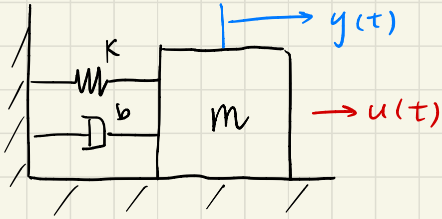


$$m\ddot{y}(t) + b\dot{y}(t) + Ky(t) = u(t)$$

1. Define State Variable

$$\left.\begin{array}{l} x_1(t) = y(t) \\ x_2(t) = \dot{y}(t) \end{array}\right\} \Rightarrow x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \in \mathbb{R}^2$$

2. ODE for each state variable

$$\dot{x}_1(t) = \dot{y}(t) = x_2(t)$$

$$\dot{x}_2(t) = \ddot{y}(t) = -\frac{b}{m}\dot{y}(t) - \frac{K}{m}y(t) + \frac{1}{m}u(t)$$

$$= -\frac{b}{m}x_2(t) - \frac{K}{m}x_1(t) + \frac{1}{m}u(t)$$

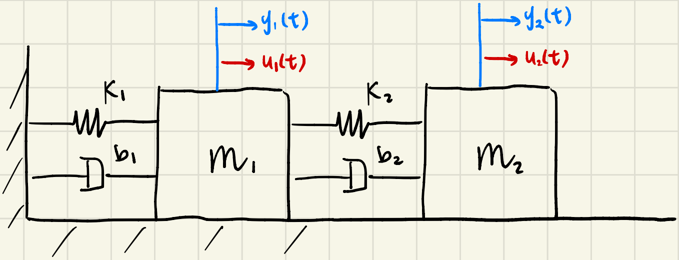
3. Vector form

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -\frac{b}{m}x_2(t) - \frac{K}{m}x_1(t) + \frac{1}{m}u(t) \end{bmatrix}$$

4. Matrix form

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0\,u(t)$$



for m_1: $m_1\ddot{y}_1(t) + b_1\dot{y}_1(t) + k_1 y_1(t) = b_2(\dot{y}_2(t) - \dot{y}_1(t)) + k_2(y_2(t) - y_1(t)) + u_1(t)$

$$\Rightarrow m_1\ddot{y}_1(t) + (b_1+b_2)\dot{y}_1(t) - b_2\dot{y}_2(t) + (k_1+k_2)y_1(t) - k_2 y_2(t) = u_1(t)$$

for m_2: $m_2\ddot{y}_2(t) + b_2(\dot{y}_2(t) - \dot{y}_1(t)) + k_2(y_2(t) - y_1(t)) = u_2(t)$

1. Define State Variable

$$\left.\begin{aligned} x_1(t) &= y_1(t) \\ x_2(t) &= y_2(t) \\ x_3(t) &= \dot{y}_1(t) \\ x_4(t) &= \dot{y}_2(t) \end{aligned}\right] \Rightarrow x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^2$$

2. ODE for each state variable

$$\dot{x}_1(t) = \dot{y}_1(t) = x_3(t)$$

$$\dot{x}_2(t) = \dot{y}_2(t) = x_4(t)$$

$$\dot{x}_3(t) = \ddot{y}_1(t) = -\frac{1}{m_1}(b_1+b_2)\dot{y}_1(t) - \frac{1}{m_1}(k_1+k_2)y_1(t) + \frac{1}{m_1}(b_2\dot{y}_2(t) + k_2 y_2(t)) + \frac{1}{m_1}u_1(t)$$

$$= -\frac{1}{m_1}(b_1+b_2)x_3(t) - \frac{1}{m_1}(k_1+k_2)x_1(t) + \frac{1}{m_1}(b_2 x_4(t) + k_2 x_2(t)) + \frac{1}{m_1}u_1(t)$$

$$\dot{x}_4(t) = \ddot{y}_2(t) = -\frac{1}{m_2}b_2(\dot{y}_2(t) - \dot{y}_1(t)) - \frac{1}{m_2}k_2(y_2(t) - y_1(t)) + \frac{1}{m_2}u_2(t)$$

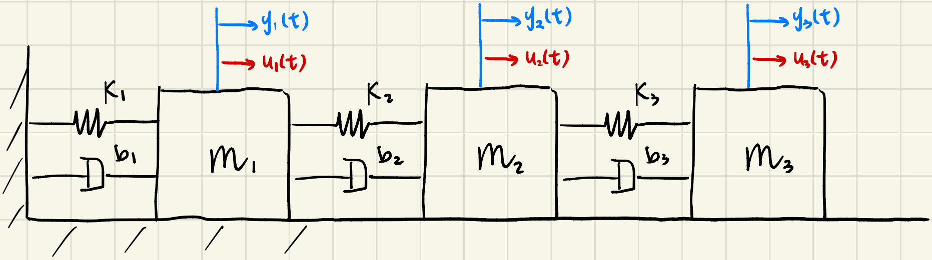
$$= -\frac{1}{m_2}b_2(x_4(t) - x_3(t)) - \frac{1}{m_2}k_2(x_2(t) - x_1(t)) + \frac{1}{m_2}u_2(t)$$

3. Vector form

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \\ \dot{X}_4(t) \end{bmatrix} = \begin{bmatrix} X_3(t) \\ X_4(t) \\ -\frac{1}{m_1}(b_1+b_2)X_3(t) - \frac{1}{m_1}(K_1+K_2)X_1(t) + \frac{1}{m_1}(b_2X_4(t) + K_2X_2(t)) + \frac{1}{m_1}u_1(t) \\ -\frac{1}{m_2}b_2(X_4(t) - X_3(t)) - \frac{1}{m_2}K_2(X_2(t) - X_1(t)) + \frac{1}{m_2}u_2(t) \end{bmatrix}$$

4. Matrix form

$$\dot{X}(t) = \begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \\ \dot{X}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(K_1+K_2)}{m_1} & \frac{K_2}{m_1} & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} \\ \frac{K_2}{m_2} & \frac{-K_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2}{m_2} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ X_4(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$



for m_1:

$$m_1\ddot{y}_1(t) + b_1\dot{y}_1(t) + K_1 y_1(t) = b_2(\dot{y}_2(t) - \dot{y}_1(t)) + K_2(y_2(t) - y_1(t)) + u_1(t)$$

$$\Rightarrow m_1\ddot{y}_1(t) + (b_1+b_2)\dot{y}_1(t) + (K_1+K_2)y_1(t) - b_2\dot{y}_2(t) - K_2 y_2(t) = u_1(t)$$

for m_2:

$$m_2\ddot{y}_2(t) + b_2(\dot{y}_2(t) - \dot{y}_1(t)) + K_2(y_2(t) - y_1(t)) = b_3(\dot{y}_3(t) - \dot{y}_2(t)) + K_3(y_3(t) - y_2(t)) + u_2(t)$$

$$\Rightarrow m_2\ddot{y}_2(t) + (b_2+b_3)\dot{y}_2(t) + (K_2+K_3)y_2(t) - b_3\dot{y}_3(t) - K_3 y_3(t) - b_2\dot{y}_1(t) - K_2 y_1(t) = u_2(t)$$

for m_3:

$$m_3\ddot{y}_3(t) + b_3(\dot{y}_3(t) - \dot{y}_2(t)) + K_3(y_3(t) - y_2(t)) = u_3(t)$$

$$\Rightarrow m_3\ddot{y}_3(t) + b_3\dot{y}_3(t) + K_3 y_3(t) - b_3\dot{y}_2(t) - K_3 y_2(t) = u_3(t)$$

$$\ddot{y}_1(t) = \frac{1}{m_1}\left[-(b_1+b_2)\dot{y}_1(t) - (K_1+K_2)y_1(t) + b_2\dot{y}_2(t) + K_2 y_2(t) + u_1(t)\right]$$

$$\ddot{y}_2(t) = \frac{1}{m_2}\left[-(b_2+b_3)\dot{y}_2(t) - (K_2+K_3)y_2(t) + b_3\dot{y}_3(t) + K_3 y_3(t) + b_2\dot{y}_1(t) + K_2 y_1(t) + u_2(t)\right]$$

$$\ddot{y}_3(t) = \frac{1}{m_3}\left[-b_3\dot{y}_3(t) - K_3 y_3(t) + b_3\dot{y}_2(t) + K_3 y_2(t) + u_3(t)\right]$$

1. Define State Variable

$$\left.\begin{aligned} x_1(t) &= y_1(t) \\ x_2(t) &= y_2(t) \\ x_3(t) &= y_3(t) \\ x_4(t) &= \dot{y}_1(t) \\ x_5(t) &= \dot{y}_2(t) \\ x_6(t) &= \dot{y}_3(t) \end{aligned}\right] \Rightarrow x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \in \mathbb{R}^2$$

2. ODE for each state variable

$$\dot{x}_1(t) = \dot{y}_1(t) = x_4(t)$$

$$\dot{x}_2(t) = \dot{y}_2(t) = x_5(t)$$

$$\dot{x}_3(t) = \dot{y}_3(t) = x_6(t)$$

$$\dot{x}_4(t) = \ddot{y}_1(t) = \frac{1}{m_1}\left[-(b_1+b_2)\dot{y}_1(t) - (k_1+k_2)y_1(t) + b_2\dot{y}_2(t) + k_2y_2(t) + u_1(t)\right]$$

$$= \frac{1}{m_1}\left[-(b_1+b_2)x_4(t) - (k_1+k_2)x_1(t) + b_2x_5(t) + k_2x_2(t) + u_1(t)\right]$$

$$\dot{x}_5(t) = \ddot{y}_2(t)$$

$$= \frac{1}{m_2}\left[-(b_2+b_3)\dot{y}_2(t) - (k_2+k_3)y_2(t) + b_3\dot{y}_3(t) + k_3y_3(t) + b_2\dot{y}_1(t) + k_2y_1(t) + u_2(t)\right]$$

$$= \frac{1}{m_2}\left[-(b_2+b_3)x_5(t) - (k_2+k_3)x_2(t) + b_3x_6(t) + k_3x_3(t) + b_2x_4(t) + k_2x_1(t) + u_2(t)\right]$$

$$\dot{x}_6(t) = \ddot{y}_3(t) = \frac{1}{m_3}\left[-b_3\dot{y}_3(t) - k_3y_3(t) + b_3\dot{y}_2(t) + k_3y_2(t) + u_3(t)\right]$$

$$= \frac{1}{m_3}\left[-b_3x_6(t) - k_3x_3(t) + b_3x_5(t) + k_3x_2(t) + u_3(t)\right]$$

3. Vector form

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \\ \dot{X}_4(t) \\ \dot{X}_5(t) \\ \dot{X}_6(t) \end{bmatrix} = \begin{bmatrix} x_4(t) \\ x_5(t) \\ x_6(t) \\ 1/m_1 \left[-(b_1+b_2)\, x_4(t) - (k_1+k_2)\, x_1(t) + b_2 x_5(t) + k_2 x_2(t) + u_1(t) \right] \\ 1/m_2 \left[-(b_2+b_3)\, x_5(t) - (k_2+k_3)\, x_2(t) + b_3 x_6(t) + k_3 x_3(t) + b_2 x_4(t) + k_2 x_1(t) + u_2(t) \right] \\ 1/m_3 \left[-b_3 x_6(t) - k_3 x_3(t) + b_3 x_5(t) + k_3 x_2(t) + u_3(t) \right] \end{bmatrix}$$

4. Matrix form

$$\dot{X}(t) = \begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \\ \dot{X}_4(t) \\ \dot{X}_5(t) \\ \dot{X}_6(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -(k_1+k_2)/m_1 & k_2/m_1 & 0 & -(b_1+b_2)/m_1 & b_2/m_1 & 0 \\ k_2/m_2 & -(k_2+k_3)/m_2 & k_3/m_2 & b_2/m_2 & -(b_2+b_3)/m_2 & b_3/m_2 \\ 0 & k_3/m_3 & -k_3/m_3 & 0 & b_3/m_3 & -b_3/m_3 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ X_4(t) \\ X_5(t) \\ X_6(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m_1 & 0 & 0 \\ 0 & 1/m_2 & 0 \\ 0 & 0 & 1/m_3 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$