Problem #2 (4.26)

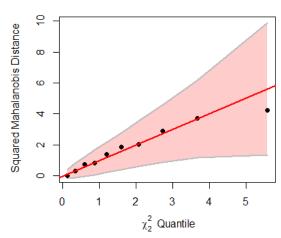
```
> x1 = c(1:3,3:6,8,9,11)
> x2 = c(18.95, 19, 17.95, 15.54, 14, 12.95, 8.94, 7.49, 6, 3.99)
> X = cbind(matrix(c(x1)), matrix(c(x2)))
> mean = rbind(mean(x1), mean(x2))
> diff1 = x1-mean[1]
> diff2 = x2-mean[2]
> Diff = t(rbind(diff1,diff2))
> S = matrix(c(var(x1), cov(x1, x2), cov(x1, x2), var(x2)), 2, 2)
> SInverse = solve(S)
> contour(X)
> chi = qchisq(0.5,2)
> distance = double()
> falling.contour = logical()
> for(i in c(1:10)){
    temp = Diff[i,]
    result = t(temp)%*%SInverse%*%temp
    if(result>chi) {
     text = 'fall'
    }else{
     text = 'not fall'
    cat("i=", i, ':\t', format.default(result,digits = 4),'
\t',text,'\n', sep = '')
    distance = rbind(distance,result)
    falling.contour = rbind(falling.contour,result>chi)
+ }
             1.875
                          fall
i=1:
i=2:
             2.02
                          fall
i=3:
             2.901
                          fall
i=4:
             0.7353
                          not fall
             0.3105
                          not fall
i=5:
```

```
i=6:
             0.01762
                          not fall
i=7:
             3.733
                          fall
i=8:
             0.8165
                          not fall
i=9:
             1.375
                          not fall
             4.215
i=10:
                          fall
> print(sort(distance))
```

[1] 0.0176162 0.3105192 0.7352659 0.8165401 1.3753379 1.87 53045 2.0203262 2.9009088 3.7329012 4.2152799

- > library(heplots)
- > cqplot(X)

Chi-Square Q-Q Plot of X



Problem #3 (4.40)

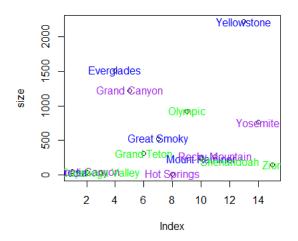
```
> park = c('Arcdia','Bruce Canyon', 'Cuyahoga Valle
y','Everglades', 'Grand Canyon', 'Grand Teton', 'Gr
eat Smoky', 'Hot Springs',
+ 'Olympic', 'Mount Raininer', 'Rocky Moun
tain', 'Shenandoah', 'Yellowstone', 'Yosemite', 'Zi
on')
> size = c(47.4,35.8,32.9,1508.5,1217.4,310.0,521.
8,5.6,922.7,235.6,265.8,199.0,2219.8,761.3,146.6)
> visitors = c(2.05,1.02,2.53,1.23,4.40,2.46,9.19,
1.34,3.14,1.17,2.80,1.09,2.84,3.30,2.59)
```

A)

Let's first observe the scatterplot of *size* (x_1): We can see that Yellowstone, Everglades, and Grand Canyon maybe potential outliers. This can be proved by the Normal Q-Q Plot.

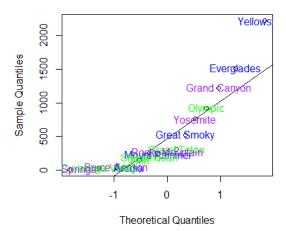
```
> plot(size, main='Scatter Plot - Size Original')
> color = c('blue', 'purple', 'green')
> text(size, park, col = color)
```

Scatter Plot - Size Original



- > plot1 = qqnorm(size, main = 'Normal Q-Q Plot, Si
 ze Original')
 > qqline(size)
- > text(plot1\$x,plot1\$y,park,col=color)

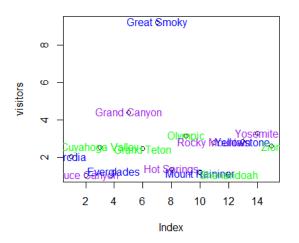
Normal Q-Q Plot, Size Original



Then, observe the scatterplot of *visitors* (*x*₂): We can see that Grand Smoky maybe a potential outlier. This can also be seen in the normal Q-Q Plot

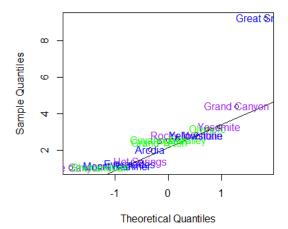
```
> plot(visitors, main='Scatter Plot - Visitors Orig
inal')
> text(visitors, park, col = color)
```

Scatter Plot - Visitors Original

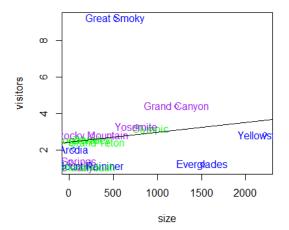


- > plot1=qqnorm(visitors, main = 'Normal Q-Q Plot, V
 isitors Original')
- > qqline(visitors)
- > text(plot1\$x,plot1\$y,park,col=color)

Normal Q-Q Plot, Visitors Original



Plot *visitors - size*, we can see that Grand Canyon, Great Smoky, Yellowstone, and Everglades can also be outliers.



B)

Using Box-cox to transform $size~(x_1)$. Lambda is 0.194947. Examine the results using a scatter plot and a normal Q-Q Plot, we can see remarkable improvements. The data is almost normal now

- > library(car)
- > trans=powerTransform(size)
- > summary(powerTransform(size))

bcPower Transformation to Normality

Est Power Rounded Pwr Wald Lwr bnd Wald Upr Bnd

size 0.1949 0 -0.129 0.5189

Likelihood ratio tests about transformation parameters

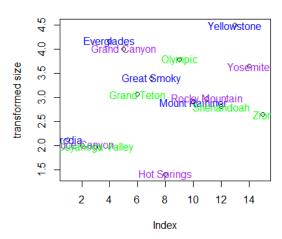
LRT df pval

LR test, lambda = (0) 1.485416 1 2.229290e-01

LR test, lambda = (1) 17.221093 1 3.327216e-05

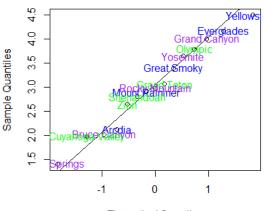
- > plot(size**trans\$lambda, ylab = 'transformed size
 ')
- > text(size**trans\$lambda, park, col = color)

Scatter Plot - Size Transformed



> plot1=qqnorm(size**trans\$lambda, main = 'Normal Q
-Q Plot, Size Transformed')
> qqline(size**trans\$lambda)
> text(plot1\$x,plot1\$y,park,col=color)

Normal Q-Q Plot, Size Transformed



Theoretical Quantiles

C) Using Box-cox to transform *visitors* (x_2). Lambda is - 0.3456574. Examine the results using a scatter plot and a normal Q-Q Plot, we can see remarkable improvements.

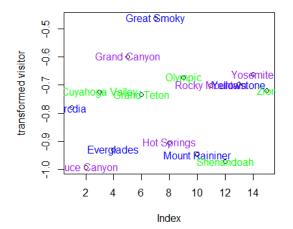
The data is almost normal now.

```
LR test, lambda = (0) 0.7365957 1 0.3907535765

LR test, lambda = (1) 12.5728818 1 0.0003913846

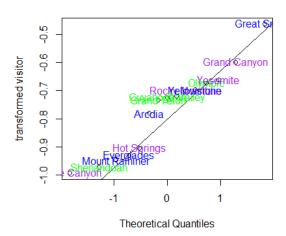
> plot(-visitors**trans$lambda, ylab = 'transformed visitor', main = 'Scatter Plot - Visitors Transformed')
> text(-visitors**trans$lambda, park, col = color)
```

Scatter Plot - Visitors Transformed



- > plot1=qqnorm(-visitors**trans\$lambda, ylab = 'tra
 nsformed visitor',main = 'Normal Q-Q Plot, Visitors
 Transformed')
- > qqline(-visitors**trans\$lambda)
- > text(plot1\$x,plot1\$y,park,col=color)

Normal Q-Q Plot, Visitors Transformed



D)

To achieve bivariate normality, we have the followings. The scatter plot shows some improvement, but it seems Great Smoky may still be an outlier. But the Bivariate Normal Q-Q Plot shows the data looks fine.

> summary(powerTransform(cbind(size, visitors)))

```
Est Power Rounded Pwr Wald Lwr bnd Wald Upr 8nd
size 0.1738 0 -0.1355 0.4830
visitors -0.3410 0 -1.1198 0.4378
```

Transformations to Multinormality

Likelihood ratio tests about transformation parameters

LRT df pval

LR test, lambda = (0 0) 2.068243 2 3.555386e-01

LR test, lambda = (1 1) 32.167033 2 1.035184e-07

> trans.visitor=-visitors**trans\$lambda[2]

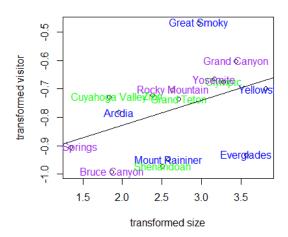
> trans.size = (size**trans\$lambda[1])

> plot(trans.size,trans.visitor,xlab = 'transformed size', ylab = 'transformed visitor', main = 'Scatt er Plot - Bivariate Transformed')

> text(trans.size,trans.visitor, park, col = color)

> abline(lm(trans.visitor~trans.size))

Scatter Plot - Bivariate Transformed



- > plot1 = qqplot(trans.size,trans.visitor)
- > text(plot1\$x,plot1\$y,park,col=color)

