

DCT / IDCT

(Based on the DCT wiki link shared on GS)

ID-DCT:-

Assume a 1D signal $\rightarrow x_0, x_1, \dots, x_{N-1}$

Then its ID-DCT transform is given by X_0, X_1, \dots, X_{N-1} ,

where,

$$X_k = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x_n \cos \left[\frac{\pi}{N} \left(n + \frac{1}{2} \right) \left(k + \frac{1}{2} \right) \right] \quad \text{--- (A)} \quad k=0, 1, \dots, N-1$$

ID-IDCT:-

Assume you are given $X_0, X_1, \dots, X_{N-1} \rightarrow$ ID-DCT

To compute 1D signal you perform inverse DCT operation,

$$x_n = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} X_k \cos \left[\frac{\pi}{N} \left(n + \frac{1}{2} \right) \left(k + \frac{1}{2} \right) \right] \quad \text{--- (B)} \quad n=0, 1, \dots, N-1$$

Note that (A) & (B) are same functions, ~~except for~~ ~~the~~ ~~variables~~.

\therefore if you have a function $y = \text{ID_dct}(x)$ implemented in MATLAB ~~that~~ that is based on eqⁿ (A). Then,

$$\boxed{X = \text{ID_dct}(x)} \quad \text{where } x \text{ is } N \times 1 \text{ vector.}$$

X is $N \times 1$ vector.

To ~~compute~~ recover input from X we can use the same function:-

$$\boxed{x_r = \text{ID_dct}(X)}$$

2D - DCT:

Assume your 2D input image is as follows:-

$$N_1 \left\{ \begin{array}{c} \left[\begin{array}{cccc} x_{0,0} & x_{0,1} & \dots & x_{0,N_2-1} \\ x_{1,0} & & & \\ \vdots & & & \\ x_{N_1-1,0} & \dots & x_{N_1-1,N_2-1} \end{array} \right] \\ N_2 \end{array} \right.$$

Then, 2D-DCT is given by :-

$$X_{k_1, k_2} = \sqrt{\frac{4}{N_1 N_2}} \sum_{n_1=0}^{N_1-1} \left(\sum_{n_2=0}^{N_2-1} x_{n_1, n_2} \cos \left[\frac{\pi}{N_2} \left(n_2 + \frac{1}{2} \right) \left(k_2 + \frac{1}{2} \right) \right] \right) \times \cos \left[\frac{\pi}{N_1} \left(n_1 + \frac{1}{2} \right) \left(k_1 + \frac{1}{2} \right) \right]$$

$$\text{for } k_1 = 0, \dots, N_1-1$$

$$k_2 = 0, \dots, N_2-1$$

2D - IDCT:-

$$x_{n_1, n_2} =$$

$$\sqrt{\frac{4}{N_1 N_2}} \sum_{k_1=0}^{N_1-1} \left(\sum_{k_2=0}^{N_2-1} X_{k_1, k_2} \cos \left[\frac{\pi}{N_2} \left(n_2 + \frac{1}{2} \right) \left(k_2 + \frac{1}{2} \right) \right] \right) \cos \left[\frac{\pi}{N_1} \left(n_1 + \frac{1}{2} \right) \left(k_1 + \frac{1}{2} \right) \right]$$

for, $n_1 = 0, \dots, N_1-1$
 $n_2 = 0, \dots, N_2-1$

Again, as in 1D-DCT case, if you have a function to implement 2D-DCT you could use the exact same function to compute 2D-IDCT. ~~and multiply by a factor~~

~~of $\frac{4}{N_1 N_2}$~~

~~det~~