가우시안 함수의 최적 파라미터 증명 2021202058 송채영

- 가우시안 함수의 pdf는 $\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x^{(k)}-\mu)^2}{2\sigma^2}}$

- 평균

$$\frac{d}{d\mu}\sum_{k=1}^{N}\left\{\log\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)-\frac{1}{2\sigma^2}(x^{(k)}-\mu)^2\right\} = -\frac{1}{2\sigma^2}\frac{d}{d\mu}\sum_{k=1}^{N}\left(x^{2(k)}-2x^{(k)}\mu+\mu^2\right)$$

$$= -\frac{1}{2\sigma^2}\frac{d}{d\mu}\sum_{k=1}^{N}\left(-2x^{(k)}\mu+\mu^2\right) = -\frac{1}{2\sigma^2}\sum_{k=1}^{N}\left(-2x^{(k)}+2\mu\right) = \frac{1}{\sigma^2}\sum_{k=1}^{N}\left(x^{(k)}-\mu\right)=0$$

$$\sum_{k=1}^{N}x^{(k)}-N\mu=0, \quad \underline{\mu=\frac{1}{N}\sum_{k=1}^{N}x^{(k)}}$$

- 분산

$$\frac{d}{d\sigma^2}\sum_{k=1}^{N}\left\{\log\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)-\frac{1}{2\sigma^2}(x^{(k)}-\mu)^2\right\} = \frac{d}{d\sigma^2}\sum_{k=1}^{N}\left\{-\log(\sqrt{2\pi}\,\sigma)-\frac{1}{2\sigma^2}(x^{(k)}-\mu)^2\right\}$$

$$= \frac{d}{d\sigma^2}\sum_{k=1}^{N}\left\{-\frac{1}{2}\log(2\pi\sigma^2)-\frac{1}{2\sigma^2}(x^{(k)}-\mu)^2\right\} = \frac{d}{d\sigma^2}\sum_{k=1}^{N}\left(-\frac{\log\sigma^2}{2}-\frac{1}{2\sigma^2}(x^{(k)}-\mu)^2\right)$$

$$= -\sum_{k=1}^{N}\left(\frac{1}{2\sigma^2}-\frac{1}{2\sigma^4}(x^{(k)}-\mu)^2\right) = -\sum_{k=1}^{N}\left(\frac{1}{\sigma^2}-\frac{1}{\sigma^4}(x^{(k)}-\mu)^2\right)=0$$

$$-\sum_{k=1}^{N}\frac{1}{\sigma^2}+\sum_{k=1}^{N}\frac{1}{\sigma^4}(x^{(k)}-\mu)^2=0 \qquad N\sigma^2=\sum_{k=1}^{N}(x^{(k)}-\mu)^2,$$

$$\underline{\sigma^2=\frac{1}{N}\sum_{k=1}^{N}(x^{(k)}-\mu)^2}$$

가우시안 분포를 따르는 데이터에 대한 MLE의 평균은 데이터 포인트 값들의 평균값이고 분산은 데이터 값들의 분산이다.