MVA HW#4

Problem 6.7

```
> S1 = matrix(c(13825.3,23823.4,23823.4,73107.4),2,2)
> S2 = matrix(c(8632.0,19616.7,19616.7,55964.5),2,2)
> n1 = 45
> n2 = 55
> p = 2
> xbar1 = c(204.4,556.6)
> xbar2 = c(130,355.0)
> TSquare.two_sample.withSummary(mean1=xbar1, mean2=xbar2,
cov1 = S1, cov2 = S2, n1=n1, n2=n2, p=p, pooled = TRUE)
------
Method: Two Population T-square Test with Equal Variance (P
ooled)
---------
Pooled covariance:
         [,1]     [,2]
[1,] 10963.69 21505.42
[2,] 21505.42 63661.31

------
T-square Value: 16.06622
P-value: 0.0006343777
------
Confident Intervals at 95% confident level:
Simontaneous CI:
       low     high
1 21.80733 126.9927
2 74.86846 328.3315

Bonferroni CI:
        low     high
1  42.64506 106.1549
2 125.08073 278.1193

-----------
The most critical linear combination to reject H0 is:
           [,1]
[1,] 0.00170252
[2,] 0.00259163

-----------
```

Problem 6.8

```
> x1 = c(6,5,8,4,7,3,1,2,2,5,3,2)
> x2 = c(7,9,6,9,9,3,6,3,3,1,1,3)
> trt = c(rep(1,5),rep(2,3),rep(3,4))
> MANOVA(cbind(x1,x2), trt)
--------
MANOVA Procedure
--------

Grand Mean:
x1 x2
 4  5

Treatment Means:
   group 1 group 2 group 3
x1       6       2       3
x2       8       4       2

Treatment Effects:
   group 1 group 2 group 3
x1       2      -2      -1
x2       3      -1      -3

Residuals:
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,1
1] [,12]
[1,]    0   -1    2   -2    1    1   -1    0   -1     2
 0    -1
[2,]   -1    1   -2    1    1   -1    2   -1    1    -1
-1     1

--------
The MANOVA Table Elements:
--------
SStrt (df = 2)
     x1 x2
[1,] 36 48
[2,] 48 84

SSerror (df = 9)
     [,1] [,2]
[1,]   18  -13
[2,]  -13   18

SStotal (df = 11)
     x1  x2
[1,] 54  35
[2,] 35 102

--------
Wilk's Test:
--------
lamba = 0.03618959       ratio = 4.256639         p-value:
0.01554857
Bartlett's correction: 28.21136       p-value: 1.13011e-05
```

Problem 6.21

```
> n1 = 20
> n2 = 20
> p = 4
> xbar1 = c(2.287, 12.6,0.347, 14.830)
> xbar2 = c(2.404,7.155,0.524,12.840)
> S1 = matrix(c(0.459,.254,-.026,-.244,.254,27.465,-.589,-.
267,-.026,-.589,.030,.102,-.244,-.267,.102,6.854),4,4)
> S2 = matrix(c(.944,-.089,.002,-.719,-.089,16.432,-.4,19.0
44,.002,-.400,.024,-.094,-.719,19.044,-.094,61.854),4,4)
> TSquare.two_sample.withSummary(mean1=xbar1, mean2=xbar2,
cov1 = S1, cov2 = S2, n1=n1, n2=n2, p=p, pooled = TRUE)
------
Method: Two Population T-square Test with Equal Variance (P
ooled)
---------
Pooled covariance:
        [,1]    [,2]    [,3]    [,4]
[1,]  0.7015  0.0825 -0.0120 -0.4815
[2,]  0.0825 21.9485 -0.4945  9.3885
```

```
[3,] -0.0120 -0.4945  0.0270  0.0040
[4,] -0.4815  9.3885  0.0040 34.3540

------
T-square Value: 15.82996
P-value: 0.01386451
------
Confident Intervals at 95% confident level:
Simontaneous CI:
         low         high
1 -1.0140653  0.780065294
2  0.4272082 10.462791829
3 -0.3529917 -0.001008335
4 -4.2876756  8.267675648

Bonferroni CI:
         low        high
1 -0.5458907  0.31189075
2  3.0459723  7.84402771
3 -0.2611424 -0.09285763
4 -1.0113835  4.99138355

-----------
The most critical linear combination to reject H0 is:
            [,1]
[1,] -0.24176170
[2,]  0.16009566
[3,] -3.73254518
[4,]  0.01122035

-----------
> TSquare.two_sample.withSummary(mean1=xbar1, mean2=xbar2,
cov1 = S1, cov2 = S2, n1=n1, n2=n2, p=p, pooled = FALSE)
------
Method: Two Population T-square Test with Unequal Variance
(Unpooled)
---------
Unpooled covariance:
         [,1]     [,2]     [,3]     [,4]
[1,]  0.07015  0.00825 -0.00120 -0.04815
[2,]  0.00825  2.19485 -0.04945  0.93885
[3,] -0.00120 -0.04945  0.00270  0.00040
[4,] -0.04815  0.93885  0.00040  3.43540

------
T-square Value: 15.82996
P-value: 0.003256029
------
Confident Intervals at 95% confident level:
Simontaneous CI:
         low        high
1 -0.9328212  0.69882118
2  0.8816523 10.00834768
3 -0.3370527 -0.01694729
4 -3.7191282  7.69912816

Bonferroni CI:
         low        high
1 -0.5355858  0.30158576
2  3.1036139  7.78638610
3 -0.2591207 -0.09487933
4 -0.9392691  4.91926909

-----------
The most critical linear combination to reject H0 is:
            [,1]
[1,]  -2.4176170
[2,]   1.6009566
[3,] -37.3254518
[4,]   0.1122035

-----------
```

Problem 6.24

```
> x1 = c(131,125,131,119,136,138,139,125,131,134,124,133,13
8,148,126,135,132,133,131,133,132,133,138,130,136,134,136,1
33,138,138)
> x2 = c(138,131,132,132,143,137,130,136,134,134,138,134,13
4,129,124,136,145,130,134,125,130,131,137,127,133,123,137,1
31,133,133)
> x3 = c(89,92,99,96,100,89,108,93,102,99,101,97,98,104,95,
98,100,102,96,94,91,100,94,99,91,95,101,96,100,91)
> x4 = c(49,48,50,44,54,56,48,48,51,51,48,48,45,51,45,52,5
4,48,50,46,52,50,51,45,49,52,54,49,55,46)
> trt = c(rep(1,10),rep(2,10),rep(3,10))
> X = cbind(x1,x2,x3,x4)
> MANOVA(X,trt)

--------
MANOVA Procedure
--------

Grand Mean:
       x1        x2        x3        x4
133.00000 133.03333  97.00000  49.63333

Treatment Means:
   group 1 group 2 group 3
x1   130.9   133.3   134.8
x2   134.7   132.9   131.5
x3    96.7    98.5    95.8
x4    49.9    48.7    50.3

Treatment Effects:
      group 1    group 2    group 3
x1 -2.1000000  0.3000000  1.8000000
x2  1.6666667 -0.1333333 -1.5333333
x3 -0.3000000  1.5000000 -1.2000000
x4  0.2666667 -0.9333333  0.6666667

Residuals:
     [,1] [,2] [,3]  [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22]
[1,]  0.1 -5.9  0.1 -11.9  5.1  7.1  8.1 -5.9  0.1   3.1  -9.3  -0.3   4.7  14.7  -7.3   1.7  -1.3  -0.3  -2.3  -0.3  -2.8  -1.8
[2,]  3.3 -3.7 -2.7  -2.7  8.3  2.3 -4.7  1.3 -0.7  -0.7   5.1   1.1   1.1  -3.9  -8.9   3.1  12.1  -2.9   1.1  -7.9  -1.5  -0.5
[3,] -7.7 -4.7  2.3  -0.7  3.3 -7.7 11.3 -3.7  5.3   2.3   2.5  -1.5  -0.5   5.5  -3.5  -0.5   1.5   3.5  -2.5  -4.5  -4.8   4.2
[4,] -0.9 -1.9  0.1  -5.9  4.1  6.1 -1.9 -1.9  1.1   1.1  -0.7  -0.7  -3.7   2.3  -3.7   3.3   5.3  -0.7   1.3  -2.7   1.7  -0.3
     [,23] [,24] [,25] [,26] [,27] [,28] [,29] [,30]
[1,]   3.2  -4.8   1.2  -0.8   1.2  -1.8   3.2   3.2
[2,]   5.5  -4.5   1.5  -8.5   5.5  -0.5   1.5   1.5
[3,]  -1.8   3.2  -4.8  -0.8   5.2   0.2   4.2  -4.8
[4,]   0.7  -5.3  -1.3   1.7   3.7  -1.3   4.7  -4.3

--------
The MANOVA Table Elements:
--------
SStrt (df = 2)
       x1         x2    x3         x4
[1,]  77.4 -63.000000 -10.8   3.600000
[2,] -63.0  51.466667  11.4  -4.533333
[3,] -10.8  11.400000  37.8 -22.800000
[4,]   3.6  -4.533333 -22.8  13.866667

SSerror (df = 27)
      [,1]  [,2]  [,3]  [,4]
```

```
[1,] 822.6  90.0 186.8 220.4
[2,]  90.0 651.5   1.6 217.9
[3,] 186.8   1.6 570.2  56.8
[4,] 220.4 217.9  56.8 277.1

SStotal (df = 29)
      x1       x2  x3       x4
[1,] 900  27.0000 176 224.0000
[2,]  27 702.9667  13 213.3667
[3,] 176  13.0000 608  34.0000
[4,] 224 213.3667  34 290.9667

--------
Wilk's Test:
--------
lamba = 0.7020877        ratio = 0.1934503        p-value:
0.9407504
Bartlett's correction: 3.006424       p-value: 0.9339537
> cor(X[1:10,])
          x1         x2         x3         x4
x1 1.0000000  0.2890107  0.3419187  0.7498376
x2 0.2890107  1.0000000 -0.3077189  0.6098596
x3 0.3419187 -0.3077189  1.0000000 -0.1044035
x4 0.7498376  0.6098596 -0.1044035  1.0000000
> cor(X[11:20,])
           x1         x2        x3        x4
x1  1.0000000 -0.1211279 0.4459786 0.2627420
x2 -0.1211279  1.0000000 0.3302588 0.6757719
x3  0.4459786  0.3302588 1.0000000 0.4532676
x4  0.2627420  0.6757719 0.4532676 1.0000000
> cor(X[21:30,])
           x1         x2          x3        x4
x1  1.0000000 0.64895539 -0.16062260 0.3521922
x2  0.6489554 1.00000000  0.05200949 0.2493121
x3 -0.1606226 0.05200949  1.00000000 0.3160608
x4  0.3521922 0.24931209  0.31606076 1.0000000
```

Define functions:

1.

```
> TSquare.two_sample.withSummary = function(mean1, mean2, cov1, cov2, n1, n2, p, mu = 0, x1_minus_x2=TRUE, alpha = 0.05, pooled =
 FALSE){
+   if(x1_minus_x2){
+     mean_diff = mean1 - mean2
+   }else{
+     mean_diff = mean1 + mean2
+   }
+   diff = mean_diff - mu
+   cat('------\n')
+   if(pooled){
+     cat("Method: Two Population T-square Test with Equal Variance (Pooled)\n---------\n")
+     cov.pooled = ((n1-1)*cov1+(n2-1)*cov2)/(n1+n2-2)
+     cat('Pooled covariance: \n')
+     print(cov.pooled)
+     T.sq = t(diff) %*% solve((1/n1+1/n2)*cov.pooled) %*% diff
+     F = TF.convert.twosample(T.sq = T.sq, n1=n1, n2=n2, p=p)
+     p_value = pf(F$F, F$df1, F$df2, lower.tail = FALSE)
+ 
+     T.sq.alpha = TF.convert.twosample(F=qf(1-alpha,p,n1+n2-p-1),n1=n1,n2=n2,p=p, T_to_F = FALSE)
+     factor = sqrt(1/n1+1/n2)* sqrt(diag(cov.pooled))
+     simontaneous.distance = sqrt(T.sq.alpha$T.sq) * factor
+     simontaneous.CI = cbind(mean_diff - simontaneous.distance, mean_diff + simontaneous.distance)
+     colnames(simontaneous.CI) = c('low', 'high')
+     rownames(simontaneous.CI) = c(1:p)
+     t = qt(1-alpha/2/p, df = n1+n2-2)
+     bonferroni.distance  = sqrt(t) * factor
+     bonferroni.CI = cbind(mean_diff - bonferroni.distance, mean_diff + bonferroni.distance)
+     colnames(bonferroni.CI) = c('low', 'high')
+     rownames(bonferroni.CI) = c(1:p)
+     cat('\n------\nT-square Value: ', T.sq, '\nP-value: ', p_value,'\n------\n', 'Confident Intervals at ', 100-alpha*100, '% c
onfident level:\n',sep = '')
+     linear_combination = solve(cov.pooled) %*% diff
+   }else{
+     cat("Method: Two Population T-square Test with Unequal Variance (Unpooled)\n---------\n")
+     cov.unpooled = 1/n1 * cov1 + 1/n2 *cov2
+     cat('Unpooled covariance: \n')
+     print(cov.unpooled)
+     T.sq = t(diff) %*% solve(cov.unpooled) %*% diff
+     p_value = pchisq(T.sq, p, lower.tail = FALSE)
+ 
+     chi.sq.alpha = qchisq(1-alpha,p)
+     factor = sqrt(diag(cov.unpooled))
+     simontaneous.distance = sqrt(chi.sq.alpha) * factor
+     simontaneous.CI = cbind(mean_diff - simontaneous.distance, mean_diff + simontaneous.distance)
+     colnames(simontaneous.CI) = c('low', 'high')
+     rownames(simontaneous.CI) = c(1:p)
```

```
+
+     z = qnorm(1-alpha/2/p)
+     bonferroni.distance  = sqrt(z) * factor
+     bonferroni.CI = cbind(mean_diff - bonferroni.distance, mean_diff + bonferroni.distance)
+     colnames(bonferroni.CI) = c('low', 'high')
+     rownames(bonferroni.CI) = c(1:p)
+     cat('\n------\nT-square Value: ', T.sq, '\nP-value: ', p_value,'\n------\n', 'Confident Intervals at ', 100-alpha*100, '% c
onfident level:\n',sep = '')
+     linear_combination = solve(cov.unpooled) %*% diff
+   }
+   cat('Simontaneous CI: \n')
+   print(simontaneous.CI)
+   cat('\nBonferroni CI: \n')
+   print(bonferroni.CI)
+   cat('\n-----------\n')
+   cat('The most critical linear combination to reject H0 is: \n')
+   print(linear_combination)
+   cat('\n-----------\n')
+ }
```

2.

```
> TF.convert.twosample = function(T.sq = 0, F=0, n1, n2, p, T_to_F = TRUE){
+   if(T_to_F){
+     F = (n1+n2-p-1)/(n1+n2-2)/p * T.sq
+     df1 = p
+     df2 = n1+n2-p-1
+     returnValue(data.frame(F,df1,df2))
+   }else{
+     T.sq = F*(n1+n2-2)*p/(n1+n2-p-1)
+     df1 = p
+     df2 = n1+n2-2
+     returnValue(data.frame(T.sq,df1,df2))
+   }
+ }
```

3.

```
> MANOVA = function(X,trt){
+   n = as.integer(table(trt))
+   N = length(trt)
+   cat('\n--------\nMANOVA Procedure\n--------\n\nGrand Mean:\n')
+   xbar = apply(X, 2, mean)
+   print(xbar)
+   cat('\nTreatment Means: \n')
+   trtmean = NULL
+   residual = NULL
+   for(i in c(1:dim(X)[2])){
+     k = 1
+     trtmean_i = NULL
+     trtmean_i_long = NULL
+     for(j in c(1:length(n))){
+       mean_ij = mean(X[k:(k+n[j]-1),i])
+       trtmean_i = cbind(trtmean_i, mean_ij)
+       trtmean_i_long = cbind(trtmean_i_long, t(rep(mean_ij,n[j])))
+       k = k+n[j]
+     }
+     trtmean = rbind(trtmean,trtmean_i)
+     residual_i = X[,i] - trtmean_i_long
+     residual = rbind(residual, residual_i)
+   }
+   groups = paste('group', c(1:length(n)))
+   vars = paste('x', c(1:dim(X)[2]), sep = '')
+   colnames(trtmean) = groups
+   rownames(trtmean) = vars
+   print(trtmean)
+
+   cat('\nTreatment Effects: \n')
```

```
+   trteffect = trtmean - xbar
+   print(trteffect)
+   cat('\nResiduals:\n')
+   print(residual)
+   SStrt = 0
+   for(i in c(1:length(n))){
+     SStrt = SStrt + n[i] * trteffect[,i] %*% t(trteffect[,i])
+   }
+   SSerror = 0
+   for(i in c(1:dim(residual)[2])){
+     temp = residual[,i]
+     SSerror = SSerror + temp %*% t(temp)
+   }
+   SStotal = SStrt + SSerror
+   cat('\n--------\nThe MANOVA Table Elements: \n--------\nSStrt (df = ',length(n)-1, ')\n', sep = '')
+   print(SStrt)
+   cat('\nSSerror (df = ',N-length(n), ')\n', sep = '')
+   print(SSerror)
+   cat('\nSStotal (df = ',N-1, ')\n', sep = '')
+   print(SStotal)
+   wilk = det(SSerror) / det(SStotal)
+   ratio = (1-sqrt(wilk))/sqrt(wilk)
+   correction = -(12-1-(2+3)/2)*log(wilk)
+   cat('\n--------\nWilk\'s Test: \n--------\nlamba = ', wilk, '\tratio = ', ratio,
+       '\tp-value: ',1-pf(ratio, 2*(length(n)-1),2*(N-length(n)-1)),
+       '\nBartlett\'s correction: ', correction, '\tp-value: ', 1-pchisq(correction, dim(X)[2]*(length(n)-1)),'\n',sep = '')
+ }
```