queue
    <int, greater<int>, thin_heap_tag> pq;
```

0C pragma

```
#pragma GCC optimize("Ofast,unroll-loops")
#pragma GCC target("avx,avx2,sse,sse2
    ,sse3,ssse3,sse4,popcnt,abm,mmx,fma,tune=native")
// chrono
    ::steady_clock::now().time_since_epoch().count()
```

0D Default Code

```
using namespace std;

#define F first
#define S second
#define pii pair<int, int>
#define pll pair<ll, ll>
#define pdd pair<double, double>
#define ll long long
#define ld long double
#define i128 __int128

#define all(x) x.begin(), x.end()
#define pb emplace_back

#ifdef LOCAL
#define px(
    args...) LKJ("\033[1;32m(" #args "):\033[0m", args)
template<class I> void LKJ(I&&x){ cerr << x << '\n'; }
template<class I, class...T> void
     LKJ(I&&x, T&&...t){ cerr << x << ' ', LKJ(t...); }
template<class I> void OI(I a, I b){ while
    (a < b) cerr << *a << " \n"[next(a) == b], ++a; }
#define pv(v) cerr
     << "\033[1;31m[" << #v << "]: \033[0m"; OI(all(v))
#else
#define px(...)
#define OI(...)
#define pv(v)
#endif

template<class A, class
     B> ostream& operator<<(ostream &os, pair<A, B> p)
     { return os << '(' << p.F << ", " << p.S << ')'; }

void solve() {}

int main() {
  cin.tie(0)->sync_with_stdio(0);
  int T = 1;

  // cin >> T;
  while (T--) solve();
}
```

0E LambdaCompare

```
auto cmp = [](int x, int y) { return x < y; };
std::set<int, decltype(cmp)> st(cmp);
```

1 Graph

1A 2SAT/SCC

```
struct SAT { // 0-base
  int low[N], dfn[N], bln[N], n, Time, nScc;
  bool instack[N], istrue[N];
  stack<int> st;
  vector<int> G[N], SCC[N];
  void init(int _n) {
    n = _n; // assert(n * 2 <= N);
    for (int i = 0; i < n + n; ++i) G[i].clear();
  }
  void add_edge(int a, int b) { G[a].emplace_back(b); }
  int rv(int a) {
    if (a >= n) return a - n;
    return a + n;
  }
  void add_clause(int a, int b) {
    add_edge(rv(a), b), add_edge(rv(b), a);
  }
  void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for (int i : G[u])
      if (!dfn[i])
        dfs(i), low[u] = min(low[i], low[u]);
      else if (instack[i] && dfn[i] < dfn[u])
        low[u] = min(low[u], dfn[i]);
    if (low[u] == dfn[u]) {
      int tmp;
      do {
        tmp = st.top(), st.pop();
        instack[tmp] = 0, bln[tmp] = nScc;
      } while (tmp != u);
      ++nScc;
    }
  }
  bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
      SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
      if (!dfn[i]) dfs(i);
    for (int i =
         0; i < n + n; ++i) SCC[bln[i]].emplace_back(i);
    bool flag = true;
    for (int i = 0; i < n; ++i) {
      if (bln[i] == bln[i + n]) flag = false;
      istrue[i] = bln[i] < bln[i + n];
      istrue[i + n] = !istrue[i];
    }
    return flag;
    // return whether there is a set of solutions
    // istrue[] are one set of solutions if any
  }
};
```

1B Vertex BCC

```
struct vertex_cc {
  struct edge {
    int to, nt;
  } e[M << 1];
  int hd[N], tot = 1;

  void add(int u, int v) {
    e[++tot] = edge{v, hd[u]}, hd[u] = tot; }
  void uadd(int u, int v) { add(u, v), add(v, u); }

  int ans, top, cnt, ord, root;
  int dfn[N], low[N], sta[N];
  bool cut[N];
  vector<int> dcc[N];

  void tarjan(int u) {
    dfn[u] = low[u] = ++ord, sta[++top] = u;
    if (u == root && hd[u] == 0) {
      dcc[++cnt].push_back(u);
      return;
    }
    int f = 0;
    for (int i = hd[u]; i; i = e[i].nt) {
      int v = e[i].to;
      if (!dfn[v]) {
        tarjan(v);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
          if (++f > 1 || u != root) cut[u] = true;
          cnt++;
          do dcc[cnt].push_back(sta[top--]);
          while (sta[top + 1] != v);
          dcc[cnt].push_back(u);
        }
      } else low[u] = min(low[u], dfn[v]);
    }
  }
  int solve
      (int L, int R) { // vertex index range [L, R)
    for (int i = L; i < R; ++i) dfn[i] = 0;
    for (int i = L; i < R; ++i)
      if (!dfn[i]) { root = i; tarjan(i); }
    return cnt;
  } // answer in dcc, BCCs are 1-based
};
```

1C Edge BCC

```
namespace bridge_cc { // vertex 0-based
  vector<int> tim, low;
  stack<int, vector<int>> st;
  int t, bcc_id;
  void dfs(int u, int p, const
        vector<vector<pii>> &edge, vector<int> &pa) {
    tim[u] = low[u] = t++;
    st.push(u);
    for (const auto &[v, id] : edge[u]) {
      if (id == p)
        continue;
      if (tim[v])
        low[u] = min(low[u], tim[v]);
      else {
        dfs(v, id, edge, pa);
        if(low[v] > tim[u]) {
          int x;
          do {
            pa[x = st.top()] = bcc_id;
            st.pop();
          } while (x != v);
          bcc_id++;
        }
        else
          low[u] = min(low[u], low[v]);
      }
    }
  }
  vector<int> solve
      (const vector<vector<pii>> &edge) { // (to, id)
    int n = edge.size();
    tim.resize(n);
    low.resize(n);
    t = bcc_id = 1;
    vector<int> pa(n);

    for (int i = 0; i < n; i++) {
      if (!tim[i]) {
        dfs(i, -1, edge, pa);
        while (!st.empty()) {
          pa[st.top()] = bcc_id;
          st.pop();
        }
        bcc_id++;
      }
    }
    return pa;
  } // return bcc id(start from 1)
};
```

1D Virtual Tree

```
// requires DFS io, lca, is_child
vector<int> tre[N];
bool cmp(int a, int b){ return in[a] < in[b]; }
void add_edge(int a, int b){
  tre[a].emplace_back(b);
  tre[b].emplace_back(a);
}
void virtual_tree(vector<int> arr, int k){
  vector<int> sta;
  sort(arr.begin(), arr.end(), cmp);
  for (int i = 1; i < k; i++)
    arr.emplace_back(lca(arr[i], arr[i - 1]));
  sort(arr.begin(), arr.end(), cmp);
  arr.resize
      (unique(arr.begin(), arr.end()) - arr.begin());
  for (auto i : arr){
    while (!sta.empty
        () && !is_child(sta.back(), i)) sta.pop_back();
    if (!sta.empty()) add_edge(sta.back(), i);
    sta.push_back(i);
  }
}
```

1E Maximum Clique

```
struct MaxClique
     { // max complete subgraph, fast when N <= 100
  bitset<N> G[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) G[i].reset();
  }
  void add_edge(int u, int v) {
    G[u][v] = G[v][u] = 1;
  }
  void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
    if (l < 4) {
      for (int i : r) d[i] = (G[i] & mask).count();
      sort(all(r)
          , [&](int x, int y) { return d[x] > d[y]; });
    }
    vector<int> c(r.size());
    int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
    cs[1].reset(), cs[2].reset();
    for (int p : r) {
      int k = 1;
      while ((cs[k] & G[p]).any()) ++k;
      if (k > rgt) cs[++rgt + 1].reset();
      cs[k][p] = 1;
      if (k < lft) r[tp++] = p;
    }
    for (int k = lft; k <= rgt; ++k)
      for (int p = cs[k]._Find_first
          (); p < N; p = cs[k]._Find_next(p))
        r[tp] = p, c[tp] = k, ++tp;
    dfs(r, c, l + 1, mask);
  }
  void dfs(vector<
      int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pop_back(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr;
      for (int i : r) if (G[p][i]) nr.emplace_back(i);
      if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
      else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pop_back(), --q;
    }
  }
  int solve() {
    vector<int> r(n);
    ans = q = 0, iota(all(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
  } // first ans elements in sol form the vertex set
};
```

1F MinimumSteinerTree

```
struct SteinerTree { // 0-base
  int n, dst[N][N], dp[1 << T][N], tdst[N];
  int vcst[N]; // the cost of vertexs
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) {
      fill_n(dst[i], n, INF);
      dst[i][i] = vcst[i] = 0;
    }
  }
  void chmin(int &x, int val) {
    x = min(x, val);
  }
  void add_edge(int ui, int vi, int wi) {
    chmin(dst[ui][vi], wi);
  }
  void shortest_path() {
    for (int k = 0; k < n; ++k)
      for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
          chmin(dst[i][j], dst[i][k] + dst[k][j]);
  }
  int solve(const vector<int>& ter) {
    shortest_path();
    int t = ter.size(), full = (1 << t) - 1;
    for (int i = 0; i <= full; ++i)
      fill_n(dp[i], n, INF);
    copy_n(vcst, n, dp[0]);
    for (int msk = 1; msk <= full; ++msk) {
      if (!(msk & (msk - 1))) {
        int who = __lg(msk);
        for (int i = 0; i < n; ++i)
          dp[msk
              ][i] = vcst[ter[who]] + dst[ter[who]][i];
      }
      for (int i = 0; i < n; ++i)
        for (int sub = (
            msk - 1) & msk; sub; sub = (sub - 1) & msk)
          chmin(dp[msk][i],
              dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
      for (int i = 0; i < n; ++i) {
        tdst[i] = INF;
        for (int j = 0; j < n; ++j)
          chmin(tdst[i], dp[msk][j] + dst[j][i]);
      }
      copy_n(tdst, n, dp[msk]);
    }
    return *min_element(dp[full], dp[full] + n);
  }
}; // O(V 3^T + V^2 2^T)
```

1G Dominator Tree

```
struct DominatorTree { // 1-base
  vector<int> G[N], rG[N];
  int n, pa[N], dfn[N], id[N], Time;
  int semi[N], idom[N], best[N];
  vector<int> tree[N]; // dominator_tree
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      G[i].clear(), rG[i].clear();
  }
  void add_edge(int u, int v) {
    G[u].emplace_back(v), rG[v].emplace_back(u);
  }
  void dfs(int u) {
    id[dfn[u] = ++Time] = u;
    for (auto v : G[u])
      if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
  }
  int find(int y, int x) {
    if (y <= x) return y;
    int tmp = find(pa[y], x);
    if (semi[best[y]] > semi[best[pa[y]]])
      best[y] = best[pa[y]];
    return pa[y] = tmp;
  }
  void tarjan(int root) {
    Time = 0;
    for (int i = 1; i <= n; ++i) {
      dfn[i] = idom[i] = 0;
      tree[i].clear();
      best[i] = semi[i] = i;
    }
    dfs(root);
    for (int i = Time; i > 1; --i) {
      int u = id[i];
      for (auto v : rG[u])
        if (v = dfn[v]) {
          find(v, i);
          semi[i] = min(semi[i], semi[best[v]]);
        }
      tree[semi[i]].emplace_back(i);
      for (auto v : tree[pa[i]]) {
        find(v, pa[i]);
        idom[v] =
          semi[best[v]] == pa[i] ? pa[i] : best[v];
      }
      tree[pa[i]].clear();
    }
    for (int i = 2; i <= Time; ++i) {
      if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
      tree[id[idom[i]]].emplace_back(id[i]);
    }
  }
};
```

1H Directed MST (slow)

```
struct DMST { // O(VE)
  struct edge {
    int u, v;
    ll w;
  };
  vector<edge> E; // 0-base
  int pe[N], id[N], vis[N];
  ll in[N];
  void init() { E.clear(); }
  void add_edge(int u, int v, ll w) {
    if (u != v) E.emplace_back(edge{u, v, w});
  }
  ll build(int root, int n) {
    ll ans = 0;
    for (;;) {
      fill_n(in, n, INF);
      for (int i = 0; i < (int)E.size(); ++i)
        if (E[i].u != E[i].v && E[i].w < in[E[i].v])
          pe[E[i].v] = i, in[E[i].v] = E[i].w;
      for (int u = 0; u < n; ++u) // no solution
        if (u != root && in[u] == INF) return -INF;
      int cntnode = 0;
      fill_n(id, n, -1), fill_n(vis, n, -1);
      for (int u = 0; u < n; ++u) {
        if (u != root) ans += in[u];
        int v = u;
        while (vis[v] != u && !~id[v] && v != root)
          vis[v] = u, v = E[pe[v]].u;
        if (v != root && !~id[v]) {
          for (int x = E[pe[v]].u; x != v;
               x = E[pe[x]].u)
            id[x] = cntnode;
          id[v] = cntnode++;
        }
      }
      if (!cntnode) break; // no cycle
      for (int u = 0; u < n; ++u)
        if (!~id[u]) id[u] = cntnode++;
      for (int i = 0; i < (int)E.size(); ++i) {
        int v = E[i].v;
        E[i].u = id[E[i].u], E[i].v = id[E[i].v];
        if (E[i].u != E[i].v) E[i].w -= in[v];
      }
      n = cntnode, root = id[root];
    }
    return ans;
  }
};
```

1I Directed MST (fast)

```
// O(E + V log V)
#define rep(i, a, b) for (int i = a; i < (b); ++i)
#define sz(x) (int)(x).size()
typedef vector<int> vi;
struct RollbackUF {
  vi e;
  vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i-- > t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b];
    e[b] = a;
    return true;
  }
};
struct Edge {
  int a, b;
  ll w;
};
struct Node { // lazy skew heap node
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() {
    prop();
    return key;
  }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node *&a) {
  a->prop();
  a = merge(a->l, a->r);
}

pair<ll, vi> dmst(int n, int r, vector<Edge> &g) {
  RollbackUF uf(n);
  vector<Node *> heap(n);
  for (Edge e : g)
    heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1, -1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s, 0, n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1, {}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) { /// found cycle, contract
        Node *cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto &[u, t, cmp] : cycs) {
    // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto &e : cmp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i, 0, n) par[i] = in[i].a;
  return {res, par};
}
```

1J Minimum Clique Cover

```
struct CliqueCover
     { // 0-base, O(n2^n), 用最少數量的團覆蓋無向圖
  int co[1 << N], n, E[N];
  int dp[1 << N];
  void init(int _n) {
    n = _n, fill_n(dp, 1 << n, 0);
    fill_n(E, n, 0), fill_n(co, 1 << n, 0);
  }
  void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
  }
  int solve() {
    for (int i = 0; i < n; ++i)
      co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
      int t = i & -i;
      dp[i] = -dp[i ^ t];
      co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
      co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1); // needs FWHT
    for (int ans = 1; ans < n; ++ans) {
      int sum = 0; // probabilistic
      for (int i = 0; i < (1 << n); ++i)
        sum += (dp[i] *= co[i]);
      if (sum) return ans;
    }
    return n;
  }
};
```

1K Count Maximal Clique

```
struct BronKerbosch { // 1-base
  int n, a[N], g[N][N];
  int S, all[N][N], some[N][N], none[N][N];
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j) g[i][j] = 0;
  }
  void add_edge(int u, int v) {
    g[u][v] = g[v][u] = 1;
  }
  void dfs(int d, int an, int sn, int nn) {
    if (S > 1000) return; // pruning
    if (sn == 0 && nn == 0) ++S;
    int u = some[d][0];
    for (int i = 0; i < sn; ++i) {
      int v = some[d][i];
      if (g[u][v]) continue;
      int tsn = 0, tnn = 0;
      copy_n(all[d], an, all[d + 1]);
      all[d + 1][an] = v;
      for (int j = 0; j < sn; ++j)
        if (g[v][some[d][j]])
          some[d + 1][tsn++] = some[d][j];
      for (int j = 0; j < nn; ++j)
        if (g[v][none[d][j]])
          none[d + 1][tnn++] = none[d][j];
      dfs(d + 1, an + 1, tsn, tnn);
      some[d][i] = 0, none[d][nn++] = v;
    }
  }
  int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
  }
};
```

1L Theorems

|max independent edge set| $=|V|-$ |min edge cover|
|max independent set| $=|V|-$ |min vertex cover|
max independent set $=$ max clique in the complement graph

2 Flow-Matching

2A Hopcroft-Karp

```
struct HopcroftKarp { // 0-based, return btoa to
    get matching, O(E sqrt(V)) bipartite graph matching
  bool dfs(int a, int L, vector<vector<int>> &g,
    vector<int> &btoa, vector<int> &A,
    vector<int> &B) {
    if (A[a] != L) return 0;
    A[a] = -1;
    for (int b : g[a])
      if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 ||
          dfs(btoa[b], L + 1, g, btoa, A, B))
          return btoa[b] = a, 1;
      }
    return 0;
  }
  int solve(vector<vector<int>> &g, int m) {
    int res = 0;
    vector<int> btoa(m, -1), A(g.size()),
      B(btoa.size()), cur, next;
    for (;;) {
      fill(all(A), 0), fill(all(B), 0);
      cur.clear();
      for (int a : btoa)
        if (a != -1) A[a] = -1;
      for (int a = 0; a < (int)g.size(); a++)
        if (A[a] == 0) cur.push_back(a);
      for (int lay = 1;; lay++) {
        bool islast = 0;
        next.clear();
        for (int a : cur)
          for (int b : g[a]) {
            if (btoa[b] == -1) {
              B[b] = lay;
              islast = 1;
            } else if (btoa[b] != a && !B[b]) {
              B[b] = lay;
              next.push_back(btoa[b]);
            }
          }
        if (islast) break;
        if (next.empty()) return res;
        for (int a : next) A[a] = lay;
        cur.swap(next);
      }
      for (int a = 0; a < (int)g.size(); a++)
        res += dfs(a, 0, g, btoa, A, B);
    }
    return res;
    // or btoa
  }
};
```

2B KM (Hungarian Alg.)

```
struct KM { // 0-base, maximum matching
  ll w[N][N], hl[N], hr[N], slk[N];
  int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
  bool vl[N], vr[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      fill_n(w[i], n, -INF);
  }
  void add_edge(int a, int b, ll wei) {
    w[a][b] = wei;
  }
  bool Check(int x) {
    if (vl[x] = 1, ~fl[x])
      return vr[qu[qr++] = fl[x]] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
  }
  void bfs(int s) {
    fill_n(slk
        , n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    for (ll d;;) {
      while (ql < qr)
        for (int x = 0, y = qu[ql++]; x < n; ++x)
          if (!vl[x] && slk
              [x] >= (d = hl[x] + hr[y] - w[x][y])) {
            if (pre[x] = y, d) slk[x] = d;
            else if (!Check(x)) return;
          }
      d = INF;
      for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !Check(x)) return;
    }
  }
  ll solve() {
    fill_n(fl
        , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
    for (int i = 0; i < n; ++i)
      hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    ll res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
  }
};
```

2C Max Flow (Dinic)

```
struct BoundedFlow { // 0-base
  struct edge { // note int!
    int to, cap, flow, rev;
  };
  vector<edge> G[N];
  int n, s, t, dis[N], cur[N], cnt[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
      G[i].clear(), cnt[i] = 0;
  }
  void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].emplace_back(
      edge{v, rcap, lcap, (int)G[v].size()});
    G[v].emplace_back(
      edge{u, 0, 0, (int)G[u].size() - 1});
  }
  void add_edge(int u, int v, int cap) {
    G[u].emplace_back(
      edge{v, cap, 0, (int)G[v].size()});
    G[v].emplace_back(
      edge{u, 0, 0, (int)G[u].size() - 1});
  }
  int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < (int)G[u].size(); ++i) {
      edge &e = G[u][i];
      if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
        int df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df, G[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (edge &e : G[u])
        if (!~dis[e.to] && e.flow != e.cap)
          q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
  }
  int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
      fill_n(cur, n + 3, 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        add_edge(n + 1, i, cnt[i]), sum += cnt[i];
      else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        G[n + 1].pop_back(), G[i].pop_back();
      else if (cnt[i] < 0)
        G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
  }
  int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
  }
};
```

2D MCMF

```
struct MinCostMaxFlow { // 0-base
  struct Edge {
    ll from, to, cap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  int inq[N], n, s, t;
  ll dis[N], up[N], pot[N];
  bool BellmanFord() {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
      if (cap > 0 && dis[u] > d) {
        dis[u] = d, up[u] = cap, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
    };
    relax(s, 0, INF, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u]) {
        ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax(
          e.to, d2, min(up[u], e.cap - e.flow), &e);
      }
    }
    return dis[t] != INF;
  }
  bool Dijkstra() {
    fill_n(dis, n, INF);
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
      if (cap > 0 && dis[u] > d) {
        dis[u] = d, up[u] = cap, past[u] = e;
        pq.push(pll(d, u));
      }
    };
    relax(s, 0, INF, 0);
    while (!pq.empty()) {
      auto [d, u] = pq.top();
      pq.pop();
      if (dis[u] != d) continue;
      for (auto &e : G[u]) {
        ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax(
          e.to, d2, min(up[u], e.cap - e.flow), &e);
      }
    }
    return dis[t] != INF;
  }
  void solve(int _s, int _t, ll &flow, ll &cost,
    bool neg = true) {
    s = _s, t = _t, flow = 0, cost = 0;
    if (neg) BellmanFord(), copy_n(dis, n, pot);
    // do BellmanFord() if time isn't tight
    for (; Dijkstra(); copy_n(dis, n, pot)) {
      for (int i = 0; i < n; ++i)
        dis[i] += pot[i] - pot[s];
      flow += up[t], cost += up[t] * dis[t];
      for (int i = t; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        e.flow += up[t], G[e.to][e.rev].flow -= up[t];
      }
    }
  }
  void init(int _n) {
    n = _n, fill_n(pot, n, 0);
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].emplace_back(
      Edge{a, b, cap, 0, cost, (int)G[b].size()});
    G[b].emplace_back(
      Edge{b, a, 0, 0, -cost, (int)G[a].size() - 1});
  }
};
```

2E General Graph Matching

```
struct Matching
    { // 0-base, O(VE^2), but somehow very fast
  queue<int> q; int n;
  vector<int> fa, s, vis, pre, match;
  vector<vector<int>> G;
  int Find(int u)
  { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
  int LCA(int x, int y) {
    static int tk = 0; tk++; x = Find(x); y = Find(y);
    for (;; swap(x, y)) if (x != n) {
      if (vis[x] == tk) return x;
      vis[x] = tk;
      x = Find(pre[match[x]]);
    }
  }
  void Blossom(int x, int y, int l) {
    for (; Find(x) != l; x = pre[y]) {
      pre[x] = y, y = match[x];
      if (s[y] == 1) q.push(y), s[y] = 0;
      for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
    }
  }
  bool Bfs(int r) {
    iota(fa.begin
        (), fa.end(), 0); fill(s.begin(), s.end(), -1);
    q = queue<int>(); q.push(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
      for (int x = q.front(); int u : G[x])
        if (s[u] == -1) {
          if (pre[u] = x, s[u] = 1, match[u] == n) {
            for (int a = u, b = x, last;
                b != n; a = last, b = pre[a])
              last =
                  match[b], match[b] = a, match[a] = b;
            return true;
          }
          q.push(match[u]); s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
          int l = LCA(u, x);
          Blossom(x, u, l); Blossom(u, x, l);
        }
    }
    return false;
  }
  Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
      (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
  void add_edge(int u, int v)
  { G[u].emplace_back(v), G[v].emplace_back(u); }
  int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
      if (match[x] == n) ans += Bfs(x);
    return ans;
  } // match[x] == n means not matched
};
```

2F Max Weight Maching

```
#define rep(i, l, r) for (int i = (l); i <= (r); ++i)
struct WeightGraph { // 1-based, note int!
  struct edge {
    int u, v, w;
  };
  int n, nx;
  vector<int> lab;
  vector<vector<edge>> g;
  vector<int> slack, match, st, pa, S, vis;
  vector<vector<int>> flo, flo_from;
  queue<int> q;
  WeightGraph(int n_)
    : n(n_), nx(n * 2), lab(nx + 1),
      g(nx + 1, vector<edge>(nx + 1)), slack(nx + 1),
      flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
    match = st = pa = S = vis = slack;
    rep(u, 1, n) rep(v, 1, n) g[u][v] = {u, v, 0};
  }
  int ED(edge e) {
    return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
  }
  void update_slack(int u, int x, int &s) {
    if (!s || ED(g[u][x]) < ED(g[s][x])) s = u;
  }
  void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
      if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
        update_slack(u, x, slack[x]);
  }
  void q_push(int x) {
    if (x <= n) q.push(x);
    else
      for (int y : flo[x]) q_push(y);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n)
      for (int y : flo[x]) set_st(y, b);
  }
  vector<int> split_flo(auto &f, int xr) {
    auto it = find(all(f), xr);
    if (auto pr = it - f.begin(); pr % 2 == 1)
      reverse(1 + all(f)), it = f.end() - pr;
    auto res = vector(f.begin(), it);
    return f.erase(f.begin(), it), res;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
    auto &f = flo[u], z = split_flo(f, xr);
    rep(i, 0, (int)z.size() - 1)
      set_match(z[i], z[i ^ 1]);
    set_match(xr, v);
    f.insert(f.end(), all(z));
  }
  void augment(int u, int v) {
    for (;;) {
      int xnv = st[match[u]];
      set_match(u, v);
      if (!xnv) return;
      set_match(xnv, st[pa[xnv]]);
      u = st[pa[xnv]], v = xnv;
    }
  }
  int lca(int u, int v) {
    static int t = 0;
    ++t;
    for (++t; u || v; swap(u, v))
      if (u) {
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
      }
    return 0;
  }
  void add_blossom(int u, int o, int v) {
    int b = find(n + 1 + all(st), 0) - begin(st);
    lab[b] = 0, S[b] = 0;
    match[b] = match[o];
    vector<int> f = {o};
    for (int x = u, y; x != o; x = st[pa[y]])
      f.emplace_back(x),
        f.emplace_back(y = st[match[x]]), q_push(y);
    reverse(1 + all(f));
    for (int x = v, y; x != o; x = st[pa[y]])
      f.emplace_back(x),
        f.emplace_back(y = st[match[x]]), q_push(y);
    flo[b] = f;
    set_st(b, b);
    for (int x = 1; x <= nx; ++x)
      g[b][x].w = g[x][b].w = 0;
    fill(all(flo_from[b]), 0);
    for (int xs : flo[b]) {
      for (int x = 1; x <= nx; ++x)
        if (g[b][x].w == 0 ||
          ED(g[xs][x]) < ED(g[b][x]))
          g[b][x] = g[xs][x], g[x][b] = g[x][xs];
      for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
      if (xs == -1) {
        xs = x;
        continue;
      }
      pa[xs] = g[x][xs].u;
      S[xs] = 1, S[x] = 0;
      slack[xs] = 0;
      set_slack(x);
      q_push(x);
      xs = -1;
    }
    for (int x : flo[b])
      if (x == xr) S[x] = 1, pa[x] = pa[b];
      else S[x] = -1, set_slack(x);
    st[b] = 0;
  }
  bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
      int nu = st[match[v]];
      pa[v] = e.u;
      S[v] = 1;
      slack[v] = slack[nu] = 0;
      S[nu] = 0;
      q_push(nu);
    } else if (S[v] == 0) {
      if (int o = lca(u, v)) add_blossom(u, o, v);
      else return augment(u, v), augment(v, u), true;
    }
    return false;
  }
  bool matching() {
    fill(all(S), -1), fill(all(slack), 0);
    q = queue<int>();
    for (int x = 1; x <= nx; ++x)
      if (st[x] == x && !match[x])
        pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
      while (q.size()) {
        int u = q.front();
        q.pop();
        if (S[st[u]] == 1) continue;
        for (int v = 1; v <= n; ++v)
          if (g[u][v].w > 0 && st[u] != st[v]) {
            if (ED(g[u][v]) != 0)
              update_slack(u, st[v], slack[st[v]]);
            else if (on_found_edge(g[u][v]))
              return true;
          }
      }
      int d = INF;
      for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1)
          d = min(d, lab[b] / 2);
      for (int x = 1; x <= nx; ++x)
        if (int s = slack[x];
            st[x] == x && s && S[x] <= 0)
          d = min(d, ED(g[s][x]) / (S[x] + 2));
      for (int u = 1; u <= n; ++u)
        if (S[st[u]] == 1) lab[u] += d;
        else if (S[st[u]] == 0) {
```

```
     if (lab[u] <= d) return false;
     lab[u] -= d;
    }
   rep(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
    lab[b] += d * (2 - 4 * S[b]);
   for (int x = 1; x <= nx; ++x)
    if (int s = slack[x]; st[x] == x && s &&
      st[s] != x && ED(g[s][x]) == 0)
     if (on_found_edge(g[s][x])) return true;
   for (int b = n + 1; b <= nx; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
     expand_blossom(b);
  }
  return false;
 }
 pair<ll, int> solve() {
  fill(all(match), 0);
  rep(u, 0, n) st[u] = u, flo[u].clear();
  int w_max = 0;
  rep(u, 1, n) rep(v, 1, n) {
   flo_from[u][v] = (u == v ? u : 0);
   w_max = max(w_max, g[u][v].w);
  }
  fill(all(lab), w_max);
  int n_matches = 0;
  ll tot_weight = 0;
  while (matching()) ++n_matches;
  rep(u, 1, n) if (match[u] && match[u] < u)
   tot_weight += g[u][match[u]].w;
  return make_pair(tot_weight, n_matches);
 }
 void add_edge(int u, int v, int w) {
  g[u][v].w = g[v][u].w = w;
 }
};
```

2G Global Min Cut

```
struct StoerWagner { // O(V^3), is it O(VE + V log V)?
 int vst[N], edge[N][N], wei[N];
 void init(int n) {
  for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
 }
 void addEdge(int u, int v, int w) {
  edge[u][v] += w;
  edge[v][u] += w;
 }
 int search(int &s, int &t, int n) {
  fill_n(vst, n, 0), fill_n(wei, n, 0);
  s = t = -1;
  int mx, cur;
  for (int j = 0; j < n; ++j) {
   mx = -1, cur = 0;
   for (int i = 0; i < n; ++i)
    if (wei[i] > mx) cur = i, mx = wei[i];
   vst[cur] = 1, wei[cur] = -1;
   s = t;
   t = cur;
   for (int i = 0; i < n; ++i)
    if (!vst[i]) wei[i] += edge[cur][i];
  }
  return mx;
 }
 int solve(int n) {
  int res = INF;
  for (int x, y; n > 1; n--) {
   res = min(res, search(x, y, n));
   for (int i = 0; i < n; ++i)
    edge[i][x] = (edge[x][i] += edge[y][i]);
   for (int i = 0; i < n; ++i) {
    edge[y][i] = edge[n - 1][i];
    edge[i][y] = edge[i][n - 1];
   } // edge[y][y] = 0;
  }
  return res;
 }
} sw;
```

2H Gomory-Hu Tree

```
// 最小割樹, 樹上兩點最小割 = 樹上兩點最小割
BoundedFlow Dinic;
int g[N];
void add_edge(int u, int v, int w); // TODO
void GomoryHu(int n) { // 0-base
 fill_n(g, n, 0);
 for (int i = 1; i < n; ++i) {
  Dinic.init(n);
  // build the graph
  add_edge(i, g[i], Dinic.maxflow(i, g[i]));
  for (int j = i + 1; j <= n; ++j)
   if (g[j] == g[i] && ~Dinic.dis[j])
    g[j] = i;
 }
}
```

2I Min Cost Circulation

```
struct MinCostCirculation { // 0-base
 struct Edge {
  ll from, to, cap, fcap, flow, cost, rev;
 } *past[N];
 vector<Edge> G[N];
 ll dis[N], inq[N], n;
 void BellmanFord(int s) {
  fill_n(dis, n, INF), fill_n(inq, n, 0);
  queue<int> q;
  auto relax = [&](int u, ll d, Edge *e) {
   if (dis[u] > d) {
    dis[u] = d, past[u] = e;
    if (!inq[u]) inq[u] = 1, q.push(u);
   }
  };
  relax(s, 0, 0);
  while (!q.empty()) {
   int u = q.front();
   q.pop(), inq[u] = 0;
   for (auto &e : G[u])
    if (e.cap > e.flow)
     relax(e.to, dis[u] + e.cost, &e);
  }
 }
 void try_edge(Edge &cur) {
  if (cur.cap > cur.flow) return ++cur.cap, void();
  BellmanFord(cur.to);
  if (dis[cur.from] + cur.cost < 0) {
   ++cur.flow, --G[cur.to][cur.rev].flow;
   for (int
      i = cur.from; past[i]; i = past[i]->from) {
    auto &e = *past[i];
    ++e.flow, --G[e.to][e.rev].flow;
   }
  }
  ++cur.cap;
 }
 void solve(int mxlg) {
  for (int b = mxlg; b >= 0; --b) {
   for (int i = 0; i < n; ++i)
    for (auto &e : G[i])
     e.cap *= 2, e.flow *= 2;
   for (int i = 0; i < n; ++i)
    for (auto &e : G[i])
     if (e.fcap >> b & 1)
      try_edge(e);
  }
 }
 void init(int _n) { n = _n;
  for (int i = 0; i < n; ++i) G[i].clear();
 }
 void add_edge(ll a, ll b, ll cap, ll cost) {
  G[a].emplace_back(Edge{a, b,
     0, cap, 0, cost, (ll)G[b].size() + (a == b)});
  G[b].emplace_back(Edge
    {b, a, 0, 0, 0, -cost, (ll)G[a].size() - 1});
```

```cpp
  }
} mcmf; // O(VE * ElogC)
```

2J Flow Models Building

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x,y,l,u), connect $x \to y$ with capacity $u-l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X,Y)
  1. Redirect every edge: $y \to x$ if $(x,y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x,y,c), connect $x \to y$ with $(cost,cap) = (c,1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost,cap) = (-c,1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost,cap) = (0,d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost,cap) = (0,-d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C+K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u,v,w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e\in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u,v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s,v) with capacity p_v; otherwise, create edge (v,t) with capacity $-p_v$.
  2. Create edge (u,v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  1. Capacity c_{uv}, Flow f_{uv}, Cost w_{uv}, Required Flow difference for vertex b_u.
  2. If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\begin{array}{ccc} & \min\sum_{uv} w_{uv} f_{uv} & \\ & -f_{uv} \geq -c_{uv} \Leftrightarrow & \min\sum_{u} b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv}) \\ \sum_{v} f_{vu} - \sum_{v} f_{uv} = -b_u & & p_u \geq 0 \end{array}$$

3 Data Struture

3A LichaoTree

```cpp
#define INF 4e18
struct Line {
    ll a, b;
    Line(ll _a = 0, ll _b = INF) : a(_a), b(_b) {}
    ll f(ll x) const { return a * x + b; }
};
struct Node {
    Line line;
    Node *left = nullptr, *right = nullptr;
    Node(Line l) : line(l) {}
};
struct LiChaoTree {
    const ll MIN_X = -1e9;
    const ll MAX_X = 1e9;
    Node* root = nullptr;
    void insert(Line new_line, ll L, ll R) {
        insert(root, MIN_X, MAX_X, new_line, L, R);
    }
    ll query(ll x) const {
        return query(root, MIN_X, MAX_X, x);
    }
private:
    void insert(Node*&
         node, ll l, ll r, Line new_line, ll L, ll R) {
        if (r < L || R <= l) return;
        if (!node) node = new Node(Line());
        // don't use (l + r) / 2, it may overflow
        ll mid = (l + r) >> 1;
        if (L <= l && r < R) {
            bool left_better
                = new_line.f(l) < node->line.f(l);
            bool mid_better
                = new_line.f(mid) < node->line.f(mid);
            if (mid_better) swap(node->line, new_line);
            if (r == l) return;
            if (left_better != mid_better) insert
                (node->left, l, mid, new_line, L, R);
            else insert(node
                ->right, mid + 1, r, new_line, L, R);
            return;
        }
        insert(node->left,  l, mid, new_line, L, R);
        insert
            (node->right, mid + 1, r, new_line, L, R);
    }
    ll query(Node* node, ll l, ll r, ll x) const {
        if (!node) return INF;
        ll res = node->line.f(x);
        if (l == r) return res;
        ll mid = (l + r) >> 1;
        if (x <= mid) return
             min(res, query(node->left, l, mid, x));
        else return min
            (res, query(node->right, mid + 1, r, x));
    }
} seg;
```

3B Treap

```cpp
mt19937 rd(1);
#define sz(t) ((t) == 0 ? 0 : (t)->size)
struct Treap {
  int pri, size;
  Treap *l, *r;
  Treap(ll val = 0)
    : pri(rd()), size(1), l(0), r(0) {};
  void push();
  void pull() { size = 1 + sz(l) + sz(r); }
};
void spilt(int k, Treap *rt, Treap *&a, Treap *&b) {
  if (!rt) return a = b = 0, void();
  rt->push();
  int lsz = 1 + sz(rt->l);
  if (k >= lsz)
```

```
    a = rt, spilt(k - lsz, a->r, a->r, b), a->pull();
  else b = rt, spilt(k, b->l, a, b->l), b->pull();
}
Treap *merge(Treap *l, Treap *r) {
  if (!l) return r;
  if (!r) return l;
  if (l->pri < r->pri) {
    l->push(), l->r = merge(l->r, r), l->pull();
    return l;
  } else {
    r->push(), r->l = merge(l, r->l), r->pull();
    return r;
  }
}
```

3C LinkCutTree

```
#define ls(x) Tree[x].son[0]
#define rs(x) Tree[x].son[1]
#define fa(x) Tree[x].fa
struct node {
  int son[2], Min, id, fa, lazy;
} Tree[N];
int n, m, q, w[N], Min;
struct Node {
  int u, v, w;
} a[N];
inline bool IsRoot(int x) {
  return (ls(fa(x)) == x || rs(fa(x)) == x) ? 0 : 1;
}
inline void PushUp(int x) {
  Tree[x].Min = w[x], Tree[x].id = x;
  if (ls(x) && Tree[ls(x)].Min < Tree[x].Min) {
    Tree[x].Min = Tree[ls(x)].Min;
    Tree[x].id = Tree[ls(x)].id;
  }
  if (rs(x) && Tree[rs(x)].Min < Tree[x].Min) {
    Tree[x].Min = Tree[rs(x)].Min;
    Tree[x].id = Tree[rs(x)].id;
  } // id: id of Min value
} // change Min to sum and remove < if summing
inline void Update(int x) {
  Tree[x].lazy ^= 1;
  swap(ls(x), rs(x));
}
inline void PushDown(int x) {
  if (!Tree[x].lazy) return;
  if (ls(x)) Update(ls(x));
  if (rs(x)) Update(rs(x));
  Tree[x].lazy = 0;
}
inline void Rotate(int x) {
  int y = fa(x), z = fa(y), k = rs(y) == x,
      w = Tree[x].son[!k];
  if (!IsRoot(y)) Tree[z].son[rs(z) == y] = x;
  fa(x) = z, fa(y) = x;
  if (w) fa(w) = y;
  Tree[x].son[!k] = y, Tree[y].son[k] = w;
  PushUp(y);
}
inline void Splay(int x) {
  stack<int> Stack;
  int y = x, z;
  Stack.push(y);
  while (!IsRoot(y)) Stack.push(y = fa(y));
  while (!Stack.empty())
    PushDown(Stack.top()), Stack.pop();
  while (!IsRoot(x)) {
    y = fa(x), z = fa(y);
    if (!IsRoot(y))
      Rotate((ls(y) == x) ^ (ls(z) == y) ? x : y);
    Rotate(x);
  }
  PushUp(x);
}
inline void Access(int root) {
  for (int x = 0; root; x = root, root = fa(root))
    Splay(root), rs(root) = x, PushUp(root);
}
inline void MakeRoot(int x) {
  Access(x), Splay(x), Update(x);
}
inline int FindRoot(int x) {
  Access(x), Splay(x);
  while (ls(x)) x = ls(x);
  return Splay(x), x;
}
inline void Link(int u, int v) {
  MakeRoot(u);
  if (FindRoot(v) != u) fa(u) = v;
} // create edge u -> v
inline void Cut(int u, int v) {
  MakeRoot(u);
  if (FindRoot(v) != u || fa(v) != u || ls(v)) return;
  fa(v) = rs(u) = 0;
} // delete edge u -> v
inline void Split(int u, int v) {
  MakeRoot(u), Access(v), Splay(v);
} // v stores the path u -> v
inline bool Check(int u, int v) {
  return MakeRoot(u), FindRoot(v) == u;
} // check u, v in the same tree
inline int LCA(int root, int u, int v) {
  MakeRoot(root), Access(u), Access(v), Splay(u);
  if (!fa(u)) {
    Access(u), Splay(v);
    return fa(v);
  }
  return fa(u);
} // LCA of u, v in tree with root
/* ETT
每次進入節點和走邊都放入一次共 3n - 2
node(u) 表示進入節點 u 放入 treap 的位置
edge(u, v) 表示 u -> v 的邊放入 treap 的位置 (push v)
Makeroot u :
  L1 = [begin, node(u) - 1], L2 = [node(u), end]
  -> L2 + L1

Insert u, v :
  Tu -> L1 = [begin, node(u) - 1], L2 = [node(u), end]
  Tv -> L3 = [begin, node(v) - 1], L4 = [node(v), end]
  -> L2 + L1 + edge(u, v) + L4 + L3 + edge(v, u)

Delect u, v :
  maybe need swap u, v
  T -> L1 + edge(u, v) + L2 + edge(v, u) + L3
  -> L1 + L3, L2
*/
```

3D CentroidDecomposition

```
struct Cent_Dec { // 1-base
  vector<pll> G[N];
  pll info[N]; // store info. of itself
  pll upinfo[N]; // store info. of climbing up
  int n, pa[N], layer[N], sz[N], done[N];
  ll dis[__lg(N) + 1][N];
  void init(int _n) {
    n = _n, layer[0] = -1;
    fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
    for (int i = 1; i <= n; ++i) G[i].clear();
  }
  void add_edge(int a, int b, int w) {
    G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
  }
  void get_cent(
    int u, int f, int &mx, int &c, int num) {
    int mxsz = 0;
    sz[u] = 1;
    for (pll e : G[u])
      if (!done[e.X] && e.X != f) {
        get_cent(e.X, u, mx, c, num);
        sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
      }
    if (mx > max(mxsz, num - sz[u]))
      mx = max(mxsz, num - sz[u]), c = u;
  }
  void dfs(int u, int f, ll d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pll e : G[u])
      if (!done[e.X] && e.X != f)
        dfs(e.X, u, d + e.Y, org);
  }
  int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pll e : G[c])
      if (!done[e.X]) {
        if (sz[e.X] > sz[c])
          lc = cut(e.X, c, num - sz[c]);
        else lc = cut(e.X, c, sz[e.X]);
        upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
      }
    return done[c] = 0, c;
  }
  void build() { cut(1, 0, n); }
  void modify(int u) {
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
      info[a].X += dis[ly][u], ++info[a].Y;
      if (pa[a])
        upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
  }
  ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
      rt += info[a].X + info[a].Y * dis[ly][u];
      if (pa[a])
        rt -=
          upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
  }
};
```

4 String

4A KMP

```
vector<int> pi(const string& s) {
  vector<int> p((int)s.size());
  for(int i=1; i<(int)s.size(); i++) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}
```

4B Z

```
vector<int> Z(const string& S) {
  vector<int> z((int)S.size());
    z[0] = (int)S.size(); // or 0
  int l = -1, r = -1;
  for(int i=1; i<(int)S.size(); i++) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i +
        z[i] < (int)S.size() && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

4C Manacher

```
vector<int> manacher(const string& s) {
  int n = s.size();
  array<vector
      <int>,2> p = {vector<int>(n+1), vector<int>(n)};
  for(int z
      =0; z<2; z++) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  vector<int> res(n*2+1);
  for(int i=0; i<n; i++){
    res[2*i]=p[0][i];
    res[2*i+1]=p[1][i];
  }
  res[2*n]=p[0][n];
  return res;
}
```

4D SuffixArray

```
struct SuffixArray {
#define add(x, k) (x + k + n) % n
  vector<int> sa, cnt, rk, tmp, lcp;
  // sa: order, rk[i]: pos of s[i..],
  // lcp[i]: LCP of sa[i], sa[i-1]
  void SA(string s) { // remember to append '\1'
    int n = (int)s.size();
    sa.resize(n), cnt.resize(n);
    rk.resize(n), tmp.resize(n);
    iota(all(sa), 0);
    sort(all(sa),
      [&](int i, int j) { return s[i] < s[j]; });
    rk[0] = 0;
    for (int i = 1; i < n; i++)
      rk[sa[i]] =
        rk[sa[i - 1]] + (s[sa[i - 1]] != s[sa[i]]);
    for (int k = 1; k <= n; k <<= 1) {
      fill(all(cnt), 0);
      for (int i = 0; i < n; i++)
        cnt[rk[add(sa[i], -k)]]++;
      for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
      for (int i = n - 1; i >= 0; i--)
        tmp[--cnt[rk[add(sa[i], -k)]]] =
          add(sa[i], -k);
      sa.swap(tmp);
      tmp[sa[0]] = 0;
      for (int i = 1; i < n; i++)
        tmp[sa[i]] = tmp[sa[i - 1]] +
          (rk[sa[i - 1]] != rk[sa[i]] ||
            rk[add(sa[i - 1], k)] !=
              rk[add(sa[i], k)]);
      rk.swap(tmp);
    }
  }
  void LCP(string s) {
    int n = (int)s.size(), k = 0;
    lcp.resize(n);
    for (int i = 0; i < n; i++)
      if (rk[i] == 0) lcp[rk[i]] = 0;
      else {
        if (k) k--;
        int j = sa[rk[i] - 1];
        while (
          max(i, j) + k < n && s[i + k] == s[j + k])
          k++;
        lcp[rk[i]] = k;
      }
  }
};
```

4E SA-IS(Induced Sorting)

```
auto sais(const auto &s) {
  const int n = (int)s.size(), z = ranges::max(s) + 1;
  if (n == 1) return vector{0};
  vector<int> c(z);
  for (int x : s) ++c[x];
  partial_sum(all(c), begin(c));
  vector<int> sa(n);
  auto I = views::iota(0, n);
  vector<bool> t(n, true);
  for (int i = n - 2; i >= 0; --i)
    t[i] =
      (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
  auto is_lms = views::filter(
    [&t](int x) { return x && t[x] && !t[x - 1]; });
  auto induce = [&] {
    for (auto x = c; int y : sa)
      if (y--)
        if (!t[y]) sa[x[s[y] - 1]++] = y;
    for (auto x = c; int y : sa | views::reverse)
      if (y--)
        if (t[y]) sa[--x[s[y]]] = y;
  };
  vector<int> lms, q(n);
  lms.reserve(n);
  for (auto x = c; int i : I | is_lms)
    q[i] = (int)lms.size(),
    lms.emplace_back(sa[--x[s[i]]] = i);
  induce();
  vector<int> ns((int)lms.size());
  for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
      int len = min({n - i, n - j, lms[q[i] + 1] - i});
      ns[q[i]] = nz += lexicographical_compare(
        begin(s) + j, begin(s) + j + len, begin(s) + i,
        begin(s) + i + len);
    }
    j = i;
  }
  fill(all(sa), 0);
  auto nsa = sais(ns);
  for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
  return induce(), sa;
}
// sa[i]: sa[i]-th suffix is the i-th lexicographically
// smallest suffix. hi[i]: LCP of suffix sa[i] and
// suffix sa[i - 1].
struct Suffix {
  int n;
  vector<int> sa, hi, ra;
  Suffix(const auto &_s, int _n)
    : n(_n), hi(n), ra(n) {
    vector<int> s(n + 1); // s[n] = 0;
    copy_n(_s, n, begin(s)); // _s shouldn't contain 0
    sa = sais(s);
    sa.erase(sa.begin());
    for (int i = 0; i < n; ++i) ra[sa[i]] = i;
    for (int i = 0, h = 0; i < n; ++i) {
      if (!ra[i]) {
        h = 0;
        continue;
      }
      for (int j = sa[ra[i] - 1];
           max(i, j) + h < n && s[i + h] == s[j + h];)
        ++h;
      hi[ra[i]] = h ? h-- : 0;
    }
  }
};
```

4F ACAutomaton

```
#define sigma 26
#define base 'a'
struct AhoCorasick { // N: sum of length
  int ch[N][sigma] = {{}}, f[N] = {-1}, tag[N],
      mv[N][sigma], jump[N], cnt[N];
  int idx = 0, t = -1;
  vector<int> E[N], q;
  pii o[N];
  int insert(string &s) {
    int j = 0;
    for (int i = 0; i < (int)s.size(); i++) {
      if (!ch[j][s[i] - base])
        ch[j][s[i] - base] = ++idx;
      j = ch[j][s[i] - base];
    }
    tag[j] = 1;
    return j;
  }
  int next(int u, int c) {
    return u < 0 ? 0 : mv[u][c];
  }
  void dfs(int u) {
    o[u].F = ++t;
    for (auto v : E[u]) dfs(v);
    o[u].S = t;
  }
  void build() {
    int k = -1;
    q.emplace_back(0);
    while (++k < (int)q.size()) {
      int u = q[k];
      for (int v = 0; v < sigma; v++) {
        if (ch[u][v]) {
          f[ch[u][v]] = next(f[u], v);
          q.emplace_back(ch[u][v]);
        }
        mv[u][v] =
          (ch[u][v] ? ch[u][v] : next(f[u], v));
      }
      if (u) jump[u] = (tag[f[u]] ? f[u] : jump[f[u]]);
    }
    reverse(q.begin(), q.end());
    for (int i = 1; i <= idx; i++)
      E[f[i]].emplace_back(i);
    dfs(0);
  }
  void match(string &s) {
    fill(cnt, cnt + idx + 1, 0);
    for (int i = 0, j = 0; i < (int)s.size(); i++)
      cnt[j = next(j, s[i] - base)]++;
    for (int i : q)
      if (f[i] > 0) cnt[f[i]] += cnt[i];
  }
} ac;
```

4G MinRotation

```
int mincyc(string s) {
  int n = (int)s.size();
  s = s + s;
  int i = 0, ans = 0;
  while (i < n) {
    ans = i;
    int j = i + 1, k = i;
    while (j < 2 * n && s[j] >= s[k]) {
      k = (s[j] > s[k] ? i : k + 1);
      ++j;
    }
    while (i <= k) i += j - k;
  }
  return ans;
}
```

4H ExtSAM

```
#define CNUM 26
struct exSAM {
  int len[N * 2], link[N * 2]; // maxlength, suflink
  int next[N * 2][CNUM], tot; // [0, tot), root = 0
  int lenSorted[N * 2]; // topo. order
  int cnt[N * 2]; // occurence
  int newnode() {
    fill_n(next[tot], CNUM, 0);
    len[tot] = cnt[tot] = link[tot] = 0;
    return tot++;
  }
  void init() { tot = 0, newnode(), link[0] = -1; }
  int insertSAM(int last, int c) {
    int cur = next[last][c];
    len[cur] = len[last] + 1;
    int p = link[last];
    while (p != -1 && !next[p][c])
      next[p][c] = cur, p = link[p];
    if (p == -1) return link[cur] = 0, cur;
    int q = next[p][c];
    if (len
        [p] + 1 == len[q]) return link[cur] = q, cur;
    int clone = newnode();
    for (int i = 0; i < CNUM; ++i)
      next[
          clone][i] = len[next[q][i]] ? next[q][i] : 0;
    len[clone] = len[p] + 1;
    while (p != -1 && next[p][c] == q)
      next[p][c] = clone, p = link[p];
    link[link[cur] = clone] = link[q];
    link[q] = clone;
    return cur;
  }
  void insert(const string &s) {
    int cur = 0;
    for (auto ch : s) {
      int &nxt = next[cur][int(ch - 'a')];
      if (!nxt) nxt = newnode();
      cnt[cur = nxt] += 1;
    }
  }
  void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
      int cur = q.front();
      q.pop();
      for (int i = 0; i < CNUM; ++i)
        if (next[cur][i])
          q.push(insertSAM(cur, i));
    }
    vector<int> lc(tot);
    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(all(lc), lc.begin());
    for (int i
        = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
  }
  void solve() {
    for (int i = tot - 2; i >= 0; --i)
      cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
  }
};
```

5 Number Theory

5A Primes

12721 13331 14341 75577 123457 222557 556679 999983
1097774749 1076767633 100102021 999997771 1001010013
1000512343 987654361 999991231 999888733 98789101
987777733 999991921 1010101333 1010102101 1000000000039
1000000000000037 2305843009213693951 4611686018427387847
9223372036854775783 18446744073709551557

5B ExtGCD

```
// beware of negative numbers!
void extgcd(ll a, ll b, ll c, ll &x, ll &y) {
  if (b == 0) x = c / a, y = 0;
  else {
    extgcd(b, a % b, c, y, x);
    y -= x * (a / b);
  }
} // |x| <= b/2, |y| <= a/2
```

5C FloorCeil

```
int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }
```

5D FloorSum

Computes

$$f(a,b,c,n)=\sum_{i=0}^{n}\left\lfloor\frac{a\cdot i+b}{m}\right\rfloor$$

Furthermore, Let $m=\left\lfloor\frac{an+b}{c}\right\rfloor$:

$$g(a,b,c,n)=\sum_{i=0}^{n}i\left\lfloor\frac{ai+b}{c}\right\rfloor$$

$$=\begin{cases}\left\lfloor\frac{a}{c}\right\rfloor\cdot\frac{n(n+1)(2n+1)}{6}+\left\lfloor\frac{b}{c}\right\rfloor\cdot\frac{n(n+1)}{2} & \\ +g(a \bmod c,b \bmod c,c,n), & a\geq c\vee b\geq c\\ 0, & n<0\vee a=0\\ \frac{1}{2}\cdot(n(n+1)m-f(c,c-b-1,a,m-1) & \\ -h(c,c-b-1,a,m-1)), & \text{otherwise}\end{cases}$$

$$h(a,b,c,n)=\sum_{i=0}^{n}\left\lfloor\frac{ai+b}{c}\right\rfloor^2$$

$$=\begin{cases}\left\lfloor\frac{a}{c}\right\rfloor^2\cdot\frac{n(n+1)(2n+1)}{6}+\left\lfloor\frac{b}{c}\right\rfloor^2\cdot(n+1) & \\ +\left\lfloor\frac{a}{c}\right\rfloor\cdot\left\lfloor\frac{b}{c}\right\rfloor\cdot n(n+1) & \\ +h(a \bmod c,b \bmod c,c,n) & \\ +2\left\lfloor\frac{a}{c}\right\rfloor\cdot g(a \bmod c,b \bmod c,c,n) & \\ +2\left\lfloor\frac{b}{c}\right\rfloor\cdot f(a \bmod c,b \bmod c,c,n), & a\geq c\vee b\geq c\\ 0, & n<0\vee a=0\\ nm(m+1)-2g(c,c-b-1,a,m-1) & \\ -2f(c,c-b-1,a,m-1)-f(a,b,c,n), & \text{otherwise}\end{cases}$$

```
ll floorsum(ll A, ll B, ll C, ll N) {
  if (A == 0) return (N + 1) * (B / C);
  if (A > C || B > C)
    return (N + 1) * (B / C) +
      N * (N + 1) / 2 * (A / C) +
      floorsum(A % C, B % C, C, N);
  ll M = (A * N + B) / C;
  return N * M - floorsum(C, C - B - 1, A, M - 1);
}
```

5E MillerRabin

```
// n < 4,759,123,141        3 : 2, 7, 61
// n < 1,122,004,669,633    4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383    6 : primes <= 13
// n < 2^64                 7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
ll mul(ll a, ll b, ll mod) {
  return (ll)(__int128(a) * b % mod);
}
bool Miller_Rabin(ll a, ll n) { // O(log(n)^3)
  if ((a = a % n) == 0) return 1;
  if (n % 2 == 0) return n == 2;
  ll tmp = (n - 1) / ((n - 1) & (1 - n));
  ll t = __lg(((n - 1) & (1 - n))), x = 1;
  for (; tmp; tmp >>= 1, a = mul(a, a, n))
    if (tmp & 1) x = mul(x, a, n);
  if (x == 1 || x == n - 1) return 1;
  while (--t)
    if ((x = mul(x, x, n)) == n - 1) return 1;
  return 0;
}
```

5F PollardRho

```
map<ll, int> cnt;
void PollardRho(ll n) { // O(n^(1/4))
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2 == 0)
    return PollardRho(n / 2), ++cnt[2], void();
  ll x = 2, y = 2, d = 1, p = 1;
#define f(x, n, p) ((mul(x, x, n) + p) % n)
  while (true) {
    if (d != n && d != 1) {
      PollardRho(n / d);
      PollardRho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p), y = f(f(y, n, p), n, p);
    d = gcd(abs(x - y), n);
  }
}
```

5G Fraction

```
struct fraction {
    ll n, d;
    fraction(const ll &_n = 0, const ll &_d = 1)
        : n(_n), d(_d) {
        ll t = __gcd(n, d);
        n /= t, d /= t;
        if (d < 0) n = -n, d = -d;
    }
    fraction
         operator-() const { return fraction(-n, d); }
    fraction operator+(const fraction &b) const {
         return fraction(n * b.d + b.n * d, d * b.d); }
    fraction operator-(const fraction &b) const {
         return fraction(n * b.d - b.n * d, d * b.d); }
    fraction operator*(const fraction &
        b) const { return fraction(n * b.n, d * b.d); }
    fraction operator/(const fraction &
        b) const { return fraction(n * b.d, d * b.n); }
    bool is_zero() const { return n == 0; }
    void print() {
        cout << n;
        if (d != 1) cout << "/" << d;
    }
};
```

5H modint

```
#include<bits/stdc++.h>
using namespace std;
#define int long long
constexpr int mod=998244353;
long long divide(long long a, long long b = mod - 2) {
  long long ret = 1;
  for (a %= mod; b; b >>= 1, a = a * a % mod)
    if (b & 1) ret = ret * a % mod;
  return ret;
}
struct modint {int x; modint(const int& _ = 0): x
     (mod + _) {x %= mod;} operator bool() {return x;}};
modint operator+(const modint
     & a, const modint& b) {return (a.x + b.x) % mod;}
modint operator-(const modint& a,
      const modint& b) {return (a.x - b.x + mod) % mod;}
modint operator*(const modint&
     a, const modint& b) {return 1LL * a.x * b.x % mod;}
modint operator/(const modint& a, const
      modint& b) {return 1LL * a.x * divide(b.x) % mod;}
modint& operator
     +=(modint& a, const modint& b) {return a = a + b;}
modint& operator
     -=(modint& a, const modint& b) {return a = a - b;}
modint& operator
     *=(modint& a, const modint& b) {return a = a * b;}
modint& operator
     /=(modint& a, const modint& b) {return a = a / b;}
istream& operator
     >>(istream& is, modint& n) {is >> n.x; return is;}
ostream& operator<<(ostream
     & os, const modint& n) {os << n.x; return os;}
```

5I ChineseRemainder

```
ll solve(ll x1, ll m1, ll x2, ll m2) {
  ll g = gcd(m1, m2);
  if ((x2 - x1) % g) return -1; // no sol
  m1 /= g; m2 /= g;
  ll x, y;
  extgcd(m1, m2, __gcd(m1, m2), x, y);
  ll lcm = m1 * m2 * g;
  ll res = x * (x2 - x1) * m1 + x1;
  // be careful with overflow
  return (res % lcm + lcm) % lcm;
}
```

5J FactorialModp^k

```
// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
  prod[0] = 1;
  for (int i = 1; i <= pk; ++i)
    if (i % p) prod[i] = prod[i - 1] * i % pk;
    else prod[i] = prod[i - 1];
  ll rt = 1;
  for (; n; n /= p) {
    rt = rt * mpow(prod[pk], n / pk, pk) % pk;
    rt = rt * prod[n % pk] % pk;
  }
  return rt;
} // (n! without factor p) % p^k
```

5K QuadraticResidue

```
// Berlekamp-Rabin, log^2(p)
ll trial(ll y, ll z, ll m) {
  ll a0 = 1, a1 = 0, b0 = z, b1 = 1, p = (m - 1) / 2;
  while (p) {
    if (p & 1)
      tie(a0, a1) =
        make_pair((a1 * b1 % m * y + a0 * b0) % m,
          (a0 * b1 + a1 * b0) % m);
    tie(b0, b1) =
      make_pair((b1 * b1 % m * y + b0 * b0) % m,
        (2 * b0 * b1) % m);
    p >>= 1;
  }
  if (a1) return inv(a1, m);
  return -1;
}
mt19937 rd(49);
ll psqrt(ll y, ll p) { // sqrt(y) mod p
  if (y == 0) return 0;
  if (fpow(y, (p - 1) / 2, p) != 1) return -1;
  for (int i = 0; i < 30; i++) {
    ll z = rd() % p;
    if (z * z % p == y) return z;
    ll x = trial(y, z, p);
    if (x == -1) continue;
    return x;
  }
  return -1;
}
```

5L MeisselLehmer

```
ll PrimeCount(ll n) { // n ~ 10^13 => < 2s
  if (n <= 1) return 0;
  int v = sqrt(n), s = (v + 1) / 2, pc = 0;
  vector<int> smalls(v + 1), skip(v + 1), roughs(s);
  vector<ll> larges(s);
  for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
  for (int i = 0; i < s; ++i) {
    roughs[i] = 2 * i + 1;
    larges[i] = (n / (2 * i + 1) + 1) / 2;
  }
  for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
      int q = p * p;
      ++pc;
      if (1LL * q * q > n) break;
      skip[p] = 1;
      for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
      int ns = 0;
      for (int k = 0; k < s; ++k) {
        int i = roughs[k];
        if (skip[i]) continue;
        ll d = 1LL * i * p;
        larges[ns] = larges[k] - (d <= v ? larges
            [smalls[d] - pc] : smalls[n / d]) + pc;
        roughs[ns++] = i;
      }
      s = ns;
      for (int j = v / p; j >= p; --j) {
        int c =
             smalls[j] - pc, e = min(j * p + p, v + 1);
        for (int i = j * p; i < e; ++i) smalls[i] -= c;
      }
    }
  }
  for (int k = 1; k < s; ++k) {
    const ll m = n / roughs[k];
    ll t = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
      int p = roughs[l];
      if (1LL * p * p > m) break;
      t -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= t;
  }
  return larges[0];
}
```

5M DiscreteLog

```
int DiscreteLog(int s, int x, int y, int m) {
  constexpr int kStep = 32000;
  unordered_map<int, int> p;
  int b = 1;
  for (int i = 0; i < kStep; ++i) {
    p[y] = i;
    y = 1LL * y * x % m;
    b = 1LL * b * x % m;
  }
  for (int i = 0; i < m + 10; i += kStep) {
    s = 1LL * s * b % m;
    if (p.find(s) != p.end()) return i + kStep - p[s];
  }
  return -1;
}
int DiscreteLog(int x, int y, int m) {
  if (m == 1) return 0;
  int s = 1;
  for (int i = 0; i < 100; ++i) {
    if (s == y) return i;
    s = 1LL * s * x % m;
  }
  if (s == y) return 100;
  int p = 100 + DiscreteLog(s, x, y, m);
  if (fpow(x, p, m) != y) return -1;
  return p;
}
```

5N Mobius Function

```
vector<int> mobius_up_to(int N) {
  vector<int> mu(N + 1, 1), prm;
  vector<bool> is_prm(N + 1, 1);
  mu[0] = 0; // place holder
  for (int i = 2; i <= N; ++i) {
    if (is_prm[i]) {
      prm.push_back(i);
      mu[i] = -1;
    }
    for (auto &p : prm) {
      if (i * p > N) break;
      is_prm[i * p] = 0;
      if (i % p == 0) {
        mu[i * p] = 0;
        break;
      } else mu[i * p] = -mu[i];
    }
  }
  return mu;
}
```

50 Theorems

- **Cramer's Rule**

$$\begin{matrix} ax+by=e \\ cx+dy=f \end{matrix} \Rightarrow \begin{matrix} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{matrix}$$

- **Vandermonde's Identity**

$$C(n+m,k)=\sum_{i=0}^{k}C(n,i)C(m,k-i)$$

- **Kirchhoff's Theorem**
  Denote L be a $n\times n$ matrix as the Laplacian matrix of graph G, where $L_{ii}=d(i)$, $L_{ij}=-c$ where c is the number of edge (i,j) in G.
  - The number of undirected spanning in G is $|\text{det}(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\text{det}(\tilde{L}_{rr})|$.
- **Tutte's Matrix**
  Let D be a $n\times n$ matrix, where $d_{ij}=x_{ij}$ (x_{ij} is chosen uniformly at random) if $i<j$ and $(i,j)\in E$, otherwise $d_{ij}=-d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.
- **Cayley's Formula**
  - Given a degree sequence $d_1,d_2,\ldots,d_n$ for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1,2,\ldots,k$ belong to different components. Then $T_{n,k}=kn^{n-k-1}$.
- **Erdős-Gallai Theorem**
  A sequence of nonnegative integers $d_1\ge\cdots\ge d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1+\cdots+d_n$ is even and $\sum\limits_{i-1}^{k}d_i\le k(k-1)+\sum\limits_{i=k+1}^{n}\text{min}(d_i,k)$ holds for every $1\le k\le n$.
- **Gale-Ryser Theorem**
  A pair of sequences of nonnegative integers $a_1\ge\cdots\ge a_n$ and $b_1,\ldots,b_n$ is bigraphic (degree seqence of bipartie graph) if and only if $\sum\limits_{i=1}^{n}a_i=\sum\limits_{i=1}^{n}b_i$ and $\sum\limits_{i=1}^{k}a_i\le\sum\limits_{i=1}^{n}\text{min}(b_i,k)$ holds for every $1\le k\le n$.
- **Fulkerson-Chen-Anstee Theorem**
  A sequence $(a_1,b_1),\ldots,(a_n,b_n)$ of nonnegative integer pairs with $a_1\ge\cdots\ge a_n$ is digraphic (in, out degree of a directed graph) if and only if $\sum\limits_{i=1}^{n}a_i=\sum\limits_{i=1}^{n}b_i$ and $\sum\limits_{i=1}^{k}a_i\le\sum\limits_{i=1}^{k}\text{min}(b_i,k-1)+\sum\limits_{i=k+1}^{n}\text{min}(b_i,k)$ holds for every $1\le k\le n$.
- **Möbius Inversion Formula**
  - $f(n)=\sum_{d|n}g(d)\Leftrightarrow g(n)=\sum_{d|n}\mu(d)f(\frac{n}{d})$
  - $f(n)=\sum_{n|d}g(d)\Leftrightarrow g(n)=\sum_{n|d}\mu(\frac{d}{n})f(d)$
- **Lagrange Multiplier**
  - Optimize $f(x_1,\ldots,x_n)$ when k constraints $g_i(x_1,\ldots,x_n)=0$.
  - Lagrangian function $\mathcal{L}(x_1,\ \ldots\ ,\ x_n,\ \lambda_1,\ \ldots\ ,\ \lambda_k)\ =\ f(x_1,\ldots,x_n)-\sum_{i=1}^{k}\lambda_i g_i(x_1,\ldots,x_n)$.
  - The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

5P Estimation

- Number of divisors

$n\le$	100	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
max$d(n)$	12	32	240	1344	6720	26880	103680

- Unordered integer partition

n	2	3	4	5	6	7	8	9	20	30	40	50	100
$p(n)$	2	3	5	7	11	15	22	30	627	5604	$4\cdot10^4$	$2\cdot10^5$	$2\cdot10^8$

- Ways of partitions of n distinct elements

n	2	3	4	5	6	7	8	9	10	11	12	13
B_n	2	5	15	52	203	877	4140	21147	115975	$7\cdot10^5$	$4\cdot10^6$	$3\cdot10^7$

5Q Numbers

- Bernoulli numbers
  $B_0-1, B_1^{\pm}=\pm\frac{1}{2}, B_2=\frac{1}{6}, B_3=0$
  $\sum_{j=0}^{m}\binom{m+1}{j}B_j=0$, EGF is $B(x)=\frac{x}{e^x-1}=\sum_{n=0}^{\infty}B_n\frac{x^n}{n!}$.
  $S_m(n)=\sum_{k=1}^{n}k^m=\frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k^+n^{m+1-k}$
- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.
  $S(n,k)=S(n-1,k-1)+kS(n-1,k), S(n,1)=S(n,n)=1$
  $S(n,k)=\frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$
  $x^n=\sum_{i=0}^{n}S(n,i)(x)_i$
- Pentagonal number theorem
  $\prod_{n=1}^{\infty}(1-x^n)=1+\sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k+1)/2}+x^{k(3k-1)/2}\right)$
- Catalan numbers
  $C_n^{(k)}=\frac{1}{(k-1)n+1}\binom{kn}{n}$
  $C^{(k)}(x)=1+x[C^{(k)}(x)]^k$
- Eulerian numbers
  Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j)>\pi(j+1)$, $k+1$ j:s s.t. $\pi(j)\geq j$, k j:s s.t. $\pi(j)>j$.
  $E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$
  $E(n,0)=E(n,n-1)=1$
  $E(n,k)=\sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$

5R GeneratingFunctions

- Ordinary Generating Function $A(x)=\sum_{i\geq 0}a_ix^i$
  - $A(rx)\Rightarrow r^na_n$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)'\Rightarrow na_n$
  - $\frac{A(x)}{1-x}\Rightarrow\sum_{i=0}^{n}a_i$
- Exponential Generating Function $A(x)=\sum_{i\geq 0}\frac{a_i}{i!}x_i$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A^{(k)}(x)\Rightarrow a_{n+k}$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}\binom{n}{i}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}\binom{n}{i_1,i_2,\ldots,i_k}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)\Rightarrow na_n$
- Special Generating Function
  - $(1+x)^n=\sum_{i\geq 0}\binom{n}{i}x^i$
  - $\frac{1}{(1-x)^n}=\sum_{i\geq 0}\binom{i}{n-1}x^i$
  - $S_k=\sum_{x=1}^{n}x^k$: $S=\sum_{p=0}^{\infty}x^p=\frac{e^x-e^{x(n+1)}}{1-e^x}$

6 Linear Algebra

6A GaussianElimination

```
struct Matrix {
  int m, n;
  vector<vector<fraction>> A;

  Matrix(int rows, int cols) : m(rows
      ), n(cols), A(rows, vector<fraction>(cols)) {}
  // Reduced Row Echelon Form (over rationals)
  void rref() {
    int row = 0;
    for (int col = 0; col < n && row < m; ++col) {
      int pivot = -1;
      for (int i = row; i < m; ++i)
        if (!A[i][col].is_zero()) {
          pivot = i;
          break;
        }
      if (pivot == -1) continue;
      swap(A[row], A[pivot]);
      // normalize pivot row
      fraction div = A[row][col];
      for (int j = 0; j < n; ++j)
        A[row][j] = A[row][j] / div;
      // eliminate other rows
      for (int i = 0; i < m; ++i) {
        if (i == row) continue;
        fraction fac = A[i][col];
        if (fac.is_zero()) continue;
        for (int j = 0; j < n; ++j)
          A[i][j] = A[i][j] - fac * A[row][j];
      }
      ++row;
    }
  }
  fraction det() { // make sure n == m
    fraction dv(1);
    int sign = 1;
    for (int col = 0; col < n; ++col) {
      int pivot = -1;
      for (int i = col; i < n; ++i)
        if (!A[i][col].is_zero()) {
          pivot = i;
          break;
        }
      if (pivot == -1)
        return fraction(0); // det = 0
      if (pivot != col) {
        swap(A[pivot], A[col]);
        sign = -sign;
      }
      fraction piv = A[col][col];
      dv = dv * piv;
      for (int i = col + 1; i < n; ++i) {
        fraction fac = A[i][col] / piv;
        for (int j = col; j < n; ++j)
          A[i][j] = A[i][j] - fac * A[col][j];
      }
    }
    if (sign == -1) dv = dv * fraction(-1);
    return dv;
  }
};
```

6B Simplex

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x}\leq\mathbf{b}$ and $\mathbf{x}\geq 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y}\geq\mathbf{c}$ and $\mathbf{y}\geq 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i\in[1,n]$, either $\bar{x}_i=0$ or $\sum_{j=1}^{m}A_{ji}\bar{y}_j=c_i$ holds and for all $i\in[1,m]$ either $\bar{y}_i=0$ or $\sum_{j=1}^{n}A_{ij}\bar{x}_j=b_j$ holds.

1. In case of minimization, let $c_i'=-c_i$
2. $\sum_{1\leq i\leq n}A_{ji}x_i\geq b_j\rightarrow\sum_{1\leq i\leq n}-A_{ji}x_i\leq -b_j$
3. $\sum_{1\leq i\leq n}A_{ji}x_i=b_j$
   - $\sum_{1\leq i\leq n}A_{ji}x_i\leq b_j$
   - $\sum_{1\leq i\leq n}A_{ji}x_i\geq b_j$
4. If x_i has no lower bound, replace x_i with x_i-x_i'

```
struct Simplex {
  using T = long double;
  static const int N = 50, M = 100;
  const T eps = 1e-7;
  int n, m;
  int Left[M], Down[N];
  T a[M][N], b[M], c[N], v, sol[N];
  bool eq(T a, T b) { return fabs(a - b) < eps; }
  bool ls (T a, T b) { return a < b && !eq(a, b); }
  void init(int _n, int _m) {
  n = _n, m = _m, v = 0;
  for(int
       i = 0; i < m; ++i) for (int j = 0; j < n; ++j) {
    a[i][j] = 0;
  }
  for(int i = 0; i < m; ++i) b[i] = 0;
  for(int i = 0; i < n; ++i) c[i] = sol[i] = 0;
  }
  void pivot(int x, int y) {
  swap(Left[x], Down[y]);
  T k = a[x][y]; a[x][y] = 1;
  vector<int> nz;
  for(int i = 0; i < n; ++i) {
    a[x][i] /= k;
    if(!eq(a[x][i], 0)) nz.push_back(i);
  }
  b[x] /= k;
  for(int i = 0 ; i < m; ++i) {
    if(i == x || eq(a[i][y], 0)) continue;
    k = a[i][y], a[i][y] = 0;
    b[i] -= k * b[x];
    for(int j : nz) a[i][j] -= k * a[x][j];
  }
  if(eq(c[y], 0)) return;
  k = c[y], c[y] = 0, v += k * b[x];
  for(int i : nz) c[i] -= k * a[x][i];
  }
  int solve() {
  for(int i = 0; i < n; ++i) Down[i] = i;
  for(int i = 0; i < m; ++i) Left[i] = n + i;
  while(1) {
    int x = -1, y = -1;
    for(int i = 0; i < m; ++i) if(
        ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
    if(x == -1) break;
    for(int i = 0; i < n; ++i) if(ls(a[x][i
        ], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;
    if(y == -1) return 1;
    pivot(x, y);
  }
  while(1) {
    int x = -1, y = -1;
    for(int i = 0; i < n; ++i) if(
        ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
    if(y == -1) break;
    for(int
         i = 0; i < m; ++i) if(ls(0, a[i][y]) && (x ==
         -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
    if (x == -1) return 2;
    pivot(x, y);
  }
  for(int i = 0;
       i < m; ++i) if(Left[i] < n) sol[Left[i]] = b[i];
  return 0;
  }
};
```

7 Polynomials

7A NTT (FFT)

Mod	g	Form
65 537	3	$2^{16}+1$
998 244 353	3	$119\cdot2^{23}+1$
1 315 962 881	3	$1255\cdot2^{20}+1$
1 711 276 033	29	$51\cdot2^{25}+1$
9 223 372 036 737 335 297	3	$549755813881\cdot2^{24}+1$

```
#define base ll // complex<double>
// const double PI = acosl(-1);
const ll mod = 998244353, g = 3;
base omega[4 * N], omega_[4 * N];
int rev[4 * N];

ll fpow(ll b, ll p);
ll inverse(ll a) { return fpow(a, mod - 2); }

void calcW(int n) {
  ll r = fpow(g, (mod - 1) / n), invr = inverse(r);
  omega[0] = omega_[0] = 1;
  for (int i = 1; i < n; i++) {
    omega[i] = omega[i - 1] * r % mod;
    omega_[i] = omega_[i - 1] * invr % mod;
  }
  // double arg = 2.0 * PI / n;
  // for (int i = 0; i < n; i++)
  // {
  //   omega[i] = base(cos(i * arg), sin(i * arg));
  //   omega_[i] = base(cos(-i * arg), sin(-i * arg));
  // }
}

void calcrev(int n) {
  int k = __lg(n);
  for (int i = 0; i < n; i++) rev[i] = 0;
  for (int i = 0; i < n; i++)
    for (int j = 0; j < k; j++)
      if (i & (1 << j)) rev[i] ^= 1 << (k - j - 1);
}

vector<base> NTT(vector<base> poly, bool inv) {
  base *w = (inv ? omega_ : omega);
  int n = (int)poly.size();
  for (int i = 0; i < n; i++)
    if (rev[i] > i) swap(poly[i], poly[rev[i]]);

  for (int len = 1; len < n; len <<= 1) {
    int arg = n / len / 2;
    for (int i = 0; i < n; i += 2 * len)
      for (int j = 0; j < len; j++) {
        base odd =
          w[j * arg] * poly[i + j + len] % mod;
        poly[i + j + len] =
          (poly[i + j] - odd + mod) % mod;
        poly[i + j] = (poly[i + j] + odd) % mod;
      }
  }
  if (inv)
    for (auto &a : poly) a = a * inverse(n) % mod;
  return poly;
}

vector<base> mul(vector<base> f, vector<base> g) {
  int sz = 1 << (__lg(f.size() + g.size() - 1) + 1);
  f.resize(sz), g.resize(sz);
  calcrev(sz);
  calcW(sz);
  f = NTT(f, 0), g = NTT(g, 0);
  for (int i = 0; i < sz; i++)
    f[i] = f[i] * g[i] % mod;
  return NTT(f, 1);
}
```

7B FHWT

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
op: 1, invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { // or
  for (int L = 2; L <= n; L <<= 1)
    for (int i = 0; i < n; i += L)
      for (int j = i; j < i + (L >> 1); ++j)
        a[j + (L >> 1)] += a[j] * op;
}
const int P = 21; // power of max N
int f[P][1 << P], g[P][1 << P], h[P][1 << P],
  ct[1 << P];
void subset_convolution(
  int *a, int *b, int *c, int L) {
  // c_k = \sum_{i | j = k, i & j = 0} a_i * b_j
  int n = 1 << L;
  for (int i = 1; i < n; ++i)
    ct[i] = ct[i & (i - 1)] + 1;
  for (int i = 0; i < n; ++i)
    f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
  for (int i = 0; i <= L; ++i)
    fwt(f[i], n, 1), fwt(g[i], n, 1);
  for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
      for (int x = 0; x < n; ++x)
        h[i][x] += f[j][x] * g[i - j][x];
  for (int i = 0; i <= L; ++i) fwt(h[i], n, -1);
  for (int i = 0; i < n; ++i) c[i] = h[ct[i]][i];
}
```

7C PolynomialOperations

```
#define poly vector<ll>
poly inv(poly A) {
  A.resize(1 << (__lg(A.size() - 1) + 1));
  poly B = {inverse(A[0])};
  for (int n = 1; n < (int)A.size(); n <<= 1) {
    poly pA(A.begin(), A.begin() + 2 * n);
    calcrev(4 * n), calcW(4 * n);
    pA.resize(4 * n), B.resize(4 * n);
    pA = NTT(pA, 0);
    B = NTT(B, 0);
    for (int i = 0; i < 4 * n; i++)
      B[i] =
        ((B[i] * 2 - pA[i] * B[i] % mod * B[i]) % mod +
          mod) %
        mod;
    B = NTT(B, 1);
    B.resize(2 * n);
  }
  return B;
}
pair<poly, poly> div(poly A, poly B) {
  if (A.size() < B.size()) return make_pair(poly(), A);
  int n = A.size(), m = B.size();
  poly revA = A, invrevB = B;
  reverse(all(revA)), reverse(all(invrevB));
  revA.resize(n - m + 1);
  invrevB.resize(n - m + 1);
  invrevB = inv(invrevB);
  poly Q = mul(revA, invrevB);
  Q.resize(n - m + 1);
  reverse(all(Q));
  poly R = mul(Q, B);
  R.resize(m - 1);
  for (int i = 0; i < m - 1; i++)
    R[i] = (A[i] - R[i] + mod) % mod;
  return make_pair(Q, R);
}
poly modulo(poly A, poly B) { return div(A, B).S; }
ll fast_kitamasa(ll k, poly A, poly C) {
  int n = A.size();
  C.emplace_back(mod - 1);
  poly Q, R = {0, 1}, F = {1};
  R = modulo(R, C);
  for (; k; k >>= 1) {
    if (k & 1) F = modulo(mul(F, R), C);
    R = modulo(mul(R, R), C);
    k >>= 1;
  }
  ll ans = 0;
  for (int i = 0; i < F.size(); i++)
    ans = (ans + A[i] * F[i]) % mod;
  return ans;
}
vector<ll> fpow(vector<ll> f, ll p, ll m) {
  int b = 0;
  while (b < f.size() && f[b] == 0) b++;
  f = vector<ll>(f.begin() + b, f.end());
  int n = f.size();
  f.emplace_back(0);
  vector<ll> q(min(m, b * p), 0);
  q.emplace_back(fpow(f[0], p));
  for (int k = 0; q.size() < m; k++) {
    ll res = 0;
    for (int i = 0; i < min(n, k + 1); i++)
      res = (res +
              p * (i + 1) % mod * f[i + 1] % mod *
                q[k - i + b * p]) %
        mod;
    for (int i = 1; i < min(n, k + 1); i++)
      res = (res -
              f[i] * (k - i + 1) % mod *
                q[k - i + 1 + b * p]) %
        mod;
    res = (res < 0 ? res + mod : res) *
      inv(f[0] * (k + 1) % mod) % mod;
    q.emplace_back(res);
  }
  return q;
}
```

7D NewtonMethod+MiscGF

Given $F(x)$ where

$$F(x)=\sum_{i=0}^{\infty}\alpha_i(x-\beta)^i$$

for β being some constant. Polynomial P such that $F(P)=0$ can be found iteratively. Denote by Q_k the polynomial such that $F(Q_k)=0 \pmod{x^{2^k}}$, then

$$Q_{k+1}=Q_k-\frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

- A^{-1}: $B_{k+1}=B_k(2-AB_k)$ $\bmod x^{2^{k+1}}$
- $\ln A$: $(\ln A)'=\frac{A'}{A}$
- $\exp A$: $B_{k+1}=B_k(1+A-\ln B_k)$ $\bmod x^{2^{k+1}}$
- $\sqrt{A}$: $B_{k+1}=\frac{1}{2}(B_k+AB_k^{-1})$ $\bmod x^{2^{k+1}}$

8 Geometry

8A Basic

```
typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double pi = acos(-1);
const double eps = 1e-8;
pll operator+(pll a, pll b)
{ return pll(a.F + b.F, a.S + b.S); }
pll operator-(pll a, pll b)
{ return pll(a.F - b.F, a.S - b.S); }
pll operator-(pll a)
{ return pll(-a.F, -a.S); }
pll operator*(pll a, ll b)
{ return pll(a.F * b, a.S * b); }
pdd operator/(pll a, double b)
{ return pdd(a.F / b, a.S / b); }
ll dot(pll a, pll b)
{ return a.F * b.F + a.S * b.S; }
ll cross(pll a, pll b)
{ return a.F * b.S - a.S * b.F; }
ll abs2(pll a)
{ return dot(a, a); }
double abs(pll a)
{ return sqrt(dot(a, a)); }
int sign(ll a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pll a, pll b, pll c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pll p1, pll p2, pll p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pll a, pll b, pll c) {
  return collinearity
      (a, b, c) && sign(dot(a - c, b - c)) <= 0;
}
bool seg_strict_intersect
    (pdd p1, pdd p2, pdd p3, pdd p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  return a123 * a124 < 0 && a341 * a342 < 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if (a123 == 0 && a124 == 0)
    return btw(p1, p2, p3) || btw(p1, p2, p4) ||
      btw(p3, p4, p1) || btw(p3, p4, p2);
  return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  double a123 = cross(p2 - p1, p3 - p1);
  double a124 = cross(p2 - p1, p4 - p1);
  return (p4
      * a123 - p3 * a124) / (a123 - a124); // C^3 / C^2
}
pdd orth(pdd p1)
{ return pdd(-p1.S, p1.F); }
```

```
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + orth(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
  pdd dp = p1 - p0
      , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
  return q0 + pdd(
      cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r
```

8B ConvexHull

```
vector<pll> hull(vector<pll> dots) { // n=1 => ans = {}
  sort(dots.begin(), dots.end());
  vector<pll> ans(1, dots[0]);
  for (int ct = 0; ct < 2; ++ct, reverse(all(dots)))
    for (int i = 1, t = (int)ans.size();
         i < (int)dots.size();
         ans.emplace_back(dots[i++]))
      while ((int)ans.size() > t &&
        ori(ans.end()[-2], ans.back(), dots[i]) <= 0)
        ans.pop_back(); // "<" for keeping collinear
  ans.pop_back();
  return ans;
}
```

8C SortByAngle

```
bool down(pll k) {
  return sign(k.S) < 0 ||
    (sign(k.S) == 0 && sign(k.F) < 0);
}
int cmp(pll a, pll b, bool same = true) {
  int A = down(a), B = down(b);
  if (A != B) return A < B;
  if (sign(cross(a, b)) == 0)
    return same ? abs2(a) < abs2(b) : -1;
  return sign(cross(a, b)) > 0;
}
```

8D Formulas

- **Rotation**

$$M(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

  90 degree: $(x,y)=(Y-y,x)$

- **Pick's theorem**
  For simple integer-coordinate polygon,

$$A=B+\frac{I}{2}-1$$

  Where A is the area; B,I is #lattice points in the interior, on the boundary.

- **Spherical Cap**
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: arcsin(a/r).
  - Volume $=\pi h^2(3r-h)/3=\pi h(3a^2+h^2)/6=\pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $=2\pi rh=\pi(a^2+h^2)=2\pi r^2(1-\cos\theta)$.

- **Nearest points of two skew lines**
  - Line 1:$\boldsymbol{v}_1=\boldsymbol{p}_1+t_1\boldsymbol{d}_1$
  - Line 2:$\boldsymbol{v}_2=\boldsymbol{p}_2+t_2\boldsymbol{d}_2$
  - $\boldsymbol{n}=\boldsymbol{d}_1\times\boldsymbol{d}_2$
  - $\boldsymbol{n}_1=\boldsymbol{d}_1\times\boldsymbol{n}$
  - $\boldsymbol{n}_2=\boldsymbol{d}_2\times\boldsymbol{n}$
  - $\boldsymbol{c}_1=\boldsymbol{p}_1+\frac{(\boldsymbol{p}_2-\boldsymbol{p}_1)\cdot\boldsymbol{n}_2}{\boldsymbol{d}_1\cdot\boldsymbol{n}_2}\boldsymbol{d}_1$
  - $\boldsymbol{c}_2=\boldsymbol{p}_2+\frac{(\boldsymbol{p}_1-\boldsymbol{p}_2)\cdot\boldsymbol{n}_1}{\boldsymbol{d}_2\cdot\boldsymbol{n}_1}\boldsymbol{d}_2$

8E TriangleHearts

```
pdd excenter(
  pdd p0, pdd p1, pdd p2) { // radius = abs(center)
  p1 = p1 - p0, p2 = p2 - p0;
  auto [x1, y1] = p1;
  auto [x2, y2] = p2;
  double m = 2. * cross(p1, p2);
  pdd center = pdd((x1 * x1 * y2 - x2 * x2 * y1 +
                   y1 * y2 * (y1 - y2)),
                 (x1 * x2 * (x2 - x1) - y1 * y1 * x2 +
                   x1 * y2 * y2)) / m;
  return center + p0;
}
pdd incenter(
  pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
  double a = abs(p2 - p3), b = abs(p1 - p3),
         c = abs(p1 - p2);
  double s = a + b + c;
  return (p1 * a + p2 * b + p3 * c) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3) {
  return (p1 + p2 + p3) / 3;
}
pdd orthcenter(pdd p1, pdd p2, pdd p3) {
  return masscenter(p1, p2, p3) * 3 -
    excenter(p1, p2, p3) * 2;
}
```

8F PointSegmentDist

```
double PointSegDist(pdd q0, pdd q1, pdd p) {
  if (abs(q0 - q1) <= eps) return abs(q0 - p);
  if (dot(q1 - q0,
       p - q0) >= -eps && dot(q0 - q1, p - q1) >= -eps)
    return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
  return min(abs(p - q0), abs(p - q1));
}
```

8G PointInCircle

```
// return q'
    s relation with circumcircle of tri(p[0],p[1],p[2])
bool in_cc(const array<pll, 3> p, pll q) {
  __int128 det = 0;
  for (int i = 0; i < 3; ++i)
    det += __int128(abs2(p[i]) - abs2(q)) *
         cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
  return det > 0; // in: >0, on: =0, out: <0
}
```

8H PointInConvex

```
bool PointInConvex
    (const vector<pll> &C, pll p, bool strict = true) {
  int a = 1, b = (int)C.size() - 1, r = !strict;
  if ((int)C.size() == 0) return false;
  if ((int)
      C.size() < 3) return r && btw(C[0], C.back(), p);
  if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
  if (ori
      (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
  }
  return ori(C[a], C[b], p) < r;
}
```

8I PointToConvexHull

```
double PointToHull(const vector<pdd>& C, pdd p) {
    bool flg
         = (cross(C[0] - C.back(), p - C.back()) > 0);
    double ans = PointSegDist(C.back(), C[0], p);
    for (int i = 1; i < (int)C.size(); ++i) {
      ans =
          min(ans, PointSegDist(C[i - 1], C[i], p));
      if ((cross(C[i
          ] - C[i - 1], p - C[i - 1]) <= 0)) flg = 0;
    }
    return (flg ? 0 : ans);
}
```

8J PointTangentConvex

```
/* The point should be strictly out of hull
  return arbitrary point on the tangent line */
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
  if (n == 1) return 0;
  int l = 0, r = n; bool rv = pred(1, 0);
  while (r - l > 1) {
    int m = (l + r) / 2;
    if (pred(0, m) ? rv: pred(m, (m + 1) % n)) r = m;
    else l = m;
  }
  return pred(l, r % n) ? l : r % n;
}
pii get_tangent(vector<pll> &C, pll p) {
  auto gao = [&](int s) {
    return cyc_tsearch((int)C.size(), [&](int x, int y)
    { return ori(p, C[x], C[y]) == s; });
  };
  return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0
```

8K CircTangentCirc

```
vector<Line> go(Cir c1, Cir c2, int sign1) {
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = abs2(c1.O - c2.O);
  if (sign(d_sq) == 0) return ret;
  double d = sqrt(d_sq);
  pdd v = (c2.O - c1.O) / d;
  double c = (c1.R - sign1 * c2.R) / d;
  if (c * c > 1) return ret;
  double h = sqrt(max(0.0, 1.0 - c * c));
  for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    pdd n = pdd(v.F * c - sign2 * h * v.S,
      v.S * c + sign2 * h * v.F);
    pdd p1 = c1.O + n * c1.R;
    pdd p2 = c2.O + n * (c2.R * sign1);
    if (sign(p1.F - p2.F) == 0 and
      sign(p1.S - p2.S) == 0)
      p2 = p1 + perp(c2.O - c1.O);
    ret.emplace_back(Line(p1, p2));
  }
  return ret;
}
```

8L LineCircleIntersect

```
vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
  pdd p
      = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
  double s = cross
      (b - a, c - a), h2 = r * r - s * s / abs2(b - a);
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  pdd h = (b - a) / abs(b - a) * sqrt(h2);
  return {p - h, p + h};
}
```

8M LineConvexIntersect

```
int cyc_tsearch(int n, auto pred); // ref: TanPointHull
int TangentDir(vector<pll> &C, pll dir) {
  return cyc_tsearch((int)C.size(), [&](int a, int b) {
    return cross(dir, C[a]) > cross(dir, C[b]);
  });
}
#define cmpL(i) sign(cross(C[i] - a, b - a))
pii lineHull(pll a, pll b, vector<pll> &C) {
  int A = TangentDir(C, a - b);
  int B = TangentDir(C, b - a);
  int n = (int)C.size();
  if (cmpL(A) < 0 || cmpL(B) > 0)
    return pii(-1, -1); // no collision
  auto gao = [&](int l, int r) {
    for (int t = l; (l + 1) % n != r;) {
      int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(t) ? l : r) = m;
    }
    return (l + !cmpL(r)) % n;
  };
  pii res = pii(gao(B, A), gao(A, B)); // (i, j)
  if (res.F == res.S) // touching the corner i
    return pii(res.F, -1);
  if (!cmpL(res.F) &&
    !cmpL(res.S)) // along side i, i+1
    switch ((res.F - res.S + n + 1) % n) {
    case 0: return pii(res.F, res.F);
    case 2: return pii(res.S, res.S);
    }
  /* crossing sides (i, i+1) and (j, j+1)
  crossing corner i is treated as side (i, i+1)
  returned in the same order as the line hits the
  convex */
  return res;
} // convex cut: (r, l]
```

8N CircIntersectCirc

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
  pdd o1 = a.O, o2 = b.O;
  double r1 =
       a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
  if(d < max
      (r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
  pdd u = (o1 + o2) * 0.5
       + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
  double A = sqrt((r1 + r2 + d) *
       (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
  pdd v
       = pdd(o1.S - o2.S, -o1.F + o2.F) * A / (2 * d2);
  p1 = u + v, p2 = u - v;
  return 1;
}
```

8O PolyIntersectCirc

```
// Divides into multiple triangle, and sum up
const double PI = acos(-1);
double _area(pdd pa, pdd pb, double r) {
  if (abs(pa) < abs(pb)) swap(pa, pb);
  if (abs(pb) < eps) return 0;
  double S, h, theta;
  double a = abs(pb), b = abs(pa), c = abs(pb - pa);
  double cosB = dot(pb, pb - pa) / a / c,
         B = acos(cosB);
  double cosC = dot(pa, pb) / a / b, C = acos(cosC);
  if (a > r) {
    S = (C / 2) * r * r;
    h = a * b * sin(C) / c;
    if (h < r && B < PI / 2)
      S -= (acos(h / r) * r * r -
        h * sqrt(r * r - h * h));
  } else if (b > r) {
    theta = PI - B - asin(sin(B) / r * a);
    S = .5 * a * r * sin(theta) +
      (C - theta) / 2 * r * r;
  } else S = .5 * sin(C) * a * b;
  return S;
}
double area_poly_circle(const vector<pdd> poly,
  const pdd &O, const double r) {
  double S = 0;
  for (int i = 0; i < (int)poly.size(); ++i)
    S += _area(poly[i] - O,
           poly[(i + 1) % (int)poly.size()] - O, r) *
      ori(
        O, poly[i], poly[(i + 1) % (int)poly.size()]);
  return fabs(S);
}
```

8P PolyUnion

```
double rat(pll a, pll b) {
  return sign
      (b.F) ? (double)a.F / b.F : (double)a.S / b.S;
} // all poly. should be ccw
double polyUnion(vector<vector<pll>> &poly) {
  double res = 0;
  for (auto &p : poly)
    for (int a = 0; a < (int)p.size(); ++a) {
      pll A = p[a], B = p[(a + 1) % (int)p.size()];
      vector
          <pair<double, int>> segs = {{0, 0}, {1, 0}};
      for (auto &q : poly) {
        if (&p == &q) continue;
        for (int b = 0; b < (int)q.size(); ++b) {
          pll C = q[b], D = q[(b + 1) % (int)q.size()];
          int sc = ori(A, B, C), sd = ori(A, B, D);
          if (sc != sd && min(sc, sd) < 0) {
            double sa = cross(D
                - C, A - C), sb = cross(D - C, B - C);
            segs.emplace_back
                (sa / (sa - sb), sign(sc - sd));
          }
          if (!sc && !sd &&
              &q < &p && sign(dot(B - A, D - C)) > 0) {
            segs.emplace_back(rat(C - A, B - A), 1);
            segs.emplace_back(rat(D - A, B - A), -1);
          }
        }
      }
      sort(all(segs));
      for (auto &s : segs) s.F = clamp(s.F, 0.0, 1.0);
      double sum = 0;
      int cnt = segs[0].second;
      for (int j = 1; j < (int)segs.size(); ++j) {
        if (!cnt) sum += segs[j].F - segs[j - 1].F;
        cnt += segs[j].S;
      }
      res += cross(A, B) * sum;
    }
  return res / 2;
}
```

8Q MinkowskiSum

```
void shift(vector<pdd>& h) {
    int p = 0; // must be convex hull
    for (int i = 1; i < h.size(); i++) {
        if (h[i].S < h[p].S) p = i;
        if (h[i].S == h[p].S && h[i].F < h[p].F) p = i;
    }
    rotate(h.begin(), h.begin() + p, h.end());
}
void Minkowski
    (vector<pdd>& a, vector<pdd>& b, vector<pdd>& c) {
    shift(a), shift(b); c = {}; // |A|,|B|>=3
    int A = a.size(), B = b.size(), i = 0, j = 0;
    a.pb(a[0]), a.pb(a[1]), b.pb(b[0]), b.pb(b[1]);
    while (i < A || j < B) {
        c.pb(a[i] + b[j]);
        ll c = cross(a[i + 1] - a[i], b[j + 1] - b[j]);
        if (c >= 0 && i < A) i++;
        if (c <= 0 && j < B) j++;
    }
    for (int
         i = 0; i < 2; i++) a.pop_back(), b.pop_back();
}
```

8R DistanceBetweenHull

```
double ConvexHullDist
    (vector<pdd>& A, vector<pdd>& B, vector<pdd>& C) {
    Minkowski(A, B, C); // assert (int)C.size() > 0
    return PointToHull(C, pdd(0, 0));
}
```

8S MinMaxEnclosingRect

```
const double qi = acos(-1) / 2 * 3;
pdd solve(vector<pll> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[i])
  hull(dots);
  double Max = 0, Min = INF, deg;
  int n = (int)dots.size();
  dots.emplace_back(dots[0]);
  for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
    pll nw = vec(i + 1);
    while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
      u = (u + 1) % n;
    while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
      r = (r + 1) % n;
    if (!i) l = (r + 1) % n;
    while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
      l = (l + 1) % n;
    Min = min(Min, (double)(dot(nw, vec(r)) - dot
        (nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
    deg = acos(dot(diff(r
        , l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
    deg = (qi - deg) / 2;
    Max = max(Max, abs(diff
        (r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
  }
  return pdd(Min, Max);
}
```

8T CircleCover

```
// N ~= 1000
struct CircleCover {
  int C;
  Cir c[N];
  bool g[N][N], overlap[N][N];
  // Area[i] : area covered by at least i circles
  double Area[ N ];
  void init(int _c){ C = _c;}
  struct Teve {
    pdd p; double ang; int add;
    Teve() {}
    Teve(pdd _a
        , double _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[N * 2];
  // strict: x = 0, otherwise x = -1
  bool disjuct(Cir &a, Cir &b, int x)
  {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
  bool contain(Cir &a, Cir &b, int x)
  {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
  bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign
        (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].
        R) == 0 && i < j)) && contain(c[i], c[j], -1);
  }
  void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
            disjuct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
      int E = 0, cnt = 1;
      for(int j = 0; j < C; ++j)
        if(j != i && overlap[j][i])
          ++cnt;
      for(int j = 0; j < C; ++j)
        if(i != j && g[i][j]) {
          pdd aa, bb;
          CCinter(c[i], c[j], aa, bb);
          double A =
               atan2(aa.S - c[i].O.S, aa.F - c[i].O.F);
          double B =
               atan2(bb.S - c[i].O.S, bb.F - c[i].O.F);
          eve[E++] = Teve
              (bb, B, 1), eve[E++] = Teve(aa, A, -1);
          if(B > A) ++cnt;
        }
      if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
      else{
        sort(eve, eve + E);
        eve[E] = eve[0];
        for(int j = 0; j < E; ++j){
          cnt += eve[j].add;
          Area[cnt
              ] += cross(eve[j].p, eve[j + 1].p) * .5;
          double theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2. * pi;
          Area[cnt] += (theta
               - sin(theta)) * c[i].R * c[i].R * .5;
        }
      }
    }
  }
};
```

8U LineCmp

```
struct lineCmp { // coordinates should be even!
  bool operator()(Line l1, Line l2) const {
    int X =
      (max(l1.F.F, l2.F.F) + min(l1.S.F, l2.S.F)) / 2;
    ll p1 =
         (X - l1.F.F) * l1.S.S + (l1.S.F - X) * l1.F.S,
       p2 =
         (X - l2.F.F) * l2.S.S + (l2.S.F - X) * l2.F.S,
       q1 = (l1.S.F - l1.F.F), q2 = (l2.S.F - l2.F.F);
    if (q1 == 0) p1 = l1.F.S + l1.S.S, q1 = 2;
    if (q2 == 0) p2 = l2.F.S + l2.S.S, q2 = 2;
    // for query a point: ask make_pair(P, P)
    if (l1.F == l2.F || l2.F == l2.S) l1 = l2;
    return make_tuple((__int128)(p1 * q2), l1) <
      make_tuple((__int128)(p2 * q1), l2);
  }
};
```

8V Trapezoidalization

```
template<class T>
struct SweepLine {
  struct cmp {
    cmp(const SweepLine &_swp): swp(_swp) {}
    bool operator()(int a, int b) const {
      if (abs(swp.get_y(a) - swp.get_y(b)) <= swp.eps)
        return swp.slope_cmp(a, b);
      return swp.get_y(a) + swp.eps < swp.get_y(b);
    }
    const SweepLine &swp;
  } _cmp;
  T curTime, eps, curQ;
  vector<Line> base;
  multiset<int, cmp> sweep;
  multiset<pair<T, int>> event;
  vector<typename multiset<int, cmp>::iterator> its;
  vector
      <typename multiset<pair<T, int>>::iterator> eits;
  bool slope_cmp(int a, int b) const {
    assert(a != -1);
    if (b == -1) return 0;
    return sign(cross(base
        [a].S - base[a].F, base[b].S - base[b].F)) < 0;
  }
  T get_y(int idx) const {
    if (idx == -1) return curQ;
    Line l = base[idx];
    if (l.F.F == l.S.F) return l.S.S;
    return ((curTime - l.F.F) * l.S.S
        + (l.S.F - curTime) * l.F.S) / (l.S.F - l.F.F);
  }
  void insert(int idx) {
    its[idx] = sweep.insert(idx);
    if (its[idx] != sweep.begin())
      update_event(*prev(its[idx]));
    update_event(idx);
    event.emplace
        (base[idx].S.F, idx + 2 * (int)base.size());
  }
  void erase(int idx) {
    assert(eits[idx] == event.end());
    auto p = sweep.erase(its[idx]);
    its[idx] = sweep.end();
    if (p != sweep.begin())
      update_event(*prev(p));
  }
  void update_event(int idx) {
    if (eits[idx] != event.end())
      event.erase(eits[idx]);
    eits[idx] = event.end();
    auto nxt = next(its[idx]);
    if (nxt ==
         sweep.end() || !slope_cmp(idx, *nxt)) return;
    auto t = intersect(base[idx].
        F, base[idx].S, base[*nxt].F, base[*nxt].S).F;
    if (t + eps < curTime || t
         >= min(base[idx].S.F, base[*nxt].S.F)) return;
    eits[idx
        ] = event.emplace(t, idx + (int)base.size());
  }
  void swp(int idx) {
    assert(eits[idx] != event.end());
    eits[idx] = event.end();
    int nxt = *next(its[idx]);
    swap((int&)*its[idx], (int&)*its[nxt]);
    swap(its[idx], its[nxt]);
    if (its[nxt] != sweep.begin())
      update_event(*prev(its[nxt]));
    update_event(idx);
  }
  // only expected to call the functions below
  SweepLine(T t, T e, vector<Line> vec): _cmp
      (*this), curTime(t), eps(e), curQ(), base(vec),
       sweep(_cmp), event(), its((int)vec.size(), sweep
      .end()), eits((int)vec.size(), event.end()) {
    for (int i = 0; i < (int)base.size(); ++i) {
      auto &[p, q] = base[i];
      if (p > q) swap(p, q);
      if (p.F <= curTime && curTime <= q.F)
        insert(i);
      else if (curTime < p.F)
        event.emplace(p.F, i);
    }
  }
  void setTime(T t, bool ers = false) {
    assert(t >= curTime);
    while (!event.empty() && event.begin()->F <= t) {
      auto [et, idx] = *event.begin();
      int s = idx / (int)base.size();
      idx %= (int)base.size();
      if (abs(et - t) <= eps && s == 2 && !ers) break;
      curTime = et;
      event.erase(event.begin());
      if (s == 2) erase(idx);
      else if (s == 1) swp(idx);
      else insert(idx);
    }
    curTime = t;
  }
  T nextEvent() {
    if (event.empty()) return INF;
    return event.begin()->F;
  }
  int lower_bound(T y) {
    curQ = y;
    auto p = sweep.lower_bound(-1);
    if (p == sweep.end()) return -1;
    return *p;
  }
};
```

8W HalfPlaneIntersect

```
pll area_pair(Line a, Line b)
{ return pll(cross(a.S
     - a.F, b.F - a.F), cross(a.S - a.F, b.S - a.F)); }
bool isin(Line l0, Line l1, Line l2) {
  // Check inter(l1, l2) strictly in l0
  auto [a02X, a02Y] = area_pair(l0, l2);
  auto [a12X, a12Y] = area_pair(l1, l2);
  if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
  return (__int128)
       a02Y * a12X - (__int128) a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> arr) {
  sort(all(arr), [&](Line a, Line b) -> int {
    if (cmp(a.S - a.F, b.S - b.F, 0) != -1)
      return cmp(a.S - a.F, b.S - b.F, 0);
    return ori(a.F, a.S, b.S) < 0;
  });
  deque<Line> dq(1, arr[0]);
  for (auto p : arr) {
    if (cmp(
        dq.back().S - dq.back().F, p.S - p.F, 0) == -1)
      continue;
    while ((int)dq.size() >= 2
        && !isin(p, dq[(int)dq.size() - 2], dq.back()))
      dq.pop_back();
    while
        ((int)dq.size() >= 2 && !isin(p, dq[0], dq[1]))
      dq.pop_front();
    dq.emplace_back(p);
  }
  while ((int)dq.size() >= 3 &&
       !isin(dq[0], dq[(int)dq.size() - 2], dq.back()))
    dq.pop_back();
  while ((int)
        dq.size() >= 3 && !isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
  return vector<Line>(all(dq));
}
```

8X RotatingSweepLine

```
void rotatingSweepLine(vector<pii> &ps) {
  int n = (int)ps.size(), m = 0;
  vector<int> id(n), pos(n);
  vector<pii> line(n * (n - 1));
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      if (i != j) line[m++] = pii(i, j);
  sort(all(line), [&](pii a, pii b) {
    return cmp(ps[a.S] - ps[a.F], ps[b.S] - ps[b.F]);
  }); // cmp(): polar angle compare
  iota(all(id), 0);
  sort(all(id), [&](int a, int b) {
    if (ps[a].S != ps[b].S) return ps[a].S < ps[b].S;
    return ps[a] < ps[b];
  }); // initial order, since (1, 0) is the smallest
  for (int i = 0; i < n; ++i) pos[id[i]] = i;
  for (int i = 0; i < m; ++i) {
    auto l = line[i];
    // do something
    tie(pos[l.F], pos[l.S], id[pos[l.F]], id[pos[l.S
        ]]) = make_tuple(pos[l.S], pos[l.F], l.S, l.F);
  }
}
```

8Y DelaunayTriangulation

```
/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle. */
struct Edge {
  int id; // oidx[id]
  list<Edge>::iterator twin;
  Edge(int _id = 0) : id(_id) {}
};
struct Delaunay { // 0-base
  int n, oidx[N];
  list<Edge> head[N]; // result udir. graph
  pll p[N];
  void init(int _n, pll _p[]) {
    n = _n, iota(oidx, oidx + n, 0);
    for (int i = 0; i < n; ++i) head[i].clear();
    sort(oidx, oidx + n,
      [&](int a, int b) { return _p[a] < _p[b]; });
    for (int i = 0; i < n; ++i) p[i] = _p[oidx[i]];
    divide(0, n - 1);
  }
  void addEdge(int u, int v) {
    head[u].push_front(Edge(v));
    head[v].push_front(Edge(u));
    head[u].begin()->twin = head[v].begin();
    head[v].begin()->twin = head[u].begin();
  }
  void divide(int l, int r) {
    if (l == r) return;
    if (l + 1 == r) return addEdge(l, l + 1);
    int mid = (l + r) >> 1, nw[2] = {l, r};
    divide(l, mid), divide(mid + 1, r);
    auto gao = [&](int t) {
      pll pt[2] = {p[nw[0]], p[nw[1]]};
      for (auto it : head[nw[t]]) {
        int v = ori(pt[1], pt[0], p[it.id]);
        if (v > 0 ||
          (v == 0 &&
            abs2(pt[t ^ 1] - p[it.id]) <
              abs2(pt[1] - pt[0])))
          return nw[t] = it.id, true;
      }
      return false;
    };
    while (gao(0) || gao(1));
    addEdge(nw[0], nw[1]); // add tangent
    while (true) {
      pll pt[2] = {p[nw[0]], p[nw[1]]};
      int ch = -1, sd = 0;
      for (int t = 0; t < 2; ++t)
        for (auto it : head[nw[t]])
          if (ori(pt[0], pt[1], p[it.id]) > 0 &&
            (ch == -1 ||
              in_cc({pt[0], pt[1], p[ch]}, p[it.id])))
            ch = it.id, sd = t;
      if (ch == -1) break; // upper common tangent
      for (auto it = head[nw[sd]].begin();
          it != head[nw[sd]].end();)
        if (seg_strict_intersect(
              pt[sd], p[it->id], pt[sd ^ 1], p[ch]))
          head[it->id].erase(it->twin),
            head[nw[sd]].erase(it++);
        else ++it;
      nw[sd] = ch, addEdge(nw[0], nw[1]);
    }
  }
} tool;
```

8Z VonoroiDiagram

```
// all coord. is even
    , you may want to call halfPlaneInter after then
vector<vector<Line>> vec;
void build_voronoi_line(int n, pll *arr) {
  tool.init(n, arr); // Delaunay
  vec.clear(), vec.resize(n);
  for (int i = 0; i < n; ++i)
    for (auto e : tool.head[i]) {
      int u = tool.oidx[i], v = tool.oidx[e.id];
      pll m = (arr[v
          ] + arr[u]) / 2LL, d = perp(arr[v] - arr[u]);
      vec[u].emplace_back(Line(m, m + d));
    }
}
```

9 Misc

9A ManhattanMST

```
#define p3i tuple<int, int, int>
struct DSU {
  vector<int> v;
  DSU(int n);
  int query(int u);
  void merge(int x, int y);
};
vector<p3i> manhattanMST(vector<pll> ps) {
  vector<int> id(ps.size());
  iota(id.begin(), id.end(), 0);
  vector<p3i> edges;
  for (int k = 0; k < 4; ++k) {
    sort(id.begin(), id.end(), [&](int i, int j) {
      return (ps[i] - ps[j]).F < (ps[j] - ps[i]).S;
    });
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].S);
           it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        pll d = ps[i] - ps[j];
        if (d.S > d.F) break;
        edges.emplace_back(d.S + d.F, i, j);
      }
      sweep[-ps[i].S] = i;
    }
    for (auto &p : ps)
      if (k & 1) p.F = -p.F;
      else swap(p.F, p.S);
  }
  return edges;
}
vector<int> MST(int n, const vector<p3i> &e) {
  vector<int> idx(e.size());
  iota(idx.begin(), idx.end(), 0);
  sort(idx.begin(), idx.end(), [&](int i, int j) {
    return get<0>(e[i]) < get<0>(e[j]);
  });
  vector<int> r;
  DSU dsu(n);
  for (int o : idx) {
    const auto &[w, i, j] = e[o];
    if (dsu.query(i) == dsu.query(j)) continue;
    r.push_back(o);
    dsu.merge(i, j);
  }
  return r;
}
```

9B SternBrocotTree

- Construction: Root $\frac{1}{1}$, left/right neighbor $\frac{0}{1}, \frac{1}{0}$, each node is sum of last left/right neighbor: $\frac{a}{b}, \frac{c}{d} \to \frac{a+c}{b+d}$
- Property: Adjacent (mid-order DFS) $\frac{a}{b}, \frac{c}{d} \Rightarrow bc - ad = 1$.
- Search known $\frac{p}{q}$: keep L-R alternative. Each step can calcaulated in $O(1) \Rightarrow$ total $O(\log C)$.
- Search unknown $\frac{p}{q}$: keep L-R alternative. Each step can calcaulated in $O(\log C)$ checks $\Rightarrow$ total $O(\log^2 C)$ checks.

9C CDQ

```
void
    solve(int l, int r, vector<Point> &a) { // 三維偏序
    if (l == r) return;
    int mid = (l+r)>>1;
    solve(l,mid,a), solve(mid+1,r,a);
    int lp = l, rp = mid+1;
    vector<Point> tmp;
    while(lp <= mid && rp <= r) {
        if (a[lp].y > a[rp].y) {
            bit.upd(a[lp].z,1);
            tmp.pb(a[lp++]);
        } else {
            ans[a[rp].id
                ] += (bit.sum(maxn)-bit.sum(a[rp].z));
            tmp.pb(a[rp++]);
        }
    }
    while(lp <= mid) {
        bit.upd(a[lp].z,1);
        tmp.pb(a[lp++]);
    }
    while(rp <= r) {
        ans[a[rp
            ].id] += (bit.sum(maxn)-bit.sum(a[rp].z));
        tmp.pb(a[rp++]);
    }
    for (int i = l; i <= mid; ++i) bit.upd(a[i].z,-1);
    for (int i = l; i <= r; ++i) a[i] = tmp[i-l];
}
```

9D SimulatedAnnealing

```
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans: answer, nw: current value
    if (exp(-(nw -
        ans) / factor) >= (double)(rd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

9E Python

```
import math
math.isqrt(2) # integer sqrt
from decimal import *
Decimal(str(0.1)) # prevent precision issue
getcontext().prec = 100
```